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Source: *Journal of Political Economy*, Vol. 84, No. 3 (Jun., 1976), pp. 449-472
Published by: The University of Chicago Press
Stable URL: https://www.jstor.org/stable/1829864
Accessed: 13-03-2020 21:50 UTC

Human Capital and Labor Supply: A Synthesis

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The joint determination of work and investment in human capital over the life cycle is analyzed. At low rates of impatience investment is decreasing throughout life, as in simpler models which assume hours of work to be fixed. The demand for leisure over the life cycle is "U shaped." Wages rise to a single peak which occurs after the peak in hours of work. Distinctly different patterns arise when the rate of impatience is high. Such individuals may prefer an increasing hours of work profile, and schooling need not be concentrated at the beginning of life. Conditions are provided to determine a critical level of time preference which is sufficient to induce a "normal" life-cycle pattern for investment and work.

1. Introduction

It is by now widely recognized that investment decisions play a major role in the determination of individual age-earnings profiles. Several studies, most notably Mincer (1974), rely on investment in human capital as a leading single hypothesis from which many of the observed regularities in earnings data are derived.

This is a condensed version of a working paper by the same title which contains many of the proofs and derivations omitted here. The longer version is available on request from the Industrial Relations Section, Princeton University, Princeton, New Jersey. The authors have benefited from seminar presentations at the University of Minnesota, Michigan State University, and the National Bureau of Economic Research, West, and from comments by Mark Killingsworth and John Driffill. Blinder's participation in this project has been supported by the National Science Foundation. Weiss's participation has been supported by the Industrial Relations Section, Princeton University, and by the National Bureau of Economic Research.

[*Journal of Political Economy*, 1976, vol. 84, no. 3]

The investment hypothesis, in its simplest form, can be summarized by the equation[1]

$$Y = F(K, \dot{K}), \tag{1}$$

where Y is observed earnings, K is potential earnings ("human capital"), and $\dot{K}$ is its rate of change. Competitive equilibrium in the labor market places restrictions on the signs of the partial derivatives. Specifically, as long as human capital contributes to earnings (i.e., $\partial F/\partial K > 0$), the individual must give up current earnings in order to enhance future earning opportunities (i.e., $\partial F/\partial \dot{K} < 0$). It is typically assumed that at each point in his life the individual faces a spectrum of earnings-investment combinations ("jobs") and selects the one which is optimal.

Equation (1) highlights the formal similarity to the investment problem of the firm in the presence of adjustment costs (see Eisner and Strotz 1963). However, the fact that human capital is embodied in the individual and cannot be sold leads to several important differences. First, since the property rights to human capital cannot be transferred, the finiteness of life plays a central role in human investment. Second, since either the utilization or formation of human capital requires the sacrifice of leisure (a specific consumption good), it is not possible, in general, to separate the consumption and investment problems as is done in the theory of the firm. Third, neither human capital nor the investment therein are normally observable, so a human-capital model is useful only if it generates testable predictions about observable variables like wage rates and earnings. Finally, lack of data on human capital makes it hard to distinguish empirically among the effects of investment and a myriad of other influences on actual earnings. The most obvious of these, of course, is the choice of work intensity, which affects both current earnings (labor-leisure choices) and the rate of human-capital accumulation (training-leisure choices).

The purpose of this paper is to present a simple life-cycle model of investment in human capital in which leisure choices are explicitly incorporated. In so doing, we integrate two previously disparate branches of life-cycle theory: models of labor supply with exogenous wages[2] and models of human-capital formation with exogenous leisure.[3] Of course, to accomplish this we must posit utility maximization as the individual's goal rather than income maximization.

[1] This precise representation of the model is due to Rosen (1973), but similar formulations are implicit in Becker (1964) and Ben-Porath (1967).

[2] See, for example, Weiss (1972), Blinder (1974, chap. 3), and Heckman (1974). The paper by Weiss allows for endogeneity of wages due to learning by doing but not for human investment.

[3] See, for example, Ben-Porath (1967), Weiszacker (1967), and Sheshinski (1968). Formally, our approach also embraces life-cycle consumption theory, but we take pains to separate this from the other two problems and have little to say about consumption.

Apart from the direct interest in the interaction of labor supply and human investment over the life cycle, such a model is needed to test the robustness of the widely used wealth-maximization models of human-capital accumulation. For example, a standard implication of these models is that a period of specialization in investment (interpreted as schooling), if it exists, will occur only at the beginning of life.[4] However, one can imagine that when schooling involves foregone leisure, the existence of pure time preference might lead a utility-maximizing individual to postpone his education. Such possibilities appear explicitly in our model. Another important implication of the wealth-maximization model is that the fraction of time spent investing falls throughout the postschooling investment period. This turns out to be generally true in our model as well, though some exceptions are noted.

To our knowledge, there have been four previous attempts to integrate human capital and labor supply as we do here. The treatment by Ghez and Becker (1975) is the most general and explores the widest variety of issues. But it is also the least ambitious, in that they generally content themselves with stating and interpreting the first-order conditions. Our model can be viewed as a special case of theirs, but a case which is pushed much farther. The three other studies[5] at least attempt to analyze the shape of the optimal plan and naturally adopt simplifying assumptions in order to do so. Typically, the rate of investment in human capital and the supply of labor are related to some key variable such as the stock of human capital or its shadow price (both unobserved). However, since the models generally do not fully determine the behavior of these endogenous variables, a disturbing and unnecessary amount of ambiguity remains. Further, except for Heckman (1976), virtually no attention is paid to periods of specialization such as schooling or retirement. This seems a major omission.

The model presented here is more general than previous work on the subject (with the exception of Ghez and Becker) and yet generates many more concrete conclusions. Among the restrictions which our model places on optimal plans are: (*a*) Several distrinct patterns in investment, work, and leisure may arise, depending on the subjective rate of impatience (i.e., the discount rate for future utilities). (*b*) We consider the case where the rate of impatience is "small" (in a sense to be defined later) to be the leading case. In this case, specialization in schooling can occur only at the beginning of life, and retirement can come only at the end. (*c*) Schooling is followed by a period of on-the-job training (OJT), during which the fraction of potential earnings and time devoted to

[4] On this, see Weiss (1971), Ishikawa (1973), and also the references cited in the preceding footnote.

[5] See Landsberger and Passy (1973), Stafford and Stephan (1973), and Heckman (1976).

human-capital formation declines monotonically. (*d*) Investment reaches zero some finite time before retirement (or death, if there is no retirement). Thus there is a finite interval of "pure work" with no investment late in life. This econometric finding of Mincer (1974) is explicitly ruled out by the Ben-Porath (1967) formulation of the problem (which is followed by Heckman [1976] and by Stafford and Stephan [1973]). (*e*) The demand for leisure over the life cycle is "U-shaped" with a tendency to decline during schooling and the early part of OJT and thereafter to rise. (*f*) Wages rise to a single peak, which occurs after the peak in hours of work.

