QUESTION 5

Theorem All Americans are the same age.

Proof:

1. Let $S(n)$ be the statement: In any group of n Americans, everyone in that group has the same age.
2. We prove $S(n)$ by induction on n.
3. Since everyone in a group of one American has the same age, $S(1)$ is true.
4. Assume $S(n)$ is true for some n.
5. We prove $S(n+1)$.
6. Let G be an arbitrary group of $n+1$ Americans.
7. We show that everyone in G has the same age.
8. We do this by showing that any two members of G have the same age.
9. Let $a, b \in G$.
10. Let G_a be the result of removing a from G.
11. Since G_a has n members, b (which is in G_a) has the same age as any other person in G_a.
12. Similarly, if G_b is G with b removed, then a has the same age as any other person in G_b.
13. Now let c be any person in G other than a and b.
14. Then $c \in G_a$ and c $\in G_b$.
15. So, a and b both have the same age as c.
16. Hence a and b have the same age.
17. This proves $S(n+1)$.
18. Hence, by induction, $S(n)$ is true for all n.
19. This implies that all Americans have the same age.