

Throughput Maximization for Delay-Sensitive Random Access Communication

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Abstract

We consider a single cell uplink in which many devices randomly transmit a data payload to the base station. Given a fixed latency constraint per device, we propose a time and frequency slotted random access scheme with retransmissions, which when necessary, are Chase combined at the receiver. We analyze the proposed setting at constant SNR. We characterize the scaling of random access throughput versus the latency, by optimizing the number of retransmissions and the number of frequency bins. For infinite block length (IBL), we conclude that at low SNR devices should completely share the time and frequency resources. For high SNR, however, the number of frequency bins should scale with altered load, and the slot duration for each retransmission is determined by the outage tolerance. Since infinite packet sizes are not possible, we extend our model to the finite block length (FBL) regime and characterize the gap versus the IBL regime. We also provide some new results for FBL capacity to bound the probability of outage. The proposed random access model gives an upper bound for the total uplink traffic that can be successfully decoded for a single receive antenna given the latency constraint, and provides insights for 5G cellular system design.

I. INTRODUCTION

Machine-type communication (MTC) applications are rapidly growing, and will become an increasingly important source of traffic and revenue in cellular networks [2]. Unlike most human-generated

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traffic, MTC is often characterized by a very large number of small transactions. The LTE air interface design for high-data-rate applications may not effectively support reliable and low-latency MTC with a vast number of devices. Emerging Internet of Things (IoT) examples include sensor [3] and time of arrival measurements [4], smart grids for power distribution automation, industrial manufacturing, control and automation applications [5], traffic safety, and monitoring of materials in industrial plants [6].

Random access (RA) is a critical challenge in IoT networks, because of the massive number of devices that wish to access the channel and their frequently very short payloads [7], [8], [9]. Furthermore, there is overhead for scheduled access and it doesn't make sense to invest if the payload size is small. In such scenarios it can be envisioned that the devices never move into a scheduled access phase [7], [8], instead just using the RA process [10], to send a small data payload. However, a large number of RA transmissions can overload the random access channel (RACH), resulting in a significant access delay. Thus, MTC will require sophisticated access management and resource allocation to prevent RACH congestion [3].

A. Related Work and Motivation

Use cases for MTC and requirements for massive and ultra-reliable MTC have been outlined in [11]. Guidelines for context-aware functionality for MTC were provided in [12]. Random access for MTC and scheduling have been extensively studied in [13], [14], [15], and access for ultra-reliable and low-latency communication was investigated in [5]. Random access overhead and congestion in MTC networks were analyzed in [16], and access with collision resolution for massive MTC was proposed in [17]. Various aspects such as power control [18], spectrum allocation [19], and energy-efficient uplink resource allocation [20], and load control [21] for MTC traffic, have been studied.

There are complex tradeoffs between the energy efficiency [7], [22], reliability (using short packets) [2], latency [9], [23], signaling overhead [24], [25], data payload size, and number of devices that can successfully complete the RA process within a time slot [26]. An information theoretic upper bound on the random access transmission rate was derived in [27], where a large number of devices with sporadic traffic communicate in the uplink. Effective capacity of retransmission schemes was studied in [28]. Throughput of hybrid ARQ (HARQ) with incremental redundancy and the effect of the transmission

rate, queuing and hard-deadline constraints on the throughput were investigated in [29]. The power and energy optimal uplink design for various access strategies was studied in [7], while an optimal uncoordinated strategy to maximize the average throughput for a time-slotted RACH was developed in [8]. A strategy that transmits both identity and data over the RACH was shown to support significantly more devices compared to the conventional approach, where transmissions are scheduled after an initial RA stage. An online delay-optimal proportionally fair scheduler for delay-sensitive MTC uplink traffic was proposed in [30]. There is also an ongoing effort to optimize cellular networks to support ultra-low complexity, power, and throughput for IoT, currently under standardization for LTE-A [31], [32].

Recently, some results on finite block length (FBL) have also been developed for some scenarios [2], [9], [33], [34], [35], [36]. FBL for interference-limited regimes was analyzed in [33], and spectrum sharing in the FBL regime using rate adaptation was proposed in [34].

Variants of slotted ALOHA have been studied to mitigate channel contention and achieve high throughput values. Contention resolution diversity is an improvement of slotted ALOHA that relies on MAC burst repetition and on interference cancellation (IC) for resolving collisions [37]. Irregular repetition slotted ALOHA techniques can achieve high-throughput performance for small MAC frame sizes when IC methods are adopted [38].

The optimal strategy in [7] for minimizing the total power consumed to transmit a fixed size payload is not achievable in practice because it assumes ideal channel knowledge and ideal successive interference cancellation (SIC). Therefore, we consider a suboptimal but practical random access strategy with retransmissions and many orthogonal frequency bins. We initially studied a random access structure with slotted ALOHA in [1] for an idealized regime that does not capture the finiteness of the number of resources. In this paper, we characterize the theoretical performance of this strategy in terms of the maximum arrival rate that can be supported for a fixed maximum delay for a given number of resource symbols. We then optimize the resource utilization (retransmissions and number of frequency bins) for different SNR regimes.

B. Contributions

We consider a single cell model in which a set of transmitters transmit to a common base station (BS). For each transmit opportunity, there are K users wishing to communicate, where K is a realization of a

Poisson distribution with mean λ . The payload size of each user is L bits. We characterize maximum Poisson arrival rate such that each device meets a fixed delay constraint, with probability at least $1 - \delta$. Our proposed RA policy is a generalization of slotted ALOHA, and has flexible slot length and memory. Each frame is of duration T , which is the delay constraint, and is evenly partitioned into $M \geq 1$ slots. The total bandwidth W is equally divided into $B \geq 1$ subbands. To prevent decoding failure, i.e. outage, each device needs to complete the RA phase within M retransmission attempts (not exceeding total duration T) by selecting one bin at random at each attempt. The resulting SINR of each transmission attempt is combined at the receiver; similar to HARQ with Chase combining [39].

We analyze the proposed setting in the infinite block length (IBL) and finite block length (FBL) regimes under low and high SNR conditions. We maximize the throughput over multiple retransmissions with respect to the deadline constraint T for a given bandwidth and outage constraint by optimizing M and B . Exploiting the scaling results for the IBL regime, we then determine the scaling of the throughput in the FBL regime. We also provide some new results for FBL block error rate hence the maximal rate achievable to bound the probability of outage.

The key design insights for the proposed strategy are as follows.

- At constant SNR, i.e. no fading, for the high SNR regime, the resources should be split sufficiently such that devices do not experience any interference provided that the total number of resources is sufficiently large. Hence, the number of frequency bins should approximately be the same as the number of device arrivals, i.e. $B \approx K$.
- At constant SNR, for the low SNR regime we show that the devices should share the resources.
- For the case of Rayleigh fading, although the variability of the channel causes a drop in the number of arrivals that can successfully complete the random access phase, we briefly discuss that similar conclusions extend to that case.
- FBL regime is the practical case for short packet sizes. Although there is a gap in the throughput of the FBL model from the IBL model, scaling results for both regimes are similar.

The insights could be applied to 5G cellular system design where delay-constrained communications will be an important use case.

II. SYSTEM MODEL

We consider a single cell uplink in which a set of devices transmits a fixed payload to the BS. We propose a slotted ALOHA-like scheme with retransmissions and multiple frequency bins. Below, we summarize the key notation and the random access model parameters used in the paper:

- There is a fixed size payload of L bits per device.
- Each device has a fixed latency constraint T .
- The total available bandwidth is W .
- The total symbol resources $N = TW$.
- The total bandwidth W is evenly partitioned into B frequency bins.
- Each payload is granted up to a maximum of $M - 1$ retransmission attempts, with M being the total number of transmissions.
- Each time frame of duration T is evenly split into M slots of duration T/M each.
- The N resources are partitioned into N/MB resources each bin per retransmission.
- The encoded block length per transmission should satisfy $n \leq N/MB$.
- The transmit rate is L/n bits per transmission attempt.
- At a transmission attempt, a given device selects one of the B bins at random and transmits.
- There is precise synchronization between the beginning and ending of transmissions and the slots, and the transmission attempts occur at the beginning of each slot.
- If a device can successfully transmit its payload in $1 \leq m \leq M$ consecutive attempts, it does not use the remaining $M - m$ slots.
- The effective channel gains are constant. We later relax this assumption to the case of Rayleigh fading (see Sect. III-B), and discuss how the constant gain results are modified by incorporating the channel power distributions.
- Devices perform open loop power control under which the received power at the BS from any device, P_r , is constant.
- The noise power at the base station is additive and constant with value σ^2 .
- The received SNR per device at the BS is $\rho = P_r/\sigma^2$.

In this section, we optimistically assume that the encoded block length of n symbols is sufficiently

large that we can exploit the IBL¹ limit to characterize the system performance. An outage occurs when a given device fails to communicate its payload on M consecutive attempts, i.e. a decoding error occurs if the signal-to-noise-plus-interference ratio (SINR) across multiple transmissions is below the threshold Γ . This threshold is given by $\Gamma = 2^{\frac{L}{n}} - 1$ for a given block length n per transmission.

Chase combining is used to aggregate the received signals across multiple transmissions at the BS, resulting in maximal ratio combining of the desired signal. After the device transmits its payload, the BS either sends ACK to the sender (device) to indicate the successful decoding of the payload if the received payload is error free, or NACK if the decoding of the received payload was unsuccessful. For tractability, we assume there are no errors or delay in the feedback (ACK/NACK), so there is immediate retransmission on the next slot after a failure. In practice, there would be a gap of at least one slot while feedback from the BS was sent and processed, which would increase the latency.

