РОССИЙСКОЙ ФЕДЕРАЦИИ
ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ
УЧРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ

**«БЕЛГОРОДСКИЙ ГОСУДАРСТВЕННЫЙ
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Лабораторная работа №4.2

по дисциплине: Дискретная математика
тема: «Циклы»

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Цель работы: изучить разновидности циклов в графах, научиться генерировать случайные графы, определять их принадлежность к множеству эйлеровых и гамильтоновых графов, находить все эйлеровы и гамильтоновы циклы в графах.

Задания

№1. Разработать и реализовать алгоритм генерации случайного графа, содержащего n вершин и m ребер.

№2. Написать программу, которая: а) в течение десяти секунд генерирует случайные графы, содержащие n вершин и m ребер; б) для каждого полученного графа определяет, является ли он эйлеровым или гамильтоновым; в) подсчитывает общее количество сгенерированных графов и количество графов каждого типа.

Код программы:

```
#include "iostream"
#include <vector>
#include <set>
#include <map>
#include <algorithm>
#include <ctime>
#include <random>
#include <chrono>

using namespace std;

using GraphRow = vector<bool>;
using Graph = set<GraphRow>;

bool isExist(const vector<int> &adjacencyVector, const int &value) {
  for (auto &adjacency: adjacencyVector)
    if (adjacency == value)
      return true;
  return false;
}

void _generateGraph(Graph &graph, const size_t &verticesNumber, const size_t
&edgesNumber) {
  if (edgesNumber > (verticesNumber * (verticesNumber - 1) / 2)) {
    std::cerr << "Unable to build a graph with the specified number of vertices and edges\n";
    exit(1);
  }
  srand(time(0));
  set<int> randVertex;
  vector<int> initAdjacency;
  for (int i = 0; i < verticesNumber; i++) {
    int randValue = rand() % 1000;
    if (randVertex.find(randValue) != randVertex.end()) {
      i--;
    } else {
      randVertex.insert(randValue);
      graph.insert({randValue, initAdjacency});
    }
  }
  for (auto &value: randVertex) {
```

```
    initAdjacency.push_back(value);
  }
  for (int i = 0; i < edgesNumber; i++) {
    size_t randIndexFirstVertex = rand() % verticesNumber;
    size_t randIndexSecondVertex = rand() % verticesNumber;
    if ((randIndexFirstVertex != randIndexSecondVertex)
        && !isExist(graph.at(initAdjacency.at(randIndexFirstVertex)),
                    initAdjacency.at(randIndexSecondVertex))) {
      graph.at(initAdjacency.at(randIndexFirstVertex)).push_back(initAdjacency.at(randIndexSecondVertex));
      graph.at(initAdjacency.at(randIndexSecondVertex)).push_back(initAdjacency.at(
            randIndexFirstVertex));
    } else {
      i--;
    }
  }
}

Graph generateGraph(const size_t &verticesNumber, const size_t &edgesNumber) {
  Graph graph;
  _generateGraph(graph, verticesNumber, edgesNumber);
  return graph;
}

bool isEulerGraph(const Graph &graph) {
  for (auto &x: graph)
    if (x.second.size() % 2 == 1)
      return false;
  return true;
}

bool isHamiltonianGraph(const Graph &graph) {
  if (graph.size() >= 3) {
    for (auto &x: graph)
      if (x.second.size() < (graph.size() / 2))
        return false;
  } else
    return false;

  return true;
}

void exercise(const size_t &verticesNumber) {
  size_t maxEdgesNumber = verticesNumber * (verticesNumber - 1) / 2;
  for (size_t i = verticesNumber; i <= maxEdgesNumber; i++) {
    auto untilMoment{chrono::high_resolution_clock::now() + chrono::seconds(10)};
    size_t hamiltonianCounter = 0;
    size_t eulerCounter = 0;
    size_t generalCounter = 0;
    while (chrono::high_resolution_clock::now() < untilMoment) {
      Graph graph = generateGraph(verticesNumber, i);
      hamiltonianCounter += isHamiltonianGraph(graph);
      eulerCounter += isEulerGraph(graph);
      generalCounter++;
    }
    cout << "\tInformation about " << i << "-edges graph\n";
    cout << "-Hamiltonian: " << hamiltonianCounter
         << "; Euler: " << eulerCounter
         << "; Total: " << generalCounter << '\n';
  }
}


int main() {
```

```
  exercise(8);
  return 0;
}
```

№3. Выполнить программу при **n** = 8, 9, 10 и сделать выводы.

