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LIFE-CYCLE LABOR SUPPLY WITH HUMAN CAPITAL ACCUMULATION*

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A dynamic model of labor supply under uncertainty with endogenous human capital accumulation is developed and estimated using 1968–81 PSID male panel data. Given a learning-by-doing technology for human capital investment and a translog utility function, structural parameters for preferences and technology are estimated from the orthogonality conditions implied by the Euler equations assuming rational expectations. The parameter estimates conform to economic theory. Simulations of the model suggest that the intertemporal labor supply elasticity with endogeneous wages will rise over the life cycle, producing different policy implications than typical models having exogenous wages and constant intertemporal elasticities.

1. INTRODUCTION

In the last fifteen years, theoretical models of the labor supply decision process have begun to incorporate both endogenous wage rates and individual uncertainty about exogeneous variables. The endogeneity of wages in the labor supply decision results from the belief that current hours of work indirectly increase future wage rates, by entering the production function of human capital investment. The inclusion of individual uncertainty regarding future realizations of exogeneous variables is important now that the labor supply decision is modelled as a life-cycle decision. This recent theoretical literature has demonstrated that the potential shape of the life-cycle profile of labor supply may diverge considerably from the profile which assumes exogeneous wages or perfect certainty.[2]

Empirical models of labor supply have lagged behind theoretical developments, despite the recent advances made possible by the availability of time-series cross-section data. Heckman and MaCurdy (1980) and MaCurdy (1981) estimated deterministic decision models in which individuals choose their life-cycle profiles of labor supply by maximizing a lifetime preference function, conditional on exogeneous wage rates. The importance of these models lies in their ability to distinguish between an agent's reactions to shifts in his wage profile versus movements along that profile. Later work by MaCurdy (1985) and by Hotz, Kydland and Sedlacek (1988) relaxed the perfect certainty assumption. MaCurdy (1985) presents an empirical model of life-cycle economic behavior in which consumers

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[2] See, for example, the theoretical results of Weiss (1972), Blinder and Weiss (1976), Heckman (1976a) and Ryder, Stafford, and Stephen (1976).

are uncertain about future realizations of prices and exogenous variables. Hotz, Kydland and Sedlacek (1988) estimate the structural parameters of a rational expections life-cycle model of economic behavior where agent's preferences follow a translog functional form specification which is time non-separable in its arguments. However, the estimation methodology employed in both these papers relies critically upon the assumption that the wage process is not controllable by the economic agent. Yet, in considering the life-cycle nature of the labor supply decision, this assumption is likely to be violated.

Labor supply models which account for the endogeneity of wage rates are difficult to implement empirically. The difficulty is caused by the analyst's inability to observe several of the agent's decision variables and some of the exogenous variables. These unobserved variables include the proportion of time spent in investment activities, the discrepancy between potential wage rates and observed wage rates, and the rental rate on the stock of human capital. In order to overcome the unobservable variables problem, and to derive analytically tractable solutions for the optimal path of the decision variables, fairly stringent assumptions on functional form are usually introduced. Heckman (1976b), Rosen (1976), Haley (1976), and Brown (1976) have produced estimates of the human capital structural parameters within a deterministic environment.[3] In Heckman's work, for example, empirical estimates of the human capital model were obtained by identifying the parameters of the utility and human capital production functions imbedded in a nonlinear earnings function.[4] Unfortunately, the recovered parameter estimates were consistently found to be very imprecise and to simulate the data poorly (Killingsworth, 1983).

The widespread belief that wages are actually endogenous to the labor supply decision has led many researchers to instrument wages in an *ad hoc* fashion as a means of making wages endogenous in their empirical work. It is easy to demonstrate that this technique will not produce unbiased estimates of supply elasticities, since labor supply equations derived under the assumption of exogenous wage rates will be misspecified when that assumption is false. The extent of this misspecification can be readily described if one recalls that in a life-cycle model the agent's first-order conditions with respect to the number of hours of market work will contain the expected present value of the marginal effect of current hours on all future wage rates weighted by future hours and the discount factor, in addition to the wage rate and the expected marginal utilty of lifetime wealth. The omission of the expected present value of the future benefits of current hours will cause an omitted variables bias. The severity of the bias depends crucially upon the current age of the specific agent under analysis. For new entrants, the

[3] Miller's (1984) paper on labor mobility, which links turnover behavior with tenure, constitutes the only attempt to estimate structural parameters of a human capital model within an uncertainty framework.

[4] When the optimal human capital investment equation and leisure equation is substituted into the earnings equations, the parameters of utility and human capital investment are established from earnings, as a function of time, initial human capital, and the marginal utility of initial assets. Heckman (1976b) uses a Cobb-Douglas utility function and neutral human capital production function in new investment time and the stock of human capital, $K(t) = c[I(t)\ K(t)]^{\eta}$.

omission of the present value variable should substantially affect estimates of the labor supply elasticities, because this variable would be largest for youth.[5] The larger the investment in human capital, the smaller the agent's reaction to any transitory change in wages. Since most human capital investment behavior should occur early in life, one might expect a systematic bias in the elasticities computed in the absence of endogenous human capital investment. Thus, the use of exogenous-wage labor supply elasticities may produce misleading results when examining the volatility of youth employment or the effects of tax changes on labor supply over the life cycle.

This paper utilizes cross-section time-series data and nonlinear instrumental variable econometric techniques to estimate structural parameters of a life-cycle labor supply model which assumes that agents are uncertain about future realizations of exogenous variables and that wages are endogenously determined. In Section 2, we develop the dynamic environment where agents rationally choose the optimal life-cycle paths for the decision variables. We assume that the individual's human capital investment function is limited to learning-by-doing, or that investment in human capital is a by-product of market work. In Section 3, we specify the functional forms for the agent's preferences and the human capital investment technology, and we propose a strategy to recover structural parameter estimates for the model. In Section 4 we present and analyze the parameter estimates obtained by fitting the proposed model to a sub-sample of data from the Panel Study of Income Dynamics. The estimated structural parameters are consistent with economic theory, producing empirical support for the role of human capital investment in altering labor supply under uncertainty. The role of endogenous human capital is amplified in Section 5, where simulations suggest that the intertemporal elasticity of substitution is not constant, but rises over time. These results are summarized in the conclusion.

2. MODEL OF LABOR SUPPLY WITH ENDOGENOUS WAGES UNDER UNCERTAINTY

The model of life-cycle labor supply developed herein is a dynamic model in which agents rationally choose the optimal life-cycle paths for their consumption of market goods, their number of hours of work, and their hourly wage rates. Agent's choices are made conditional on the information sets available to them at the time of the decision process. We assume that all past and current realizations of exogenous and control variables are known to the agents at the beginning of each time period, but that they are uncertain about future realizations of the exogenous variables. The agent's information set at the beginning of time period t is denoted Γ_t.

At a given time period t, the agent's life-cycle utility is a time-separable concave function of the number of hours of non-market time, L_t (often referred to below as leisure), and the consumption of market goods, C_t. Agents choose the

[5] Elasticities are affected if increases in current hours of work cause future wages to increase; we present evidence below that they do. See the paper by Shaw (1982) for further evidence and the derivation of the model described above.

optimal time paths for their consumption and leisure by maximizing their discounted expected life-cycle utility function over the working horizon T, or they maximize

$$E_t \sum_{\tau=t}^{T} \beta^{\tau} U(L_{\tau}, C_{\tau}) \tag{1}$$

subject to the asset accumulation constraint, the wage determination process, and conditional on the information set Γ_t. The utility function, $U(L_{\tau}, C_{\tau})$, is a concave twice differentiable function increasing in C_{τ} and L_{τ} and the discount factor is $\beta \equiv 1/(1+\rho)$ where $\rho \equiv$ the rate of time preference.

The per-period asset accumulation constraint is

$$A_{t+1} = (1 + r_t)(A_t + W_t h_t - C_t), \tag{2}$$

where A_t is the net real assets (or wealth) at the beginning of period t, r_t is the market interest rate, h_t is the number of hours allocated to market activities, and W_t is the real wage rate, assuming that the consumption good is acting as the numeraire. Given finite lifetimes, we also assume no bequests, so that $A_{T+1} = 0$.

