

A stochastic model for Meningococcal Disease in the African Meningitis belt

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1 Introduction

The mathematical description of populations and disease dynamics is a field which can be extremely complex. It is a subject of much research, relevant to policy makers and medical researchers. An accurate model that can predict future dynamics can be of immense value, both to prepare for and deal with epidemics, and to evaluate different vaccination policies. However, accuracy in itself is not enough. The model must also be precise enough, as well as capable of evaluating several possible scenarios within a reasonable time frame. This leads to a need for a balance between accuracy, which means several different interactions must be considered, and swiftness, which calls for approximations and numerical efficiency.

A common practise for population dynamics is to approximate a discrete and random system, according to the central limit theorem and the law of large numbers, by a continuous and deterministic system. It is obvious that in a given population, the number of individuals is a discrete number. Furthermore, it is intuitively clear that events such as contagion and infection are fundamentally random. However, models that incorporate this randomness into their structure are in general significantly slower to use, as a single evaluation will take longer and more evaluations are needed to increase precision and accuracy (in correspondence to the Monte Carlo framework). Consequently, an approximation is made to a deterministic system governed by differential equations, and letting the populations vary continuously. By the law of large numbers and the central limit theorem, this is no issue if the population is large enough and a continuous population will, while not entirely accurate, be at least as specific as a realisation of a discrete random system. With this framework standard numerical procedures can be used, and as the model is deterministic only one evaluation is needed. However, the law of large numbers only holds as long as there is indeed a large number of individuals; whenever a population (or subpopulation) faces extinction the rule no longer holds. These and other factors must all be considered when determining what framework and what model is the most appropriate for the system at hand, with respect to accuracy, speed and likely system states.

Most classical approaches to population dynamics describes the system in terms of the relative number of individuals in a given sub-population, but there are some limitations to this method and alternatives should be considered. SIR-models and other compartmental models are examples of the aforementioned, where the system state is recorded based on how large the different population compartments are. However, in some cases this will not provide sufficient information. Consider for example the case where a model is used to estimate what capacity a maternity ward should have some years into the future. It is then not sufficient to consider only the population growth over time, but the raw number of births are the relevant factor. A compartmental model is not suited for this specific problem, and a model more focused on the specific dynamics should be developed.

The complexity in modelling population dynamics arises from several factors. Firstly, the modelling of a population in itself, disregarding external factors, can be made increasingly complex by considering each individual and the exact changes they are subject to. Secondly, external factors should be considered so as to make the modelling relevant for policy makers. What external factors to include and how precisely they should be modelled, as well as their precise effects on the population, introduces a whole new layer of complexity. For example, one external factor that may be interesting to consider is weather, and currently there is no reasonable way to reliably model the weather. It can even be argued that to accurately model a population, the whole system that is the earth itself must be considered. Consequently, some approximations and assumptions will be made, even when choosing the stochastic approach to modelling. As the stochastic approach is in general more complex, it can be argued that more of these assumptions and approximations will be made than in a corresponding deterministic model to compensate for the increased complexity otherwise. This should also be considered when evaluating different options for modelling.

The goals of modelling diseases can be many. There is obviously an interest in understanding and explaining the dynamics of the disease itself, to increase knowledge about the disease and potentially understanding the underlying biological mechanisms. It can also be used as a validation tool to confirm or dismiss theories about a disease, if the model and simulation method is proven sound beforehand. One of the more relevant and obvious goals of these models is to help policy makers determine what the consequences of different actions can be in different circumstances. This will help showing whether a vaccination policy is efficient, or what the best cause of action during an epidemic may be. Furthermore, as there are frequently issues in gathering real-world data, a reliable model can provide more and higher-quality data that can later be used to identify dynamics that could, for example, be used as a predictor for future epidemics.

For the reasons mentioned above, it is important that a given model is flexible. The framework must not only support different scenarios in the disease dynamics itself, but it should also support variations in external policies, corresponding to several different policies but also to variations in the resources controlled by the policy makers. This is especially important when modelling populations in areas of the world with poor infrastructure, as this can affect the data available as well as the possible responses from medical personnel and governments.

A flexible and reliable model should also be able to adapt to changes within the community themselves. For many diseases, including for example measles, there is a clear pattern of disease corresponding to population movements and population mixing[Spatial patterns of...]. These mechanisms can lead to an increased rate of

transmission, as populations grow, or the introduction of new disease variations to groups with lacking immunity or poor preparations in other ways.

