Half-wormhole solutions & Black Hole singularity - A brief review

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Abstract

In AdS/CFT, the partition functions of decoupled boundary systems factorize in the "boundary" description, but not in the "bulk" description due to gravitational configurations that connect the two systems. These space-time Wormhole explanations give rise to the so-called "Factorizing Puzzle". Wormhole toy models such as SYK Model/JT Gravity, by studying the averaging over ensembles of the boundary systems, get rid of the puzzle. However, a current study (arXiv:2103.16754 [hep-th]) unveils a version of the puzzle from such models by considering single elements of the ensemble with fixed fermion couplings. It pointed out that new Half-wormhole saddles, which are non-self-averaging, exist and restore the factorization of semiclassical description. In this talk, we review Half-wormhole solutions and their possible realizations from Black Hole singularity.

Theoretical background

Sachdev–Ye–Kitaev (SYK) models

SYK models: Toy models for Wormhole calculation of boundary systems. It's a (large) N Majorana fermions theory satisfying Clifford algebra described by the Lagrangian

$$\mathcal{L}_{\mathrm{SYK}} = \frac{1}{2}\sum_{i=1}^{N} \chi_i \dot{\chi}_i - \frac{1}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l\,.$$

with Gaussian-distributed random couplings,

$$\overline{J_{ijkl}} = 0, \quad \overline{J_{ijkl}^2} = \frac{3!J^2}{N^3}\,.$$

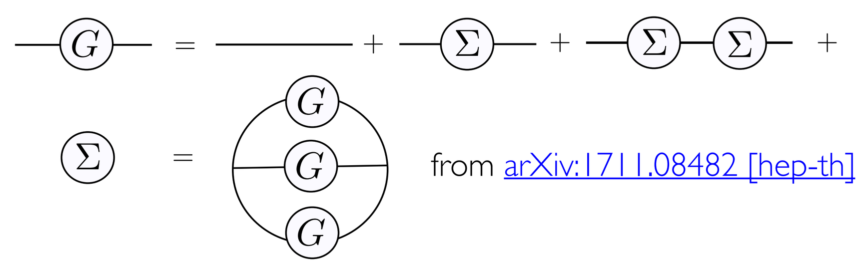
Bi-local collective fields,

$$G_0(\tau,\tau') \equiv \frac{1}{N}\sum_{i=1}^{N}\langle \mathcal{T}\chi_i(\tau)\chi_i(\tau')\rangle\,,$$

$$G(\tau,\tau') = G_0(\tau,\tau') + \int d\tau d\tau' G_0(\tau,\tau_1)\Sigma(\tau_1-\tau_2)G(\tau_2-\tau'),$$

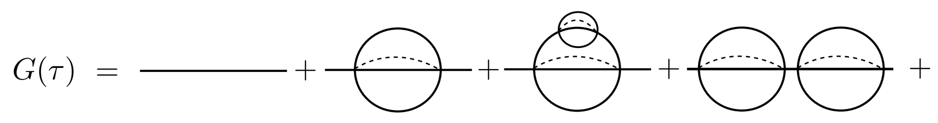
$$\Sigma(\tau,\tau') \equiv J^2 G(\tau,\tau')^3.$$

Iterative structure:



from arXiv:1711.08482 [hep-th]

Geometric series can be reduced as Dyson - Swingcher Equation:



from arXiv:1711.08482 [hep-th]

$$G = G_0 + G_0\Sigma G_0 + G_0\Sigma G_0\Sigma G_0 + \ldots$$

$$= \frac{1}{G_0^{-1}-\Sigma}.\ (\textbf{Dyson-Schwinger Equation})$$

Effective action

$$\frac{I_{eff}}{N} = -\frac{1}{2}\log\det\left(-\delta(\tau-\tau')\partial_\tau - \Sigma(\tau,\tau')\right) + \frac{1}{2}\int d\tau d\tau'\left(\Sigma(\tau,\tau')G(\tau,\tau') - \frac{J^2}{q}G(\tau,\tau')^q\right).$$

Saddle points when we varies action w.r.t. collective fields

$$\delta_G \text{ yields } \Sigma = J^2G^3 \text{ (as above)},$$

$$\delta_\Sigma \text{ yields } G^{-1} = G_0^{-1} - \Sigma \text{ (as DS eq)}.$$

Jackiw–Teitelboim (JT) Gravity

JT Gravity: (SYK with $h=2$ mode) 2d dilaton gravity theory defined by Euclidean action, comprise of Einstein - Hilbert action + Boundary + Matter + Dilaton term which can be summed as,