Proposition 1. Chase combiner output SNR. *If $M > 1$, a device is allowed to retransmit if the preceding one fails, for a total of M transmissions. In general, if the received signal vector during transmission $i = 1, \dots, M$ is $\mathbf{r}_i = a_i \mathbf{s} + \mathbf{n}_i$, where $\mathbf{s} \in \mathbb{C}^n$ is the desired signal, a_i is the complex amplitude, and \mathbf{n}_i is an n -dimensional complex Gaussian noise vector with zero mean and covariance $\sigma_i^2 \mathbf{I}_n$, then the SNR at the output of the Chase combiner is [1]*

$$\left(\sum_{i=1}^M |a_i|^2 \right)^2 / \sum_{i=1}^M |a_i|^2 \sigma_i^2. \quad (1)$$

Given M , denote the set of retransmission indices by $\mathcal{M} = \{1, \dots, M\}$. For ease of notation, the number of arrivals on the m^{th} attempt where $m \leq M$ is denoted by k_m , and the set of arrivals up to and including m^{th} attempt is denoted by $\mathcal{K}_m := \{k_1, \dots, k_m\}$. If there are $k_m \geq 1$ devices arriving during transmission $m \in \mathcal{M}$, each with receiver SNR ρ , in a given resource bin, then the SINR during transmission m is $\rho / (1 + (k_m - 1)\rho)$. Hence, from Prop. 1, the Chase combiner output SINR from (1) as a result of $m \in \mathcal{M}$ transmissions is

$$\text{SINR}(\mathcal{K}_m) = \frac{\rho m^2}{m + \rho \left(\sum_{i=1}^m k_i - m \right)}, \quad m \in \mathcal{M}. \quad (2)$$

¹We relax this assumption in Sect. V to the finite block length (FBL) regime.

A device fails on the m^{th} attempt if the combined output SINR up to and including the m^{th} attempt, i.e. $\text{SINR}(\mathcal{K}_m)$, is below the threshold Γ . Given a target SINR outage rate δ per device, outage occurs if more than a certain number of users share the same resources.

We define $P_{\text{Fail}}(m)$ to be the probability of outage up to and including the m^{th} attempt for a given SINR threshold Γ , which is given as

$$P_{\text{Fail}}(m) = \mathbb{P}[\text{SINR}(\mathcal{K}_1) < \Gamma, \text{SINR}(\mathcal{K}_2) < \Gamma, \dots, \text{SINR}(\mathcal{K}_m) < \Gamma]. \quad (3)$$

In other words, the joint probability in (3) is the intersection of the decoding failure events up to and including the m^{th} attempt. We emphasize that these m events are dependent, since the Chase combiner output on the m^{th} attempt, i.e. $\text{SINR}(\mathcal{K}_m)$, is a function of all previous receptions as well, i.e. $\mathcal{K}_m = \{k_1, \dots, k_m\}$.

For the proposed setting, our objective is to optimize the random access strategy by calculating the values of the number of bins B and the number of retransmission attempts M that maximize the average user arrival rate λ that can be supported per slot, denoted by λ_{opt} . At the same time, for a given number of symbol resources N (T sec, W Hz), the probability of outage given up to a maximum of M transmission attempts should be less than or equal to δ . We then characterize the tradeoffs for random access between throughput in terms of the average number of devices supported and latency.

The system model for $M = 5$ retransmissions is shown in Fig. 1. In this example, a device arriving during slot 1 has until slot 5 to transmit. Another device arriving during slot 2 has until slot 6 to transmit. This emphasizes the fact that devices can arrive during each slot. In the same figure, we also illustrate a typical device that uses only $m = 3$ time slots out of 5 to retransmit its payload to the BS by choosing frequency bins at random at each time slot. The first 2 attempts combined at the BS cannot be decoded and the BS sends NACK back to the device after each failure. The payload is successfully decoded after 2 retransmission attempts and the BS sends an ACK to the device.

Given the number of frequency bins B , the average number of arrivals per bin per transmission interval is Poisson distributed with an average arrival rate of $\lambda_B = \lambda/B$. Hence, the probability of $k \geq 1$ devices choosing a given resource bin out of B bins for transmission, given that there is at

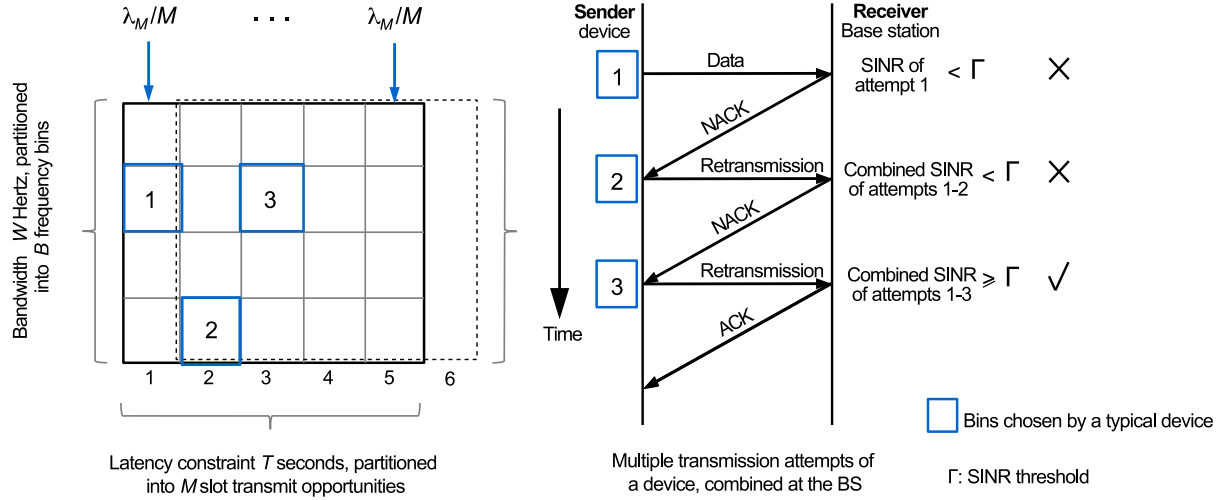


Fig. 1: An example RA scenario. The bandwidth is partitioned into $B(= 4)$ frequency bins and time is partitioned into $M(= 5)$ transmit opportunities. The average arrival rate of devices for each slot is λ_M/M . We illustrate a typical device that is unsuccessful on the first transmission attempt, and retransmits 2 times by choosing frequency bins at random at each time slot. The transmission attempts are combined at the BS, yielding success in slot $m = 3$.

least one arrival, is given by the following conditional probability:

$$D(k, \lambda_B) = \frac{\mathbb{P}(k \text{ device arrivals per bin, } k \geq 1)}{\mathbb{P}(k \geq 1)} = \frac{\lambda_B^k \exp(-\lambda_B)}{k!(1 - \exp(-\lambda_B))}, \quad k \geq 1, \quad (4)$$

where we note that the probability of at least one device arrival is $\mathbb{P}(k \geq 1) = 1 - \exp(-\lambda_B)$.

Denote the average aggregate device arrival rate with up to M total transmissions per frame by λ_M , which is the sum of the rates of the original arrivals per slot, i.e. λ , and the arrivals occurring as a result of failed transmissions up to a maximum of $M - 1$ consecutive attempts. It is given by

$$\lambda_M = \lambda \left[1 + \sum_{m=1}^{M-1} P_{\text{Fail}}(m) \right], \quad (5)$$

where $P_{\text{Fail}}(m)$ as given in (3) denotes the fraction of devices that fail up to and including the m^{th} attempt. Hence, given maximum number of retransmission attempts M per time frame, the number of arrivals at any time slot $1 \leq m \leq M$ is Poisson distributed² with an average arrival rate of $\frac{\lambda_M}{M}$.

²For tractability, we inherently have the Poisson distribution assumption for the composite arrival process. From [8] and [40], this assumption is justifiable when the number of retransmissions is not too large.

The optimization problem is

$$\begin{aligned}
\lambda_{\text{opt}} &= \max_{B, M \in \mathbb{Z}^+} \lambda \\
\text{s.t.} \quad & P_{\text{Fail}}(M) \leq \delta, \quad \Gamma = 2^{\frac{L}{n}} - 1, \\
& C(\text{SINR}(\mathcal{K}_m)) \geq \frac{L}{n}, \quad m \in \mathcal{M}, \quad n \leq \frac{TW}{MB},
\end{aligned} \tag{6}$$

where $P_{\text{Fail}}(M)$ is given in (3), and $C(\text{SINR}(\mathcal{K}_m))$ is the channel capacity of a typical device in the uplink as a function of (2).

III. INFINITE BLOCK LENGTH

Shannon's channel capacity is achievable at an arbitrarily low error rate when coding is performed in the infinite block length (IBL) regime, i.e. using a code block of infinite length. However, since the number of resources N is finite, the ratio L/N is always finite. Therefore, given L , the IBL scheme gives an upper bound on the achievable transmit rate, hence a lower bound on n .

In this section, we assume that the encoded block length n symbols is sufficiently large so that we can exploit the Shannon limit. The capacity of an additive white Gaussian noise (AWGN) channel where interference is treated as noise in that case is

$$C(\text{SINR}) = \log_2(1 + \text{SINR}). \tag{7}$$

For some number of retransmission attempts, combining the capacity constraint in (6) with the Chase combiner output (2), we investigate the outage probability for both constant SNR and Rayleigh fading.

A. Constant SNR

At constant SNR, the Chase combiner output SINR as a result of $m \in \mathcal{M}$ transmissions, i.e. $\text{SINR}(\mathcal{K}_m)$ as a function of \mathcal{K}_m , is given in (2). Hence, incorporating (7) in the capacity constraint in (6), the maximum number of arrivals that can be supported at time slot $m \in \mathcal{M}$ can be determined as a function of \mathcal{K}_{m-1} . We denote this maximum by $k_m^* := k_m(\mathcal{K}_{m-1})$.

For a given maximum number of retransmission attempts M , an outage occurs if there are more than k_m^* devices sharing the same bin on attempt $m \in \mathcal{M}$, due to limited number of symbol resources N/MB . Using (4), the outage probability at slot $m \in \mathcal{M}$ is calculated adding up the probabilities

$D(k_m, \lambda_B)$ over the number of arrivals that cannot be supported. Given B and N , Prop. 2 gives the outage probability at constant SNR up to a maximum of M transmissions.

