Sampling Systems

Scope and Background Reading

This session is an introduction to sampling theory. It reviews the important ideas that pertain to sampling but leaves the detailed mathematics for your further study.

The material in this presentation and notes is based on Chapter 15 of Benoit Boulet, Fundamentals of Signals and Systems from the **Recommended Reading List** and you'll find the mathematical treatments there. There is much more detail in Chapter 9 of Steven T. Karris, Signals and Systems: with Matlab Computation and Simulink Modelling, 5th Edition from the **Required Reading List**.

Agenda

- Sampling of Continuous-Time Signals
- Signal Reconstruction
- Discrete-time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Systems

Acknowledgements

We will be using an adaptation of a pair of demo scripts to illustrate *alialising*. These scripts were published by Prof. Charles A. Bouman, School of Electrical and Computer Engineering, Purdue University as part of the course materials for ECE438: Digital Signal Processing.

Introduction

- The *sampling process* provides the bridge between continuous-time (CT) and discrete-time (DT) signals
- Sampling records discrete values of a CT signal at periodic instants of time.
- Sampled data can be used in *real-time* or *off-line* processing
- Sampling opens up possibility of processing CT signals through *finite impulse response* (FIR) and *infinite impulse response* IIR filters.

A Real Example

Sound sampling

I need a volunteer to provide a sound sample ....

1. I will use this script sampling_demo.m to sample your voice.
2. I will then playback the recording.
3. I will the plot the data.

Technical Details

- **Sampling rate**: 8000 samples per second (fs = 8 kHz)
- **Resolution**: 8 bits per sample
- **Channels**: 1 channel.
- **Reconstruction**: Matlab plays the audio back at 8192 samples per second.

Question

What will the bit-rate be for playback?

Answer

bit rate $=$ [number of samples per second] x [number of bits per sample] x [number of channels]

bit rate $= 8192 \times 8 \times 1$ bits/second [baud]

bit rate $= 65,536$ bits/second

Sampling CT Signals

Sampling CT Signals

What is going on here?

Time domain

Sampling can be modelled as the multiplication of a continuous-time signal by a sequence of periodic impulses as illustrated here. T_s is the period of the periodic sampling function.

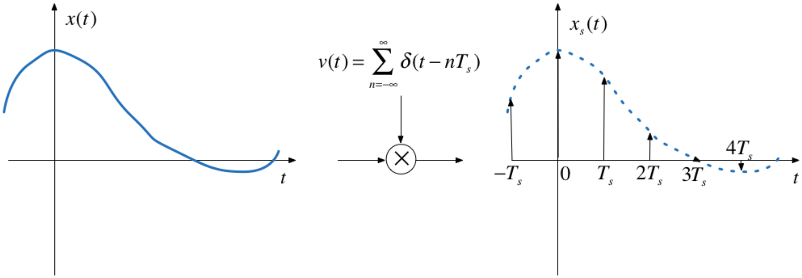

Figure 1:Sampling

This is a form of **modulation**

Frequency domain

Multiplication in time domain is *convolution* in the frequency domain

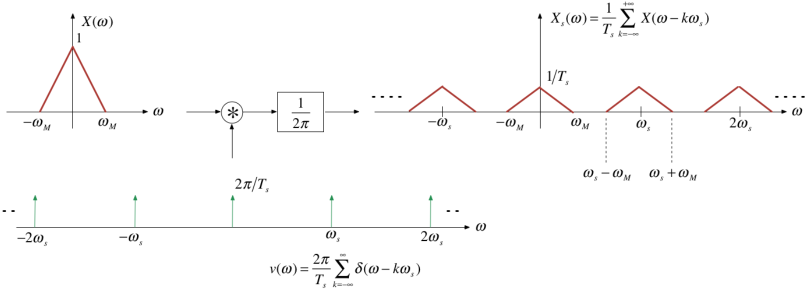

Figure 2:Frequency domain

ω_s is the frequency of the periodic sampling function $= 2\pi/T_s$.

The Mathematics

The Sampled signal:

$$x_s(t) = \sum_{n=-\infty}^{+\infty} x(nT_s)\delta(t - nT_s)$$

Frequency convolution:

$$X_s(\omega) = \frac{1}{T_s} \int_{-\infty}^{+\infty} X(\upsilon) \sum_{n=-\infty}^{+\infty} \delta(t - \upsilon - k\omega_s)\, d\upsilon$$

The Mathematics (continued)

Sampling property:

$$X_s(\omega) = \frac{1}{T_s} \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} X(\omega - k\omega_s)\delta(t - \upsilon - k\omega_s)\, d\upsilon$$

Sifting property:

$$X_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(\omega - k\omega_s)$$

Nyquist-Shannon Sampling Theorem

Gives a sufficient condition to recover a continuous time signal from its samples $x(nT_s)$, n is an integer.