The observed wage, W_t, is defined as the product of the human capital stock, K_t, times the rental rate on a unit of human capital, R_t:

$$W_t \equiv R_t K_t. \tag{3}$$

The rental rate, R_t, is the market price of the services of a unit of human capital. It is the market clearing price at which the aggregate supply and aggregate demand for human capital services are in equilibrium.

Agents are capable of increasing their future wages by engaging in activities in the present which enhance their future quantities of productive skills. Thus, wage rates are endogenous to the agent's consumption-leisure decision. Given the objective of deriving an empirically tractable human capital production model, we assume that the individual's human capital investment function is limited to *learning-by-doing*, or that investment in human capital is a by-product of market work. The more time an agent spends in the market, the higher his current earnings, and the higher his future productivity when engaged in market activities.

In general, the stock of human capital investment at time $t+1$, K_{t+1}, is equal to the previous stock of capital minus depreciation plus new investment:

$$K_{t+1} = (1 - \delta)K_t + g(h_t, K_t) \tag{4}$$

where δ is the depreciation rate and $g(h_t, K_t)$ is the human capital production function, which is concave, twice differentiable and increasing in h_t and K_t, the hours of work and the current stock of human capital respectively. Equation (4) states that the initial stock of human capital will depreciate over the course of the year at rate δ and human capital will simultaneously be augmented by combining the current capital stock and hours of work to form new human capital.

The parameters of the human capital production function can be identified by inserting the production function into the wage equation, W_{t+1}, or equation (4) into (3). However, without knowledge of the true functional form of $g(h_t, K_t)$, the distinction between new investment and depreciation can only be made by choosing a convenient functional form. In other words, in estimating the wage equation, the separation of the growth of the human capital stock into a depreciation and an investment component is either unidentified or very sensitive to functional form. Depreciation is unidentified if, for example, the depreciation rate and human capital production function are linear in K_t. Alternatively, if the production function is Cobb-Douglas, the depreciation rate and the parameters of the production function are identified. However, the identification of the true δ depends entirely upon the assumption that Cobb-Douglas technology represents the true human capital production function. To avoid the arbitrary choice of a specific functional form, we rewrite (4) as a general function, $f(h_t, K_t)$, assumed to incorporate both the growth of investment and the depreciation of the human capital stock:

$$K_{t+1} = f(h_t, K_t). \tag{5}$$

A quadratic approximation for $f(h_t, K_t)$ will be used in the empirical work.

Inserting the final production function (5) into wage equation (3), the human capital production function can be rewritten in terms of the rental rates on the human capital stock and observed variables:

$$W_{t+1}/R_{t+1} = f(h_t, W_t/R_t). \tag{6}$$

Because human capital develops in a learning-by-doing process, as a by-product of hours of work on the job, there is no difference between the potential wage rate and the observed wage rate, an assumption which is necessary to estimate a closed-form solution for labor supply. We do, however, add an important atypical feature to the model—we permit year to year variation in rental rates, R_t.

Wages, or the logarithm of wages, are typically regressed on a set of individual characteristics and measures of workers' experience to estimate fixed parameters which impose the restriction of time stationarity. In other words, in fitting wage equations to cross-section time-series observations, the parameter space is usually fixed over the sample's time period. However, earnings function (6) implies that changes in the rental rate for human capital will affect the earnings function in a non-trivial way, or that the failure to account for rental price changes over time will lead to a misspecified earnings function and potentially uninterpretable parameter estimates. Parameter estimates obtained by fitting wage functions on individual cross-sectional data can be expected to reflect the value of the rental rate on the human capital stock prevailing in that particular year. Comparing parameter estimates obtained by fitting wage equations to cross-sectional data collected in different years, one would expect these parameters to shift from year to year as the market rental rate on human capital changes over the business cycle. In analyzing individual data from 1968–1981 below, we find strong evidence that time shifts in earnings functions do indeed occur.

At time period t, the agent's decision process can now be formulated as the maximization of the value function

$$(7) \qquad V^t(A_t, W_t, R_t) = \max_{h_t, C_t} \{U(L - h_t, C_t) + \beta E_t V^{t+1}[(1 + r_t)(A_t + W_t h_t - C_t), R_{t+1} f(h_t, W_t/R_t), R_{t+1}]\}$$

where V^t denotes the maximum level of lifetime utility the agent can expect to attain if he optimally allocates all the decision variables, C_t and h_t, for $t = 1, \ldots, T$. V^t is a function of the asset stock, A_t, the hourly wage rate, W_t, and the rental value of human capital, R_t. Leisure, L_t, is replaced by $(L - h_t)$ in the utility function, where L is the maximum potential hours of work. E_t is the expectation operator conditional on the current information set, Γ_t, and r_t is the market rate of interest prevailing in time period t.

First-order conditions for the maximization of (7) are:

$$(8) \qquad E_t[-U_{1,t} + \beta(1 + r_t)W_t V_A^{t+1} + \beta R_{t+1} f_{1,t} V_W^{t+1}] = 0$$

and

$$(9) \qquad E_t[-U_{2,t} + \beta(1 + r_t)V_A^{t+1}] = 0,$$

where

$$U_{1,t} \equiv \partial U(L - h_t, C_t)/\partial h_t,$$
$$U_{2,t} \equiv \partial U(L - h_t, C_t)/\partial C_t,$$
$$f_{1,t} \equiv \partial f(h_t, K_t)/\partial h_t,$$
$$V_A^{t+1} \equiv \partial V^{t+1}/\partial A_{t+1}, \quad \text{and}$$
$$V_W^{t+1} \equiv \partial V^{t+1}/\partial W_{t+1}.$$

V_A^{t+1} is the marginal utility of wealth at time $t + 1$ and V_W^{t+1} is the change in the agent's welfare due to a change in wages at time period $t + 1$.

Conditional on the information set Γ_t, the agent's optimal allocation of resources over time implies that

$$(10) \qquad V_A^t = \beta(1 + r_t)E_t V_A^{t+1}$$

and

$$(11) \qquad V_W^t = E_t[\beta(1 + r_t)h_t V_A^{t+1} + \beta(R_{t+1}/R_t)f_{2,t} V_W^{t+1}]$$

where $f_{2,t} \equiv \partial f(h_t, K_t)/\partial K_t$.

Using equation (9), rewrite (8) and (11) as

$$(12) \qquad E_t[-U_{1,t} + W_t U_{2,t} + \beta R_{t+1} f_{1,t} V_W^{t+1}] = 0$$

and

$$(13) \qquad V_W^t = E_t[h_t U_{2,t} + \beta(R_{t+1}/R_t)f_{2,t} V_W^{t+1}].$$

The first-order equation for leisure has been rewritten to more readily examine the effect of human capital investment on the optimal labor-leisure choice. To analyze this effect, decompose condition (12) into three parts: $-U_{1,t}$, which denotes the loss in current utility due to an increase in the number of hours worked; $W_t U_{2,t}$, which is the utility gain derived from the effect of increased earnings on the consumption of market goods; and $\beta R_{t+1} f_{1,t} V_W^{t+1}$, which measures the discounted increase in consumer welfare due to the higher level of wages in all future periods (where $f_{1,t}$ measures increases in the human capital stock and V_W^{t+1} measures the increase in consumer welfare derived from an extra unit of human capital). In the absence of endogenous human capital investment, condition (12) would collapse to

$$U_{1,t} = W_t U_{2,t}.$$

This is the traditional model wherein agents choose the optimal combination of consumption and leisure time which sets the ratio of the marginal substitution between consumption and leisure equal to relative prices at each time period. Thus, the bias in the parameter estimates based on this latter model may be quite significant if the third term in (12), $\beta R_{t+1} f_{1,t} V_W^{t+1}$, is large. The third term will be large if current on-the-job training significantly affects future wage rates, as we would expect. Thus, human capital investment may well play a prominent role in the labor-consumption decision.

