

РОССИЙСКОЙ ФЕДЕРАЦИИ  
ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ  
УЧРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ

**«БЕЛГОРОДСКИЙ ГОСУДАРСТВЕННЫЙ  
ТЕХНОЛОГИЧЕСКИЙ УНИВЕРСИТЕТ им. В. Г. ШУХОВА»**  
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Кафедра программного обеспечения вычислительной техники и автоматизированных систем

**Лабораторная работа №1.4**  
по дисциплине: Дискретная математика  
тема: «Теоретико-множественные уравнения»

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**Цель занятия:** научиться решать теоретико-множественные уравнения с применением ЭВМ.

Вариант 7

**Задания**

1. Преобразовать исходное уравнение в уравнение с пустой правой частью.

Вариант 7.

$$(X \cup A) \cap (X - B) \cup C = A \cap \overline{X} \Delta (C - X)$$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 9, 10\}$$

$$B = \{1, 2, 8, 9, 10\}$$

$$C = \{1, 3, 6, 7\}$$

$$X = ?$$

2. Преобразовать левую часть уравнения к виду  $X \cap \varphi^0 \cup X \cap \varphi^1$ , используя разложение Шеннона по неизвестному множеству  $X$ .

$$\text{№2. } \overline{X} \cap \varphi^0 \cup X \cap \varphi^1 = \emptyset, \text{ где}$$

$$\varphi^0 = (A - (\emptyset \cap B) \cup (C \cap U)) \Delta ((X - A) \cap (X - B) - C)$$

$$\varphi^1 = (A - (U \cap B) \cup (C \cap \emptyset)) \Delta ((U - A) \cap (U - B) - C)$$

3. Написать программу, вычисляющую значения множеств  $\phi\emptyset$  и  $\phi U$  при заданных исходных множествах.

```
class InToPost {
private:
    stack<char> stackOperators;
    string postfixNotation;
    string input;

public:
    explicit InToPost(string input) :
        input(std::move(input)) {};

    void gotOper(char oper, int priorityNew) {
        while (!this->stackOperators.empty()) {
            char topOperator = this->stackOperators.top();
            if (topOperator != '(') {
                int priorityTop;
                if (topOperator == '!'')
                    priorityTop = 1;
                else if (topOperator == '&')
                    priorityTop = 2;
                else priorityTop = 3;

                if (priorityNew >= priorityTop) {
                    this->postfixNotation += topOperator;
                    this->stackOperators.pop();
                } else
                    break;
            } else {
                break;
            }
        }
        this->stackOperators.push(oper);
    }

    void gotParen() {
        while (!this->stackOperators.empty()) {
            char i = this->stackOperators.top();
            this->stackOperators.pop();
            if (i == u'(')
                break;
            else
                this->postfixNotation += i;
        }
    }

    void makePostfixNotation(const string &infixNotation) {
        for (auto i: infixNotation) {
            switch (i) {
                case 'u':
                case '^':
                case '-':{
                    gotOper(i, 3);
                    break;
                }
                case '&':{
                    gotOper(i, 2);
                    break;
                }
                case '!'': {
                    gotOper(i, 1);
                    break;
                }
            }
        }
    }
}
```

```

        case '(': {
            this->stackOperators.push(i);
            break;
        }
        case ')': {
            gotParen();
            break;
        }
        default:
            postfixNotation += i;
    }

}

while (!this->stackOperators.empty()) {
    this->postfixNotation += this->stackOperators.top();
    this->stackOperators.pop();
}

string getPostfixNotation() {
    makePostfixNotation(input);
    return this->postfixNotation;
}
};

class SolverPost {
private:
    int countSets;
    string postExpression;
    unordered_array_set X;
    int a[3] = {2, 9, 10};
    int b[5] = {1, 2, 8, 9, 10};
    int c[4] = {1, 3, 6, 7};
    int d[10] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};

    unordered_array_set universum = UOAS_create_from_array(d, 10);
    vector<unordered_array_set> sets = {
        UOAS_create_from_array(a, 3),
        UOAS_create_from_array(b, 5),
        UOAS_create_from_array(c, 4),
    };
    stack<unordered_array_set> executionStack;

public:
    void inputX(unordered_array_set x) {
        this->X = x;
    }

    unordered_array_set getUniversum() {
        return universum;
    }

    SolverPost(int numSets, string postExpression) :
        countSets(numSets), postExpression(std::move(postExpression)) {};

    unordered_array_set fi() {
        while (!this->executionStack.empty())
            this->executionStack.pop();

        int curIndex = 0;
        while (curIndex < postExpression.size()) {