Where the infrastructure is poor, a plethora of new issues arise. There are often issues with distributing drugs and vaccines, meaning that any policy will be less efficient. This also increases the need for an early warning, as the time it takes to react to a given scenario increases. Furthermore, there can be issues considering the gathering of information itself. If sick individuals can not reach a medical institution in time, or if the information from the medical institutions take too long to spread, the true situation will be unknown. These are factors that should be considered when using models as an aid for political decisions, but it should also be considered when constructing the model itself.

In this project, a stochastic and discrete model for the dynamics of Meningococcal disease in the African Meningitis belt is constructed and evaluated. The model is then used to examine dynamics of the disease, as well as to examine some possible explanations for the dynamics in question.

Meningococcal disease are any diseases caused by *Neisseria Meningitidis*. The infection of *N. Meningitidis* can take many forms, of which two common (and severe) are meningococcal septicaemia and meningitis [source]. Meningitis is an infection of the thin lining surrounding the brain and the spinal cord, and is a serious condition that untreated leads to death in about 50% of cases. Early diagnosis and adequate treatment reduces the fatality rate to about 5-10%, with permanent disabilities in about 10-20% of cases. The incubation time varies from 2-10 days, but is on average 4 days [source: WHO]. Meningococcal septicaemia is the infection of the bloodstream, and is often even more severe than meningitis. The infection damages the walls of blood vessels and leads to bleeding into the skin and other organs. The treatment is similar, but possible consequences include amputation[source]. In rare cases, *N. Meningitidis* can cause arthritis and similar diseases[source].

Humans are the only known reservoir for *N. Meningitidis*. At a given time, about 2-50% of the population are likely to be carriers of the bacterium, and it is spread by salivary droplets. Carriage can last both for a very short period of time as well as for several months, during which the bacterium is present in the nasopharynx of the carrier [4]. In general, carriage does not lead to invasive disease, but it has been linked to a subsequent immune response to the bacterium. There are several mechanisms that can participate in the immune response, and research has shown that a significant part of the adult population have an immune response that is putatively protective towards the disease [source: human immunity to the meningococcus]. Another bacterium frequently theorized as potentially leading to immunity to the disease is *Neisseria Lactamica*, often present in the nasopharynx of young children [source]. While immunity protects towards invasive disease, it has not been shown that there is any reduction of carriage caused by the immune response. [sources]

N. Meningitidis is genetically variable. It is most commonly classified based on the composition of its polysaccharide capsule into 13 different serogroups. While it is rather common, world-wide, for carriage strains to not have a capsule (around 50% of carriage cases), and thus be non-serogroupable, disease is primarily caused by encapsulated strains. Serogroups A, B, C, W135 and X account for more than 90% of cases of invasive disease world wide. The production of a capsule can be switched on and off with high frequency, and also non-serogroupable carriage strains can be pathogenic. [NM an overview..].

The African meningitis belt is a region in Sub-Saharan Africa with an increased rate of meningococcal disease as well as recurring epidemics. The belt was first described by Lapeyssonnie in 1968 [1]. This region is shown in ?? and has been characterized by semi-regular epidemics documented since the early 20th century. Its specific geographical area is defined by the WHO as the region in Africa between Senegal in the west and Ethiopia in the east [source], which coincides fairly well with the area described by Lapeyssonnie.

Meningitis dynamics in the region have, among others, these characteristics. Firstly, there are recurring epidemics with intervals of about 5-15 years. Secondly, the incidence rates of disease is higher than in other regions of the world. Thirdly, the disease is most commonly caused by a serogroup which does not usually cause epidemics in the rest of the world. All of these properties are also affected by a strong seasonality, so that there are three distinguishable states for the disease in a population. There is the hypoendemic state, with a low incidence rate of about λ which occurs during the wet season. During the dry season, the system falls into a hyperendemic state with increased incidence rate, or occasionally an epidemic state. An epidemic state is defined as a rate of disease above λ . [3]

The reason for the specific dynamics in this region is often considered to be related to climatic factors. One such factor frequently considered is the total precipitation. The region coincides fairly well with the area between the 300mm and 1100mm isohyets [2]. Another common climate factor for the area, is that it is affected by the Harmattan, a dry wind from the Sahara desert. As there are links between factors that irritate the airways and subsequent infection by *N. Meningitidis* [sources on smoking, colds], a link between the Harmattan and the hyperendemic meningitis season would seem plausible. Is is an area of active research [?], and finding any climactic factors that could be driving the epidemics is outlined as an area of priority by the WHO [source: research priorities].