To facilitate the estimation of equation (12), the number of unobservables in the first-order condition may be further reduced. Since the first-order conditions are expected to hold in period $t + 1$, along with equations (10) and (11), rewrite (12) by substituting for V_W^{t+1} from (13) and substituting for V_W^{t+2} from the first-order condition. After rearranging, (12) becomes:

$$(14) \qquad E_t\{f_{2,t+1}[-U_{1,t+1} + W_{t+1}U_{2,t+1}] - [f_{1,t+1}/(\beta f_{1,t})][-U_{1,t} + W_t U_{2,t}] - R_{t+1} f_{1,t+1} h_{t+1} U_{2,t+1}\} = 0.$$

Note that the marginal rate of substitution in consumption over time must equal one plus the rate of time preference divided by one plus the market rate of interest, or

$$(15) \qquad E_t[U_{2,t+1}/U_{2,t}] = 1/[\beta(1 + r_t)].$$

An alternative representation of (14) can be obtained by dividing (14) by $U_{2,t}$ and substituting in the marginal rate of substitution from equation (15), to get

$$(16) \qquad E_t\{f_{2,t+1}\{(-U_{1,t+1}/U_{2,t} + W_{t+1}/[\beta(1 + r_t)]) - [f_{1,t+1}/(\beta f_{1,t})][-U_{1,t}/U_{2,t} + W_t] - R_{t+1} f_{1,t+1} h_{t+1}/[\beta(1 + r_t)]\}\} = 0.$$

Although condition (16) has expection zero at time period t, actual realizations of the future random variables imply that

$$(17) \quad f_{2,t+1}(-U_{1,t+1}/U_{2,t} + W_{t+1}/[\beta(1 + r_t)]) - [f_{1,t+1}/(\beta f_{1,t})][-U_{1,t}/U_{2,t} + W_t] - R_{t+1} f_{1,t+1} h_{t+1}/[\beta(1 + r_t)] = u_{t+1}$$

where u_{t+1} is the forecast error associated with equation (16) in period t. Since the individual choice process is assumed to be rational, it must be the case that

$$E_t\{u_{t+1}\} = 0$$

for all t, or that u_{t+1} is orthogonal to all the elements of the information set Γ_t. This property will be utilized in the next section to recover the structural parameter estimates for this model.

3. ECONOMETRIC SPECIFICATION

In this section, the methods of estimating the structural parameters underlying the agent's preferences and the structural parameters of the human capital production technology are described. The properties of the specific functional forms chosen for the agent's utility functions are also discussed, as well as those for the human capital production function.

We propose a two-stage estimation strategy to estimate the Euler equation (17). As originally proposed by Hansen and Singleton (1982) and by MaCurdy (1985), estimators of the structural parameters of the first-order conditions can be derived by exploiting the implications of economic rationality in the agent's behavior under uncertainty. Their work forms the basis of the estimation. However, the estimation of the structural parameters of the human capital labor supply model differs fundamentally from the estimation of parameters for the standard labor supply model which lacks investment in human capital. Parameter estimators based on equation (17) cannot be implemented directly, because the rental rates on human capital stock, R_t, are not observed and enter the model nonlinearly. Thus, a two-stage estimation strategy can be utilized to take advantage of the cross-section time-series aspect of the available data. In the first step the wage equation is estimated. The wage equation is a technological relationship relating current hours of work and current wages to future wages. By assuming that all agents within each cross-section are facing identical prices, the yearly unobserved rental rates on the human capital stock can be estimated jointly with the structural parameters of the human capital production function.[6] The second step incorporates these estimates into the labor-leisure condition, (17), and the remaining parameters of the utility function are estimated.

The human capital production function, (5), which is defined above for a general functional form of the production function, $f(\ )$, is now assumed to be quadratic in its arguments. This specification of the human capital production function is selected to represent the likely concave nature of the life-cycle earnings profile, arising from the combined influence of the depreciation of the stock of human capital and new investment. The specific functional form is

$$K_{i,t+1} = \alpha_1 K_{it} + \alpha_2 K_{it}^2 + \alpha_3 K_{it} h_{it} + \alpha_4 h_{it} + \alpha_5 h_{it}^2 + \zeta_t + \varepsilon_{it} \tag{18}$$

where α_1, α_2, α_3, α_4, and α_5 are the structural parameters of the human capital

[6] Heckman and Sedlacek (1985) and Sedlacek (1983) have utilized a similar procedure to recover estimates for latent sector-specific skill prices.

production function, ζ_t is an exogenous time-specific growth rate of human capital which is common across all agents, and ε_{it} represents the exogenous individual-specific component of human capital growth.[7] ζ_t and ε_{it} are assumed to be unobserved by the econometrician, but are known to the economic agent at the beginning of time period t. ε_{it} is modelled as a mean zero random variable defined to be orthogonal to all observed arguments in the human capital production function. In addition, ε_{it} is also assumed to be orthogonal to the rental rates for the human capital stock in time period $t + 1$, R_{t+1}.

As noted before, equation (18) cannot be estimated directly since the human capital stock K_t is not observed by the econometrician. However, wages are observed, and equation (18) can be rewritten in terms of observables, as was done to produce equation (6) above:

$$W_{i,t+1}/R_{t+1} = \alpha_1 W_{it}/R_t + \alpha_2 W_{it}^2/R_t^2 + \alpha_3 h_{it} W_{it}/R_t + \alpha_4 h_{it} + \alpha_5 h_{it}^2 + \zeta_t + \varepsilon_{it} \tag{19}$$

where the definition $W_t \equiv R_t K_{it}$ has been utilized to substitute for K_{it} ($\equiv W_{it}/R_t$). Equation (19) is a description of the individual wage growth process which values human capital at the rental rate R_t. To introduce individual heterogeneity in the productivity of investment, (19) may be estimated as a varying parameters model

$$\alpha_j = \alpha_{j1} + \alpha_{j2} \mathbf{D}_i \qquad \text{for} \qquad j = 1, \ldots, 5 \tag{20}$$

where $\mathbf{D}$ is a vector of the agent's observed demographic characteristics, such as education, which remain constant over the sample period.

The parameters to be estimated in (19) are the α_j, for $j = 1, \ldots, 5$, and the rates of return R_t, for $t = 1, \ldots, T + 1$. It is also necessary to normalize one of the rental rates to a constant, chosen to be $R_1 = 1.0$, to fix the units in which human capital (an unobservable) is measured. It is then easy to demonstrate that the identification of all the structural parameters in wage function (19) requires a minimum of three years of longitudinal data. Conditional on the assumption that ε_{it} has an expected value equal to zero and is orthogonal to all observed variables in $f(\ )$, standard instrumental variables procedures can be utilized to recover estimates for all structural parameters, $\alpha = (\alpha_1, \ldots, \alpha_5)$, and for unobserved rental rates, R_t, $t = 1, \ldots, T + 1$.

The next step is the estimation of condition (17) to recover the preference parameters, conditional on the parameter estimates obtained in the first stage and conditional on the functional form of the utility function $U(\ )$. The translog utility function is selected for its well-known properties. The translog, or transcendental logarithmic utility, is quadratic in logarithms of the quantities consumed and therefore represents a local second-order approximation to any utility function. As such, it does not impose the restrictions of additivity and homo-

[7] Note that ε_{it} is not specified to include a fixed individual-specific effect in (18), or in the equivalent wage equation (19). The common reason for individual-specific effects is that wage equations omit unobserved ability. This is not a major concern here. To see this, note that K_t is a function of the initial stock of human capital, and thus is a function of individual ability. Or, the fixed effect enters the wage equation through the lagged wage rate.

theticity associated with many common utility functions, such as the Cobb-Douglas or CES.[8] The exact specification of the utility function is

$$U_t = \ln L_t + \gamma_1 \ln C_t + \gamma_2(\ln L_t)(\ln C_t) + \gamma_3(\ln L_t)^2 + \gamma_4(\ln C_t)^2 \tag{21}$$

where ln is the natural log and the coefficient on $\ln L_t$ is 1.0 for normalization.[9] Individual observed heterogeneity in preferences are introduced by specifying a varying parameters model wherein

$$\gamma_j = \gamma_{j1} + \gamma_{j2}\mathbf{D} \qquad \text{for} \qquad j = 1, \ldots, 4, \tag{22}$$

analogous to (20) above.

Differentiating the translog utility function, the resulting marginal utility of nonmarket time, $U_{1,t}$, and the marginal utility of consumption, $U_{2,t}$, are now

$$U_{1,t} = [1 + \gamma_2 \ln C_t + 2\gamma_3 \ln L_t]/L_t \tag{23}$$

and

$$U_{2,t} = [\gamma_1 + \gamma_2 \ln L_t + 2\gamma_4 \ln C_t]/C_t \tag{24}$$

for $t = 1, \ldots, T$, and where $\gamma_j = \gamma_{j1} + \gamma_{j2}\mathbf{D}$ for $j = 1, \ldots, 4$.