```

```

char curSymbol = this->postExpression[curIndex];
if ('A' <= curSymbol && curSymbol <= 'Z') {
    if (curSymbol == 'U')
        this->executionStack.push(this->universum);
    else if (curSymbol == 'X')
        this->executionStack.push(this->X);
    else
        this->executionStack.push(this->sets[curSymbol - 'A']);
} else {
    if (curSymbol == 'u') {
        unordered_array_set set1 = this->executionStack.top();
        this->executionStack.pop();
        unordered_array_set set2 = this->executionStack.top();
        this->executionStack.pop();

        this->executionStack.push(UOAS_union(set1, set2));

    } else if (curSymbol == '&') {
        unordered_array_set set1 = this->executionStack.top();
        this->executionStack.pop();
        unordered_array_set set2 = this->executionStack.top();
        this->executionStack.pop();

        this->executionStack.push(
            UOAS_intersection(set1, set2));

    } else if (curSymbol == '^') {
        unordered_array_set set1 = this->executionStack.top();
        this->executionStack.pop();
        unordered_array_set set2 = this->executionStack.top();
        this->executionStack.pop();

        this->executionStack.push(
            UOAS_symmetricDifference(set1, set2));

    } else if (curSymbol == '-') {
        unordered_array_set set1 = this->executionStack.top();
        this->executionStack.pop();
        unordered_array_set set2 = this->executionStack.top();
        this->executionStack.pop();

        this->executionStack.push(UOAS_difference(set2, set1));

    } else if (curSymbol == '!') {
        unordered_array_set set1 = this->executionStack.top();
        this->executionStack.pop();

        this->executionStack.push(
            UOAS_complement(set1, this->universum));
    }
}
curIndex++;
}
return executionStack.top();
};

```

```

int main() {
#ifdef FIRST
    string input = "(A-(X&B)u(C&!X))^((X-A)&(X-B)-C)";
    InToPost translator(input);
    SolverPost solver(3, translator.getPostfixNotation());
    int a[0];
    cout << translator.getPostfixNotation() << '\n';
    solver.inputX(UOAS_create_from_array(a, 0));
    unordered_array_set set = solver.fi();

    qsort(set.data, set.size, sizeof(int), compare_ints);
    for (int i = 0; i < set.size; i++) {
        cout << set.data[i] << " ";
    }
    cout << endl;

    InToPost translator2(input);
    SolverPost solver2(3, translator2.getPostfixNotation());
    int b[] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
    solver2.inputX(UOAS_create_from_array(b, 10));
    unordered_array_set set2 = solver2.fi();
    set2 = UOAS_complement(set2, UOAS_create_from_array(b, 10));

    qsort(set2.data, set2.size, sizeof(int), compare_ints);
    for (int i = 0; i < set2.size; i++) {
        cout << set2.data[i] << " ";
    }
#endif

    return 0;
}

```

4. Вычислить значения множеств  $\varphi^\emptyset$  и  $\varphi^U$  и сделать вывод о существовании решения уравнения. Если решения уравнения не существует, то выполнить п.п. 1—4 для следующего (предыдущего) варианта.

$$\begin{aligned} \text{НЧ. } \varphi^\emptyset &= A - (\emptyset \cap B) \cup (C \cap \emptyset) \Delta ((\emptyset - A) \cap (\emptyset - B) - C) = \\ &= (A - \emptyset \cup C) \Delta (\emptyset \cap \emptyset - C) = \\ &= (A \cup C) \Delta \emptyset = A \cup C = \{1; 2; 3; 6; 7; 9; 10\} \end{aligned}$$

$$\begin{aligned} \varphi^U &= (A - (U \cap B) \cup (C \cap \emptyset)) \Delta ((U - A) \cap (U - B) - C) = \\ &= (A - B \cup \emptyset) \Delta (\bar{A} \cap \bar{B} - C) = \\ &= (A \cap \bar{B}) \Delta (\bar{A} \cap \bar{B} \cap \bar{C}) = \\ &= (\{2; 9; 10\} \cap \{3; 4; 5; 6; 7\}) \Delta (\{1; 3; 4; 5; 6; 7; 8\} \cap \\ &\cap \{3; 4; 5; 6; 7\} \cap \{2; 4; 5; 8; 9; 10\}) = \emptyset \Delta \{4; 5\} = \\ &= \{4; 5\} \end{aligned}$$

$$\overline{\varphi^U} = \{1; 2; 3; 6; 7; 8; 9; 10\}$$

$\varphi^\emptyset$  - подмножество  $\overline{\varphi^U} \Rightarrow$  ур-е имеет решение

5. Определить мощность общего решения, найти некоторые (или все) частные решения, в том числе частные решения наименьшей и наибольшей мощности.