Of course, we do not pull these rabbits out of the proverbial hat. Like other investigators, we have to make some simplifying assumptions, of which two seem most crucial. First, (1) is given the special form

$$Y = F(K, \dot{K}) = KG(\dot{K}/K), \; G' < 0, \tag{2}$$

so that $G(\cdot)$ is the fraction of potential earnings which is actually realized. The nature of this important function is described briefly in the next section. Here we only wish to point out that the formulation is quite similar to the early work of Rosen (1973) who dealt with the special case $F(K, \dot{K}) = K - C(\dot{K})$, $C'(\dot{K}) > 0$. In both specifications, K is defined as earnings capacity, that is, the amount the individual would earn in the absence of investment. While Rosen orders jobs by their absolute rate of increase of earning potential ($\dot{K}$) and deducts costs additively, we order jobs by their proportionate rate of growth in potential earnings ($\dot{K}/K$) and deduct costs multiplicatively.[6] In principle, either representation is as good as the other, but the multiplicative version leads more naturally to the logarithmic wage functions encountered so frequently in empirical work (Weiss 1975).

Our second important assumption is that, for a given input of time, the quantity of human capital created is proportional to the stock of human capital. This specification, which is suggested by Mincer's work (1974), simplifies the mathematics considerably. Together, these two assumptions enable us to portray the rather complicated optimal control problem on a two-dimensional phase diagram.

2. The Earnings-Investment Frontier

Basic to any model of human capital is the market constraint delimiting the combinations of current earnings and human-capital formation which are obtainable from a given stock of human capital. This function, which we call the "earnings-investment frontier," is defined implicitly by (1). It is clearly downward sloping. Our special form (2) enables us to normalize

[6] In his later work, Rosen (1975) also adopts the multiplicative form.

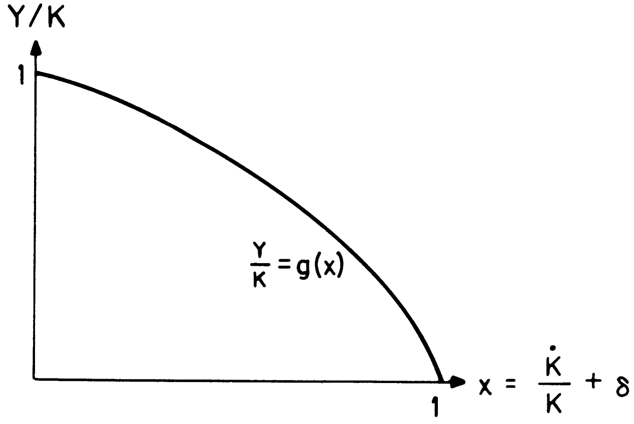

FIG. 1—The earnings-investment frontier

this relation so that a single frontier, applicable to any K, can be drawn; thus it plays for us the role that constant returns to scale plays in growth theory. The normalized frontier is sketched in figure 1, where the new symbol, x, is defined as $\dot{K}/K + \delta$, where δ is the rate of depreciation of earning capacity. As indicated in the introduction, we index jobs by their proportionate rate of growth; x is this index, and it runs from zero to unity. Then $x = 1$ indicates the job that yields the maximum feasible rate of growth. It makes sense to call this "going to school" and to specify that $g(1) = 0$—that is, that all earnings are sacrificed during schooling. Conversely, $x = 0$ is the job where potential earnings are fully realized, so $g(0) = 1$.

Throughout the analysis we shall assume that $g(x)$ is a continuous, decreasing, and concave function of x. Since $g(x)$ is really a description of an equilibrium wage structure, the assumption of concavity requires some justification.

Strict convexity is ruled out by allowing workers to divide their time between jobs or between work and school without incurring significant transactions costs. This leaves open the possibility that the frontier might be linear, which would imply that combining training and work on the job is no better than dividing one's time between working and attending school. Though possible, we find this notion somewhat unrealistic. The essence of learning from experience, as distinct from formal schooling, is that the work environment facilitates the acquisition of productive skills. Accordingly, we shall focus on the case in which $g(x)$ is strictly concave, that is, $g''(x) < 0$.[7] However, an almost equivalent model can be developed under the alternative assumption that $g(x)$ is linear.

[7] For a more detailed derivation of the earnings-investment frontier as a general equilibrium problem, see Rosen (1972).

3. Statement of the Problem

The individual is assumed to derive utility from three sources: the stream of lifetime real consumption, $c(t)$; the fraction of time devoted to leisure, $l(t)$; and the bequest, or terminal value of real (nonhuman) assets, $A(T)$.[8] Here t denotes the individual's age and runs from zero to T, the length of life which is assumed known and exogenous. Specifically, lifetime utility is assumed to be additively separable with a constant rate of time discounting,

$$\int_0^T U(c, l)e^{-\rho t}\, dt + B[A(T)], \tag{3}$$

where $U(c, l)$ and $B[A(T)]$ are assumed to be twice-differentiable, strictly concave functions of their arguments, and ρ is what we call the rate of impatience.[9] We rule out optimal paths with segments of zero consumption or zero leisure by assuming

$$\lim_{c \to 0} U_c(c, l) = \infty \text{ for all } l,$$

$$\lim_{l \to 0} U_l(c, l) = \infty \text{ for all } c.$$

The further stipulation that $U_l(c, 1) > 0$ for any c allows us to consider retirement $[l(t) = 1]$ as an endogenous decision.

What are the constraints on this maximization problem? Letting $h(t)$ denote the fraction of time devoted to market activity (including work and education), the time budget requires

$$h(t) + l(t) = 1 \tag{4}$$

$$h(t) \geq 0. \tag{5}$$

As noted above, the occupational index is bounded between zero and unity:

$$0 \leq x(t) \leq 1. \tag{6}$$

Nonhuman capital is generated by the differential equation

$$\dot{A} = rA + g(x)hK - c, \tag{7}$$

where r is the real rate of interest.[10] And human capital is generated by the human-capital production function $\dot{K} = \Phi(x, h, K)$, which can be

[8] It will be assumed that a person's human wealth dies with him, that is, that it cannot be bequeathed.

[9] A variable rate of impatience could be accommodated without too much difficulty. But it would make the notation more cumbersome, and qualitative results would depend on the time profile of $\rho(t)$.

[10] Letting r denote the real rate of interest implicitly incorporates changes in the price of consumer goods.

simplified to

$$\dot{K} = (axh - \delta)K \tag{8}$$

by one assumption and a convenient definition. (In this expression, a and δ are constants.)

The assumption was mentioned earlier: that the human-capital production function is homogeneous of degree one in K, namely, $\Phi(x, h, k) = \phi(x, h)K$. The convenient definition is to measure the occupational index, x, in terms of equivalent schooling time, so that working h hours at job x is equivalent (in the production of human capital) to spending xh hours in school.[11]

There are two initial conditions corresponding to the individual's endowment of financial and human wealth,[12] $A(0) = A_0 \geq 0$ and $K(0) = K_0 > 0$, and both terminal stocks are to be chosen optimally. Under the assumption that

$$\lim_{A_T \to 0} B'(A_T) = \infty,$$

the optimal bequest will be strictly positive.

To set up this problem in a form suitable for the application of Pontryagin's maximum principle, substitute (4) into (3), define shadow prices $p(t)e^{-\rho t}$ for human capital and $\mu(t)e^{-\rho t}$ for financial wealth, and write the Hamiltonian function $H(h, x, c, K, A, p, \mu) = e^{-\rho t}\{U(c, 1 - h) + pK(axh - \delta) + \mu[rA + g(x)hK - c]\}$.