The marginal product of hours of work, $f_{1,t}$, and the marginal product of the stock of human capital, $f_{2,t}$, are obtained by differentiating the quadratic human capital production function specified in equation (19):

$$f_{1,t} = \alpha_4 + \alpha_3 W_t/R_t + 2\alpha_5 h_t \tag{25}$$

and

$$f_{2,t} = \alpha_1 + \alpha_3 h_t + 2\alpha_2 W_t/R_t \tag{26}$$

for $t = 1, \ldots, T$, and where $\alpha_j = \alpha_{j1} + \alpha_{j2}\mathbf{D}$ for $j = 1, \ldots, 5$.

To simplify the notation in equation (17), define $h(\mathbf{X}_t, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1})$ as

$$\begin{aligned} h(\mathbf{X}_t, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1}) \equiv\, & f_{2,t+1}[-U_{1,t+1}/U_{2,t} + W_{t+1}/(\beta(1 + r_t))] \\ & - [f_{1,t+1}/(\beta f_{1,t})][-U_{1,t}/U_{2,t} + W_t] - R_{t+1} f_{1,t+1} h_{t+1}/[\beta(1 + r_t)], \end{aligned} \tag{27}$$

where $\mathbf{X}_t$ denotes the vector of variables which enter condition (17), $\boldsymbol{\theta}_0$ is a vector of population parameters, $\boldsymbol{\theta}_0 = (\beta, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$, and the function $h(\ )$ is conditional on the true parameter vector from the human capital production function, $\boldsymbol{\alpha}_0 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$, and the rental rates of human capital for periods t and $t + 1$, R_t and R_{t+1}. Condition (17) can now be rewritten as

$$h(\mathbf{X}_t, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1}) = u_{t+1}. \tag{28}$$

Assuming that individuals behave rationally, we have noted that

$$E_t\{u_{t+1}\} = 0$$

[8] See Christensen, Jorgensen, and Lau (1975) for a discussion of these properties.

[9] The budget shares resulting from the maximization of the translog utility are homogeneous of degree zero in the γ parameters, so a normalization is required. See Christensen, Jorgensen, and Lau (1975).

for all t, or u_{t+1} is orthogonal to all elements of the individual information set Γ_t. If the information set at time t is denoted $\Gamma_t = (z_{1t}, \ldots, z_{kt})$ then

$$E_t\{u_{t+1} z_{jt}\} = 0 \tag{29}$$

for $j = 1, \ldots, k$.

The orthogonality between $h(\mathbf{X}_t, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1})$ and Γ_t, in equation (29), can be exploited for the estimation of $\boldsymbol{\theta}_0$. Assume that we have panel data for T years for each individual. The population orthogonality conditions for these years can then be rewritten as

$$E \sum_{t=1}^{T} h(\mathbf{X}_t, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1}) \mathbf{z}_{jt} = EM(\mathbf{X}_t, \mathbf{z}_{jt}, \boldsymbol{\theta}_0; \boldsymbol{\alpha}_0, R_t, R_{t+1}) = 0 \tag{30}$$

over $j = 1, \ldots, k$. To convert to sample orthogonality conditions, assume that $\mathbf{u}_i \equiv (u_{i1}, \ldots, u_{iT})$ is independently distributed across individuals. The sum of (30) over a random sample of N individuals should therefore approach zero for large values of N, or

$$\mathbf{O}_N(\boldsymbol{\theta}) = 1/N \sum_{i=1}^{N} M(\mathbf{X}_i, \mathbf{z}_i, \boldsymbol{\theta}; \boldsymbol{\alpha}_0, R_t, R_{t+1}) \tag{31}$$

should approach zero.

Estimates of θ are then obtained by minimizing the quadratic form

$$\mathbf{O}_N(\boldsymbol{\theta})' \mathbf{W}_N \mathbf{O}_N(\boldsymbol{\theta}) \tag{32}$$

where $\mathbf{W}_N$ is a symmetric positive definite weighting matrix which may be a function of sample information. Define $\boldsymbol{\theta}_N$ as the estimator from minimizing (32). Consistency and asymptotic normality are established by Hansen (1982) under the assumption that $\boldsymbol{\alpha}_0$, R_t and R_{t+1} are fixed at their true values.

There are several advantages to the non-linear instrumental variables procedure proposed above. The first is that the computation of $\boldsymbol{\theta}_N$ does not require the specification of the underlying distributions of the random variables. Secondly, the variance-covariance matrix of u_t is not assumed to be homoscedastic. Lastly, note that aggregate time-specific shocks which may affect u_t will not affect the consistency of the estimator when properly accounted for in the specification of the Euler equations.

4. EMPIRICAL RESULTS

The data source utilized in this study consists of a subsample of 526 randomly selected white males derived from the Panel Study of Income Dynamics (PSID) data base. The subsample covers 18–64 year-old individuals surveyed annually from 1968 to 1981.[10] The actual calendar years for the data are 1967 to 1980. All

[10] Thus, the total sample size is 5873 (526 * 14 minus missing values). Due to the cost and capacity constraints involved in estimating a highly nonlinear model, the entire PSID data set was not used. A random sample of 600 males age 18–64 was selected and then non-whites were dropped from the sample to yield a sample size of 526.

results are presented according to calendar year instead of survey year. The PSID procedure of interviewing families, not individuals, introduces new cohorts into the data set as the panel ages. As children leave their parents home they enter the data base as new heads of households. When data are missing for an individual during a particular year, either because he enters the sample late or due to no response, that year is dropped from the data set. An individual is included in the sample as long as he has non-missing data for at least two years during the sample period.

The variable definitions and summary statistics are reported in the Appendix. The labor supply variable is annual hours of work divided by 1000 and the wage rate variable is that which is derived from reported income deflated by the 1967 base CPI.

Table 1 presents the estimates of the structural parameters of the human capital production functions (19) and (20). As discussed previously, equation (19) specifies the human capital production function to be quadratic in hours of work and in the stock of human capital. The structural parameters of the human capital production function are allowed to vary linearly with education according to equation (20). The instrumental variable set utilized in the nonlinear instrumental variables procedure consists of; the current annual values of wages, of hours of work, schooling, age, the local unemployment rate, a south dummy, year dummies, an interaction between hours and wage rates, and squared values of wages.

Column 1 of Table 1 presents the estimates of the structural coefficients of the human capital production function based on the assumption that these parameters are invariant to changes in the education level of a given agent. These estimates indicate that current wage rates are a strong predictor of future wages, decreasing in predictive power as wages increase. Interpreting this in terms of human capital production, under the assumption that $K_t \equiv W_t/R_t$, the results indicate that human capital investment increases with the stock of current human capital at a decreasing rate. Decreasing marginal productivity of the stock of human capital is in accordance with most human capital production functions specified in the literature.

The "learning-by-doing" human capital investment hypothesis implies that current hours of work should play a causal role in increasing future wage rates. The results in Table 1 demonstrate that this is indeed the case. Though the HOURS variable has a negative linear effect on future wages, it has very strong positive interactions with current wages and hours. All coefficients are significantly different from zero. The positive WAGE * HOURS interaction indicates that current hours of work augment the stock of human capital in the investment function, or that hours of work and the current stock of human capital (represented by the current wage) are complements in the production of human capital.[11]

[11] The positive effect of hours on wages may reflect tied wage-hours offers as well as the impact of on-the-job learning. However, the hours effect herein would represent an unusual form of tied offers:

