

РОССИЙСКОЙ ФЕДЕРАЦИИ
ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ
УЧРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ

**«БЕЛГОРОДСКИЙ ГОСУДАРСТВЕННЫЙ
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Лабораторная работа №4.2
по дисциплине: Дискретная математика
тема: «Циклы»

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Цель работы: изучить разновидности циклов в графах, научиться генерировать случайные графы, определять их принадлежность к множеству эйлеровых и гамильтоновых графов, находить все эйлеровы и гамильтоновы циклы в графах.

Задания

№1. Разработать и реализовать алгоритм генерации случайного графа, содержащего n вершин и m ребер.

№2. Написать программу, которая: а) в течение десяти секунд генерирует случайные графы, содержащие n вершин и m ребер; б) для каждого полученного графа определяет, является ли он эйлеровым или гамильтоновым; в) подсчитывает общее количество сгенерированных графов и количество графов каждого типа.

Код программы:

```
#include "iostream"
#include <vector>
#include <set>
#include <map>
#include <algorithm>
#include <ctime>
#include <random>

bool isExist(const std::vector<int> &adjacencyVector, const int &value) {
    for (auto &adjacency: adjacencyVector) {
        if (adjacency == value) {
            return true;
        }
    }
    return false;
}

void _generateGraph(Graph &graph, const size_t &verticesNumber, const size_t
&edgesNumber) {
    if (edgesNumber > (verticesNumber * (verticesNumber - 1) / 2)) {
        std::cerr << "Unable to build a graph with the specified number of
vertices and edges\n";
        exit(1);
    }
    std::srand(std::time(0));
    std::set<int> randVertex;
    std::vector<int> initAdjacency;
    for (int i = 0; i < verticesNumber; i++) {
        int randValue = std::rand() % 1000;
        if (randVertex.find(randValue) != randVertex.end()) {
            i--;
        } else {
            randVertex.insert(randValue);
            graph.insert({randValue, initAdjacency});
        }
    }
    for (auto &value: randVertex) {
        initAdjacency.push_back(value);
    }
    for (int i = 0; i < edgesNumber; i++) {
```

```

size_t randIndexFirstVertex = std::rand() % verticesNumber;
size_t randIndexSecondVertex = std::rand() % verticesNumber;
if ((randIndexFirstVertex != randIndexSecondVertex)
    && !isExist(graph.at(initAdjacency.at(randIndexFirstVertex)),
        initAdjacency.at(randIndexSecondVertex))) {
    graph.at(initAdjacency.at(randIndexFirstVertex)).push_back(initAdjacency.at(r
        andIndexSecondVertex));
    graph.at(initAdjacency.at(randIndexSecondVertex)).push_back(initAdjacency.at(
        randIndexFirstVertex));
} else {
    i--;
}
}
}

```

```

Graph generateGraph(const size_t &verticesNumber, const size_t &edgesNumber){
    Graph graph;
    _generateGraph(graph, verticesNumber, edgesNumber);
    return graph;
}

```

```

bool isEulerGraph(const Graph &graph) {
    for (auto &x: graph) {
        if (x.second.size() % 2 == 1) {
            return false;
        }
    }
    return true;
}

```

```

bool isHamiltonianGraph(const Graph &graph) {
    if (graph.size() >= 3) {
        for (auto &x: graph) {
            if (x.second.size() < (graph.size() / 2)) {
                return false;
            }
        }
    } else {
        return false;
    }
    return true;
}

```

```

void exercise(const size_t &verticesNumber) {
    size_t maxEdgesNumber = verticesNumber * (verticesNumber - 1) / 2;
    for (size_t i = verticesNumber; i <= maxEdgesNumber; i++) {
        auto untilMoment {std::chrono::high_resolution_clock::now() +
            std::chrono::seconds(10)};
        size_t hamiltonianCounter = 0;
        size_t eulerCounter = 0;
        size_t generalCounter = 0;
        while (std::chrono::high_resolution_clock::now() < untilMoment) {
            Graph graph = generateGraph(verticesNumber, i);
            hamiltonianCounter += isHamiltonianGraph(graph);
            eulerCounter += isEulerGraph(graph);
            generalCounter++;
        }
        std::cout << "\tInformation about " << i << "-edges graph\n";
        std::cout << "-Hamiltonian: " << hamiltonianCounter
            << "; Euler: " << eulerCounter
            << "; Total: " << generalCounter << '\n';
    }
}

```

```
int main() {  
    exercise(8);  
    return 0;  
}
```

№3. Выполнить программу при $n = 8, 9, 10$ и сделать выводы.