Proposition 2. *Given B , M , symbol resources N , the outage probability in the IBL regime in the case of constant SNR is given by the following expression:*

$$P_{\text{Fail}}(M) = \sum_{k_m > k_m^*(\mathcal{K}_{m-1})} \prod_{m \in \mathcal{M}} D\left(k_m, \frac{\lambda_M}{B}\right), \quad (8)$$

where the lower limit of the summation in (8) is

$$k_m^*(\mathcal{K}_{m-1}) = \max \left\{ \left\lfloor m + \frac{m^2}{2^{\frac{L}{n}} - 1} - \frac{m}{\rho} - \sum_{i=1}^{m-1} k_i \right\rfloor, 1 \right\},$$

$$m \in \mathcal{M}, \quad n \leq N/(BM). \quad (9)$$

The aggregate arrival rate λ_M with up to M total transmissions is given by (5).

Proof. See Appendix A. □

B. Small Scale (Rayleigh) Fading

For the case of small scale Rayleigh fading with mean $1/\mu$, let k_m arrivals choose a given frequency bin, each having SNR ρ , during transmission $m \in \mathcal{M}$. Using (1) and incorporating the channel power distributions, the Chase combiner output SINR as a result of M transmissions is obtained as

$$\text{SINR}(\mathcal{K}_M) \stackrel{(a)}{=} \frac{\rho \left(\sum_{m \in \mathcal{M}} h_m \right)^2}{\sum_{m \in \mathcal{M}} h_m (1 + I_{k_m})} \stackrel{(b)}{=} \frac{M^2 \rho h_M}{M + \sum_{m \in \mathcal{M}} I_{k_m}}, \quad (10)$$

where $h_m, g_{i,m} \sim \exp(\mu)$, $m \in \mathcal{M}$ are independent and identically distributed (i.i.d.) channel power distributions of the desired device and the interferers, respectively, where $i_m \in \{1, \dots, k_m - 1\}$ is the interferer index at retransmission attempt m , (a) follows from letting $I_{k_m} = \sum_{i=1}^{k_m-1} \rho g_{i_m}$ denote the total interference seen at transmission attempt m , and (b) is based on the assumption that h_M is unchanged within a time slot. This is consistent with the Rayleigh block fading model [26] in which the power fading coefficients remain static over each time slot, and are temporally (and spatially) independent with exponential distribution with parameter $\mu = 1$.

The outage probability in the case of small scale Rayleigh fading is derived next.

Proposition 3. *In the case of Rayleigh fading with mean $1/\mu$, the outage probability as a function of B, M, L, ρ, N , and SINR threshold Γ is characterized as*

$$P_{\text{Fail}}(M) = \sum_{l_m \in \{l_{m-1}, l_{m-1}+1\}} (-1)^{l_M} \prod_{n \in \mathcal{M}} f\left(\mu, \frac{\lambda_M}{B}, l_n \Gamma\right), \quad (11)$$

where $l_0 = 0$ and the term $f(\mu, \alpha, \Gamma)$ is expressed as

$$f(\mu, \alpha, \Gamma) = e^{-\mu \Gamma \rho^{-1}} (\Gamma + 1) \frac{e^{\alpha/(\Gamma+1)} - 1}{e^\alpha - 1}, \quad (12)$$

and the relation between λ_{opt} and λ_M is given by (5).

Proof. See Appendix B. □

Corollary 1. *For the case of no retransmissions, the outage probability can be derived as*

$$P_{\text{Fail}}(1) = 1 - f\left(\mu, \frac{\lambda_{\text{opt}}}{B}, \Gamma\right). \quad (13)$$

In the next section, we investigate throughput scaling laws for different SNR regimes exploiting the outage probability given in (8). We only focus on the constant SNR case, for which the analytical derivations are more tractable than the Rayleigh fading case. However, we do not present the technical details of the Rayleigh fading results since they complicate the analysis without providing any additional insights. We discussed in detail how to derive scaling results for the Rayleigh fading case exploiting the constant SNR results in [1]. In Sect. VI, we illustrate scaling results both for the constant SNR and Rayleigh fading cases and provide numerical comparisons.

IV. OPTIMAL DESIGNS FOR HIGH AND LOW SNR

The goal of this section is to determine what the optimal choices of the number of bins B and the number of retransmissions M , and the corresponding maximum average throughput λ_{opt} . For the proposed random access setting, we solve the throughput optimization problem in (6) to maximize λ with respect to the deadline constraint T for high and low SNR regimes, since this is not tractable for

general SNR settings. The total number of resources $N = TW$ is evenly split into M retransmissions and B bins.

Let B_{opt} and M_{opt} denote the optimal number of bins and the optimal number of retransmissions, respectively, and λ_{opt} be the maximum allowed arrival rate for a fixed deadline constraint T . We investigate how B_{opt} , M_{opt} and λ_{opt} scale with the number of resource symbols N at high and low SNR. Resource allocation is significantly different for these regimes. At one extreme, the combined SINR at the BS can be made high by splitting the resources into many resource bins and at the same time satisfying the outage and capacity constraints in (6). At another extreme, however, the combined SINR will be very low no matter how the resources are split, and the resources have to be shared in order not to violate the constraints of (6).

A. High SNR Regime

At high SNR, resources can be utilized more efficiently because we have more freedom in selecting B and M in solving the optimization formulation in (6). From (8), $P_{\text{Fail}}(M)$ can be made arbitrarily small by increasing B . We can also observe from (9) that given $1 \leq k_m \leq k_m^*(\mathcal{K}_{m-1})$ arrivals choose a given frequency bin, each having SNR ρ , the value of $k_m^*(\mathcal{K}_{m-1})$ for $m \in \mathcal{M}$ can be made arbitrarily small by decreasing the block length n , i.e. increasing B and/or increasing M . However, B or M may not be increased arbitrarily because the capacity constraint of (6) may not be satisfied for small n .

We first study the SNR threshold ρ_T for which $\rho \geq \rho_T$ such that $k_m^*(\mathcal{K}_{m-1}) = 1$ during transmission $m \in \mathcal{M}$, i.e. no other interferer is allowed in the same frequency bin. Using (9) we have

$$k_m^*(\mathcal{K}_{m-1}) = 1 = \max \left\{ \left\lfloor m + \frac{m^2}{\Gamma} - \frac{m}{\rho} - \sum_{i=1}^{m-1} k_i \right\rfloor, 1 \right\} = \max \left\{ \left\lfloor \frac{m^2}{\Gamma} - \frac{m}{\rho} + 1 \right\rfloor, 1 \right\},$$

where $\Gamma = 2^{\frac{L}{n}} - 1$. Hence, for $\rho_T = \frac{\Gamma}{m}$, the maximum number of devices per bin is $k_m^*(\mathcal{K}_{m-1}) = 1$. In this case, devices experience no interference and the outage constraint is eliminated from (6).

We next aim to provide an upper bound on the number of frequency bins. The following condition is required in the IBL regime for a typical device to successfully transmit L bits, given \mathcal{K}_m :

$$\min_{m \in \mathcal{M}} \{C(\text{SINR}(\mathcal{K}_m))\} \geq \frac{L}{n}, \quad n \leq \frac{N}{MB}. \quad (14)$$

At high SNR, we have $k_m = 1$ for $m \in \mathcal{M}$, and from (2) the Chase combiner output SINR linearly scales with M . Hence, from (14) we require $C(\text{SINR}(\mathcal{K}_m)) = \log_2(1 + \rho m) \geq \frac{L}{n}$, for all $m \in \mathcal{M}$. Equivalently, we need to satisfy $\rho \geq \Gamma = 2^{\frac{L}{n}} - 1$ where $n \leq N/(MB)$. This yields the following upper bound on B :

$$B \leq \frac{N}{ML} \log_2(1 + \rho). \quad (15)$$

The outage constraint is satisfied by choosing B sufficiently large so that there is no other interferer in the same resource bin. In order to maximize λ , the number of bins B should be maximized given the resource constraints.

Proposition 4. *Given δ, L, ρ, N , the optimal number of bins, B_{opt} , for the high SNR regime equals*

$$B_{\text{opt}} = \frac{N}{M_{\text{opt}}L} \log_2(1 + \rho), \quad (16)$$

and the maximum average user arrival rate per frame is $\lambda_{M_{\text{opt}}} = \alpha_{M_{\text{opt}}}(\delta)B_{\text{opt}}$, where $\alpha_M(\delta)$ satisfies

$$\alpha_M(\delta) = -W((\delta^{\frac{1}{M}} - 1)e^{\delta^{\frac{1}{M}-1}}) + \delta^{\frac{1}{M}} - 1, \quad (17)$$

where W is the Lambert function and M_{opt} is given as

$$M_{\text{opt}} = \arg \max_{M \geq 1} B_{\text{opt}} \alpha_M(\delta) (1 - \delta^{1/M}). \quad (18)$$

Proof. See Appendix C. □

At high SNR, from Prop. 4, we can infer that the optimal number of bins $B_{\text{opt}} \geq 1$ linearly scales with the number of resource symbols N . Therefore, resources should be split into as many frequency bins as possible such that the devices do not experience any interference. The value of the optimal number of retransmission attempts $M_{\text{opt}} \geq 1$ however, depends on the desired outage δ , should decrease as the outage tolerance decreases, and can be determined from (18) as a function of the outage constraint δ . In this regime, the maximum average arrival rate scales with the optimal number of bins B_{opt} and hence with N .

B. Low SNR Regime

At low SNR, the Chase combiner output SINR will also be low and it might not be possible to improve the output SINR via Chase combining. Due to low SINR, the capacity constraint of the optimization formulation in (6) may only be satisfied by choosing a sufficiently large block length n . In that case, $B_{\text{opt}} = M_{\text{opt}} = 1$. In addition to that, due to the interference we always have $\text{SINR} \leq \rho$. Given the total number of resources N , when SNR ρ is such that $\rho < 2^{\frac{2L}{N}} - 1$, using the rate constraint in (6) $\frac{N}{L} \log_2(1 + \text{SINR}) \geq MB$, we have $B_{\text{opt}} = M_{\text{opt}} = 1$. Therefore, the resource utilization will be mainly constrained by the capacity requirement. The maximum average rate of device arrivals can be computed using the outage probability in (8).

