Echo-aware signal processing for audio scene analysis

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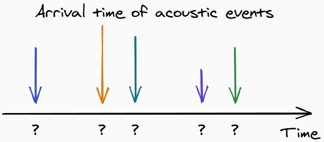
Acoustic Echo Estimation

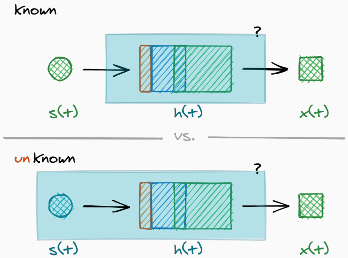
Acoustic Echo Retrieval

Estimating early (strong) reflections for microphones recordings, i.e.,

$$\{\tilde{x}_i\}_i \longrightarrow \{\tau_i^{(r)}, \alpha_i^{(r)}\}_{i,r}$$



Scenarios: the source signal is



Our case: signal source and passive system of (I microphones)

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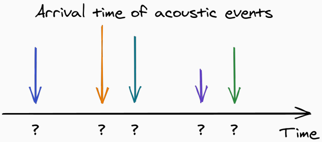

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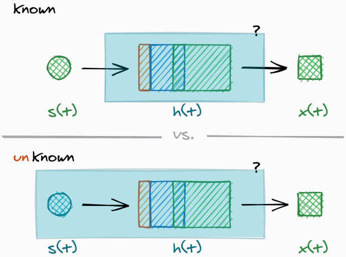

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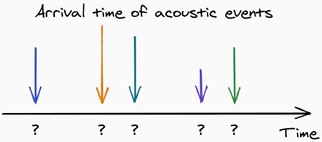

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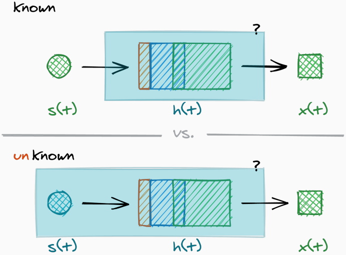

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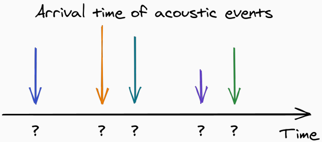

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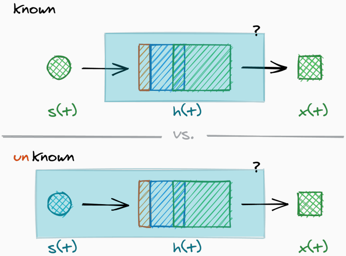

Active

- **non-blind** problem
- **intrusive** or specific setups
  (**Application:** sonar, calibration, measurements, etc.)

Passive

- **blind inverse** problem (harder)
- **passive** and more common setups
  (**Applications:** recording on smart speakers, laptop, etc.)

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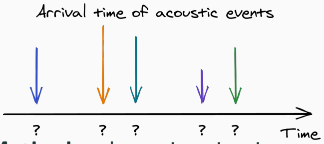

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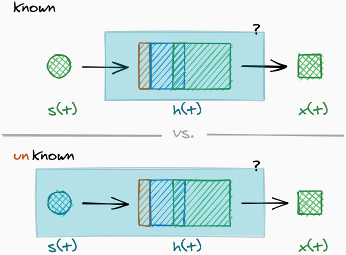
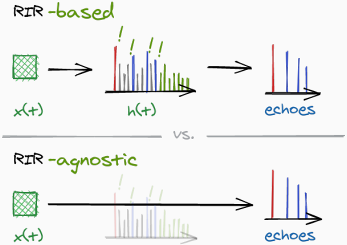


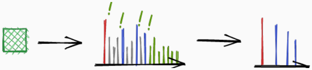
Methods: the estimation is



Our case: signal source and passive system of (I microphones)

Passive Acoustic Echo Retrieval

RIR-based approaches



1. "BCE"[1] problem $\Longrightarrow$ RIRs
2. Peak picking $\Longrightarrow$ Echoes

✓ BCE is well and known studied

✓ it is the state of the art
[Crocco and Del Bue, 2016]

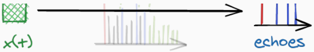
✓ reasonably good for some application

✗ Full RIRs need to be estimated

✗ Peak picking has hyperparameters

✗ Issues due to *on-grid* estimation

RIR-agnostic approaches



1. Direct off-grid estimation of $\left\{\tau_i^{(r)}, \alpha_i^{(r)}\right\}$ e.g., with maximum-likelihood

✓ No full RIRs & no peak picking
- lower complexity
- less hyperparameters

✓ Echo properties are respected
(e.g. Sparsity, Non-negativity)

✗ exploratory
(no standard solver, few works on audio)