First-order necessary conditions for a maximum are: (i) at each instant, $x(t)$, $h(t)$, and $c(t)$ are chosen to maximize H, given K, A, p, and μ and subject to the constraints (5) and (6); (ii) $\partial H/\partial K = -(d/dt)(e^{-\rho t}p)$ for all t; (iii) $\partial H/\partial A = -(d/dt)(e^{-\rho t}\mu)$ for all t; (iv) $K(T)p(T) = 0$; (v) $\mu(T) = B'(A_T)$.

As the reader may verify, conditions i–iii can be written:

$$U_c(c, l) = \mu \tag{9}$$

$$\left.\begin{array}{lll} U_l(c, l) = \mu K g(x) + apKx & \text{if} & 0 < l < 1 \\ U_l(c, l) \geq \mu K g(x) + apKx & \text{if} & l = 1 \end{array}\right\} \tag{10}$$

$$\left.\begin{array}{lll} h[apK + \mu K g'(x)] = 0 & \text{if} & 0 < x < 1 \\ h[apK + \mu K g'(0)] \leq 0 & \text{if} & x = 0 \\ h[apK + \mu K g'(1)] \geq 0 & \text{if} & x = 1 \end{array}\right\} \tag{11}$$

$$\dot{\mu}/\mu = \rho - r \tag{12}$$

$$\dot{p}/p = \rho + \delta - axh - g(x)h\mu/p. \tag{13}$$

[11] This convention means $\phi(x, h) = f(xh)$. The further stipulation that $f(\cdot)$ is linear is innocuous as long as $g(x)$ is concave. An equivalent model arises if $f(\cdot)$ is concave while $g(\cdot)$ is linear. Equation (8) is suggested by Mincer's work because xh is the change in accumulated years of schooling, $\dot{S}$, so (8) integrates to $\log K = aS - \delta t$.

[12] In interpreting these, it should be noted that K_0 includes both the initial endowment of education and "ability," or whatever else it is that determines productivity.

4. Interpretation of the Optimality Conditions

The optimality conditions, equations (9)–(13), have straightforward economic interpretations. Since $p(t)$ and $\mu(t)$ are, respectively, the shadow prices (in current utils) of human and nonhuman capital, $V(t) \equiv [p(t)/\mu(t)]K(t)$ is the value of the human-capital stock in terms of the numeraire good (consumption). This can be verified by substituting this definition into (13), using (8) and (12), and integrating to obtain

$$V(t) = \int_t^T e^{-r(\tau - t)} h(\tau) g[x(\tau)] K(\tau)\, d\tau, \tag{14}$$

which states that $V(t)$ is the present value, in time t dollars, of future earnings along the optimal path—a very natural interpretation of the value of the human-capital stock.[13]

Using this variable, we can rewrite the optimality condition (11) in a form which is readily interpretable:

$$\left.\begin{array}{lll} -g'(x)K = aV & \text{if} & 0 < x < 1 \\ -g'(0)K \geq aV & \text{if} & x = 0 \\ -g'(1)K \leq aV & \text{if} & x = 1. \end{array}\right\} \tag{15}$$

Here $-g'(x)K$ is the marginal cost in terms of foregone earnings per hour of raising x, and it can be shown that aV measures the marginal benefits in higher future earnings.[14]

Condition (15) can also be given the usual rate-of-return interpretation. Let π be the rate of interest that equates the discounted benefits to the costs; that is, π is implicitly defined by

$$-g'(x)K = a \int_t^T e^{-\pi(\tau - t)} h(\tau) g(x) K(\tau)\, d\tau$$

as the marginal internal rate of return. Then an equivalent statement of (15) is $\pi = r$ if $0 < x < 1$, $\pi \leq r$ if $x = 0$, $\pi \geq r$ if $x = 1$. Thus, during the schooling period ($x = 1$), the internal rate of return on human investment is strictly greater than the interest rate, despite the fact that hours of study are variable.[15] During the postschool investment period, the marginal rate of return is always equal to the rate of interest. Since these results hold for any arbitrary leisure profile, they are independent of tastes. Therefore, one can verify ex post whether individuals have behaved optimally by computing the appropriate rates of return. Of course, when labor supply is a choice variable, the rate of return is ill-defined as an ex ante concept.

[13] The integration that yields (14) requires the terminal condition $V(T) = 0$. This follows from the transversality condition (iv) since human capital cannot be cashed in at the end of life—i.e., $K(T) > 0$.

[14] The proof can be found in our longer working paper.

[15] Equality holds at the instant of leaving school.

We now turn to the conditions for optimal work effort (10). When there is no investment ($x = 0$), wages are exogenous to the individual,[16] and, in view of (9), (10) simply states that the marginal rate of substitution (MRS) is equal to the (potential and actual) wage. Analysis of the age-hours profile is exactly as in our previous models of dynamic labor supply with exogenous wages. Ghez and Becker (1975) have claimed that this marginal condition also holds when wages are endogenous, and that therefore for the analysis of time-versus-goods substitution in consumption we need not worry about why wages change. Two remarks can be made about this finding. First, when there is investment, the observed wage and the potential wage are very different. Nor is their behavior over time identical: during OJT the ratio of observed wage/potential wage is monotonically rising. Second, the MRS is equated to the potential wage only in the special case of a linear earnings-investment frontier. To see this, divide the left-hand side of (10) by $U_c(c, l)$ and the right-hand side by μ (they are equal by [9]) to get $U_l/Uc = a\,(p/\mu)\,Kx + Kg(x) = aVx + Kg(x)$. Then substitute for aV from (15) (assuming an interior solution) to get

$$\mathrm{MRS} \equiv \frac{U_l}{Uc} = K[g(x) - xg'(x)]. \tag{16}$$

It is easy to verify that the function in square brackets is equal to one for all x only in the case $g(x) = 1 - x$. Otherwise, it is always strictly greater than one (as long as $0 < x < 1$). We call the right-hand side of (16) the "full wage." Clearly, the first component, $g(x)K$, measures the current benefits per hour of work. That the second component, $-xg'(x)K$, measures the future benefits per hour of today's work follows from (15). This second component of the "full wage" prices x at its marginal (not average) value in computing the future benefits. The gap between the full wage and the potential wage represents the "pure profits" from OJT.

5. The Temporal Succession of Life-Cycle Phases

Due to the possibility of corner solutions, four qualitatively distinct phases might occur in an individual's life cycle. Whether each phase occurs, how long each phase lasts, and the temporal order of the phases form the subject of this section. We give the four phases descriptive names and numbers which seem to indicate a sensible ordering as follows: Phase I, schooling: $x = 1$, $h > 0$; Phase II, OJT: $0 < x < 1$, $h > 0$; Phase III, Work: $x = 0$, $h > 0$; Phase IV, Retirement: $h = 0$. Note that when $h = 0$ the value of x is arbitrary, as is clear from (11).

[16] Of course, part of the problem is to determine when the individual will opt for leaving his wage exogenous.