Sampling Theoreom

Let $x(t)$ be a band-limited signal with $X(\omega) = 0$ for $|\omega| > \omega_M$.

Then $x(t)$ is uniquely determined by its samples $x(nT_s)$, $\infty < n < +\infty$ if

$$\omega_s > 2\omega_M,$$

where $\omega_s = 2\pi/T_s$ is the sampling frequency.

Recovery of signal by filtering

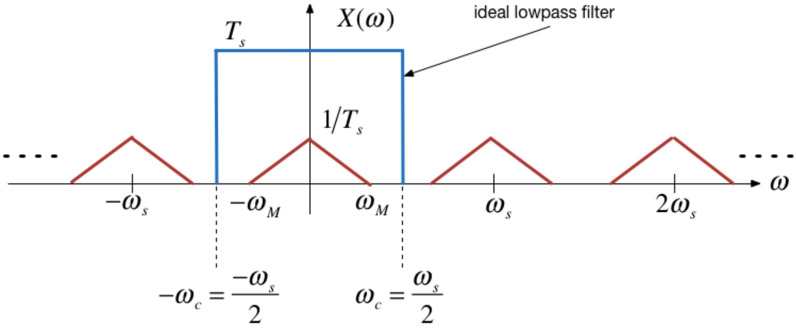

Figure 3:Signal recovery

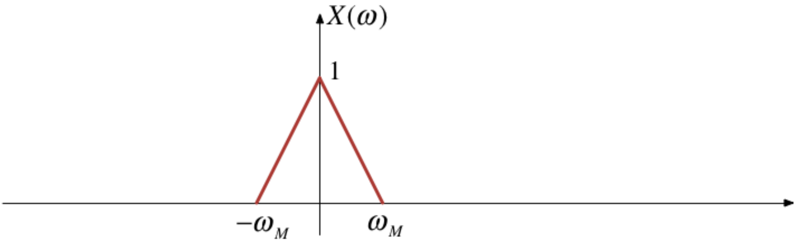

Figure 4:Recovered signal

Ideal Lowpass Filter for CT Recovery from DT Sampled Signal

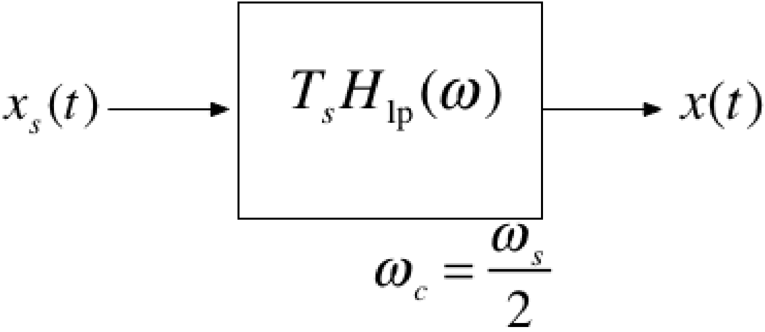

Figure 5:Ideal low-pass filter

This is of course theoretical only!

