Fourier Transforms for Circuit and LTI Systems Analysis

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Digital Technium 123

Office Hours: 12:00-13:00 Mondays

You can view the notes for this presentation in HTML and PDF.

The source code of this presentation is available in Markdown format from GitHub: ft2.md.

The GitHub repository EG-247 Resources also contains the source code for all the Matlab/Simulink examples and the Laboratory Exercises.

Scope and Background Reading

This session we will apply what we have learned about Fourier transforms to some typical cicuit problems. After a short introduction, this session will be an examples class.

The material in this presentation and notes is based on Chapter 8 of Steven T. Karris, Signals and Systems: with Matlab Computation and Simulink Modelling, 5th Edition. from the **Required Reading List**. I also used Benoit Boulet, Fundamentals of Signals and Systems from the **Recommended Reading List**.

Agenda

- The system function
- Examples

The System Function

System response from system impulse response

Recall that the convolution integral of a system with impulse response $h(t)$ and input $u(t)$ is

$$h(t) * u(t) = \int_{-\infty}^{\infty} h(t-\tau)u(\tau)\,d\tau.$$

We let

$$g(t) = h(t) * u(t)$$

Then by the time convolution property

$$h(t) * u(t) = g(t) \Leftrightarrow G(\omega) = H(\omega).U(\omega)$$

The System Function

We call $H(\omega)$ the *system function*.

We note that the system function $H(\omega)$ and the impulse response $h(t) form the Fourier transform pair

$$h(t) \Leftrightarrow H(\omega)$$

Obtaining system response

If we know the impulse resonse $h(t)$, we can compute the system response $g(t)$ of any input $u(t)$ by multiplying the Fourier transforms of $H(\omega)$ and $U(\omega)$ to obtain $G(\omega)$. Then we take the inverse Fourier transform of $G(\omega)$ to obtain the response $g(t)$.

1. Transform $h(t) \to H(\omega)$
2. Transform $u(t) \to U(\omega)$
3. Compute $G(\omega) = H(\omega).U(\omega)$
4. Find $\mathcal{F}^{-1}\left\{G(\omega)\right\} \to g(t)$

Example 1

Karris example 8.8: for the linear network shown below, the impulse response is $h(t) = 3e^{-2t}$. Use the Fourier transform to compute the response $y(t)$ when the input

$$u(t) = 2[u_0(t) - u_0(t-3)].$$

Verify the result with Matlab.

Matlab verification

See ft3_ex1.m

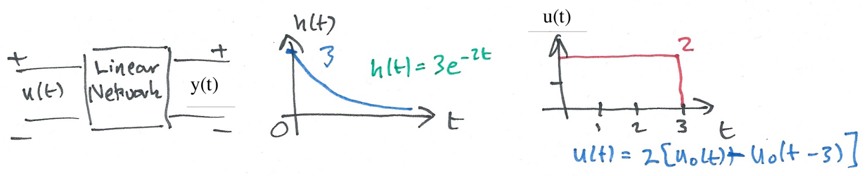

Figure 1: Example 1

Result:

```
y = 3*heaviside(t) - 3*heaviside(t - 3) + 3*heaviside(t - 3)*exp(6 - 2*t)
 - 3*exp(-2*t)*heaviside(t)
```

Which after gathering terms gives

$$y(t) = 3(1 - 3e^{-2t})u_0(t) - 3(1 - 3e^{-2(t-3)})u_0(t - 3)$$

And here's a plot:

Example 2

Karris example 8.9: for the circuit shown below, use the Fourier transfrom method, and the system function $H(\omega)$ to compute $V_L(t)$. Assume $i_L(0^-) = 0$. Verify the result with Matlab.

