

KTH Royal Institute of Technology

Omogen Heap

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G	$\frac{1}{2}$ raphs (1)	
	imHeap.cpp scription: desc	8d29d7, 40 line
us:	ing edge = pair <int, int="">; // (v. cost)</int,>	

```
using item = pair<int, int>; // (cost, v)
/\!/\operatorname{Prim's\ algorithm\ in\ }O(E*log\left(V\right))\ time:\ https://cp-algorithms.
     com/graph/mst\_prim.html
tuple<long long, vector<int>> prim_mst(const vector<vector<edge</pre>
    >> &g) {
    size_t n = q.size();
    vector<int> pred(n, -1);
    vector<int> prio(n, numeric_limits<int>::max());
    priority_queue<item, vector<item>, greater<>> q;
    q.emplace(prio[0] = 0, 0);
    long long tree_cost = 0;
    while (!q.empty()) {
        auto [c, u] = q.top();
        q.pop();
        if (prio[u] != c)
            continue;
        tree cost += c;
        for (auto [v, cost] : g[u]) {
            if (prio[v] > cost) {
                prio[v] = cost;
                pred[v] = u;
                q.emplace(cost, v);
    return {tree_cost, pred};
```

```
// usage example
   int main() {
       vector<vector<edge>> graph = {{edge{1, 10}, edge{2, 5}}, {
            edge{0, 10}, edge{2, 10}}, {edge{1, 10}, edge{0, 5}}};
0
       auto [tree_weight, pred] = prim_mst(graph);
       cout << tree_weight << endl;</pre>
       for (int x : pred)
           cout << x << " ";
       cout << endl;
   CycleDetection.cpp
   Description: desc
                                                         bf7a7c, 51 lines
  int dfs(const vector<vector<int>> &graph, int u, vector<int> &
2
        color, vector<int> &next) {
       color[u] = 1;
       for (int v : graph[u]) {
\mathbf{2}
           next[u] = v;
           if (color[v] == 0) {
6
                int cycleStart = dfs(graph, v, color, next);
                if (cycleStart != -1) {
                   return cycleStart;
6
           } else if (color[v] == 1) {
               return v:
       color[u] = 2;
       return -1;
   vector<int> find_cycle(const vector<vector<int>> &graph) {
       int n = graph.size();
       vector<int> color(n);
       vector<int> next(n);
       for (int u = 0; u < n; u++) {
           if (color[u] != 0)
               continue;
           int cycleStart = dfs(graph, u, color, next);
           if (cycleStart != -1) {
               vector<int> cvcle;
                cycle.push_back(cycleStart);
                for (int i = next[cycleStart]; i != cycleStart; i =
                     next[i]) {
                    cycle.push_back(i);
               cycle.push_back(cycleStart);
               return cycle;
       return {};
   // usage example
   int main() {
       vector<vector<int>> graph{{1}, {2}, {0}};
       auto cycle = find_cycle(graph);
       cout << cycle.size() << endl;</pre>
       for (int x : cycle)
           cout << x << " ";
       cout << endl;
       graph = \{\{1\}, \{2\}, \{\}\};
       cycle = find_cycle(graph);
       cout << cycle.size() << endl;</pre>
```

```
TreeCenters.cpp
```

```
Description: desc
                                                      aea483, 21 lines
// returns vertex that has all its subtrees sizes \leq n/2
int find_tree_centroid(const vector<vector<int>> &tree, int u,
     int p) {
    int n = tree.size();
    int cnt = 1;
    bool goodCenter = true;
    for (int v : tree[u]) {
        if (v == p)
            continue;
        int res = find_tree_centroid(tree, v, u);
        if (res >= 0)
            return res;
        int size = -res;
        goodCenter &= size <= n / 2;</pre>
        cnt += size;
    goodCenter &= n - cnt <= n / 2;
    return goodCenter ? u : -cnt;
// usage example
int main() {}
TreeIsomorphism.cpp
Description: desc
                                                     13d630, 150 lines
vector<vector<int>> children, subtreeLabels, tree, L;
vector<int> pred, mapping;
int n;
auto compare = [](int a, int b) -> bool { return subtreeLabels[
     a) < subtreeLabels[b]; };</pre>
auto equals = [](int a, int b) -> bool { return subtreeLabels[a
     ] == subtreeLabels[b]; };
void generate_mapping(int r1, int r2) {
    mapping.resize(n);
    mapping[r1] = r2 - n;
    sort(children[r1].begin(), children[r1].end(), compare);
    sort(children[r2].begin(), children[r2].end(), compare);
    for (int i = 0; i < children[r1].size(); i++) {</pre>
        int u = children[r1][i];
        int v = children[r2][i];
        generate_mapping(u, v);
vector<int> find_center(int offset = 0) {
    int cnt = n;
    vector<int> a;
    vector<int> deg(n);
    for (int i = 0; i < n; i++) {</pre>
        deg[i] = tree[i + offset].size();
        if (deg[i] <= 1) {
            a.push_back(i + offset);
            --cnt;
    while (cnt > 0) {
        vector<int> na;
        for (int i = 0; i < a.size(); i++) {</pre>
            int u = a[i];
            for (int j = 0; j < tree[u].size(); j++) {</pre>
                int v = tree[u][j];
```

if (--deg[v - offset] == 1) {

na.push_back(v);

--cnt;

int main() {
 n = 5;

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```
a = na;
    return a;
int dfs(int u, int p = -1, int depth = 0) {
    L[depth].push_back(u);
    int h = 0;
    for (int i = 0; i < tree[u].size(); i++) {</pre>
        int v = tree[u][i];
        if (v == p)
            continue;
        pred[v] = u;
        children[u].push_back(v);
       h = max(h, dfs(v, u, depth + 1));
    return h + 1;
bool rooted tree isomorphism(int r1, int r2) {
    L.assign(n, vector<int>());
    pred.assign(2 * n, -1);
    children.assign(2 * n, vector<int>());
    int h1 = dfs(r1);
    int h2 = dfs(r2);
    if (h1 != h2)
        return false;
    int h = h1 - 1;
    vector<int> label(2 * n);
    subtreeLabels.assign(2 * n, vector<int>());
    for (int i = h - 1; i >= 0; i--) {
        for (int j = 0; j < L[i + 1].size(); j++) {
            int v = L[i + 1][j];
            subtreeLabels[pred[v]].push_back(label[v]);
        sort(L[i].begin(), L[i].end(), compare);
        for (int j = 0, cnt = 0; j < L[i].size(); j++) {</pre>
            if (j && !equals(L[i][j], L[i][j - 1]))
            label[L[i][j]] = cnt;
    if (!equals(r1, r2))
        return false;
    generate_mapping(r1, r2);
    return true;
bool treeIsomorphism() {
    vector<int> c1 = find center();
    vector<int> c2 = find center(n);
    if (c1.size() == c2.size()) {
        if (rooted_tree_isomorphism(c1[0], c2[0]))
            return true;
        else if (c1.size() > 1)
            return rooted_tree_isomorphism(c1[1], c2[0]);
    return false;
```

```
vector<vector<int>> t1(n);
    t1[0].emplace_back(1);
    t1[1].emplace_back(0);
    t1[1].emplace_back(2);
    t1[2].emplace_back(1);
    t1[1].emplace_back(3);
    t1[3].emplace_back(1);
    t1[0].emplace_back(4);
    t1[4].emplace_back(0);
    vector<vector<int>> t2(n);
    t2[0].emplace_back(1);
    t2[1].emplace_back(0);
    t2[0].emplace_back(4);
    t2[4].emplace_back(0);
    t2[4].emplace_back(3);
    t2[3].emplace_back(4);
    t2[4].emplace_back(2);
    t2[2].emplace_back(4);
    tree.assign(2 * n, vector<int>());
    for (int u = 0; u < n; u++) {
        for (int i = 0; i < t1[u].size(); i++) {</pre>
            int v = t1[u][i];
            tree[u].emplace back(v);
        for (int i = 0; i < t2[u].size(); i++) {</pre>
            int v = t2[u][i];
            tree[u + n].emplace_back(v + n);
    }
    bool res = treeIsomorphism();
    cout << res << endl;
    if (res)
        for (int i = 0; i < n; i++)</pre>
            cout << mapping[i] << endl;</pre>
LcaRmqSchieberVishkin.cpp
Description: desc
                                                     2f8390, 125 lines
#ifdef MSC VER
int __builtin_clz(unsigned x) {
    int bit = 31;
    while (bit >= 0 && (x & (1 << bit)) == 0)
       --bit;
    return 31 - bit;
#endif
const int MAX_NODES = 200'000;
int parent [MAX_NODES];
unsigned pre_order[MAX_NODES];
unsigned I[MAX NODES];
int head[MAX_NODES];
unsigned A[MAX_NODES];
unsigned Time;
unsigned lowest_one_bit(unsigned x) {
    return x \& -x;
unsigned highest_one_bit(unsigned x) {
    return x ? 1u << (31 - __builtin_clz(x)) : 0;
```

```
void dfs1(const vector<vector<int>> &tree, int u, int p) {
    parent[u] = p;
    I[u] = pre_order[u] = Time++;
    for (int v : tree[u]) {
        if (v == p)
            continue;
        dfs1(tree, v, u);
        if (lowest_one_bit(I[u]) < lowest_one_bit(I[v])) {</pre>
            I[u] = I[v];
    head[I[u]] = u;
void dfs2(const vector<vector<int>> &tree, int u, int p,
    unsigned up) {
    A[u] = up | lowest_one_bit(I[u]);
    for (int v : tree[u]) {
        if (v == p)
            continue;
        dfs2(tree, v, u, A[u]);
// initialization in O(n)
void init_lca(const vector<vector<int>> &tree, int root) {
    Time = 0:
    dfs1(tree, root, -1);
    dfs2(tree, root, -1, 0);
int enter_into_strip(int x, int hz) {
    if (lowest_one_bit(I[x]) == hz)
    int hw = highest_one_bit(A[x] & (hz - 1));
    return parent[head[(I[x] & -hw) | hw]];
// lca in O(1)
int lca(int x, int v) {
    int hb = I[x] == I[y] ? lowest_one_bit(I[x]) :
        highest one bit(I[x] ^ I[v]);
    int hz = lowest_one_bit(A[x] & A[y] & -hb);
    int ex = enter_into_strip(x, hz);
    int ey = enter into strip(y, hz);
    return pre_order[ex] < pre_order[ey] ? ex : ey;
void init_rmq(const vector<int> &values) {
    // build Cartesian Tree
    int n = values.size();
    int root = 0;
    vector<int> p(n, -1);
    for (int i = 1; i < n; i++) {
        int prev = i - 1;
        int next = -1;
        while (values[prev] > values[i] && p[prev] != -1) {
            next = prev;
            prev = p[prev];
        if (values[prev] > values[i]) {
            p[prev] = i;
            root = i;
       } else {
            p[i] = prev;
            if (next != -1) {
                p[next] = i;
```

BellmanFord Dijkstra DijkstraCustomHeap

```
vector<vector<int>> tree(n);
    for (int i = 0; i < n; ++i)
       if (p[i] != -1)
           tree[p[i]].push_back(i);
    init_lca(tree, root);
// random test
int main() {
   mt19937 rng(1);
    for (int step = 0; step < 1000; ++step) {
        int n = uniform_int_distribution<int>(1, 10)(rng);
        vector<int> v(n);
        for (int i = 0; i < n; ++i) {
            v[i] = uniform_int_distribution<int>(0, 5)(rng);
        int a = uniform_int_distribution<int>(0, n - 1)(rng);
        int b = uniform_int_distribution<int>(0, n - 1)(rng);
       if (a > b)
            swap(a, b);
        init_rmq(v);
       int res1 = v[lca(a, b)];
        int res2 = *min_element(v.begin() + a, v.begin() + b +
       if (res1 != res2) {
            for (int i = 0; i < n; ++i)
               cout << v[i] << " ";
            cout << endl;
            cout << a << " " << b << " - " << res1 << " " <<
                res2 << endl;
            assert (res1 != res2);
1.1 Shortest Paths
BellmanFord.cpp
Description: desc
                                                    e48dec, 74 lines
// https://cp-algorithms.com/graph/bellman_ford.html
typedef pair<int, int> edge;
const int INF = numeric limits<int>::max() / 3;
tuple<bool, vector<int>, vector<int>> bellman_ford(const vector
    <vector<edge>> &g, int s) {
    size_t n = g.size();
   vector<int> prio(n, INF);
   prio[s] = 0;
   vector<int> pred(n, -1);
   bool was_changed = true;
    for (int k = 0; k < n; k++) {
        was_changed = false;
        for (int u = 0; u < n; u++) {</pre>
            for (auto [v, cost] : g[u]) {
                if (prio[v] > prio[u] + cost) {
                    prio[v] = prio[u] + cost;
                    pred[v] = u;
                    was_changed = true;
        if (!was_changed)
            break;
```

```
// was_changed is true iff graph has a negative cycle
    return {was_changed, prio, pred};
vector<int> find_negative_cycle(const vector<vector<edge>> &g)
    size_t n = g.size();
    vector<int> pred(n, -1);
    vector<int> prio(n, INF);
   prio[0] = 0;
    int last = 0;
    for (int k = 0; k < n; k++) {
        last = -1;
        for (int u = 0; u < n; u++) {
            for (auto [v, cost] : g[u]) {
                if (prio[v] > prio[u] + cost) {
                    prio[v] = prio[u] + cost;
                    pred[v] = u;
                    last = v;
        if (last == -1)
            return {};
    vector<int> path(n);
    vector<int> pos(n, -1);
    for (int i = 0; ; i++) {
        if (pos[last] !=-1)
            return vector<int>(path.rend() - i, path.rend() -
        path[i] = last;
        pos[last] = i;
        last = pred[last];
    vector<vector<edge>> g(4);
    g[0].emplace_back(1, 1);
    q[1].emplace back(0, 1);
    q[1].emplace_back(2, 1);
    g[2].emplace back(3, -10);
    g[3].emplace_back(1, 1);
    vector<int> cycle = find_negative_cycle(g);
    for (int u : cycle)
        cout << u << " ";
Dijkstra.cpp
Description: desc
                                                     9a8489, 76 lines
typedef pair<int, int> edge;
typedef pair<int, int> item;
// https://cp-algorithms.com/graph/dijkstra\_sparse.html
// O(E*log(V)) time and O(E) memory
tuple<vector<int>, vector<int>> dijkstra_heap(const vector<
    vector<edge>> &g, int s) {
    size_t n = q.size();
    vector<int> prio(n, numeric_limits<int>::max());
    vector<int> pred(n, -1);
    priority_queue<item, vector<item>, greater<>> q;
    q.emplace(prio[s] = 0, s);
```

while (!q.empty()) {

```
auto [d, u] = q.top();
        q.pop();
        if (d != prio[u])
            continue;
        for (auto [v, len] : g[u]) {
            int nprio = prio[u] + len;
            if (prio[v] > nprio) {
                prio[v] = nprio;
                pred[v] = u;
                q.emplace(nprio, v);
       }
    return {prio, pred};
// O(E*log(V)) time and O(V) memory
tuple<vector<int>, vector<int>> dijkstra_set(const vector<</pre>
    vector<edge>> &g, int s) {
    size_t n = g.size();
    vector<int> prio(n, numeric_limits<int>::max());
    vector<int> pred(n, -1);
    set<item> q;
    q.emplace(prio[s] = 0, s);
    while (!q.empty()) {
        int u = q.begin()->second;
        q.erase(q.begin());
        for (auto [v, len] : g[u]) {
            int nprio = prio[u] + len;
            if (prio[v] > nprio) {
                q.erase({prio[v], v});
                prio[v] = nprio;
                pred[v] = u;
                g.emplace(prio[v], v);
       }
    return {prio, pred};
int main() {
    vector<vector<edge>> q(3);
    q[0].emplace back(1, 10);
    g[1].emplace_back(2, -5);
    q[0].emplace_back(2, 8);
    auto [prio1, pred1] = dijkstra_heap(g, 0);
    auto [prio2, pred2] = dijkstra_set(g, 0);
    for (int x : prio1)
        cout << x << " ";
    cout << endl;
    for (int x : prio2)
        cout << x << " ";
    cout << endl;
DijkstraCustomHeap.cpp
Description: desc
                                                    9e60b8, 118 lines
const int maxnodes = 200'000;
```

const int maxedges = 1000'000;

StronglyConnectedComponents TopologicalSort

```
// graph
int edges;
int last[maxnodes], head[maxedges], previous[maxedges], len[
int prio[maxnodes], pred[maxnodes];
void reset_graph() {
    fill(last, last + maxnodes, -1);
    edges = 0:
void add_edge(int u, int v, int length) {
    head[edges] = v;
    len[edges] = length;
    previous[edges] = last[u];
    last[u] = edges++;
// heap
int h[maxnodes];
int pos2Id[maxnodes];
int id2Pos[maxnodes];
int hsize:
void hswap(int i, int j) {
    swap(h[i], h[j]);
    swap(pos2Id[i], pos2Id[j]);
    swap(id2Pos[pos2Id[i]], id2Pos[pos2Id[j]]);
void move_up(int pos) {
    while (pos > 0) {
        int parent = (pos - 1) >> 1;
        if (h[pos] >= h[parent]) {
            break;
       hswap(pos, parent);
        pos = parent;
void add(int id, int prio) {
    h[hsize] = prio;
    pos2Id[hsize] = id;
    id2Pos[id] = hsize;
    move up(hsize++);
void decrease_value(int id, int prio) {
    int pos = id2Pos[id];
    h[pos] = prio;
    move_up(pos);
void move down(int pos) {
    while (pos < (hsize >> 1)) {
        int child = 2 * pos + 1;
        if (child + 1 < hsize && h[child + 1] < h[child]) {</pre>
            ++child;
        if (h[pos] <= h[child]) {</pre>
            break;
       hswap(pos, child);
        pos = child;
```

```
int remove min() {
    int res = pos2Id[0];
    int lastNode = h[--hsize];
    if (hsize > 0) {
        h[0] = lastNode;
        int id = pos2Id[hsize];
        id2Pos[id] = 0;
        pos2Id[0] = id;
        move_down(0);
    return res:
void dijkstra(int s) {
    fill (pred, pred + maxnodes, -1);
    fill(prio, prio + maxnodes, numeric_limits<int>::max());
    prio[s] = 0;
    hsize = 0;
    add(s, prio[s]);
    while (hsize) {
        int u = remove_min();
        for (int e = last[u]; e >= 0; e = previous[e]) {
            int v = head[e];
            int nprio = prio[u] + len[e];
            if (prio[v] > nprio) {
                if (prio[v] == numeric_limits<int>::max())
                    add(v, nprio);
                    decrease_value(v, nprio);
                prio[v] = nprio;
                pred[v] = u;
    }
int main() {
    reset graph();
    add_edge(0, 1, 10);
    add_edge(1, 2, -5);
    add edge(0, 2, 8);
    dijkstra(0);
    for (int i = 0; i < 3; i++)
        cout << prio[i] << endl;</pre>
1.2 DFS
StronglyConnectedComponents.cpp
Description: desc
                                                     bf930c, 71 lines
// https://en.wikipedia.org/wiki/Kosaraju%27s_algorithm
void dfs(const vector<vector<int>> &graph, vector<bool> &used,
    vector<int> &res, int u) {
    used[u] = true;
    for (int v : graph[u])
        if (!used[v])
            dfs(graph, used, res, v);
    res.push_back(u);
// Get strongly connected components
vector<vector<int>> scc(const vector<vector<int>> &graph) {
    int n = graph.size();
    vector<bool> used(n);
    vector<int> order;
```

```
for (int i = 0; i < n; i++)
        if (!used[i])
            dfs(graph, used, order, i);
    vector<vector<int>> reverseGraph(n);
    for (int i = 0; i < n; i++)
        for (int j : graph[i])
            reverseGraph[j].push_back(i);
    vector<vector<int>> components;
    fill(used.begin(), used.end(), false);
    reverse(order.begin(), order.end());
    for (int u : order)
        if (!used[u]) {
            vector<int> component;
            dfs(reverseGraph, used, component, u);
            components.push_back(component);
    return components;
// DAG of strongly connected components
vector<vector<int>> scc_graph(const vector<vector<int>> &graph,
      const vector<vector<int>> &components) {
    vector<int> comp(graph.size());
    for (int i = 0; i < components.size(); i++)</pre>
        for (int u : components[i])
            comp[u] = i;
    vector<vector<int>> g(components.size());
    unordered_set<long long> edges;
    for (int u = 0; u < graph.size(); u++)</pre>
        for (int v : graph[u])
            if (comp[u] != comp[v] && edges.insert(((long long)
                 comp[u] \ll 32) + comp[v]).second)
                g[comp[u]].push_back(comp[v]);
    return q;
// usage example
int main() {
    vector<vector<int>> graph = {{1}, {0}, {0, 1}};
    vector<vector<int>> components = scc(graph);
    for (auto &component : components) {
        for (int v : component)
            cout << v << " ";
        cout << endl;
    vector<vector<int>> sg = scc_graph(graph, components);
    for (auto &a : sq) {
        for (int v : a)
            cout << v << " ";
        cout << endl:
TopologicalSort.cpp
Description: desc
// https://en.wikipedia.org/wiki/Topological_sorting
void dfs(const vector<vector<int>> &graph, vector<bool> &used,
    vector<int> &order, int u) {
    used[u] = true;
    for (int v : graph[u])
        if (!used[v])
```

w[i] += cap[sel][i];

prev = sel;

```
KTH
            dfs(graph, used, order, v);
    order.push_back(u);
vector<int> topological_sort(const vector<vector<int>> &graph)
    size_t n = graph.size();
    vector<bool> used(n);
    vector<int> order;
    for (int i = 0; i < n; i++)</pre>
        if (!used[i])
            dfs(graph, used, order, i);
    reverse(order.begin(), order.end());
    return order;
// usage example
int main() {
    vector<vector<int>> g = \{\{0\}, \{\}, \{0, 1\}\};
    vector<int> order = topological_sort(g);
    for (int v : order)
       cout << v << " ";
    cout << endl;
1.3 Flows
GlobalMinCutStoerWagner.cpp
Description: desc
// https://en.wikipedia.org/wiki/Stoer%E2%80%93Wagner_algorithm
pair<int, vector<int>> min cut(vector<vector<int>> &cap) {
    int best cap = numeric limits<int>::max();
    vector<int> best cut;
    int n = cap.size();
    vector<vector<int>> v(n);
    for (int i = 0; i < n; ++i)
       v[i].push_back(i);
    vector<int> w(n);
    vector<bool> exist(n, true);
    vector<bool> in a(n);
    for (int ph = 0; ph < n - 1; ++ph) {
        fill(in_a.begin(), in_a.end(), false);
        fill(w.begin(), w.end(), 0);
        for (int it = 0, prev; it < n - ph; ++it) {</pre>
            int sel = -1;
            for (int i = 0; i < n; ++i)
                if (exist[i] && !in_a[i] && (sel == -1 || w[i]
                    > w[sel])) {
                    sel = i;
            if (it == n - ph - 1) {
                if (w[sel] < best_cap) {</pre>
```

best_cap = w[sel];

best_cut = v[sel];

for (int i = 0; i < n; ++i)

cap[i][prev] += cap[sel][i];

cap[prev][i] += cap[sel][i];

[sel].end()); for (int i = 0; i < n; ++i) {</pre>

exist[sel] = false;

in_a[sel] = true;

} else {

```
return {best_cap, best_cut};
                                                   // usage example
                                                   int main() {
                                                       vector<vector<int>>> capacity{{0, 1, 1, 0}, {1, 0, 1, 1}, {
                                                            1, 1, 0, 1}, {0, 1, 1, 0}};
                                                       auto [cap, cut] = min_cut(capacity);
                                                       cout << cap << endl;</pre>
                                                       for (int v : cut)
                                                           cout << v << " ";
                                                       cout << endl;
                                                   MaxFlowDinic.cpp
                                                   Description: desc
                                                   "max_flow_dinic.h"
                                                                                                         eeb74b, 12 lines
                                                   // usage example
                                                   int main() {
                                                       int capacity[][3] = {{0, 3, 2}, {0, 0, 2}, {0, 0, 0}};
                                                       max flow dinic flow(n);
                                                       for (int i = 0; i < n; i++)
                                                           for (int j = 0; j < n; j++)</pre>
                                                               if (capacity[i][j] != 0)
                                                                   flow.add_edge(i, j, capacity[i][j]);
                                                       cout << (4 == flow.max flow(0, 2)) << endl;
                                                   MaxFlowDinic.h
                                                   Description: desc
                                                                                                        cc3b04, 83 lines
                                                   // https://en.wikipedia.org/wiki/Dinic%27s_algorithm in O(V^2 *
                                                   struct Edge {
                                                       int to, rev, cap, f;
                                                   struct max_flow_dinic {
                                                       vector<vector<Edge>> graph;
                                                       vector<int> dist;
                                                       max_flow_dinic(int nodes) : graph(nodes), dist(nodes) {}
                                                       void add_edge(int s, int t, int cap) {
                                                           Edge a = {t, (int)graph[t].size(), cap, 0};
                                                           Edge b = {s, (int)graph[s].size(), cap, 0};
                                                           graph[s].emplace_back(a);
                                                           graph[t].emplace_back(b);
                                                       bool dinic bfs(int src, int dest) {
                                                           fill(dist.begin(), dist.end(), -1);
v[prev].insert(v[prev].end(), v[sel].begin(), v
                                                           dist[src] = 0;
                                                           vector<int> q(graph.size());
                                                           int qt = 0;
                                                           q[qt++] = src;
                                                           for (int qh = 0; qh < qt; qh++) {
                                                               int u = q[qh];
                                                               for (auto &e : graph[u]) {
                                                                   int v = e.to;
                                                                   if (dist[v] < 0 && e.f < e.cap) {</pre>
```

```
dist[v] = dist[u] + 1;
                    q[qt++] = v;
        return dist[dest] >= 0;
    int dinic_dfs(vector<int> &ptr, int u, int dest, int f) {
        if (u == dest)
            return f:
        for (int &i = ptr[u]; i < (int)graph[u].size(); i++) {</pre>
            Edge &e = graph[u][i];
            if (e.cap <= e.f)
                continue;
            int v = e.to;
            if (dist[v] == dist[u] + 1) {
                int df = dinic_dfs(ptr, v, dest, min(f, e.cap -
                      e.f));
                if (df > 0) {
                    e.f += df;
                    graph[v][e.rev].f -= df;
                    return df;
        return 0;
    int max_flow(int src, int dest) {
        int flow = 0;
        while (dinic_bfs(src, dest)) {
            vector<int> ptr(graph.size());
            while (int delta = dinic_dfs(ptr, src, dest,
                 numeric limits<int>::max()))
                flow += delta;
        return flow;
    // invoke after max_flow()
    vector<bool> min cut() {
        vector<bool> cut(graph.size());
        for (size t i = 0; i < cut.size(); ++i) {</pre>
            cut[i] = dist[i] != -1;
        return cut;
    void clear flow() {
        for (auto &v : graph)
            for (Edge &e : v)
                e.f = 0;
};
MinCostCirculation.cpp
Description: desc
                                                     53df05, 91 lines
// Negative cycles canceling algorithm in O(E^2 * V *
    MAXEDGEFLOW * MAXEDGECOST)
// negative-cost edges are allowed
// negative-cost cycles are allowed
struct Edge {
    int to, rev, f, cap, cost;
```

struct min cost circulation {

MinCostFlowBf MinCostFlowDijkstra