Sample-and-hold

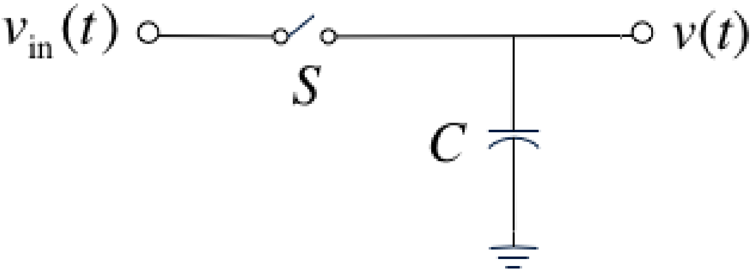

Figure 6:Sample and hold

Sample-and-hold operator

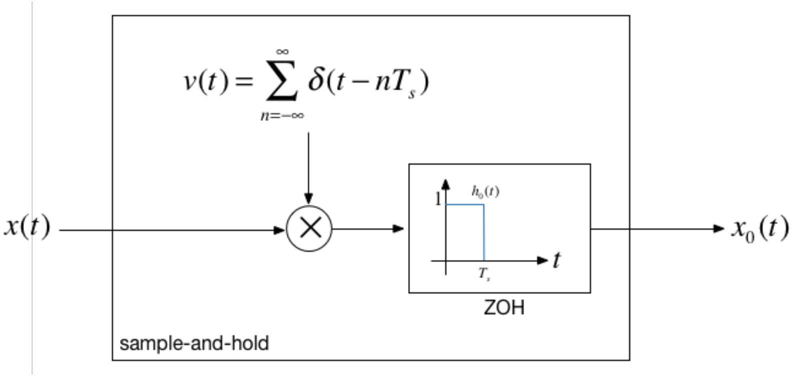

Figure 7:Zero-order hold

Example: CT Signal

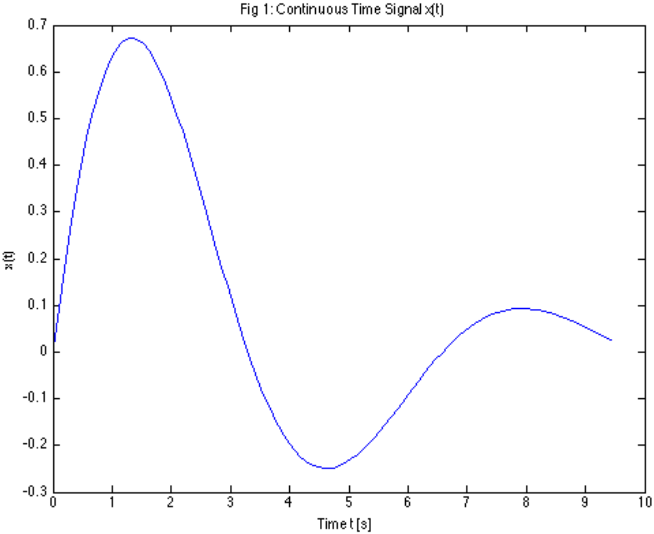

Figure 8:CT Signal

Example: After sampling

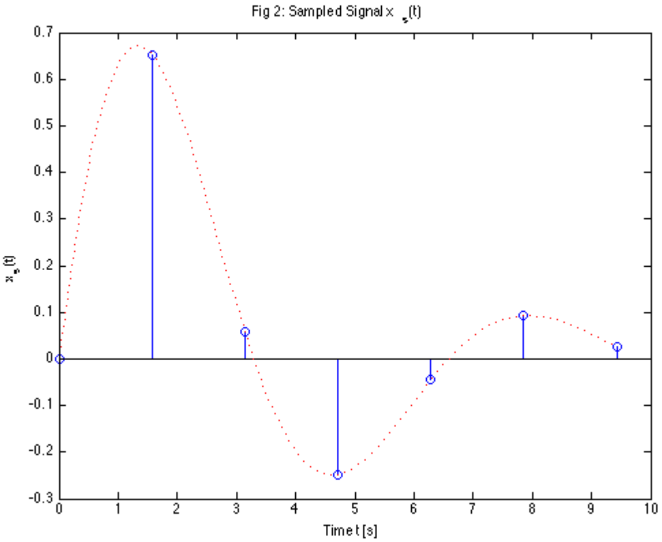

Figure 9:Sampled signal

Example: Reconstructed with sample and hold

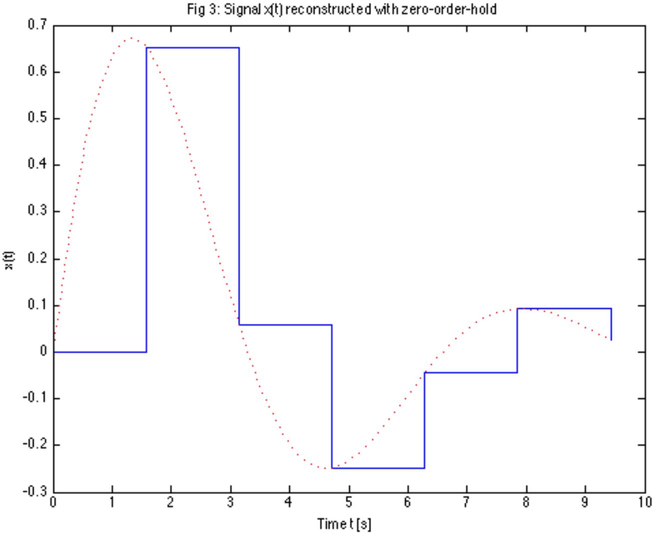

Figure 10:After sample-and-hold (E.G. ADC Output)

Signal Reconstruction

Signal Reconstruction

Problem

- We have a bandlimited signal that is sampled at the Nyquist-Shannon sampling frequency $\omega_s = 2\pi/T_s$.
- We therefore have a discrete-time (DT) signal $x(nT_s)$ from which we want to reconstruct the original signal.