при n = 8

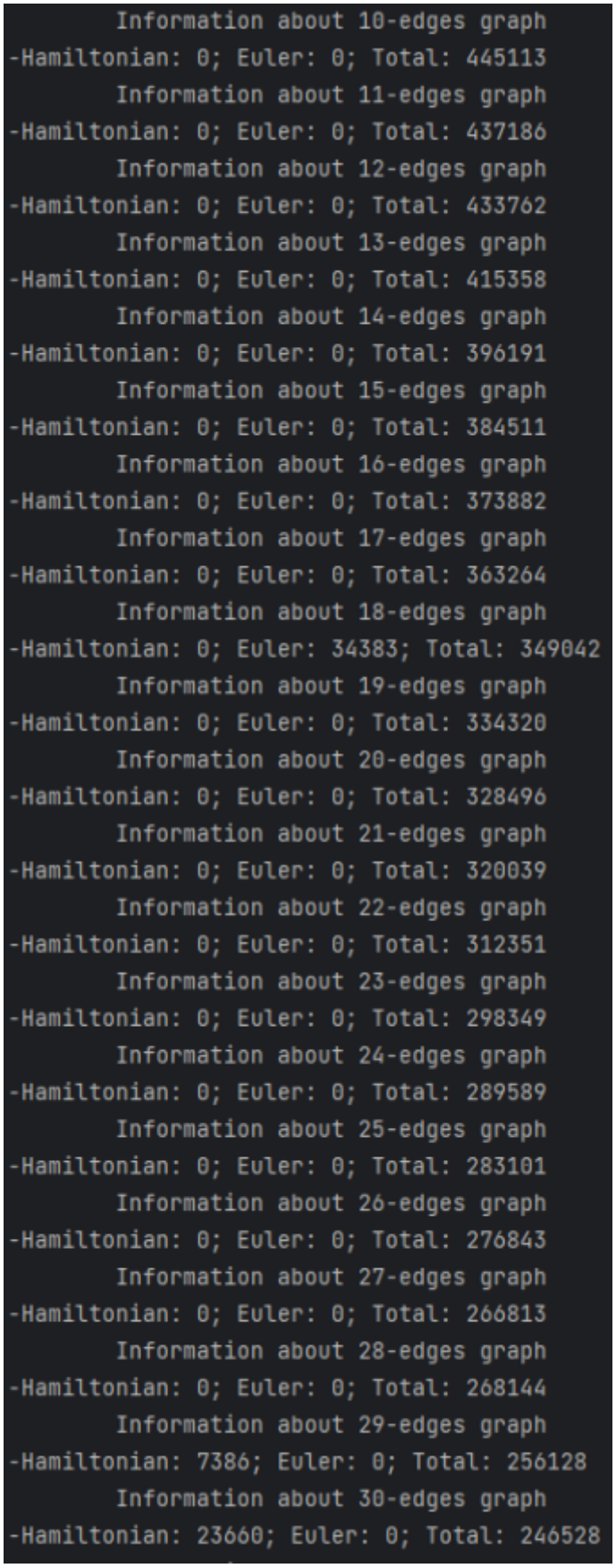
```
        Information about 8-edges graph
-Hamiltonian: 0; Euler: 0; Total: 583664
        Information about 9-edges graph
-Hamiltonian: 0; Euler: 0; Total: 546046
        Information about 10-edges graph
-Hamiltonian: 0; Euler: 0; Total: 495352
        Information about 11-edges graph
-Hamiltonian: 0; Euler: 0; Total: 474414
        Information about 12-edges graph
-Hamiltonian: 0; Euler: 0; Total: 465235
        Information about 13-edges graph
-Hamiltonian: 0; Euler: 0; Total: 446247
        Information about 14-edges graph
-Hamiltonian: 0; Euler: 0; Total: 427249
        Information about 15-edges graph
-Hamiltonian: 0; Euler: 0; Total: 415332
        Information about 16-edges graph
-Hamiltonian: 0; Euler: 0; Total: 393552
        Information about 17-edges graph
-Hamiltonian: 0; Euler: 0; Total: 380852
        Information about 18-edges graph
-Hamiltonian: 36120; Euler: 0; Total: 366501
        Information about 19-edges graph
-Hamiltonian: 73770; Euler: 0; Total: 362836
        Information about 20-edges graph
-Hamiltonian: 126600; Euler: 0; Total: 331860
        Information about 21-edges graph
-Hamiltonian: 298091; Euler: 0; Total: 331521
        Information about 22-edges graph
-Hamiltonian: 316639; Euler: 0; Total: 316639
        Information about 23-edges graph
-Hamiltonian: 304840; Euler: 0; Total: 304840
        Information about 24-edges graph
-Hamiltonian: 297186; Euler: 0; Total: 297186
        Information about 25-edges graph
-Hamiltonian: 278230; Euler: 0; Total: 278230
        Information about 26-edges graph
-Hamiltonian: 251322; Euler: 0; Total: 251322
        Information about 27-edges graph
-Hamiltonian: 248415; Euler: 0; Total: 248415
        Information about 28-edges graph
-Hamiltonian: 230636; Euler: 0; Total: 230636
```