```
vector<vector<Edge>> graph;
min cost circulation(int nodes) : graph(nodes) {}
void add_edge(int s, int t, int cap, int cost) {
    Edge a = {t, (int)graph[t].size(), 0, cap, cost};
    Edge b = \{s, (int)graph[s].size(), 0, 0, -cost\};
    graph[s].emplace_back(a);
    graph[t].emplace_back(b);
int calc_min_cost_circulation() {
    int circulation_cost = 0;
    int n = graph.size();
    constexpr int INF = 1000'000'000;
    for (bool found = true; found;) {
        found = false;
        vector<int> dist(n, INF);
        vector<int> p(n, -1);
        for (int i = 0; i < n; ++i) {
            if (dist[i] == INF) {
                dist[i] = 0;
                vector<int> q;
                g.emplace_back(i);
                for (int j = 0; j < n && !q.empty(); ++j) {</pre>
                    vector<int> nq;
                    for (int u : q)
                        for (auto &e : graph[u])
                            if (e.f < e.cap)
                                if (dist[e.to] > dist[u] +
                                     e.cost) {
                                    dist[e.to] = dist[u] +
                                         e.cost;
                                    p[e.to] = u;
                                    nq.push_back(e.to);
                    sort(q.begin(), q.end());
                    g.erase(unique(g.begin(), g.end()), g.
                if (!q.emptv()) {
                    int leaf = q[0];
                    vector<int> path;
                    for (int u = leaf; u != -1; u = p[u])
                        if (find(path.begin(), path.end(),
                             u) == path.end())
                            path.push_back(u);
                            path.erase(path.begin(), find(
                                 path.begin(), path.end(),
                                 u));
                            break;
                    for (size_t j = 0; j < path.size(); ++j</pre>
                        int to = path[j];
                        int u = path[(j + 1) % path.size()
                        for (auto &e : graph[u]) {
                            if (e.to == to) {
                                e.f += 1;
                                graph[u][e.rev].f -= 1;
                                circulation_cost += e.cost;
                    found = true;
```

```
return circulation cost;
};
// Usage example
int main() {
    int capacity[][3] = \{\{0, 1, 0\}, \{0, 0, 1\}, \{1, 0, 0\}\};
    int cost[][3] = \{\{0, -1, 0\}, \{0, 0, 1\}, \{-1, 0, 0\}\};
    int nodes = 3;
    min_cost_circulation mcf(nodes);
    for (int i = 0; i < nodes; i++)</pre>
        for (int j = 0; j < nodes; j++)
            if (capacity[i][j] != 0)
                mcf.add_edge(i, j, capacity[i][j], cost[i][j]);
    int circulation_cost = mcf.calc_min_cost_circulation();
    cout << circulation_cost << endl;</pre>
MinCostFlowBf.cpp
Description: desc
// https://en.wikipedia.org/wiki/Minimum-cost_flow_problem in O
     (E * V * FLOW)
// negative-cost edges are allowed
// negative-cost cycles are not allowed
struct Edge {
    int to, rev, cap, f, cost;
};
struct min cost flow {
    vector<vector<Edge>> graph;
    min_cost_flow(int nodes) : graph(nodes) {}
    void add_edge(int s, int t, int cap, int cost) {
        Edge a = {t, (int)graph[t].size(), cap, 0, cost};
        Edge b = \{s, (int)graph[s].size(), 0, 0, -cost\};
        graph[s].emplace_back(a);
        graph[t].emplace_back(b);
    void bellman_ford(int s, vector<int> &curflow, vector<int>
         &dist, vector<int> &prevnode, vector<int> &prevedge) {
        int n = graph.size();
        vector<int> q(n);
        vector<bool> inqueue(n);
        fill(dist.begin(), dist.end(), numeric_limits<int>::max
             ());
        dist[s] = 0;
        curflow[s] = numeric limits<int>::max();
        int qt = 0;
        q[qt++] = s;
        for (int qh = 0; qh != qt; qh++) {
            int u = q[qh % n];
            inqueue[u] = false;
            for (size_t i = 0; i < graph[u].size(); i++) {</pre>
                Edge &e = graph[u][i];
                if (e.cap <= e.f)
                    continue;
                int v = e.to;
                int ndist = dist[u] + e.cost;
                if (dist[v] > ndist) {
                    dist[v] = ndist;
                    prevnode[v] = u;
```

```
prevedge[v] = i;
                    curflow[v] = min(curflow[u], e.cap - e.f);
                    if (!inqueue[v]) {
                        inqueue[v] = true;
                        q[qt++ % n] = v;
                }
        }
    tuple<int, int> calc min cost flow(int s, int t, int maxf)
        int flow = 0;
        int flow_cost = 0;
        size_t n = graph.size();
        vector<int> curflow(n), dist(n), prevnode(n), prevedge(
             n);
        while (flow < maxf) {</pre>
            bellman_ford(s, curflow, dist, prevnode, prevedge);
            if (dist[t] == numeric_limits<int>::max())
            int df = min(curflow[t], maxf - flow);
            flow += df;
            for (int v = t; v != s; v = prevnode[v]) {
                Edge &e = graph[prevnode[v]][prevedge[v]];
                e.f += df;
                graph[v][e.rev].f -= df;
                flow_cost += df * e.cost;
        return {flow, flow_cost};
};
// Usage example
int main() {
    int capacity[][3] = {{0, 3, 2}, {0, 0, 2}, {0, 0, 0}};
    int nodes = 3;
    min_cost_flow mcf(nodes);
    for (int i = 0; i < nodes; i++)</pre>
        for (int j = 0; j < nodes; j++)
            if (capacity[i][j] != 0)
                mcf.add_edge(i, j, capacity[i][j], 1);
    int s = 0;
    auto [flow, flow_cost] = mcf.calc_min_cost_flow(s, t,
         numeric limits<int>::max());
    cout << (4 == flow) << endl;
    cout << (6 == flow cost) << endl;
MinCostFlowDijkstra.cpp
Description: desc
"../../structures/binary_heap_indexed.h"
// https://cp-algorithms.com/graph/min_cost_flow.html in O(E *
    V + E * log V * FLOW
// negative-cost edges are allowed
// negative-cost cycles are not allowed
struct Edge {
    int to, rev, cap, f, cost;
struct min_cost_flow {
    vector<vector<Edge>> graph;
```

```
min_cost_flow(int nodes) : graph(nodes) {}
void add_edge(int s, int t, int cap, int cost) {
    Edge a = {t, (int)graph[t].size(), cap, 0, cost};
    Edge b = \{s, (int)graph[s].size(), 0, 0, -cost\};
    graph[s].emplace_back(a);
    graph[t].emplace_back(b);
void bellman_ford(int s, vector<int> &dist) {
    int n = graph.size();
    vector<int> q(n);
    vector<bool> inqueue(n);
    fill(dist.begin(), dist.end(), numeric_limits<int>::max
    dist[s] = 0;
    int qt = 0;
    q[qt++] = s;
    for (int qh = 0; qh != qt; qh++) {
        int u = q[qh % n];
        inqueue[u] = false;
        for (auto &e : graph[u]) {
            if (e.cap <= e.f)
                continue;
            int v = e.to;
            int ndist = dist[u] + e.cost;
            if (dist[v] > ndist) {
                dist[v] = ndist;
                if (!inqueue[v]) {
                    inqueue[v] = true;
                    q[qt++ % n] = v;
           }
void dijkstra(int s, int t, vector<int> &pot, vector<int> &
    dist, vector < bool > & finished, vector < int > & curflow,
              vector<int> &prevnode, vector<int> &prevedge)
   binary_heap_indexed<int> h(graph.size());
    fill(dist.begin(), dist.end(), numeric limits<int>::max
         ());
    dist[s] = 0;
    curflow[s] = numeric limits<int>::max();
    fill(finished.begin(), finished.end(), false);
    while (!finished[t] && h.size != 0) {
        int u = h.remove min();
        finished[u] = true;
        for (size_t i = 0; i < graph[u].size(); i++) {</pre>
            Edge &e = graph[u][i];
            if (e.f \ge e.cap)
                continue;
            int v = e.to;
            int nprio = dist[u] + e.cost + pot[u] - pot[v];
            if (dist[v] > nprio) {
                if (dist[v] == numeric limits<int>::max())
                    h.add(v, nprio);
                    h.change_value(v, nprio);
                dist[v] = nprio;
                prevnode[v] = u;
                prevedge[v] = i;
                curflow[v] = min(curflow[u], e.cap - e.f);
```

```
tuple<int, int> cal_min_cost_flow(int s, int t, int maxf) {
        size_t n = graph.size();
        vector<int> pot(n), curflow(n), dist(n), prevnode(n),
            prevedge(n);
        vector<bool> finished(n);
        bellman_ford(s, pot); // this can be commented out if
             edges costs are non-negative
        int flow = 0;
        int flow_cost = 0;
        while (flow < maxf) {</pre>
            dijkstra(s, t, pot, dist, finished, curflow,
                 prevnode, prevedge);
            if (dist[t] == numeric_limits<int>::max())
                break:
            for (size_t i = 0; i < n; i++)</pre>
                if (finished[i])
                    pot[i] += dist[i] - dist[t];
            int df = min(curflow[t], maxf - flow);
            flow += df;
            for (int v = t; v != s; v = prevnode[v]) {
                Edge &e = graph[prevnode[v]][prevedge[v]];
                e.f += df;
                graph[v][e.rev].f -= df;
                flow_cost += df * e.cost;
        return {flow, flow_cost};
};
// Usage example
int main() {
    int capacity[][3] = {{0, 3, 2}, {0, 0, 2}, {0, 0, 0}};
    int nodes = 3;
    min_cost_flow mcf(nodes);
    for (int i = 0; i < nodes; i++)</pre>
        for (int j = 0; j < nodes; j++)
            if (capacity[i][j] != 0)
                mcf.add_edge(i, j, capacity[i][j], 1);
    int s = 0;
    auto [flow, flow_cost] = mcf.cal_min_cost_flow(s, t,
        numeric limits<int>::max());
    cout << (4 == flow) << endl;
    cout << (6 == flow_cost) << endl;
1.4 Matchings
MaxBipartiteMatchingEv.cpp
Description: desc
// https://en.wikipedia.org/wiki/Matching_(graph_theory)#
     In\_unweighted\_bipartite\_graphs
// time complexity: O(E * V)
bool findPath(const vector<vector<int>> &graph, int u1, vector<
    int> &matching, vector<bool> &vis) {
    vis[u1] = true;
    for (int v : graph[u1]) {
        int u2 = matching[v];
        if (u2 == -1 || (!vis[u2] && findPath(graph, u2,
            matching, vis))) {
            matching[v] = u1;
            return true;
```

```
return false;
tuple<int, vector<int>> max_matching(const vector<vector<int>>
     &graph, int n2) {
    int n1 = graph.size();
    vector<int> matching(n2, -1);
    int matches = 0;
    for (int u = 0; u < n1; u++) {</pre>
        vector<bool> vis(n1);
        if (findPath(graph, u, matching, vis))
            ++matches;
    return {matches, matching};
// usage example
int main() {
    vector<vector<int>> q(2);
    g[0].push_back(0);
    g[0].push_back(1);
    q[1].push_back(0);
    auto [max_matching_cardinality, mapping] = max_matching(q,
    cout << (2 == max_matching_cardinality) << endl;</pre>
    for (int x : mapping)
        cout << x << " ";
    cout << endl;
MaxBipartiteMatchingHopcroftKarpEsqrtv.cpp
Description: desc
Time: \mathcal{O}\left(E * sqrt(V)\right)
                                                       227fd0, 71 lines
```

```
void bfs(const vector<vector<int>> &graph, vector<bool> &used,
    vector<int> &mapping, vector<int> &dist) {
    fill(dist.begin(), dist.end(), -1);
    size_t n1 = graph.size();
    vector<int> Q(n1);
    int sizeQ = 0;
    for (int u = 0; u < n1; ++u) {</pre>
        if (!used[u]) {
            Q[sizeQ++] = u;
            dist[u] = 0;
    for (int i = 0; i < sizeQ; i++) {</pre>
        int u1 = Q[i];
        for (int v : graph[u1]) {
            int u2 = mapping[v];
            if (u2 >= 0 && dist[u2] < 0) {</pre>
                dist[u2] = dist[u1] + 1;
                O[sizeO++] = u2;
bool dfs(const vector<vector<int>> &graph, vector<bool> &vis,
    vector<bool> &used, vector<int> &matching,
         vector<int> &dist, int u1) {
```

vis[u1] = true;

for (int v : graph[u1]) {

```
int u2 = matching[v];
        if (u2 < 0 || (!vis[u2] && dist[u2] == dist[u1] + 1 &&</pre>
            dfs(graph, vis, used, matching, dist, u2))) {
            matching[v] = u1;
            used[u1] = true;
            return true;
    return false;
tuple<int, vector<int>> max_matching(const vector<vector<int>>
    &graph, int n2) {
    vector<int> mapping(n2, -1);
    size_t n1 = graph.size();
    vector<int> dist(n1);
    vector<bool> used(n1);
    for (int res = 0;;) {
       bfs(graph, used, mapping, dist);
       vector<bool> vis(n1);
       int f = 0;
        for (int u = 0; u < n1; ++u)
            if (!used[u] && dfs(graph, vis, used, mapping, dist
                ++f;
        if (f == 0)
            return {res, mapping};
        res += f;
// usage example
int main() {
    vector<vector<int>> g(3);
    q[0].push_back(0);
    g[0].push_back(1);
    q[1].push_back(1);
    g[2].push_back(1);
    auto [max_matchinq_cardinality, mapping] = max_matching(q,
    cout << (2 == max_matching_cardinality) << endl;</pre>
    for (int x : mapping)
       cout << x << " ";
    cout << endl;
```

KTH

MinBipartiteWeightedMatchingHungarian.cpp

```
Description: desc
                                                      18b15d, 57 lines
// https://en.wikipedia.org/wiki/Hungarian_algorithm in O(n^2 *
pair<int, vector<int>> min_weight_perfect_matching(const vector
    <vector<int>> &a) {
    int n = a.size();
    int m = a[0].size();
   vector<int> u(n);
   vector<int> v(m);
   vector<int> p(m);
    vector<int> way(m);
    for (int i = 1; i < n; ++i) {</pre>
        vector<int> minv(m, numeric_limits<int>::max());
        vector<bool> used(m);
        p[0] = i;
        int j0 = 0;
        while (p[j0] != 0) {
```

```
used[j0] = true;
            int i0 = p[j0];
            int delta = numeric limits<int>::max();
            int i1 = 0;
            for (int j = 1; j < m; ++ j)
                if (!used[j]) {
                    int d = a[i0][j] - u[i0] - v[j];
                    if (minv[j] > d) {
                        minv[j] = d;
                        way[j] = j0;
                    if (delta > minv[j]) {
                        delta = minv[j];
                        j1 = j;
            for (int j = 0; j < m; ++j)
                if (used[j]) {
                    u[p[j]] += delta;
                    v[j] -= delta;
                } else
                    minv[j] -= delta;
            j0 = j1;
        while (j0 != 0) {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
    vector<int> matching(n);
    for (int i = 1; i < m; ++i)
        matching[p[i]] = i;
    return {-v[0], matching};
// usage example
int main() {
    // row1 and col1 should contain 0
    vector<vector<int>> a{{0, 0, 0}, {0, 1, 2}, {0, 1, 2}};
    auto [cost, matching] = min_weight_perfect_matching(a);
    cout << boolalpha << (cost == 3) << endl;
StableMatching.cpp
Description: desc
                                                      6538f<u>6</u>, 34 lines
vector<int> stable_matching(vector<vector<int>> prefer_m,
    vector<vector<int>> prefer_w) {
    int n = prefer_m.size();
    vector<int> pair_m(n, -1);
    vector<int> pair_w(n, -1);
    vector<int> p(n);
    for (int i = 0; i < n; i++) {</pre>
        while (pair_m[i] < 0) {
            int w = prefer_m[i][p[i]++];
            int m = pair_w[w];
            if (m == -1) {
                pair_m[i] = w;
                pair_w[w] = i;
            } else if (prefer_w[w][i] < prefer_w[w][m]) {</pre>
                pair_m[m] = -1;
                pair_m[i] = w;
                pair_w[w] = i;
                i = m;
    return pair_m;
```

```
// usage example
int main() {
    vector<vector<int>> prefer_m{{0, 1, 2}, {0, 2, 1}, {1, 0, 2}
    vector<vector<int>> prefer_w{{0, 1, 2}, {2, 0, 1}, {2, 1, 0}
    vector<int> matching = stable_matching(prefer_m, prefer_w);
    for (int x : matching)
        cout << x << " ";
    cout << endl;
```

Linear Algebra (2)

Gauss.cpp

Description: desc

```
"../numbertheory/modint.h", "matrix.h"
vector<double> gauss (vector<vector<double>> A, vector<double> b
    int n = A.size();
    for (int i = 0; i < n; i++) {</pre>
        int pivot = i;
        for (int j = i + 1; j < n; j++)
            if (abs(A[pivot][i]) < abs(A[j][i]))</pre>
                pivot = j;
        swap(A[i], A[pivot]);
        swap(b[i], b[pivot]);
        for (int j = i + 1; j < n; j++)
            A[i][j] /= A[i][i];
        b[i] /= A[i][i];
        for (int j = 0; j < n; j++)
            if (j != i && A[j][i] != 0) {
                for (int k = i + 1; k < n; k++)
                    A[j][k] -= A[i][k] * A[j][i];
                b[j] -= b[i] * A[j][i];
    return b:
template <class T>
vector<T> gauss(vector<vector<T>> A, vector<T> b) {
    int n = A.size();
    for (int i = 0; i < n; i++) {</pre>
        int pivot = i;
        for (int j = i; j < n; j++)
            if (A[j][i] != 0) {
                pivot = j;
                break;
        assert(A[pivot][i] != 0);
        swap(A[i], A[pivot]);
        swap(b[i], b[pivot]);
        for (int j = i + 1; j < n; j++)
            A[i][j] /= A[i][i];
        b[i] /= A[i][i];
        for (int j = 0; j < n; j++)
            if (j != i && A[j][i] != 0) {
                for (int k = i + 1; k < n; k++)
                    A[j][k] -= A[i][k] * A[j][i];
                b[j] -= b[i] * A[j][i];
```

```
return b:
template <class T>
vector<vector<T>> inverse(vector<vector<T>> A) {
    int n = A.size():
    vector < vector < T >> B(n, vector < T > (n));
    for (int i = 0; i < n; ++i)
        B[i][i] = 1;
    for (int i = 0; i < n; i++) {</pre>
        int pivot = i;
        for (int j = i; j < n; j++)
            if (A[j][i] != 0) {
                pivot = j;
                break:
        assert(A[pivot][i] != 0);
        swap(A[i], A[pivot]);
        swap(B[i], B[pivot]);
        T Aii = A[i][i];
        for (int j = 0; j < n; j++) {</pre>
            A[i][j] /= Aii;
            B[i][i] /= Aii;
        for (int j = 0; j < n; j++) {
            T Aji = A[j][i];
            if (j != i && Aji != 0)
                for (int k = 0; k < n; k++) {
                    A[j][k] -= A[i][k] * Aji;
                    B[j][k] -= B[i][k] * Aji;
    return B;
constexpr int mod = (int)1e9 + 7;
using mint = modint<mod>;
// usage example
int main() {
        vector<vector<double>> A{{4, 2, -1}, {2, 4, 3}, {-1, 3,
        vector<double> b = \{1, 0, 0\};
        vector<double> x = gauss(A, b);
        vector<vector<double>> v = A * transpose(vector<vector<</pre>
             double>>{x});
        for (int i = 0; i < b.size(); i++)</pre>
            assert (abs (b[i] - y[i][0]) < 1e-9);
        vector<vector<mint>> A{{4, 2, -1}, {2, 4, 3}, {-1, 3, 5}
             }};
        vector<mint> b = \{1, 2, 3\};
        vector < mint > x = gauss(A, b);
        vector<vector<mint>> y = A * transpose(vector<vector<</pre>
             mint >> {x});
        assert(transpose(y)[0] == b);
        vector<vector<mint>> B = inverse(A);
        vector<vector<mint>> E = A * B:
        for (int i = 0; i < A.size(); ++i)</pre>
            for (int j = 0; j < A.size(); ++j)</pre>
                assert(E[i][j] == (i == j ? 1 : 0));
```

KTH

```
Description: desc
"matrix.h", "../numbertheory/modint.h"
                                                      99cd9f, 19 lines
constexpr int mod = (int)1e9 + 7;
using mint = modint<mod>;
// usage example
int main() {
    // Fibonacci numbers
    vector<mint> f{1, 1};
    vector<mint> a{1, 1};
    for (int i = 0; i < 60; ++i) {</pre>
        cout << (int)nth_element_of_recurrence(a, f, i) << endl</pre>
    vector<vector<mint>> A(2, vector<mint>(2));
    A[0][0] = 1;
    A[0][1] = 1;
   A[1][0] = 1;
    matrix_print(matrix_pow_sum(A, 4));
    matrix print (A + (A^2) + (A^3) + (A^4));
Matrix.h
Description: desc
                                                      35ec5c, 113 lines
template <class T>
vector<vector<T>> matrix unit(int n) {
    vector<vector<T>> res(n, vector<T>(n));
    for (int i = 0; i < n; i++)
        res[i][i] = 1;
    return res;
template <class T>
vector<vector<T>> &operator+=(vector<vector<T>> &a, const
    vector<vector<T>> &b) {
    for (size t i = 0; i < a.size(); i++)</pre>
        for (size_t j = 0; j < a[0].size(); j++)</pre>
            a[i][j] += b[i][j];
    return a:
template <class T>
vector<vector<T>> operator+(vector<vector<T>> a, const vector<</pre>
    vector<T>> &b) {
    a += b;
    return a;
template <class T>
vector<vector<T>> operator*(const vector<vector<T>> &a, const
     vector<vector<T>> &b) {
    int n = a.size();
    int m = a[0].size();
    int k = b[0].size();
    vector<vector<T>> res(n, vector<T>(k));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < k; j++)
            for (int p = 0; p < m; p++)
                res[i][j] += a[i][p] * b[p][j];
    return res;
template <class T>
vector<vector<T>> &operator *= (vector < vector < T>> &a, const
     vector<vector<T>> &b) {
    a = a * b;
```

Matrix.cpp

```
return a:
template <class T>
vector<vector<T>> operator^(const vector<vector<T>> &a, long
    long p) {
    vector<vector<T>> res = matrix_unit<T>(a.size());
    int highest_one_bit = -1;
    while (1LL << (highest_one_bit + 1) <= p)</pre>
        ++highest_one_bit;
    for (int i = highest_one_bit; i >= 0; i--) {
        res *= res;
        if (p >> i & 1) {
            res *= a;
    return res;
template <class T>
vector<vector<T>> transpose(const vector<vector<T>> &a) {
    int n = a.size();
    int m = a[0].size();
    vector<vector<T>> b(m, vector<T>(n));
    for (int i = 0; i < n; ++i) {</pre>
        for (int j = 0; j < m; ++j) {
            b[j][i] = a[i][j];
    return b;
// a + a^2 + ... + a^p
template <class T>
vector<vector<T>> matrix_pow_sum(const vector<vector<T>> &a,
    long long p) {
    int n = a.size();
    vector<vector<T>> res = vector<vector<T>> (n, vector<T> (n));
    vector<vector<T>> b = matrix unit<T>(n);
    int highest_one_bit = -1;
    while (1LL << (highest_one_bit + 1) <= p)</pre>
        ++highest one bit;
    for (int i = highest_one_bit; i >= 0; i--) {
        res = res * (matrix unit<T>(n) + b);
        b \star = b;
        if (p >> i & 1) {
            b *= a;
            res = res * a + a;
    return res;
// returns f[n] = f[n-1]*a[k-1] + ... + f[n-k]*a[0], where f
     [0], \ldots, f[k-1] are provided
// O(k^3*log(n)) complexity
template <class T>
T nth_element_of_recurrence(const vector<T> &a, const vector<T>
     &f, long long n) {
    int k = f.size();
    if (n < k)
        return f[n];
    vector<vector<T>> A(k, vector<T>(k));
    A[k-1] = a;
    for (int i = 0; i < k - 1; ++i) {
        A[i][i + 1] = 1;
    vector<vector<T>> F = transpose(vector<vector<T>>{f});
    return ((A ^ n) * F)[0][0];
```

const vector<int> $p = nsv({1, 1, 3, 2});$

for (int x : p)

cout << x << " ";

```
cout << endl;
template <class T>
void matrix_print(const vector<vector<T>> &a) {
    for (auto &row : a) {
                                                                   Binary Exponentiation.cpp
        for (T x : row)
                                                                   Description: desc
           cout << (int)x << " ";
                                                                   // https://en.wikipedia.org/wiki/Exponentiation_by_squaring
       cout << endl;
                                                                   // https://cp-algorithms.com/algebra/binary-exp.html
                                                                   int pow_mod(int x, int n, int mod) {
                                                                       int res = 1;
                                                                       for (long long p = x; n > 0; n >>= 1, p = (p * p) % mod)
Tridiagonal.cpp
                                                                           if ((n & 1) != 0)
Description: desc
                                                     bd75dc, 26 lines
                                                                               res = (int) (res * p % mod);
                                                                       return res;
//\ https://en.wikipedia.org/wiki/Tridiagonal\_matrix\_algorithm
template <class T>
vector<T> tridiagonal_solve(vector<T> diag, const vector<T> &
                                                                   // usage example
    super, const vector<T> &sub, vector<T> b) {
                                                                   int main() {
    for (int i = 0; i < b.size() - 1; ++i) {</pre>
                                                                       const int MOD = 1000'000'007;
       diag[i + 1] -= super[i] * sub[i] / diag[i];
                                                                       int x = pow_mod(2, 10, MOD);
       b[i + 1] = b[i] * sub[i] / diag[i];
                                                                       cout << x << endl;
    for (int i = b.size() - 1; i > 0; --i) {
       b[i] /= diag[i];
       b[i - 1] -= b[i] * super[i - 1];
                                                                   BinarySearch.cpp
                                                                   Description: desc
    b[0] /= diag[0];
                                                                                                                        32e434, 44 lines
    return b;
                                                                   int binary search(bool (*f)(int) /* function<bool(int)> f */,
                                                                        int from inclusive, int to inclusive) {
                                                                       // invariant: f[lo] = false, f[hi] = true
// usage example
                                                                       int lo = from inclusive - 1;
int main() {
                                                                       int hi = to_inclusive + 1;
    // -1 \quad 1 \quad 0 \quad x/0 5
                                                                       // while there are some elements between lo and hi
    // 3 -1 2 * x[1] = 6
                                                                       while (hi - lo > 1) {
    \frac{1}{1} 0 4 -1 x[2] 7
                                                                           // invariant: lo < mid < hi
    vector<double> x = tridiagonal_solve<double>({-1, -1, -1},
                                                                           int mid = (lo + hi) / 2;
         \{1, 2\}, \{3, 4\}, \{5, 6, 7\}\};
                                                                           if (!f(mid)) {
                                                                               lo = mid;
    for (double v : x)
                                                                           } else {
       cout << fixed << setprecision(5) << v << " ";</pre>
                                                                               hi = mid;
    cout << endl;
                                                                       // here lo + 1 == high
                                                                       return hi:
Misc (3)
                                                                   // binary_search(new bool[5]{false, false, false, true, true},
AllNearestSmallerValues.cpp
                                                                        0.4) = 3
Description: desc
                                                    48d499, 22 lines
                                                                   int binary_search(bool a[], int from_inclusive, int
                                                                        to_inclusive) {
// https://en.wikipedia.org/wiki/All_nearest_smaller_values
                                                                       // invariant: f[lo] = false, f[hi] = true
vector<int> nsv(const vector<int> &a) {
                                                                       int lo = from_inclusive - 1;
    int n = a.size();
                                                                       int hi = to_inclusive + 1;
    vector<int> p(n);
    for (int i = 0; i < n; i++) {</pre>
                                                                       // while there are some elements between lo and hi
        int j = i - 1;
                                                                       while (hi - lo > 1) {
                                                                           // invariant: lo < mid < hi
        while (j != -1 \&\& a[j] >= a[i]) {
                                                                           int mid = (lo + hi) / 2;
            j = p[j];
                                                                           if (!a[mid]) {
                                                                               lo = mid;
       p[i] = j;
                                                                           } else {
    return p;
                                                                               hi = mid;
// usage example
                                                                       // here lo + 1 == high
int main() {
                                                                       return hi:
```

// usage example

int main() {

