The power of exponential growth

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There's a famous riddle: If the water in a lakebed doubles in volume every day, and the lakebed started filling on January 1, and is half full on June 17, when will it be full?

The answer is of course June 18—if it doubles every day, it will go from half full to full in a single day.

But most people assume that half the work takes about half the time, so they usually give answers in December. Others try to correct, but don't go far enough, and say something like October.

Human brains are programmed to understand linear processes. We expect things to come in direct proportion: If you work twice as hard, you expect to get twice as much done. If you study twice as long, you expect to learn twice as much. If you pay twice as much, you expect to get twice as much stuff.

We tend to apply this same intuition to situations where it does not belong, processes that are not actually linear but exponential. As a result, when we extrapolate the slow growth early in the process, we wildly underestimate the total growth in the long run.

For example, suppose we have two countries. Arcadia has a GDP of $100 billion per year, and they grow at 4% per year. Berkland has a GDP of $200 billion, and they grow at 2% per year. Assuming that they maintain these growth rates, how long will it take for Arcadia's GDP to exceed Berkland's?

If we do this intuitively, we might sort of guess that at 4% you'd add 100% in 25 years, and at 2% you'd add 100% in 50 years; so it should be something like 75 years, because then Arcadia will have added $300 million while Berkland added $200 million. You might even just fudge the numbers in your head and say "about a century".

In fact, it is only 35 years. You could solve this exactly by setting $(100)(1.04^x) = (200)(1.02^x)$; but I have an intuitive method that I think may help you to estimate exponential processes in the future.

Divide the percentage into 69. (For some numbers it's easier to use 70 or 72; remember, these are just to be approximate. The exact figure is 100*ln(2) = 69.3147… and then it wouldn't be the percentage p but 100*ln(1+p/100); try plotting those and you'll see why using p works.) This is the time it will take to double.

So at 4%, Arcadia will double in about 17.5 years, quadrupling in 35 years. At 2%, Berkland will double in about 35 years. Thus, in 35 years, Arcadia will quadruple and Berkland will double, so their GDPs will be equal.

Economics is full of exponential processes: Compound interest is exponential, and over moderately long periods GDP and population both tend to grow exponentially. (In fact they grow *logistically*, which is similar to exponential until it gets very large and begins to slow down. If you smooth out our recessions, you can get a sense that since the 1940s, US GDP growth has slowed down from about 4% per year to about 2% per year (http://www.tradingeconomics.com/united-states/gdp-growth-annual).) It is therefore quite important to understand how exponential growth works.

Let's try another one. If one account has \$1 million, growing at 5% per year, and another has \$1,000, growing at 10% per year, how long will it take for the second account to have more money in it?

69/5 is about 14, so the first account doubles in 14 years. 69/10 is about 7, so the second account doubles in 7 years. A factor of 1000 is about 10 doublings (2^10 = 1024), so the second account needs to have doubled 10 times more than the first account. Since it doubles twice as often, this means that it must have doubled 20 times while the other doubled 10 times. Therefore, it will take about 140 years.

In fact, it takes 141—so our quick approximation is actually remarkably good.

This example is instructive in another way; 141 years is a pretty long time, isn't it? You can't just assume that exponential growth is "as fast as you want it to be". Once people realize that exponential growth is very fast, they often overcorrect, assuming that exponential growth automatically means growth that is absurdly—or arbitrarily—fast. (XKCD made a similar point in this comic (http://xkcd.com/947/).)

I think the worst examples of this mistake are among Singularitarians (https://en.wikipedia.org/wiki/Singularitarianism). They—correctly—note that computing power has become exponentially greater and cheaper over time, doubling about every 18 months, which has been dubbed **Moore's Law** (https://en.wikipedia.org/wiki/Moore's_law). They assume that this will continue into the indefinite future (this is already problematic; the growth rate seems to be already slowing down). And therefore they conclude there will be a sudden moment, a **technological singularity** (https://en.wikipedia.org/wiki/Technological_singularity), at which computers will suddenly outstrip humans in every way and bring about a new world order of artificial intelligence basically overnight. They call it a "hard takeoff"; here's a direct quote (http://slatestarcodex.com/2015/12/17/should-ai-be-open/):

> But many thinkers in this field including Nick Bostrom and Eliezer Yudkowsky worry that AI won't work like this at all. Instead there could be a "hard takeoff", a huge subjective discontinuity in the function mapping AI research progress to intelligence as measured in ability-to-get-things-done. If on January 1 you have a toy AI as smart as a cow, one which can identify certain objects in pictures and navigate a complex environment, and on February 1 it's proved the Riemann hypothesis and started building a ring around the sun, that was a hard takeoff.

