Flatter experience-wage profiles and declining labor force nonparticipation

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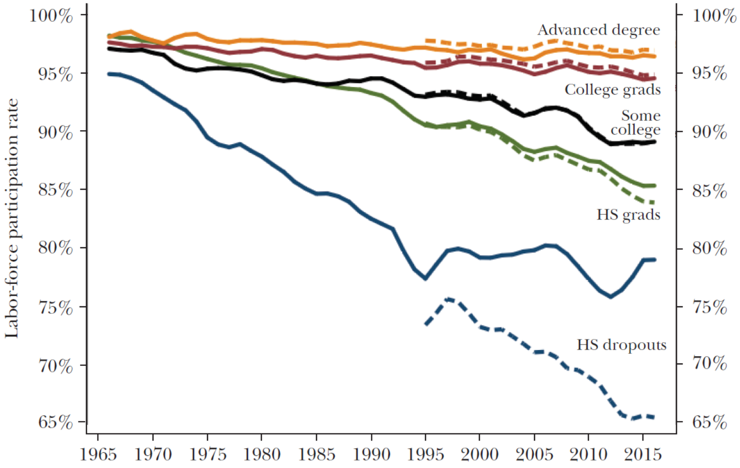
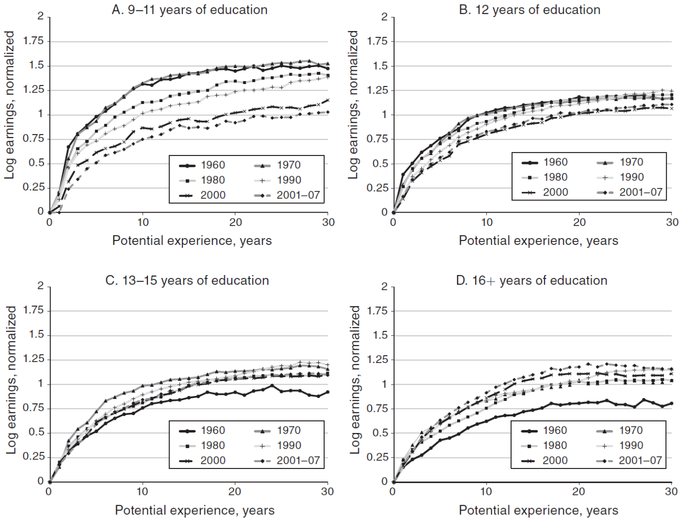
Motivation

Declining labor force participation among prime-aged low-skilled men (Binder & Bound JEP 2019)



Motivation

Flattening of experience-income profile of low-skilled relative to high-skilled (Elsby & Shapiro AER 2012)



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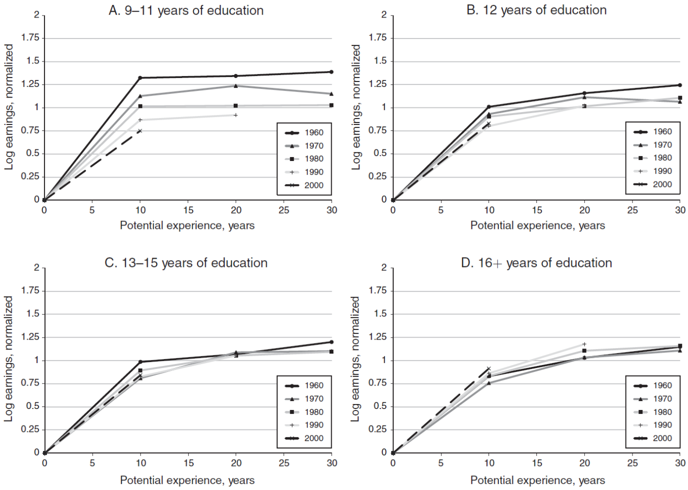
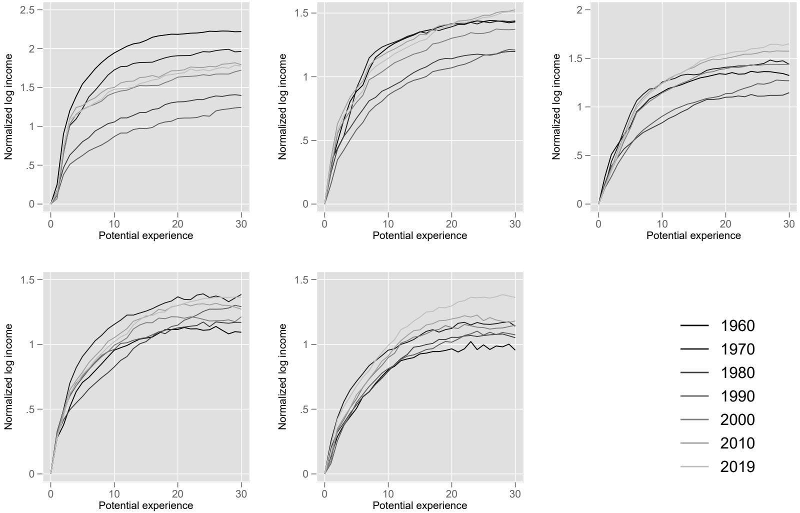


Motivation

Experience-income profile of lowest-skilled has actually steepened recently!



