Econ 281 final project: Solving Blinder and Weiss (1976) numerically and estimating with time use data

Churn Ken Lee

I aim to numerically solve a discrete-time version of the model in Blinder and Weiss (1976), and then estimate the model using time use and labor market data.

1 Discrete-time version of Blinder and Weiss (1976)

Agents with finite lifespan T maximize lifetime utility

$$\sum_{t=0}^{T} \beta^t u(c_t, l_t) + B(A_{T+1}) \tag{1}$$

where $u(c_t, l_t)$ is their period utility over consumption and leisure, and $B(A_{T+1})$ is their preference for bequest of financial assets at end of life, A_{T+1}. They choose consumption path $\{c_t\}_{t=0}^{T}$, leisure path $\{l_t\}_{t=0}^{T}$, and earnings-investment path $\{x_t\}_{t=0}^{T}$ subject to the evolution of financial assets,

$$A_{t+1} = (1+r) * A_t + (1-l_t) * g(x_t) * K_t - c_t, \tag{2}$$

and human capital,

$$K_{t+1} = (1-\delta) * K_t + a * x_t * (1-l_t) * K_t, \tag{3}$$

period time constraint

$$l_t \in [0, 1], \tag{4}$$

and initial conditions A_0, K_0.

The function $g(\cdot)$ has to satisfy several properties:

1. $g(\cdot)$ is a concave, continuous, and decreasing function over the interval $[0, 1]$
2. $g'(0) < 0$ and $g'(1) > -\infty$
3. $g(0) = 1$ and $g(1) = 0$

Property 2 allows for the existence of corners $x_t = 0$ and $x_t = 1$ in the optimal path. For example, if $g'(0) = 0$, then no agent would choose $x = 0$ as a slightly higher x would entail no decrease in wages but a positive amount of human capital accumulation. Property 3 allows for interpreting $g(x)$ as the proportion of potential earnings capacity, K, that is realized with the choice of x.

I chose a functional form for $g(x)$:

$$g(x) = \frac{5}{4} - \left[x * \left(\sqrt{\frac{5}{4}} - \frac{1}{2}\right)\right]^2. \tag{5}$$

This form was chosen for simplicity, and not as an actual representation of the labor market equilibrium.

2 Solution strategy

I know the terminal value of the agent at $t = T$ given state A_T, K_T:

$$V_T(A_T, K_T) = \max_{c_T, l_T} \{u(c_T, l_T) + B(A_{T+1})\} \tag{6}$$

subject to

$$A_{T+1} = (1 + r) * A_T + (1 - l_T) * K_T - c_T. \tag{7}$$

Since human capital after death, K_{T+1}, is not relevant for the agent, we know that $x_T = 0$ regardless of the choice of l_T. I compute $V_T(A_T, K_T)$ over a grid of values for A_T and K_T and obtain policy functions $c_T(A_T, K_T), l_T(A_T, K_T)$. I then interpolated $V_T(A_T, K_T)$ over those grid points using a bivariate cubic spline to obtain a continuous function of $V_T(A_T, K_T)$.

The value of the agent at $t = T - 1$ given state A_{T-1}, K_{T-1} is

$$V_{T-1}(A_{T-1}, K_{T-1}) = \max_{c_{T-1}, l_{T-1}, x_{T-1}} \{u(c_{T-1}, l_{T-1}) + \beta * V_T(A_T, K_T)\} \tag{8}$$

subject to

$$A_T = (1 + r) * A_{T-1} + (1 - l_{T-1}) * g(x_{T-1}) * K_{T-1} - c_{T-1}, \tag{9}$$

$$K_T = (1 - \delta) * K_{T-1} + a * x_{T-1} * (1 - l_{T-1}) * K_{T-1}, \tag{10}$$

$$l_{T-1} \in [0, 1]. \tag{11}$$

Since I have $V_T(A_T, K_T)$, I can compute $V_{T-1}(A_{T-1}, K_{T-1})$, $c_{T-1}(A_{T-1}, K_{T-1})$, $l_T(A_{T-1}, K_{T-1})$, $x_T(A_{T-1}, K_{T-1})$ over a grid of values of A_{T-1} and K_{T-1}. I interpolate $V_{T-1}(A_{T-1}, K_{T-1})$ over those grid points to obtain a continuous function $V_{T-1}(A_{T-1}, K_{T-1})$. I can then repeat the process for $t = T - 2$ and so on, to eventually obtain the value function and policy functions for all t.

References

Blinder, Alan S. and Yoram Weiss (1976). "Human Capital and Labor Supply: A Synthesis". In: *Journal of Political Economy* 84.3, pp. 449–472. ISSN: 00223808, 1537534X. URL: `http://www.jstor.org/stable/1829864`.