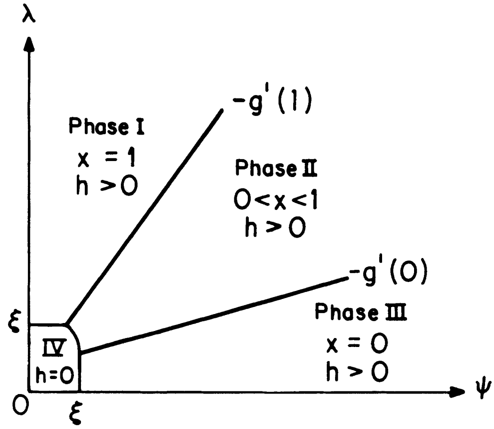

FIG. 2—The four phases in (λ, ψ)-space

Economically, this says that a retired person can be considered as holding any job at all (and working zero hours).

To facilitate the exposition, we define two new state variables: $\lambda(t) \equiv ap(t)K(t)$ = the value (in utils) of human capital, multiplied by a, and $\psi(t) \equiv \mu(t)K(t)$ = the potential wage converted to utils, and adopt the assumption that the utility function is separable, namely, $U(c, l) = u(c) + v(l)$. Due to our simplifying assumptions, a phase diagram can be drawn in (λ, ψ) space, in which the four phases are defined by (see fig. 2):

Phase I (schooling)

$$\frac{\lambda}{\psi} > -g'(1) \tag{17.1}$$

$$v'(1 - h) = \lambda \tag{18.1}$$

Phase II (OJT)

$$\frac{\lambda}{\psi} = -g'(x) \tag{17.2}$$

$$v'(1 - h) = \lambda x + \psi g(x) \tag{18.2}$$

Phase III (work)

$$\frac{\lambda}{\psi} < -g'(0) \tag{17.3}$$

$$v'(1 - h) = \psi \tag{18.3}$$

Phase IV (retirement)

$$v'(1) \geq \lambda x + \psi g(x) \qquad \text{for all} \qquad x \varepsilon [0, 1]. \tag{18.4}$$

It is clear from equations (17) that the precise shape of $g(x)$ strongly colors the likelihood and length of each phase. For example, if $-g'(1) = \infty$, then the individual will never specialize in schooling. The reason is that there are jobs (x's very near unity) which are nearly as effective as schooling in imparting skills, but which pay a positive wage. Since finiteness of $-g'(1)$ is a minimal requirement for schooling to be possible, we assume it to be true.

As another extreme, if $g'(0) = 0$, then Phase III (work without training) is impossible because there would be a job with a wage arbitrarily close to the no-training job, but which gave a finite amount of training.

In this space, the optimal trajectory can, in principle, begin anywhere. But, since the transversality condition is $p(T) = 0$, it must terminate on the horizontal axis. This trivial observation already establishes that anyone who ever works will indeed have a Phase III—that is, a finite period of work without OJT. In other words, as long as we do not rule Phase III out of court by assuming $-g'(0) = 0$, Phase III must be the last phase before retirement (or death) in any optimal plan. We believe this makes sense and is an argument against using the Ben-Porath (1967) specification.

One other interesting observation can be made already. If ψ, the potential wage, is less than $\xi \equiv v'(1)$, the marginal utility of complete leisure, then the work activity is unattractive. Similarly, if $\lambda < \xi$, the schooling activity is unattractive. However, figure 2 shows that there is a region where both of these inequalities hold, and yet the individual chooses not to retire. This means that, for some values of ψ and ξ, the option of combining work and schooling by OJT keeps the individual on the job when he would otherwise retire. It can be shown that this is a direct implication of strict concavity of the earning-investment frontier.[17]

To determine the nature of optimal trajectories in figure 2, it is necessary to know how λ and ψ change over time. Using the definitions of λ and ψ and equations (8), (12), and (13), it follows that

$$\dot{\lambda} = \rho\lambda - a\psi g(x)h \tag{19}$$

$$\dot{\psi} = (ahx - \gamma)\psi, \tag{20}$$

where $\gamma \equiv r - \rho + \delta$.

The sign of γ is a major determinant of the qualitative behavior of the model. To save space, we present a detailed analysis only of the case $\gamma > 0$, which is probably more realistic (being consistent with an

[17] The northeast boundary of Phase IV is concave because it is defined by

$$Q(\lambda, \psi) \equiv \max_{0 \leq x \leq 1} [\lambda x + \psi g(x)] = \xi.$$

In the case $g(x) = 1 - x$, the retirement region expands to fill the entire square.

increasing consumption profile)[18] and also more interesting analytically. As we shall see, a complication arises from a phenomenon which we call "cycling," that is, the recurrence of a specific phase more than once in the optimal life cycle. It turns out that there is little room for such cycling in the case $\gamma \leq 0$.

Let us, therefore, first deal briefly with people with "high impatience," that is, with $\rho > r + \delta$.[19] The following propositions, which we state without proof, rather sharply delimit the kinds of life cycles that could ever be optimal for such a person: (*a*) The only possibility of cycling is that an optimal path might include two disjoint periods of work separated by a period of OJT. (*b*) If there is a period of schooling, it comes either at the beginning of life or immediately after retirement. A period of OJT follows schooling. (*c*) If there is a period of retirement, it comes at the beginning of life. (*d*) If there is a period of OJT (and there will be one unless the optimal path includes no training whatever), it is followed by a period of work. (*e*) The last years of life are a period of work.

Thus the typical life cycle for persons with high impatience, assuming most people find it optimal to take some schooling and some retirement, is roughly as follows: an initial period of retirement is followed by schooling, then by OJT, and then by pure work until death.

Of course, life cycles with early retirement are rarely observed in practice. But this is not because such programs are irrational. Individuals with very high impatience (or very high positive exogenous wage growth) will want to bunch their leisure early in life. To do so they will have to work very hard when they are old, since consumption depends on lifetime discounted earnings. We may surmise that it is the absence of perfect capital markets that precludes all but inheritors of large fortunes from pursuing such a program.

Hereafter we focus on what we take to be the leading case: $\gamma > 0$. In order to analyze the behavior of λ and ψ over time, it is necessary to locate the stationary loci, $\dot{\lambda} = 0$ and $\dot{\psi} = 0$, in the four regions.

Because $\rho > 0$, equation (19) implies that the $\dot{\lambda} = 0$ locus can never enter Phase I ($x = 1$) or Phase IV ($h = 0$), for then $\dot{\lambda}$ would be positive. In Phase III ($x = 0$), the $\dot{\lambda} = 0$ locus is $\lambda = (a/\rho)\psi h$, where h is implicitly defined as a function of ψ by (18.3). It can be shown (proof omitted) that the $\dot{\lambda} = 0$ line begins at the point $\lambda = 0$, $\psi = \xi$ and slopes upward until it intersects the $-g'(0)$ ray at point A in figure 3.

Analogously, when $\gamma > 0$ the $\dot{\psi} = 0$ locus cannot enter Phase III ($x = 0$) or Phase IV ($h = 0$), for then $\dot{\psi}$ would have to be negative. In

[18] This simple identification of rising or falling consumption paths with $r - \rho$ is true only under separable utility. For the more general case, see Weiss (1972). Our point is that this specific model behaves most like the real world when $r > \rho$.

[19] Note that if the rate of growth of wages in the absence of investment, $-\delta$, is positive (due to economy-wide productivity changes) and exceeds the real rate of interest, then all persons are in this class.