TABLE 1
HUMAN CAPITAL PRODUCTION

		1 Without Observed Heterogeneity	2 With Observed Heterogeneity
A. Structural Coefficients			
	WAGE	.479 (.081)	2.56 (.773)
	WAGE * SC		−.141 (.052)
	WAGE^2	−.021 (.005)	−.064 (.063)
	$\text{WAGE}^2 * \text{SC}$		.002 (.003)
	WAGE * HOURS	.299 (.050)	−.766 (.335)
	WAGE * HOURS * SC		.070 (.022)
	HOURS	−3.550 (1.84)	−.783 (1.16)
	HOURS^2	.686 (.382)	.162 (.249)
B. Estimated Annual Human Capital Rental Rates (in natural log)			
	1968	.154 (.144)	.472 (.421)
	1969	.279 (.230)	.711 (.542)
	1970	.042 (.241)	.417 (.579)
	1971	.079 (.273)	.518 (.623)
	1972	.135 (.295)	.553 (.684)
	1973	.224 (.306)	.735 (.685)
	1974	.013 (.319)	.695 (.713)
	1975	−.106 (.313)	.934 (.771)
	1976	.127 (.286)	.824 (.658)
	1977	.063 (.311)	.765 (.670)
	1978	.051 (.343)	.725 (.691)
	1979	−.121 (.373)	.631 (.726)
	1980	−.496 (.529)	.438 (.800)
C. Annual Time Intercepts			
	1967	4.31 (2.04)	1.45 (2.33)
	1968	4.96 (1.98)	2.70 (1.89)
	1969	5.12 (1.41)	2.91 (1.37)
	1970	4.46 (1.31)	2.47 (1.36)
	1971	4.64 (1.35)	2.73 (1.51)
	1972	4.96 (1.47)	2.92 (1.66)
	1973	4.84 (1.18)	2.84 (1.69)
	1974	4.13 (1.01)	2.78 (2.09)
	1975	3.97 (1.34)	3.45 (1.72)
	1976	4.77 (1.48)	3.22 (1.78)
	1977	4.53 (1.37)	3.11 (1.68)
	1978	4.11 (1.10)	2.66 (1.58)
	1979	3.82 (.70)	2.88 (1.09)
Chi-Square Statistic	42.665	13.9595	
	Degrees of Freedom	14	11
	Level of Significancy	0.999	0.7647

Asymptotic standard errors are in parentheses. Column (1) estimates text equation (19) and column (2) adds text equation (20).

rather than the usual cov $(W_t, H_t) > 0$, ours implies that cov $(W_t, H_{t-1}) \mid W_{t-1}) > 0$, or greater hours are tied to a higher growth rate of wages, not a higher wage level. This hours effect on the wage growth rate could represent greater worker motivation, or unobserved heterogeneity in growth rates. The schooling interactions are attempts to partially control for such worker heterogeneity, but would not control for it completely.

The net effect of current hours of work on future wage rates is strongly positive when all interactions are included. A temporary 25 percent increase in hours of work increases next year's wages by 12.8 percent, starting from initial mean values of all variables.[12] This empirical result, that current hours of work exert a significant positive effect on future wages, is an important first test of the human capital model proposed in this paper. It is evidence in favor of the hypothesis that wages are endogenous to the agent's decision process.

The results presented in column 2 of Table 1 introduce observed individual heterogeneity into the parameters of the human capital production function, as described by equation (20).[13] In that specification, schooling is interacted with the WAGE, WAGE * HOURS, and with WAGE^2. The first two of the interactions are significantly different from zero, and the joint Chi-square test for the inclusion of all schooling interactions is $\chi^2(3) = 28.71$, strongly rejecting the restriction that the interactions are jointly zero. Interactions of schooling with HOURS and HOURS^2 are not added, because an F-test based on unrestricted ordinary wage regressions strongly rejected their inclusion. The estimated parameters of the human capital production function suggest that schooling augments post-school human capital production, or that the two are complements. A ten percent increase in schooling, at mean values of wages and hours, will increase wage rates by 1.78 percent.[14]

The estimated rental rates on human capital, R_t, are presented in the middle of Table 1. Rental rates are estimated in log form to constrain them to be positive (though from the results below, it is clear that this was not a binding constraint). Translating the estimated logarithm of the rental rate presented in Table 1 into the estimated rental rate levels, we get:

Year	1967	1968	1969	1970	1971	1972	1973
No Heterogeneity	1.00	1.12	1.32	1.04	1.08	1.14	1.25
With Heterogeneity	1.00	1.60	2.04	1.52	1.68	1.74	2.08

Year	1974	1975	1976	1977	1978	1979	1980
No Heterogeneity	1.01	.90	1.14	1.06	1.05	.89	.61
With Heterogeneity	2.00	2.54	2.28	2.15	2.06	1.88	1.56

where heterogeneity refers to the introduction of schooling effects in the parameters of the human capital production function. Comparing the estimated rental

[12] In this calculation, and others reported below, the mean values of the prices and time intercepts are utilized, so the reported effect does not refer to any one year.

[13] Several alternative specifications were also estimated: additional lag variables; more interactions between wages, hours, and education; and serially correlated residuals. In all cases, the inclusion of last year's wage eliminates these secondary effects.

[14] The level of schooling is constant for each individual over the sample period, so in analyzing the effect of an increase in schooling we are making comparisons across individuals. This implies that the positive effect may well represent the unobserved characteristics of individuals that are correlated with schooling.

prices presented in row 1 to those presented in row 2 we find that the estimated rates are somewhat sensitive to the specification of the human capital production function, as would be expected. The set of returns from row 1 have a lower mean value of 1.04 compared to 1.86 for row 2, and the two sets have a simple correlation of 0.18. Refering to the standard errors in Table 1, it is clear that these rental rate parameters are not precisely estimated—none are significantly different from zero. However, the Chi-square test for the unrestricted rental rates versus constant rates is $\chi^2(13) = 26.408$, which strongly rejects the restrictions at probability .018.[15]

These rental rate results indicate that standard wage equations should control for year-to-year variation in the returns to human capital investment. The inclusion of rental rates is likely to be important in producing human capital production function parameter estimates which are not proxies for business cycle effects. For example, hours of work may be positively correlated with wage rates over the business cycle because individuals with high firm-specific human capital, and therefore high wage rates, are the least likely to experience hours reductions during recessions. Controlling for variation in rates of return over the cycle reduces this potential spurious correlation between hours and wages.

The correct specification of the human capital investment equation and the leisure equation depend upon the interpretation of the estimated wage deflators as consistent estimates of the rental rates on the human capital stock. To provide a market interpretation to these estimated rental rates on human capital, we compute several simple correlations between the rental rates and price shocks and cyclical indicators. The correlations between the rental rate and five variables over 1967–1980 are: the annual producer price index for food, $-.592$; a fuel index, $-.815$; an index for the cost of capital, .011; the growth of real GNP, .574; and the change in the unemployment rate, $-.478$.[16] The very strong negative correlations with food and fuel indices suggest that supply shocks, and thus shocks to aggregate output, caused the wage rates of more skilled workers to adjust more than those of less skilled workers. The common perception that hours of work adjust more than wages for less skilled workers over the cycle, and that union wages rose relative to nonunion wages during the oil shocks of the 1970's, would both account for the negative correlations that are obtained. On the basis of this evidence, we suggest that the interpretation of the estimated rental rates as proxies for human capital rental rates appears reasonable.

The bottom section of Table 1 presents the estimates of time intercepts which are added to equation (19) and also presents the overall Chi-square statistics. The additive time intercepts may represent omitted variables which are common across all individuals at each point in time, and which alter the aggregate time

[15] This statistic refers to the wage equation with heterogeneity, but the value is very similar, at 26.61 for the linear wage equation.

[16] The producer price indices for food and fuel are those for the crude state of processing and the measure of the cost of capital is the real rental rate for producers durable equipment and nonresidential construction, calculated by the Federal Reserve Board to reflect changes in real interest rates, the rate of return on equity, and economic depreciation.

profile of wages.[17] An example of one such omitted variable is the fluctuation in aggregate labor productivity due to technological change. The Chi-square statistics for the fit of the overall model indicate that the model fits the data very well.

Moving to the second stage of the estimation procedure, we recover the structural parameters associated with the leisure equation (17), conditional on the consistent parameter estimates obtained by fitting the wage function. The parameter estimates of the wage equation, which are the estimates of the rental rates and the technology of human capital production, are now fixed at the values reported in Table 1 column 2. These parameter estimates for human capital production are those which included observed heterogeneity in the parameters.[18] The leisure equation is based on utility function (21). Estimation of (17) also requires data on interest rates and on consumption. The real interest rate utilized is the nominal prime rate of interest adjusted with the 1967 base CPI. Consumption data is available from the PSID data set, though only for food expenditures. The annual value of food expenditures for food consumed at home plus food eaten out is deflated by the food component of the CPI. Because the food expenditure question was not asked in 1967, 1968 and in 1975, the maximum number of time periods for the leisure equation falls to ten. We define the amount of non-market time consumed by an agent, L_t, as equal to the total number of potential annual hours to be utilized in any activity $(365 * 24)$ minus the observed annual hours of work. The instruments used in estimating (17) are; the current values of leisure, food, the wage rate, schooling, age, the local unemployment rate, a south dummy, annual time dummies, interactions between leisure and food, between leisure and wages, and between wages and food, and squared values of wages, food, and leisure.