- Topological term consisting of Einstein-Hilbert action and part of boundary term over extrinsic curvature, which only depend topologically on Euler character of the manifold (via Gauss-Bonnet Theorem)

$$S_{\mathrm{Topo.}} = -\frac{S_0}{2\pi}\left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}R + \int_{\partial\mathcal{M}}\sqrt{h}K\right].$$

- The term which we integrate out dilaton yields E.O.M. of negative constant curvature $R=-2$,

$$S_{\mathrm{Curv.}} = -\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2).$$

- Action for boundary to regularize the assymptoptic boundary.

$$S_{\mathrm{Bdry}} = -\frac{1}{2}\int_{\partial\mathcal{M}}\sqrt{h}\phi(\mathcal{K}+2).$$

Spectral Form Factor (SFF)

Considering JT Gravity is a matrix integral. **SFF** is considered as a Fourier transform of energy level spacing, with β is the inverse temperature. We start with the (analytically continued) thermal partition function,

$$Z(\beta+it) = \mathrm{Tr}\, e^{-\beta H - itH}.$$

In JT Gravity, we cannot diagonalize gravity Hamiltonian to calculate energy level directly, hence, we use path integral to study gravity through modulus square of thermal partition function as,

$$Z(\beta+it)Z(\beta-it) = \sum_{n,m} e^{-\beta(E_m+E_n)+it(E_m-E_n)}.$$

The SFF is the Fourier transform of the eigenvalue pair correlation function which for GUE we can write schematically as

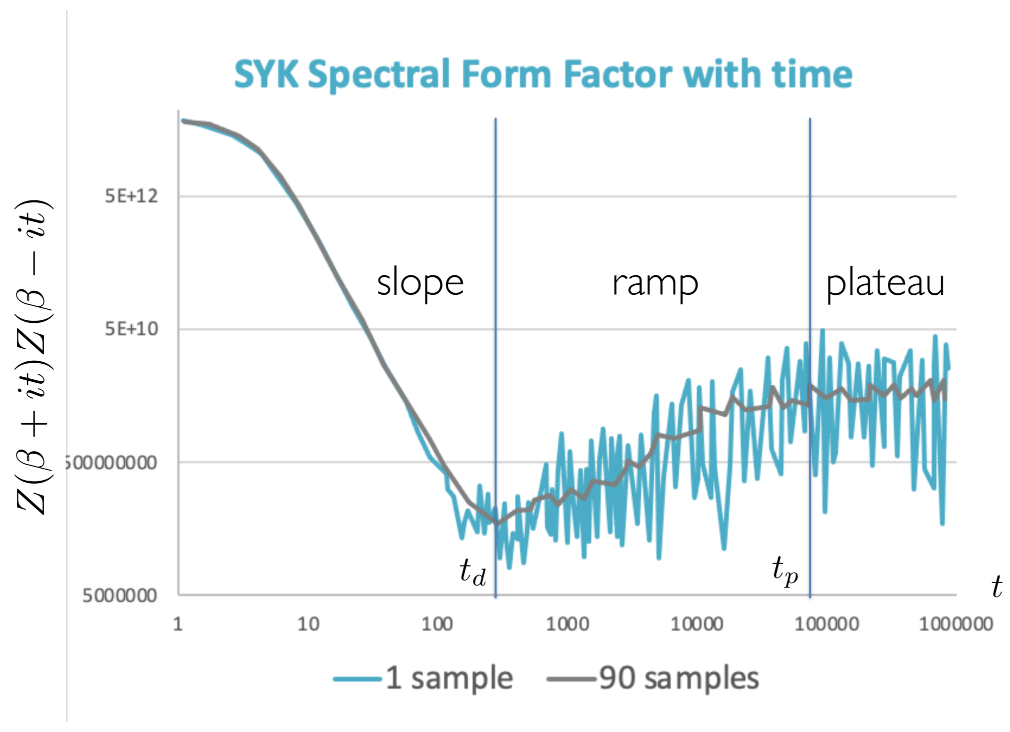
$$\langle Z(\beta+it)Z(\beta-it)\rangle = \int dE_1 dE_2 \langle\rho(E_1)\rho(E_2)\rangle e^{-\beta(E_1+E_2)}e^{-it(E_1-E_2)}.$$

