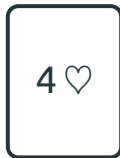# Practice Counting

Alexander S. Kulikov

Steklov Mathematical Institute at St. Petersburg, Russian Academy of Sciences
and
University of California, San Diego

Number of Hands

Question

What is the number of 5-card hands dealt off of a standard 52-card deck?

A♣   2♦   7♠   5♣   4♡

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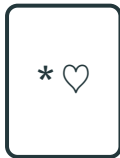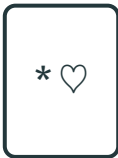
| A♣ | 2♢ | 7♠ | 5♣ | 4♡ |

Answer

$$\binom{52}{5} = \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2\,598\,960$$

Two Hearts and Three Spades

Question

What is the number of 5-card hands with two hearts and three spades?

| * ♡ | * ♡ | * ♠ | * ♠ | * ♠ |

Two Hearts and Three Spades

Question

What is the number of 5-card hands with two hearts and three spades?

* ♡	* ♡	* ♠	* ♠	* ♠

Answer

$$\binom{13}{2}\binom{13}{3} = 22\,308$$

4-Digit Numbers Containing 7

Question

What is the number of non-negative integers with at most four digits at least one of which is equal to 7?

4-Digit Numbers Containing 7

Question

What is the number of non-negative integers with at most four digits at least one of which is equal to 7?

Answer

$$10^4 - 9^4 = 3\,439$$

Code

```python
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if 7 in d:
        count += 1

print(count)
print(10**4 - 9**4)
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```
3439
3439
```

4-Digit Numbers with Increasing Digits

Question

What is the number of four-digit sequences whose digits are increasing?

4-Digit Numbers with Increasing Digits

Question

What is the number of four-digit sequences whose digits are increasing?

Answer

$$\binom{10}{4} = 210$$

Code

```python
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
        count += 1
        print(d)

print(count)
```

Code

```python
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
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print(count)
```
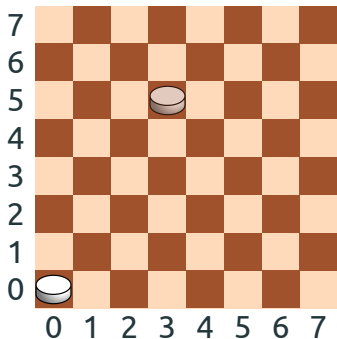
```
(0, 1, 2, 3)          (4, 6, 7, 8)
(0, 1, 2, 4)          (4, 6, 7, 9)
(0, 1, 2, 5)          (4, 6, 8, 9)
(0, 1, 2, 6)          (4, 7, 8, 9)
(0, 1, 2, 7)          (5, 6, 7, 8)
(0, 1, 2, 8)          (5, 6, 7, 9)
(0, 1, 2, 9)          (5, 6, 8, 9)
(0, 1, 3, 4)          (5, 7, 8, 9)
(0, 1, 3, 5)          (6, 7, 8, 9)
(0, 1, 3, 6)          210
```

Piece on a Chessboard

A piece can move one step up or one step to the right. What is the number of ways of getting from the cell [0, 0] (bottom left corner) to the cell [5, 3]?

Solution

- There are exactly eight moves

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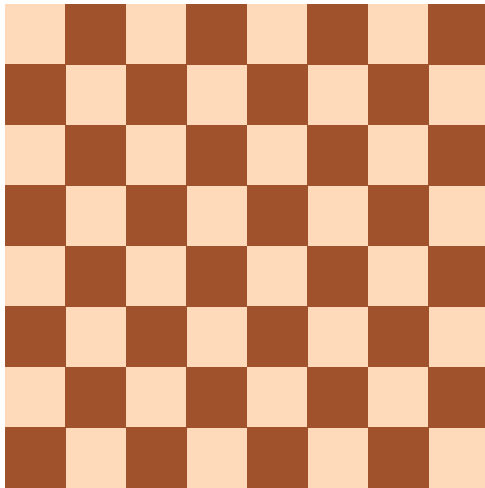- Three of them should be to the right, while the remaining five should go up

Solution

- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell $[5, 3]$

Solution

- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell $[5, 3]$
- Hence, the answer is $\binom{8}{3} = 56$

Solution Using Pascal's Triangle

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