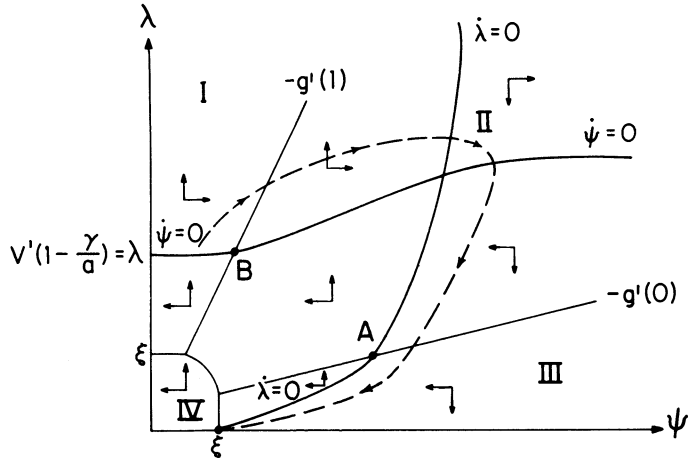

FIG. 3—Phase diagram when $\gamma > 0$

Phase I, $\dot{\psi}$ is stationary only when $h = \gamma/a$. Since h is implicitly defined as a function of λ by (18.1), this means that only one specific value of λ makes $\dot{\psi} = 0$ in Phase I (see fig. 3).

The positions of the two stationary loci in Phase II can be derived by simultaneously manipulating equations (17.2) and (18.2). As this task is rather arduous and uninteresting, it will not be described here. Instead we confine ourselves to stating without proof the following three basic properties: (*a*) The $\dot{\lambda} = 0$ locus in Phase II begins at point A in figure 3 and slopes upward from there. (*b*) The $\dot{\psi} = 0$ locus in Phase II begins at point B in figure 3 and slopes upward from there. (*c*) While both loci must begin with positive slopes, either or both slopes might become negative somewhere in the region in contrast to the way they are depicted in figure 3. This is not important. What is important is that the two loci must have a unique intersection in Phase II with the $\dot{\lambda} = 0$ line cutting the $\dot{\psi} = 0$ line from below.[20]

6. Labor Supply and Human Investment in a Normal Life Cycle

It can be seen from figure 3 that if the life plan includes schooling and has no cycles, the only possibility is that schooling comes first, followed by OJT, work, and then retirement.[21] We therefore call this the "normal" life cycle and proceed to examine its properties.

[20] The rather lengthy proof of this assertion appears in the appendix to our longer working paper.

[21] This is actually not quite so obvious, since the diagram makes it look as though retirement might come first. However, we show in Section 7 that the same conditions that preclude cycling also preclude early retirement.

Phase I: Schooling

Since λ is rising through time during Phase I, it is clear from (18.1) that the amount of time devoted to schooling rises steadily as education progresses—a prediction to which all former graduate students will doubtless attest. The intuition behind this is simply that the value (in utils) of the human-capital stock $\lambda(t)$ rises, making leisure more expensive.[22]

We may also make some rough judgments on the concavity of the hours profile, $h(t)$. Differentiating (18.1) logarithmically gives

$$\left[\frac{-v''(l)}{v'(l)}\right]\dot{h} = \frac{\dot{\lambda}}{\lambda} = \rho.$$

Denote the expression in brackets by $R(h) > 0$ and take the time derivative again to get

$$R'(h)(\dot{h})^2 + R(h)\ddot{h} = 0. \tag{21}$$

It is clear from (21) that $\ddot{h}$ and $R'(h)$ have opposite signs. What is the likely sign of $R'(h)$? Noting that the time budget makes $dh/dl = -1$ everywhere, we can write $R'(h)$ as

$$R'(h) = \frac{d}{dl}\frac{v''(l)}{v'(l)}.$$

In the case of utility functions for choices involving risk, the ratio $v''(l)/v'(l)$ is called the degree of "absolute risk aversion" (Pratt 1964) and is generally thought to be an increasing function of l (i.e., to fall in absolute value as l rises). While the present model does not discuss risk, if our $v(l)$ function also has this property, then $\ddot{h}(t) < 0$—that is, h is a concave function of time during schooling.[23] We take this to be the leading case. And since $\dot{K}/K = ah - \delta$ in Phase I, the rate of growth of potential wages would also be an increasing and concave function of time.

Phase III: Work

Since there are some formal similarities between Phase I and Phase III, it is convenient to take up next the case of work with no training. Here labor supply is governed by the usual condition: equation (18.3) states that the marginal rate of substitution between leisure and consumption

[22] Marginal calculations during this phase involve only leisure and schooling. The work activity is dominated.

[23] If, instead, $(d/dl)[v''(l)/v'(l)] < 0$, h will be a convex function of time.

goods is equated to the real wage. Since, by (20), $\dot{\psi}/\psi = -\gamma$ in Phase III, logarithmic differentiation of (18.3) yields

$$\frac{-v''(l)}{v'(l)}\dot{h} = -\gamma = \rho - r - \delta. \tag{22}$$

So people with "normal" time discount rates have diminishing labor supply in Phase III. By contrast, people with high impatience offer increasing amounts of labor to the market despite wages which are falling if $\delta > 0$. It cannot be stressed too much that these contrasting behavior patterns have absolutely nothing to do with competing income and substitution effects, although cross-sectional studies of labor supply might possibly confound the two phenomena.

From (22) it is clear that the concavity issue is precisely as it was in Phase I. The sign of $\ddot{h}$ depends only on the behavior of "absolute risk aversion" as l rises, and the more attractive utility functions imply $\ddot{h} < 0$.

The behavior of wage rates (actual and potential coincide) and earnings in Phase III is also of interest. Letting W be the observed wage, we have $\dot{W}/W = \dot{K}/K = -\delta$, a constant. So logarithmic age-wage profiles should be straight lines in Phase III, which are falling, flat, or rising according as δ is greater than, equal to, or less than zero.

Finally, consider earnings, $Y(t) = h(t)W(t)$. Since $(\dot{Y}/Y) = (\dot{h}/h) + (\dot{W}/W) = (\dot{h}/h) - \delta$, if $\delta > 0$ (i.e., if depreciation outweighs economy-wide wage growth), earnings will surely decline. However, if δ is sufficiently negative, they may not. But regardless of the slope of the logarithmic earnings profile, it will certainly be concave if $h(t)$ is for

$$\frac{d}{dt}\left(\frac{\dot{Y}}{Y}\right) = \frac{d}{dt}\left(\frac{\dot{h}}{h}\right) = \frac{\ddot{h}h - (\dot{h})^2}{h^2} < 0 \qquad \text{if} \qquad \ddot{h} < 0.$$

Phase II: On-the-Job-Training

To analyze the behavior of hours of work and investment in Phase II, it is convenient to transform the differential equations so that x and h rather than λ and ψ are the variables. In (x, h) space, Phase II is the open unit square, Phase I is the vertical line $x = 1$, Phase III is the vertical line $x = 0$, and Phase IV is the horizontal axis (see fig. 4). In a life cycle without cycling, we enter from Phase I and exit to Phase III.

The first-order conditions for an interior maximum are (17.2) and (18.2) which, since the Hamiltonian is strictly concave in (x, h) for given (λ, ψ), define x and h as unique functions of λ and ψ.[24] Given these, the

[24] Uniqueness of λ and ψ for given x and h is obvious since (17.2) and (18.2) are linear in λ and ψ.