Given a target SINR outage rate δ per device, we evaluate the outage probability for $B_{\text{opt}} = M_{\text{opt}} = 1$ using (8) and $k_m^*(\mathcal{K}_{m-1})$ using (9). Then, we want to determine the optimal value of λ that satisfies

$$\delta = P_{\text{Fail}}(1) = \sum_{k > k^*} D\left(k, \frac{\lambda}{B_{\text{opt}}}\right) = \sum_{k > k^*} D(k, \lambda),$$

where λ is increasing in k^* . For sufficiently large values of λ , ($\lambda > 1000$), the normal distribution with mean λ and standard deviation $\sqrt{\lambda}$ is an excellent approximation to the Poisson distribution. Using the Gaussian approximation for the Poisson arrivals, we can approximate the maximum device arrival rate λ_{opt} using

$$\delta \approx Q\left((k^* - \lambda_{\text{opt}})/\sqrt{\lambda_{\text{opt}}}\right), \quad (19)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-u^2/2) du$ is the tail probability of the standard normal distribution, and

$$k^* = \left\lfloor \frac{1}{\Gamma} - \frac{1}{\rho} + 1 \right\rfloor \approx Q^{-1}(\delta) \sqrt{\lambda_{\text{opt}}} + \lambda_{\text{opt}}, \quad (20)$$

where $\Gamma = 2^{\frac{L}{N}} - 1$. Thus, at low SNR, if ρ is in a range such that

$$\left(1 + \frac{1}{\Gamma} - k^*\right)^{-1} \leq \rho < \left(\frac{1}{\Gamma} - k^*\right)^{-1},$$

it yields $B_{\text{opt}} = 1$ and λ_{opt} that satisfies (20). Using (19), the maximum average arrival rate that can be supported at low SNR can be determined using Prop. 5.

Proposition 5. *At low SNR, $B_{\text{opt}} = 1$, $M_{\text{opt}} = 1$, and the maximum arrival rate λ_{opt} can be*

<p>Low SNR/Constant SNR</p> $\lambda_{\text{opt}} \approx \left(\sqrt{k_1^* + \frac{(\mathcal{Q}^{-1}(\delta))^2}{4}} - \frac{\mathcal{Q}^{-1}(\delta)}{2} \right)^2$ $M_{\text{opt}} = 1^*, B_{\text{opt}} = 1$ $k_1^* \approx \left\lfloor 1 + \frac{1}{2^{\frac{N}{M}} - 1} - \frac{1}{\rho} \right\rfloor$	<p>Low SNR/Rayleigh fading</p> $\lambda_{\text{opt}} = \{ \lambda_{\text{opt}} \delta = 1 - f(\mu, \frac{\lambda_{\text{opt}}}{B_{\text{opt}}}, \Gamma_{\text{opt}}) \}$ $M_{\text{opt}} = 1^*, B_{\text{opt}} = 1$ $\Gamma_{\text{opt}} = 2^{\frac{M_{\text{opt}}L}{N}} - 1$
<p>High SNR/Constant SNR</p> $\lambda_{\text{opt}} = \lambda_{M_{\text{opt}}} \left(\frac{1 - \delta^{1/M_{\text{opt}}}}{1 - \delta} \right)$ $M_{\text{opt}} = \arg \max_{M \geq 1} \lambda_M (1 - \delta^{1/M})$ $B_{\text{opt}} = \frac{N}{M_{\text{opt}}L} \log_2(1 + \rho)$	<p>High SNR/Rayleigh fading</p> $\lambda_{\text{opt}} = \lambda_{M_{\text{opt}}} / \left[1 + \sum_{m=1}^{M_{\text{opt}}-1} \text{P}_{\text{Fail}}(m) \right]$ $M_{\text{opt}} = \arg \max_{M \geq 1} \frac{\lambda_M}{1 + \sum_{m=1}^{M-1} m! (\mu \Gamma_{\text{opt}} \rho^{-1})^m}$ $B_{\text{opt}} \approx \frac{N}{M_{\text{opt}}L} \log_2(1 + \Gamma_{\text{opt}})$ $\Gamma_{\text{opt}} = -M_{\text{opt}} \log(1 - \delta) \rho$

TABLE I: Scaling results for different SNR regimes for constant SNR and with Rayleigh fading [1] in the IBL regime.

approximated by

$$\lambda_{\text{opt}} \approx \left(\sqrt{k^* + (\mathcal{Q}^{-1}(\delta))^2/4} - \mathcal{Q}^{-1}(\delta)/2 \right)^2. \quad (21)$$

Proof. See Appendix D. □

At low SNR, from Prop. 5, we can infer that all devices should share the resources in order to maximize the average arrival rate that can be supported. This is because the combined SINR at the BS will also be low, and for successful decoding the block length should be made as large as possible to decrease the threshold Γ . Hence, we require $B_{\text{opt}} = M_{\text{opt}} = 1$.

We summarize the scaling of parameters λ_{opt} , B_{opt} and M_{opt} in the IBL regime in Table I.

V. FINITE BLOCK LENGTHS

In this section, we assume that the encoded block length n symbols is finite, which is the practical case for short packet sizes. We exploit the finite block length (FBL) model of Polyanskiy *et al.* [35] to characterize the performance of the FBL regime by optimizing the number of retransmission attempts and the required number of bins.

The maximum achievable rate with error probability ε , as a function of the block length n , and the Chase combiner output SINR given in (2), is characterized by [35]

$$C_{\text{FBL}}(n, \text{SINR}, \varepsilon) = C(\text{SINR}) - \sqrt{\frac{V(\text{SINR})}{n}} Q^{-1}(\varepsilon) + \frac{0.5 \log_2(n)}{n} + o\left(\frac{1}{n}\right), \quad (22)$$

where $V(\text{SINR})$ is the channel dispersion given by

$$V(\text{SINR}) = \left[1 - \frac{1}{(1 + \text{SINR})^2} \right] \log_2^2(e). \quad (23)$$

Note that the maximal rate achievable with error probability ε is closely approximated by (22) for block lengths n as short as 100 bits [35].

Based on an asymptotic expansion of the maximum achievable rate in (22), we can calculate the block error rate for the FBL regime given the payload size L bits and encoded block length n symbols, as

$$\epsilon_{\text{FBL}}(n, L, \text{SINR}) \approx Q(f(n, L, \text{SINR})), \quad (24)$$

which is shown to be valid for packet sizes as small as $L = 50$ bits [35], where

$$f(n, L, \text{SINR}) = \frac{nC(\text{SINR}) + 0.5 \log_2 n - L}{\sqrt{nV(\text{SINR})}}, \quad (25)$$

which is non-decreasing in SINR.

We define $G_{\text{FBL}} = C - C_{\text{FBL}}$, which is the gap between the maximum achievable rate for the FBL and the Shannon capacity. Note from (23) that the channel dispersion $V(\text{SINR})$ increases with SINR. From (22), we can infer that as the block length n approaches ∞ , the gap G_{FBL} decreases to 0. We next study throughput scaling as function of the delay constraint in the FBL regime.

The throughput – characterized by Poisson arrival rate – optimization problem in the FBL regime can be written as

$$\begin{aligned} \lambda_{\text{opt}} &= \max_{B, M \in \mathbb{Z}^+} \lambda \\ \text{s.t.} \quad & P_{\text{Fail, FBL}}(M) \leq \delta, \\ & C_{\text{FBL}}(n, \text{SINR}(\mathcal{K}_m), \varepsilon) \geq \frac{L}{n}, \quad m \in \mathcal{M}, \quad n \leq \frac{TW}{MB}, \end{aligned} \quad (26)$$

where $P_{\text{Fail,FBL}}(M)$ is the probability of outage up to and including the M^{th} retransmission attempt for the FBL regime, and is given by Prop. 6. Since we have a retransmission based model, the block error rate ε can be larger than the target SINR outage rate δ , and the outage requirement may be met after Chase combining for general M . When $M = 1$, the block error rate has to satisfy $\varepsilon \leq \delta$.

Proposition 6. *Given B , M , symbol resources N , the outage probability in the FBL regime is given by*

$$P_{\text{Fail,FBL}}(M) = \sum_{k_m \geq 1} \prod_{m \in \mathcal{M}} D\left(k_m, \frac{\lambda_M}{B}\right) \epsilon_{\text{FBL}}\left(n, L, \text{SINR}(\mathcal{K}_m)\right), \quad (27)$$

where

$$\lambda_M = \lambda \left[1 + \sum_{m=1}^{M-1} P_{\text{Fail,FBL}}(m) \right]. \quad (28)$$

Proof. See Appendix G. □

Note that (28) is a fixed point equation in the form of $\lambda_M = g(\lambda_M)$, where $g(\lambda_M)$ is determined by (27). The solution λ_M is unique because (27) is a monotonically decreasing function of λ_M .

Before we characterize the FBL regime at low and high SNR, we provide the following Lemma.

Lemma 1. *The function $f(n, L, X)$ is monotonically increasing, positive, and concave in X for $X > 0$.*

Proof. See Appendix E. □

Proposition 7. *Given a random variable X , the function $f(n, L, X)$ satisfies the following relation:*

$$\mathbb{E}[\mathbb{Q}(f(n, L, X))] \geq \mathbb{Q}(\mathbb{E}[f(n, L, X)]) \geq \mathbb{Q}(f(n, L, \mathbb{E}[X])). \quad (29)$$

Proof. See Appendix F. □

Prop. 7 will be used in Sect. V-B to derive bounds for the probability of outage for the FBL regime.

The goal of the rest of this section is to figure out what the optimal choices of the number of bins B and the number of retransmissions M should be, and what this gives for the maximum average arrival rate λ for the FBL regime. For the proposed random access setting, we solve the throughput optimization problem in (26) to maximize λ with respect to the deadline constraint T for high and low

SNR regimes, since this is not tractable for general SNR settings as encountered in the IBL regime. The total number of resources $N = TW$ is evenly split into M retransmissions and B bins.