при $n = 8$

```
Information about 8-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 583664  
Information about 9-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 546046  
Information about 10-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 495352  
Information about 11-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 474414  
Information about 12-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 465235  
Information about 13-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 446247  
Information about 14-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 427249  
Information about 15-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 415332  
Information about 16-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 393552  
Information about 17-edges graph  
-Hamiltonian: 0; Euler: 0; Total: 380852  
Information about 18-edges graph  
-Hamiltonian: 36120; Euler: 0; Total: 366501  
Information about 19-edges graph  
-Hamiltonian: 73770; Euler: 0; Total: 362836  
Information about 20-edges graph  
-Hamiltonian: 126600; Euler: 0; Total: 331860  
Information about 21-edges graph  
-Hamiltonian: 298091; Euler: 0; Total: 331521  
Information about 22-edges graph  
-Hamiltonian: 316639; Euler: 0; Total: 316639  
Information about 23-edges graph  
-Hamiltonian: 304840; Euler: 0; Total: 304840  
Information about 24-edges graph  
-Hamiltonian: 297186; Euler: 0; Total: 297186  
Information about 25-edges graph  
-Hamiltonian: 278230; Euler: 0; Total: 278230  
Information about 26-edges graph  
-Hamiltonian: 251322; Euler: 0; Total: 251322  
Information about 27-edges graph  
-Hamiltonian: 248415; Euler: 0; Total: 248415  
Information about 28-edges graph  
-Hamiltonian: 230636; Euler: 0; Total: 230636
```

При $n = 9$:

```
Information about 9-edges graph
-Hamiltonian: 0; Euler: 0; Total: 457247
Information about 10-edges graph
-Hamiltonian: 0; Euler: 0; Total: 496223
Information about 11-edges graph
-Hamiltonian: 0; Euler: 0; Total: 472507
Information about 12-edges graph
-Hamiltonian: 0; Euler: 0; Total: 453032
Information about 13-edges graph
-Hamiltonian: 0; Euler: 0; Total: 436792
Information about 14-edges graph
-Hamiltonian: 0; Euler: 0; Total: 413971
Information about 15-edges graph
-Hamiltonian: 0; Euler: 40957; Total: 400529
Information about 16-edges graph
-Hamiltonian: 0; Euler: 0; Total: 388130
Information about 17-edges graph
-Hamiltonian: 0; Euler: 0; Total: 363612
Information about 18-edges graph
-Hamiltonian: 0; Euler: 0; Total: 361561
Information about 19-edges graph
-Hamiltonian: 0; Euler: 0; Total: 346502
Information about 20-edges graph
-Hamiltonian: 0; Euler: 0; Total: 336055
Information about 21-edges graph
-Hamiltonian: 131929; Euler: 0; Total: 327085
Information about 22-edges graph
-Hamiltonian: 11679; Euler: 0; Total: 312754
Information about 23-edges graph
-Hamiltonian: 123468; Euler: 0; Total: 307654
Information about 24-edges graph
-Hamiltonian: 119384; Euler: 0; Total: 299046
Information about 25-edges graph
-Hamiltonian: 223368; Euler: 0; Total: 282014
Information about 26-edges graph
-Hamiltonian: 226259; Euler: 0; Total: 281663
Information about 27-edges graph
-Hamiltonian: 271773; Euler: 28307; Total: 271773
Information about 28-edges graph
-Hamiltonian: 263260; Euler: 0; Total: 263260
Information about 29-edges graph
-Hamiltonian: 248221; Euler: 0; Total: 248221
```

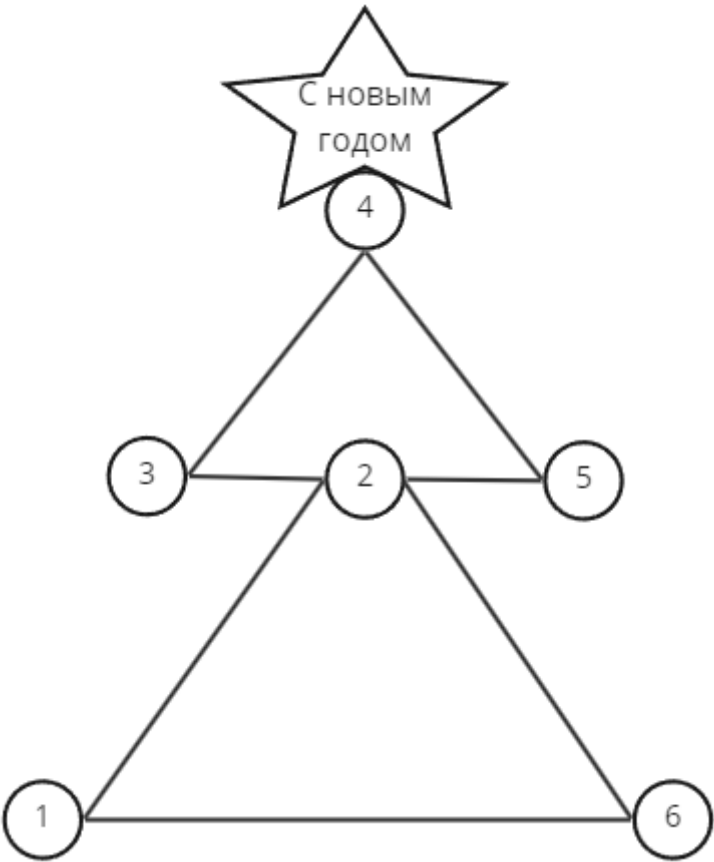
```
Information about 30-edges graph
-Hamiltonian: 242095; Euler: 0; Total: 242095
Information about 31-edges graph
-Hamiltonian: 239667; Euler: 0; Total: 239667
Information about 32-edges graph
-Hamiltonian: 227266; Euler: 0; Total: 227266
Information about 33-edges graph
-Hamiltonian: 223215; Euler: 0; Total: 223215
Information about 34-edges graph
-Hamiltonian: 197652; Euler: 0; Total: 197652
Information about 35-edges graph
-Hamiltonian: 194589; Euler: 0; Total: 194589
Information about 36-edges graph
-Hamiltonian: 153075; Euler: 153075; Total: 153075
```