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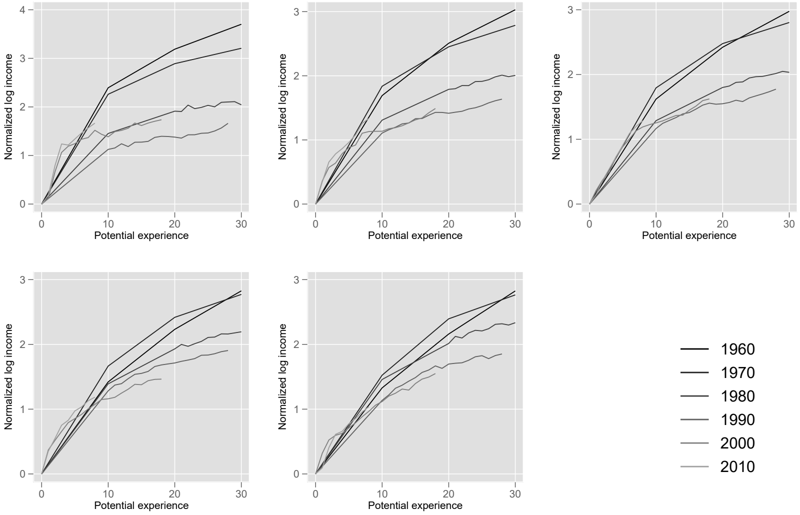
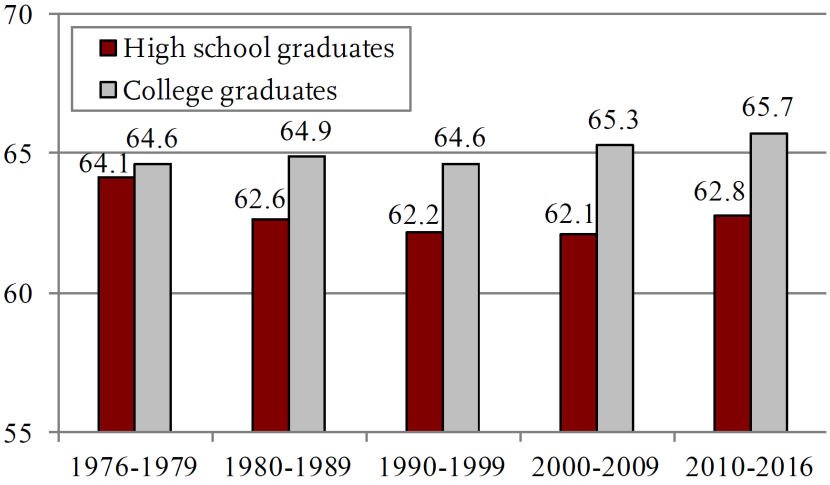


Motivation

Increase in gap of retirement age between high and low-skilled (Rutledge 2018)



Idea

Declining returns to accumulation of human capital leads to

- less human capital accumulation
- lower participation
- and earlier retirement

among low-skilled, and lower human capital level leads to

- higher sensitivity and persistence to shocks

Literature

Many explanations for declining LFP of prime-aged men:

- Skill-biased technical change (Card & Dinardo 2002, Acemoglu & Autor 2010)
- Job polarization (Foote & Ryan 2015)
- Improvements in leisure technology (Aguiar et. al. 2018)
- Disability and SSDI (Autor & Duggan QJE 2003, Krueger 2017)
- Incarceration (Binder & Bound JEP 2019)