Let $B_{\text{opt}}^{\text{FBL}}$ and $M_{\text{opt}}^{\text{FBL}}$ denote the optimal number of bins and the optimal number of retransmissions, respectively, and $\lambda_{\text{opt}}^{\text{FBL}}$ be the maximum allowed arrival rate for a fixed deadline constraint T . We next investigate how $B_{\text{opt}}^{\text{FBL}}$, $M_{\text{opt}}^{\text{FBL}}$ and $\lambda_{\text{opt}}^{\text{FBL}}$ scale with the number of resource symbols N at high and low SNR for the FBL regime.

A. High SNR Regime

At high SNR, similar to the IBL regime, resource utilization can be optimized by splitting the resources, hence using multiple frequency bins, i.e. $B_{\text{opt}}^{\text{FBL}} > 1$, and Chase combining to aggregate the received signals across multiple transmissions, i.e. $M_{\text{opt}}^{\text{FBL}} > 1$. We need to determine $B_{\text{opt}}^{\text{FBL}}$ and $M_{\text{opt}}^{\text{FBL}}$ in this regime by carefully incorporating the constraints of the optimization formulation for the FBL model in (26).

Proposition 8. *At high SNR, for a given device transmission, the probability of outage for the FBL regime can be approximated as*

$$P_{\text{Fail,FBL}}(M) \approx D\left(1, \frac{\lambda_M}{B}\right)^M \prod_{m \in \mathcal{M}} Q(f(n, L, \rho m)). \quad (30)$$

Proof. See Appendix I. □

Comparing the FBL regime to the IBL regime, we can observe that there will a decrease in the throughput of the multiuser channel which is mainly due to the capacity of the FBL regime (see the second constraint in (26)). For the FBL regime, the achievable rate is lower than the IBL regime when the SNR at the BS is fixed. Hence, to achieve the same rate, it is required to increase the received SNR ρ per device at the BS.

B. Low SNR Regime

At low SNR, since the Chase combiner output SINR will be low, the capacity constraint of the optimization formulation in (26) may only be satisfied by choosing a sufficiently large block length n . Hence, the devices should share the resources such that $B_{\text{opt}}^{\text{FBL}} = M_{\text{opt}}^{\text{FBL}} = 1$. In addition to that,

<p style="text-align: center;">Low SNR/Constant SNR</p> $\lambda_{\text{opt}}^{\text{FBL}} = \left\{ \lambda \middle \delta = \sum_{k \geq 1} D\left(k, \frac{\lambda}{B}\right) \epsilon_{\text{FBL}}(n, L, \text{SINR}(\mathcal{K})) \right\}$ $M_{\text{opt}}^{\text{FBL}} = 1^*, B_{\text{opt}}^{\text{FBL}} = 1$
<p style="text-align: center;">High SNR/Constant SNR</p> $\lambda_{\text{opt}}^{\text{FBL}} = \lambda_{M_{\text{opt}}^{\text{FBL}}}^{\text{FBL}} / \left[1 + \sum_{m=1}^{M_{\text{opt}}^{\text{FBL}}-1} P_{\text{Fail}}(m) \right]$ $M_{\text{opt}}^{\text{FBL}} = \arg \max_{M \geq 1} \lambda_M^{\text{FBL}} / \left[1 + \sum_{m=1}^{M-1} P_{\text{Fail}}(m) \right]$ $B_{\text{opt}}^{\text{FBL}} = \left\{ B \middle \max_{B n=\frac{N}{BM_{\text{opt}}^{\text{FBL}}}} \bigcap_{m=1}^{M_{\text{opt}}^{\text{FBL}}} \frac{L}{n} \leq C_{\text{FBL}} \left(m\rho, \frac{N}{Bm}, \delta^{1/M_{\text{opt}}^{\text{FBL}}} \right) \right\}$

TABLE II: Scaling results for constant SNR in the FBL regime.

due to the interference we have $\text{SINR} \leq \rho$. Given the total number of resources N , when SNR ρ is such that $\rho < 2^{\frac{2L}{N}} - 1$ (see Sect. IV-B), since the rate constraint in (26) is stricter than the constraint $\frac{N}{L} \log_2(1 + \text{SINR}) \geq MB$ of (6), it is guaranteed that we have $B_{\text{opt}}^{\text{FBL}} = M_{\text{opt}}^{\text{FBL}} = 1$.

Given N , Prop. 9 gives an approximation for the outage probability for the FBL regime at high SNR up to a maximum of M transmissions.

Proposition 9. *At low SNR, for a given device transmission, the probability of outage for the FBL model can be approximated as*

$$P_{\text{Fail},\text{FBL}}(1) \approx \mathbb{E}_{\mathcal{K}} \left[Q(f(n, L, \text{SINR}(\mathcal{K}))) \right], \quad (31)$$

where \mathcal{K} is a random variable that denotes the number of arrivals on the 1st attempt given that there is at least one arrival. Its distribution is given by (4).

Proof. At low SNR, for the FBL regime, the outage probability can be derived as

$$P_{\text{Fail},\text{FBL}}(1) = \sum_{k=1}^{\infty} D\left(k, \frac{\lambda}{B}\right) \epsilon_{\text{FBL}}(n, L, \text{SINR}(\mathcal{K})) = \mathbb{E}_{\mathcal{K}} \left[\epsilon_{\text{FBL}}(n, L, \text{SINR}(\mathcal{K})) \right],$$

where the final result follows from the approximation in (24). □

We next provide a lower and upper bound on the probability of outage at low SNR for FBL.

Proposition 10. *At low SNR, for a given device transmission, the probability of outage for the FBL model can be lower and upper bounded as follows*

$$\begin{aligned} P_{\text{Fail,FBL}}(1) &\geq Q(f(n, L, \mathbb{E}_{\mathcal{K}}[\text{SINR}(\mathcal{K})])), \\ P_{\text{Fail,FBL}}(1) &\leq \sum_{k=1}^{\infty} D\left(k, \frac{\lambda}{B}\right) \exp(-f(n, L, \text{SINR}(\mathcal{K}))^2). \end{aligned} \quad (32)$$

Proof. See Appendix H. □

At low SNR, similar to the IBL regime, all devices should share the resources in order to maximize the average number of arrivals that can be supported. We illustrate the scaling of parameters $\lambda_{\text{opt}}^{\text{FBL}}$, $B_{\text{opt}}^{\text{FBL}}$ and $M_{\text{opt}}^{\text{FBL}}$ for constant SNR at high and low SNR in the FBL regime in Table II. These results can be extended to characterize the scaling of throughput versus latency for Rayleigh fading case, which is left as future work.

VI. PERFORMANCE COMPARISONS

We investigate the scaling of the throughput λ_{opt} in (6), with respect to W , ρ and T for constant SNR and Rayleigh fading cases. In the plots, solid and dashed curves denote the analytical approximations for the scaling results, while simulation results are squares.

Throughput scaling with respect to bandwidth W . At high SNR, there is a linear relationship between λ_{opt} and W because from Prop. 4, λ_{opt}/B is fixed for given δ and B linearly scales with W . Given the fixed parameters L , T , and δ , the slope λ_{opt}/W for the high SNR regime as a function of ρ can be found using Table I, which yields

$$\frac{\lambda_{\text{opt}}}{W} \stackrel{(a)}{\approx} \alpha_{M_{\text{opt}}}(\delta) \frac{T}{M_{\text{opt}}L} \left(\frac{1 - \delta^{1/M_{\text{opt}}}}{1 - \delta} \right) 0.332 \cdot \rho \text{ (dB)},$$

where (a) follows from the high SNR approximation.

For the low SNR regime, we approximate λ_{opt} using (21). Since we have $B_{\text{opt}} = 1$, and using $2^x - 1 \approx x \log(2)$, we can observe that $k^* = 1 + 1/(2^{L/N} - 1) - 1/\rho \approx \lfloor 1 + TW/(L \log(2)) - 1/\rho \rfloor$, hence λ_{opt} scales sublinearly in W , and k^* changes when $W > L \log(2)/T$, implying that there is a minimum W amount that yields a change in k^* hence λ_{opt} due to the granularity of k^* , and as T

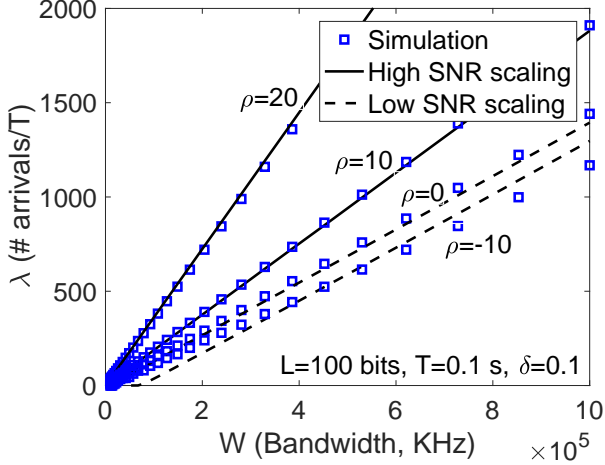


Fig. 2: λ_{opt} versus bandwidth W (kHz) for different ρ (dB) with constant SNR, in the IBL regime.