- при $n = 10$:

```
Information about 10-edges graph
-Hamiltonian: 0; Euler: 0; Total: 445113
Information about 11-edges graph
-Hamiltonian: 0; Euler: 0; Total: 437186
Information about 12-edges graph
-Hamiltonian: 0; Euler: 0; Total: 433762
Information about 13-edges graph
-Hamiltonian: 0; Euler: 0; Total: 415358
Information about 14-edges graph
-Hamiltonian: 0; Euler: 0; Total: 396191
Information about 15-edges graph
-Hamiltonian: 0; Euler: 0; Total: 384511
Information about 16-edges graph
-Hamiltonian: 0; Euler: 0; Total: 373882
Information about 17-edges graph
-Hamiltonian: 0; Euler: 0; Total: 363264
Information about 18-edges graph
-Hamiltonian: 0; Euler: 34383; Total: 349042
Information about 19-edges graph
-Hamiltonian: 0; Euler: 0; Total: 334320
Information about 20-edges graph
-Hamiltonian: 0; Euler: 0; Total: 328496
Information about 21-edges graph
-Hamiltonian: 0; Euler: 0; Total: 320039
Information about 22-edges graph
-Hamiltonian: 0; Euler: 0; Total: 312351
Information about 23-edges graph
-Hamiltonian: 0; Euler: 0; Total: 298349
Information about 24-edges graph
-Hamiltonian: 0; Euler: 0; Total: 289589
Information about 25-edges graph
-Hamiltonian: 0; Euler: 0; Total: 283101
Information about 26-edges graph
-Hamiltonian: 0; Euler: 0; Total: 276843
Information about 27-edges graph
-Hamiltonian: 0; Euler: 0; Total: 266813
Information about 28-edges graph
-Hamiltonian: 0; Euler: 0; Total: 268144
Information about 29-edges graph
-Hamiltonian: 7386; Euler: 0; Total: 256128
Information about 30-edges graph
-Hamiltonian: 23660; Euler: 0; Total: 246528
```

```
Information about 31-edges graph
-Hamiltonian: 71529; Euler: 0; Total: 242453
Information about 32-edges graph
-Hamiltonian: 108040; Euler: 0; Total: 232200
Information about 33-edges graph
-Hamiltonian: 136543; Euler: 0; Total: 222988
Information about 34-edges graph
-Hamiltonian: 145643; Euler: 0; Total: 219803
Information about 35-edges graph
-Hamiltonian: 145627; Euler: 0; Total: 210815
Information about 36-edges graph
-Hamiltonian: 181554; Euler: 0; Total: 202582
Information about 37-edges graph
-Hamiltonian: 203486; Euler: 0; Total: 203486
Information about 38-edges graph
-Hamiltonian: 193615; Euler: 0; Total: 193615
Information about 39-edges graph
-Hamiltonian: 189290; Euler: 0; Total: 189290
Information about 40-edges graph
-Hamiltonian: 179346; Euler: 0; Total: 179346
Information about 41-edges graph
-Hamiltonian: 178863; Euler: 0; Total: 178863
Information about 42-edges graph
-Hamiltonian: 156716; Euler: 0; Total: 156716
Information about 43-edges graph
-Hamiltonian: 156870; Euler: 0; Total: 156870
Information about 44-edges graph
-Hamiltonian: 145200; Euler: 0; Total: 145200
Information about 45-edges graph
-Hamiltonian: 118853; Euler: 0; Total: 118853
```

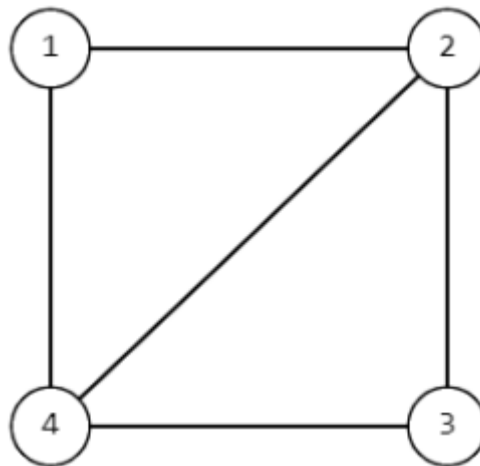

№4. Привести пример диаграммы графа, который является эйлеровым, но не гамильтоновым. Найти в нем все эйлеровы циклы.