The distribution of genetic variations of *N. Meningitidis* is different in the meningitis belt compared to the rest of the world. There is a higher ratio of encapsulated virulent strains carried in the population, most notably of serogroup A. This serogroup has also been the cause of most epidemics in the region. Furthermore,

there is significantly less genetic variation in the population than in other regions. The temporal distribution of the bacterium is also unstable, leading to a clonal wave pattern, where one strain is dominant at a given time. [clonal waves]. It has been theorized that this lack of genetic variation can contribute to the epidemic patterns, as an epidemic strain can quickly become dominant and out-compete less virulent strains. The reasons for the lack of stability and lack of variation is unclear.

There is a clear relationship between age and the distribution of carriage and disease caused by *N. Meningitidis*. Globally, the carriage rates are around 10% in an endemic state, but the rate is significantly lower ($< 3\%$) for young children, and significantly higher for teenagers and young adults, especially for military recruits and university students ($> 24\%$). Other factors that correlate with increased carriage are smoking and crowding, for example at bars or during the Hajj [Epidemic meningitis...]. The significant increase in carriage in closed communities such as the military, the Hajj or to some extent new university students suggest that population mixing may be related to the transmission of disease.

The group at most risk for disease are young children, in connection with waning maternal immunity. There is also an increased risk in teenagers and young adults, similar to the increase in carriage rates for these groups.

Vaccines against the bacterium have historically been focused primarily on antigens found in the capsule, called polysaccharide vaccines. These vaccines have been used to prevent invasive disease in individuals older than infants, but does not induce an immunological memory. As such, preventative vaccine policies can not efficiently be carried out using these types of vaccines. Recently, polysaccharide conjugate vaccines have been introduced, which are capable of inducing immunological memory as well as immunizing infants. These conjugate vaccines currently exist for serogroups A and C. There have been indications that carriage of the targeted strain decreases following a conjugate vaccine intervention. However, it is unclear whether this is caused by a decrease in carriage of the strain itself or only prevents the production of the capsule. [NM an overview]

Since 2010, a WHO project has been undertaken to combat epidemic meningitis in the region. A polysaccharide-conjugate vaccine effective against serogroup A has been introduced, and since its introduction, no new epidemics has been recorded from that specific strain. There are also indications that the vaccine affects carriage rates of the specific strain, which could lead to herd immunity. These types of effects has been seen in other vaccination projects against *N. Meningitidis* [sources], but the effects on the specific dynamics in the Meningitis belt is yet unclear. One concern is whether the eradication of serogroup A could lead to an increase in other serogroups, specifically those with a higher virality.

A [recent] conference on the Meningitis belt outlined a few priorities for future research, where it was determined that attempts to construct mathematical models should include factors that may affect the spatial, seasonal and year-to-year variability in risk for meningococcal disease. The model constructed in this project focuses on the seasonal variability, and examines two different explanatory models for the variations. The year-to-year variability will also be examined to a certain extent, as variations in the population distribution over time could lead to long-term variability in risks for disease and epidemics.

This model is also the first, to my knowledge, stochastic model for Meningococcal meningitis in the African Meningitis belt. As much of the region has a low urbanization rate, a model suitable for smaller populations is appropriate. Furthermore, as the incidence rates during the endemic season can be very low there are clearly situations where the law of large numbers may not hold. This model could be used to verify previous modelling attempts, as well as evaluate current theories on the underlying causes of epidemiological dynamics.

1.1 Main goals of this project

The primary goal of this project has been to create and implement stochastic model for the population and disease dynamics in the African Meningitis belt. The model should be evaluated, and its calculations should be compared to real-world data and other models, to determine whether there is an improvement in the accuracy and specificity of the model.

If the model and its implementation seems sound, some further questions are to be considered. The primary consideration of this thesis is to attempt to determine whether the difference between the endemic and the hyperendemic states of the disease can be explained by behavioural changes or whether the explanation must be found somewhere else, for example in biochemical factors of the bacterium itself.

To further determine the seasonality of the disease, also the epidemic state should be considered. While epidemic outbreaks are connected to the introduction of a more invasive strain, the epidemics themselves also display a certain seasonality. Using the seasonal variations found to produce the most accurate results in the previous section, the results of introducing a more invasive strain during different seasons will be examined. If the introduction leads to an epidemic also during the wet season, or fails to lead to an epidemic in the dry season, the suggested explanation does not hold.

