A study on the Moog 904b high pass filter

Diego Di Carlo, Mattia Paterna

Aalborg Universitet København

June 14, 2016

Outcome

Goal of the project

- Analyse the original analog circuit
- Get tuned-frequency behaviour given control voltage E
- Replicate the circuit in discrete-time

General overview

What is a Moog 904b?

- A voltage controlled filter, or **VCF**
- Built on Moog's late-60s filter patent
- Exhibits a high pass behaviour

General overview

Inside the module

- 4-pole high pass filter configuration
- 24 dB/oct (80dB/dec) attenuation for frequencies below ω_c
- frequency range switch to tuned the whole frequency band

Specifics

US patent

- adder stage
- inverter stage
- four identical stages of PNP-NPN coupled transistors, working as voltage control resistor

Specifics

The audio part

- ~ 40 dB attenuation stage
- 4 RC-circuit filtering stage
- output amplifier (+42 dB)

Specifics

Sziklay pair

- unity-gain op-amp
- make each stage independent

Frequency range switch

- decoupling capacitors
- $1 + \frac{1}{2}$ octaves shifting

Circuit Analysis

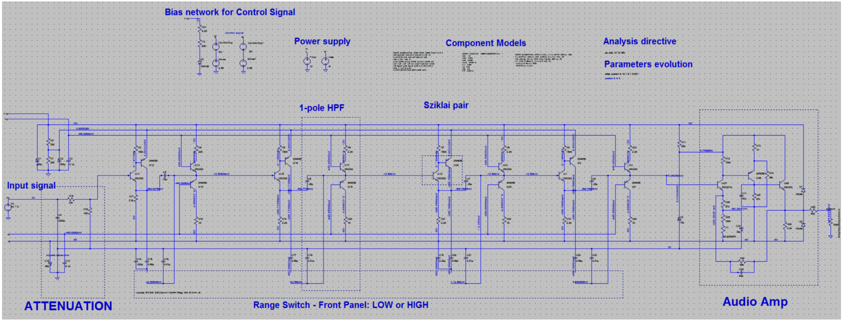


Figure: SPICE model version of only the audio part of the circuit subtitled with "DATE 6/23/70 DWG. NO 1118".

DC Analysis

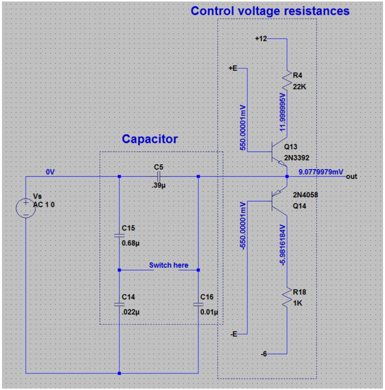
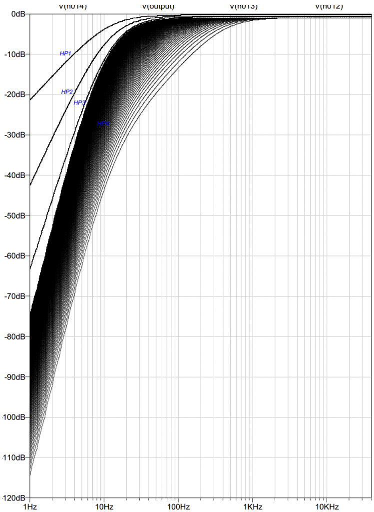


Figure: SPICE model version of only one high pass module

AC Analysis


V(n014)
V(output)
V(n013)
V(n012)
0dB
-10dB
-20dB
-30dB
-40dB
-50dB
-60dB
-70dB
-80dB
-90dB
-100dB
-110dB
-120dB
HP1
HP2
HP3
1Hz
10Hz
100Hz
1KHz
10KHz

AC Analysis

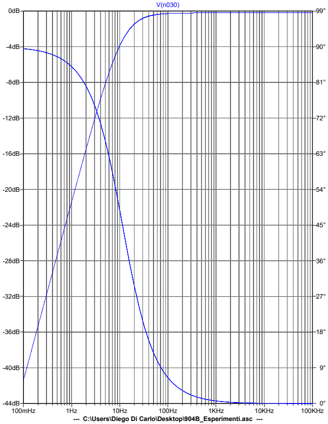


Figure: The transfer function (magnitude and phase) of a one *single-pole HPF stage*

Laplace Analysis

Generalized *Ohm's Law*:

$$V(s) = \dot{Z}(s)I(s) \tag{1}$$

The quantity $\dot{Z}(s) = \frac{V(s)}{I(s)}$ is called *impedance* of the circuit element. It assumes values in $\mathbb{C}$ and in *Cartisian form* is defined as

$$\dot{Z}(s) = R(s) + jX(s) \tag{2}$$

where $R(s)$ is called *resistance* and $X(s)$ *reactance*.