The structural preference parameters which can be identified from the leisure condition are reported in Table 2. Though all parameter estimates are significantly different from zero, the Chi-square for the overall fit of the basic model indicates a very weak fit for the model. Observed heterogeneity is added to the parameter specification in column 2 by interacting all parameters with schooling, as specified in equation (22) of Section 3. Introducing heterogeneity improves the fit of the model: the Chi-square statistic for the null hypothesis that the structural parameters of preferences are not a linear function of schooling yields a Chi-Squared statistic of 12.419 (with 5 degrees of freedom), which implies a level of significancy of 0.03 percent.

Turning to an examination of the point estimates of the parameters, the first estimated parameter represents $\beta = 1/(1 + \rho)$, or one over one plus the rate of time preference. The implied discount rate in column 1 is an unreasonable nega-

[17] This addition changes the structure of the error term specified in equation (17) by assuming that u_{t+1} is the error term conditional on a time-specific intercept.

[18] To test for sensitivity to the human capital production function specification, all results for the leisure equation are duplicated with the simplified human capital production function of column 1 of Table 1. They are presented in Table A2 in the Appendix. The differences are relatively minor, some of which are discussed in subsequent footnotes.

TABLE 2
LEISURE EQUATION
(Conditional on the Wage Equation Parameters reported in column 2 of Table 1.)

	1 Without Observed Heterogeneity	2 With Observed Heterogeneity	3 With Observed Heterogeneity in All Parameters But β
β_{11}	1.117 (.151)	.0111(2.32)	.958 (.224)
β_{12}		.0693(.177)	
γ_{11}	.0155 (.0031)	.128 (.105)	.128 (.138)
γ_{12}		−.0075(.0076)	−.0076 (.0096)
γ_{21}	−.008 (.0015)	−.0638(.052)	−.0637 (.0679)
γ_{22}		.0037(.0039)	.0037 (.0048)
γ_{31}	−.270 (.0030)	−.269 (.022)	−.2690 (.0366)
γ_{32}		.0009(.0018)	.0010 (.0025)
γ_{41}	−.00014(.00012)	.0015(.0037)	.0007 (.0027)
γ_{42}		−.0001(.0002)	−.00005(.0002)
Chi-Square	6.314	0.236	0.5195
Degrees of Freedom	8	3	4
Level of Significancy	0.387	0.010	0.0284

Asymptotic standard errors are in parentheses. The symbols for the parameters are defined in equations (22)–(24) of the text, for the estimation of equation (26). The equations also contain annual time intercepts, which are presented in Appendix Table A3.

tive value. In column 2 the parameter estimate is interacted with schooling, but the schooling interaction is not significantly different from zero (t-statistic = 0.39). Thus, in column 3, the discount rate is again restricted to be constant across all individuals while all other parameters vary with schooling. The Chi-square for this restriction is $\chi^2(1) = .275$, which does not reject the restriction.[19] This specification produces a very reasonable annual discount rate of 4.2 percent.

The estimates of the parameters of the utility function, γ_j for $j = 1, \ldots, 4$, are all significantly different from zero and have plausible values. The coefficients on the logarithms of market goods and hours of leisure are positive, while their interactions and squared terms exhibit some curvature. The result is that both the marginal utility of leisure and the marginal utility of consumption are positive and both are subject to diminishing returns, when evaluated at the mean values of the variables.[20] In addition, the cross-partial between leisure and consumption is negative, indicating that hours of non-market time and market goods are substitutes in preferences.

[19] Note that when the leisure equation is specified under the assumption of no observed heterogeneity in the wage equation the comparable Chi-square statistic is $\chi^2(1) = 2.518$, indicating that the hypothesis is rejected at the one percent level of significance.

[20] Evaluating the marginal utility of leisure and the marginal utility of consumption at mean sample values we get: $U_{1,t} = .0079$, $U_{2,t} = .00073$. At the same mean sample values, the second-partials of the utility function take the values $\partial U_{1,t}/\partial L_t = -.061$, $\partial U_{2,t}/\partial C_t = -.00012$.

5. SIMULATIONS

The dynamic model estimated herein has important implications for life-cycle changes in hours of work, or for the intertemporal substitution of labor supply. Define the intertemporal substitution elasticity as the response of an individual's hours of work to evolutionary changes in wages over the life cycle. Previous work has treated these evolutionary wage changes as exogenous to the hours decision, resulting in a constant value of the intertemporal substitution elasticity.[21] The intertemporal substitution elasticity is not likely to be constant—it will vary over the life cycle if the wage rate is not the only marginal benefit of an additional hour of work. In particular, as described in Section 2, the first-order condition for work contains an additional term equal to the discounted increase in consumer welfare resulting from the higher level of wages in all future periods due to greater current hours of work. Or, the marginal benefit also includes the effect of current hours on all future wage rates. This additional term causes the intertemporal substitution elasticity to rise over time.

Given the estimates of the labor supply model with endogenous wage rates, the wage and hours profiles can be simulated to examine the variation in the intertemporal elasticity over the life cycle, assuming no stochastic shocks.[22] Conditional on the wage equation estimates from column 2 of Table 1, two sets of simulations are provided for the leisure equations corresponding to columns 1 and 3 of Table 2. Hours and wage profiles are simulated for varying levels of education for a worker with average initial earnings capacity, or human capital endowment, equal to \$3.30 per hour.[23]

[21] MaCurdy (1981) and Altonji (1986). Wages are treated as exogenous in the theory, but are acknowledged to be endogenous in the estimation, due in part to the role of human capital investment.

[22] There is no analytical solution for hours as a function of wages in equation (17), so point estimates of the intertemporal elasticity cannot be calculated. MaCurdy (1981) and Altonji (1986) are able to provide these point estimates by assuming exogenous wages and a utility function which is additively separable in consumption and leisure, so the hours equation reduces to a simple linear functional form. In a world of uncertainty and endogenous wages, it is not possible to express labor supply as a function of initial assets and a constant intertemporal elasticity, because these change over time, as hours move and realizations do not equal expectations.

[23] The simulations were done using the IMSL routine ZSPOW suggested by Zvi Eckstein. The preferred method of simulation is to simulate backward from terminal time T using starting values that maximize last period's utility. This proved to be infeasible. As is done for all labor supply papers, the sample used here is restricted to prime-age working males. As a result, maximization of terminal utility, with a corner solution of full-time leisure, produced predictions of small negative consumption. Another option is to start the simulation at the final year of full-time employment. The difficulty with this approach is that utility maximization requires a final wage rate. Because the wage rate is endogenous, choosing an arbitrary final wage rate constrains the optimal hours of work over the life cycle, and thereby makes the comparison of simulations of different levels of hours of work meaningless for different educational groups. Thus, the most advantageous simulation begins at time period zero, conditional on initial hours and wages. Wage rates can be set equal to initial human capital endowments, and alternative values of initial hours of work are set exogenously in a more clear cut fashion than would result from assuming a terminal wage rate. The hours and wage profiles are then estimated from the intertemporal relationships of the model, illustrating the life cycle variation in hours.

Before turning to these simulations, consider the determinants of the age variation in hours of work. The age-hours profile is a function of the value of the individual's rate of time preference and of life cycle changes in the intertemporal elasticity and wages. Regarding the rate of time preference, if the individual's discount rate exceeds the market interest rate, the individual prefers to consume more leisure now and work more later. The parameter estimates for the leisure equation, from column 3 of Table 2, estimate the discount rate to be 4.2 percent given an average real interest rate of less than 3 percent. Thus, the higher discount rate alone produces an upward sloping age-hours profile. If the life-cycle wage profile is also upward sloping, as empirical evidence suggests, a constant intertemporal substitution elasticity would also produce an upward sloping age-hours profile. Neither effect would cause the hours profile to slope downward with age.