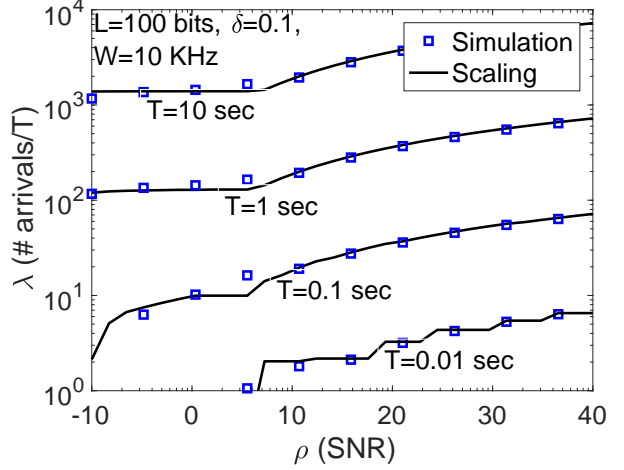
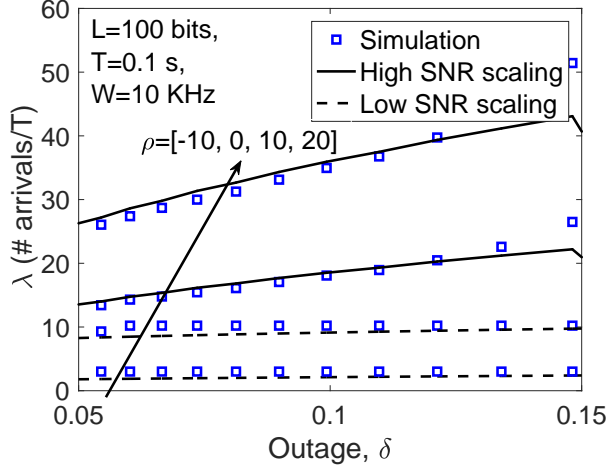
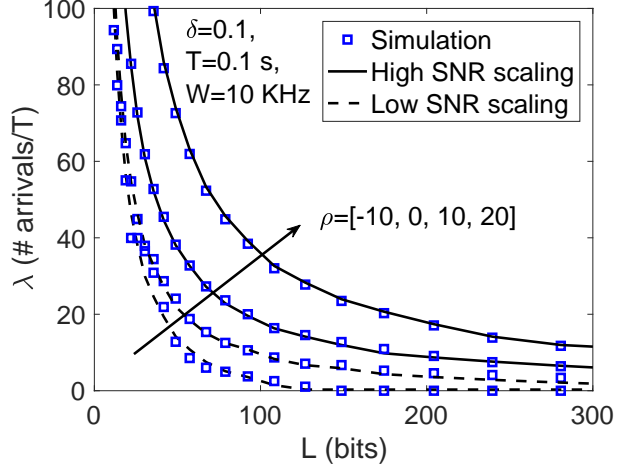


Fig. 3: λ_{opt} versus SNR ρ (dB) for different delay constraints T (s) with constant SNR, in the IBL regime.

becomes smaller or L becomes larger, a linear scaling between λ_{opt} and W is not possible to achieve. From (21) of Prop. 5, we have $\lambda_{\text{opt}} \approx (\sqrt{k^* + (Q^{-1}(\delta))^2/4} - Q^{-1}(\delta)/2)^2$. At low SNR, when δ is sufficiently large (i.e. $Q^{-1}(\delta)$ is very small), $\lambda_{\text{opt}} \approx k^*$. The trend of λ_{opt} with respect to W is illustrated in Fig. 2.

Throughput scaling with respect to SNR. For high SNR, the capacity formula can be approximated as $C(\text{SINR}) \approx 0.332 \cdot \text{SINR}(\text{dB})$, where $\text{SINR}(\text{dB}) = 10 \log_{10}(\text{SINR})$. Using this approximation, and the expression for λ_{opt} in Table I, we have $\lambda_{\text{opt}} \approx \alpha_M(\delta) 3.32 \frac{N}{ML} \log_{10}(\rho) \frac{(1-\delta^{1/M})}{(1-\delta)}$. Thus, we have $\frac{\lambda_{\text{opt}}}{10 \log_{10}(\rho)} \approx \alpha_M(\delta) \frac{WT}{ML} 0.332 \frac{1-\delta^{1/M}}{1-\delta}$. For different latency constraints ranging between $T_{\min} = 10$ msec and $T_{\max} = 10$ sec, we illustrate the trend of λ_{opt} with respect to ρ in Fig. 3.

Throughput scaling with respect to outage constraint δ . From (6), we can observe the tradeoff between the throughput and delay such that T is lower bounded by $T \geq \frac{LBM}{W \log_2(1 + \text{SINR}(\mathcal{K}_M))}$. Note that λ_{opt} increases in δ , i.e. as the outage constraint becomes loose. Large δ implies a rate constrained system and there is a lot of devices per bin. Small δ implies an SINR constrained model with fewer number of devices per bin. In Fig. 4, we illustrate the trend of λ_{opt} with respect to δ for $\rho = [-10, 0, 10, 20]$ dB. At low SNR, $B_{\text{opt}} = M_{\text{opt}} = 1$, and k^* increases with λ_{opt} . However, it is constrained by the outage requirement in (6) (see Table I), using which λ_{opt} can be obtained. For the high SNR regime, $B_{\text{opt}}, M_{\text{opt}} \geq 1$, and $k^* = 1$, λ_{opt} changes linearly in B , and it can be computed using the rate constraint in (6).

Fig. 4: λ_{opt} versus outage constraint δ for different ρ (dB).Fig. 5: λ_{opt} versus payload L (bits) for different ρ (dB).

Throughput scaling with respect to packet size L . In Fig. 5, we illustrate the trend with respect to L for $\rho = [-10, 0, 10, 20]$ dB, and for fixed $\delta = 0.1$, deadline $T = 0.1$ sec, and bandwidth $W = 10$ KHz. It is intuitive that the maximum average supportable rate λ_{opt} decays in payload size L .

At low SNR, since Γ increases in L , to satisfy a given outage constraint in (6), λ_{opt} decays in L . Similarly, at high SNR, from Table I, B_{opt} is inversely proportional to L and from

$$\lambda_{\text{opt}} \approx \alpha_{M_{\text{opt}}}(\delta) B_{\text{opt}} (1 - \delta^{1/M_{\text{opt}}}) / (1 - \delta),$$

which justifies the inversely proportional relation between λ_{opt} and L . We can also observe that B_{opt} and M_{opt} decrease in L due to the capacity constraint in (6).

Throughput scaling with respect to delay constraint T . We consider the SNR range $\rho = [-10, 20]$ dB. At high SNR, the trend of λ_{opt} versus T (or N) is similar to trend between λ_{opt} and W . For the low SNR regime, we can observe that k^* changes when T is a multiple of $L \log(2)/W$. Hence, it can be approximated as linear for sufficiently large W . The variation of λ_{opt} with T is shown in Fig. 6-(a). In the same figure, for a few points along the curves, the optimal values of B and M are also illustrated.

Although the variability of the channel causes a drop in the number of arrivals that can successfully complete the random access phase, we showed that similar conclusions in the constant SNR case extend to the case of Rayleigh fading in Fig. 6-(b).

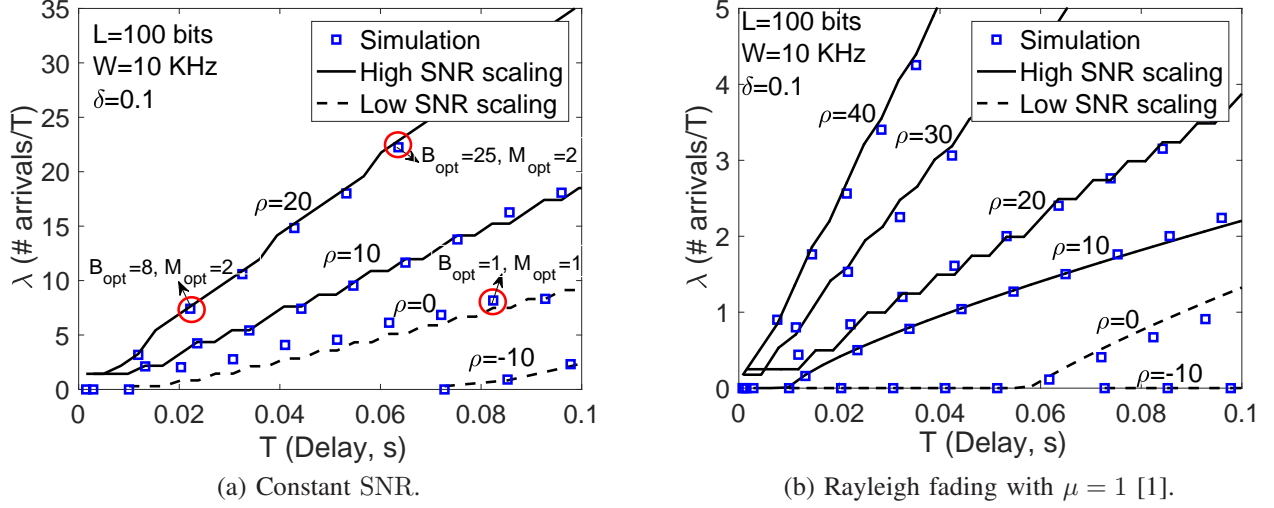


Fig. 6: λ_{opt} versus delay constraint T (s) for different ρ (dB) in the IBL regime.

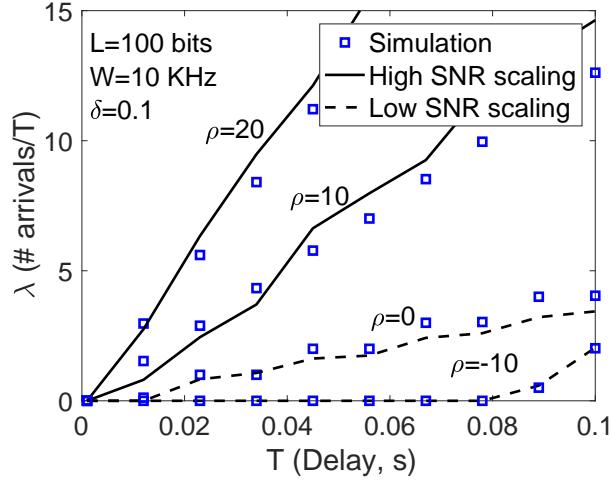


Fig. 7: λ_{opt} versus delay constraint T (s) at constant SNR for different ρ (dB) in the FBL regime.

Finally, we consider the λ_{opt} versus T trend of the FBL, which is illustrated in Fig. 7 at constant SNR, and compare it to the IBL case. For a given L , T , and W , the IBL throughput is an upper bound to the FBL throughput. This is mainly due to the stricter capacity constraint of the FBL model as given in (26). Note from (23) that $V(\text{SINR})$ increases with SINR. Therefore, from (22), we can infer that this gap G_{FBL} increases as ρ increases for a given block length n and block error rate ε .