Perfect Signal Interpolation Using sinc Functions

- In the *frequency domain*, the ideal way to reconstruct the signal would be to construct a chain of impulse $x_s(t)$ and then to filter this signal with an ideal lowpass filter.
- In the *time domain*, this is equivalent to interpolating the samples using time-shifted sinc functions with zeros at nT_s for $\omega_c = \omega_s$.

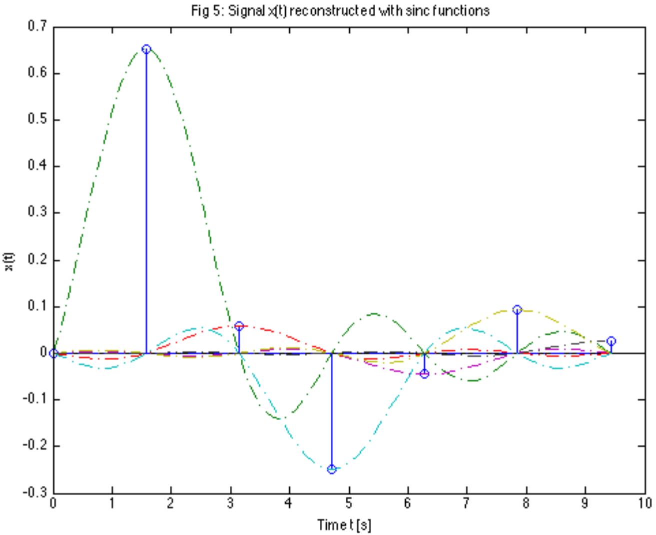

Figure 11:Perfect Signal Interpolation Using sinc Functions

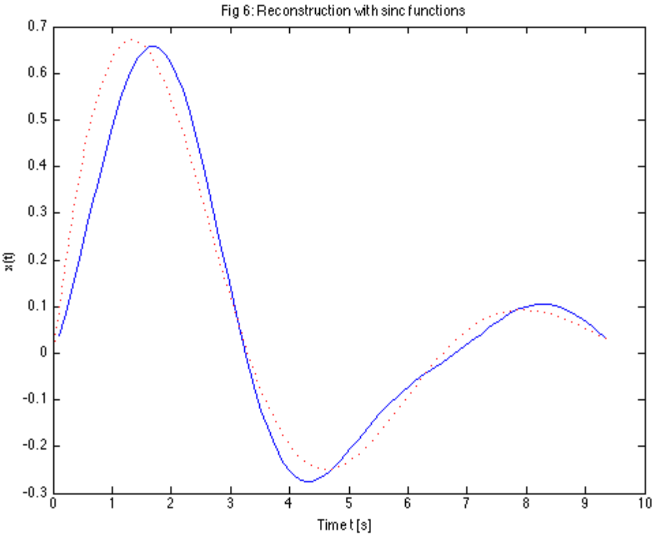

Figure 12:fig

Zero-Order-Hold

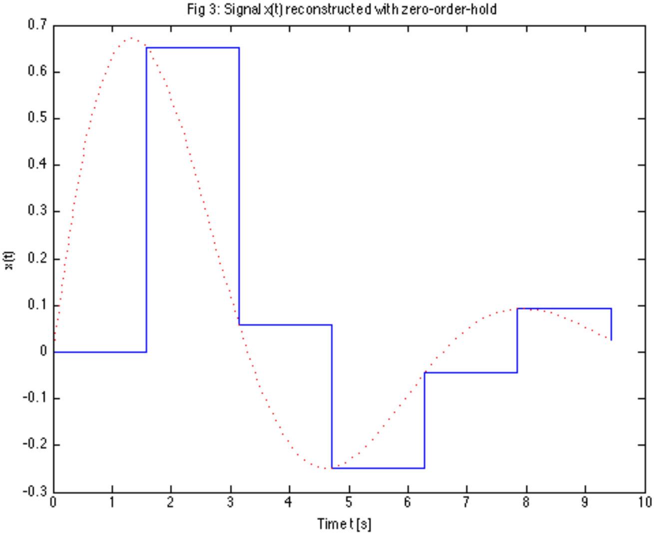

Figure 13:Zero-Order-Hold

First-order Hold

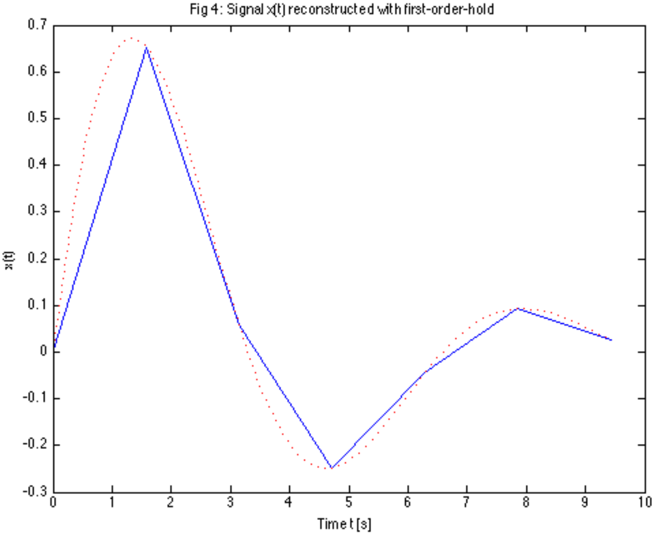

Figure 14:First-order Hold

Aliasing

Aliasing

- Aliasing Occurs when the sampling frequency is too low to ovoid overlapping between the spectra.
- When aliasing oucus, we have violated the sampling theorem that is $\omega_s < 2\omega_m$.
- When aliasing occurs, the original signal cannot be recovered by lowpass filtering.

An Aliased Signal

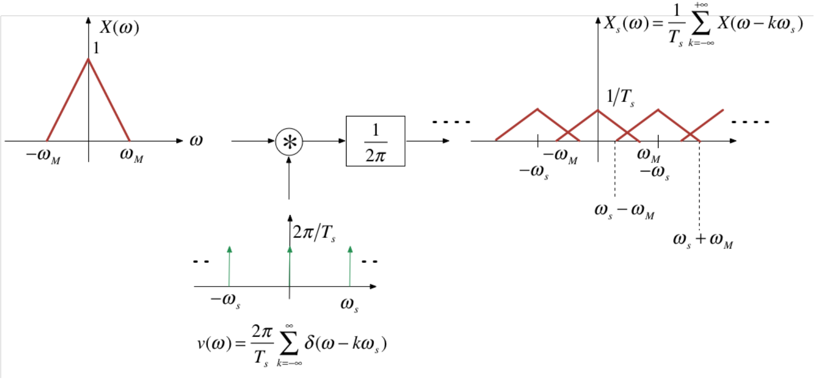

Figure 15:An Aliased Signal