При n = 9:

```
        Information about 9-edges graph
-Hamiltonian: 0; Euler: 0; Total: 457247
        Information about 10-edges graph
-Hamiltonian: 0; Euler: 0; Total: 496223
        Information about 11-edges graph
-Hamiltonian: 0; Euler: 0; Total: 472507
        Information about 12-edges graph
-Hamiltonian: 0; Euler: 0; Total: 453032
        Information about 13-edges graph
-Hamiltonian: 0; Euler: 0; Total: 436792
        Information about 14-edges graph
-Hamiltonian: 0; Euler: 0; Total: 413971
        Information about 15-edges graph
-Hamiltonian: 0; Euler: 40957; Total: 400529
        Information about 16-edges graph
-Hamiltonian: 0; Euler: 0; Total: 388130
        Information about 17-edges graph
-Hamiltonian: 0; Euler: 0; Total: 363612
        Information about 18-edges graph
-Hamiltonian: 0; Euler: 0; Total: 361561
        Information about 19-edges graph
-Hamiltonian: 0; Euler: 0; Total: 346502
        Information about 20-edges graph
-Hamiltonian: 0; Euler: 0; Total: 336055
        Information about 21-edges graph
-Hamiltonian: 131929; Euler: 0; Total: 327085
        Information about 22-edges graph
-Hamiltonian: 11679; Euler: 0; Total: 312754
        Information about 23-edges graph
-Hamiltonian: 123468; Euler: 0; Total: 307654
        Information about 24-edges graph
-Hamiltonian: 119384; Euler: 0; Total: 299046
        Information about 25-edges graph
-Hamiltonian: 223368; Euler: 0; Total: 282014
        Information about 26-edges graph
-Hamiltonian: 226259; Euler: 0; Total: 281663
        Information about 27-edges graph
-Hamiltonian: 271773; Euler: 28307; Total: 271773
        Information about 28-edges graph
-Hamiltonian: 263260; Euler: 0; Total: 263260
        Information about 29-edges graph
-Hamiltonian: 248221; Euler: 0; Total: 248221
```

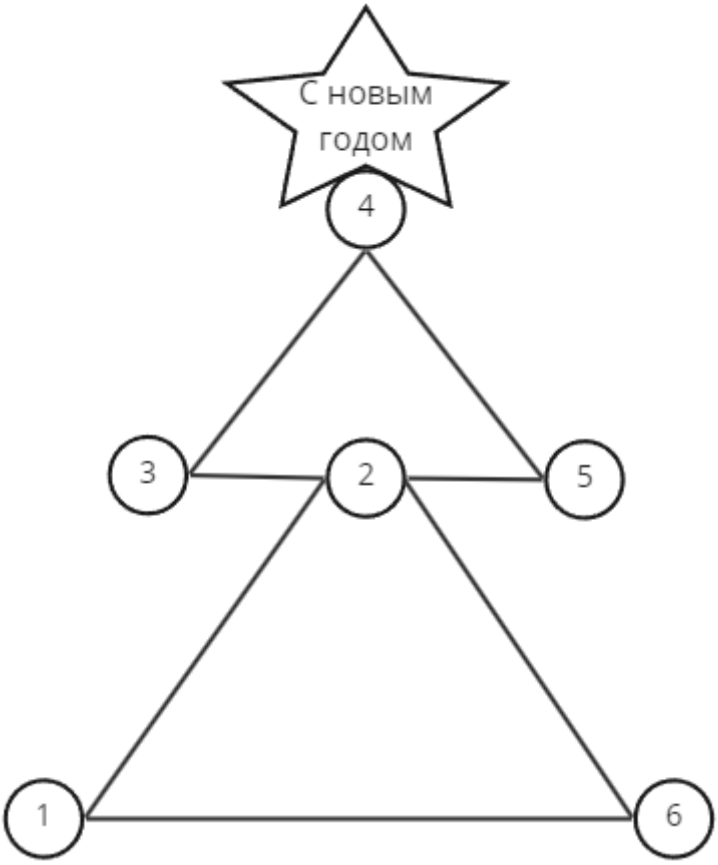
```
        Information about 30-edges graph
-Hamiltonian: 242095; Euler: 0; Total: 242095
        Information about 31-edges graph
-Hamiltonian: 239667; Euler: 0; Total: 239667
        Information about 32-edges graph
-Hamiltonian: 227266; Euler: 0; Total: 227266
        Information about 33-edges graph
-Hamiltonian: 223215; Euler: 0; Total: 223215
        Information about 34-edges graph
-Hamiltonian: 197652; Euler: 0; Total: 197652
        Information about 35-edges graph
-Hamiltonian: 194589; Euler: 0; Total: 194589
        Information about 36-edges graph
-Hamiltonian: 153075; Euler: 153075; Total: 153075
```