1.2 Terminology

2 Methods

The development of a stochastic model for population dynamics aims at determining the probability for the system to be in a given state at a given time. Typically, there will not be a closed-form analytical solution for this problem, and the Monte Carlo framework is employed to produce estimates. This is also the basic method used in this project. This means that a model is developed for the general, stochastic behaviour of the system, and a multitude of realisations are simulated. The aggregate of these realisations should then be a viable approximation of the probabilistic behaviour of the system.

For this problem, it was determined that a model should not focus only on the population distribution over time but also the number of times any individual was infected. There are several reasons for this; One is that the rate of permanent sequelae is significant, meaning that while an individual may recover from the disease itself the history of their disease is still relevant. Another reason is that the information used to determine when an epidemic is, is the number of cases that has occurred and not the specific number of sick individuals at a given time. These factors leads us to construct a model that focuses on the dynamics of the system.

In this section, we first explain the model constructed for this problem and the theory behind the algorithm used to construct realisations. Then follows a section on the implementation, both from a mathematical standpoint and a computational, and finally a section discussing the real-world data and parameters used in the project.

2.1 Model

According to the comments made above, a stochastic model focusing on the relevant dynamics was created. We begin by defining what dynamics should be considered, and constructing classes of events corresponding to each type of dynamic. For example, we will consider dynamics such as transmission and recovery and define one event class for each of these. Using this framework, we can record the number of times an event of each class occurs and take it as the system state variable. Given m event classes, the system state variable at time t is the vector $\mathbf{N}(t) = [N_1(t), \dots, N_m(t)]$, each $N_\mu \in \mathbb{N}$. The ultimate goal is to determine the probability $P(\mathbf{N}(t) = \mathbf{n} | X(0))$ or: The probability that precisely $\mathbf{n} = [n_1, \dots, n_m]$ events of each class has occurred at time t , given an initial population space X_0 .

The population space is constructed by choosing a particular population and compartmentalising it into k sub-populations, similarly to the methodology of classical SIR-models. The compartmentalisation should be such that no individual can be included in more than one sub-population at a time, and no event should increase or decrease a given sub-population by more than one. The compartmentalisation used in this project is shown in ?? and further discussed in section 2.2.1. The population space at time t , X_t is a k -dimensional stochastic variable. Later we will discuss its dependence on the event counts $N(t)$.

The event $N(t)$ counts form an m -dimensional stochastic process, which under the following assumptions is a density dependent Markov Jump process.

Assumption 1 *For a sufficiently small time interval h , the following assumptions hold:*

- (i) *Event occurrences in disjoint time-intervals are independent*
- (ii) *The Chapman-Kolmogorov equation [?, ?] holds.*
- (iii) $P(N_\mu(t+h) - N_\mu(t) = 1 | X_t) = W_\mu(X_t)h + \mathcal{O}(h)$.
- (iv) $P(N_\mu(t+h) = N_\mu(t) | X_t) = 1 - W_\mu(X_t)h + \mathcal{O}(h)$.
- (v) $P(N_\mu(t+h) - N_\mu(t) = k | X_t) = \mathcal{O}(h)$, for all $k \geq 2$.

Items (iii) to (iv) in assumption 1 implies that the process formed by the event counts for a single class is a Poisson process with rate W_μ . We assume each W_μ is a smooth function of the population. Assumption (v) ensures that no more than one event can occur simultaneously. Note also that conditioning the probabilities on the population X_t at t is sufficient, although conditioning based on the event counts $N(t)$ and the initial population state would also hold in this context.

The Chapman-Kolmogorov equation is as follows: Consider a stochastic process ξ_1, \dots, ξ_k , with a joint probability density function $p_{\xi_1, \dots, \xi_k}(x_1, \dots, x_k)$. Then the probability density

$$p_{\xi_1, \dots, \xi_{k-1}}(x_1, \dots, x_{k-1}) = \int_{-\infty}^{\infty} p_{\xi_1, \dots, \xi_k}(x_1, \dots, x_k) dx_k,$$

corresponding to a straight-forward marginalisation of the joint probability density function.