```
int first_true = binary_search([](int x) { return x >= 4; }
         , 0, 10);
    cout << (first true == 4) << endl;
    cout << (binary_search(new bool[3]{false, true, true}, 0,</pre>
         2) == 1) << end1;
Comparator.cpp
Description: desc
                                                      bd5409, 42 lines
struct point {
    int x, y;
struct point2 {
    int x, y;
    bool operator<(const point2 &b) const { return x == b.x ? y</pre>
          < b.v : x < b.x; }
    bool operator<(int qx) const { return x < qx; }</pre>
};
int main() {
    point a[] = \{\{2, 3\}, \{1, 2\}\};
    auto cmp = [](auto &a, auto &b) { return a.x < b.x || (a.x</pre>
         == b.x \&\& a.y < b.y); };
    sort(a, a + 2, cmp);
    set<point, decltype(cmp) > s(a, a + 2, cmp);
    for (auto &it : s) {
        cout << it.x << " " << it.v << endl;
    point2 b[] = \{\{2, 3\}, \{1, 2\}\};
    set<point2, less<>> s2(b, b + 2);
    for (auto &it : s2) {
        cout << it.x << " " << it.y << endl;
    cout << s2.lower_bound(1)->y << endl;</pre>
    map<point, int, decltype(cmp) > m({{point{2, 3}, 1}}, cmp);
    for (auto &entry : m) {
        cout << entry.first.x << " " << entry.first.y << " " <<
              entry.second << endl;
    priority_queue<point, vector<point>, decltype(cmp)> q(a, a
         + 2, cmp);
    while (!q.empty()) {
        auto item = q.top();
        q.pop();
        cout << item.x << " " << item.y << endl;
Hashcode.cpp
Description: desc
                                                      5218c1, 15 lines
struct point {
    int x, y;
    bool operator == (const point &p) const { return x == p.x &&
         y == p.y; }
struct hasher {
```

p.y; }

cout << endl;

```
};
int main() {
    unordered_map<point, int, hasher> m;
    m[{1, 2}] = 1;
HashcodeUnorderedMap.cpp
Description: desc
                                                      f01337, 18 lines
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        //\ http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
             steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED RANDOM);
};
    unordered map<long long, int, custom hash> safe map;
Knapsack.cpp
Description: desc
                                                      1565fd, 34 lines
vector<bool> knapsack(const vector<int> &w, int W) {
    vector<int> cnt(W + 1);
    for (int x : w) {
        if (x <= W)
            ++cnt[x];
    vector<bool> can(W + 1);
    can[0] = true;
    for (int v = 1; v \leftarrow W; ++v) {
        int am = cnt[v];
        if (am == 0)
            continue;
        for (int from = 0; from < v; ++from) {</pre>
            int counter = -1;
            for (int got = from; got <= W; got += v) {</pre>
                --counter;
                if (can[got])
                    counter = am;
                else if (counter >= 0) {
                     can[got] = true;
    return can;
// usage example
int main() {
    auto b = knapsack(\{2, 3, 6, 7\}, 10);
    for (auto x : b)
       cout << x;
```

size_t operator()(const point &p) const { return p.x * 37 +

```
MaxZeroSubMatrix.cpp
Description: desc
                                                      70401a, 39 lines
int maximum_zero_submatrix(const vector<vector<int>> &a) {
    int R = a.size();
    int C = a[0].size();
    int res = 0;
    vector<int> d(C, -1);
    vector<int> d1(C);
    vector<int> d2(C);
    vector<int> st(C);
    for (int r = 0; r < R; ++r) {
        for (int c = 0; c < C; ++c)
            if (a[r][c] == 1)
                d[c] = r;
        int size = 0;
        for (int c = 0; c < C; ++c) {</pre>
            while (size > 0 && d[st[size - 1]] <= d[c])</pre>
            d1[c] = size == 0 ? -1 : st[size - 1];
            st[size++] = c;
        size = 0;
        for (int c = C - 1; c >= 0; --c) {
            while (size > 0 && d[st[size - 1]] <= d[c])</pre>
                --size:
            d2[c] = size == 0 ? C : st[size - 1];
            st[size++] = c;
        for (int j = 0; j < C; ++j)
            res = \max(res, (r - d[j]) * (d2[j] - d1[j] - 1));
    return res;
// usage example
int main() {
    vector<vector<int>> m{{1, 0, 0, 1}, {1, 0, 0, 1}, {1, 1, 0,
    int area = maximum_zero_submatrix(m);
    cout << area << endl;
Scanner.cpp
Description: desc
<memory.h>, <cstdio>
                                                     b1b6d8, 50 lines
const int BUF_SIZE = 65536;
char input[BUF_SIZE];
struct scanner {
    char* curPos;
    scanner() {
        fread(input, 1, sizeof(input), stdin);
        curPos = input;
    void ensureCapacity() {
        int size = input + BUF_SIZE - curPos;
        if (size < 100) {
            memcpy(input, curPos, size);
            fread(input + size, 1, BUF_SIZE - size, stdin);
            curPos = input;
```

```
int nextInt() {
        ensureCapacity();
        while (*curPos <= ' ')</pre>
            ++curPos:
        bool sign = false;
        if (*curPos == '-') {
            sign = true;
            ++curPos:
        int res = 0;
        while (*curPos > ' ')
            res = res * 10 + (*(curPos++) & 15);
        return sign ? -res : res;
    char nextChar() {
        ensureCapacity();
        while (*curPos <= ' ')</pre>
            ++curPos:
        return *(curPos++);
};
int main() {
    scanner sc;
    int a = sc.nextInt();
    char b = sc.nextChar();
    printf("%d %c\n", a, b);
```

Number Theory (4)

DiscreteLog.cpp Description: desc

__ ____

```
0badf8, 34 lines
// returns any x such that a^x = b \pmod{m}
// O(m^0.5) complexity
int discrete_log(int a, int b, int m) {
    assert (\gcd(a, m) == 1);
    int n = (int) sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)</pre>
        an = ((long long) an * a) % m;
    unordered_map<int, int> vals;
    for (int i = 1, cur = an; i <= n; ++i) {</pre>
        if (!vals.count(cur))
            vals[cur] = i;
        cur = ((long long)cur * an) % m;
    for (int i = 0, cur = b; i <= n; ++i) {
        if (vals.count(cur)) {
            int res = (long long) vals[cur] * n - i;
            if (res < m)
                return res;
        cur = ((long long)cur * a) % m;
    return -1;
// usage example
int main() {
    // 2^x = 3 \pmod{5}, x = 3
```

cout << discrete_log(2, 3, 5) << endl;</pre>

DiscreteRoot Euclid Factorization LinearRecurrence

```
DiscreteRoot.cpp
Description: desc
                                                      a6e798, 63 lines
// https://cp-algorithms.com/algebra/discrete-root.html
int pow mod(int x, int n, int mod) {
    int res = 1;
    for (long long p = x; n > 0; n >>= 1, p = (p * p) % mod)
       if ((n & 1) != 0)
            res = (int) (res * p % mod);
    return res;
int calc_generator(int m) {
    if (m == 2)
        return 1:
    vector<int> factors;
    int phi = m - 1;
    int n = phi;
    for (int i = 2; i * i <= n; ++i)</pre>
       if (n % i == 0) {
            factors.emplace_back(i);
            while (n % i == 0)
                n /= i:
    if (n > 1)
        factors.emplace back(n);
    for (int res = 2; res <= m; ++res) {</pre>
        if (gcd(res, m) != 1)
            continue;
        bool ok = true;
        for (size t i = 0; i < factors.size() && ok; ++i)</pre>
            ok &= pow_mod(res, phi / factors[i], m) != 1;
       if (ok)
            return res:
    return -1;
// returns any x such that x^a = b \pmod{m}
// precondition: m is prime
int discrete_root(int a, int b, int m) {
    if (a == 0)
        return -1:
    int g = calc_generator(m);
    int sq = (int)sqrt(m) + 1;
    vector<pair<int, int>> dec(sq);
    for (int i = 1; i <= sq; ++i)
        dec[i-1] = \{pow_mod(q, (long long)i * sq * b % (m -
             1), m), i};
    sort(dec.begin(), dec.end());
    for (int i = 0; i < sq; ++i) {</pre>
        int my = pow_mod(g, (long long) i * b % (m - 1), m) * (
             long long) a % m;
        auto it = lower_bound(dec.begin(), dec.end(), make_pair
             (mv, 0));
        if (it != dec.end() && it->first == my) {
            int x = it -> second * sq - i;
            int delta = (m - 1) / gcd(b, m - 1);
            return pow_mod(q, x % delta, m);
    return -1;
// usage example
```

```
int main() {
    cout << discrete_root(3, 3, 5) << endl;</pre>
Euclid.cpp
Description: desc
                                                       c2ffc8, 75 lines
// precondition: mod > 1 \& gcd(a, mod) = 1
int mod inverse(int a, int mod) {
    int u = 0, v = 1, m = mod;
    while (a != 0) {
        int t = m / a;
        m -= t * a;
        swap(a, m);
        u -= t * v;
        swap(u, v);
    assert (m == 1);
    return u < 0 ? u + mod : u;
// returns { qcd(a,b), x, y } such that qcd(a,b) = a*x + b*y
template <class T>
tuple<T, T, T> euclid(T a, T b) {
    T x = 1, y = 0, x1 = 0, y1 = 1;
    // invariant: a=a\_orig*x+b\_orig*y, b=a\_orig*x1+b\_orig*y1
    while (b != 0) {
        Tq = a / b;
        T \times 1 = \times 1;
        T v1 = v1;
        T b = b;
        x1 = x - q * x1;
        y1 = y - q * y1;
        b = a - q * b;
        x = x1:
        y = _y1;
        a = _b;
    return a > 0 ? tuple{a, x, y} : tuple{-a, -x, -y};
//\ https://cp-algorithms.com/algebra/linear\_congruence\_equation
     .html
// solves a * x = b \pmod{m}
int solve1(int a, int b, int m) {
    int g = gcd(a, m);
    if (b % q)
        return -1;
    a /= q;
    b /= q;
    m /= q;
    int x = (long long)b * mod_inverse(a, m) % m;
    return x:
    // all solutions: x[i]=x+i*m, i=0..g-1
// https://cp-algorithms.com/algebra/linear-diophantine-
     equation.html
// solves a * x + b * y = c \pmod{m}
optional<tuple<int, int>> solve2(int a, int b, int c) {
    auto [g, x0, y0] = euclid(abs(a), abs(b));
    if (c % q != 0)
        return {};
    x0 \star = c / q;
    y0 \star = c / q;
    return make_optional(tuple{a > 0 ? x0 : -x0, b > 0 ? y0 : -
    // \ all \ solutions: x=x0+k*b/g, y=y0-k*a/g, k \ in \ Z
```

```
// usage example
int main() {
    auto [gcd, x, y] = euclid(6, 9);
    cout << gcd << " = 6 * " << x << " + 9 * " << y << endl;
    cout << "x=" << solve1(2, 3, 5) << endl;
    auto res = solve2(4, 6, 2);
    if (res) {
        auto [xx, yy] = *res;
        cout << "x=" << xx << " y=" << yy << endl;
    cout << mod_inverse(3, 10) << endl;</pre>
Factorization.cpp
Description: desc
                                                     4fd5d8, 23 lines
// returns prime_divisor -> power
// O(sqrt(n)) complexity
map<long long, int> factorize(long long n) {
    map<long long, int> factors;
    for (int d = 2; (long long) d * d \le n; d++) {
        while (n % d == 0) {
            ++factors[d];
            n /= d;
    if (n > 1) {
        ++factors[n];
    return factors;
// usage example
int main() {
    map<long long int, int> factors = factorize(4 * 3 *
         1000000000039);
    for (auto e : factors) {
        cout << e.first << "^" << e.second << " " << endl;
LinearRecurrence.cpp
Description: desc
"polynom.h"
                                                    2b8a71, 68 lines
// returns f[n] = f[n-1]*a[k-1] + ... + f[n-k]*a[0], where f
     [0], \ldots, f[k-1] are provided
// O(k*log(k)*log(n)) complexity
template <class T>
T nth_element_of_recurrence(vector<T> a, const vector<T> &f,
    long long n) {
    if (n < f.size())
        return f[n];
    a = -a;
    a.emplace_back(1);
    vector<T> xn = power({0, 1}, n, a);
    return inner_product(f.begin(), f.begin() + min(f.size(),
         xn.size()), xn.begin(), T{0});
// https://en.wikipedia.org/wiki/Berlekamp%E2%80%93
     Massey\_algorithm
template <typename M>
vector<M> berlekamp_massey(const vector<M> &a) {
    int n = a.size();
    vector<M> C(n), B(n);
```

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Modint Modint Polynom Polynom

```
C[0] = B[0] = 1;
   M b = 1;
    int L = 0;
    for (int i = 0, m = 1; i < n; ++i) {
       M d = a[i];
        for (int j = 1; j <= L; ++j)</pre>
            d = d + C[j] * a[i - j];
        if (d == 0) {
            ++m:
            continue;
        vector < M > T = C;
       M coef = d / b;
        for (int j = m; j < n; ++j)
            C[j] -= coef * B[j - m];
        if (2 * L > i) {
            ++m;
            continue;
        L = i + 1 - L;
       B = T;
       b = d;
        m = 1;
    C.resize(L + 1);
    C.erase(C.begin());
    reverse(C.begin(), C.end());
    return -C;
// usage example
constexpr int mod = (int)1e9 + 7;
using mint = modint<mod>;
int main() {
        // Fibonacci numbers
        vector<mint> f{1, 1};
        vector<mint> a{1, 1};
        for (int i = 0; i < 10; ++i) {</pre>
            cout << (int)nth_element_of_recurrence(a, f, i) <<</pre>
        cout << endl;
        vector<mint> f = berlekamp_massey(vector<mint>({1, 1,
             3, 5, 11}));
        for (auto v : f)
            cout << (int) v << " ";
        cout << endl;
Modint.cpp
Description: desc
"modint.h"
                                                      afddbe, 12 lines
constexpr int mod = (int)1e9 + 7;
using mint = modint<mod>;
// usage example
int main() {
   mint a = 1;
   mint b = 1;
   b += 1;
   mint c = 1000'000'000;
   mint d = a / b * c / c;
    cout << ((int)d) << endl;
```

```
Modint.h
Description: desc
                                                     7aa503, 62 lines
template <int mod>
struct modint {
   int value;
    modint(long long x = 0) { value = normalize(x); }
    int normalize(long long x) {
        if (x < -mod \mid | x >= mod)
            x %= mod;
        if (x < 0)
            x += mod;
        return static_cast<int>(x);
    explicit operator int() const { return value; }
    modint operator-() const { return modint(-value); }
    modint &operator+= (modint rhs) {
        if ((value += rhs.value) >= mod)
            value -= mod;
        return *this;
    modint & operator -= (modint rhs) {
        if ((value -= rhs.value) < 0)</pre>
            value += mod;
        return *this;
    modint &operator*=(modint rhs) {
        value = normalize(static_cast<long long>(value) * rhs.
            value):
        return *this;
    modint &operator/=(modint rhs) { return *this *= modint(
         inverse(rhs.value, mod)); }
    int inverse(int a, int m) {
        int u = 0, v = 1;
        while (a != 0) {
            int t = m / a;
            m -= t * a;
            swap(a, m);
            u -= t * v;
            swap(u, v);
        assert (m == 1);
        return u;
    bool operator==(modint rhs) const { return value == rhs.
         value; }
    bool operator!=(modint rhs) const { return !(*this == rhs);
    friend modint operator+(modint lhs, modint rhs) { return
         lhs += rhs; }
    friend modint operator-(modint lhs, modint rhs) { return
         lhs -= rhs; }
    friend modint operator * (modint lhs, modint rhs) { return
         lhs *= rhs; }
```

```
friend modint operator/(modint lhs, modint rhs) { return
         lhs /= rhs; }
};
Polynom.cpp
Description: desc
"polynom.h"
                                                      e4cd25, 49 lines
// usage example
constexpr int mod = (int)1e9 + 7;
using mint = modint<mod>;
int main() {
        vector<mint> poly{3, 2, 1};
        vector<mint> x{1, 2, 3, 4, 5};
        vector<mint> values = evaluate(poly, x);
        for (size_t i = 0; i < x.size(); ++i) {</pre>
            cout << (int)values[i] << " " << (int)evaluate(poly</pre>
                 , x[i]) << endl;
        cout << endl;
        vector<mint> x{7, 2, 1, 3, 5, 6};
        vector<mint> y;
        for (size t i = 0; i < x.size(); ++i) {</pre>
            y.emplace_back(x[i] * x[i] * x[i] + 5 * x[i] * x[i]
                  + x[i] + 3);
        vector<mint> poly = interpolate(x, y);
        for (size_t i = 0; i < poly.size(); ++i) {</pre>
            cout << (int)poly[i] << " ";
        cout << endl << endl;
        for (int i = 0; i < 10; ++i) {
            vector<mint> s = sequence<mint>(i);
            for (auto x : s)
                cout << (int)x << " ";
            cout << endl:
        cout << endl:
        for (int i = 0; i < 10; ++i) {
            cout << (int) factorial<mint>(i) << endl;</pre>
        auto t1 = chrono::high_resolution_clock::now();
        cout << (int) factorial<mint>(1000'000'000) << endl;</pre>
        auto t2 = chrono::high_resolution_clock::now();
        chrono::duration<double, milli> duration = t2 - t1;
        cout << duration.count() << " ms" << endl;</pre>
        cout << endl;
Polynom.h
Description: desc
"../numeric/fft.h", "modint.h"
//\ https://cp-algorithms.com/algebra/polynomial.html
template <class T>
vector<T> &operator+=(vector<T> &a, const vector<T> &b) {
    if (a.size() < b.size()) {
```

```
a.resize(b.size());
    for (size t i = 0; i < b.size(); i++) {</pre>
       a[i] += b[i];
    return a;
template <class T>
vector<T> operator+(vector<T> a, const vector<T> &b) {
    a += b:
    return a:
template <class T>
vector<T> &operator == (vector<T> &a, const vector<T> &b) {
    if (a.size() < b.size()) {
        a.resize(b.size());
    for (int i = 0; i < b.size(); i++) {</pre>
       a[i] -= b[i];
    return a;
template <class T>
vector<T> operator-(vector<T> a, const vector<T> &b) {
   a -= b;
    return a:
template <class T>
vector<T> operator-(vector<T> a) {
    for (int i = 0; i < a.size(); i++) {</pre>
       a[i] = -a[i];
    return a;
// fast multiplication for modint in O(n*log(n))
template <int mod>
vector<modint<mod>> operator*(const vector<modint<mod>> &a,
    const vector<modint<mod>> &b) {
    if (a.emptv() || b.emptv()) {
        return {};
    if (min(a.size(), b.size()) < 150) {</pre>
        vector<modint<mod>> c(a.size() + b.size() - 1, 0);
        for (size t i = 0; i < a.size(); i++) {</pre>
            for (size_t j = 0; j < b.size(); j++) {</pre>
                c[i + j] += a[i] * b[j];
        return c:
    vector<int> a int(a.size());
    for (size_t i = 0; i < a.size(); i++) {</pre>
        a int[i] = static cast<int>(a[i]);
    vector<int> b int(b.size());
    for (size t i = 0; i < b.size(); i++) {</pre>
       b_int[i] = static_cast<int>(b[i]);
   vector<int> c_int = multiply_mod(a_int, b_int, mod);
    vector<modint<mod>> c(c int.size());
    for (size_t i = 0; i < c.size(); i++) {</pre>
       c[i] = c_{int}[i];
    return c;
```

```
// fallback multiplication in O(n*m)
template <class T>
vector<T> operator*(const vector<T> &a, const vector<T> &b) {
    if (a.empty() || b.empty()) {
        return {};
   vector<T> c(a.size() + b.size() - 1, 0);
    for (int i = 0; i < a.size(); i++) {</pre>
        for (int j = 0; j < b.size(); j++) {</pre>
            c[i + j] += a[i] * b[j];
    return c;
template <class T>
vector<T> &operator*=(vector<T> &a, const vector<T> &b) {
   a = a * b;
    return a;
template <class T>
vector<T> inverse(const vector<T> &a) {
   assert(!a.empty());
   int n = a.size();
   vector < T > b = {1 / a[0]};
   while (b.size() < n) {</pre>
        vector<T> a_cut(a.begin(), a.begin() + min(a.size(), b.
             size() << 1));
       vector<T> x = b * b * a_cut;
       b.resize(b.size() << 1);
        for (size_t i = b.size() >> 1; i < min(x.size(), b.size</pre>
             ()); i++) {
            b[i] = -x[i];
   b.resize(n);
    return b;
template <class T>
vector<T> &operator/=(vector<T> &a, vector<T> b) {
    int n = a.size();
    int m = b.size();
    if (n < m) {
        a.clear();
    } else {
        reverse(a.begin(), a.end());
        reverse(b.begin(), b.end());
       b.resize(n - m + 1);
       a \star = inverse(b);
       a.erase(a.begin() + n - m + 1, a.end());
        reverse(a.begin(), a.end());
   return a;
template <class T>
vector<T> operator/(vector<T> a, const vector<T> &b) {
   a /= b;
    return a;
template <class T>
vector<T> &operator%=(vector<T> &a, const vector<T> &b) {
   int n = a.size();
    int m = b.size();
```

```
if (n \ge m) {
        vector<T> c = (a / b) * b;
        a.resize(m - 1);
        for (int i = 0; i < m - 1; i++) {</pre>
            a[i] -= c[i];
    return a;
template <class T>
vector<T> operator%(vector<T> a, const vector<T> &b) {
    a %= b;
    return a:
template <class T>
vector<T> power(const vector<T> &a, long long b, const vector<T</pre>
     > &mod) {
    assert (b >= 0);
    vector<T> res = vector<T>{1} % mod;
    int highest_one_bit = -1;
    while (1LL << (highest one bit + 1) <= b)</pre>
        ++highest_one_bit;
    for (int i = highest_one_bit; i >= 0; i--) {
        res = res * res % mod;
        if (b >> i & 1) {
            res = res * a % mod;
    return res;
template <class T>
vector<T> derivative(vector<T> a) {
    for (size_t i = 0; i < a.size(); i++) {</pre>
        a[i] *= i;
    if (!a.empty()) {
        a.erase(a.begin());
    return a;
template <class T>
vector<T> integrate(vector<T> a) {
    a.insert(a.begin(), 0);
    for (int i = 1; i < a.size(); i++) {</pre>
        a[i] /= i;
    return a;
template <class T>
vector<T> logarithm(const vector<T> &a) {
    assert(!a.emptv() && a[0] == 1);
    vector<T> res = integrate(derivative(a) * inverse(a));
    res.resize(a.size());
    return res:
template <class T>
vector<T> exponent(const vector<T> &a) {
    assert(!a.empty() && a[0] == 0);
    int n = a.size();
    vector<T> b = {1}:
    while (b.size() < n) {</pre>
        vector<T> x(a.begin(), a.begin() + min(a.size(), b.size
             () << 1);
```

```
x[0] += 1;
        vector<T> old b = b;
       b.resize(b.size() << 1);</pre>
       x -= logarithm(b);
       x *= old_b;
        for (int i = b.size() >> 1; i < min(x.size(), b.size())</pre>
            ; i++) {
           b[i] = x[i];
       }
   b.resize(n);
   return b;
template <class T>
vector<T> sqrt (const vector<T> &a) {
    assert(!a.empty() && a[0] == 1);
    int n = a.size();
    vector < T > b = \{1\};
    while (b.size() < n) {
        vector<T> x(a.begin(), a.begin() + min(a.size(), b.size
             () << 1));
       b.resize(b.size() << 1);</pre>
       x *= inverse(b);
       T inv2 = 1 / static_cast<T>(2);
        for (int i = b.size() >> 1; i < min(x.size(), b.size())</pre>
            b[i] = x[i] * inv2;
        }
   b.resize(n);
    return b;
template <class T>
vector<T> multiply(const vector<vector<T>> &a) {
    if (a.empty()) {
        return {0};
    function<vector<T>(int, int)> mult = [&](int 1, int r) {
        if (1 == r) {
            return a[1];
        int m = (1 + r) >> 1;
        return mult(1, m) * mult(m + 1, r);
    return mult(0, (int)a.size() - 1);
template <class T>
T evaluate (const vector<T> &a, const T &x) {
   T res = 0;
    for (int i = (int) a.size() - 1; i >= 0; i--) {
       res = res * x + a[i];
   return res;
template <class T>
vector<T> evaluate(const vector<T> &a, const vector<T> &x) {
    if (x.empty()) {
        return {};
   if (a.empty()) {
        return vector<T>(x.size());
   int n = x.size();
   vector<vector<T>> t(2 * n - 1);
```

```
function<void(int, int, int)> build = [&](int v, int 1, int
         r) {
        if (1 == r) {
            t[v] = vector<T>{-x[1], 1};
        } else {
            int m = (1 + r) >> 1;
            int y = v + ((m - 1 + 1) << 1);
            build(v + 1, 1, m);
            build(y, m + 1, r);
            t[v] = t[v + 1] * t[y];
    };
    build(0, 0, n - 1);
    vector<T> res(n);
    function<void(int, int, int, vector<T>)> eval = [&](int v,
        int 1, int r, vector<T> p) {
        p %= t[v];
        if (p.size() < 150) {
            for (int i = 1; i <= r; i++) {</pre>
                res[i] = evaluate(p, x[i]);
            return;
        if (1 == r) {
            res[1] = p[0];
        } else {
            int m = (1 + r) >> 1;
            int z = v + ((m - 1 + 1) << 1);
            eval(v + 1, 1, m, p);
            eval(z, m + 1, r, p);
    };
    eval(0, 0, n - 1, a);
    return res;
template <class T>
vector<T> interpolate(const vector<T> &x, const vector<T> &y) {
    if (x.emptv()) {
        return {}:
    assert(x.size() == v.size());
    int n = x.size();
    vector < vector < T >> t(2 * n - 1);
    function<void(int, int, int) > build = [&] (int v, int 1, int
        if (1 == r) {
            t[v] = vector < T > \{-x[1], 1\};
        } else {
            int m = (1 + r) >> 1;
            int z = v + ((m - 1 + 1) << 1);
            build(v + 1, 1, m);
            build(z, m + 1, r);
            t[v] = t[v + 1] * t[z];
    };
    build(0, 0, n - 1);
    vector<T> val(n);
    function<void(int, int, int, vector<T>)> eval = [&](int v,
        int 1, int r, vector<T> p) {
        p %= t[v];
        if (p.size() < 150) {
            for (int i = 1; i <= r; i++) {</pre>
                val[i] = evaluate(p, x[i]);
            return;
        if (1 == r) {
            val[1] = p[0];
```