Matlab verification

See ft3_ex2.m

Result:

```
vout = -5*exp(-3*t)*heaviside(t)*(2*exp(t) - 3)
```

Which after gathering terms gives

$$v_{\text{out}} = 5\left(3e^{-3t} - 2e^{-2t}\right) u_0(t)$$

And here's a plot:

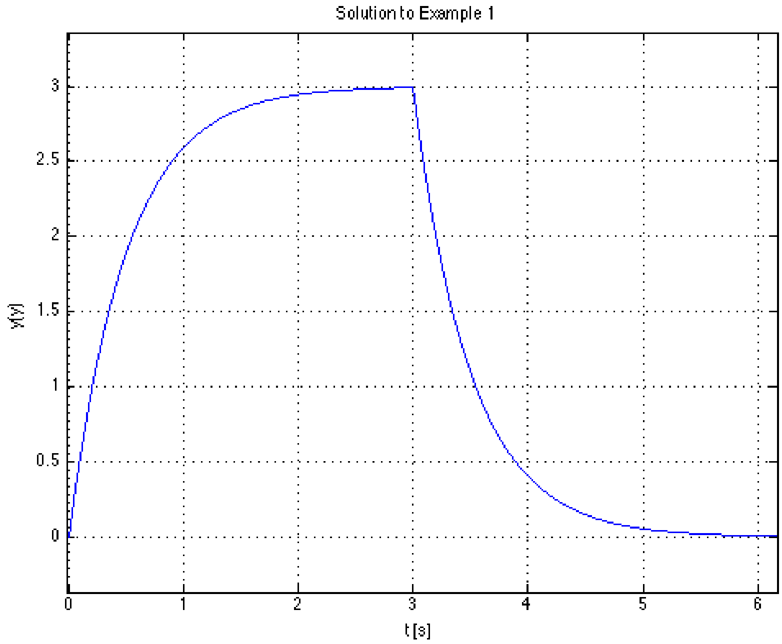

Figure 2: Solution for example 1

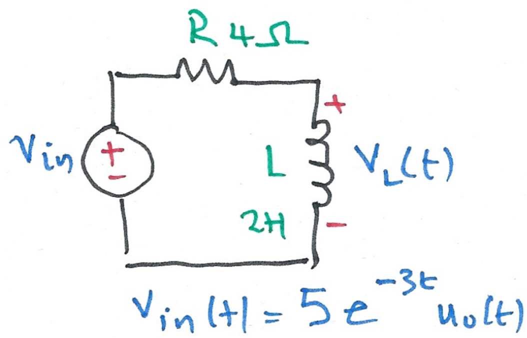

Figure 3: Example 2

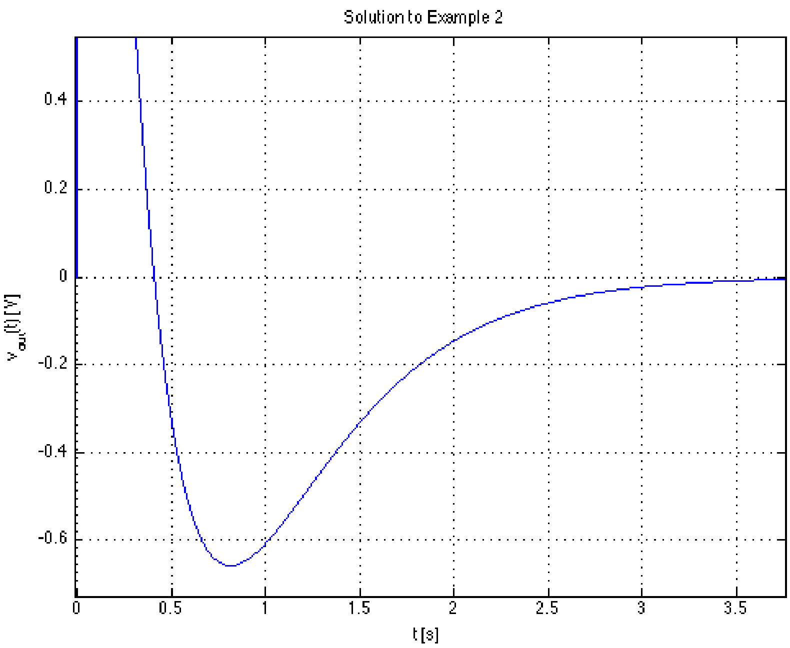

Figure 4: Solution for example 2

Example 3

Karris example 8.10: for the linear network shown below, the input-output relationship is:

$$\frac{d}{dt}v_{\text{out}} + 4v_{\text{out}} = 10v_{\text{in}}$$

where $v_{\text{in}} = 3e^{-2t}$. Use the Fourier transform method, and the system function $H(\omega)$ to compute the output v_{out}. Verify the result with Matlab.