Now, we consider the dependence of the population space on the event space. For each event class μ we define a vector δ^μ , where δ_κ^μ describes the effects of event μ on population κ . The eventcounts can then easily be mapped to the population space by constructing a matrix $\delta = [\delta^1 \dots \delta^m]^T$ as $X_t = X_0 + \delta \mathbf{N}(t)$, but note that some information is lost in this projection. Using this description, there are no changes in the population outside of the events, which should match the initial idea that all relevant dynamics should be modelled using the event framework.

Earlier we mentioned that the process constructed is a density dependent Markov jump process. The jump aspect of the process is in the discrete nature of the event counts, meaning that the system state jumps from one state to another with the instantaneous occurrence of an event. The density dependence is introduced via the probability rates $W_\mu(X_t)$, as they describe the future evolution of the process based on the current population and the current system state. Then only the Markov property should receive some further consideration. Again, we look to the W_μ . They depend on the population state (and consequently the system state) X at time t , but not on any earlier state. This is precisely the Markov property.

In the comments on assumption 1, we mentioned that each event count formed a Poisson process, so that the aggregate event counts forms a multivariate Poisson process. A well-known result for these is the following:

Lemma 1 *The waiting time H to the next event is exponentially distributed with a rate $W = \sum_{\mu=1}^m W_\mu$*

This result will be used later, and so we will prove it here:

PROOF The waiting time to the next event is given by the time until the very next event occurs. Let H_μ be the waiting time until event μ . Then $H = \min\{H_1, \dots, H_m\}$ or equivalently $H_\mu \geq H$ for all μ . Given the system state at time t , the events can be regarded as independent. This gives us the following probability density function for H , where we use that $H_\mu \sim \exp(W_\mu)$:

$$f_H(h) = \prod_{\mu=1}^m P(H_\mu \geq h) = \prod_{\mu=1}^m 1 - F_{H_\mu}(h) \stackrel{h \geq 0}{=} \prod_{\mu=1}^m e^{-W_\mu h} = e^{-h \sum_{\mu=1}^m W_\mu}$$

This is precisely the exponential probability density function, with parameter $\sum_{\mu=1}^m W_\mu$.

Another result concerns the time-evolution of the probabilities governing the system state:

Theorem 1 *Under the previous assumptions, the dynamics in the event space obeys the Kolmogorov Forward equation.*

$$\frac{d}{dt} P(N(t+s) = \mathbf{n} | X_t) = \sum_{\mu=1}^m P(N(t+s) = [n_1 \dots n_\mu - 1 \dots n_m]) W_\mu(X_{t+s} - \delta^\mu) - \quad (1)$$

$$- \sum_{\mu=1}^m P(N(t+s) = \mathbf{n}) W_\mu(X_{t+s}) \quad (2)$$

where we have used X_{t+s} to mean $X_0 + \delta \mathbf{n}$, or the population state given the event counts \mathbf{n} . The interpretation of the equation is as follows: The probability that a function should be in a given state at a given time increases as the probability that it should enter the state increases, and decreases as the probability that it should leave that state increases. The first term corresponds to the probability that it should enter the given state, as it is the probability that it should be in an immediately preceding state and enter the given state. The second term is interpreted similarly.

Theorem 1 provides us with a differential equation describing the probability distribution evolution for the event counts, but in general there are no simple solutions to it. Instead we look to an approximate solution using the Monte Carlo-framework, meaning we construct many artificial "samples" of the stochastic variable $N(t)$, and assume that the distribution of the samples can be used as an approximation for the distribution itself. This requires us to find an effective method of simulating a system as the one we have described, and one such method is the Feller-Kendall algorithm, also known as the Gillespie algorithm [citations].

To construct a realisation of the system, we will work iteratively and use the fact that the system state is constant between events. This means that the only times we are interested in are the times where an event occurs. Furthermore, we are only interested in the very next event, as that event may change the probability distributions for future events. Consequently, in order to move the realisation forward in time, the stochastic variables we would like to sample from are the time of the next event, H and what event it is that occurs, M . An initial idea for sampling these variables together is the following:

Algorithm 1 *Let the system be in state N at time t , with populations X .*

- (i) *Calculate the probability rates $W_\mu(X)$*
- (ii) *Sample times H_μ for the time of the next event of each event class*
- (iii) *Choose the event class M so that $H_M = \min\{H_1, \dots, H_m\}$*
- (iv) *Increase the event count of M by one:*

$$N := [N_1(t) \dots N_M(t) + 1 \dots N_m]$$

- (v) *Update the time and the population:*

$$X := X + \delta^M$$

$$t := t + H_M$$

- (vi) *Repeat*

This algorithm will work, but is not very efficient. In each step, m random numbers are generated, of which $m - 1$ are discarded. If the event count is high, this can lead to a significant increase in simulation time on each step. Instead, we will use the following result:

Lemma 2 *M and H are independent.*

In order to prove this, we will use the definition of independence: $P(A \cap B) = P(A) \cdot P(B)$. We will determine the joint probability density function of (H, M) and show that this is the product of the marginal distributions.