```
} else {
            int m = (1 + r) >> 1;
            int z = v + ((m - 1 + 1) << 1);
            eval(v + 1, 1, m, p);
            eval(z, m + 1, r, p);
    };
    vector<T> d = derivative(t[0]);
    eval(0, 0, n - 1, d);
    for (int i = 0; i < n; i++) {</pre>
        val[i] = y[i] / val[i];
    function<vector<T>(int, int, int)> calc = [&](int v, int 1,
          int r) {
        if (1 == r) {
            return vector<T>{val[1]};
        int m = (1 + r) >> 1;
        int z = v + ((m - 1 + 1) << 1);
        return calc(v + 1, 1, m) * t[z] + calc(z, m + 1, r) * t
             [v + 1];
    return calc(0, 0, n - 1);
// f[i] = 1^n + 2^n + \dots + up^n
// O(n*log(n)) complexity
template <class T>
vector<T> faulhaber(const T &up, int n) {
    vector<T> ex(n + 1);
    T e = 1;
    for (int i = 0; i <= n; i++) {</pre>
        ex[i] = e;
        e /= i + 1;
    vector<T> den = ex;
    den.erase(den.begin());
    for (auto &d : den) {
        d = -d;
    vector<T> num(n);
    T p = 1:
    for (int i = 0; i < n; i++) {
        p *= up + 1;
        num[i] = ex[i + 1] * (1 - p);
    vector<T> res = num * inverse(den);
    res.resize(n);
    for (int i = 0; i < n; i++) {</pre>
        res[i] *= f;
        f *= i + 1;
    return res:
//(x+1)*(x+2)*...*(x+n)
// (can be optimized with precomputed inverses)
template <class T>
vector<T> sequence(int n) {
    if (n == 0) {
        return {1};
    if (n % 2 == 1) {
        return sequence<T>(n - 1) * vector<T>{n, 1};
    vector<T> c = sequence<T>(n / 2);
    vector<T> a = c;
    reverse(a.begin(), a.end());
```

PrimesAndDivisors PrimitiveRoot

if ((int)c.size() <= i + (int)c mul.size()) {</pre>

vector<T> b mul(b.end() - z, b.end());

vector<T> c_mul = a_mul * b_mul;

```
T f = 1;
    for (int i = n / 2 - 1; i >= 0; i--) {
        f *= n / 2 - i;
        a[i] *= f;
    vector<T> b(n / 2 + 1);
   b[0] = 1;
    for (int i = 1; i <= n / 2; i++) {</pre>
       b[i] = b[i - 1] * (n / 2) / i;
    vector<T> h = a * b;
   h.resize(n / 2 + 1);
    reverse(h.begin(), h.end());
    f = 1;
    for (int i = 1; i <= n / 2; i++) {</pre>
       f /= i;
       h[i] \star = f;
    vector<T> res = c * h;
    return res;
template <class T>
T factorial (long long n) {
    if (n == 0)
        return 1:
    int m = min((long long)(sqrt(n) * 2), n);
    vector<T> a = sequence<T>(m);
    vector<T> x(n / m);
    for (size_t i = 0; i < x.size(); ++i) {</pre>
       x[i] = i;
        x[i] *= m;
    vector<T> b = evaluate(a, x);
    T res = 1:
    for (auto v : b)
        res \star= v;
    for (long long i = n / m * m + 1; i <= n; ++i) {
        res *= i;
    return res;
template <class T>
T binomial(int n, int m) {
    return factorial<T>(n) / factorial<T>(m) / factorial<T>(n -
template <class T>
class OnlineProduct {
  public:
    const vector<T> a;
    vector<T> b:
    vector<T> c;
    OnlineProduct(const vector<T> &a_) : a(a_) {}
    T add(const T &val) {
        int i = (int)b.size();
        b.push back(val);
        if ((int)c.size() <= i) {
            c.resize(i + 1);
        c[i] += a[0] * b[i];
        int z = 1;
        while ((i \& (z - 1)) == z - 1 \&\& (int)a.size() > z) {
            vector<T> a_mul(a.begin() + z, a.begin() + min(z <<</pre>
                  1, (int)a.size()));
```

```
c.resize(i + c mul.size() + 1);
            for (int j = 0; j < (int)c_mul.size(); j++) {</pre>
                c[i + 1 + j] += c_mul[j];
            z <<= 1;
        return c[i];
   }
};
PrimesAndDivisors.cpp
Description: desc
                                                      2aae08, 87 lines
// https://cp-algorithms.com/algebra/sieve-of-eratosthenes.html
vector<int> get_primes(int n) {
   if (n <= 1)
        return {};
    vector<bool> prime(n + 1, true);
    prime[0] = prime[1] = false;
    for (int i = 2; i * i <= n; i++)</pre>
        if (prime[i])
            for (int j = i * i; j <= n; j += i)</pre>
                prime[j] = false;
    vector<int> primes;
    for (int i = 0; i < prime.size(); ++i)</pre>
        if (prime[i])
            primes.push back(i);
    return primes;
// Generates prime numbers up to n in O(n) time
vector<int> generate_primes_linear_time(int n) {
    vector<int> lp(n + 1);
    vector<int> primes;
    for (int i = 2; i <= n; ++i) {</pre>
        if (lp[i] == 0) {
            lp[i] = i;
            primes.push_back(i);
        for (int j = 0; j < primes.size() && primes[j] <= lp[i]</pre>
              && i * primes[j] <= n; ++j)
            lp[i * primes[j]] = primes[j];
    return primes;
vector<int> number of prime divisors(int n) {
    vector<int> divisors(n + 1);
    fill(divisors.begin() + 2, divisors.end(), 1);
    for (int i = 2; i * i <= n; ++i)</pre>
        if (divisors[i] == 1)
            for (int j = i; j * i <= n; j++)</pre>
                divisors[i * j] = divisors[j] + 1;
    return divisors;
// Generates minimum prime divisor of all numbers up to n in O(
vector<int> generate_min_divisors(int n) {
    vector<int> lp(n + 1);
    lp[1] = 1;
```

```
vector<int> primes;
    for (int i = 2; i <= n; ++i) {</pre>
        if (lp[i] == 0) {
            lp[i] = i;
            primes.push_back(i);
        for (int j = 0; j < primes.size() && primes[j] <= lp[i]</pre>
              && i * primes[j] <= n; ++j)
            lp[i * primes[j]] = primes[j];
    return lp;
// Generates prime divisor of all numbers up to n
vector<int> generate_divisors(int n) {
    vector<int> divisors(n + 1);
    iota(divisors.begin(), divisors.end(), 0);
    for (int i = 2; i * i <= n; i++)
        if (divisors[i] == i)
            for (int j = i * i; j <= n; j += i)
                divisors[j] = i;
    return divisors;
// usage example
int main() {
    auto print = [](const vector<int> &a) {
        for (int x : a)
            cout << x << " ";
        cout << endl;
    };
    int n = 32;
    print(get_primes(n));
    print(generate_primes_linear_time(n));
    print(generate_min_divisors(n));
    print(generate_divisors(n));
PrimitiveRoot.cpp
Description: desc
                                                     4cabe8, 59 lines
// https://cp-algorithms.com/algebra/primitive-root.html
int pow_mod(int x, int n, int mod) {
    int res = 1;
    for (long long p = x; n > 0; n >>= 1, p = (p * p) % mod)
        if ((n & 1) != 0)
            res = (int) (res * p % mod);
    return res;
int totient function(int n) {
    int res = n;
    for (int i = 2; i * i <= n; ++i)</pre>
        if (n % i == 0) {
            while (n % i == 0)
               n /= i;
            res -= res / i;
    if (n > 1)
        res -= res / n;
    return res;
// returns g such that g^i runs through all numbers from 1 to m
     -1 modulo m
// q exists for m = 2,4,p^a,2*p^a, where p > 2 is a prime
```

Rational Rational SchreierSims

```
// O(m^0.5) complexity
int calc_generator(int m) {
    if (m == 2)
        return 1:
    vector<int> factors;
    int phi = totient_function(m);
    int n = phi;
    for (int i = 2; i * i <= n; ++i)</pre>
        if (n \% i == 0) {
            factors.emplace_back(i);
            while (n % i == 0)
                n /= i;
    if (n > 1)
        factors.emplace_back(n);
    for (int res = 2; res <= m; ++res) {</pre>
        if (gcd(res, m) != 1)
            continue;
       bool ok = true;
        for (size_t i = 0; i < factors.size() && ok; ++i)</pre>
            ok &= pow_mod(res, phi / factors[i], m) != 1;
        if (ok)
            return res;
    return -1;
// usage example
int main() {
    for (int i = 0; i < 15; ++i) {
        cout << "generator(" << i << ") = " << calc_generator(i</pre>
    cout << "generator(" << 998244353 << ") = " <<
         calc_generator(998244353) << endl;
Rational.cpp
Description: desc
                                                      320125, 12 lines
"rational.h"
using rt = rational<long long /*__int128*/>;
// usage example
int main() {
    rt x{1, 2};
    rt y\{2, 3\};
    if (y > x)
        cout << "y > x" << endl;
   x -= y;
    x = x.abs();
    cout << x.a << "/" << x.b << endl;
Rational.h
Description: desc
                                                       eff5c5, 70 lines
template <class T>
struct rational {
    Ta, b;
    rational(T a, T b = 1) : a{a}, b{b} { normalize(); }
    void normalize() {
        if (b < 0) {
            a = -a;
            b = -b;
        auto g = gcd(::abs(a), b);
        if (g != 0) {
```

```
a /= q;
            b /= q;
   }
    bool operator == (rational rhs) const { return a == rhs.a &&
        b == rhs.b; }
    bool operator!=(rational rhs) const { return !(*this == rhs
    bool operator<(rational rhs) const { return a * rhs.b < b *</pre>
          rhs.a; }
    bool operator<=(rational rhs) const { return ! (rhs < *this)</pre>
    bool operator>(rational rhs) const { return rhs < *this; }</pre>
    bool operator>=(rational rhs) const { return ! (*this < rhs)</pre>
    rational operator-() const { return rational(-a, b); }
    rational abs() const { return rational(::abs(a), b); }
    rational & operator += (rational rhs) {
        a = a * rhs.b + b * rhs.a;
        b \star = rhs.b;
        normalize();
        return *this;
    rational & operator -= (rational rhs) {
        a = a * rhs.b - b * rhs.a;
        b \star = rhs.b;
        normalize();
        return *this;
    rational &operator *= (rational rhs) {
        a *= rhs.a;
        b \star = rhs.b;
        normalize();
        return *this;
    rational & operator /= (rational rhs) {
        a \star = rhs.b;
        b *= rhs.a;
        normalize();
        return *this;
    friend rational operator+(rational lhs, rational rhs) {
         return lhs += rhs; }
    friend rational operator-(rational lhs, rational rhs) {
         return lhs -= rhs; }
    friend rational operator*(rational lhs, rational rhs) {
         return lhs *= rhs; }
    friend rational operator/(rational lhs, rational rhs) {
         return lhs /= rhs; }
};
```

```
SchreierSims.cpp
Description: desc
                                                     019312, 115 lines
// https://en.wikipedia.org/wiki/Schreier%E2%80%93
     Sims\_algorithm
// time: O(n^2 \lg^3 |G| + t n \lg |G|)
// mem : O(n^2 \lg |G| + tn)
// t : number of generator
namespace SchreierSimsAlgorithm {
using vi = vector<int>;
using pii = pair<int, int>;
vi inv(const vi &p) {
    vi res(p.size());
    for (int i = 0; i < p.size(); i++) {</pre>
        res[p[i]] = i;
    return res;
vi operator* (const vi &a, const vi &b) {
    vi res(a.size());
    for (int i = 0; i < a.size(); i++)</pre>
        res[i] = b[a[i]];
    return res:
int n, m;
vector<vector<vector<int>>> bkts, bktsInv;
vector<vector<int>> lookup;
int fast_filter(const vi &g, bool addToG = 1) {
    n = bkts.size();
    vi p = g;
    for (int i = 0; i < n; i++) {</pre>
        int res = lookup[i][p[i]];
        if (res == -1) {
            if (addToG) {
                bkts[i].push_back(p);
                bktsInv[i].push_back(inv(p));
                lookup[i][p[i]] = (int)bkts[i].size() - 1;
            return i;
        p = p * bktsInv[i][res];
    return -1;
long long calc_total_size() {
    long long ret = 1;
    for (int i = 0; i < n; i++)</pre>
        ret *= bkts[i].size();
    return ret;
bool in_group(const vi &g) {
    return fast_filter(g, false) == -1;
void solve(const vector<vector<int>> &perms, int _n) {
    m = perms.size();
    bkts.clear();
    bktsInv.clear();
    lookup.clear();
    lookup.resize(n);
    for (int i = 0; i < n; i++) {
```

```
lookup[i].resize(n);
        fill(lookup[i].begin(), lookup[i].end(), -1);
    vi id(n);
    iota(id.begin(), id.end(), 0);
    bkts.resize(n);
    bktsInv.resize(n);
    for (int i = 0; i < n; i++) {</pre>
        bkts[i].push_back(id);
        bktsInv[i].push_back(id);
        lookup[i][i] = 0;
    for (int i = 0; i < m; i++)</pre>
        fast filter(perms[i]);
    queue<pair<pii, pii>> toUpd;
    for (int i = 0; i < n; i++)</pre>
        for (int j = i; j < n; j++)
            for (int k = 0; k < bkts[i].size(); k++)</pre>
                for (int 1 = 0; 1 < bkts[j].size(); 1++)</pre>
                    toUpd.push({pii(i, k), pii(j, l)});
    while (!toUpd.empty()) {
        pii a = toUpd.front().first;
        pii b = toUpd.front().second;
        toUpd.pop();
        int res = fast_filter(bkts[a.first][a.second] * bkts[b.
             first][b.second]);
        if (res == -1)
            continue;
        pii newPair(res, (int)bkts[res].size() - 1);
        for (int i = 0; i < n; i++)</pre>
            for (int j = 0; j < bkts[i].size(); ++j) {</pre>
                if (i <= res)
                     toUpd.push({pii(i, j), newPair});
                if (res <= i)
                     toUpd.push({newPair, pii(i, j)});
} // namespace SchreierSimsAlgorithm
// usage example
int main() {
    vector<vector<int>> perms{{0, 1, 3, 2}, {1, 0, 2, 3}};
    SchreierSimsAlgorithm::solve(perms, 4);
    cout << SchreierSimsAlgorithm::in_group({0, 1, 2, 3}) <</pre>
    cout << SchreierSimsAlgorithm::in_group({1, 0, 2, 3}) <</pre>
    cout << SchreierSimsAlgorithm::in_group({0, 1, 3, 2}) <</pre>
    cout << SchreierSimsAlgorithm::in_group({1, 0, 3, 2}) <</pre>
    cout << SchreierSimsAlgorithm::in_group({2, 3, 0, 1}) <</pre>
         endl;
```

Numeric (5)

```
Bigint.cpp
Description: desc
```

52f441, 410 lines "fft.h" constexpr int digits(int base) noexcept { **return** base <= 1 ? 0 : 1 + digits(base / 10); constexpr int base = 1000'000'000; constexpr int base_digits = digits(base);

```
constexpr int fft_base = 10'000; // fft_base^2 * n /
    fft base digits <= 10^15 for double
constexpr int fft_base_digits = digits(fft_base);
struct bigint {
   // value == 0 is represented by empty z
   vector<int> z; // digits
    // sign == 1 <==> value >= 0
    // sign == -1 <==> value < 0
   int sign;
   bigint (long long v = 0) { *this = v; }
   bigint &operator=(long long v) {
       sign = v < 0 ? -1 : 1;
       v *= sign;
       z.clear();
        for (; v > 0; v = v / base)
            z.push_back((int)(v % base));
       return *this;
   bigint(const string &s) { read(s); }
    bigint &operator+=(const bigint &other) {
       if (sign == other.sign) {
            for (int i = 0, carry = 0; i < other.z.size() ||
                carry; ++i) {
                if (i == z.size())
                    z.push_back(0);
                z[i] += carry + (i < other.z.size() ? other.z[i</pre>
                    ] : 0);
                carry = z[i] >= base;
                if (carry)
                    z[i] -= base;
       } else if (other != 0 /* prevent infinite loop */) {
            *this -= -other:
        return *this;
    friend bigint operator+(bigint a, const bigint &b) {
        return a;
   bigint &operator = (const bigint &other) {
        if (sign == other.sign) {
            if ((sign == 1 && *this >= other) || (sign == -1 &&
                  *this <= other)) {
                for (int i = 0, carry = 0; i < other.z.size()
                     || carry; ++i) {
                    z[i] -= carry + (i < other.z.size() ? other</pre>
                         .z[i] : 0);
                    carry = z[i] < 0;
                    if (carry)
                        z[i] += base;
               trim();
            } else {
                *this = other - *this;
                this->sign = -this->sign;
       } else {
            *this += -other;
```

```
return *this;
friend bigint operator-(bigint a, const bigint &b) {
   return a;
bigint &operator *= (int v) {
   if (v < 0)
        sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < z.size() \mid \mid carry; ++i)
        if (i == z.size())
            z.push_back(0);
        long long cur = (long long)z[i] * v + carry;
        carry = (int)(cur / base);
        z[i] = (int)(cur % base);
   trim();
   return *this;
bigint operator*(int v) const { return bigint(*this) *= v;
friend pair < bigint, bigint > divmod(const bigint &al, const
    bigint &b1) {
    int norm = base / (b1.z.back() + 1);
   bigint a = a1.abs() * norm;
   bigint b = b1.abs() * norm;
   bigint q, r;
   q.z.resize(a.z.size());
    for (int i = (int)a.z.size() - 1; i >= 0; i--) {
       r *= base;
        r += a.z[i];
        int s1 = b.z.size() < r.z.size() ? r.z[b.z.size()]
        int s2 = b.z.size() - 1 < r.z.size() ? r.z[b.z.size
        int d = (int)(((long long)s1 * base + s2) / b.z.
             back()):
        r -= b * d;
        while (r < 0)
            r += b, --d;
        q.z[i] = d;
   q.sign = a1.sign * b1.sign;
   r.sign = al.sign;
   g.trim();
   r.trim():
    return {q, r / norm};
friend bigint sqrt(const bigint &al) {
   bigint a = a1;
   while (a.z.empty() \mid \mid a.z.size() % 2 == 1)
        a.z.push back(0);
   int n = a.z.size();
   int firstDigit = (int)::sgrt((double)a.z[n - 1] * base
        + a.z[n - 21);
   int norm = base / (firstDigit + 1);
   a *= norm:
   a \star = norm;
    while (a.z.empty() \mid \mid a.z.size() % 2 == 1)
```

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```
a.z.push back(0);
    bigint r = (long long)a.z[n - 1] * base + a.z[n - 2];
    firstDigit = (int)::sqrt((double)a.z[n - 1] * base + a.
        z[n - 2]);
    int q = firstDigit;
    bigint res;
    for (int j = n / 2 - 1; j >= 0; j--) {
        for (;; --q) {
            bigint r1 = (r - (res * 2 * base + q) * q) *
                 base * base +
                         (j > 0 ? (long long)a.z[2 * j - 1]
                              * base + a.z[2 * j - 2] : 0);
            if (r1 >= 0) {
                r = r1:
                break;
            }
        res *= base;
        res += q;
        if (i > 0) {
            int d1 = res.z.size() + 2 < r.z.size() ? r.z[
                 res.z.size() + 2] : 0;
            int d2 = res.z.size() + 1 < r.z.size() ? r.z[
                 res.z.size() + 1] : 0;
            int d3 = res.z.size() < r.z.size() ? r.z[res.z.
                 size()] : 0;
            q = (int)(((long long)d1 * base * base + (long long)d1)
                 long)d2 * base + d3) / (firstDigit * 2));
    }
    res.trim();
    return res / norm;
bigint operator/(const bigint &v) const { return divmod(*
     this, v).first; }
bigint operator% (const bigint &v) const { return divmod(*
     this, v).second; }
bigint & operator /= (int v) {
    if (v < 0)
        sign = -sign, v = -v;
    for (int i = (int)z.size() - 1, rem = 0; i >= 0; --i) {
        long long cur = z[i] + rem * (long long)base;
        z[i] = (int)(cur / v);
        rem = (int)(cur % v);
    trim():
    return *this:
bigint operator/(int v) const { return bigint(*this) /= v;
int operator%(int v) const {
    if (v < 0)
        v = -v:
    int m = 0;
    for (int i = (int)z.size() - 1; i >= 0; --i)
        m = (int)((z[i] + m * (long long)base) % v);
    return m * sign;
bigint &operator *= (const bigint &v) {
```

```
*this = *this * v;
    return *this:
bigint &operator/=(const bigint &v) {
    *this = *this / v;
    return *this;
bigint &operator%=(const bigint &v) {
    *this = *this % v;
    return *this;
bool operator<(const bigint &v) const {
    if (sign != v.sign)
        return sign < v.sign;
    if (z.size() != v.z.size())
        return z.size() * sign < v.z.size() * v.sign;
    for (int i = (int)z.size() - 1; i >= 0; i--)
        if (z[i] != v.z[i])
            return z[i] * sign < v.z[i] * sign;
    return false;
bool operator>(const bigint &v) const { return v < *this; }
bool operator <= (const bigint &v) const { return ! (v < *this
    ); }
bool operator>=(const bigint &v) const { return !(*this < v
bool operator == (const bigint &v) const { return sign == v.
     sign && z == v.z; }
bool operator!=(const bigint &v) const { return ! (*this ==
void trim() {
    while (!z.empty() \&\& z.back() == 0)
        z.pop back();
    if (z.empty())
        sign = 1;
bool isZero() const { return z.empty(); }
friend bigint operator-(bigint v) {
    if (!v.z.empty())
        v.sign = -v.sign;
    return v:
}
bigint abs() const { return sign == 1 ? *this : -*this; }
long long longValue() const {
    long long res = 0;
    for (int i = (int)z.size() - 1; i >= 0; i--)
       res = res * base + z[i];
    return res * sign;
friend bigint gcd(const bigint &a, const bigint &b) {
     return b.isZero() ? a : gcd(b, a % b); }
friend bigint lcm(const bigint &a, const bigint &b) {
     return a / gcd(a, b) * b; }
```

```
void read(const string &s) {
    sign = 1;
    z.clear();
    int pos = 0;
    while (pos < s.size() && (s[pos] == '-' || s[pos] == '+
        ')) {
        if (s[pos] == '-')
            sign = -sign;
        ++pos;
    for (int i = (int)s.size() - 1; i >= pos; i -=
        base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j <= i;</pre>
            x = x * 10 + s[j] - '0';
        z.push_back(x);
    trim();
}
friend istream &operator>>(istream &stream, bigint &v) {
    string s;
    stream >> s;
    v.read(s);
    return stream;
friend ostream & operator << (ostream & stream, const bigint &v
    if (v.sign == -1)
        stream << '-';
    stream << (v.z.empty() ? 0 : v.z.back());
    for (int i = (int)v.z.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0') << v.z[</pre>
            i];
    return stream;
static vector<int> convert_base(const vector<int> &a, int
    old_digits, int new_digits) {
    vector<long long> p(max(old_digits, new_digits) + 1);
    p[0] = 1;
    for (int i = 1; i < p.size(); i++)
        p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int v : a) {
        cur += v * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back(int(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
    res.push back((int)cur);
    while (!res.empty() && res.back() == 0)
        res.pop_back();
    return res;
bigint operator* (const bigint &v) const {
    if (min(z.size(), v.z.size()) < 150)
        return mul_simple(v);
    bigint res;
    res.sign = sign * v.sign;
```

```
res.z = multiply_bigint(convert_base(z, base_digits,
            fft_base_digits),
                                convert_base(v.z, base_digits,
                                     fft_base_digits), fft_base
        res.z = convert_base(res.z, fft_base_digits,
            base digits);
        res.trim();
        return res;
   bigint mul simple(const bigint &v) const {
        bigint res;
        res.sign = sign * v.sign;
        res.z.resize(z.size() + v.z.size());
        for (int i = 0; i < z.size(); ++i)
            if (z[i])
                for (int j = 0, carry = 0; j < v.z.size()
                     carry; ++j) {
                    long long cur = res.z[i + j] + (long long)z
                         [i] * (j < v.z.size() ? v.z[j] : 0) +
                         carry;
                    carry = (int)(cur / base);
                    res.z[i + j] = (int)(cur % base);
        res.trim();
        return res;
};
mt19937 rng(1);
bigint random_bigint(int n) {
    string s;
    for (int i = 0; i < n; i++) {
        s += uniform_int_distribution<int>('0', '9')(rng);
    return bigint(s);
// random tests
int main() {
    bigint x = bigint("120");
    bigint v = bigint("5");
    cout << x / y << endl;
    for (int i = 0; i < 1000; i++) {
        int n = uniform_int_distribution<int>(1, 100) (rng);
        bigint a = random bigint(n);
       bigint res = sgrt(a);
        bigint xx = res * res;
       bigint yy = (res + 1) * (res + 1);
        if (xx > a | | yy \le a)  {
            cout << i << endl;
            cout << a << " " << res << endl;
            break:
        int m = uniform int distribution<int>(1, n)(rng);
       bigint b = random_bigint(m) + 1;
        res = a / b;
        xx = res * b;
       yy = b * (res + 1);
        if (xx > a \mid \mid yy \le a) {
            cout << i << endl;
            cout << a << " " << b << " " << res << endl;
            break;
```

```
bigint a = random_bigint(10'000);
        bigint b = random_bigint(2000);
        auto t1 = chrono::high resolution clock::now();
        bigint c = a / b;
        auto t2 = chrono::high_resolution_clock::now();
        chrono::duration<double, milli> duration = t2 - t1;
        cout << duration.count() << " ms" << endl;</pre>
        cout << endl;
    bigint a = random_bigint(200'000);
    bigint b = random_bigint(200'000);
   bigint c1, c2;
        auto t1 = chrono::high_resolution_clock::now();
        c1 = a * b;
        auto t2 = chrono::high_resolution_clock::now();
        chrono::duration<double, milli> duration = t2 - t1;
        cout << duration.count() << " ms" << endl;</pre>
        auto t1 = chrono::high_resolution_clock::now();
        c2 = a.mul_simple(b);
        auto t2 = chrono::high resolution clock::now();
        chrono::duration<double, milli> duration = t2 - t1;
        cout << duration.count() << " ms" << endl;</pre>
    cout << (c1 == c2) << endl;
Fft.cpp
Description: desc
"fft.h"
                                                       5777cc, 9 lines
// usage example
int main() {
    vector<int> a{9, 9};
    vector<int> b{8, 9};
    vector<int> mul = multiply_bigint(a, b, 10);
    for (int x : mul)
       cout << x << " ";
    cout << endl;
Fft.h
Description: desc
                                                      2f0e77, 116 lines
// Fast Fourier transform
// https://cp-algorithms.com/algebra/fft.html
// https://drive.google.com/file/d/1
     B9BIfATnI_qL6rYiE5hY9bh20SMVmHZ7/view
using cpx = complex<double>;
const double PI = acos(-1);
vector < cpx > roots = \{\{0, 0\}, \{1, 0\}\};
void ensure_capacity(int min_capacity) {
    for (int len = roots.size(); len < min_capacity; len *= 2)</pre>
        for (int i = len >> 1; i < len; i++) {</pre>
            roots.emplace back(roots[i]);
            double angle = 2 * PI * (2 * i + 1 - len) / (len * len) / (len * len)
             roots.emplace_back(cos(angle), sin(angle));
```

```
void fft(vector<cpx> &z, bool inverse) {
    int n = z.size();
    assert((n & (n - 1)) == 0);
    ensure_capacity(n);
    for (unsigned i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1)
           j -= bit;
        j += bit;
        if (i < j)
            swap(z[i], z[j]);
    for (int len = 1; len < n; len <<= 1) {</pre>
        for (int i = 0; i < n; i += len * 2) {
            for (int j = 0; j < len; j++) {
                cpx root = inverse ? conj(roots[j + len]) :
                     roots[j + len];
                cpx u = z[i + j];
                cpx v = z[i + j + len] * root;
                z[i + j] = u + v;
                z[i + j + len] = u - v;
        }
    if (inverse)
        for (int i = 0; i < n; i++)</pre>
            z[i] /= n;
vector<int> multiply_bigint(const vector<int> &a, const vector<</pre>
    int> &b, int base) {
    int need = a.size() + b.size();
    int n = 1:
    while (n < need)
        n <<= 1;
    vector<cpx> p(n);
    for (size t i = 0; i < n; i++) {</pre>
        p[i] = cpx(i < a.size() ? a[i] : 0, i < b.size() ? b[i]
              : 0);
    fft(p, false);
    // a[w/k] = (p[w/k] + conj(p[w/n-k])) / 2
    // b[w[k]] = (p[w[k]] - conj(p[w[n-k]])) / (2*i)
    vector<cpx> ab(n);
    cpx r(0, -0.25);
    for (int i = 0; i < n; i++) {</pre>
        int j = (n - i) & (n - 1);
        ab[i] = (p[i] * p[i] - conj(p[j] * p[j])) * r;
    fft(ab, true);
    vector<int> result(need);
    long long carry = 0;
    for (int i = 0; i < need; i++) {</pre>
        long long d = (long long) (ab[i].real() + 0.5) + carry;
        carry = d / base;
        result[i] = d % base;
    return result;
vector<int> multiply_mod(const vector<int> &a, const vector<int</pre>
    > &b, int m) {
    int need = a.size() + b.size() - 1;
    int n = 1;
    while (n < need)
        n <<= 1;
    vector<cpx> A(n);
```

fft(A, false);

vector<cpx> B(n);

for (size_t i = 0; i < a.size(); i++) {</pre>

for (size_t i = 0; i < b.size(); i++) {</pre>

int x = (b[i] % m + m) % m;