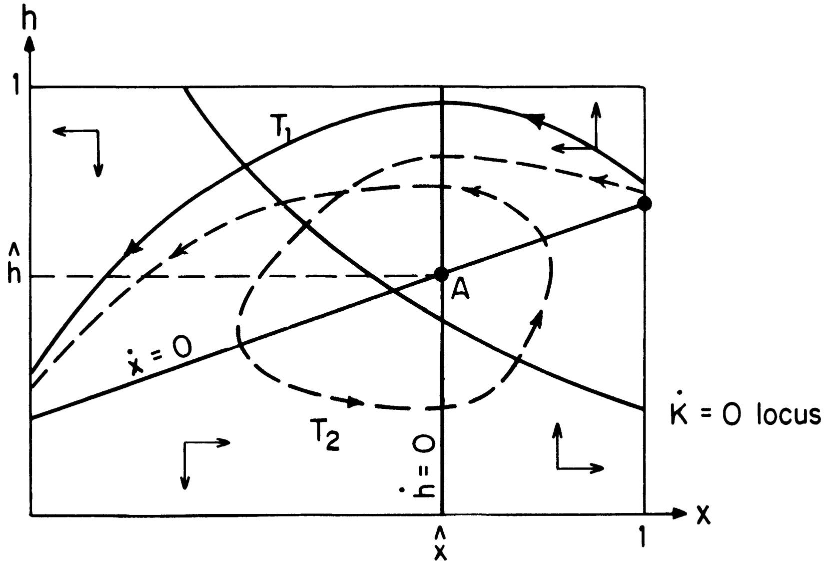

FIG. 4—Alternative phase diagram in (x, h)-space

two differential equations in $\dot{\lambda}$ and $\dot{\psi}$ can be transformed into a pair of differential equations in h and x. The results are

$$\frac{-v''(l)}{v'(l)}\,\dot{h} = \rho - \frac{r + \delta}{1 + \eta(x)} \tag{23}$$

$$\frac{g''(x)}{g'(x)}\,\dot{x} = r + \delta - ahx\,\frac{1 + \eta(x)}{\eta(x)}, \tag{24}$$

where $\eta(x) \equiv -xg'(x)/g(x)$ is the (sign-corrected) elasticity of the $g(x)$ function.

Using (23) and (24), it is easy to partition figure 4 into four regions by the $\dot{x} = 0$ and $\dot{h} = 0$ loci. The $\dot{h} = 0$ locus has the simple form

$$\eta(x) = \gamma/\rho, \tag{25}$$

which defines a unique value of x.[25] Call this $\hat{x}$. Using (24), the end points of the $\dot{x} = 0$ locus, which is defined by

$$r + \delta = ahx\left(\frac{1 + \eta(x)}{\eta(x)}\right), \tag{26}$$

[25] This is because $\eta(x)$ rises monotonically from $\eta(0) = 0$ to $\eta(1) = +\infty$.

can easily be determined. When $x = 1$, $h = (r + \delta)/a$, and when $x = 0$, $h = -g'(0)(r + \delta)/a < (r + \delta)/a$. It can also be shown that the locus is upward sloping.[26]

It is clear from figure 4, then, that h can be no less than $(r + \delta)/a$ at the start of Phase II.[27] And the trajectory must proceed smoothly from $x = 1$ to $x = 0$ with $\dot{x} < 0$ everywhere, like path T_1 in figure 4, rather than form "mini cycles" like path T_2 in figure 4. This is because (23) and (24) define $\dot{x}$ and $\dot{h}$ as functions of x and h only—that is, because $\dot{x}$ and $\dot{h}$ do not depend directly on either the state variables or the costate variables. It therefore follows that $(\dot{x}, \dot{h})$ must be the same whenever (x, h) are. But a path like T_2 would have to cross itself—include two distinct points in time with the same (x, h) and different $(\dot{x}, \dot{h})$—which is impossible. This is an important result, since it shows that within Phase II, $x(t)$ is monotonically declining—a property which holds in the simple Ben-Porath (1967) model and which is vital if human-capital theory is to account for the gross facts. It also implies that within Phase II labor supply rises to a single peak and then declines.

Nothing can be said in general about the concavity of the $x(t)$ profile. However, we can use equation (23) to determine the concavity of $h(t)$. Letting $R(h) \equiv -v''(l)/v'(l)$ as before, its time derivative is

$$R(h)\ddot{h} = \frac{(r + \delta)\eta'(x)}{[1 + \eta(x)]^2}\dot{x} - (\dot{h})^2 R'(h). \tag{27}$$

Since the first term on the right-hand side is negative, $\ddot{h} < 0$ for utility functions with $R'(h) \geq 0$.[28]

The behavior of potential wages, $K(t)$, can be displayed conveniently on the (x, h) diagram. Since $\dot{K}/K = ahx - \delta$, the $\dot{K} = 0$ locus (which exists only if $\delta > 0$) is the rectangular hyberbola, $hx = \delta/a$, shown in figure 4. By showing that the intersection of the $\dot{x} = 0$ and $\dot{h} = 0$ loci (labeled point A in fig. 4) lies above this hyperbola, we will establish that the peak in labor supply precedes the peak in human capital. Point A is defined by $\hat{x}$ and the $\hat{h}$, which satisfies

$$r + \delta = a\hat{h}\hat{x}\left(\frac{1 + \eta(\hat{x})}{\eta(\hat{x})}\right).$$

[26] Proof: Using the definition of $\eta(x)$ and differentiating (26) yields

$$0 = ax\frac{1 + \eta(x)}{\eta(x)}\frac{dh}{dx} + ah\frac{g(x)g''(x)}{g'(x)^2}.$$

Since $g''(x) < 0$, we have $dh/dx \mid_{\dot{x}=0} > 0$.

[27] We must assume $a > r + \delta$ if there is to be any training at all.

[28] When $R'(h) < 0$, (27) is ambiguous a priori. However, we already know that there is a unique maximum (where $\dot{h} = 0$), and it is clear that $\ddot{h} < 0$ in the neighborhood of this maximum.

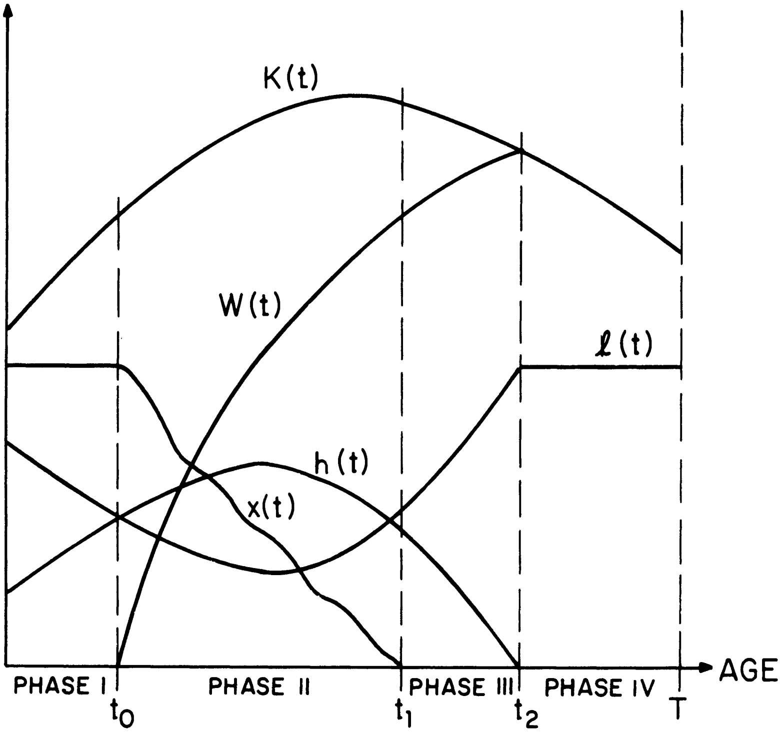

Fig. 5—The age profiles of human capital, wages, leisure, work, and investment in a "normal" life cycle.