VII. CONCLUSIONS

We proposed a random access model for a single cell uplink where the objective is to characterize the fundamental limits of supported arrival rate by optimizing the number of retransmissions and resource allocation under a maximum latency constraint. We evaluate the performance of the model with respect to total bandwidth, latency constraint, payload, outage constraint, and SNR. We obtain the following design insights for the infinite block length (IBL) and finite block length (FBL) models for different SNR, with different packet sizes and outage constraints:

- Independent of the payload size L , at low SNR, the resources should be shared, implying small B_{opt} and M_{opt} . As SNR increases, the resources should be split, yielding large B_{opt} and M_{opt} .
- As the outage constraint δ decreases, B_{opt} and M_{opt} increase.

The insights can be applied to the design of narrowband IoT systems for 5G with ultra-reliability, ultra-low latency, and a large number of connected devices. Possible extensions include the modeling and analysis of the random access in a cellular network setting, by capturing the scaling of throughput using realistic power control and fading models, and characterizing the uplink interference. They also include the analysis of heterogeneous random access strategies to accommodate a wide range of device-chosen strategies with different capabilities and requirements. The overhead caused by channel-estimation is significant and different diversity combining or multiplexing techniques can improve the system performance, especially when there is deep fading. Another direction would be to incorporate the FBL model in a cellular network setting, and characterize the finite block-error probability by capturing the effect of the uplink interference at low and high contention.

APPENDIX

A. Proof of Proposition 2

The probability of outage given up to a maximum of M transmission attempts is given as

$$P_{\text{Fail}}(M) = \sum_{k_1 > k_1^*} \sum_{k_2 > k_2^*(k_1)} \cdots \sum_{k_M^* > k_M^*(\mathcal{K}_{M-1})} \prod_{m \in \mathcal{M}} D\left(k_m, \frac{\lambda_M}{B}\right),$$

where k_m^* is a function of $\mathcal{K}_{m-1} = \{k_1, \dots, k_{m-1}\}$ for $2 \leq m \leq M$ to be calculated based on the rate requirement. Note that $P_{\text{Fail}}(m)$ monotonically decreases in m .

The aggregate arrival rate λ_M with up to M total transmissions is given by

$$\lambda_M = \lambda_{\text{opt}} \left[1 + \sum_{m=1}^{M-1} P_{\text{Fail}}(m) \right].$$

For successful transmission, we require that $\frac{L}{n} \leq C(\text{SINR}(\mathcal{K}_m))$ for $1 \leq k_m \leq k_m^*(\mathcal{K}_{m-1})$ for $m \in \mathcal{M}$.

Using (2), we derive

$$\sum_{i=1}^m k_i \leq \left\lfloor m + \frac{m^2}{\Gamma} - \frac{m}{\rho} \right\rfloor, m \in \mathcal{M},$$

where $\Gamma = 2^{\frac{L}{n}} - 1$, yielding the lower limit of the summation in (8), as given by (9).

B. Proof of Proposition 3

The outage probability given up to a maximum of M transmissions with Rayleigh fading with mean $1/\mu$ equals

$$\begin{aligned} P_{\text{Fail}}(M) &= \mathbb{E} \left[\mathbb{P} \left(\max_{m \in \mathcal{M}} \{C(\text{SINR}(\mathcal{K}_m))\} < \frac{L}{n} \middle| \mathcal{K}_M \right) \right] \\ &\stackrel{(a)}{=} \mathbb{E}_{\mathcal{K}_M} \left[\mathbb{E}_{I_{\mathcal{K}_M}} \left[\mathbb{P} \left(h < \frac{\Gamma}{\rho m^2} \left(m + \sum_{i=1}^m I_{k_i} \right) \middle| I_{\mathcal{K}_M}, \mathcal{K}_M \right) \right] \right] \\ &\stackrel{(b)}{=} \mathbb{E}_{\mathcal{K}_M} \left[\mathbb{E}_{I_{\mathcal{K}_M}} \left[\prod_{m=1}^M 1 - e^{-\frac{\mu \Gamma}{\rho m^2} \left(m + \sum_{i=1}^m I_{k_i} \right)} \middle| I_{\mathcal{K}_M}, \mathcal{K}_M \right] \right] \\ &= 1 - e^{-\mu \Gamma \rho^{-1}} \mathbb{E}_{\mathcal{K}_1} \left[\mathcal{L}_{I_{k_1}}(\mu \Gamma \rho^{-1}) \middle| \mathcal{K}_1 \right] - e^{-2\mu \Gamma \rho^{-1}} \mathbb{E}_{\mathcal{K}_2} \left[\mathcal{L}_{I_{k_1}}(\mu \Gamma \rho^{-1}) \mathcal{L}_{I_{k_2}}(\mu \Gamma \rho^{-1}) \middle| \mathcal{K}_2 \right] \\ &\quad + e^{-3\mu \Gamma \rho^{-1}} \mathbb{E}_{\mathcal{K}_2} \left[\mathcal{L}_{I_{k_1}}(2\mu \Gamma \rho^{-1}) \mathcal{L}_{I_{k_2}}(\mu \Gamma \rho^{-1}) \middle| \mathcal{K}_2 \right] + \dots \\ &\stackrel{(c)}{=} \sum_{l_1=0}^1 \sum_{l_2=l_1}^{l_1+1} \dots \sum_{l_M=l_{M-1}}^{l_{M-1}+1} (-1)^{l_M} \prod_{n \in \mathcal{M}} f\left(\mu, \frac{\lambda_M}{B}, l_n \Gamma\right), \end{aligned} \tag{33}$$

where we used the shorthand notation for interference as $I_{\mathcal{K}_m} := \{I_{k_1}, \dots, I_{k_m}\}$, and (a) follows from the definition of SINR in (10) and (b) from that $h \sim \exp(\mu)$, and (c) from the definition of $D(k, \frac{\lambda_M}{B})$, independence of I_{k_i} 's, and $\mathcal{L}_{I_{k_i}}(s) = \mathcal{L}_{I_{k_1}}(s)$ for $i \in \mathcal{M}$. Using the Laplace transform of the interference given k arrivals, which is denoted by $\mathcal{L}_{I_k}(s)$ and expressed as

$$\mathcal{L}_{I_k}(s) = \mathbb{E}[e^{-sI_k}] = \mathbb{E} \left[e^{-s\rho \sum_{i=1}^{k-1} g_{i_m}} \right] = \mathbb{E} \left[\prod_{i=1}^{k-1} e^{-s\rho g_i} \right] = \left(\frac{\mu}{\mu + s\rho} \right)^{k-1}.$$

Averaging $\mathcal{L}_{I_k}(s)$ using the distribution $D(k, \lambda_B)$, we obtain

$$\mathcal{L}_{\text{ICI}}(s) = \mathbb{E}_k[\mathcal{L}_{I_k}(s)] = \sum_{k=1}^{\infty} D(k, \lambda_B) \left(\frac{\mu}{\mu + s\rho} \right)^{k-1} = (s\mu^{-1}\rho + 1) \frac{e^{\lambda_B/(s\mu^{-1}\rho+1)} - 1}{e^{\lambda_B} - 1}. \quad (34)$$

To simplify the notation, we let $s = \mu\Gamma\rho^{-1}$ and define $f(\mu, \alpha, \Gamma) = e^{-\mu\Gamma\rho^{-1}}(\Gamma + 1) \frac{e^{\alpha/(\Gamma+1)} - 1}{e^{\alpha} - 1}$, using which the final expression (c) is evaluated.

C. Proof of Proposition 4

At high SNR regime, since $k^* = 1$, the SINR becomes $\text{SINR} = \rho$. Since the total number of resources N is split into M retransmissions and B bins to successfully transmit L bits, in the IBL regime, we have $B \approx \frac{N}{ML} \log_2(1 + \rho)$.

For the high SNR regime, the relation between the target outage δ and $\alpha_M(\delta) = \lambda_M/B$ is given as

$$\text{P}_{\text{Fail}}(M) = \delta = \left(1 - \frac{\alpha_M(\delta)}{e^{\alpha_M(\delta)} - 1} \right)^M = \text{P}_{\text{Fail}}(m)^{\frac{M}{m}}, \quad m \in \mathcal{M},$$

where $\alpha_M(\delta) = \alpha_1(\delta^{1/M}) = \alpha(\delta^{1/M})$. The relation between λ and the aggregate arrival rate is

$$\lambda = \lambda_M / \left[1 + \sum_{m=1}^{M-1} \delta^{\frac{m}{M}} \right] = \lambda_M (1 - \delta^{1/M}) / (1 - \delta).$$

The optimal number of transmissions can be found solving

$$M_{\text{opt}} = \arg \max_{M \geq 1} \lambda_M (1 - \delta^{1/M}),$$

which depends on δ at high SNR, and $B \approx \frac{N}{ML} \log_2(1 + \rho)$.

D. Proof of Proposition 5

We have $B \leq \frac{N}{L} \log_2(1 + \text{SINR}_k)$, and $k^* \leq 1 + \frac{1}{\frac{L}{N} - 1} - \frac{1}{\rho}$, and $\delta = \sum_{k > k^*}^{\infty} D(k, \frac{\lambda}{B})$. At low SNR, as $k_{\text{max}} \rightarrow \infty$, $\frac{\lambda}{B} \rightarrow \infty$. However, we also require $B = 1$ because we cannot achieve arbitrarily high $\frac{\lambda}{B}$ picking B arbitrarily small since $B \in \mathbb{Z}^+$.

The proof follows from approximating $D(k, \frac{\lambda}{B})$ by a Gaussian distribution that has the same mean and variance as the Poisson distribution, i.e. $\mathcal{N}(\frac{\lambda}{B}, \frac{\lambda}{B})$ for high $\frac{\lambda}{B}$. Hence, $\delta \approx \text{Q} \left(\frac{k^* - \lambda/B}{\sqrt{\lambda/B}} \right)$. Therefore, $\sqrt{\frac{\lambda}{B}} \approx \sqrt{k^* + \frac{(\text{Q}^{-1}(\delta))^2}{4} - \frac{\text{Q}^{-1}(\delta)}{2}}$. Thus, we can infer that $\frac{\lambda}{B}$ scales linearly with k^* . However, since $B \leq$

$\frac{N}{L} \log_2 \left(1 + \frac{\rho}{1+\rho(k^*-1)} \right)$, the number of bins, B , decays sub-linearly with k^* . Therefore, their product λ increases as k^* increases. The above approximation with $B = 1$ gives the maximum supportable rate λ_{opt} for the low SNR regime.