Elements I need in my model

- Human capital accumulation
- Education
- Labor supply
- Retirement

Blinder-Weiss 1976

Agents with finite lifespan T maximize lifetime utility

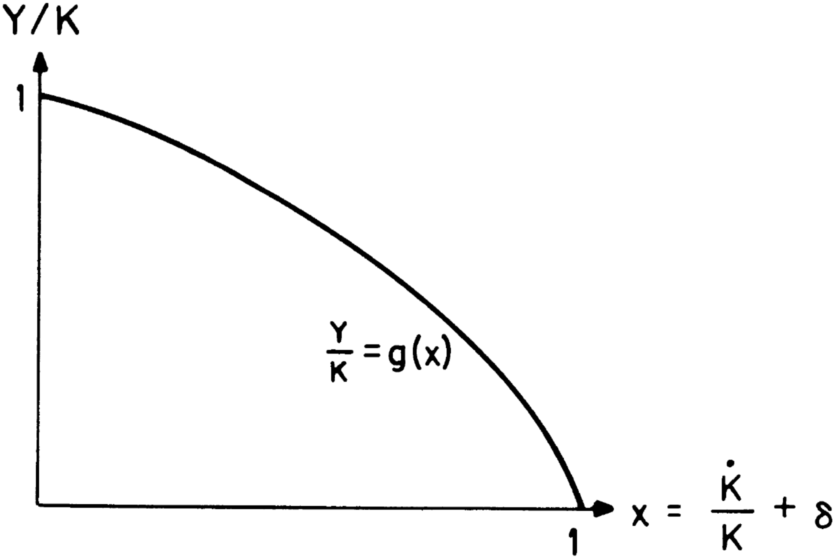
$$\max_{\{c_t\}_{t=0}^T, \{h_t\}_{t=0}^T, \{x_t\}_{t=0}^T} \sum_{t=0}^{T} \beta^t u(c_t, 1 - h_t) + B(A_{T+1})$$

subject to

$$A_{t+1} = (1 + r)A_t + h_t g(x_t) K_t - c_t,$$
$$K_{t+1} = (1 - \delta)K_t + x_t h_t K_t,$$
$$x_t, h_t \in [0, 1],$$

- x_t and $g(x_t)$ governs tradeoff between accumulating human capital and earnings
- B is bequest, A is assets, and K is human capital

$g(x)$

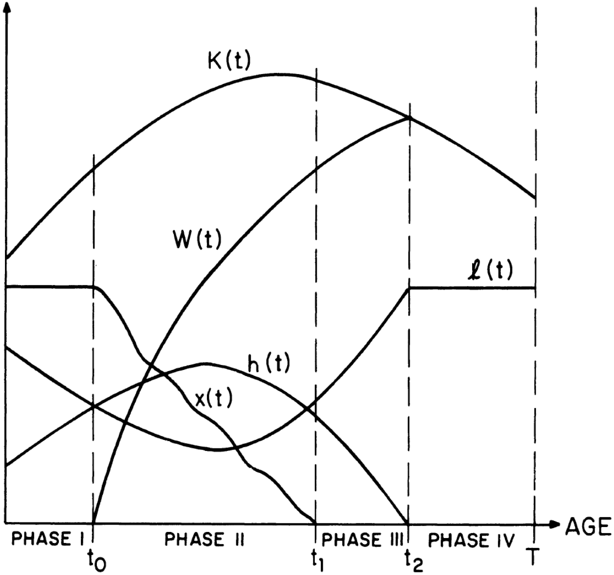


Endogenous life phases

Four phases:

- Education: $x = 1$
- Work + learning: $0 < x < 1$
- Work + no learning: $x = 0$
- Retirement: $h = 0$

Endogenous life phases



Challenges

- Both shooting method (continuous time model) and backward induction of value function (discrete time model) not working
- Possible way forward: discretize labor supply and investment decisions (Keane & Wolpin 1994)
- What is $g(x)$?
- What is causing the flattening and steepening of the experience-income profile?
  - Changes in labor markets; incorporate into the $g(x)$ function?
  - Changes in monopsony power?
  - Endogenize changes in the slope of experience-income profile

HJB

The HJB equation for the agent's problem is

$$\begin{aligned}\rho V(A,K,t) = \max_{c,h,x} \quad & u(c,1-h) \\ & + \partial_A V(A,K,t)\left[rA + hg(x)K - c\right] \\ & + \partial_K V(A,K,t)\left[-\delta K + xhK\right] \\ & + \partial_t V(A,K,t)\end{aligned}$$

Finite difference approximation of the HJB

- nA and nK are the number of grid points for A and K
- Denote the index for A and K as i and j respectively
- Denote $V(A_i, K_j, t) = V_{ij}^t$
- Denote drift in A as μ_A, drift in K as μ_K

Forward and backward difference approximation

- Forward difference:

$$\partial_A^F V_{i,j}^t = \frac{V_{i+1,j}^t - V_{i,j}^t}{\Delta A}$$
$$\partial_K^F V_{i,j}^t = \frac{V_{i,j+1}^t - V_{i,j}^t}{\Delta K}$$