PROOF First, we consider the marginal distributions. In lemma 1, we showed that $H \sim \exp\left(\sum_{\mu=1}^m W_\mu\right)$. We now consider the distribution of M , by first studying the case where there are two exponentially distributed independent variables and generalising. Let $S \sim \exp(\lambda_S), T \sim \exp(\lambda_T)$ be independent. We will determine the probability that S occurs before T , or $P(S < T)$. Using independence and the exponential distribution, we get:

$$P(S < T) = \int_0^\infty P(S = s)P(T > s)ds = \int_0^\infty \lambda_S e^{-\lambda_S s} e^{-\lambda_T s} ds = \quad (3)$$

$$= \frac{\lambda_S}{\lambda_S + \lambda_T} \int_0^\infty (\lambda_S + \lambda_T) e^{-(\lambda_S + \lambda_T)s} ds = \frac{\lambda_S}{\lambda_S + \lambda_T}, \quad (4)$$

showing that the probability that it should be a particular event is proportional to its rate. Now for the generalization: Consider a set of variables $T_j \sim \exp(\lambda_j), j = 1, \dots, I$. Let $S = T_i$ and U the minimum of the remaining variables. By the result from lemma 1, $U \sim \exp(\sum_{j \neq i} \lambda_j)$. Combining this with eq. (4), we have

$$P(T_i = \min\{T_1, \dots, T_I\}) = P(S < U) = \frac{\lambda_i}{\sum_{j=1}^I \lambda_j}, \quad (5)$$

implying that the marginal distribution for M is proportional to the individual rates W_μ .

Now to compute the joint probability distribution for (H, M) .

$$\begin{aligned} P(H = h \cap M = \mu) &= P(H_\mu = h, H_n > h \text{ for } n \neq \mu) = W_\mu e^{-W_\mu h} \prod_{n \neq \mu} e^{-W_n h} = \\ &= W_\mu e^{-h \sum_{n=1}^m W_n} = \frac{W_\mu}{\sum_{n=1}^m W_n} \left(\sum_{n=1}^m W_n \right) e^{-h \sum_{n=1}^m W_n}, \end{aligned}$$

which is precisely the product of the probability density and distribution of H, M . Hence they are independent.

This result allows us to sample H and M separately from the marginal distributions defined above. The new algorithm is:

Algorithm 2 *Let the system be in state N at time t , with populations X :*

- (i) *Calculate the probability rates $W_\mu(X)$ and use this to determine the distributions of H and M*
- (ii) *Sample H and M*
- (iii) *Increase the event count for class M by one,*

$$N := [N_1(t) \dots N_M(t) + 1 \dots N_m(t)]$$

- (iv) *Update the time and the population:*

$$\begin{aligned} t &:= t + H \\ X &:= X + \delta_M \end{aligned}$$

- (v) *Repeat*

This method only requires two random variables to be sampled in each iteration, and is therefore significantly faster than the option presented in algorithm 1. This is the basic Feller-Kendall algorithm, and also the one used in this project.

2.2 Implementation

The first part of this section focused on how the model described above was applied to the specific problem discussed in this paper. The population compartmentalisation is explained, and an example of an equation for calculating the rates is given. Furthermore, the introduction of a time dependency is discussed. The second part focuses on the work to implement a computer program suitable for generating realisations of the system.

2.2.1 Events and populations

When constructing a compartmentalisation suitable for this problem, we start by considering what dynamics we want to model as events. We then construct the sub-populations in such a way that each event will increase one sub-population by one, decrease a sub-population by one, or both (moving a single individual from one population to another).

The types of dynamics that we want to model in this project can roughly be divided into two categories, dynamics relating to meningococcal disease and dynamics present in the healthy population. Due to the rather long intervals between the large-scale epidemics, it was determined that changes in the population distribution, as well as ageing individuals, could affect the model. This is the reason for including some dynamics of a healthy population, and finding a suitable approximation for the large-scale population variation over time was the first step.