But $\eta(\hat{x}) = \gamma/\rho$, so $\hat{h}\hat{x} = \gamma/a = (r - \rho)/a + \delta/a > \delta/a$ when $r > \rho$. So $r \geq \rho$ is sufficient though certainly not necessary for the peak in labor supply to precede the peak in human capital.

What about the peak in observed wages, $W(t) = K(t)g[x(t)]$? Since $\dot{W}(t) = Kg'(x)\dot{x} + g(x)\dot{K}$, and since $g'(x)$ and $\dot{x}$ are negative while $g(x)$ is positive, the peak in observed wages (if any) must follow the peak in potential wages (if any). Of course, if $\delta = 0$, both actual and potential wages reach their peak at the Phase II–Phase III switch point and are level thereafter.

Figure 5 summarizes our results. It depicts the time profiles of labor supply, human capital, and observed wage rates in the four phases for the case $\gamma > 0$, $\delta > 0$. The peak in observed earnings, $h(t)W(t)$, must come between the peak in h and the peak in W, though its relation to the peak in K is unclear. If the $\delta = 0$ case is a good benchmark, earnings peak earlier than human capital.

Figure 5 shows wages and hours of work rising together at first (when wages are low), then moving in opposite directions (when wages are higher), and finally falling together (when wages are again low). Once again, we stress that this has nothing whatsoever to do with income and substitution effects—although cross-sectional studies over diverse age cohorts might mistakenly identify this phenomenon as a backward-bending labor-supply function.

7. The Question of Cycling

We now address ourselves to two important questions which have thus far been avoided: (*a*) When cycling arises, what broad characterizations of the optimal path can be made? (*b*) What meaningful conditions can be derived which exclude the possibility of cycling?

In Section 6, we proved that the optimal trajectory can never cross itself.[29] This result enables us to answer the first question, since it implies that cycles must be "expanding" as in figure 6 rather than "contracting." Thus a cycling path can be broken down into several "quasi life cycles," each beginning with a period of schooling (with the possible exception of the first). Since the contours of constant work effort look like blowups of the border of the retirement region (which is the special case $h = 0$), with higher contours connoting higher h, we see that the individual works harder during his second "quasi cycle" than in his first. Also, he spends more time in Phase II.

We can develop strong conditions which rule out cycles by considering the minimum length of time required to complete a quasi cycle. If this time exceeds the available life, T, then cycles are impossible. In particular, we focus on the last quasi cycle and let t_0 = age of starting last schooling period, t_1 = age of starting last OJT period, t_2 = age of starting last work period, and t_3 = age of ending last work period (possibly $t_3 = T$). These four points on the optimal trajectory are indicated in figure 6.

Consider first the length of the schooling period, $t_1 - t_0$. Since we are considering a cycling path, we know that[30]

$$h(t_0) \leq \gamma/a = \frac{r + \delta - \rho}{a} \tag{28}$$

(see fig. 6). Furthermore, we know from figure 4 that

$$h(t_1) \geq (r + \delta)/a. \tag{29}$$

Finally, we know that optimal behavior implies that $v'(1 - h)$ grows at the exponential rate ρ in Phase I (see eqq. [18.1] and [19]). Thus we can derive a minimal time required to pass through Phase I which will depend on the rate of impatience, the elasticity of the marginal

[29] Actually, we only proved that it could never cross itself in Phase II. It is trivial to prove this for the other phases as well.

[30] Note that no such statement can be made about the schooling period in a trajectory without cycling. Thus the bound to be derived will not apply to "normal" life cycles.

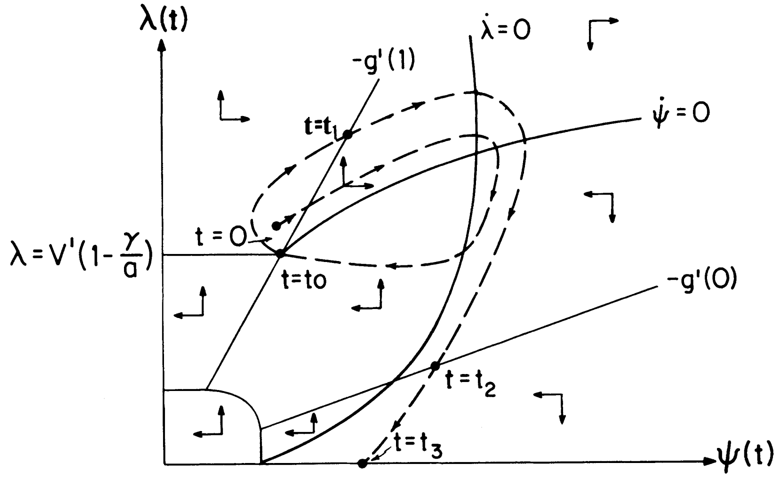

FIG. 6—A possible cycle when $\gamma > 0$

utility function, and the parameters a, r, and δ. Specifically, since $\lambda(t_0) \leq v'(1 - [\gamma/a])$ by (28), and $\lambda(t_1) \geq v'[1 - (r + \delta)/a]$ by (29), we have

$$e^{\rho(t_1 - t_0)} \geq \frac{v'\left(\dfrac{a - r - \delta}{a}\right)}{v'\left(\dfrac{a - r - \delta + \rho}{a}\right)} \tag{30}$$

or

$$t_1 - t_0 \geq \frac{1}{\rho} \log \frac{v'\left(\dfrac{a - r - \delta}{a}\right)}{v'\left(\dfrac{a - r - \delta + \rho}{a}\right)} > -\frac{1}{a} \frac{v''\left(1 - \dfrac{\gamma}{a}\right)}{v'\left(1 - \dfrac{\gamma}{a}\right)} \equiv S^*. \tag{31}$$

The inequality in (31) follows if we assume, as in Section 6, that $d/dl\ [v''(l)/v'(l)] > 0$ and provides us with a lower bound, S^*, for the interval $t_1 - t_0$. This lower bound increases as ρ falls.