**Общее решение:**  $\{\{\}, \{1\}, \{2\}, \{1, 2\}, \{3\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{6\}, \{1, 6\}, \{2, 6\}, \{1, 2, 6\}, \{3, 6\}, \{1, 3, 6\}, \{2, 3, 6\}, \{1, 2, 3, 6\}, \{7\}, \{1, 7\}, \{2, 7\}, \{1, 2, 7\}, \{3, 7\}, \{1, 3, 7\}, \{2, 3, 7\}, \{1, 2, 3, 7\}, \{6, 7\}, \{1, 6, 7\}, \{2, 6, 7\}, \{1, 2, 6, 7\}, \{3, 6, 7\}, \{1, 3, 6, 7\}, \{2, 3, 6, 7\}, \{1, 2, 3, 6, 7\}, \{8\}, \{1, 8\}, \{2, 8\}, \{1, 2, 8\}, \{3, 8\}, \{1, 3, 8\}, \{2, 3, 8\}, \{1, 2, 3, 8\}, \{6, 8\}, \{1, 6, 8\}, \{2, 6, 8\}, \{1, 2, 6, 8\}, \{3, 6, 8\}, \{1, 3, 6, 8\}, \{2, 3, 6, 8\}, \{1, 2, 3, 6, 8\}, \{7, 8\}, \{1, 7, 8\}, \{2, 7, 8\}, \{1, 2, 7, 8\}, \{3, 7, 8\}, \{1, 3, 7, 8\}, \{2, 3, 7, 8\}, \{1, 2, 3, 7, 8\}, \{6, 7, 8\}, \{1, 6, 7, 8\}, \{2, 6, 7, 8\}, \{1, 2, 6, 7, 8\}, \{3, 6, 7, 8\}, \{1, 3, 6, 7, 8\}, \{2, 3, 6, 7, 8\}, \{1, 2, 3, 6, 7, 8\}, \{9\}, \{1, 9\}, \{2, 9\}, \{1, 2, 9\}, \{3, 9\}, \{1, 3, 9\}, \{2, 3, 9\}, \{1, 2, 3, 9\}, \{6, 9\}, \{1, 6, 9\}, \{2, 6, 9\}, \{1, 2, 6, 9\}, \{3, 6, 9\}, \{1, 3, 6, 9\}, \{2, 3, 6, 9\}, \{1, 2, 3, 6, 9\}, \{7, 9\}, \{1, 7, 9\}, \{2, 7, 9\}, \{1, 2, 7, 9\}, \{3, 7, 9\}, \{1, 3, 7, 9\}, \{2, 3, 7, 9\}, \{1, 2, 3, 7, 9\}, \{6, 7, 9\}, \{1, 6, 7, 9\}, \{2, 6, 7, 9\}, \{1, 2, 6, 7, 9\}, \{3, 6, 7, 9\}, \{1, 3, 6, 7, 9\}, \{2, 3, 6, 7, 9\}, \{1, 2, 3, 6, 7, 9\}, \{8, 9\}, \{1, 8, 9\}, \{2, 8, 9\}, \{1, 2, 8, 9\}, \{3, 8, 9\}, \{1, 3, 8, 9\}, \{2, 3, 8, 9\}, \{1, 2, 3, 8, 9\}, \{6, 8, 9\}, \{1, 6, 8, 9\}, \{2, 6, 8, 9\}, \{1, 2, 6, 8, 9\}, \{3, 6, 8, 9\}, \{1, 3, 6, 8, 9\}, \{2, 3, 6, 8, 9\}, \{1, 2, 3, 6, 8, 9\}, \{7, 8, 9\}, \{1, 7, 8, 9\}, \{2, 7, 8, 9\}, \{1, 2, 7, 8, 9\}, \{3, 7, 8, 9\}, \{1, 3, 7, 8, 9\}, \{2, 3, 7, 8, 9\}, \{1, 2, 3, 7, 8, 9\}, \{6, 7, 8, 9\}, \{1, 6, 7, 8, 9\}, \{2, 6, 7, 8, 9\}, \{1, 2, 6, 7, 8, 9\}, \{3, 6, 7, 8, 9\}, \{1, 3, 6, 7, 8, 9\}, \{2, 3, 6, 7, 8, 9\}, \{1, 2, 3, 6, 7, 8, 9\}, \{10\}, \{1, 10\}, \{2, 10\}, \{1, 2, 10\}, \{3, 10\}, \{1, 3, 10\}, \{2, 3, 10\}, \{1, 2, 3, 10\}, \{6, 10\}, \{1, 6, 10\},$