Laplace Analysis

Impedance of Resistor and Capacitor

The **resistor** is a linear time-invariant and frequency-constant component:

$$\dot{Z}_R(s) = R, \forall s \in \mathbb{C}, \forall t \in \mathbb{R}. \tag{3}$$

The **capacitors** is a non-linear component:

$$i(t) = C\frac{dv(t)}{dt} \Leftrightarrow v(t) = \frac{1}{C}\int_0^{\infty} i(t)dt \tag{4}$$

where C is the capacitance of the capacitor.
If we apply the Laplace transform,

$$V(s) = \frac{1}{C}\frac{1}{s}I(s) \Rightarrow \dot{Z}_C(s) = \frac{1}{sC} \tag{5}$$

We find the impedance of the capacitor.

Laplace Analysis

Complex Signal Voltage Divider

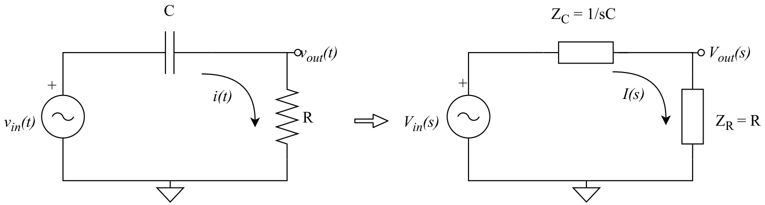


Figure: RC circuit as a voltage divider with complex impedences in Laplace domain

Laplace Analysis

The RC circuit can be seen as a voltage divider of complex impedences for complex signals:

$$V_{\text{out}}(s) = \frac{\dot{Z}_R}{\dot{Z}_R + \dot{Z}_C} V_{\text{in}}(s) = \frac{R}{R + \frac{1}{sC}} V_{\text{in}}(s) \quad (6)$$

Rearranging the equation we obtain:

$$V_{\text{out}}(s) = \frac{s}{s + \frac{1}{RC}} V_{\text{in}}(s) \quad (7)$$

which is the well-know relation for an first order high pass filter (HPF) with cut-off frequency $\omega_c = 1/RC$

Laplace Analysis

Total transfer functions

The i-th stage single-pole HPF transfer function:

$$H_i(s) = \frac{V_{i,\text{out}}(s)}{V_{i,\text{in}}(s)} = \frac{s}{s+\omega_c} \tag{8}$$

Thus, overall transfer function $H_{\text{HPF}}(s)$ considering all the 4 stages is:

$$\begin{aligned} H_{\text{HPF}}(s) &= H_1(s)H_2(s)H_3(s)_4H(s) \\ &= \frac{s^4}{s^4+4\omega_c s^3+6\omega_c^2 s^2+4\omega_c^3 s+\omega_c^4} \end{aligned} \tag{9}$$

R and C derivation

Transistor as variable resistor

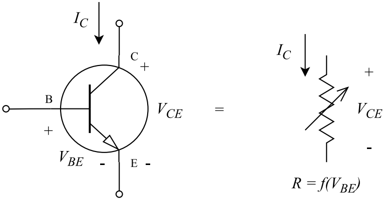
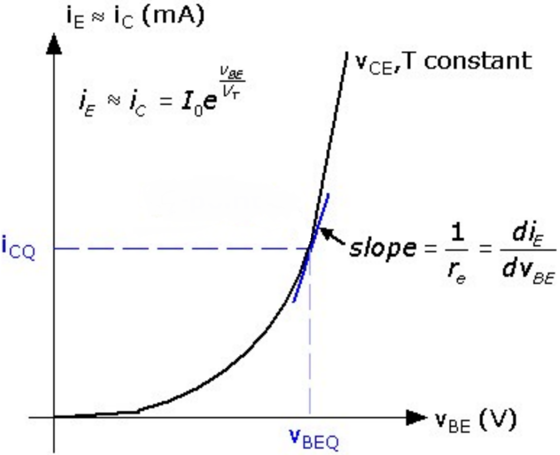


Figure: BJT NPN transistor as voltage-controlled resistor. The equivalent resistance R is a non-linear function of the voltage V_{BE}