It is also intuitively clear that there must be some minimal period of work during which the benefits from schooling are realized. This is the idea of our lower bounds on the lengths of Phases II and III, which depend basically on the trade-offs embodied in the earnings-investment frontier. To derive these bounds, we examine the behavior of the ratio $\theta(t) \equiv p(t)/\mu(t)$, which is the price of human capital in terms of consumption goods. Using this definition, t_1, t_2, and t_3 are defined by (see fig. 6):

$$a\theta(t_1) = -g'(1) \tag{32}$$

$$a\theta(t_2) = -g'(0) \tag{33}$$

$$\theta(t_3) = 0. \tag{34}$$

It is easy to show that

$$\dot{\theta} = (r + \delta)\theta - h[g(x) - xg'(x)] \text{ in Phase II} \tag{35}$$

$$\dot{\theta} = (r + \delta)\theta - h \text{ in Phase III.} \tag{36}$$

First consider Phase II, where the solution to (35) is

$$\theta(t)e^{-(r+\delta)(t-t_1)} = \theta(t_1) - \int_{t_1}^{t} e^{-(r+\delta)(\tau-t_1)}h[g(x) - xg'(x)]\, d\tau.$$

Setting $t = t_2$ and using (32) and (33), we have

$$-g'(0)e^{-(r+\delta)(t_2-t_1)} = -g'(1) - a\int_{t_1}^{t_2} e^{-(r+\delta)(\tau-t_1)}h[g(x) - xg'(x)]\, d\tau.$$

Now, replacing $h(t)$ in the integral by 1.0 and replacing $g(x) - xg'(x)$ by $-g'(1)$ makes the integral strictly larger, so we have the inequality

$$-g'(1)a\int_{t_1}^{t_2} e^{-(r+\delta)(\tau-t_1)}\, d\tau > -g'(1) + g'(0)e^{-(r+\delta)(t_2-t_1)}$$

or

$$a\,\frac{1 - e^{-(r+\delta)(t_2-t_1)}}{r + \delta} > 1 - \frac{g'(0)}{g'(1)}\, e^{-(r+\delta)(t_2-t_1)}.$$

Our lower bound on the length of Phase II, call it J^*, therefore satisfies[31]

$$J^* = \frac{1}{r + \delta} \log \frac{a - \dfrac{g'(0)}{g'(1)}(r + \delta)}{a - r - \delta}. \tag{37}$$

The bound depends on the parameters a, r, and δ and also on the degree of concavity of the earnings-investment frontier.[32] Tastes are not involved since we obtained the bound by assuming the individual works as hard as he can.

Almost the same reasoning can be used to place a lower bound, W^*, on the length of the last Phase III. Solving (36) explicitly gives

$$\theta(t)e^{-(r+\delta)(t-t_2)} = \theta(t_2) - \int_{t_2}^{t} e^{-(r+\delta)(\tau-t_2)}h(\tau)\, d\tau.$$

[31] It should be clear that the bound applies equally well to the OJT period of a normal life cycle, since the only things assumed are that Phase II is preceded by Phase I and precedes Phase III.

[32] That is, on how much smaller $-g'(0)$ is than $-g'(1)$.

Letting $t = t_3$ and using (33) and (34), we obtain

$$-g'(0) = a\int_{t_2}^{t_3} e^{-(r+\delta)(\tau-t_2)}h(\tau)\,d\tau < a\int_{t_2}^{t_3} e^{-(r+\delta)(\tau-t_2)}\,d\tau,$$

since $h(\tau) < 1$. Thus $-g'(0) < [a/(r+\delta)]\,[1 - e^{-(r+\delta)(t_3-t_2)}]$, or

$$W^* = \frac{1}{r+\delta}\log\frac{a}{a+g'(0)(r+\delta)}. \tag{38}$$

Again, the bound depends on a, r, δ, and the $g(x)$ function.

Putting these results together, we find that there are bounds on the minimum lengths of Phases II and III which are independent of ρ, and there is a bound for the minimal schooling period which depends on ρ. Let ρ^* be the value of ρ, satisfying $S^*(\rho^*) = T - J^* - W^*$. Then cycling is certainly impossible for persons with $\rho \leq \rho^*$. Since we know that cycling is only a problem when $\rho < r + \delta$, a sufficient (but far from necessary) condition to rule out cycling is $r + \delta \leq \rho^*$. The reader should note that the condition $\rho \leq \rho^*$ rules out early retirement in the $\gamma > 0$ case as well, since the bounds $S^*(\rho^*)$, J^*, and W^* apply here as well. In a word, we will never get cycles if either ρ is "large" (i.e., $\rho \geq r + \delta$) or ρ is "small" (i.e., $\rho \leq \rho^*$). If $\rho^* < r + \delta$, there will be an intermediate range in which cycling is possible.

It may be of interest to work out a numerical example of these bounds. Suppose a, which would be the rate of return on human capital in the case of an infinite lifetime, is 8 percent, while $r + \delta$ is 6 percent. A simple form for the earnings-investment frontier is the quadratic: $g(x) = 1 - \frac{1}{2}x - \frac{1}{2}x^2$. Given these choices, (37) and (38) can be used to compute $J^* = 18.3$, $W^* = 7.8$. From (31) it is clear that a facile choice of utility function is $v'(l) = e^{-l}$. For a rate of impatience of $\rho = .02$, (31) gives $S^* = 12.5$ as the minimal schooling period. Adding these, we find that a new cycle cannot start unless there are more than 38.6 years remaining.[33]

8. In Conclusion

We have presented a life-cycle model of the behavior of a utility-maximizing individual free to allocate his daily time budget among leisure, work, and education. The main substantive conclusions are listed in Section 1 and depicted in figure 5. They need not be rehashed here.

[33] The reader is reminded that these are all weak lower bounds. The actual amount of time taken by the last quasi cycle is thus strictly greater (and probably considerably greater) than 38.6 years.

Several generalizations of the analysis immediately come to mind. More general forms of the human-capital production function could be tried, as could a nonseparable utility function. Indeed, the list of arguments of the utility function could be expanded to include human capital or, indeed, nonhuman capital. More important than all these, we imagine, would be allowance for the capital-market imperfections that severely constrain the choices of those poorly endowed with financial wealth.

Still, as long as we interpret the results as a benchmark around which there will surely be deviations (some systematic, some random), there are a number of interesting uses for the model.

First, it may be possible to do the usual kinds of comparative-dynamic exercises. How would an increase in initial financial wealth alter the optimal plan? What about an increase in initial human wealth? Is the optimal plan continuous with respect to these endowments? How sensitive is the optimal policy to the rate of impatience—a taste parameter that presumably differs across people? The reader can no doubt think of many similar questions. While we suspect that most of these can only be answered under specific functional forms for $v(l)$ and $g(x)$, some weak results may hold in greater generality.

Second, the model can provide the micro-foundation for a simulation model of the income distribution along the lines of Blinder (1974). The chief conclusion of that work is that the distribution of wage rates, taken there to be exogenous, is the principal contributor to income inequality. The present model can generate that distribution endogenously, given assumed distributions of tastes and endowments, and thus can fill what is perhaps the major gap in the positive theory of income distributions.

Third, the long-run incidence of various taxes in a world with human-capital accumulation is virtually unexplored territory.[34] It seems feasible to incorporate some simple taxes—such as a linear income or wage tax—into the model and examine, either analytically or through simulation, the effects of these programs on the acquisition of human capital. It could be that human investment responds to taxation more substantially than do hours of work.

In a word, there are a host of interesting and important questions which simply cannot be addressed by a life-cycle model which considers either labor-leisure choices or labor-education choices but not both.[35] By demonstrating the feasibility of handling both decisions together, we hope to have hastened the day when the powerful tools of life-cycle economic theory will be brought to bear on these issues.

[34] A preliminary discussion is offered by Boskin (1976).

[35] For example, if income maximization is the posited goal, a proportional tax (subsidy) on wages cannot possibly alter behavior.

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