A[i] = cpx(x & ((1 << 15) - 1), x >> 15);

int x = (a[i] % m + m) % m;

FftSlow Polynom-roots ExpressionParserShuntingYard

```
B[i] = cpx(x & ((1 << 15) - 1), x >> 15);
    fft(B, false);
   vector<cpx> fa(n);
    vector<cpx> fb(n);
    for (int i = 0, j = 0; i < n; i++, j = n - i) {
       cpx al = (A[i] + conj(A[j])) * cpx(0.5, 0);
        cpx \ a2 = (A[i] - conj(A[j])) * cpx(0, -0.5);
       cpx b1 = (B[i] + conj(B[j])) * cpx(0.5, 0);
       cpx b2 = (B[i] - conj(B[j])) * cpx(0, -0.5);
        fa[i] = a1 * b1 + a2 * b2 * cpx(0, 1);
        fb[i] = a1 * b2 + a2 * b1;
    fft(fa, true);
    fft(fb, true);
    vector<int> res(need);
    for (int i = 0; i < need; i++) {</pre>
       long long aa = (long long) (fa[i].real() + 0.5);
       long long bb = (long long) (fb[i].real() + 0.5);
       long long cc = (long long) (fa[i].imag() + 0.5);
        res[i] = (aa % m + (bb % m << 15) + (cc % m << 30)) % m
    return res;
FftSlow.cpp
Description: desc
                                                     f1b9c4, 71 lines
#pragma GCC optimize("Ofast")
// Fast Fourier transform, simple implementation
// https://cp-algorithms.com/algebra/fft.html
using cpx = complex<double>;
const double PI = acos(-1);
void fft(vector<cpx> &z, bool inverse) {
    size t n = z.size();
   assert ((n & (n - 1)) == 0);
   if (n == 1)
        return;
    vector<cpx> z0(n / 2);
    vector<cpx> z1(n / 2);
    for (int i = 0; i < n / 2; i++) {</pre>
       z0[i] = z[2 * i];
       z1[i] = z[2 * i + 1];
    fft(z0, inverse);
    fft(z1, inverse);
    for (int i = 0; i < n / 2; ++i) {
        double ang = 2 * PI * i / n * (inverse ? -1 : 1);
       cpx w(cos(ang), sin(ang));
       z[i] = z0[i] + w * z1[i];
       z[i + n / 2] = z0[i] - w * z1[i];
       if (inverse) {
            z[i] /= 2;
            z[i + n / 2] /= 2;
```

```
}
vector<int> multiply_bigint(const vector<int> &a, const vector<
    int need = a.size() + b.size();
    int n = 1:
    while (n < need)
       n <<= 1;
    vector<cpx> fa(a.begin(), a.end());
    vector<cpx> fb(b.begin(), b.end());
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++) {</pre>
        fa[i] *= fb[i];
    fft(fa, true);
    vector<int> result (need);
    for (int i = 0, carry = 0; i < need; i++) {</pre>
        result[i] = (int)(fa[i].real() + 0.5) + carry;
        carry = result[i] / 10;
        result[i] %= 10;
    return result;
// usage example
int main() {
    vector<int> a{5, 1};
    vector<int> b{2, 1};
    vector<int> res = multiply_bigint(a, b);
    for (int x : res)
       cout << x << " ";
    cout << endl:
Polynom-roots.cpp
Description: desc
                                                     ee25b3, 88 lines
// https://en.wikipedia.org/wiki/Laguerre%27s_method
typedef complex<double> cdouble;
typedef vector<cdouble> poly;
pair<poly, cdouble> horner(const poly &a, cdouble x0) {
    int n = a.size();
   poly b = poly(max(1, n - 1));
    for (int i = n - 1; i > 0; i--)
       b[i-1] = a[i] + (i < n-1 ? b[i] * x0 : 0);
    return {b, a[0] + b[0] * x0};
cdouble eval(const poly &p, cdouble x) {
    return horner(p, x).second;
poly derivative (const poly &p) {
    int n = p.size();
    poly r = poly(max(1, n - 1));
    for (int i = 1; i < n; i++)</pre>
       r[i - 1] = p[i] * cdouble(i);
    return r;
```

```
const double EPS = 1e-9;
int cmp(cdouble x, cdouble y) {
    double diff = abs(x) - abs(y);
    return diff < -EPS ? -1 : (diff > EPS ? 1 : 0);
cdouble find_one_root(const poly &p0, cdouble x) {
    int n = p0.size() - 1;
    poly p1 = derivative(p0);
    poly p2 = derivative(p1);
    for (int step = 0; step < 10'000; step++) {</pre>
        cdouble y0 = eval(p0, x);
        if (cmp(y0, 0) == 0)
            break;
        cdouble G = eval(p1, x) / y0;
        cdouble H = G * G - eval(p2, x) - y0;
        cdouble R = sqrt(cdouble(n - 1) * (H * cdouble(n) - G *
              G));
        cdouble D1 = G + R;
        cdouble D2 = G - R;
        cdouble a = cdouble(n) / (cmp(D1, D2) > 0 ? D1 : D2);
        x -= a;
        if (cmp(a, 0) == 0)
            break:
    return x;
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
uniform_real_distribution<double> uniform(0, 1);
vector<cdouble> find_all_roots(const poly &p) {
    vector<cdouble> res:
    poly q = p;
    while (q.size() > 2) {
        cdouble z(uniform(rng), uniform(rng));
        z = find_one_root(q, z);
        z = find_one_root(p, z);
        q = horner(q, z).first;
        res.push_back(z);
    res.push_back(-q[0] / q[1]);
    return res;
int main(int argc, char *argv[]) {
    // x^3 - 8x^2 - 13x + 140 = (x+4)(x-5)(x-7)
    poly p = \{140, -13, -8, 1\};
    vector<cdouble> roots = find all roots(p);
    for (size_t i = 0; i < roots.size(); i++) {
        if (abs(roots[i].real()) < EPS)</pre>
            roots[i] -= cdouble(roots[i].real(), 0);
        if (abs(roots[i].imag()) < EPS)</pre>
            roots[i] -= cdouble(0, roots[i].imag());
        cout << setprecision(3) << roots[i] << endl;</pre>
    return 0;
```

Parsing (6)

Sort Aho-corasick Hashing

ExpressionParserShuntingYard.cpp Description: desc

dd89fe, 63 lines

```
stack<int> values;
stack<char> ops;
void process op() {
    int r = values.top();
    values.pop();
    int 1 = values.top();
    values.pop();
    char op = ops.top();
    ops.pop();
    switch (op) {
        case '+':
            values.push(1 + r);
        case '-':
            values.push(1 - r);
            break;
        case ' *':
            values.push(l * r);
            break;
        case '/':
            values.push(1 / r);
            break;
int priority(char op) {
    switch (op) {
        case '+':
        case '-':
            return 1;
        case ' *':
        case '/':
            return 2:
    return 0;
int eval(string s) {
   s = '(' + s + ')';
    for (char c : s)
       if ('0' <= c && c <= '9')
            values.push(c - '0');
        else if (c == '(')
            ops.push(c);
        else if (c == ')') {
            while (ops.top() != '(')
                process_op();
            ops.pop();
        } else {
            while (!ops.empty() && priority(ops.top()) >=
                 priority(c))
                process_op();
            ops.push(c);
    return values.top();
// usage example
int main() {
    cout << eval("2*2+2") << endl;
    cout << eval("2+2*2") << endl;
    cout << eval("(3+2) *2") << endl;
```

Sort (7)

```
Sort.cpp
Description: desc
                                                      4a3cd0, 48 lines
void merge_sort(vector<int> &a, int low, int high) {
    if (high - low < 2)
        return;
    int mid = (low + high) >> 1;
    merge sort(a, low, mid);
    merge_sort(a, mid, high);
    vector<int> b;
    copy(a.begin() + low, a.begin() + mid, back_inserter(b));
    for (int i = low, j = mid, k = 0; k < b.size(); i++) {</pre>
        if (j == high || b[k] <= a[j]) {</pre>
            a[i] = b[k++];
        } else {
            a[i] = a[j++];
    }
void counting sort(vector<int> &a) {
    int max = *max_element(a.begin(), a.end());
    vector<int> cnt(max + 1);
    for (int x : a) {
        ++cnt[x];
    for (int i = 1; i < cnt.size(); i++) {</pre>
        cnt[i] += cnt[i - 1];
    int n = a.size();
    vector<int> b(n);
    for (int i = 0; i < n; i++) {</pre>
        b[--cnt[a[i]]] = a[i];
    a = b;
// usage example
int main() {
    vector<int> a{4, 1, 2, 3};
    merge_sort(a, 0, a.size());
    for (int x : a)
        cout << x << " ";
    cout << endl;
    a = \{4, 1, 2, 3\};
    counting_sort(a);
    for (int x : a)
        cout << x << " ";
    cout << endl;
```

Strings (8)

```
Aho-corasick.cpp
Description: desc
```

b06f55, 55 lines

```
constexpr int ALPHABET_SIZE = 26;
constexpr int MAX_STATES = 200'000;
int transitions[MAX_STATES][ALPHABET_SIZE];
int sufflink [MAX_STATES];
int escape[MAX_STATES];
int states = 1;
int addString(const string &s) {
```

```
int v = 0;
    for (char c : s) {
        c -= 'a';
        if (transitions[v][c] == 0) {
            transitions[v][c] = states++;
        v = transitions[v][c];
    escape[v] = v;
    return v:
void buildLinks() {
    vector<int> q(MAX_STATES);
    for (int s = 0, t = 1; s < t;) {
        int v = q[s++];
        int u = sufflink[v];
        if (escape[v] == 0) {
            escape[v] = escape[u];
        for (int c = 0; c < ALPHABET_SIZE; c++) {
            if (transitions[v][c] != 0) {
                g[t++] = transitions[v][c];
                sufflink[transitions[v][c]] = v != 0 ?
                     transitions[u][c]: 0;
            } else {
                transitions[v][c] = transitions[u][c];
int main() {
    addString("a");
    addString("aa");
    addString("abaaa");
    buildLinks();
    string s = "abaa";
    int state = 0;
    for (size t i = 0; i < s.length(); i++) {
        state = transitions[state][s[i] - 'a'];
        if (escape[state] != 0)
            cout << i << endl;
Hashing.cpp
Description: desc
                                                     6b1f33, 53 lines
// see https://codeforces.com/blog/entry/60442 for analysis
struct hashing {
    static constexpr int dimensions = 4;
    static constexpr int mod = (1u << 31) - 1;</pre>
    vector<vector<int>> hashes, p;
    static const vector<int> &get_bases() {
        static mt19937 rng(chrono::steady_clock::now().
             time_since_epoch().count());
        static vector<int> bases;
        while (bases.size() < dimensions) {</pre>
            bases.emplace_back(uniform_int_distribution<int>((
                 int)1e9, mod - 1)(rng));
        return bases;
```

) {

int n = s.size();

Kmp Manacher MinRotation Suffix-array-sa-is

```
p[d].resize(n + 1);
            p[d][0] = 1;
            long long base = bases[d];
            for (int i = 0; i < n; i++) {</pre>
                hashes[d][i + 1] = (hashes[d][i] * base + s[i])
                     % mod;
                p[d][i + 1] = p[d][i] * base % mod;
       }
    vector<int> get_hash(int i, int len) {
       vector<int> res;
        for (int d = 0; d < dimensions; ++d) {</pre>
            int hash = (int) ((hashes[d][i + len] + (long long)
                hashes[d][i] * (mod - p[d][len])) % mod);
            res.emplace back(hash);
        return res;
};
// usage example
int main() {
    string a = "abc123";
    string b = "abc";
    auto ha = hashing(a);
    auto hb = hashing(b);
    vector<int> ha1 = ha.get_hash(0, 3);
    vector<int> ha2 = ha.get_hash(3, 3);
    vector<int> hb1 = ha.get_hash(0, 3);
    cout << (ha1 == hb1) << " " << (ha1 == ha2) << endl;
    cout << hashing::mod << endl;
Description: Knuth-Morris-Pratt String Matching
                                                     02d85e, 34 lines
vector<int> prefix_function(string s) {
    vector<int> p(s.length());
    int k = 0:
    for (int i = 1; i < s.length(); i++) {</pre>
        while (k > 0 \&\& s[k] != s[i])
            k = p[k - 1];
       if (s[k] == s[i])
            ++k;
       p[i] = k;
    return p;
int find_substring(string haystack, string needle) {
    int m = needle.length();
    if (m == 0)
        return 0;
    vector<int> p = prefix_function(needle);
    for (int i = 0, k = 0; i < haystack.length(); i++) {
        while (k > 0 && needle[k] != haystack[i])
            k = p[k - 1];
        if (needle[k] == haystack[i])
            ++k;
        if (k == m)
            return i + 1 - m;
```

hashing (const string &s) : hashes (dimensions), p (dimensions

const vector<int> &bases = get_bases();

for (int d = 0; d < dimensions; ++d) {</pre>

hashes[d].resize(n + 1);

```
return -1:
// usage example
int main() {
    int pos = find_substring("acabc", "ab");
    cout << pos << endl;
Manacher.cpp
Description: desc
                                                     39033f, 52 lines
// Manacher's algorithm: https://cp-algorithms.com/string/
     manacher.html
// d1[i] - how many palindromes of odd length with center at i
vector<int> odd palindromes(const string &s) {
    size_t n = s.size();
    vector<int> d1(n);
    int 1 = 0, r = -1;
    for (int i = 0; i < n; ++i) {</pre>
        int len = i > r ? 1 : min(d1[1 + r - i], r - i + 1);
        while (i - len >= 0 \&\& i + len < n \&\& s[i - len] == s[i
              + lenl)
            ++len;
        d1[i] = len;
        if (r < i + len - 1) {
            r = i + len - 1;
            1 = i - (len - 1);
    return d1;
// d2[i] - how many palindromes of even length with center at i
vector<int> even palindromes(const string &s) {
    size_t n = s.size();
    vector<int> d2(n);
    int 1 = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int len = i > r ? 0 : min(d2[1 + r - i + 1], r - i + 1)
        while (i - len - 1 >= 0 && i + len < n && s[i - len -
            1] == s[i + len])
            ++len;
        d2[i] = len;
        if (r < i + len - 1) {
            r = i + len - 1;
            1 = i - len;
    return d2;
// usage example
int main() {
    string text = "abbba";
    auto d1 = odd_palindromes(text);
    for (int d : d1)
       cout << d << " ";
    cout << endl;
    auto d2 = even_palindromes(text);
    for (int d : d2)
       cout << d << " ";
    cout << endl;
```

```
MinRotation.cpp
Description: desc
                                                     7b7f92, 25 lines
string min_cyclic_shift(string s) {
    s += s;
    int n = s.length();
    int i = 0;
    int pos = 0;
    while (i < n / 2) {
        pos = i;
        int j = i + 1, k = i;
        while (j < n \&\& s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i:
            else
                ++k:
            ++ 1;
        while (i \le k)
            i += j - k;
    return s.substr(pos, n / 2);
// usage example
int main() {
    cout << min_cyclic_shift("bcabab") << endl;</pre>
Suffix-array-sa-is.cpp
Description: Suffix array and lcp in O(n)
Time: \mathcal{O}(n)
#define tget(i) ((t[(i) / 8] & (1 << ((i) % 8))) ? 1 : 0)
#define tset(i, b) t[(i) / 8] = (b) ? ((1 << ((i) % 8)) | t[(i)
      /81): ((\sim(1 << ((i) % 8))) & t[(i) / 81)
#define chr(i) (cs == sizeof(int) ? ((int *)s)[i] : ((unsigned
    char *)s)[i])
#define isLMS(i) (i > 0 && tget(i) && !tget(i - 1))
// find the start or end of each bucket
void getBuckets(unsigned char *s, int *bkt, int n, int K, int
    cs, bool end) {
    int i, sum = 0;
    for (i = 0; i <= K; i++)
        bkt[i] = 0; // clear all buckets
    for (i = 0; i < n; i++)
        bkt[chr(i)]++; // compute the size of each bucket
    for (i = 0; i <= K; i++) {
        sum += bkt[i];
        bkt[i] = end ? sum : sum - bkt[i];
// compute SAl
void induceSAl (unsigned char *t, int *SA, unsigned char *s, int
      *bkt, int n, int K, int cs, bool end) {
    int i, j;
    getBuckets(s, bkt, n, K, cs, end); // find starts of
    for (i = 0; i < n; i++) {
        j = SA[i] - 1;
        if (j >= 0 && !tget(j))
            SA[bkt[chr(j)]++] = j;
// compute SAs
void induceSAs (unsigned char *t, int *SA, unsigned char *s, int
      *bkt, int n, int K, int cs, bool end) {
```

```
int i, j;
   getBuckets(s, bkt, n, K, cs, end); // find ends of buckets
   for (i = n - 1; i >= 0; i--) {
       j = SA[i] - 1;
       if (j >= 0 && tget(j))
           SA[--bkt[chr(j)]] = j;
// find the suffix array SA of s[0..n-1] in \{1..K\}^n
// require s[n-1]=0 (the sentinel!), n>=2
// use a working space (excluding s and SA) of at most 2.25n+O
    (1) for a constant alphabet
void SA_IS(unsigned char *s, int *SA, int n, int K, int cs) {
   int i, j;
   unsigned char *t = (unsigned char *) malloc(n / 8 + 1); //
        LS-type array in bits
   // Classify the type of each character
   tset(n - 2, 0);
   tset (n - 1, 1); // the sentinel must be in s1, important
        !!!
   for (i = n - 3; i >= 0; i--)
       tset(i, (chr(i) < chr(i + 1) || (chr(i) == chr(i + 1)
            && tget(i + 1) == 1)) ? 1 : 0);
    // stage 1: reduce the problem by at least 1/2
    // sort all the S-substrings
   int *bkt = (int *) malloc(sizeof(int) * (K + 1)); // bucket
         array
   getBuckets(s, bkt, n, K, cs, true);
                                                      // find
        ends of buckets
    for (i = 0; i < n; i++)
       SA[i] = -1;
    for (i = 1; i < n; i++)
       if (isLMS(i))
            SA[--bkt[chr(i)]] = i;
    induceSAl(t, SA, s, bkt, n, K, cs, false);
   induceSAs(t, SA, s, bkt, n, K, cs, true);
   free (bkt);
   // compact all the sorted substrings into the first n1
        items of SA
   // 2*n1 must be not larger than n (proveable)
   int n1 = 0;
   for (i = 0; i < n; i++)
       if (isLMS(SA[i]))
            SA[n1++] = SA[i];
    // find the lexicographic names of all substrings
    for (i = n1; i < n; i++)
       SA[i] = -1; // init the name array buffer
   int name = 0, prev = -1;
   for (i = 0; i < n1; i++) {</pre>
       int pos = SA[i];
       bool diff = false;
       for (int d = 0; d < n; d++)
           if (prev == -1 || chr(pos + d) != chr(prev + d) ||
                tget(pos + d) != tget(prev + d)) {
               diff = true:
           } else if (d > 0 && (isLMS(pos + d) || isLMS(prev +
                 d)))
               break;
       if (diff) {
           name++:
           prev = pos;
       pos = (pos % 2 == 0) ? pos / 2 : (pos - 1) / 2;
       SA[n1 + pos] = name - 1;
   for (i = n - 1, j = n - 1; i >= n1; i--)
       if (SA[i] >= 0)
```

```
SA[j--] = SA[i];
    // stage 2: solve the reduced problem
    // recurse if names are not yet unique
    int *SA1 = SA, *s1 = SA + n - n1;
    if (name < n1)
        SA_IS((unsigned char *)s1, SA1, n1, name - 1, sizeof(
        // generate the suffix array of s1 directly
        for (i = 0; i < n1; i++)</pre>
            SA1[s1[i]] = i;
    // stage 3: induce the result for the original problem
    bkt = (int *)malloc(sizeof(int) * (K + 1)); // bucket
    // put all left-most S characters into their buckets
    getBuckets(s, bkt, n, K, cs, true); // find ends of
         buckets
    for (i = 1, j = 0; i < n; i++)
        if (isLMS(i))
           s1[j++] = i; // get p1
    for (i = 0; i < n1; i++)</pre>
        SA1[i] = s1[SA1[i]]; // get index in s
    for (i = n1; i < n; i++)
        SA[i] = -1; // init SA[n1..n-1]
    for (i = n1 - 1; i >= 0; i--) {
       j = SA[i];
        SA[i] = -1;
        SA[--bkt[chr(j)]] = j;
    induceSAl(t, SA, s, bkt, n, K, cs, false);
    induceSAs(t, SA, s, bkt, n, K, cs, true);
    free (bkt);
    free(t);
vector<int> suffix_array(const string &s) {
    int n = s.size();
    if (n == 0)
        return {};
    if (n == 1)
        return {0};
    vector<int> sa(n + 1);
    SA_IS((unsigned char *)s.c_str(), sa.data(), n + 1, 256, 1)
    sa.erase(sa.begin());
    return sa;
// usage example
int main() {
    string s = "abcab";
    vector<int> sa = suffix_array(s);
    for (int v : sa)
       cout << v << " ";
    cout << endl;
    mt19937 rng(1);
    s.clear();
    for (int i = 0; i < 1000'000; ++i) {</pre>
        char c = uniform_int_distribution<int>('a', 'd')(rng);
        s.push_back(c);
    auto t1 = chrono::high_resolution_clock::now();
    sa = suffix_array(s);
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    cout << duration.count() << " ms" << endl;</pre>
```

```
Suffix-array.cpp
Description: Build suffix array
Time: \mathcal{O}(nlogn)
                                                     1dc346, 93 lines
vector<int> suffix_array(const string &S) {
    int n = S.length();
    // Stable sort of characters.
    // Same characters are sorted by their position in
         descending order.
    // E.q. last character which represents suffix of length 1
         should be ordered first among same characters.
    vector<int> sa:
    for (int i = n - 1; i >= 0; --i) {
        sa.push back(i);
    stable sort(sa.begin(), sa.end(), [&](int a, int b) {
         return S[a] < S[b]; });</pre>
    vector<int> classes(n);
    for (int i = 0; i < n; ++i) {</pre>
        classes[i] = S[i];
    // sa[i] - suffix on i'th position after sorting by first
         len characters
    // classes[i] - equivalence class of the i'th suffix after
         sorting by first len characters
    for (int len = 1; len < n; len *= 2) {</pre>
        // Calculate classes for suffixes of length len * 2
        vector<int> c = classes;
        for (int i = 0; i < n; i++) {</pre>
            // Condition sa[i-1] + len < n emulates 0-symbol
                 at the end of the string.
            // A separate class is created for each suffix
                 followed by emulated 0-symbol.
            classes[sa[i]] =
                i > 0 \&\& c[sa[i-1]] == c[sa[i]] \&\& sa[i-1]
                     + len < n \&\& c[sa[i - 1] + len / 2] == c[
                     sa[i] + len / 2]
                    ? classes[sa[i - 1]]
                    : i;
        // Suffixes are already sorted by first len characters
        // Now sort suffixes by first len * 2 characters
        vector<int> cnt(n);
        iota(cnt.begin(), cnt.end(), 0);
        vector<int> s = sa;
        for (int i = 0; i < n; i++) {</pre>
            // s[i] - order of suffixes sorted by first len
                  characters
            // (s[i] - len) - order of suffixes sorted only by
                 second len characters
            int s1 = s[i] - len;
            // sort only suffixes of length > len, others are
                 already sorted
            if (s1 >= 0)
                sa[cnt[classes[s1]]++] = s1;
    return sa;
// https://en.wikipedia.org/wiki/LCP_array
vector<int> lcp_array(const string &s) {
    int n = s.size();
    vector<int> sa = suffix_array(s);
```

vector<int> rank(n);

for (int i = 0; i < n; i++)

24

for (; p != -1 && st[p].next[c] == -1; p = st[p].