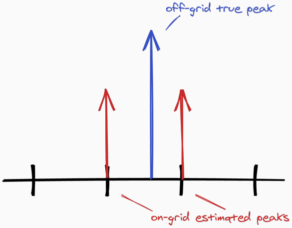
next topic

[1] Blind Channel Estimation

Limitations & bottleneck

Estimation is on-grid

- Sparsity and non-negativity not true "on grid"
- *Body guard* effect [Duval and Peyré, 2017]
  - $\rightarrow$ low recall $\implies$ low accuracy
  - $\rightarrow$ slow convergence
- Pick Picking
  - $\rightarrow$ manually tuned / corrected
  - $\rightarrow$ echo labeling (NP-hard problem)



How about higher sampling rate F_s?

$\rightarrow$ Increase Precision

But, computational bottleneck!

- Bigger vectors and matrices
  - $\longrightarrow$ memory usage
- the higher the sampling frequency
  - $\longrightarrow$ more ill-conditioned

How to solve this?

RIR-agnostic + off-grid

1. Learning-based
2. Analytical

Proposed approach: learning-based & off-grid

Idea: (Deep) Learning-based AER

1. Extend virtually learning-based SSL to AER
2. Estimate first echo estimation (simple but important)
3. Only 2 microphones

Observations:

- This *direct* mapping is difficult, the *inverse* "is not"
  → acoustic simulators: mic/src/room $\longrightarrow$ $\tau_i^{(r)}, \alpha_i^{(r)}, \quad \tilde{h}_i, \quad \tilde{x}_i$
- Acoustic simulator are "simple", versatile and fast
  → many data
- This approach is successfully in *Sound Source Localization*
  → position is related to echoes
  [Nguyen et al., 2018, Perotin et al., 2019] ⚠ Not only DNN

Models

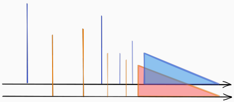
Inputs: Interchannel level and phase difference features[1] from

$$R[f] = \underset{t}{\text{avg.}} \frac{X_2[f,t]}{X_1[f,t]} \approx \underset{t}{\text{avg.}} \frac{H_2[f]S[f,t]}{H_1[f]S[f,t]}$$

$\approx$ the relative transfer function

$\rightarrow$ remove source dependency

Output: Inter and intra arrival delays



4 TOA

$\downarrow$

3 Time Difference of Arrivals (**TDOAs**)[1]

HP: first $\Leftrightarrow$ strongest echo (close-surface)

- Architecture: `MLP`, `CNN` [Chakrabarty and Habets, 2017, Nguyen et al., 2018]
- Loss Function:
  1. RMSE (Multi-label regression) $\rightarrow$ TDOAs
  2. Gaussian log-likelihood $\rightarrow \{\mu_\tau, \sigma^2_\tau\}\ \forall \tau \in$ TDOAs
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  } for data fusion similar to **MDN** [Bishop, 1994]
- Data:
  - Virtually-supervised learning (= data from acoustic simulator)
  - white-noise as source signal + noise

[1] See appendix for detailed computation

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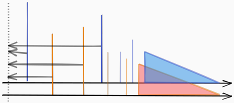

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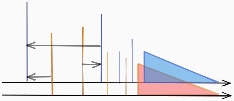

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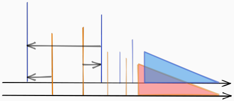

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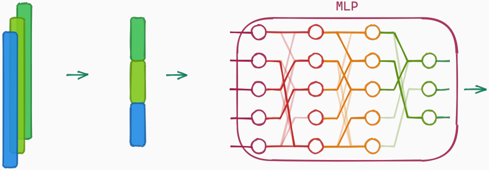

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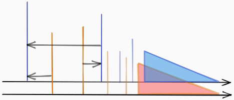

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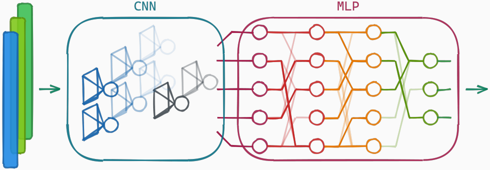

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Experimental results

Metrics: normalized RMSE (0 = best fit, 1 = random fit)

Proposed Method: MLP, CNN, $\mathtt{CNN}_{\mathcal{N}}$, $\mathtt{CNN}_{\mathcal{T}}$

Baseline: GCC PHAT[1]

Are better than baseline?

(only TDOA on direct path)

→ yes ✓

Echoes' TDOAs?

→ yes ✓

→ CNN better than MLP

Are robust to noise?