- Backward difference:

$$\partial_A^B V_{i,j}^t = \frac{V_{i,j}^t - V_{i-1,j}^t}{\Delta A}$$
$$\partial_K^B V_{i,j}^t = \frac{V_{i,j}^t - V_{i,j-1}^t}{\Delta K}$$

Choice of forward or backward difference

- At any grid point i, j at time t, solve for c, h, x:

$$\partial_c u(c_{i,j}^t, h_{i,j}^t) = \partial_A V_{i,j}^t \tag{1}$$

$$\partial_h u(c_{i,j}^t, h_{i,j}^t) = \partial_A V_{i,j}^t \left[g(x_{i,j}^t)K_j\right] + \partial_K V_{i,j}^t \left[x_{i,j}^t K_j\right] \quad \text{if} \quad 0 < h < 1 \tag{2}$$

$$\partial_h u(c_{i,j}^t, h_{i,j}^t) \geq \partial_A V_{i,j}^t \left[g(x_{i,j}^t)K_j\right] + \partial_K V_{i,j}^t \left[x_{i,j}^t K_j\right] \quad \text{if} \quad h = 0 \tag{3}$$

$$\partial_K V_{i,j}^t \left[hK_j\right] = -\partial_A V_{i,j}^t \left[hg'(x_{i,j}^t)K_j\right] \quad \text{if} \quad 0 < x < 1 \tag{4}$$

$$\partial_K V_{i,j}^t \left[hK_j\right] \geq -\partial_A V_{i,j}^t \left[hg'(x_{i,j}^t)K_j\right] \quad \text{if} \quad x = 1 \tag{5}$$

$$\partial_K V_{i,j}^t \left[hK_j\right] \leq -\partial_A V_{i,j}^t \left[hg'(x_{i,j}^t)K_j\right] \quad \text{if} \quad x = 0 \tag{6}$$

- Problem: Do we use ∂_A^F or ∂_A^B, and ∂_K^F or ∂_K^B?

Choice of forward or backward difference

- Backwards in A, forwards in K: $\partial_A^B + \partial_K^F \Rightarrow \mu_A^{BF}, \mu_K^{BF}$
- Choose approximation scheme that is consistent with computed drift
- e.g., $\partial_A^B + \partial_K^F$ requires $\mu_A^{BF} < 0, \mu_K^{BF} > 0$
- Consistent scheme may not always exist:
  - Boundaries
  - Steady-states
- May have more than 1 consistent scheme due to non-concavity of V

Discretizing HJB

- Stack all $V_{i,j}^{t+1}$ into V^{t+1}, a vector of length $nA * nK$
- V^{t+1} has associated optimal c^t, h^t, x^t, flow utility $U(V^{t+1})$, and drifts μ^t
- At each i, j, there is an associated upwind direction for ∂V
- Matrix $A(V^{t+1})$ encodes these directions and drifts μ^{t+1}, so $A(V^{t+1}) * V^{t+1} = \partial V^{t+1} * \mu^{t+1}$

Discretizing HJB

Recall HJB:

$$
\begin{aligned}
\rho V(A, K, t) = \max_{c,h,x} \quad & u(c, 1-h) \\
& + \partial_A V(A, K, t)\left[rA + hg(x)K - c\right] \\
& + \partial_K V(A, K, t)\left[-\delta K + xhK\right] \\
& + \partial_t V(A, K, t)
\end{aligned}
$$

Discretized and stacked version:

$$
\rho V^t = U(V^{t+1}) + A(V^{t+1}) * V^t + \frac{V^{t+1} - V^t}{\Delta t} \tag{7}
$$

- Solve backwards from terminal bequest motive: $V^T = B(A^T)$

Boundaries

- What do we do at A_{min}, A_{max}, K_{min}, and K_{max}?
- Problem: need $\mu_A \geq 0$ at A_{min} and $\mu_A \leq 0$ at A_{max}
- Possible solution: make A_{max} very large so that $\mu_A^{BF}, \mu_A^{BB} < 0$ naturally at A_{max}
- Choose BF or BB according to consistency with μ_K
- Cannot make A_{min} very small, e.g., borrowing constraint $\bar{A} = A_{min}$
- If $\mu_A^{FF}, \mu_A^{FB} < 0$ at A_{min}, then set ∂_A at A_{min} so that $\mu_A^{FF}, \mu_A^{FB} = 0$
- Note: possible to have steady-state in K when at A boundary, and vice versa

