Chapter 4 homework

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1 Theoretical questions

1.1 I

result $= 1.11011101 \times 2^8$

1.2 II

result $= 1.001001001 \cdots \times 2^1$

1.3 III

$x = 1.0000 \cdots 00 \times \beta^e$
$x_L = (\beta - 1).(\beta - 1) \cdots (\beta - 1) \times \beta^{e-1}$ $x_R = 1.000 \cdots 1 \times \beta^{e-1}$
So, $x_R - x = \beta^{e-p}, x - x_L = \beta^{e-p-1}$
Then we have $x_R - x = \beta(x - x_L)$

1.4 IV

$x_L = 1.00100100100100100100100 \times 2^{-1}, x_R = 1.00100100100100100100101 \times 2^{-1}$
$fl(x) = x_R$,roundoff error $= \frac{|x_R - x|}{|x|} \approx 2^{-25}$

1.5 V

$\epsilon_u = \beta^{-p} = 2^{-23}$

1.6 VI

$cos(0.25) = 1.111100000001 \cdots \times 2^{-1}$
$1 - cos(0.25) = 0.00001 \cdots \times 2^{-1}$, so it has 5 bits of precision lost.

1.7 VII

1. $1 - cos(x) = 1 - (1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots) = \frac{x^2}{2!} - \frac{x^4}{4!} \cdots$
2. $1 - cos(x) = 2sin^2(\frac{x}{2})$

1.8 VIII

1. $f(x) = (x-1)^\alpha, C_f(x) = |\frac{\alpha x(x-1)^{\alpha-1}}{(x-1)^\alpha}|$,if $\alpha \neq 0$,C_f is large when $x \to 0$
2. $f(x) = ln(x), C_f(x) = |\frac{1}{ln(x)}|$,$C_f$ is large when $x \to 0$
3. $f(x) = e^x, C_f(x) = |x|$,C_f is large when $x \to \infty$
4. $f(x) = arccos(x), C_f(x) = |\frac{x}{\sqrt{1-x^2} arccos(x)}|$,$C_f$ is large when $x \to \pm 1$

1.9 IX

1. $C_f(x) = \frac{x}{1+e^x}, \forall x \in [0,1]$
2. $C_A(x) = \frac{1}{\epsilon_u} \inf_{x_A} \frac{|x_A - x|}{|x|}$

   $f^{'}(x) = e^{-x} \geq \frac{1}{e}$

   $|f(x_A) - f(x)| = f^{'}(\xi)|x_A - x| \leq \epsilon_u$

   So,$|x_A - x| \leq e\epsilon_u$

   Then $C_A(x) \leq \frac{e}{|x|}$
3. $C_f(x)$ and the estimated upper bound of $C_A(x)$ as a function of x on [0, 1] is showed by the picture:

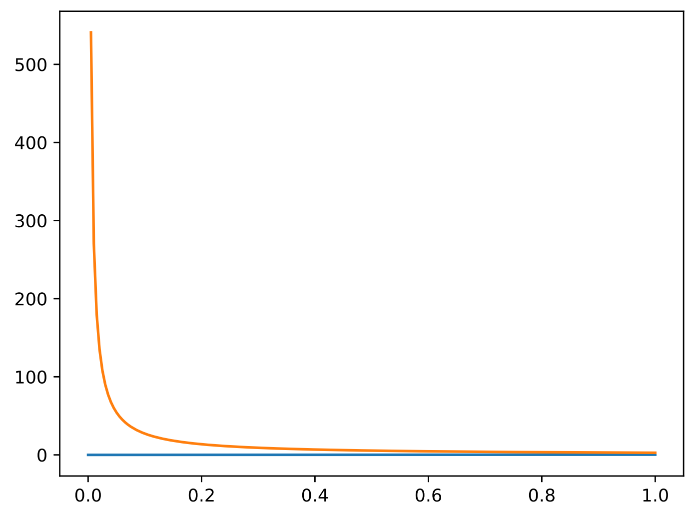

图 1: C_f and C_A on $[0,1]$

when $x = 0,\ f(0) = 0, f_A(x) = f(x)(1 + \delta(x)),\ \delta(x) \to \infty$.
So $x \to 0, C_A(x) \to \infty$

1.10 X

$Cond_1 = |\frac{1}{r}|\Sigma_{i=0}^{n-1}|a_i \frac{\partial r}{\partial a_i}| = \frac{\Sigma_{i=0}^{n-1}|a_i r^i|}{r(\Sigma_{i=1}^{n}(n-i+1)a_i r^{n-i})}$
Let $r = n, f(x) = \Pi_{i=1}^{n}(x - i)$, then we have $Cond_1 \geq \frac{n^n}{n!}$
When n is large ,$Cond_1$ also become very large, which is consistent with the Wilkinson.

1.11 XI

In FPN system(10,1,-1,1),$\frac{4}{9} = 0.44444 = 0$
$\frac{0-\frac{4}{9}}{\frac{4}{9}} = 1 > \epsilon_u$

1.12 XII

对 $x \in [128, 129]$,$x = m \times 2^e, e = 7.$
所以区间内两个相邻浮点数的距离为 $2^7 \epsilon_M = 2^{-16} > 10^{-6}$
所以不能将计算根的精度降低到 $1e - 6$

1.13 XIII

设 x_i, x_{i+1} 的距离很近，即 $|x_i - x_{i+1}| \le \delta$
则计算三次样条 $p(x) = a + b(x - x_i) + c(x - x_i)^2 + d(x - x_i)^3$ 的方程为：

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \delta & \delta^2 & \delta^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2\delta & 3\delta \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} f(x_i) \\ f(x_{i+1}) \\ f^{'}(x_i) \\ f^{'}(x_{i+1}) \end{pmatrix}$$

当 δ 小时，方程组的条件数极大，故此时三次样条结果不稳定。