• при n = 10:


Information about 10-edges graph
-Hamiltonian: 0; Euler: 0; Total: 445113
Information about 11-edges graph
-Hamiltonian: 0; Euler: 0; Total: 437186
Information about 12-edges graph
-Hamiltonian: 0; Euler: 0; Total: 433762
Information about 13-edges graph
-Hamiltonian: 0; Euler: 0; Total: 415358
Information about 14-edges graph
-Hamiltonian: 0; Euler: 0; Total: 396191
Information about 15-edges graph
-Hamiltonian: 0; Euler: 0; Total: 384511
Information about 16-edges graph
-Hamiltonian: 0; Euler: 0; Total: 373882
Information about 17-edges graph
-Hamiltonian: 0; Euler: 0; Total: 363264
Information about 18-edges graph
-Hamiltonian: 0; Euler: 34383; Total: 349042
Information about 19-edges graph
-Hamiltonian: 0; Euler: 0; Total: 334320
Information about 20-edges graph
-Hamiltonian: 0; Euler: 0; Total: 328496
Information about 21-edges graph
-Hamiltonian: 0; Euler: 0; Total: 320039
Information about 22-edges graph
-Hamiltonian: 0; Euler: 0; Total: 312351
Information about 23-edges graph
-Hamiltonian: 0; Euler: 0; Total: 298349
Information about 24-edges graph
-Hamiltonian: 0; Euler: 0; Total: 289589
Information about 25-edges graph
-Hamiltonian: 0; Euler: 0; Total: 283101
Information about 26-edges graph
-Hamiltonian: 0; Euler: 0; Total: 276843
Information about 27-edges graph
-Hamiltonian: 0; Euler: 0; Total: 266813
Information about 28-edges graph
-Hamiltonian: 0; Euler: 0; Total: 268144
Information about 29-edges graph
-Hamiltonian: 7386; Euler: 0; Total: 256128
Information about 30-edges graph
-Hamiltonian: 23660; Euler: 0; Total: 246528

```
        Information about 31-edges graph
-Hamiltonian: 71529; Euler: 0; Total: 242453
        Information about 32-edges graph
-Hamiltonian: 108040; Euler: 0; Total: 232200
        Information about 33-edges graph
-Hamiltonian: 136543; Euler: 0; Total: 222988
        Information about 34-edges graph
-Hamiltonian: 145643; Euler: 0; Total: 219803
        Information about 35-edges graph
-Hamiltonian: 145627; Euler: 0; Total: 210815
        Information about 36-edges graph
-Hamiltonian: 181554; Euler: 0; Total: 202582
        Information about 37-edges graph
-Hamiltonian: 203486; Euler: 0; Total: 203486
        Information about 38-edges graph
-Hamiltonian: 193615; Euler: 0; Total: 193615
        Information about 39-edges graph
-Hamiltonian: 189290; Euler: 0; Total: 189290
        Information about 40-edges graph
-Hamiltonian: 179346; Euler: 0; Total: 179346
        Information about 41-edges graph
-Hamiltonian: 178863; Euler: 0; Total: 178863
        Information about 42-edges graph
-Hamiltonian: 156716; Euler: 0; Total: 156716
        Information about 43-edges graph
-Hamiltonian: 156870; Euler: 0; Total: 156870
        Information about 44-edges graph
-Hamiltonian: 145200; Euler: 0; Total: 145200
        Information about 45-edges graph
-Hamiltonian: 118853; Euler: 0; Total: 118853
```