R and C derivation

BJT's Voltage-current characteristic



R and C derivation

Resistance definition:

$$R = \frac{V}{I} \Rightarrow \frac{1}{R} = \frac{I}{V} \tag{10}$$

But for non linear voltage-current characteristics:

$$\frac{1}{R} = \frac{di(t)}{dv(t)} \tag{11}$$

The collector current value:

$$i_C = I_S \cdot e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CB}}{V_A}\right) \tag{12}$$

which usually is simplified as

$$i_C = I_S \cdot e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CE}}{V_A}\right) \tag{13}$$

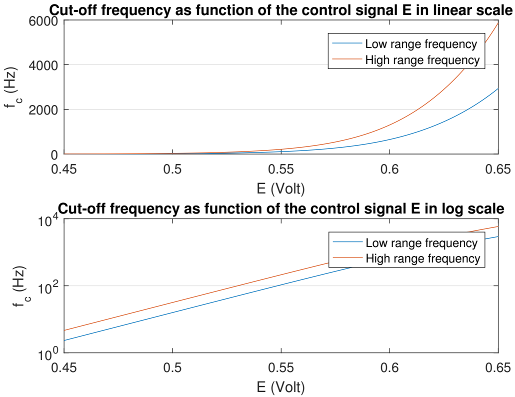
R and C derivation

Dynamic (emitter) resistance evaluation

In our case, we have that $v_{BE} = +E$. To evaluate the dynamic equivalent resistance of this component, we must compute the derivative now.

$$\frac{1}{R} = \frac{\partial i_C}{\partial v_{CE}} = \frac{I_S \cdot e^{\frac{V_{+E}}{V_T}}}{V_A} \tag{14}$$

R and C derivation



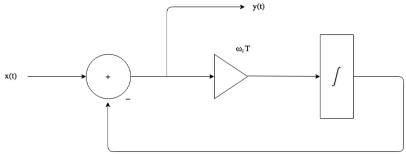
Discretization

Criteria

- topology preservation
- good transfer function replacement

Integrators

- ω_c express $1/RC$
- the output signal is the resistor voltage
- the integrator is a gain element with factor $\frac{1}{s}$

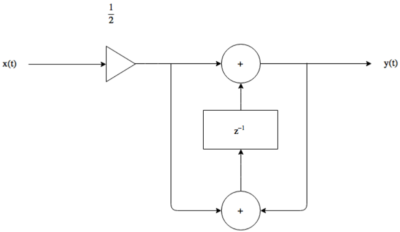


Integrators

- several models for digital integrators
- different models yield to different transfer function
- the integrator is replaced by a unit-delay
- trapezoidal integration

Trapezoidal integrator

- precise mapping of analog frequency response
- preserve the original structure
- named **topology-preserved** transform



Trapezoidal integrator

Its **transfer function** is

$$H(s) = \frac{\omega_c}{s} \Rightarrow H(z) = \frac{\omega_c}{2}\frac{1+z^{-1}}{1-z^{-1}}$$

that is the **bilinear** transform.

Implementation

The proposed implementation consists of

- four identical trapezoidal integrators
- frequency provided in terms of **voltage tension**
- control for frequency range switch
- gentle saturation before the filtering stage

Implementation

We've closely followed this models:

- Zavalishin's implementation of trapezoidal integrator
- Pirkle's general structure of Ladder Filter
- Välimäki and Huovilanen's saturation stage

Evaluation

- square wave used

- the 904b could be considered as a ladder filter without recursion
- for $E = 0.05$ we get 1954.2 Hz for he *High* setting and a frequency value of 651.39 Hz for the *Low* one, that is exactly a $1 + \frac{1}{2}$ octaves shifting

Evaluation

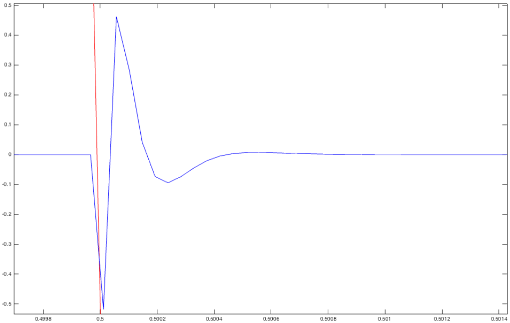


Figure: Effect of the discretized 904B on a pulse wave with duty cycle of 0.5. It can be noticed how the filter has a *smoothly* action on the square wave.

That's all, folks!