The pair density correlator $\langle\tilde{\rho}(E_1)\tilde{\rho}(E_2)\rangle$ can be decomposed into Disconnected term $\langle\tilde{\rho}_0(E_1)\tilde{\rho}_0(E_2)\rangle$, Sine kernel and Contact term. The **Slope** come from Fourier transform of semi circle of the disconnected term, which yields smooth decay from L^2 (no imaginary part) to Thouless time $t_d \sim \sqrt{L}$, The disconnected part of SFF dominate at this stage. We can say that the continuity of density of states lead to decay.

After Thouless time, the connected part of SFF dominate, which come from the sine kernel part related to discreteness of spectrum.

$$\langle\delta\tilde{\rho}(E_1)\delta\tilde{\rho}(E_2)\rangle_{\mathrm{GUE}} = -\frac{\sin^2[L(E_1-E_2)]}{[\pi L(E_1-E_2)]^2} + \frac{1}{L\pi}\delta(E_1-E_2).$$

The **Ramp** and **Plateau** comes from fast-perturbative part of the Sine kernel term. They are exponentially lower than the slope. The Ramp linearly grows $\sim t/4\pi\beta$ until Heisenberg time $t_p \sim L$, then it comes to the Plateau, which is proportional to $1/L$.



Half Worm-hole solution

based on arXiv:2103.16754 [hep-th]

Factorial problems: For correlation function across field theory, the boundary should factorize e.g partition function $Z_{LR} = Z_L Z_R$, while in the gravity perspective (bulk), it fails. There is an additional term for spacetime Worm-hole.

In SFF, Worm-hole raises the factorial problem, and we see that the noise at late-time is at the same size as wormhole's contribution.

$\longrightarrow$ What cause the noise can resolve the factorization problems.

We consider SYK model in a particular ensemble (one fixed coupling), we derive the two replicas:

$$z_L z_R = \int d^{2N}\psi \exp\left\{ i^{q/2} \sum_{1\le i_1<\cdots<i_1\le N} J_{i_1\ldots i_q}\left(\psi^L_{i_1\ldots i_q} + \psi^R_{i_1\ldots i_q}\right)\right\}.$$

which become

$$z_L z_R = \int_{\mathbb{R}} d\sigma\, \Psi(\sigma)\Phi(\sigma),$$

with

$$\Psi(\sigma) = \int_{\mathbb{R}}\frac{dg}{(2\pi/N)}\exp\left\{N\Big(-i\sigma g - \frac{1}{q}g^q\Big)\right\}.$$

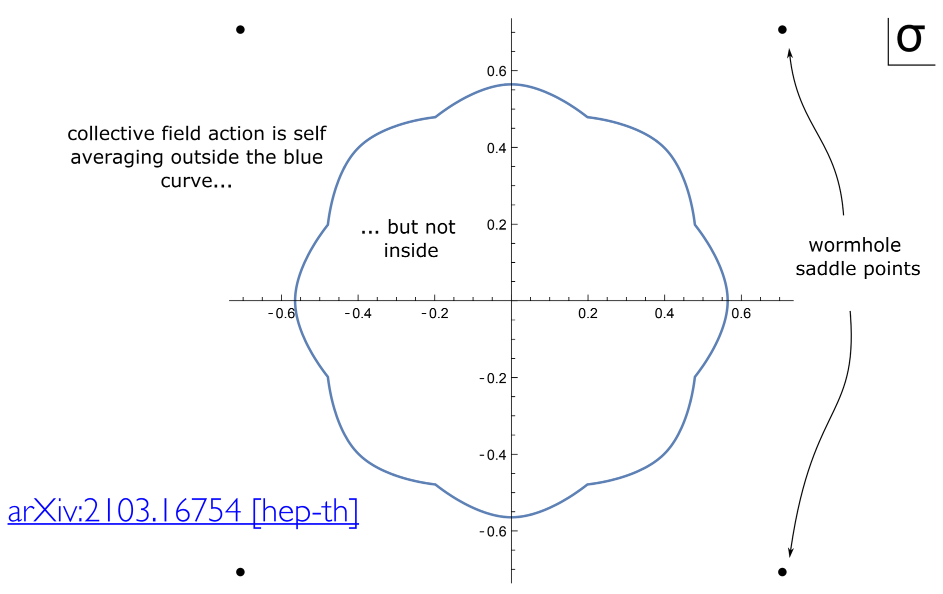
$$\Phi(\sigma) = \int d^{2N}\psi \exp\left\{ie^{-\frac{i\pi}{q}}\sigma\psi_i^L\psi_i^R + i^{q/2}J_{i_1\ldots i_q}(\psi^L_{i_1\ldots i_q} + \psi^R_{i_1\ldots i_q}) - \frac{N}{q}\left(\tfrac{1}{N}\psi_i^L\psi_i^R\right)^q\right\}.$$