suffLink) {

st[p].next[c] = cur;

```
rank[sa[i]] = i;
    vector<int> lcp(n - 1);
    for (int i = 0, h = 0; i < n; i++) {
        if (rank[i] < n - 1) {
            for (int j = sa[rank[i] + 1]; s[i + h] == s[j + h];
            lcp[rank[i]] = h;
            if (h > 0)
                --h;
    return lcp;
// usage example
int main() {
    string s = "abcab";
    vector<int> sa = suffix_array(s);
    for (int v : sa)
       cout << v << " ";
    cout << endl;
    vector<int> lcp = lcp_array(s);
    for (int v : lcp)
        cout << v << " ";
    cout << endl;
    mt19937 rng(1);
    for (int i = 0; i < 1000'000; ++i) {</pre>
        char c = uniform_int_distribution<int>('a', 'd')(rng);
    auto t1 = chrono::high_resolution_clock::now();
    sa = suffix_array(s);
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    cout << duration.count() << " ms" << endl;</pre>
Suffix-automaton.cpp
Description: desc
// https://cp-algorithms.com/string/suffix-automaton.html
struct state {
    int length:
    int suffLink;
    vector<int> inv_sufflinks;
    int firstPos = -1;
    vector<int> next = vector<int>(128, -1);
constexpr int MAXLEN = 100'000;
state st[MAXLEN * 2];
int sz;
void build_suffix_automaton(const string &s) {
    st[0].suffLink = -1;
    int last = 0;
    sz = 1;
    for (int i = 0; i < s.length(); i++) {
       char c = s[i];
       int cur = sz++;
       st[cur].length = i + 1;
```

st[cur].firstPos = i;

int p = last;

```
if (p == -1) {
            st[cur].suffLink = 0;
            int q = st[p].next[c];
            if (st[p].length + 1 == st[q].length) {
                st[cur].suffLink = q;
                int clone = sz++;
                st[clone].length = st[p].length + 1;
                st[clone].next = st[q].next;
                st[clone].suffLink = st[q].suffLink;
                for (; p != -1 && st[p].next[c] == q; p = st[p
                    ].suffLink) {
                    st[p].next[c] = clone;
                st[q].suffLink = clone;
                st[cur].suffLink = clone;
        last = cur;
    for (int i = 1; i < sz; i++) {
        st[st[i].suffLink].inv_sufflinks.push_back(i);
// usage example
int main() {
   build_suffix_automaton("ababcc");
SuffixTreeBreslauerItaliano.cpp
Description: desc
                                                    ce7<u>743</u>, 49 lines
// See https://codeforces.com/blog/entry/17956?#comment-228040
    for description
const int MAXLEN = 1e6;
string s:
int pos[MAXLEN], len[MAXLEN], par[MAXLEN];
unordered_map<char, int> to[MAXLEN], Link[MAXLEN];
void attach(int child, int parent, char c, int child_len) {
    to[parent][c] = child;
    len[child] = child_len;
    par[child] = parent;
void extend(char c) {
    int v;
    int i = s.size();
    int old = sz - 1;
    for (v = old; !Link[v].count(c); v = par[v])
       i -= len[v];
    int w = Link[v][c];
    if (to[w].count(s[i])) {
       int u = to[w][c];
        for (pos[sz] = pos[u] - len[u]; s[pos[sz]] == s[i]; pos
            [sz] += len[v], i += len[v])
            v = to[v][s[i]];
        attach(sz, w, s[pos[u] - len[u]], len[u] - (pos[u] -
             pos[sz]));
        attach(u, sz, s[pos[sz]], pos[u] - pos[sz]);
        w = Link[v][c] = sz++;
```

```
Link[old][c] = sz;
    attach(sz, w, s[i], s.size() - i);
    pos[sz++] = s.size();
void init tree() {
    len[1] = 1;
    pos[1] = 0;
    par[1] = 0;
    for (int c = 0; c <= 255; c++)
        to[0][c] = Link[0][c] = 1;
int main() {
    init_tree();
    s = "aabababbbbadcasdf#";
    for (int i = s.size() - 1; i >= 0; i--)
        extend(s[i]);
SuffixTreeUkkonen.cpp
Description: desc
// See http://codeforces.ru/blog/entry/16780 for description
const int inf = 1e9;
const int maxn = 1e5;
char s[maxn];
unordered_map<char, int> to[maxn];
int len[maxn], f_pos[maxn], Link[maxn];
int node, pos:
int sz = 1, n = 0;
int make_node(int _pos, int _len) {
    f pos[sz] = pos;
    len[sz] = _len;
    return sz++;
void go edge() {
    while (pos > len[to[node][s[n - pos]]]) {
        node = to[node][s[n - pos]];
        pos -= len[node];
void add_letter(char c) {
    s[n++] = c;
    pos++;
    int last = 0;
    while (pos > 0) {
        go_edge();
        int edge = s[n - pos];
        int &v = to[node][edge];
        int t = s[f_pos[v] + pos - 1];
        if (v == 0) {
            v = make_node(n - pos, inf);
            Link[last] = node;
            last = 0;
        } else if (t == c) {
            Link[last] = node;
            return:
        } else {
            int u = make_node(f_pos[v], pos - 1);
            to[u][c] = make_node(n - 1, inf);
            to[u][t] = v;
            f_pos[v] += pos - 1;
```

len[v] -= pos - 1;

Link[last] = u;

v = 11:

823ead, 12 lines

```
last = u;
        if (node == 0)
            pos--;
        else
            node = Link[node];
int main() {
    len[0] = inf;
    string s = "abracadabra";
    int ans = 0;
    for (int i = 0; i < s.size(); i++)</pre>
        add_letter(s[i]);
    for (int i = 1; i < sz; i++)</pre>
       ans += min((int)s.size() - f_pos[i], len[i]);
    cout << ans << "\n";
TandemRepeats.cpp
Description: desc
                                                     ae84b2, 75 lines
vector<int> z_function(const string &s) {
    int n = (int)s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - 1]);
        while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            1 = i, r = i + z[i] - 1;
    return z:
void output_tandem(const string &s, int shift, bool left, int
    cntr, int 1, int 11, int 12) {
    int pos:
   if (left)
        pos = cntr - 11;
   else
        pos = cntr - 11 - 12 - 11 + 1;
    cout << "[" << shift + pos << ".." << shift + pos + 2 * 1 -
         1 << "] = " << s.substr(pos, 2 * 1) << endl;
void output_tandems(const string &s, int shift, bool left, int
    cntr, int 1, int k1, int k2) {
    for (int 11 = 1; 11 <= 1; ++11) {</pre>
        if (left && 11 == 1)
            break;
        if (11 <= k1 && 1 - 11 <= k2)
            output_tandem(s, shift, left, cntr, 1, 11, 1 - 11);
inline int get_z(const vector<int> &z, int i) {
    return 0 <= i && i < (int) z.size() ? z[i] : 0;
void find_tandems(string s, int shift = 0) {
    int n = (int)s.length();
    if (n == 1)
        return;
```

```
int nu = n / 2;
    int nv = n - nu;
    string u = s.substr(0, nu);
    string v = s.substr(nu);
    string ru = string(u.rbegin(), u.rend());
    string rv = string(v.rbegin(), v.rend());
    find_tandems(u, shift);
    find_tandems(v, shift + nu);
    vector<int> z1 = z_function(ru);
    vector<int> z2 = z_function(v + '#' + u);
    vector<int> z3 = z_function(ru + '#' + rv);
    vector<int> z4 = z_function(v);
    for (int cntr = 0; cntr < n; ++cntr) {</pre>
        int 1, k1, k2;
        if (cntr < nu) {
            1 = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            1 = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        if (k1 + k2 >= 1)
            output_tandems(s, shift, cntr < nu, cntr, 1, k1, k2
                );
// usage example
int main() {
    find_tandems("abcabczz");
Z-function.cpp
Description: desc
                                                     e89fcd, 23 lines
// z[i] = lcp(s[0..], s[i..])
vector<int> z_function(const string &s) {
    vector<int> z(s.length());
    for (int i = 1, l = 0, r = 0; i < z.size(); ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - 1]);
        while (i + z[i] < z.size() \&\& s[z[i]] == s[i + z[i]])
            ++z[i];
        if (r < i + z[i] - 1) {
            1 = i;
            r = i + z[i] - 1;
    return z;
// usage example
int main() {
    vector<int> z = z_function("abcababc");
    for (int x : z)
        cout << x << " ";
    cout << endl;
Structures (9)
```

```
// usage example
int main() {
    binary_heap_indexed<int> h(5);
    h.add(0, 50);
    h.add(1, 30);
    h.add(2, 40);
    h.change_value(0, 20);
    h.remove(1);
    while (h.size) {
        cout << h.remove_min() << endl;</pre>
BinaryHeapIndexed.h
Description: desc
                                                     568d07, 76 lines
template <class T>
struct binary heap indexed {
    vector<T> heap;
    vector<int> pos2Id;
    vector<int> id2Pos;
    int size;
    binary_heap_indexed() : size(0) {}
    binary_heap_indexed(int n) : heap(n), pos2Id(n), id2Pos(n),
          size(0) {}
    void add(int id, T value) {
        heap[size] = value;
        pos2Id[size] = id;
        id2Pos[id] = size;
        up(size++);
    int remove min() {
        int removedId = pos2Id[0];
        heap[0] = heap[--size];
        pos2Id[0] = pos2Id[size];
        id2Pos[pos2Id[0]] = 0;
        down(0);
        return removedId;
    void remove(int id) {
        int pos = id2Pos[id];
        pos2Id[pos] = pos2Id[--size];
        id2Pos[pos2Id[pos]] = pos;
        change_value(pos2Id[pos], heap[size]);
    void change_value(int id, T value) {
        int pos = id2Pos[id];
        if (heap[pos] < value) {</pre>
            heap[pos] = value;
            down (pos);
        } else if (heap[pos] > value) {
            heap[pos] = value;
            up (pos);
    void up(int pos) {
        while (pos > 0) {
            int parent = (pos - 1) / 2;
            if (heap[pos] >= heap[parent])
```

BinaryHeapIndexed.cpp

Description: desc

"binary_heap_indexed.h"

```
break;
                                                                        int centroid = find tree centroid(tree, size, deleted, u,
            exchange(pos, parent);
                                                                            -1, total);
            pos = parent;
                                                                        deleted[centroid] = true;
                                                                        // process centroid vertex here
    void down(int pos) {
                                                                        cout << centroid << endl;</pre>
        while (true) {
            int child = 2 * pos + 1;
                                                                        for (int v : tree[centroid]) {
            if (child >= size)
                                                                           if (deleted[v])
                break;
                                                                                continue;
            if (child + 1 < size && heap[child + 1] < heap[</pre>
                 child])
                ++child;
            if (heap[pos] <= heap[child])</pre>
                break;
                                                                    // usage example
            exchange (pos, child);
                                                                   int main() {
            pos = child;
                                                                        int n = tree.size();
                                                                        vector<int> size(n);
    void exchange(int i, int j) {
                                                                       vector<bool> deleted(n);
        swap(heap[i], heap[j]);
        swap(pos2Id[i], pos2Id[j]);
        id2Pos[pos2Id[i]] = i;
        id2Pos[pos2Id[j]] = j;
                                                                    DisjointSets.cpp
                                                                   Description: desc
};
CentroidDecomposition.cpp
                                                                    vector<int> create_sets(int n) {
Description: desc
                                                      efbefe, 54 lines
                                                                        vector<int> res(n);
                                                                        iota(res.begin(), res.end(), 0);
void calc sizes(const vector<vector<int>> &tree, vector<int> &
                                                                        return res;
     size, vector<bool> &deleted, int u, int p) {
    size[u] = 1;
    for (int v : tree[u]) {
                                                                    int root(vector<int> &p, int x) {
        if (v == p || deleted[v])
        calc_sizes(tree, size, deleted, v, u);
       size[u] += size[v];
                                                                       a = root(p, a);
                                                                       b = root(p, b);
                                                                       p[b] = a;
int find_tree_centroid(const vector<vector<int>> &tree, vector<</pre>
     int> &size, vector<bool> &deleted, int u, int p,
                       int vertices) {
                                                                    // usage example
    for (int v : tree[u]) {
                                                                   int main() {
       if (v == p || deleted[v])
                                                                       vector<int> p = create_sets(3);
            continue;
                                                                       unite(p, 0, 2);
       if (size[v] > vertices / 2) {
            return find_tree_centroid(tree, size, deleted, v, u
                 , vertices);
    return u;
                                                                    DisjointSetsRanked.cpp
                                                                    Description: desc
// void dfs(const vector<vector<int>>> &tree, vector<bool> &
     deleted, int u, int p) {
       for (int v: tree[u]) {
                                                                         . html
           if (v = p \mid | deleted[v]) continue;
           dfs(tree, deleted, v, u);
                                                                        vector<int> res(n);
                                                                        iota(res.begin(), res.end(), 0);
```

void decompose(const vector<vector<int>> &tree, vector<int> &

size, vector<bool> &deleted, int u, int total) {

calc_sizes(tree, size, deleted, u, -1);

```
// dfs(tree, deleted, centroid, -1);
        decompose(tree, size, deleted, v, size[v]);
   vector<vector<int>> tree = {{3}, {3}, {3}, {0, 1, 2}};
    decompose(tree, size, deleted, 0, n);
                                                    d68061, 26 lines
// https://cp-algorithms.com/data_structures/disjoint_set_union
    return x == p[x] ? x : (p[x] = root(p, p[x]));
void unite(vector<int> &p, int a, int b) {
   cout << (0 == root(p, 0)) << endl;
   cout << (1 == root(p, 1)) << endl;
   cout << (0 == root(p, 2)) << endl;
                                                    b68d21, 33 lines
// https://cp-algorithms.com/data_structures/disjoint_set_union
tuple<vector<int>, vector<int>> create_sets(int n) {
    vector<int> rank(n);
    return tuple{res, rank};
```

```
int root(vector<int> &p, int x) {
    return x == p[x] ? x : (p[x] = root(p, p[x]));
void unite(vector<int> &p, vector<int> &rank, int a, int b) {
    a = root(p, a);
    b = root(p, b);
    if (a == b)
        return;
    if (rank[a] < rank[b])</pre>
        swap(a, b);
    if (rank[a] == rank[b])
        ++rank[a];
    p[b] = a;
// usage example
int main() {
    auto [p, rank] = create_sets(3);
    unite(p, rank, 0, 2);
    cout << (0 == root(p, 0)) << endl;
    cout << (1 == root(p, 1)) << endl;
    cout << (0 == root(p, 2)) << endl;
FenwickTree.cpp
Description: desc
                                                       ff305c 53 lines
// https://cp-algorithms.com/data_structures/fenwick.html
template <class T>
struct fenwick {
    vector<T> t;
    fenwick(int n) : t(n) {}
    void add(int i, T value) {
        for (; i < t.size(); i |= i + 1)</pre>
            t[i] += value;
    // sum[0..i]
    T sum(int i)
        T res{};
        for (; i >= 0; i = (i & (i + 1)) - 1)
            res += t[i];
        return res;
    // returns min(p | sum[0..p] >= sum)
    // requires non-negative tree values
    int lower_bound(T sum) {
        int highest_one_bit = 1;
        while (highest_one_bit << 1 <= t.size())</pre>
            highest_one_bit <<= 1;
        int pos = 0;
        for (size_t blockSize = highest_one_bit; blockSize !=
             0; blockSize >>= 1) {
            int p = pos + blockSize - 1;
            if (p < t.size() && t[p] < sum) {</pre>
                sum -= t[p];
                pos += blockSize;
        return pos;
};
```

// usage example

```
int main() {
    fenwick<int> f(3);
    f.add(0, 4);
    f.add(1, 5);
    f.add(2, 5);
   f.add(2, 5);
    cout << boolalpha;
    cout << (4 == f.sum(0)) << endl;
    cout << (19 == f.sum(2)) << endl;
    cout << (2 == f.lower_bound(19)) << endl;</pre>
   cout << (3 == f.lower_bound(20)) << endl;</pre>
FenwickTreeInterval.cpp
Description: desc
                                                     651be8, 44 lines
// https://cp-algorithms.com/data_structures/fenwick.html
template <class T>
class fenwick_interval {
   vector<T> t1, t2:
   void add(vector<T> &t, int i, T value) {
        for (; i < t.size(); i |= i + 1)</pre>
           t[i] += value;
   T sum(vector<T> &t, int i) {
       T res = 0:
        for (; i \ge 0; i = (i \& (i + 1)) - 1)
           res += t[i];
        return res;
   fenwick_interval(int n) : t1(n), t2(n) {}
   void add(int a, int b, T value) {
       add(t1, a, value);
       add(t1, b, -value);
       add(t2, a, -value * (a - 1));
        add(t2, b, value * b);
    // sum[0..i]
    T sum(int i) { return sum(t1, i) * i + sum(t2, i); }
// usage example
int main() {
    fenwick interval<int> f(3);
   f.add(0, 0, 4);
    f.add(1, 1, 5);
   f.add(2, 2, 5);
   f.add(2, 2, 5);
   cout << boolalpha;
   cout << (4 == f.sum(0)) << endl;
    cout << (19 == f.sum(2)) << endl;
FenwickTreeOnMap.cpp
Description: desc
                                                     54c78e, 28 lines
const int n = 2000'000'000;
void add(unordered_map<int, int> &t, int i, int value) {
    for (; i < n; i | = i + 1)
```

```
t[i] += value;
// sum[0,i]
int sum(unordered_map<int, int> &t, int i) {
    int res = 0;
    for (; i >= 0; i = (i & (i + 1)) - 1)
        if (t.count(i))
            res += t[i];
    return res;
// Usage example
int main() {
    unordered_map<int, int> t;
    add(t, 0, 4);
    add(t, 1, 5);
    add(t, 2, 5);
    add(t, 2, 5);
    cout << (4 == sum(t, 0)) << endl;
    cout << (19 == sum(t, 2)) << endl;
    cout << (19 == sum(t, 1000'000'000)) << endl;
HeavyLightDecomposition.cpp
Description: desc
"segment_tree.h"
                                                      677f5c, 88 lines
class HeavyLight {
    vector<vector<int>> tree;
    bool valuesOnVertices; // true - values on vertices, false
         - values on edges
    segtree segment tree;
    vector<int> parent;
    vector<int> depth;
    vector<int> pathRoot;
    vector<int> in;
    HeavyLight(const vector<vector<int>> &t, bool
        valuesOnVertices)
        : tree(t).
          valuesOnVertices(valuesOnVertices).
          segment_tree(t.size()),
          parent(t.size()),
          depth(t.size()),
          pathRoot(t.size()),
          in(t.size()) {
        int time = 0;
       parent[0] = -1;
        function<int(int)> dfs1 = [&](int u) {
            int size = 1;
            int maxSubtree = 0;
            for (int &v : tree[u]) {
                if (v == parent[u])
                    continue;
                parent[v] = u;
                depth[v] = depth[u] + 1;
                int subtree = dfs1(v);
                if (maxSubtree < subtree) {</pre>
                    maxSubtree = subtree;
                    swap(v, tree[u][0]);
                size += subtree;
            return size;
```

```
function<void(int)> dfs2 = [&](int u) {
            in[u] = time++;
            for (int v : t[u]) {
                if (v == parent[u])
                    continue;
                pathRoot[v] = v == t[u][0] ? pathRoot[u] : v;
                dfs2(v);
        };
        dfs1(0);
        dfs2(0);
    segtree::node get(int u, int v) {
        segtree::node res;
        process_path(u, v, [this, &res](int a, int b) { res =
             segtree::unite(res, segment_tree.get(a, b)); });
        return res;
    void modify(int u, int v, long long delta) {
        process_path(u, v, [this, delta](int a, int b) {
             segment_tree.modify(a, b, delta); });
    void process_path(int u, int v, const function<void(int x,</pre>
         int v) > &op) {
        for (; pathRoot[u] != pathRoot[v]; v = parent[pathRoot[
            if (depth[pathRoot[u]] > depth[pathRoot[v]])
                swap(u, v);
            op(in[pathRoot[v]], in[v]);
        if (u != v || valuesOnVertices)
            op(min(in[u], in[v]) + (valuesOnVertices ? 0 : 1),
                 max(in[u], in[v]));
};
// usage example
int main() {
    vector<vector<int>> tree{{1, 2}, {0, 3, 4}, {0}, {1}, {1}};
    HeavyLight hl_v(tree, true);
    hl_v.modify(3, 2, 1);
    hl v.modify(1, 0, -1);
    cout << hl_v.get(4, 2).sum << endl;
    HeavyLight hl_e(tree, false);
    hl_e.modify(3, 2, 1);
    hl_e.modify(1, 0, -1);
    cout << hl_e.get(4, 2).sum << endl;</pre>
KdTree.cpp
Description: desc
                                                     853a9d, 91 lines
using pii = pair<int, int>;
const int maxn = 100'000;
int tx[maxn];
int ty[maxn];
bool divX[maxn];
void build_tree(int left, int right, pii *points) {
    if (left >= right)
    int mid = (left + right) >> 1;
```

```
// sort(points + left, points + right + 1, divX ? cmpX :
        cmpY);
    int minx = numeric_limits<int>::max();
    int maxx = numeric_limits<int>::min();
    int miny = numeric_limits<int>::max();
    int maxy = numeric_limits<int>::min();
    for (int i = left; i < right; i++) {
       minx = min(minx, points[i].first);
        maxx = max(maxx, points[i].first);
       miny = min(miny, points[i].second);
       maxy = max(maxy, points[i].second);
    divX[mid] = (maxx - minx) >= (maxy - miny);
   bool (*cmpX) (pii, pii) = [] (pii a, pii b) { return a.first
        < b.first; };
    bool (*cmpY) (pii, pii) = [] (pii a, pii b) { return a.second
         < b.second; };
    nth_element(points + left, points + mid, points + right,
        divX[mid] ? cmpX : cmpY);
    tx[mid] = points[mid].first;
    tv[mid] = points[mid].second;
    if (left + 1 == right)
        return:
    build_tree(left, mid, points);
   build_tree(mid + 1, right, points);
long long closest_dist;
int closest_node;
void find_nearest_neighbour(int left, int right, int x, int y)
    if (left >= right)
        return;
    int mid = (left + right) >> 1;
    int dx = x - tx[mid];
    int dy = y - ty[mid];
    long long d = dx * (long long) dx + dy * (long long) dy;
    if (closest dist > d && d) {
        closest_dist = d;
        closest node = mid;
    if (left + 1 == right)
        return;
    int delta = divX[mid] ? dx : dv;
    long long delta2 = delta * (long long)delta;
    int 11 = left;
    int r1 = mid;
    int 12 = mid + 1;
    int r2 = right;
    if (delta > 0)
       swap(11, 12), swap(r1, r2);
    find_nearest_neighbour(11, r1, x, y);
    if (delta2 < closest dist)
        find_nearest_neighbour(12, r2, x, y);
int find_nearest_neighbour(int n, int x, int y) {
    closest_dist = LLONG_MAX;
    find_nearest_neighbour(0, n, x, y);
    return closest_node;
// usage example
```

```
int main() {
   vector<pii> p;
   p.emplace back(0, 2);
   p.emplace_back(0, 3);
   p.emplace_back(-1, 0);
   p.resize(unique(p.begin(), p.end()) - p.begin());
   int n = p.size();
   build_tree(0, n - 1, &p[0]);
   int res = find_nearest_neighbour(n, 0, 0);
    cout << p[res].first << " " << p[res].second << endl;</pre>
    return 0;
LinkCutTree.cpp
Description: desc
                                                    583380, 191 lines
// LinkCut tree with path queries. Query complexity is O(\log(n))
    ) amortized.
// Based on Daniel Sleator's implementation http://www.
    codeforces.com/contest/117/submission/860934
struct Node {
    long long node value;
    long long sub_tree_sum;
    long long add;
   bool revert;
    int size;
   Node *left;
   Node *right;
   Node *parent;
   Node (long long value)
        : node value(value),
         sub_tree_sum(value),
         add(0),
         revert (false),
         size(1),
         left(nullptr),
         right (nullptr),
         parent(nullptr) {}
    // tests whether x is a root of a splay tree
   bool isRoot() { return parent == nullptr || (parent->left
         != this && parent->right != this); }
    void apply(long long v) {
       node value += v;
       sub_tree_sum += v * size;
       add += v;
   }
    void push() {
       if (revert) {
            revert = false;
            Node *t = left;
           left = right;
            right = t;
            if (left != nullptr)
                left->revert = !left->revert;
            if (right != nullptr)
                right->revert = !right->revert;
       if (add != 0) {
            if (left != nullptr)
                left->apply(add);
```

```
if (right != nullptr)
                right->apply(add);
            add = 0;
        }
    void pull() {
        sub_tree_sum = node_value + get_sub_tree_sum(left) +
             get_sub_tree_sum(right);
        size = 1 + get_size(left) + get_size(right);
    static long long get_sub_tree_sum(Node *root) { return root
          == nullptr ? 0 : root->sub tree sum; }
    static int get_size(Node *root) { return root == nullptr ?
         0 : root->size; }
};
void connect(Node *ch, Node *p, int is_left_child) {
    if (ch != nullptr)
        ch->parent = p;
    if (is left child != 2) {
        if (is_left_child)
            p->left = ch;
        else
            p->right = ch;
// rotates edge (x, x.parent)
      x p.r
// x.l x.r
                     x.rp.r
void rotate(Node *x) {
    Node *p = x->parent;
    Node *g = p->parent;
    bool isRootP = p->isRoot();
    bool left_child_x = (x == p->left);
    // create 3 edges: (x.r(l),p), (p,x), (x,q)
    connect(left_child_x ? x->right : x->left, p, left_child_x)
    connect(p, x, !left_child_x);
    connect(x, q, isRootP ? 2 : (p == q->left ? 1 : 0));
    p->pull();
// brings x to the root, balancing tree
// zig-zig case
        p \quad g.r \quad rot(p)
      x p.r
                                              x, r = a
                     x.l x.r p.r g.r
// x.l x.r
                                                p.r.g.r
// zig-zag case
      p \quad g.r \quad rot(x) \qquad x \quad g.r \quad rot(x)
// p.l x
```

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```
p.l.x.l.x.r.q.r
                p.l.x.l
     x.l x.r
void splay(Node *x) {
    while (!x->isRoot()) {
        Node *p = x->parent;
        Node *g = p->parent;
        if (!p->isRoot())
            q->push();
        p->push();
        x->push();
        if (!p->isRoot())
            rotate((x == p->left) == (p == g->left) ? p /*zig-
                 zig*/: x /*zig-zag*/);
        rotate(x);
    x->push();
    x \rightarrow pull();
// makes node x the root of the virtual tree, and also x
     becomes the leftmost node in its splay tree
Node *expose(Node *x) {
    Node *last = nullptr;
    for (Node *y = x; y != nullptr; y = y->parent) {
        splay(y);
        y->left = last;
        last = y;
    splay(x);
    return last;
void make_root(Node *x) {
    expose(x);
    x->revert = !x->revert;
bool connected (Node *x, Node *y) {
    if (x == y)
        return true;
    expose(x);
    // now x.parent is null
    expose(y);
    return x->parent != nullptr;
void link(Node *x, Node *v) {
    assert(!connected(x, y));
    make root(x);
    x->parent = y;
void cut (Node *x, Node *y) {
    make_root(x);
    expose(y);
    // check that exposed path consists of a single edge (y,x)
    assert(y->right == x && x->left == nullptr);
    y->right->parent = nullptr;
    y->right = nullptr;
long long query(Node *from, Node *to) {
    make_root(from);
    expose (to);
    return Node::get_sub_tree_sum(to);
void modify(Node *from, Node *to, long long delta) {
    make root(from);
```