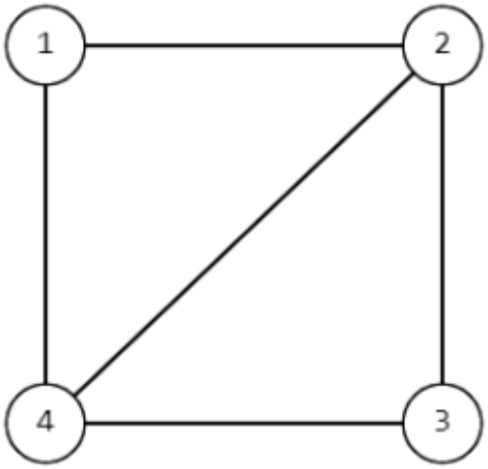
№4. Привести пример диаграммы графа, который является эйлеровым, но не гамильтоновым. Найти в нем все эйлеровы циклы.



Все эйлеровы циклы:

1 2 3 4 5 2 6 1	3 4 5 2 1 6 2 3
1 2 5 4 3 2 6 1	3 4 5 2 6 1 2 3
1 6 2 3 4 5 2 1	4 3 2 1 6 2 5 4
1 6 2 5 4 3 2 1	4 3 2 6 1 2 5 4
2 1 6 2 3 4 5 2	4 5 2 1 6 2 3 4
2 1 6 2 5 4 3 2	4 5 2 6 1 2 3 4
2 3 4 5 2 1 6 2	5 2 1 6 2 3 4 5
2 3 4 5 2 6 1 2	5 2 6 1 2 3 4 5
2 5 4 3 2 1 6 2	5 4 3 2 1 6 2 5
2 5 4 3 2 6 1 2	5 4 3 2 6 1 2 5
2 6 1 2 3 4 5 2	6 1 2 3 4 5 2 6
2 6 1 2 5 4 3 2	6 1 2 5 4 3 2 6
3 2 1 6 2 5 4 3	6 2 3 4 5 2 1 6
3 2 6 1 2 5 4 3	6 2 5 4 3 2 1 6

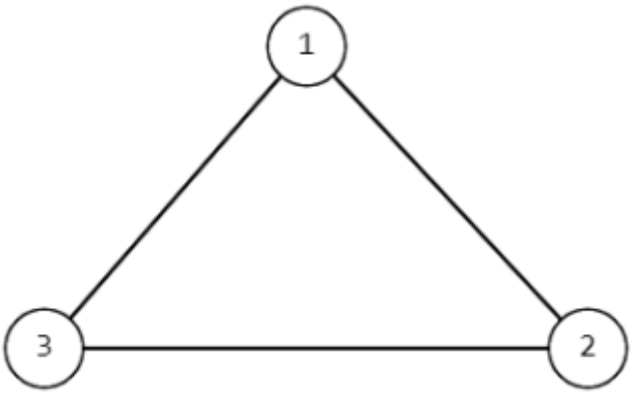
№5. Привести пример диаграммы графа, который является гамильтоновым, но не эйлеровым. Найти в нем все гамильтоновы циклы.



Все гамильтоновы циклы:

1 2 3 4 1
1 4 3 2 1
2 1 4 3 2
2 3 4 1 2
3 2 1 4 3
3 4 1 2 3
4 1 2 3 4
4 3 2 1 4

№6. Привести пример диаграммы графа, который является эйлеровым и гамильтоновым. Найти в нем все эйлеровы и гамильтоновы циклы.



Все и гамильтоновы, и эйлеровы циклы:

1 2 3 1

1 3 2 1

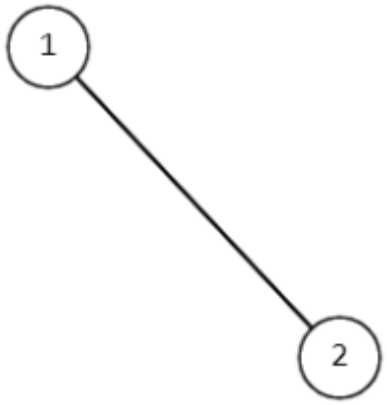
2 1 3 2

2 3 1 2

3 1 2 3

3 2 1 3

№7. Привести пример диаграммы графа, который не является ни эйлеровым, ни гамильтоновым



Вывод: в ходе лабораторной работы изучили разновидности циклов в графах, научились генерировать случайные графы, определять их принадлежность к множеству эйлеровых и гамильтоновых графов, находить все эйлеровы и гамильтоновы циклы в графах