Hence, the average read,

$$\langle\Phi(\sigma)\rangle = \int d^{2N}\psi \exp\left\{ie^{-\frac{i\pi}{q}}\sigma\psi_i^L\psi_i^R\right\} = (ie^{-\frac{i\pi}{q}}\sigma)^N.$$

To determine the "self-average" property, we compare average square to the second moment

$$\langle\Phi(\sigma)^2\rangle = \sum_{n_1+n_2+n_3=\frac{N}{q},\ n_i\ge 0} \frac{N!}{N^{2q(n_2+n_3)}}\left(\frac{N}{q}\right)^{2(n_2+n_3)} \frac{\sigma^{2qn_1}}{(qn_1)!}\frac{(qn_2)!(qn_3)!}{(n_2!)^2(n_3!)^2}.$$



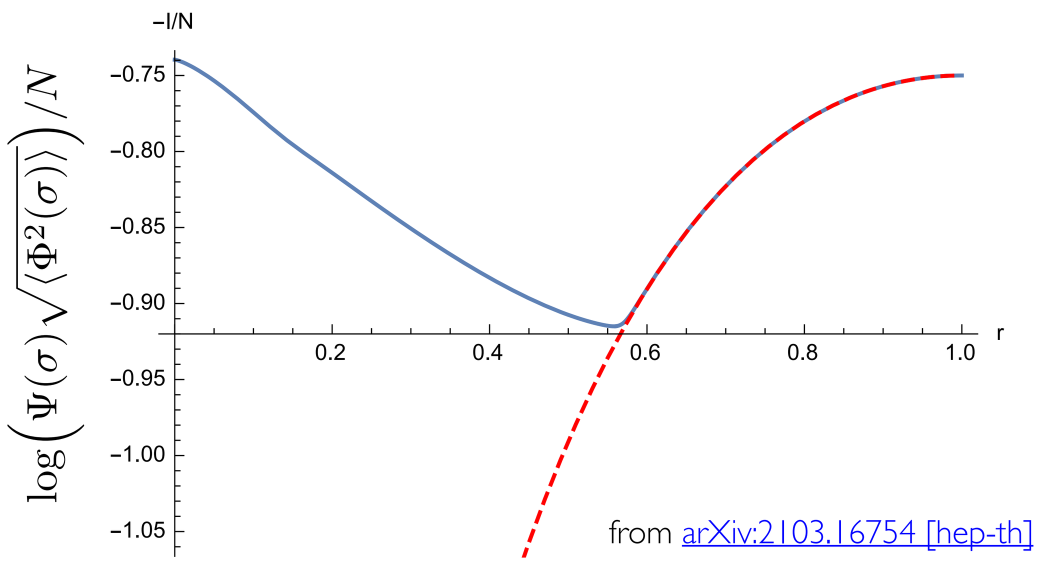
from arXiv:2103.16754 [hep-th]

$\langle\Phi(\sigma)^2\rangle$ and $\langle\Phi(\sigma)\rangle^2$ are the same outside the curve but different inside.

Outside: Self-averaging. Wormhole persist, and weakly depends on couplings.

Inside: Non self-averaging. New Half-wormhole solutions exist & strongly depends on the choice of couplings $\longrightarrow$ Contribute to the noise.

We slide along the 45^0 ray of the complex σ plane,



from arXiv:2103.16754 [hep-th]

Wormhole saddle is at $|\sigma| = 1$. Going back to the origin, the signal is exponentially suppressed, it reach the minimum, then get back to the comparable value to the Wormhole saddle at origin.