```
expose(to);
    to->apply(delta);
// usage example
int main() {
    Node *n1 = new Node(1);
    Node *n2 = new Node(2);
    link(n1, n2);
    long long q = query(n1, n2);
    cout << q << endl;
    cut(n1, n2);
MosWithUpdates.cpp
Description: desc
                                                    a50c59, 109 lines
struct query {
    int 1, r, index, t;
struct update_query {
    int pos, value, prev;
struct data structure {
    int m[26];
    int cnt;
    data structure(): m{}, cnt{0} {}
    void add(int x) {
        if (m[x]++==0)
            ++cnt;
    void remove(int x)
        if (--m[x] == 0)
            --cnt;
    int get() { return cnt; }
};
void apply(data_structure &ds, vector<int> &a, int 1, int r,
    int i, int x) { // Change s[i] to x
    if (1 <= i && i <= r) {
       ds.remove(a[i]);
       a[i] = x:
        ds.add(a[i]);
    } else {
        a[i] = x;
   }
void solve(istream &in, ostream &out) {
    string s;
    in >> s;
    int n = s.size();
    vector<int> a(n);
    for (int i = 0; i < n; ++i)
       a[i] = s[i] - 'a';
    vector<int> prev = a;
    vector<query> queries;
    vector<update_query> update_queries;
    int m;
    in >> m;
```

```
for (int i = 0; i < m; ++i) {</pre>
        int type;
        in >> type;
        if (type == 1) {
            int pos;
            char c;
            in >> pos >> c;
            --pos;
            c -= 'a';
            update_queries.emplace_back(update_query{pos, c,
                 prev[pos]});
            prev[pos] = c;
        } else {
            int 1, r;
            in >> 1 >> r;
            queries.emplace_back(query{1 - 1, r - 1, (int)
                 queries.size(), (int)update_queries.size() - 1
                 });
    int block = (int)pow(n, 2 / 3.);
    sort(queries.begin(), queries.end(), [block](auto &q1, auto
        if (q1.1 / block != q2.1 / block)
            return q1.1 < q2.1;
        if (q1.r / block != q2.r / block)
            return q1.r < q2.r;</pre>
        return q1.t < q2.t;</pre>
    });
    int 1 = 0;
    int r = -1;
    int t = -1:
    vector<int> ans(queries.size());
    data_structure ds;
    for (auto &g : gueries) {
        while (t < q.t)
            ++t, apply(ds, a, l, r, update_queries[t].pos,
                 update_queries[t].value);
        while (t > q.t)
            apply(ds, a, l, r, update gueries[t].pos,
                 update_queries[t].prev), --t;
        while (r < q.r)
            ds.add(a[++r]);
        while (1 > q.1)
            ds.add(a[--1]);
        while (r > q.r)
            ds.remove(a[r--]);
        while (1 < q.1)
            ds.remove(a[1++1);
        ans[q.index] = ds.get();
    for (size_t i = 0; i < queries.size(); i++)</pre>
        out << ans[i] << endl;
// usage example
int main() {}
OrderedSet.cpp
Description: desc
<ext/pb_ds/assoc_container.hpp>, <ext/pb_ds/tree_policy.hpp>
#if defined(__GNUC__) && !defined(__clang__)
```

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```
using namespace __gnu_pbds;
using ordered_set = tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update>;
using ordered_map = tree<int, int, less<>, rb_tree_tag,
    tree order statistics node update>;
struct tree2d {
    vector<ordered set> sets:
    tree2d(int n) : sets(n) {}
    void add(int x, int v) {
        for (; x < sets.size(); x |= x + 1)
            sets[x].insert(v);
    void remove(int x, int v) {
        while (x < sets.size()) {</pre>
            sets[x].erase(v);
            x += x & -x;
    int query(int x, int y1, int y2) {
        int res = 0;
        for (; x \ge 0; x = (x & (x + 1)) - 1)
            res += sets[x].order_of_key(y2 + 1) - sets[x].
                 order_of_key(y1);
        return res;
};
// usage example
int main() {
    ordered_set set;
    ordered_map map;
    map.insert(make_pair(1, 2));
    set.insert(1);
    set.insert(2);
    set.insert(4);
    set.insert(8);
    set.insert(16);
    cout << *set.find_by_order(1) << endl;</pre>
    cout << *set.find by order(2) << endl;</pre>
                                                            1/4
    cout << *set.find by order(4) << endl;</pre>
                                                            // 16
    cout << (set.end() == set.find_by_order(6)) << endl;</pre>
    cout << set.order of key(-5) << endl;
    cout << set.order of kev(1) << endl;
    cout << set.order of key(3) << endl;
    cout << set.order of kev(4) << endl;
    cout << set.order_of_key(400) << endl;</pre>
    cout << endl;
    tree2d t(10);
   t.add(1, 5);
   t.add(3, 6);
    cout << t.query(3, 5, 6) << endl;
#else
int main() {}
#endif
```

```
QueueMin.cpp
Description: desc
                                                      a42544, 44 lines
// https://cp-algorithms.com/data_structures/
     stack\_queue\_modification.html
template <class T>
struct queue min {
    stack<pair<T, T>> s1;
    stack<pair<T, T>> s2;
        return std::min(s1.empty() ? numeric_limits<T>::max() :
              s1.top().second,
                         s2.empty() ? numeric limits<T>::max() :
                               s2.top().second);
    void add last(T x) {
        T \min_{x \in \mathbb{R}} s1.empty() ? x : std::min(x, s1.top().
        s1.emplace(x, min value);
   T remove first() {
        if (s2.empty()) {
            while (!sl.empty()) {
                T x = s1.top().first;
                s1.pop();
                T \min_{value} = s2.empty() ? x : std::min(x, s2.
                     top().second);
                s2.emplace(x, min_value);
        T x = s2.top().first;
        s2.pop();
        return x;
};
// usage example
int main() {
    queue_min<int> q;
    g.add last(2);
    q.add_last(3);
    cout << boolalpha;
    cout << (2 == q.min()) << endl;
    q.remove_first();
    cout << (3 == q.min()) << endl;
    q.add_last(1);
    cout << (1 == q.min()) << endl;
SegmentTree.cpp
Description: desc
"segment_tree.h"
                                                      3cfd60, 11 lines
// usage example
int main() {
    segtree t(10);
    t.modify(2, 3, 1);
    t.modify(3, 4, 2);
    cout << t.get(2, 3).mx << endl;
    vector<long long> a{1, 2, 10, 20};
    segtree tt(a);
    cout << sum_lower_bound(tt, 0, tt.n - 1, 12) << endl;</pre>
```

```
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SegmentTree.h
Description: desc
                                                    12a024, 172 lines
struct segtree {
    struct node {
        // initial values for leaves
        long long mx = 0;
        long long sum = 0;
        long long add = 0;
        // set initial value for a leave
        void initialize(long long v) { mx = v; }
        // apply aggregate operation to the node
        void apply(int 1, int r, long long v) {
            mx += v:
            sum += (r - 1 + 1) * v;
            add += v:
    };
    // construct a node from its children
    static node unite (const node &a, const node &b) {
        node res:
        res.mx = max(a.mx, b.mx);
        res.sum = a.sum + b.sum;
        return res;
    void push(int x, int 1, int r) {
        int m = (1 + r) >> 1;
        int y = x + ((m - 1 + 1) << 1);
        if (tree[x].add != 0) {
            tree[x + 1].apply(l, m, tree[x].add);
            tree[y].apply(m + 1, r, tree[x].add);
            tree[x].add = 0;
    void pull(int x, int y) { tree[x] = unite(tree[x + 1], tree
         [y]); }
```

int n:

vector<node> tree;

if (1 == r) {

pull(x, y);

template <class T>

if (1 == r) {

pull(x, y);

return;

int m = (1 + r) >> 1;

build(x + 1, 1, m, v); build(y, m + 1, r, v);

return;

void build(int x, int 1, int r) {

int y = x + ((m - 1 + 1) << 1);

tree[x].initialize(v[1]);

int y = x + ((m - 1 + 1) << 1);

void build(int x, int 1, int r, const vector<T> &v) {

int m = (1 + r) >> 1;

build(x + 1, 1, m); build(y, m + 1, r); int find first (int 11, int rr, const function <bool (const

node &)> &f, int x, int 1, int r) {
if (11 <= 1 && r <= rr && !f(tree[x])) {

return -1:

```
node get(int x, int 1, int r, int 11, int rr) {
    if (11 <= 1 && r <= rr) {
        return tree[x];
    int m = (1 + r) >> 1;
    int y = x + ((m - 1 + 1) << 1);
   push(x, l, r);
   node res;
   if (rr <= m) {
        res = get(x + 1, 1, m, 11, rr);
   } else {
        if (11 > m) {
            res = get(y, m + 1, r, ll, rr);
       } else {
            res = unite(get(x + 1, 1, m, 11, rr), get(y, m)
                 + 1, r, ll, rr));
    pull(x, y);
    return res;
template <class T>
void modify(int x, int 1, int r, int 11, int rr, const T &v
    if (11 <= 1 && r <= rr) {
        tree[x].apply(1, r, v);
        return:
    int m = (1 + r) >> 1;
    int y = x + ((m - 1 + 1) << 1);
    push(x, l, r);
    if (11 <= m) {
        modify(x + 1, 1, m, 11, rr, v);
    if (rr > m) {
        modify(y, m + 1, r, ll, rr, v);
    pull(x, v);
segtree(int n) : n(n) {
    assert (n > 0);
    tree.resize(2 * n - 1);
   build(0, 0, n - 1);
template <class T>
segtree(const vector<T> &v) {
   n = v.size();
   assert (n > 0);
   tree.resize(2 * n - 1);
   build(0, 0, n - 1, v);
node get(int ll, int rr) {
   assert(0 <= 11 && 11 <= rr && rr <= n - 1);
    return get (0, 0, n - 1, 11, rr);
node get (int p) {
   assert (0 <= p && p <= n - 1);
    return get(0, 0, n - 1, p, p);
template <class T>
void modify(int 11, int rr, const T v) {
    assert(0 <= 11 && 11 <= rr && rr <= n - 1);
    modify(0, 0, n - 1, 11, rr, v);
```

```
if (1 == r) {
            return 1;
        push(x, l, r);
        int m = (1 + r) >> 1;
        int y = x + ((m - 1 + 1) << 1);
        int res = -1;
        if (11 <= m) {
            res = find_first(ll, rr, f, x + 1, l, m);
        if (rr > m \&\& res == -1) {
            res = find_first(ll, rr, f, y, m + 1, r);
        pull(x, y);
        return res;
    // calls all FALSE elements to the left of the sought
         position exactly once
    int find_first(int 11, int rr, const function<bool(const</pre>
         node &) > &f) {
        assert(0 <= 11 && 11 <= rr && rr <= n - 1);
        return find_first(ll, rr, f, 0, 0, n - 1);
};
// Returns min(p \mid p \le rr \&\& sum[ll..p] = sum). If no such p
     exists. returns -1
int sum_lower_bound(segtree &t, int 11, int rr, long long sum)
    long long sumSoFar = 0;
    return t.find first(ll, rr, [&] (const segtree::node &node)
        if (sumSoFar + node.sum >= sum)
            return true;
        sumSoFar += node.sum;
        return false;
    });
SegmentTreeWithoutRecursion.cpp
Description: desc
                                                     841166, 30 lines
int get(const vector<int> &t, int i) {
    return t[i + t.size() / 2];
void add(vector<int> &t, int i, int value) {
    i += t.size() / 2;
    t[i] += value;
    for (; i > 1; i >>= 1)
        t[i >> 1] = min(t[i], t[i ^ 1]);
int min(const vector<int> &t, int a, int b) {
    int res = numeric_limits<int>::max();
    for (a += t.size() / 2, b += t.size() / 2; a <= b; a = (a +</pre>
         1) >> 1, b = (b - 1) >> 1) {
        if ((a & 1) != 0)
            res = min(res, t[a]);
        if ((b & 1) == 0)
            res = min(res, t[b]);
```

```
return res:
// usage example
int main() {
    int n = 10;
    vector<int> t(n + n);
    add(t, 0, -1);
    add(t, 9, -2);
    cout << (-2 == min(t, 0, 9)) << endl;
Sparse-segment-tree.cpp
Description: desc
                                                     d3487d, 50 lines
struct Node {
    Node *1 = nullptr;
    Node *r = nullptr;
    int left;
    int right;
    int nsum;
    Node (int lo, int hi, int val) : left(lo), right(hi), nsum(
         val) {}
    void add(int pos, int val) {
        if (pos < left || pos > right) {
            return;
        nsum += val;
        if (right - left == 1) {
            return;
        int mid = (left + right) / 2;
        if (pos < mid) {</pre>
            if (1 == nullptr) {
                1 = new Node(left, mid, 0);
            1->add(pos, val);
        } else {
            if (r == nullptr) {
                r = new Node(mid + 1, right, 0);
            r->add(pos, val);
    int sum(int from, int to) {
        if (to < left || right < from) {</pre>
            return 0;
        if (from <= left && right <= to) {
            return nsum:
        return (1 ? 1->sum(from, to) : 0) + (r ? r->sum(from,
             to) : 0);
};
// usage example
int main() {
    Node t(0, 1000, 0);
    t.add(1, 1);
    t.add(100, 2);
    cout << t.sum(0, 10) << endl;
    cout << t.sum(0, 200) << endl;
```

```
Sparse-table.cpp
```

```
Description: desc
                                                     79d271, 37 lines
#ifdef MSC VER
int __builtin_clz(unsigned x) {
    int bit = 31;
    while (bit >= 0 && (x & (1 << bit)) == 0)
        --bit:
    return 31 - bit;
#endif
template <class T, class F = function<T(const T &, const T &)>>
struct SparseTable {
    vector<vector<T>> t;
    F func:
    SparseTable(const vector<T> &a, F f) : t(32 - __builtin_clz
         (a.size())), func(std::move(f)) {
       t[0] = a;
        for (size_t i = 1; i < t.size(); i++) {</pre>
            t[i].resize(a.size() - (1 << i) + 1);
            for (size t j = 0; j < t[i].size(); j++)</pre>
                t[i][j] = func(t[i-1][j], t[i-1][j+(1 <<
                     (i - 1)));
    T get(int from, int to) const {
        assert (0 <= from && from <= to && to <= (int)t[0].size
        int k = 31 - __builtin_clz(to - from + 1);
        return func(t[k][from], t[k][to - (1 << k) + 1]);
};
// usage example
int main() {
    vector<int> a{3, 2, 1};
    SparseTable<int> st(a, [](int i, int j) { return min(i, j);
    cout << st.get(0, 2) << endl;
    cout << st.get(0, 1) << endl;
Treap.cpp
Description: desc
"treap.h"
                                                     5b0a7a, 18 lines
// usage example
int main() {
    pTreap t = nullptr;
    int pos = 0;
    for (int a : {1, 2, 7, 4, 5})
       insert(t, pos++, a);
    int n = t->size;
    for (int i = 0; i < n; ++i)
        cout << query(t, i, i).node_value << endl;</pre>
    modify(t, 1, 3, 10);
    for (int i = 0; i < n; ++i)
        cout << query(t, i, i).node_value << endl;</pre>
    for (int i = 0; i < n; ++i)
       remove(t, 0);
```

cout << Treap::get_size(t) << endl;

clear(t);

```
Treap.h
Description: desc
                                                    5645e3, 137 lines
// https://cp-algorithms.com/data_structures/treap.html
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().
struct Treap {
    long long node value;
    long long mx;
    long long sum;
    long long add;
    long long key; // keys should be unique
    int size;
    long long prio;
    Treap *1, *r;
    Treap(long long key, long long value)
        : node_value(value), mx(value), sum(value), add(0), key
             (key), size(1), prio(rng()), l(nullptr), r(nullptr
            ) {}
    void apply(long long v) {
        node value += v;
        mx += v:
        sum += v * size;
        add += v:
    void push() {
        if (add != 0) {
            if (1 != nullptr)
                1->apply(add);
            if (r != nullptr)
                r->apply(add);
            add = 0;
    void pull() {
       mx = max(node_value, max(get_mx(l), get_mx(r)));
       sum = node_value + get_sum(1) + get_sum(r);
        size = 1 + get_size(1) + get_size(r);
    static long long get_mx(Treap *root) { return root ==
        nullptr ? numeric_limits<long long>::min() : root->mx;
    static long long get_sum(Treap *root) { return root ==
         nullptr ? 0 : root->sum; }
    static int get_size(Treap *root) { return root == nullptr ?
          0 : root->size; }
using pTreap = Treap *;
void split(pTreap t, long long min_right, pTreap &1, pTreap &r)
    if (!t) {
       1 = r = nullptr;
    } else {
        t->push();
        if (t->key >= min_right) {
            split(t->1, min_right, 1, t->1);
            r = t;
```

```
} else {
            split(t->r, min_right, t->r, r);
            1 = t;
        t->pull();
void merge(pTreap &t, pTreap &l, pTreap &r) {
    if (!l || !r) {
        t = 1 ? 1 : r;
    } else {
       1->push();
        r->push();
        if (1->prio > r->prio) {
            merge(1->r, 1->r, r);
            t = 1;
        } else {
            merge(r->1, 1, r->1);
            t = r;
        t->pull();
void insert(pTreap &t, long long key, long long value) {
    pTreap 1, r;
    split(t, key, l, r);
    auto node = new Treap(key, value);
    merge(t, 1, node);
    merge(t, t, r);
void remove(pTreap &t, long long key) {
    pTreap left1, right1, left2, right2;
    split(t, key, left1, right1);
    split(right1, key + 1, left2, right2);
    delete left2;
    merge(t, left1, right2);
void modify(pTreap &t, long long 11, long long rr, long long
    pTreap left1, right1, left2, right2;
    split(t, rr + 1, left1, right1);
    split(left1, 11, left2, right2);
    if (right2 != nullptr)
        right2->apply(delta);
    merge(t, left2, right2);
    merge(t, t, right1);
Treap query(pTreap &t, long long ll, long long rr) {
    pTreap left1, right1, left2, right2;
    split(t, rr + 1, left1, right1);
    split(left1, 11, left2, right2);
    Treap res(0, 0);
    if (right2)
        res = *right2;
    merge(t, left2, right2);
    merge(t, t, right1);
    return res:
void clear(pTreap &t) {
    if (!t)
        return:
    clear(t->1);
    clear(t->r);
```

delete t;

t = nullptr;

void print(pTreap t) {
 if (!t)

return;

TreapBst TreapBstSharedPtr

```
print(t->1);
    cout << t->node_value << endl;
    print(t->r);
TreapBst.cpp
Description: desc
// https://cp-algorithms.com/data_structures/treap.html
mt19937 64 rng(chrono::steady clock::now().time since epoch().
    count());
struct Treap {
    int key;
    long long prio;
    int size;
    Treap *1, *r;
    Treap(int key) : key(key), prio(rng()), size(1), l(nullptr)
         , r(nullptr) {}
    void update() { size = 1 + get_size(l) + get_size(r); }
    static int get_size(Treap *node) { return node ? node->size
         : 0; }
using pTreap = Treap *;
void split(pTreap t, int key, pTreap &1, pTreap &r) {
    if (!t)
       1 = r = nullptr;
    else if (key < t->key)
        split(t->1, key, 1, t->1), r = t, t->update();
    else
        split(t->r, key, t->r, r), l = t, t->update();
void merge(pTreap &t, pTreap 1, pTreap r) {
    if (!1 || !r)
        t = 1 ? 1 : r;
    else if (l->prio > r->prio)
        merge(1->r, 1->r, r), t = 1, t->update();
       merge(r->1, 1, r->1), t = r, t->update();
void insert(pTreap &t, pTreap it) {
    if (!t)
        t = it;
    else if (it->prio > t->prio)
        split(t, it->key, it->l, it->r), t = it, t->update();
    else
        insert (it->key < t->key ? t->l : t->r, it), t->update()
void erase (pTreap &t, int key) {
    if (t->key == key) {
        pTreap l = t -> 1;
        pTreap r = t->r;
```

```
delete t;
        merge(t, l, r);
    } else {
        erase(key < t->key ? t->l : t->r, key), t->update();
pTreap unite(pTreap 1, pTreap r) {
    if (!1 || !r)
        return 1 ? 1 : r;
    if (l->prio < r->prio)
        swap(1, r);
    pTreap lt, rt;
    split(r, 1->key, lt, rt);
    1->1 = unite(1->1, lt);
    1->r = unite(1->r, rt);
    return 1;
int kth(pTreap t, int k) {
    if (k < Treap::get_size(t->1))
        return kth(t->1, k);
    else if (k > Treap::get_size(t->1))
        return kth(t->r, k - Treap::get_size(t->1) - 1);
    return t->key;
void print(pTreap t) {
    if (!t)
        return;
    print(t->1);
    cout << t->key << endl;
    print(t->r);
void clear(pTreap &t) {
    if (!t)
        return:
    clear(t->1);
    clear(t->r);
    delete t;
    t = nullptr;
// usage example
int main() {
    pTreap t1 = nullptr;
    int a1[] = {1, 2};
    for (int x : a1)
        insert(t1, new Treap(x));
    pTreap t2 = nullptr;
    int a2[] = \{7, 4, 5\};
    for (int x : a2)
        insert(t2, new Treap(x));
    pTreap t = nullptr;
    merge(t, t1, t2);
    for (int i = 0; i < t->size; ++i) {
        cout << kth(t, i) << endl;</pre>
    clear(t);
```

```
Description: desc
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().
struct Treap;
using pTreap = shared_ptr<Treap>;
struct Treap {
    int kev;
    long long prio;
    int size;
    pTreap 1, r;
    Treap(int key) : key(key), prio(rng()), size(1), l(nullptr)
         , r(nullptr) {}
    void update() { size = 1 + get_size(l) + get_size(r); }
    static int get_size(pTreap node) { return node ? node->size
          : 0; }
          \sim item() {
              cout << "item " << key << " disposed" << endl;
};
void split(pTreap t, int key, pTreap &1, pTreap &r) {
    if (!t)
        1 = r = nullptr;
    else if (kev < t->kev)
        split(t->1, key, 1, t->1), r = t, t->update();
        split(t->r, key, t->r, r), l = t, t->update();
void merge(pTreap &t, pTreap &l, pTreap &r) {
    if (!1 || !r)
        t = 1 ? 1 : r;
    else if (l->prio > r->prio)
        merge(1->r, 1->r, r), t = 1, t->update();
        merge(r->1, 1, r->1), t = r, t->update();
void insert(pTreap &t, pTreap it) {
    if (!t)
        t = it;
    else if (it->prio > t->prio)
        split(t, it->key, it->l, it->r), t = it, t->update();
        insert(it->key < t->key ? t->l : t->r, it), t->update()
void erase(pTreap &t, int key) {
    if (t->key == key)
        merge(t, t->1, t->r);
        erase(key < t->key ? t->l : t->r, key), t->update();
int kth(pTreap &t, int k) {
    if (k < Treap::get_size(t->1))
        return kth(t->1, k);
    else if (k > Treap::get_size(t->1))
        return kth(t->r, k - Treap::get size(t->1) - 1);
```

TreapBstSharedPtr.cpp

```
return t->kev;
void print(pTreap &t) {
    if (!t)
        return:
    print(t->1);
    cout << t->key << endl;
    print(t->r);
// usage example
int main() {
    pTreap t1 = nullptr;
    int a1[] = {1, 2};
    for (int x : a1)
       insert(t1, make_shared<Treap>(x));
    pTreap t2 = nullptr;
    int a2[] = \{7, 4, 5\};
    for (int x : a2)
        insert(t2, make_shared<Treap>(x));
    pTreap t = nullptr;
    merge(t, t1, t2);
    for (int i = 0; i < t->size; ++i) {
       cout << kth(t, i) << endl;</pre>
TreapIndexed.cpp
Description: desc
                                                    0039b2, 201 lines
// https://cp-algorithms.com/data_structures/treap.html
mt19937 64 rng(chrono::steady clock::now().time since epoch().
    count());
struct Treap {
    long long node_value;
    long long mx;
    long long sum;
   long long add;
    int size;
    long long prio;
    Treap *1, *r;
    Treap(long long value)
        : node_value(value), mx(value), sum(value), add(0),
             size(1), prio(rng()), l(nullptr), r(nullptr) {}
    void apply(long long v) {
       node value += v;
       mx += v:
       sum += v * size;
       add += v;
    void push() {
        if (add != 0) {
            if (1 != nullptr)
                1->apply(add);
            if (r != nullptr)
                r->apply(add);
            add = 0;
```

```
void pull() {
       mx = max(node value, max(get mx(1), get mx(r)));
       sum = node_value + get_sum(1) + get_sum(r);
       size = 1 + get_size(l) + get_size(r);
    static long long get_mx(Treap *root) { return root ==
        nullptr ? numeric_limits<long long>::min() : root->mx;
    static long long get_sum(Treap *root) { return root ==
         nullptr ? 0 : root->sum; }
    static int get_size(Treap *root) { return root == nullptr ?
         0 : root->size; }
using pTreap = Treap *;
void split(pTreap t, int min_right, pTreap &1, pTreap &r) {
   if (!t) {
        1 = r = nullptr;
    } else {
       t->push();
        if (Treap::get_size(t->1) >= min_right) {
            split(t->1, min_right, 1, t->1);
            r = t;
        } else {
            split(t->r, min_right - Treap::get_size(t->1) - 1,
                t->r, r);
            1 = t;
       t->pull();
void merge(pTreap &t, pTreap &l, pTreap &r) {
    if (!l || !r) {
       t = 1 ? 1 : r;
    } else {
       1->push();
        r->push();
       if (1->prio > r->prio) {
            merge(1->r, 1->r, r);
            t = 1;
        } else {
            merge(r->1, 1, r->1);
            t = r;
        t->pull();
void insert(pTreap &t, int index, long long value) {
   pTreap l, r:
    split(t, index, l, r);
    auto node = new Treap(value);
   merge(t, 1, node);
   merge(t, t, r);
void remove(pTreap &t, int index) {
   pTreap left1, right1, left2, right2;
    split(t, index, left1, right1);
   split(right1, 1, left2, right2);
    delete left2:
    merge(t, left1, right2);
```

```
void modify(pTreap &t, int ll, int rr, long long delta) {
    pTreap left1, right1, left2, right2;
    split(t, rr + 1, left1, right1);
    split(left1, 11, left2, right2);
    if (right2 != nullptr)
        right2->apply(delta);
    merge(t, left2, right2);
    merge(t, t, right1);
Treap query(pTreap &t, int ll, int rr) {
    pTreap left1, right1, left2, right2;
    split(t, rr + 1, left1, right1);
    split(left1, l1, left2, right2);
    Treap res = *right2;
    merge(t, left2, right2);
    merge(t, t, right1);
    return res;
int find_first(pTreap t, int 11, int rr, const function<bool(</pre>
    const Treap &)> &f, int 1, int r) {
    if (ll <= 1 && r <= rr && !f(*t)) {
        return -1;
    if (1 == r) {
        return 1;
    t->push();
    int m = Treap::get_size(t->1);
    int res = -1;
   if (11 < m) {
        res = find_first(t->1, 11, rr, f, 1, 1 + m - 1);
    if (res == -1) {
        auto single = new Treap(0);
        single -> size = 1;
        single->apply(t->node value);
        res = find_first(single, ll, rr, f, l + m, l + m);
    if (rr > m \&\& res == -1) {
        res = find_first(t->r, ll, rr, f, l + m + 1, r);
    t->pull();
    return res;
// calls all FALSE elements to the left of the sought position
     exactly once
int find_first(pTreap t, int 11, int rr, const function<bool(</pre>
    const Treap &) > &f) {
    assert(0 <= 11 && 11 <= rr && rr <= Treap::get size(t) - 1)
    return find_first(t, 11, rr, f, 0, Treap::get_size(t) - 1);
// Returns min(p \mid p \le rr \&\& sum[ll..p] > sum). If no such p
     exists. returns -1
int sum lower bound(pTreap t, int ll, int rr, long long sum) {
    long long sumSoFar = 0;
    return find_first(t, ll, rr, [&](const Treap &node) {
        if (sumSoFar + node.sum >= sum)
            return true;
        sumSoFar += node.sum;
        return false;
    });
```