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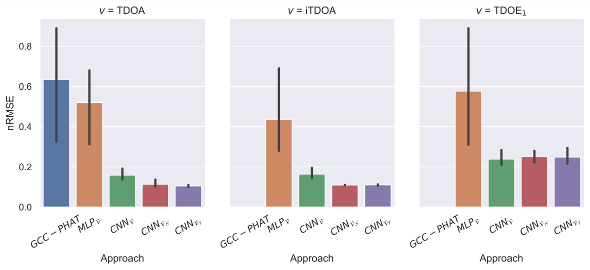
→ $\mathtt{CNN}_{\mathcal{N}}$/$\mathtt{CNN}_{\mathcal{T}}$ better than CNN



⚠Only 2 echoes ⚠work in progress (mor echoes and[6] generalizatior

[1] [Knapp and Carter, 1976]

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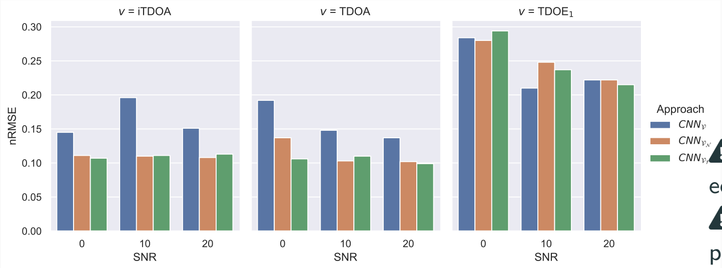

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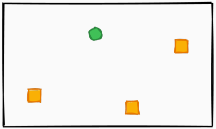
State of the Art

(common knowledge)

Key ingredient – *Cross relation identity*



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Ideas:

- Sampled version of $\tilde{x}_1, \tilde{x}_2$ are available (x_1, x_2)

1. Echoes' $\tau_i^{(r)} \propto$ sampling frequency
2. Find echoes $\rightarrow$ **find sparse vectors** h_1, h_2 of length L
3. Modeled as **Lasso**-like problem

$= \mathtt{Toeplitz}(x_i)h_j \in \mathcal{O}(L^2)$

$$\hat{h}_1, \hat{h}_2 \in \underset{h_1, h_2 \in \mathbf{R}^n}{\arg\min} \; \|x_1 * h_2 - x_2 * h_1\|_2^2 + \lambda \mathcal{P}(h_1, h_2) \quad \text{s.t.} \quad \mathcal{C}(h_1, h_2)$$

$\mathcal{P}(h_1, h_2) \longrightarrow$ sparse promoting regularizer

$\mathcal{C}(h_1, h_2) \longrightarrow$ constraints e.g. nonnegativity, anchor

✓ [Tong et al., 1994] ✓ [Lin et al., 2008] ✓ [Aissa-El-Bey and Abed-Meraim, 2008]
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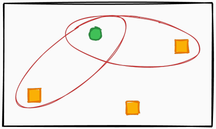

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Proposed approach: analytical & off-grid

Observation 1: the cross relation remains true in the frequency domain

$$\mathcal{F}x_1 \cdot \mathcal{F}h_2(n/F_s) = \mathcal{F}x_2 \cdot \mathcal{F}h_1(n/F_s) \qquad n = 0 \dots N-1$$

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Idea: Recover echoes by matching a finite number of frequencies

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$\sim$ **Lasso** problem, but $\mathcal{F}h_2(f)$ is a continuous function.

Instance of a **BLasso** problem [Bredies and Carioni, 2020]

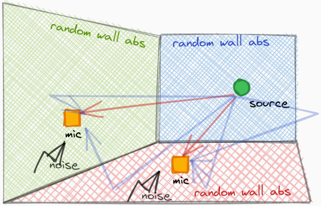
Solved with Sliding Frank-Wolfe algorithm [Denoyelle et al., 2019]

Experimental results

Methods

- BSN — SIMO
  BCE[Lin et al., 2007]
- IL1C: iteratively-weighted ℓ_1 constraint SIMO BCE [Crocco and Del Bue, 2015]
- Blaster: Proposed off-grid approach

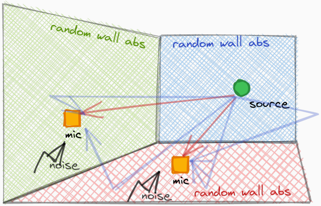
Baseline method are xvalidated on other dataset



Experimental results

Methods

- BSN — SIMO BCE[Lin et al., 2007]
- IL1C: iteratively-weighted ℓ_1 constraint SIMO BCE [Crocco and Del Bue, 2015]
- `Blaster`: Proposed off-grid approach



Baseline method are xvalidated on other dataset

Dataset

- $\mathcal{D}^{\text{SNR}}$: $SNR \in [0, 20]$ dB, $\text{RT}_{60} = 400$ ms
- $\mathcal{D}^{\text{RT60}}$: $\text{RT}_{60} = [100, 1000]$ ms, $SNR = 20$ dB