Все эйлеровы циклы:

| | |
|-----------------|-----------------|
| 1 2 3 4 5 2 6 1 | 3 4 5 2 1 6 2 3 |
| 1 2 5 4 3 2 6 1 | 3 4 5 2 6 1 2 3 |
| 1 6 2 3 4 5 2 1 | 4 3 2 1 6 2 5 4 |
| 1 6 2 5 4 3 2 1 | 4 3 2 6 1 2 5 4 |
| 2 1 6 2 3 4 5 2 | 4 5 2 1 6 2 3 4 |
| 2 1 6 2 5 4 3 2 | 4 5 2 6 1 2 3 4 |
| 2 3 4 5 2 1 6 2 | 5 2 1 6 2 3 4 5 |
| 2 3 4 5 2 6 1 2 | 5 2 6 1 2 3 4 5 |
| 2 5 4 3 2 1 6 2 | 5 4 3 2 1 6 2 5 |
| 2 5 4 3 2 6 1 2 | 5 4 3 2 6 1 2 5 |
| 2 6 1 2 3 4 5 2 | 6 1 2 3 4 5 2 6 |
| 2 6 1 2 5 4 3 2 | 6 1 2 5 4 3 2 6 |
| 3 2 1 6 2 5 4 3 | 6 2 3 4 5 2 1 6 |
| 3 2 6 1 2 5 4 3 | 6 2 5 4 3 2 1 6 |

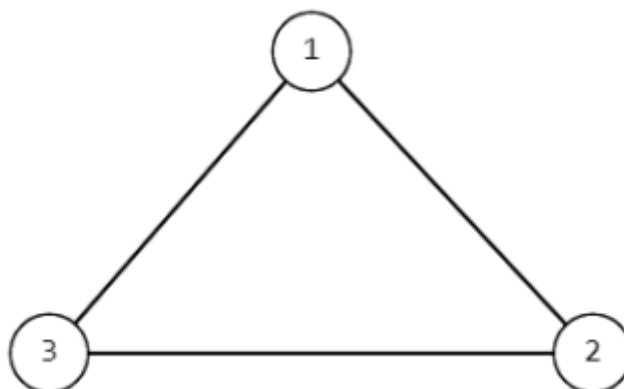
№5. Привести пример диаграммы графа, который является гамильтоновым, но не эйлеровым. Найти в нем все гамильтоновы циклы.



Все гамильтоновы циклы:

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 |
| 1 | 4 | 3 | 2 | 1 |
| 2 | 1 | 4 | 3 | 2 |
| 2 | 3 | 4 | 1 | 2 |
| 3 | 2 | 1 | 4 | 3 |
| 3 | 4 | 1 | 2 | 3 |
| 4 | 1 | 2 | 3 | 4 |
| 4 | 3 | 2 | 1 | 4 |

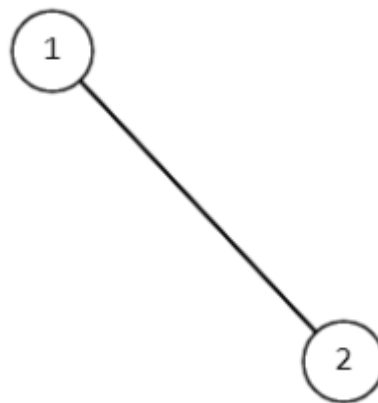
№6. Привести пример диаграммы графа, который является эйлеровым и гамильтоновым. Найти в нем все эйлеровы и гамильтоновы циклы.



Все и гамильтоновы, и эйлеровы циклы:

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 1 |
| 1 | 3 | 2 | 1 |
| 2 | 1 | 3 | 2 |
| 2 | 3 | 1 | 2 |
| 3 | 1 | 2 | 3 |
| 3 | 2 | 1 | 3 |

№7. Привести пример диаграммы графа, который не является ни эйлеровым, ни гамильтоновым



Вывод: в ходе лабораторной работы изучили разновидности циклов в графах, научились генерировать случайные графы, определять их принадлежность к множеству эйлеровых и гамильтоновых графов, находить все эйлеровы и гамильтоновы циклы в графах