x += t.size() / 2;

```
clear(t->r);
    delete t;
    t = nullptr;
void print(pTreap t) {
    if (!t)
        return;
    print(t->1);
    cout << t->node_value << endl;
    print(t->r);
// usage example
int main() {
    pTreap t = nullptr;
    int pos = 0;
    for (int a : {1, 2, 7, 4, 5})
        insert(t, pos++, a);
    int n = t->size;
    for (int i = 0; i < n; ++i)
        cout << query(t, i, i).node_value << endl;</pre>
    modify(t, 1, 3, 10);
    for (int i = 0; i < n; ++i)
        cout << query(t, i, i).node_value << endl;</pre>
    for (int i = 0; i < n; ++i)
        remove(t, 0);
    cout << Treap::get_size(t) << endl;</pre>
    for (int v : {2, 1, 10, 20}) {
        insert(t, Treap::get_size(t), v);
    cout << (2 == sum_lower_bound(t, 0, Treap::get_size(t) - 1,</pre>
    clear(t);
Tree2d.cpp
Description: desc
                                                      948270, 38 lines
"treap.h"
struct tree_2d {
    vector<pTreap> t;
    tree_2d(int n) : t(2 * n) {}
   long long query(int x1, int x2, int y1, int y2) {
        long long res = 0;
        for (x1 += t.size() / 2, x2 += t.size() / 2; x1 <= x2;</pre>
             x1 = (x1 + 1) >> 1, x2 = (x2 - 1) >> 1) {
            if ((x1 & 1) != 0)
                res += ::query(t[x1], y1, y2).sum;
            if ((x2 \& 1) == 0)
                res += ::query(t[x2], y1, y2).sum;
        return res;
    void insert(int x, int y, int value) {
       x += t.size() / 2;
        for (; x > 0; x >>= 1)
            ::insert(t[x], y, value);
    void remove(int x, int y) {
```

void clear(pTreap &t) {

return;

clear(t->1);

if (!t)

```
for (; x > 0; x >>= 1)
            ::remove(t[x], y);
};
// usage example
int main() {
    tree_2d t(10);
    t.insert(1, 5, 3);
   t.insert(3, 3, 2);
   t.insert(2, 6, 1);
   t.remove(2, 6);
    cout << t.query(0, 9, 0, 9) << endl;
Wavelet Tree.cpp
Description: desc
                                                     538158, 82 lines
struct wavelet_tree {
    int lo, hi;
    wavelet_tree *1 = nullptr;
    wavelet tree *r = nullptr;
    vector<int> b:
    static wavelet tree create(int *from, int *to) {
        int min = *min_element(from, to);
        int max = *max element(from, to);
        return wavelet_tree(from, to, min, max);
    // nos are in range [x,y]
    // array indices are [from, to]
    wavelet tree(int *from, int *to, int x, int v) {
        10 = x;
        hi = y;
        if (lo == hi || from >= to)
            return;
        int mid = (lo + hi) / 2;
        auto f = [mid] (int x) { return x <= mid; };</pre>
        b.reserve(to - from + 1);
        b.push back(0);
        for (auto it = from; it != to; it++) {
            b.push back(b.back() + f(*it));
        // see how lambda function is used here
        auto pivot = stable_partition(from, to, f);
        1 = new wavelet_tree(from, pivot, lo, mid);
        r = new wavelet_tree(pivot, to, mid + 1, hi);
    // kth smallest element in [l, r]
    int kth(int 1, int r, int k) {
        if (1 > r)
            return 0;
        if (lo == hi)
            return lo;
        int inLeft = b[r] - b[1 - 1];
        int lb = b[1 - 1]; // amt of nos in first (l-1) nos
             that go in left
        int rb = b[r];
                            // amt of nos in first (r) nos that
              go in left
        return k <= inLeft ? this->l->kth(lb + 1, rb, k) : this
             ->r->kth(l - lb, r - rb, k - inLeft);
    // count of nos in [l, r] Less than or equal to k
    int LTE(int 1, int r, int k) {
        if (1 > r || k < 10)
```

```
return 0;
        if (hi <= k)
            return r - 1 + 1;
        int lb = b[1 - 1];
        int rb = b[r];
        return this->1->LTE(lb + 1, rb, k) + this->r->LTE(l -
             lb, r - rb, k);
    // count of nos in [l, r] equal to k
    int count(int 1, int r, int k) {
        if (1 > r || k < 10 || k > hi)
            return 0:
        if (lo == hi)
            return r - 1 + 1;
        int lb = b[1 - 1];
        int rb = b[r];
        int mid = (lo + hi) / 2;
        return k <= mid ? this->1->count(lb + 1, rb, k) : this
             ->r->count(1 - lb, r - rb, k);
    ~wavelet tree() {
        delete 1:
        delete r;
};
// usage example
int main() {
    int a[] = \{3, 1, 4, 2\};
    wavelet_tree wtree = wavelet_tree::create(a, a + 4);
    cout << wtree.kth(1, 3, 2) << endl;
    cout << wtree.LTE(1, 3, 2) << endl;
    cout << wtree.count(1, 4, 2) << endl;</pre>
Backtrack (10)
Description: desc
```

```
Mis.cpp
```

e20033 44 lines

```
#ifdef MSC VER
int __builtin_ctzll(unsigned long long x) {
    int bit = 0;
    while (bit < 64 && (x & (1LL << bit)) == 0)
        ++hit:
    return bit;
int __builtin_popcountll(unsigned long long x) {
    int bits = 0;
    for (; x; x &= x - 1, ++bits)
    return bits;
#endif
using 11 = long long;
int ctz(ll x) {
    return x == 0 ? 64 : __builtin_ctzll(x);
// maximum independent set in O(3^{(n/3)})
int mis(const vector<11> &g, 11 unused) {
    if (unused == 0)
        return 0;
```

```
int v = -1;
    for (int u = __builtin_ctzll(unused); u < q.size(); u +=</pre>
         ctz(unused >> (u + 1)) + 1)
        if (v == -1 || __builtin_popcountll(g[v] & unused) >
             __builtin_popcountll(g[u] & unused))
            v = 11:
    int res = 0;
   11 \text{ nv} = q[v] \& unused;
    for (int y = __builtin_ctzll(nv); y < g.size(); y += ctz(nv</pre>
          >> (y + 1) + 1
        res = max(res, 1 + mis(g, unused & \sim g[y]));
    return res;
// usage example
int main() {
    vector<ll> g{0b110, 0b101, 0b011};
    for (int i = 0; i < q.size(); ++i) {</pre>
        q[i] |= 1LL << i;
    cout << mis(q, (1LL << q.size()) - 1) << endl;
```

Combinatorics (11)

Binomial.cpp **Description:** desc

7b4420, 11 lines

```
int binomial(int n, int k) {
    int res = 1;
    for (int i = 0; i < k; i++) {
        res = res * (n - i) / (i + 1);
    }
    return res;
}
int main() {
    cout << binomial(5, 3) << endl;
}</pre>
```

EnumeratingCombinations.cpp Description: desc

a39dff, 22 lines

```
a39dff, 22 line
bool next_combination(vector<int> &comb, int n) {
   int k = comb.size();
   for (int i = k - 1; i >= 0; i--) {
      if (comb[i] < n - k + i) {
            ++comb[i];
            while (++i < k) {
                 comb[i] = comb[i - 1] + 1;
            }
        return true;
      }
}

int main() {
   vector<int> comb{0, 1, 2};
   do {
      for (int v : comb)
            cout << v + 1 << " ";
        cout << endl;
      } while (next_combination(comb, 5));
}</pre>
```

Geometry (12)

```
AngleAreaOrientationSortRotationPerpendicular.cpp
Description: desc
                                                    351302, 72 lines
using 11 = long long;
#define PI acos(-1)
// pay attention to case ax==0 && ay==0 or bx==0 && by == 0
double angle_between(ll ax, ll ay, ll bx, ll by) {
    double a = atan2(ax * by - ay * bx, ax * bx + ay * by);
    return a < 0 ? a + 2 * PI : a;
// pay attention to case ax==0 EE ay==0 or bx==0 EE by==0
double angle_between2(ll ax, ll ay, ll bx, ll by) {
    double a = atan2(by, bx) - atan2(ay, ax);
    return a < 0 ? a + 2 * PI : a;
11 double_signed_area(const vector<int> &x, const vector<int> &
    int n = x.size();
   11 area = 0;
    for (int i = 0, j = n - 1; i < n; j = i++) {
        area += (ll) (x[i] - x[j]) * (y[i] + y[j]); // area +=
             (long) \ x[i] * y[j] - (long) \ x[j] * y[i];
    return area;
// Returns -1 for clockwise, 0 for straight line, 1 for
    counterclockwise orientation
int orientation(ll ax, ll ay, ll bx, ll by, ll cx, ll cy) {
   bx -= ax:
   by -= ay;
    cx -= ax;
    cy -= ay;
    11 det = bx \star cy - by \star cx;
    return (det > 0) - (det < 0);
bool is middle(ll a, ll m, ll b) {
    return min(a, b) <= m && m <= max(a, b);
bool is_middle(11 ax, 11 ay, 11 mx, 11 my, 11 bx, 11 by) {
    return orientation(ax, ay, mx, my, bx, by) == 0 &&
         is_middle(ax, mx, bx) && is_middle(ay, my, by);
struct Point {
   int x, y;
    bool operator<(const Point &o) const {</pre>
        bool up1 = y > 0 \mid | (y == 0 && x >= 0);
        bool up2 = o.y > 0 || (o.y == 0 && o.x >= 0);
        if (up1 != up2)
            return up1;
        11 \text{ cmp} = (11) \text{ o.x } * \text{ y } - (11) \text{ o.y } * \text{ x};
        if (cmp != 0)
            return cmp < 0;
        // return atan2(y, x) < atan2(o.y, o.x);
};
pair<double, double> rotate_ccw(pair<double, double> p, double
     angle) {
```

```
return {p.first * cos(angle) - p.second * sin(angle), p.
         first * sin(angle) + p.second * cos(angle)};
struct Line {
    11 a, b, c;
Line perpendicular (Line line, ll x, ll y) {
    return {-line.b, line.a, line.b * x - line.a * v};
// usage example
int main() {}
ConvexHull.cpp
Description: desc
                                                     dc0b85, 34 lines
// Convex hull construction in O(n*log(n)): https://cp-
     algorithms.com/geometry/grahams—scan—convex—hull.html
struct point {
    int x, y;
bool isNotRightTurn(const point &a, const point &b, const point
    long long cross = (long long)(a.x - b.x) * (c.y - b.y) - (
         long long) (a.y - b.y) * (c.x - b.x);
    long long dot = (long long) (a.x - b.x) * (c.x - b.x) + (
         long long) (a.v - b.v) * (c.v - b.v);
    return cross < 0 || (cross == 0 && dot <= 0);
vector<point> convex_hull(vector<point> points) {
    sort(points.begin(), points.end(), [](auto a, auto b) {
         return a.x < b.x || (a.x == b.x && a.y < b.y); });
    int n = points.size();
    vector<point> hull;
    for (int i = 0; i < 2 * n - 1; i++) {
        int j = i < n ? i : 2 * n - 2 - i;
        while (hull.size() >= 2 && isNotRightTurn(hull.end()
             [-2], hull.end()[-1], points[j]))
            hull.pop_back();
        hull.push back(points[i]);
    hull.pop_back();
    return hull:
// usage example
int main() {
    vector<point> hull1 = convex_hull(\{\{0, 0\}, \{3, 0\}, \{0, 3\},
         {1, 1}});
    cout << (3 == hull1.size()) << endl;
    vector<point> hull2 = convex_hull({{0, 0}, {0, 0}});
    cout << (1 == hull2.size()) << endl;</pre>
Diameter.cpp
Description: desc
                                                     6fd903, 57 lines
typedef pair<double, double> point;
bool cw(const point &a, const point &b, const point &c) {
    return (b.first - a.first) * (c.second - a.second) - (b.
         second - a.second) * (c.first - a.first) < 0;
```

DynamicUpperEnvelope FindSegmentsIntersection

```
vector<point> convexHull(vector<point> p) {
   int n = p.size();
   if (n \le 1)
       return p;
   int k = 0:
   sort(p.begin(), p.end());
   vector<point> q(n * 2);
   for (int i = 0; i < n; q[k++] = p[i++])
       for (; k \ge 2 \&\& !cw(q[k - 2], q[k - 1], p[i]); --k)
   for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
       for (; k > t \&\& !cw(q[k - 2], q[k - 1], p[i]); --k)
   q.resize(k - 1 - (q[0] == q[1]));
   return q;
double area(const point &a, const point &b, const point &c) {
   return abs((b.first - a.first) * (c.second - a.second) - (b
        .second - a.second) * (c.first - a.first));
double dist(const point &a, const point &b) {
   return hypot (a.first - b.first, a.second - b.second);
double diameter(const vector<point> &p) {
   vector<point> h = convexHull(p);
   int m = h.size();
   if (m == 1)
       return 0;
   if (m == 2)
       return dist(h[0], h[1]);
   int k = 1:
   1], h[0], h[k]))
       ++k;
   double res = 0;
   for (int i = 0, j = k; i \le k \& \& j \le m; i++) {
       res = max(res, dist(h[i], h[j]));
       while (j < m \&\& area(h[i], h[(i + 1) % m], h[(j + 1) %
            m]) > area(h[i], h[(i + 1) % m], h[j])) {
            res = \max(\text{res}, \text{dist}(h[i], h[(i + 1) % m]));
            ++j;
   return res;
// usage example
int main() {
   double d = diameter(\{\{0, 0\}, \{3, 0\}, \{0, 3\}, \{1, 1\}\});
   cout << d << endl:
```

DynamicUpperEnvelope.cpp

Description: desc

using 11 = long long;

```
// Container where you can add lines of the form ax+b, and
// get_max maximum values at points x. For each line, also
    keeps a value p,
// which is the last (maximum) point for which the current line
      is dominant.
// (obviously, for the last line, p is infinity) Useful for
    dynamic programming.
```

```
struct Line {
   11 a. b:
    mutable 11 p;
    bool operator<(const Line &o) const { return a < o.a; }</pre>
    bool operator<(11 x) const \{ return p < x; \}
};
struct LineContainer : multiset<Line, less<>>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const 11 inf = numeric_limits<11>::max();
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b);
    bool isect(iterator x, iterator y) {
        if (y == end()) {
            x->p = inf;
            return false;
        if (x->a == y->a)
            x->p = x->b > y->b ? inf : -inf;
            x->p = div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    void add_line(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z))
           z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, erase(y));
        while ((y = x) != begin() \&\& (--x)->p >= y->p)
            isect(x, erase(y));
    11 get_max(ll x) {
       assert(!empty());
        auto line = *lower bound(x);
        return line.a * x + line.b;
};
// usage example
int main() {
    LineContainer lc;
    lc.add line(1, 3);
    lc.add_line(2, 1);
    cout << lc.get_max(1) << endl;
FindSegmentsIntersection.cpp
Description: desc
<utility>
                                                     d1b7c0, 98 lines
typedef pair<int, int> pii;
int cross(int ax, int ay, int bx, int by, int cx, int cy) {
    return (bx - ax) * (cy - ay) - (by - ay) * (cx - ax);
int cross(pii a, pii b, pii c) {
    return cross (a.first, a.second, b.first, b.second, c.first,
          c.second);
struct segment {
```

```
pii a, b;
    int id:
    segment(pii a, pii b, int id) : a(std::move(a)), b(std::
         move(b)), id(id) {}
    bool operator<(const segment &o) const {</pre>
        if (a.first < o.a.first) {</pre>
            int s = cross(a, b, o.a);
            return (s > 0 || (s == 0 \&\& a.second < o.a.second)
        } else if (a.first > o.a.first) {
            int s = cross(o.a, o.b, a);
            return (s < 0 || (s == 0 && a.second < o.a.second))
        return a.second < o.a.second;</pre>
};
struct event {
    pii p;
    int id;
    int type;
    event(pii p, int id, int type) : p(std::move(p)), id(id),
         type(type) {}
    bool operator<(const event &o) const {</pre>
        return p.first < o.p.first ||</pre>
                (p.first == o.p.first && (type > o.type || (type
                     == o.type && p.second < o.p.second)));
};
bool intersect (segment s1, segment s2) {
    int x1 = s1.a. first, y1 = s1.a. second, x2 = s1.b. first, y2
         = s1.b.second;
    int x3 = s2.a. first, y3 = s2.a. second, x4 = s2.b. first, y4
         = s2.b.second;
    if (\max(x1, x2) < \min(x3, x4) \mid | \max(x3, x4) < \min(x1, x2)
         | | \max(v1, v2) < \min(v3, v4) | |
        \max(y3, y4) < \min(y1, y2)) {
        return false;
    int z1 = (x3 - x1) * (y2 - y1) - (y3 - y1) * (x2 - x1);
    int z2 = (x4 - x1) * (y2 - y1) - (y4 - y1) * (x2 - x1);
    if ((z1 < 0 && z2 < 0) || (z1 > 0 && z2 > 0)) {
        return false;
    int z3 = (x1 - x3) * (y4 - y3) - (y1 - y3) * (x4 - x3);
    int z4 = (x2 - x3) * (y4 - y3) - (y2 - y3) * (x4 - x3);
    if ((z3 < 0 && z4 < 0) || (z3 > 0 && z4 > 0)) {
        return false:
    return true;
pii findIntersection(vector<segment> s) {
    int n = s.size();
    vector<event> e;
    for (int i = 0; i < n; ++i) {</pre>
        if (s[i].a > s[i].b)
            swap(s[i].a, s[i].b);
        e.emplace_back(s[i].a, i, 1);
        e.emplace_back(s[i].b, i, -1);
    sort(e.begin(), e.end());
```

set<segment> q;

for (int i = 0; i < n * 2; ++i) {

```
int id = e[i].id;
        if (e[i].type == 1) {
            auto it = q.lower_bound(s[id]);
            if (it != q.end() && intersect(*it, s[id]))
                return {it->id, s[id].id};
            if (it != q.begin() && intersect(*--it, s[id]))
                return {it->id, s[id].id};
            q.insert(s[id]);
       } else {
            auto it = q.lower_bound(s[id]), next = it, prev =
            if (it != q.begin() && it != --q.end()) {
                ++next, --prev;
                if (intersect(*next, *prev))
                    return {next->id, prev->id};
            q.erase(it);
    return {-1, -1};
// usage example
int main() {}
LiChaoTree.cpp
Description: desc
                                                     30d496, 73 lines
// https://cp-algorithms.com/geometry/convex_hull_trick.html
using T = long long;
struct Line {
   Ta, d;
    T eval(T x) { return a * x + d; }
};
struct Node {
    Line line:
    Node *left = nullptr:
   Node *right = nullptr;
   Node (Line line) : line(line) {}
    void add_line(Line nline, T l, T r) {
       T m = (1 + r) / 2:
       bool left smaller = nline.eval(1) < line.eval(1);</pre>
       bool mid_smaller = nline.eval(m) < line.eval(m);</pre>
        if (mid_smaller) {
            swap(line, nline);
        if (r - 1 == 1) {
            return:
        if (left smaller != mid smaller) {
            if (left == nullptr)
                left = new Node(nline);
            el se
                left->add_line(nline, 1, m);
        } else {
            if (right == nullptr)
                right = new Node(nline);
                right->add_line(nline, m, r);
```

```
T get min(T x, T l, T r) {
        if (r - 1 > 1) {
            T m = (1 + r) / 2;
            if (x < m && left != nullptr) {</pre>
                return min(line.eval(x), left->get_min(x, 1, m)
            if (x >= m && right != nullptr) {
                return min(line.eval(x), right->get_min(x, m, r
        return line.eval(x);
};
struct LiChaoTree {
   T minx;
    T maxx:
   Node *root;
    LiChaoTree (T minx, T maxx) : minx(minx), maxx(maxx) { root
         = new Node({0, numeric_limits<T>::max() / 2}); }
    void add_line(Line line) { root->add_line(line, minx, maxx
        + 1); }
    T get_min(T x) { return root->get_min(x, minx, maxx + 1); }
// usage example
int main() {
   LiChaoTree t = LiChaoTree(1, 1e9);
    t.add_line({1, 3});
    t.add_line({2, 1});
    cout << t.get_min(1) << endl;</pre>
PointClassification.cpp
Description: desc
                                                      2866f2, 30 lines
using 11 = long long;
enum class Position { Left, Right, Behind, Beyond, Origin,
    Destionation, Between };
// Classifies position of point p against vector a
Position classify(ll px, ll py, ll ax, ll ay) {
    11 cross = px * ay - py * ay;
    if (cross > 0) {
        return Position::Left;
    if (cross < 0) {
        return Position::Right;
    if (px == 0 && py == 0) {
        return Position::Origin;
    if (px == ax && py == ay) {
        return Position::Destionation;
    if (ax * px < 0 || ay * py < 0) {</pre>
        return Position::Beyond;
    if (ax * ax + ay * ay < px * px + py * py) {</pre>
        return Position::Behind;
```

```
// usage example
int main() {}
PointInPolygon.cpp
Description: desc
                                                     ce59b6, 27 lines
using 11 = long long;
int pointInPolygon(int qx, int qy, const vector<int> &x, const
    vector<int> &y) {
    int n = x.size();
    int cnt = 0;
    for (int i = 0, j = n - 1; i < n; j = i++) {
        if (y[i] == qy \&\& (x[i] == qx || (y[j] == qy \&\& (x[i]
             = qx \mid | x[j] = qx) & (x[i] = qx \mid | x[j] = qx)
            return 0; // boundary
        if ((y[i] > qy) != (y[j] > qy)) {
            11 \det = ((11)x[i] - qx) * ((11)y[j] - qy) - ((11)x
                 [j] - qx) * ((ll)y[i] - qy);
            if (det == 0)
                return 0; // boundary
            if ((det > 0) != (v[i] > v[i]))
                ++cnt:
    return cnt % 2 == 0 ? -1 /* exterior */: 1 /* interior */;
// usage example
int main() {
    vector<int> x{0, 0, 2, 2};
    vector<int> y{0, 2, 2, 0};
    cout << (1 == pointInPolygon(1, 1, x, y)) << endl;</pre>
    cout << (0 == pointInPolygon(0, 0, x, y)) << endl;
    cout << (-1 == pointInPolygon(0, 3, x, y)) << endl;
PointToSegmentDistance.cpp
Description: desc
                                                     467780 31 lines
using 11 = long long;
double fastHypot(double x, double y) {
    return sqrt(x * x + y * y);
double point_to_segment_distance(int x, int y, int x1, int y1,
    int x2, int v2) {
    11 dx = x2 - x1;
    11 dy = y2 - y1;
    11 px = x - x1;
    11 py = y - y1;
    ll squaredLength = dx * dx + dy * dy;
    11 dotProduct = dx * px + dy * py;
    if (dotProduct <= 0 || squaredLength == 0)</pre>
        return fastHypot(px, py);
    if (dotProduct >= squaredLength)
        return fastHypot(px - dx, py - dy);
    double q = (double) dotProduct / squaredLength;
    return fastHypot(px - q * dx, py - q * dy);
double point_to_line_distance(ll x, ll y, ll a, ll b, ll c) {
    return abs(a * x + b * y + c) / fastHypot(a, b);
```

return Position::Between;

```
// usage example
int main() {
    cout << fixed << setprecision(10);</pre>
    cout << point_to_segment_distance(0, 0, 0, 1, 1, 0) << endl</pre>
    cout << point_to_line_distance(0, 0, 1, 1, 1) << endl;</pre>
SegmentsIntersection.cpp
Description: desc
<optional>
                                                      a14bb2, 45 lines
using ll = long long;
bool is_cross_intersect(ll x1, ll y1, ll x2, ll y2, ll x3, ll
    y3, 11 x4, 11 y4) {
    11 \ z1 = (x2 - x1) * (y3 - y1) - (y2 - y1) * (x3 - x1);
    11 z2 = (x2 - x1) * (y4 - y1) - (y2 - y1) * (x4 - x1);
    11 z3 = (x4 - x3) * (y1 - y3) - (y4 - y3) * (x1 - x3);
    11 z4 = (x4 - x3) * (y2 - y3) - (y4 - y3) * (x2 - x3);
    return (z1 < 0 || z2 < 0) && (z1 > 0 || z2 > 0) && (z3 < 0
         | | z4 < 0 | \&\& (z3 > 0 | | z4 > 0 );
bool is_cross_or_touch_intersect(11 x1, 11 y1, 11 x2, 11 y2, 11
      x3, 11 y3, 11 x4, 11 y4) {
    if (\max(x1, x2) < \min(x3, x4) \mid | \max(x3, x4) < \min(x1, x2)
        | | \max(y1, y2) < \min(y3, y4) | |
        \max(y3, y4) < \min(y1, y2)
        return false;
    11 \ z1 = (x2 - x1) * (y3 - y1) - (y2 - y1) * (x3 - x1);
    11 \ z2 = (x2 - x1) * (y4 - y1) - (y2 - y1) * (x4 - x1);
    11 z3 = (x4 - x3) * (y1 - y3) - (y4 - y3) * (x1 - x3);
    11 z4 = (x4 - x3) * (y2 - y3) - (y4 - y3) * (x2 - x3);
    return (z1 <= 0 || z2 <= 0) && (z1 >= 0 || z2 >= 0) && (z3
         <= 0 \mid \mid z4 <= 0) && (z3 >= 0 \mid \mid z4 >= 0);
optional<pair<double, double>> get_lines_intersection(ll x1, ll
     y1, 11 x2, 11 y2, 11 x3, 11 y3, 11 x4, 11 y4) {
    11^{-} a1 = y2 - y1;
    11 b1 = x1 - x2;
    11 c1 = -(x1 * y2 - x2 * y1);
    11 a2 = v4 - v3;
    11 b2 = x3 - x4;
    11 c2 = -(x3 * y4 - x4 * y3);
    11 det = a1 * b2 - a2 * b1;
    if (det == 0)
        return {};
    double x = -(c1 * b2 - c2 * b1) / (double) det;
    double y = -(a1 * c2 - a2 * c1) / (double) det;
    return optional{make_pair(x, y)};
// usage example
int main() {
    optional<pair<double, double>> intersection =
         get_lines_intersection(0, 0, 4, 2, 2, -1, 2, 5);
    cout << (bool)intersection << endl;</pre>
    cout << intersection->first << " " << intersection->second
         << endl:
    optional<pair<double, double>> no_intersection =
         get_lines_intersection(0, 0, 0, 1, 1, 0, 1, 1);
    cout << (bool) no_intersection << endl;</pre>
```