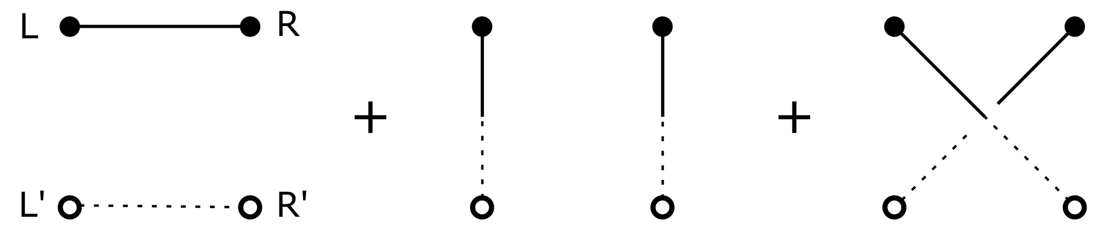
The new contribution here is non-self averaging and called "Half-wormhole"

$$z_L z_R \approx \Big(\text{wormhole saddles with } |\sigma| = 1\Big) + \Big(\text{"half-wormholes" saddle at } \sigma = 0\Big).$$

Red dashed line is averaged integrand $\langle\Phi(\sigma)\rangle \propto \sigma^N$, varnishing at origin. Half wormhole varnishes after averaging.

We introduce a second auxiliary system L', R' and compute second moment

$$\langle z_L z_R z_{L'} z_{R'}\rangle = \int_{\mathbb{R}} d\sigma_{LR} d\sigma_{L'R'}\Psi(\sigma_{LR})\Psi(\sigma_{L'R'})\langle\Phi(\sigma_{LR})\Phi(\sigma_{L'R'})\rangle.$$



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In first diagram, only LR & L'R' - Wormhole saddle.

In second & third diagram, LR varnishes, only wormhole connecting physical system to auxiliary system. This remnant in the physical system are called "Half-wormhole" (nothing to do with the geometry).

Contribution from Half-wormhole is ensure to be the same size with Wormhole by the permutation symmetry. It allows the Half-wormhole contribution cancel the non-factorizing effect of the Wormhole.

Black Hole singularity realizations

based on arXiv:2107.13130 [hep-th], arXiv:1806.06840 [hep-th]

We introduce $Y(T)$, a smoothed microcanonical transformation of $Z(\beta+iT)$ of SYK models and the its SFF counterpart:

$$Y_{E,\Delta E}(T) \equiv \int_{\gamma+i\mathbb{R}} d\beta\, e^{\beta E+\beta^2\Delta E^2} Z(\beta+iT)$$

$$|Y_{E,\Delta E}(T)|^2 = \int_{\gamma+i\mathbb{R}} d\beta_L\, e^{\beta_L E+\beta_L^2\Delta E^2} Z(\beta_L+iT)\int_{\gamma+i\mathbb{R}} d\beta_R\, e^{\beta_R E+\beta_R^2\Delta E^2} Z(\beta_R-iT).$$

The double cone solution can be constructed from eternal black hole geometry with periodic Schwarzschild time, which has a canonical singularity at bifurcate horizon. This singularity can be treated by complexifying the r coordinate.

We can do the same thing with Half-Wormhole solutions. For each copy of $Y_E(T)$, with the eternal BHs, we take r coordinate from infinity to zero, going around the singularity. The closed universe evolves in time like r direction, ending at the BHs singularity. The geometry required boundary condition to $Y_E(T)$, and is solutions to Einstein's equations away from origin. However, UV-sensitive spacetime action near singularity needed first to be evaluated for this calculation.

As BHs singularity must be resolved in Quantum Gravity, the Half-Wormhole solutions is a candidate for theory and cause the noise to $Y_E(T)$, which results from UV physics near the singularity.

The singularity can be seen as the projection of a single closed universe onto a state $|\Psi\rangle$, which relate to $Y_E(T)$,

At singularity, the $|\Psi\rangle$ must be random to create the noise from SFF. For Gaussian random state, averaging over Half-wormhole's pairs will glue them together to form a double cone.

Hence, BHs' singularity may play the role of random $|\Psi\rangle$ boundaries.

References

1. arXiv:2103.16754 [hep-th]
2. arXiv:2107.13130 [hep-th]
3. arXiv:1806.06840 [hep-th]
4. arXiv:1711.08482 [hep-th]
5. arXiv:1611.04650 [hep-th]

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