Experimental results

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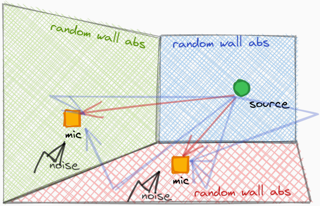

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Metrics

Performance per # of echoes

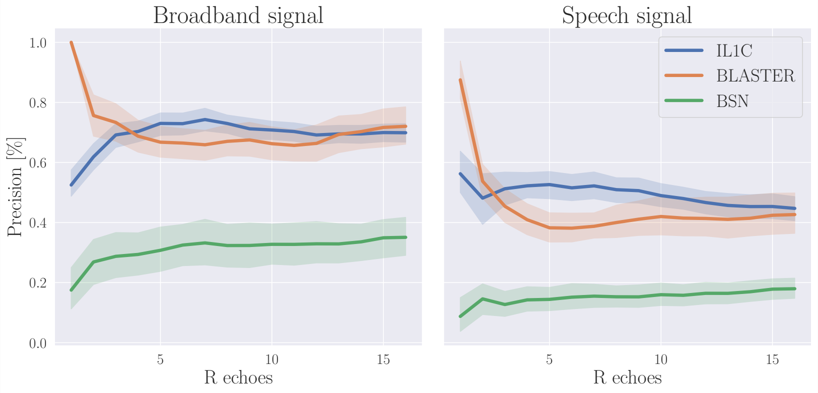


Figure 1: $\mathrm{RT}_{60} = 400$ ms and SNR $= 20$ dB.

✗ Sensitive to # echoes

✗ Sensitive source signal

✓ Good for 2 echoes

Performance per # of echoes

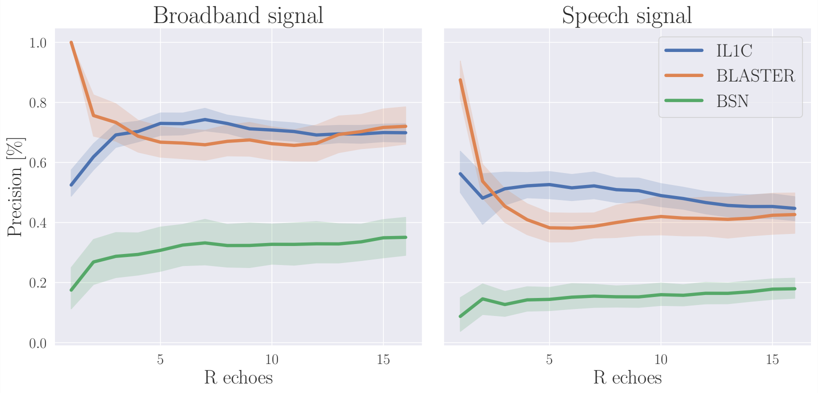


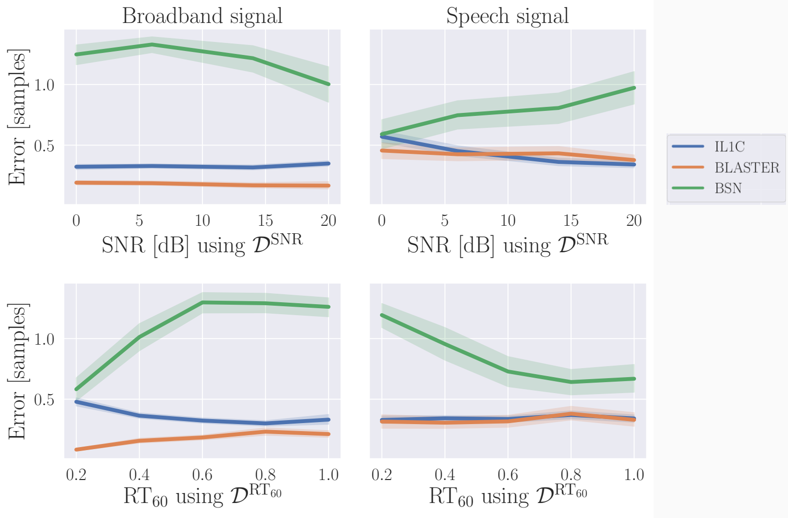
Figure 1: $\mathrm{RT}_{60} = 400$ ms and SNR $= 20$ dB.

✗ Sensitive to # echoes

✗ Sensitive source signal

✓ Good for 2 echoes [Scheibler et al., 2018,

Error per Dataset/Signal while recovering 7 echoes



✓Lower RMSE ✓ Robustness to SNR and RT_{60} ✗ Source signal dependent

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