Steady-state in one dimension

- What do we do when one of the state variables is never consistent?
- e.g., $\mu_A^{FF}, \mu_A^{FB} > 0$, so A is consistent in the F direction
- but $\mu_K^{FF} < 0$ and $\mu_K^{FB} > 0$
- I set K to be at steady-state
- Do so by setting ∂_K so that $\mu_K = 0$
- We will get $\partial_A^F, \partial_K \Rightarrow \mu_A > 0, \mu_K = 0$

Steady-state in both dimensions

- Steady-state in both dimension, so both A and K are inconsistent
- We don't have a consistent $\mu_A^{FF}, \mu_A^{FB} > 0$, or $\mu_A^{BF}, \mu_A^{BB} < 0$
- Neither do we have a consistent $\mu_K^{FF}, \mu_K^{BF} > 0$, or $\mu_K^{FB}, \mu_K^{BB} < 0$
- Then set ∂_A and ∂_K such that $\mu_A = 0$ and $\mu_K = 0$

Non-concavity of V

- With concave V, $\partial_A^F < \partial_A^B$
- $\Rightarrow \mu_A^{FF} < \mu_A^{BF}$
- We will not have A consistent in both F and B:
  - $\mu_A^{BF} < 0 < \mu_A^{FF}$ contradicts $\mu_A^{FF} < \mu_A^{BF}$
- However, this may happen with non-concave V
- Solution: choose approximation that leads to larger Hamiltonian value

Progress since last meeting

- Improved algorithm explanation slides
- Met with Johannes:
  - He suggested I fix terminal condition spike by increasing bequest motive (completed, seems to work, not sure if best solution, will try others)
  - He thinks my modified algorithm to impose boundary/steady state/boundary + steady state conditions seem reasonable
  - He suggested a modification for rare cases where boundary/steady state coincide with non-concavity ambiguity: add tie-breaking (will need to wrap boundary/steady state condition checkers inside functions; 40% done)
  - Create 1-slide explainer (what does Titan think?)
- Not completed: Look at data and calibrate (will do after algorithm is faster)

One-slide explainer

- Discretize HJB over grids of A, K indexed by i, j:
- Approximate derivatives of V using finite difference
- Forward difference:

$$\partial_A V(A_i, K_j, t) = \frac{V(A_{i+1}, K_j, t) - V(A_i, K_j, t)}{\Delta A} \tag{8}$$

- Backward difference:

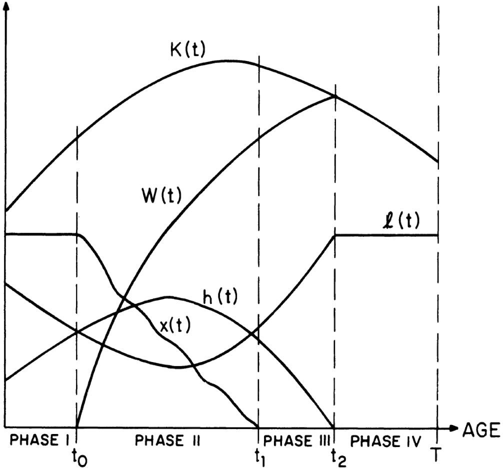
$$\partial_A V(A_i, K_j, t) = \frac{V(A_i, K_j, t) - V(A_{i-1}, K_j, t)}{\Delta A} \tag{9}$$

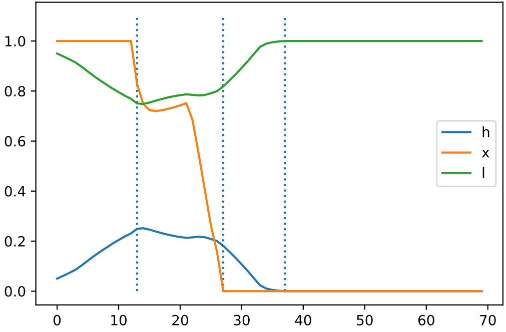
- Choose direction and optimal choice via "upwind scheme": direction is consistent with drift of state variable implied by optimal choice
- Once we know V^{t+1}, only unknown in discretized HJB is V^t
- Solve backwards from terminal V^T

Current computational issues

- Proving my algorithm satisfies the three conditions of Barles and Souganidis (1991) (monotonicity, consistency, stability)
- Speed up computation by consolidating ambiguous case checkers within a single function (goes hand-in-hand with Johannes' suggestion)
- Length of time in schooling depends on initial wealth due to credit constraint
  - With too much assets, agents end up not accumulating much human capital or assets
  - With too little assets, $x = 1$ period is very short
  - May need to allow for debt? Will require modified bequest motive

Path of choice variables





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