QUESTION 5

The *Fibonacci sequence* is defined iteratively by setting $F_1 = F_2 = 1$ and thereafter letting $F_{n+2} = F_n + F_{n+1}$.

Theorem For any natural number n,

$$\sum_{k=1}^{n} F_k = F_{n+2}$$

Proof: By induction.

For $n = 1$, the left-hand side is $F_1 = 1$ and the right-hand side is $F_2 = 1$, so the identity is valid for $n = 1$.

Assume the identity holds for n. Then:

$$\begin{aligned}\sum_{k=1}^{n+1} F_k &= \sum_{k=1}^{n} F_k \ + \ F_{n+1} \\ &= F_{n+2} \ + \ F_{n+1}, \ \text{by the induction hypothesis} \\ &= F_{n+3}, \ \text{by the definition of } F_{n+3}\end{aligned}$$

which is the identity for $n + 1$. The proof is complete.