Wait… what? For someone like me who understands exponential growth, the last part is a baffling *non sequitur.* If computers start half as smart as us and double every 18 months, in 18 months, they will be as smart as us. In 36 months, they will be twice as smart as us. Twice as smart as us literally means that two people working together perfectly can match them—certainly a few dozen working realistically can. We're not in danger of total AI domination from that. With millions of people working against the AI, we should be able to keep up with it for at least another 30 years. So are you assuming that this trend is continuing or not? (Oh, and by the way, we've had AIs that can identify objects and navigate complex environments (http://ais.informatik.uni-freiburg.de/publications/papers/kuemmerle13icra.pdf) for a couple years now, and so far, no ringworld around the Sun.)

That same essay make a biological argument, which misunderstands human evolution in a way that is surprisingly subtle yet ultimately fundamental:

> If you were to come up with a sort of objective zoological IQ based on amount of evolutionary work required to reach a certain level, complexity of brain structures, etc, you might put nematodes at 1, cows at 90, chimps at 99, homo erectus at 99.9, and modern humans at 100. The difference between 99.9 and 100 is the difference between "frequently eaten by lions" and "has to pass anti-poaching laws to prevent all lions from being wiped out".

No, actually, what makes humans what we are is *not* that we are 1% smarter than chimpanzees.

First of all, we're actually more like 200% smarter than chimpanzees, measured by encephalization quotient (http://serendip.brynmawr.edu/bb/kinser/Int3.html); they clock in at 2.49 while we hit 7.44. If you simply measure by raw volume, they have about 400 mL to our 1300 mL, so again roughly 3 times as big. But that's relatively unimportant; with Moore's Law, tripling only takes about 2.5 years.

But even having triple the brain power is not what makes humans different. It was a necessary condition, but not a sufficient one. Indeed, it was so insufficient that for about 200,000 years we had brains just as powerful as we do now and yet we did basically *nothing* in technological or economic terms—total, complete stagnation on a global scale. This is a conservative estimate of when we had brains of the same size and structure as we do today.

What makes humans what we are? *Cooperation.* We are what we are because we are *together*.
The capacity of human intelligence today is *not* 1300 mL of brain. It's more like 1.3 *gigaliters* of brain, where a gigaliter, a billion liters, is about the volume of the Empire State Building (http://www.esbnyc.com/sites/default/files/esb_fact_sheet_4_9_14_4.pdf). We have the intellectual capacity we do not because we are individually geniuses, but because we have built institutions of research and education that combine, synthesize, and share the knowledge of billions of people who came before us. Isaac Newton didn't understand the world as well as the average third-grader in the 21st century does today. Does the third-grader have more brain? Of course not. But they absolutely do have more *knowledge*.

(I recently finished my first playthrough of *Legacy of the Void* (http://us.battle.net/sc2/en/legacy-of-the-void/), in which a central point concerns whether the Protoss should detach themselves from the Khala, a psychic union which combines all their knowledge and experience into one. I won't spoil the ending, but let me say this: I can understand their hesitation, for it is basically our equivalent of the Khala—first literacy, and now the Internet—that has made us what we are. It would no doubt be the Khala that made them what they are as well.)

Is AI still dangerous? Absolutely. There are all sorts of damaging effects AI could have, culturally, economically, militarily—and some of them are already beginning to happen. I even agree with the basic conclusion of that essay that OpenAI is a bad idea because the cost of making AI available to people who will abuse it or create one that is dangerous is higher than the benefit of making AI available to everyone. But exponential growth not only isn't the same thing as instantaneous takeoff, it isn't even compatible with it.

The next time you encounter an example of exponential growth, try this. Don't just fudge it in your head, don't overcorrect and assume everything will be fast—just divide the percentage into 69 to see how long it will take to double.

2 thoughts on "**The power of exponential growth**"

1. **The real Existential Risk we should be concerned about | Human Economics** says:
   2016-03-09 at 12:02

   […] it's the overlap with Singularitarians, and their inability to understand that exponentials are not arbitrarily fast; if you just keep projecting the growth in computing power as growing forever, surely eventually […]

   Reply (https://patrickjuli.us/2016/01/02/the-power-of-exponential-growth/?replytocom=297#respond)
2. **What is the processing power of the human brain? | Human Economics** says:
   2016-04-06 at 12:03

   […] are a lot of reasons why this prediction keeps failing so miserably. One is an apparent failure to grasp the limitations of exponential growth. I actually think the most important is that a lot of AI fans don't seem to understand how […]

   Reply (https://patrickjuli.us/2016/01/02/the-power-of-exponential-growth/?replytocom=350#respond)