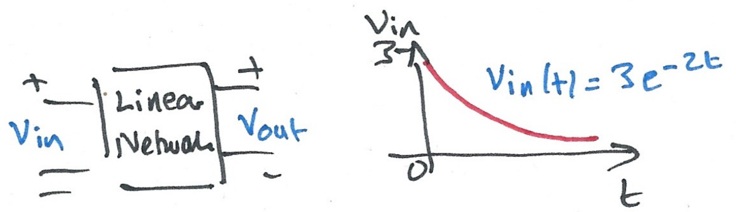

Figure 5: Example 3

Matlab verification

See ft3_ex3.m

```
Result:

    15*exp(-4*t)*heaviside(t)*(exp(2*t) - 1)
```

Which after gathering terms gives

$$v_{\text{out}}(t) = 15\left(e^{-2t}\right) - e^{-4t}\big)\, u_0(t)$$

And here's a plot:

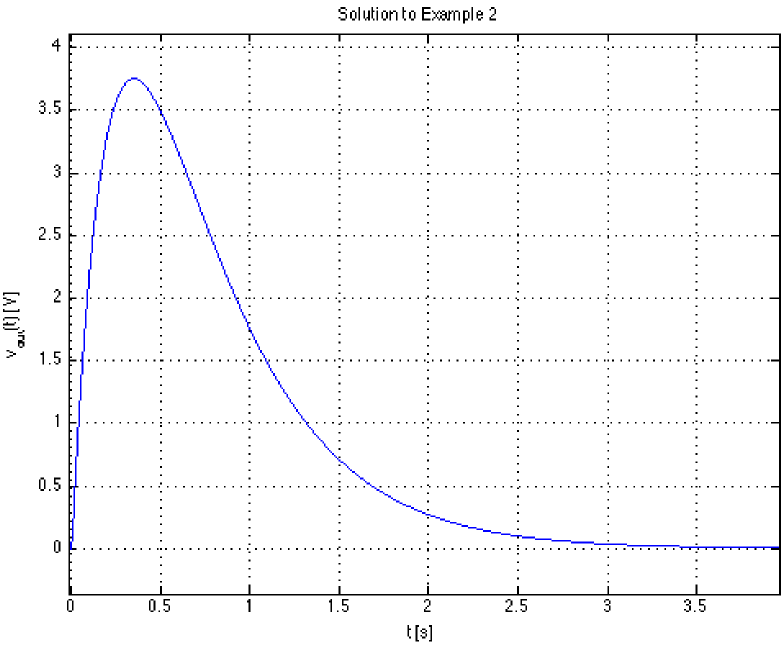

Figure 6: Solution of example 3

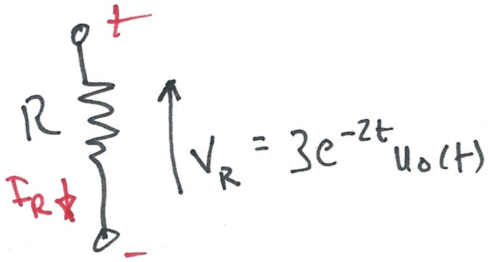

Figure 7: Example 4

Example 4

Karris example 8.11: the voltage across a 1 Ω resistor is known to be $V_R(t) = 3e^{-2t}u_0(t)$. Compute the energy dissipated in the resistor for $0 < t < \infty$, and verify the result using Parseval's theorem. Verify the result with Matlab.

Note from tables of integrals

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a} \arctan \frac{x}{a} + C.$$

Matlab verification

See ft3_ex4.m

```
Result:

    Wr = (516074502530039 31*pi)/72057594037927936 = 2.25
```

Homework

Attempt the end of the chapter exercises 7-11 (Section 8.10) from Karris. Don't look at the answers until you have attempted the problems.

Lab Work

We will verify the results and examine the frequency responses of selected examples from this session.