{2,6,10},{1,2,6,10},{3,6,10},{1,3,6,10},{2,3,6,10},{1,2,3,6,10},{7,10},{1,7,10},{2,7,10},  
 {1,2,7,10},{3,7,10},{1,3,7,10},{2,3,7,10},{1,2,3,7,10},{6,7,10},{1,6,7,10},{2,6,7,10},  
 {1,2,6,7,10},{3,6,7,10},{1,3,6,7,10},{2,3,6,7,10},{1,2,3,6,7,10},{8,10},{1,8,10},{2,8,10},  
 {1,2,8,10},{3,8,10},{1,3,8,10},{2,3,8,10},{1,2,3,8,10},{6,8,10},{1,6,8,10},{2,6,8,10},{1,2,6,8,10},  
 {3,6,8,10},{1,3,6,8,10},{2,3,6,8,10},{1,2,3,6,8,10},{7,8,10},{1,7,8,10},{2,7,8,10},  
 {1,2,7,8,10},{3,7,8,10},{1,3,7,8,10},{2,3,7,8,10},{1,2,3,7,8,10},{6,7,8,10},{1,6,7,8,10},  
 {2,6,7,8,10},{1,2,6,7,8,10},{3,6,7,8,10},{1,3,6,7,8,10},{2,3,6,7,8,10},{1,2,3,6,7,8,10},{9,10},  
 {1,9,10},{2,9,10},{1,2,9,10},{3,9,10},{1,3,9,10},{2,3,9,10},{1,2,3,9,10},{6,9,10},{1,6,9,10},  
 {2,6,9,10},{1,2,6,9,10},{3,6,9,10},{1,3,6,9,10},{2,3,6,9,10},{1,2,3,6,9,10},{7,9,10},  
 {1,7,9,10},{2,7,9,10},{1,2,7,9,10},{3,7,9,10},{1,3,7,9,10},{2,3,7,9,10},{1,2,3,7,9,10},{6,7,9,10},  
 {1,6,7,9,10},{2,6,7,9,10},{1,2,6,7,9,10},{3,6,7,9,10},{1,3,6,7,9,10},{2,3,6,7,9,10},  
 {1,2,3,6,7,9,10},{8,9,10},{1,8,9,10},{2,8,9,10},{1,2,8,9,10},{3,8,9,10},{1,3,8,9,10},{2,3,8,9,10},  
 {1,2,3,8,9,10},{6,8,9,10},{1,6,8,9,10},{2,6,8,9,10},{1,2,6,8,9,10},{3,6,8,9,10},{1,3,6,8,9,10},  
 {2,3,6,8,9,10},{1,2,3,6,8,9,10},{7,8,9,10},{1,7,8,9,10},{2,7,8,9,10},{1,2,7,8,9,10},{3,7,8,9,10},  
 {1,3,7,8,9,10},{2,3,7,8,9,10},{1,2,3,7,8,9,10},{6,7,8,9,10},{1,6,7,8,9,10},{2,6,7,8,9,10},  
 {1,2,6,7,8,9,10},{3,6,7,8,9,10},{1,3,6,7,8,9,10},{2,3,6,7,8,9,10},{1,2,3,6,7,8,9,10}

**Мощность общего решения:** 256.

**Минимальное по мощности частное решение:**  $\emptyset$ .

**Максимальное по мощности частное решение:** {1,2,3,6,7,8,9,10}.

6. Написать программу для проверки найденных решений.

```

int main() {
#ifdef SECOND
    int a[] = {1,2,3,6,7,8,9,10};
    unordered_array_set x = UOAS_create_from_array(a, 8);
    string input = "(A-(X&B)u(C&!X))^((X-A)&(X-B)-C)";
    InToPost translator(input);
    SolverPost solver(8, translator.getPostfixNotation());
    solver.inputX(x);
    unordered_array_set set = solver.fi();

    if (set.size == 0)
        cout << "Is this solution";
    else
        cout << "Is not a solution";
    cout << endl;
#endif

    return 0;
}

```