E. Proof of Lemma 1

We rewrite $f(n, L, X)$ as $f(n, L, X) = \frac{C(X)+b}{\sqrt{V(X)/n}}$, and let $a = \frac{\sqrt{n}}{\log_2 e}$, $b = \frac{0.5 \log_2 n - L}{n}$, $f_1(x) = \frac{(1+x)}{\sqrt{x(x+2)}}$ and $f_2(x) = \log_2(1+x)$. Then, we can express $f(n, L, x)$ as $f(n, L, x) = a f_1(x)[f_2(x) + b]$. Then, we can derive the identities $\frac{\partial}{\partial x} f_1(x) = -\frac{1}{(x(x+2))^{3/2}}$, $\frac{\partial^2}{\partial x^2} f_1(x) = \frac{3(1+x)}{(x(x+2))^{5/2}}$, $\frac{\partial}{\partial x} f_2(x) = \frac{1}{\log(2)} \frac{1}{1+x}$ and $\frac{\partial^2}{\partial x^2} f_2(x) = -\frac{1}{\log(2)} \frac{1}{(1+x)^2}$. We also assume that $b \leq 0$. From $b = \frac{0.5 \log_2 n - L}{n} \leq 0$ that is true when $n \leq 2^{2L}$, which is a realistic assumption for the FBL regime.

Monotonically increasing. To prove that $f(n, L, x)$ is increasing in x , we take its partial derivative:

$$\begin{aligned} \frac{\partial f(n, L, x)}{\partial x} &= a \left[\frac{\partial}{\partial x} f_1(x)[f_2(x) + b] + f_1(x) \frac{\partial}{\partial x} f_2(x) \right] \\ &= \sqrt{n} \frac{1}{(x^2 + 2x)^{1/2}} \left[1 - \frac{\log_2(1+x) + b}{\log_2(e)(x^2 + 2x)} \right] \stackrel{(a)}{>} 0, \quad x \geq 0. \end{aligned} \quad (35)$$

where (a) follows from that $x \geq 0$ and $1 - \frac{\log(1+x)}{x^2+2x} > 0$ and the assumption that $b \leq 0$.

Note that $f_1(x)$ and $f_2(x)$ are increasing in $x \geq 0$. Hence, $f(n, L, x)$ is also increasing in x . Let $x \leq y$. Then, $f(n, L, x) = a f_1(x)[f_2(x) + b] \leq a f_1(y)[f_2(y) + b] = f(n, L, y)$. Hence, $f(n, L, x)$ is a monotonically increasing function of x .

Positive. From definition of C_{FBL} , it is required to satisfy

$$C_{\text{FBL}}(n, \text{SINR}, \varepsilon) = C(\text{SINR}) - \sqrt{\frac{V(\text{SINR})}{n}} Q^{-1}(\varepsilon) + \frac{0.5 \log_2(n)}{n} + o\left(\frac{1}{n}\right) \geq \frac{L}{n}, \quad (36)$$

which yields the condition that $f(n, L, \text{SINR}) = \frac{C(X)+b}{\sqrt{V(X)/n}} \geq Q^{-1}(\varepsilon)$. Note that since $Q^{-1}(\varepsilon) > 0$ as long as $\varepsilon < 0.5$. This condition guaranties the positivity of $f(n, L, x)$.

Concave. To prove that $f(n, L, x)$ is concave, the second partial derivative of $f(n, L, x)$ with respect to x is derived as

$$\begin{aligned} \frac{\partial^2}{\partial x^2} f(n, L, x) &= a \left[\frac{\partial^2}{\partial x^2} f_1(x)[f_2(x) + b] + 2 \frac{\partial}{\partial x} f_1(x) \frac{\partial}{\partial x} f_2(x) + f_1(x) \frac{\partial^2}{\partial x^2} f_2(x) \right] \\ &\stackrel{(a)}{=} \sqrt{n} \frac{x+1}{(x^2 + 2x)^{3/2}} \left[\frac{3[\log_2(1+x) + b]}{\log_2(e)(x^2 + 2x)} - 1 - \frac{1}{(1+x)^2} \right] < 0, \quad x \geq 0, \end{aligned}$$

where (a) follows from the fact that $\frac{3 \log_2(1+x)}{\log_2(e)(x^2+2x)} - 1 - \frac{1}{(1+x)^2} < 0$ for $x \geq 0$ and from $b \leq 0$.

F. Proof of Proposition 7

Since the second derivative of $Q(x)$ is always greater than or equal to zero, $Q(x)$ is convex and increasing on $x \geq 0$. This implies that for a random variable X , we have that

$$\mathbb{E}[Q(X)] \geq Q(\mathbb{E}[X]).$$

Note also that from Lemma 1, $f(n, L, X)$ is monotonically increasing, positive, and concave for $X \geq 0$ and $Q(f(n, L, X))$ is decreasing in X . Then, $\mathbb{E}[f(n, L, X)] \leq f(n, L, \mathbb{E}[X])$. Note also that $Q(X)$ is decreasing, monotone, and convex for $X > 0$. Thus, $\mathbb{E}[Q(X)] \geq Q(\mathbb{E}[X])$. Then,

$$\mathbb{E}[Q(f(n, L, X))] \stackrel{(a)}{\geq} Q(\mathbb{E}[f(n, L, X)]) \stackrel{(b)}{\geq} Q(f(n, L, \mathbb{E}[X])), \quad (37)$$

where (a) follows from the convexity of Q and (b) follows from that $f(n, L, X)$ is concave and Q is decreasing in X .

G. Proof of Proposition 6

The outage probability for the FBL model at constant SNR with M retransmissions equals

$$\begin{aligned} P_{\text{Fail}}(M) &= \mathbb{P} \left(\max_{m \in \mathcal{M}} \{C_{\text{FBL}}(n, \text{SINR}_m, \varepsilon)\} < \frac{L}{n} \middle| \mathcal{K}_M \right) \\ &= \mathbb{E} \left[\mathbb{P} \left(\max_{m \in \mathcal{M}} \{C_{\text{FBL}}(n, \text{SINR}(\mathcal{K}_m), \varepsilon)\} < \frac{L}{n} \middle| \mathcal{K}_M \right) \right] \\ &\stackrel{(a)}{=} \mathbb{E} \left[\mathbb{P} \left(\frac{n C_{\text{FBL}}(n, \text{SINR}(\mathcal{K}_m), \varepsilon) - L}{\sqrt{n V(\text{SINR}(\mathcal{K}_m))}} < 0, m \in \mathcal{M} \middle| \mathcal{K}_M \right) \right] \\ &\stackrel{(b)}{=} \mathbb{E} \left[\mathbb{P} \left(\mathcal{Z} < -f(n, L, \text{SINR}(\mathcal{K}_m)), m \in \mathcal{M} \middle| \mathcal{K}_M \right) \right] \\ &\stackrel{(c)}{\approx} \sum_{k_1 \geq 1} \cdots \sum_{k_m \geq 1} \prod_{m \in \mathcal{M}} D \left(k_m, \frac{\lambda_M}{B} \right) f(n, L, \text{SINR}(\mathcal{K}_m)) \\ &\stackrel{(d)}{\approx} \sum_{k_1 \geq 1} \cdots \sum_{k_m \geq 1} \prod_{m \in \mathcal{M}} D \left(k_m, \frac{\lambda_M}{B} \right) \epsilon_{\text{FBL}}(n, L, \text{SINR}(\mathcal{K}_m)), \end{aligned}$$

where (a) follows from the definition of C_{FBL} using (22) and (23), (b) from the normal distribution approximation such that \mathcal{Z} is standard normal random variable $\mathcal{N}(0, 1)$, and (c) from the approximation based on standard Gaussian cdf $Q(x)$ and (d) from $Q(x) = 1 - Q(-x)$ and (24).

H. Proof of Proposition 10

At low SNR, for the FBL regime, the outage probability can be bounded as

$$P_{\text{Fail,FBL}}(1) \stackrel{(a)}{\geq} Q(f(n, L, \mathbb{E}_{\mathcal{K}}[\text{SINR}(\mathcal{K})])), \quad (38)$$

where (a) follows from (29) and (31), and the mean SINR is given by

$$\begin{aligned} \mathbb{E}_{\mathcal{K}}[\text{SINR}(\mathcal{K})] &= \sum_{k=1}^{\infty} D\left(k, \frac{\lambda}{B}\right) \frac{\rho}{1 + \rho(k-1)} \\ &= \frac{1}{\rho^{-1} - 1} \left[1 + \left(-\frac{\lambda}{B} \right)^{1-\rho^{-1}} \frac{1}{e^{\lambda/B} - 1} \left(\Gamma_0(\rho^{-1}) - \Gamma_0\left(\rho^{-1}, -\frac{\lambda}{B}\right) \right) \right]. \end{aligned}$$

Using the Chernoff bound of the Q-function, which is $Q(x) \leq e^{-x^2/2}$ for $x > 0$, we can obtain an upper bound for the probability of outage as follows:

$$\begin{aligned} P_{\text{Fail,FBL}}(1) &\leq \mathbb{E}_{\mathcal{K}} \left[\exp \left(-\frac{1}{2} f(n, L, \text{SINR}(\mathcal{K}))^2 \right) \right] \\ &= \sum_{k=1}^{\infty} D\left(k, \frac{\lambda}{B}\right) \exp(-f(n, L, \text{SINR}(\mathcal{K}))^2). \end{aligned}$$

I. Proof of Proposition 8

At high SNR, the resource symbols have to be shared such that $k_m = 1$, for all $m \in \mathcal{M}$, and hence, from (27) we have

$$P_{\text{Fail,FBL}}(M) = \prod_{m \in \mathcal{M}} D\left(1, \frac{\lambda_M}{B}\right) \epsilon_{\text{FBL}}(n, L, \rho m) \approx D\left(1, \frac{\lambda_M}{B}\right)^M \prod_{m \in \mathcal{M}} Q(f(n, L, \rho m)). \quad (39)$$

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