The dynamics concerning a healthy population includes death by causes other than meningococcal disease, ageing and birth. The compartmentalisation and the events for a healthy population is shown in ???. The population is divided into three age groups, infants (0-4 years), young (5-14 years) and adult (≥ 15 years), based on available data for the population and the disease. Death and birth was included in a slightly simplified manner. It was assumed that death only occurs in the oldest age group, and that only the oldest age group bear children. In order to compensate for the lack of infant mortality, the birth rate was based on the number of children that survive the first year.

Modelling meningococcal disease is the primary goal of this project, and as such much care must be put in determining what events and populations to include. The compartmentalisation and events relating to disease (for a single age group) are shown in ??. A central part of the disease dynamics in this case is the presence of non-symptomatic carriers, and how this relates to transmission and immunity. Here, we choose to model the acquisition of carriage (on a susceptible individual) as a two-step process. We consider the event of transmission to a susceptible individual. This moves one individual from the sub-population "Susceptible" into a sub-population we call "Infected". Subsequently there is a second event, either one where the individual becomes sick or one where they become a carrier. This event occurs on average 10 days after the transmission, corresponding to the incubation time for meningococcal disease. Carriers can then recover in one of two ways, either becoming immune to invasive disease or becoming susceptible. When it comes to transmission, we assume that carriers can transmit the bacterium to other individuals. We consider immunity to be protective against

invasive disease, but not against carriage, so immune individuals can become carriers. In this case they never pass through the infected sub-population but are instead moved to the carrier population. This type of immune individuals can also lose immunity. However, when we consider individuals that recover from invasive disease they are always immune for the rest of their lives.

When we construct the events and the populations, the equations governing the events is also created. We assume that the rates follows the law of mass-action, so that the rate is proportional to the concentration of the relevant population. This also means we assume the population is perfectly mixed at all times, or (equivalently) that the population is evenly and randomly distributed in space. The equations for each event rate can be found in ??, but we will explain the basic idea behind them here using a (simplified) example. We will consider the transmission event to a susceptible individual, but for now we will not care about the different age groups. The generalisation is however straight-forward.

Consider an individual I that is infective. In order for this individual to transmit disease, they must encounter a susceptible individual. However, not all contacts will lead to the disease being transmitted, and we will call β

2.2.2 Time dependency

As one of the main goals of this project is to examine the seasonality of the disease, some sort of time dependency must be included. So far, only probability rates that depend exclusively on the population has been considered. In this section, I will make some comments on probability rates that can vary with time.

We consider events for which the probability rate λ may depend on time.

To match the questions posed in the goals for this thesis, two types of time dependency was introduced and compared. They are intended to model hypothetical explanations for the seasonal variations in the disease pattern between dry and wet season, by changing one or more of the equations governing the event probability rates.

The dry and wet season are here modelled as discrete and deterministic periods. The dry season occurs between days 65 and 335 each year, lasting for about nine months. The rest is considered wet season.

As the modelling procedure requires the system state and probability rates to be constant between events, the changes brought by the seasonal shift is introduced immediately after the first event after the season change. The new probability rates are calculated based on the updated equations and the updated system state. Provided events are frequent enough, this method should not introduce significant errors. The uncertainty in the exact and discrete onset of the seasons should be a greater source of uncertainty and error.

The changes in the equations are different based on what type of time dependency is introduced. One is designed to model a change in the population behaviour, namely two separate groups coming in contact over the dry season. Practically, this is done by introducing events corresponding to the biological events introduced by this contact, i.e transmission from one population group to the other, and letting the probability rate from that event be zero during the wet season.

The second time dependent behaviour considered here is simply a change in the invasiveness of the disease itself, and is modelled by a change in one of the internal parameters.

2.2.3 Computer implementation

The model was implemented in C++, and could be easily adapted to a different problem.

2.3 Acquiring data and parameters

The implementation discussed in this project is designed to approximately match a theoretical population in northern Nigeria. The region was chosen as there is reliable data.

2.3.1 Population and climate data

The only climate data currently used is when the wet season occurs, which in this region is approximately from July though September. The year start is set to the first of August, to ensure a simulation start and end with as few interesting events as possible.

The distribution of the population was sourced from Gapminder, and the population growth resulting from a model disregarding the disease corresponds well to a 2% population growth, which is the current value in Nigeria.

2.3.2 Disease data

3 Results and analysis

4 Conclusions

References

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