In the first set of simulations, the more educated workers choose intertemporal labor supply paths which are downward sloping in the early years, while less educated choose upward sloping paths (Figure 1).[24] These results conform to the predictions of the model which permit life-cycle changes in the intertemporal substitution elasticity. In the estimates of the human capital production function, human capital endowments and post-school training are complements, as reflected in the strong positive interaction between education and hours in the wage equation. As a result, more educated workers have a greater incentive to intertemporally distribute their lifetime hours of work towards the early years of their career, because they earn a larger return in the form of higher future wages. The actual shape of the age-hours profile depends on the relative importance of on-the-job learning versus the preference to increase hours with age (because the rate of time preference exceeds the interest rate). In these simulations, the learning effect dominates the preference to work more with age, producing a downward sloping hours profile for the more educated.[25] A model with exogenous wages and a constant elasticity of intertemporal substitution would predict that hours rise and fall corresponding to the concavity of the wage profile. Thus, these simulations support the hypothesis that the intertemporal substitution elasticity rises over time, as the marginal effect of current hours on future wages falls.

A second set of simulations examine the magnitude of the impact of initial hours of work on future wage rates and hours of work. To do this, assume that

[24] Actual hours profiles generally slope upwards after labor force entry, in contrast to the downward sloping simulation for the more highly educated. Actual hours will differ from simulated hours if individual preferences for work rise, or opportunities for training rise, in the early years after labor force entry. The simulations omit this heterogeneity, which could enter as shocks to preferences or technology. Estimates for higher levels of education, 16 years and above, are not shown because we could not obtain convergence in the simulations for this leisure equation (the results for the alternative leisure education are in Figure A1).

[25] In the simulations based on the leisure equation of column 1 in Table 2, the interest rate exceeds the rate of time preference. As expected, the resulting hours profiles are all downward sloping, but once again, the changing intertemporal substitution elasticity causes hours to fall much faster for more highly educated workers (see Appendix, Figure A1).

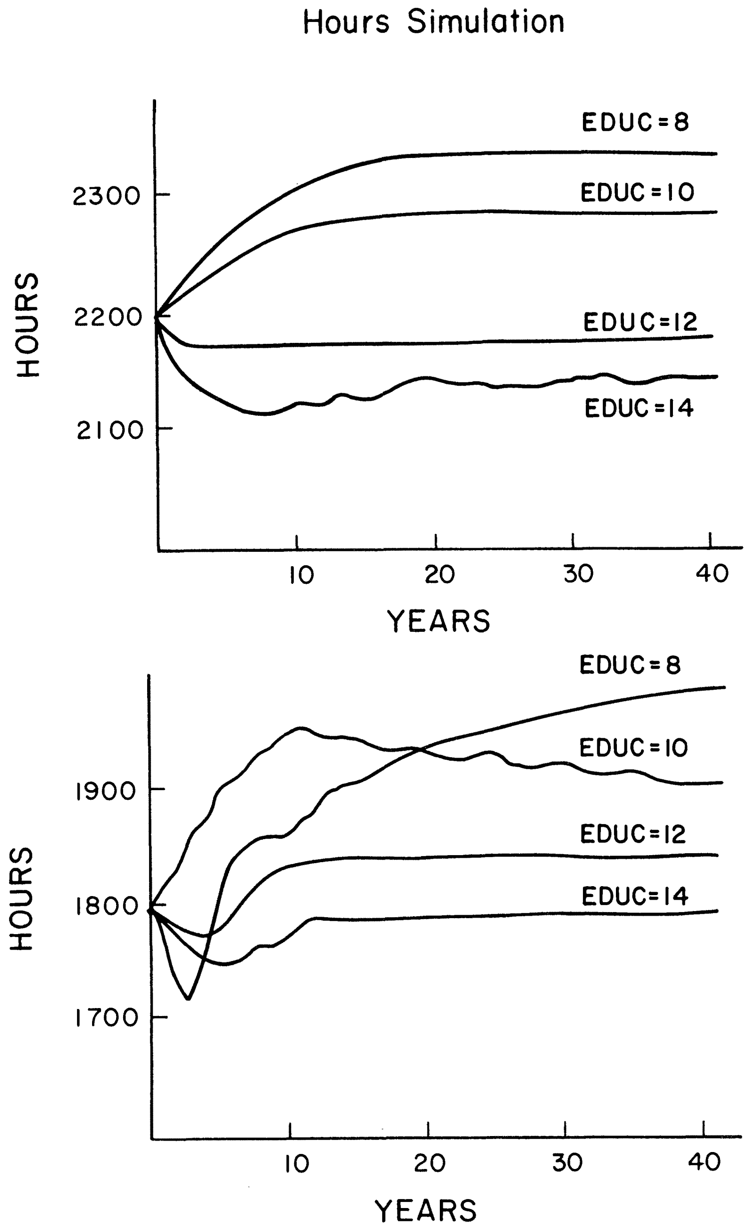

FIGURE 1

several workers experience temporary shocks to preferences which result in different initial hours of work. The resulting profiles for average high school graduates are displayed in Figure 2. Given high initial hours, the preference for postponing work is offset by the large marginal impact of current hours on future wages. This

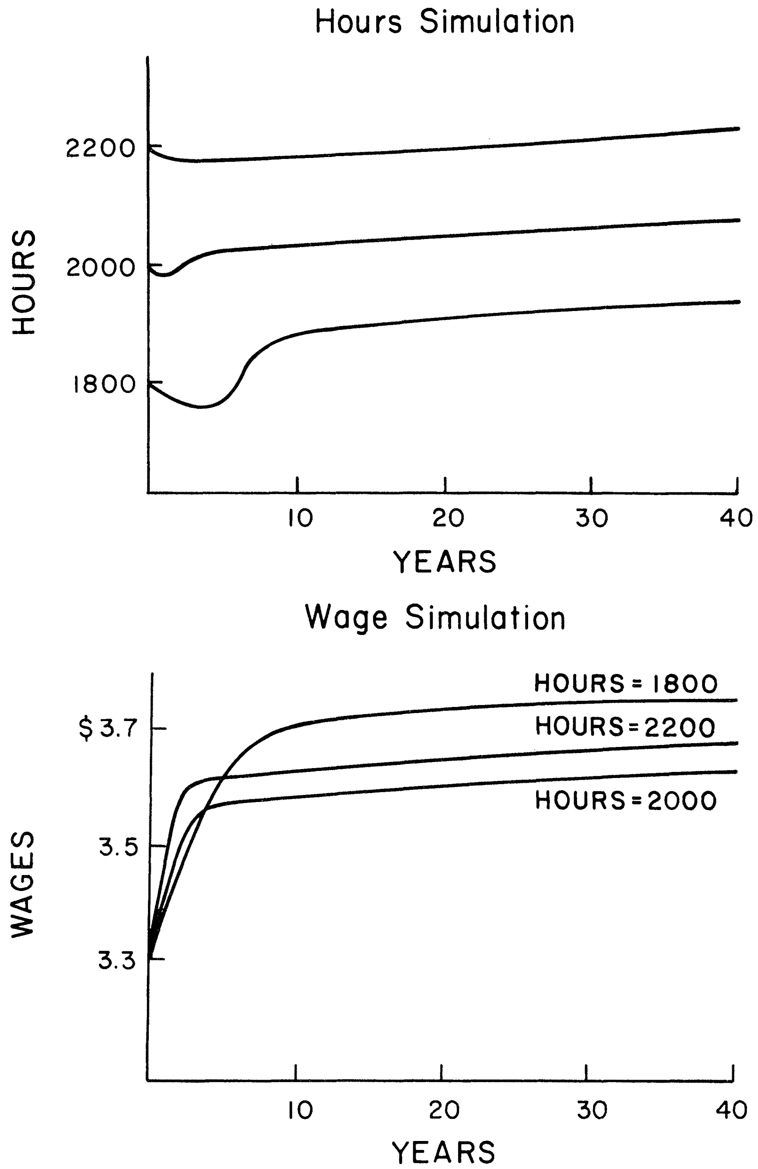

FIGURE 2
EDUCATION = 12

dampens future hours modestly. In contrast, given low initial hours, hours fall and then rise over the life cycle, as the preference of postponing work dominates.[26] These different hours profiles are also a function of different income effects. In all cases, wages grow rapidly early in the life cycle, then flatten to a

[26] Note that there is wider hours variation for some less educated groups, producing age-hours paths that cross—see Figure 1.

narrow range (\$3.54 to \$3.70), as the curvature in the hours effect on wages eventually limits the wage impact of high hours.

These simulations clearly suggest that policy recommendations based on the assumption of constant intertemporal elasticity of labor supply may be erroneous. While no point estimates for elasticities are provided here, the evidence suggests that the intertemporal elasticity evolves from negative to positive over the life cycle. As a result, policy changes which alter the net benefits of working, such as tax changes, will have significant distributional consequences.[27] Younger workers would be less responsive to the lowering of tax rates, relative to older workers.[28]

6. CONCLUSION

The primary hypothesis of this paper is that empirical labor supply models should account for the endogeneity of human capital investment in the agent's decision process. The importance of human capital investment in shaping labor supply decisions is grounded in the expectation that investment will affect labor supply differentially as individuals age. Assuming that on-the-job training is an important form of human capital investment, the finiteness of the agent's life cycle implies that the optimal investment strategy calls for investment early in the life cycle to reap the maximum return. This may well induce an increase in labor supply for younger workers, when wage rates are very low and unlikely to stimulate labor supply.

We derive and estimate a dynamic life-cycle rational expectations model of the joint determination of labor supply and human capital investment. The data set utilized in the analysis is the longitudinal data of the Panel Study of Income Dynamics. Estimation proceeds in two stages.

In the first stage, we estimate the parameters of the wage equation which represent the technological evolution of wage rates, caused by human capital investment and by changes in the rates of return to human capital. Two important results are forthcoming. First, the estimated rates of return to the stock of human capital vary significantly from year-to-year. This variation in rental rates follows a pattern which is inversely related to several proxies for alternative aggregate input prices, potentially indicative of input substitution. Very little previous empirical research on wage determination permits variation in rental rates. The second important result is that current hours of work have a significant effect on future wage rates, holding constant current wage rates. The implication is that hours of work are an important input into the production of new human capital. This result is crucial to our analysis, since it supports the hypoth-

[27] The heterogeneity in the intertemporal substitution elasticity may also result in biased average estimates of this elasticity, but no quantitative evidence on this bias is presented here.

[28] This statement refers to employed men working in jobs that have some human capital investment potential. Unemployed teenagers searching for unskilled jobs may be extremely responsive to changes in net wage rates. However, their low tax brackets also make their wage rates much less responsive to tax changes. If younger workers are liquidity constrained, this too may make them more responsive to tax changes.

esis that hours of work and human capital investment are jointly determined within the agent's decision process.

In the second stage, we estimate the structural preference parameters which can be identified from the first-order condition for labor supply. These parameters are estimated conditional on the estimated wage parameters. The resulting parameter estimates are significantly different from zero and exhibit positive and diminishing marginal utility in leisure and consumption.

The implications of these results are twofold. First, simulations of the estimated model suggest that the intertemporal elasticity of labor supply is not constant, but rises over the life cycle. As a result, policy changes, such as tax changes, will have significant distributional consequences, as younger workers are less responsive than are older workers. Secondly, these results suggest that models which estimate life-cycle labor supply decisions with endogenous wages can now be fruitfully approached, by specifying the agent's decision process within the rational expectations framework. Two logical extensions of this model include the development of alternative specifications of human capital production functions and the modelling of female labor supply behavior.

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APPENDIX

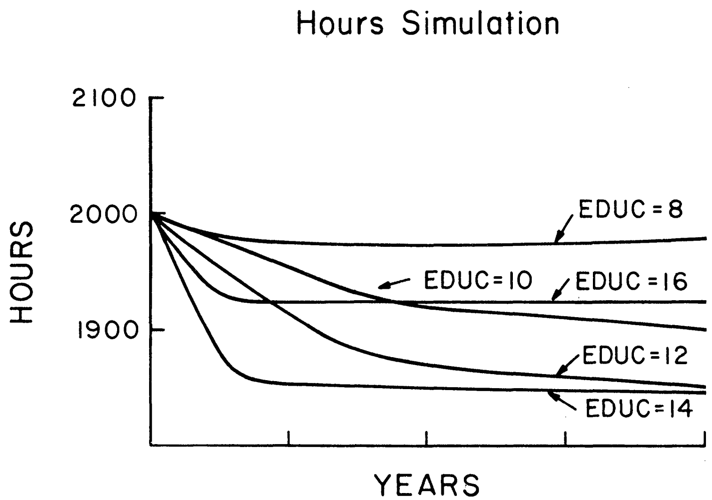

FIGURE A1

TABLE A1
MEANS AND STANDARD DEVIATIONS

WAGE	3.91 (6.73)	Annual wage rate divided by 1967–base CPI
HOURS	2.164 (.692)	Annual hours of work divided by 1000
SC	12.54 (2.85)	Years of formal schooling
AGE	37.84 (10.07)	Current age
UNEMP	9.69 (8.52)	Local area annual unemployment rate
REGION	.305 (.421)	Dummy = 1 if region = south
FOOD	1.632 (.927)	Annual food expenditures divided by CPI · 1000

TABLE A2
LEISURE EQUATION
(Based on Wage Equation Parameter Estimates of Column 1 of Table 1.)

	1 Without Observed Heterogeneity	2 With Observed Heterogeneity	3 With Observed Heterogeneity in All Parameters But β
β_{11}	1.167 (.087)	−2.026 (2.157)	1.214 (0.135)
β_{12}		.247 (.176)	
γ_{11}	−.0092 (.0043)	−.078 (.112)	−0.028 (0.068)
γ_{12}		.0055 (.0086)	0.0017 (0.0052)
γ_{21}	.0047 (.0022)	.041 (.058)	0.012 (0.033)
γ_{22}		−.0029 (.0044)	−0.0007 (0.0025)
γ_{31}	−.262 (.0016)	−.232 (.035)	−0.271 (0.026)
γ_{32}		−.0024 (.0029)	0.0004 (0.0018)
γ_{41}	.00030(.00016)	.0028 (.0025)	0.0005 (0.0012)
γ_{42}		−.00021(.0002)	−0.00004(0.00008)
Chi-square	6.480	0.0713	2.589
Degrees of Freedom	8	3	4
Level of Significancy	0.406	0.005	0.371

Asymptotic standard errors are in parentheses. The symbols for the parameters are defined in equations (22)–(24) of the text, for the estimation of equation (26). The equations also contain annual time intercepts, which are presented in Appendix Table A3.

TABLE A3
ESTIMATED ANNUAL INTERCEPTS IN LEISURE EQUATION
CORRESPONDING TO EQUATIONS OF TABLES 2 AND A2

	2.1	2.2	2.3
1969	.0006(.0011)	−.0022(.0024)	−.0001(.0027)
1970	−.0019(.0033)	−.0019(.0031)	−.0016(.0028)
1971	.0001(.0036)	.0008(.0029)	.0008(.0030)
1972	.0055(.0064)	.0040(.0051)	.0046(.0052)
1973	.0023(.0023)	.0031(.0029)	.0026(.0020)
1975	.0003(.0008)	.0003(.0020)	.0002(.0018)
1976	.0005(.0010)	−.0002(.0023)	−.0004(.0017)
1977	−.0020(.0020)	.0004(.0018)	.0004(.0016)
1978	−.0017(.0019)	.0001(.0014)	−.0001(.0014)
1979	−.0020(.0004)	−.0028(.0015)	−.0032(.0014)
	A2.1	**A2.2**	**A2.3**
1969	.0018(.0010)	−.0020(.0044)	.0004(0.0021)
1970	.0009(.0015)	−.0020(.0033)	.0001(0.0016)
1971	.0030(.0013)	.0004(.0043)	.0012(0.0029)
1972	.0024(.0022)	.0053(.0074)	.0069(0.0056)
1973	.0104(.0056)	−.0070(.0168)	.0059(0.0106)
1975	.0026(.0011)	−.0004(.0043)	.0017(0.0014)
1976	.0016(.0015)	−.0031(.0081)	−.0008(0.0045)
1977	.0025(.0006)	.0015(.0021)	.0022(0.0008)
1978	−.0006(.0008)	−.0014(.0018)	−.0002(0.0014)
1979	.0130(.0107)	−.0039(.0335)	.0101(0.0172)

The column headings refer to the table number and column number for the estimated equation. For example, 2.1 is Table 2, column 1. Asymptotic standard errors are in parentheses.

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