Example 1

We use the recording made at the start and run it through a script that effectively aliases the original signal be reducing the sampling frequency to less than half the original sampling frequency.

Here's the script: aliaseg1.m that I'll be using.

Example 2

Assume signal $x(t) = \cos(\omega_0 t)$ is sampled at a rate of $\omega_s = 1.5\omega_s$, violating the sampling theorem.

We can see the effect on the plot below:

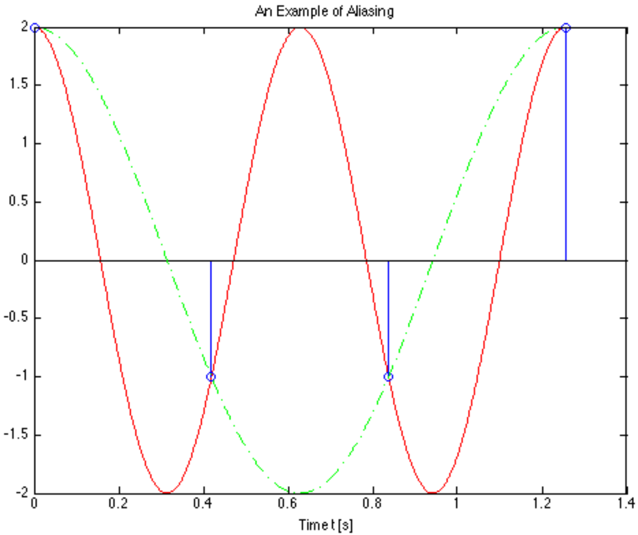

Figure 16:Aliasing

Image generated by aliasing.m

Antialising Filters

- Most real signals are not band-limited so we have to artificially make them bandlimited using an *anti-aliasing filter*.
- An antialiasing filter is a low-pass filter whose cutoff frequency is lower than half the sampling frequency.
- This can produce some distortion at high-frequencies but this is often better than the distortion that would occus at low frequencies if aliasing was allowed to happen.
- For more on this topic see Pages 551—552 of Boulet.

Example 3

This example uses anti-aliasing to downsample the audio. You should hear that the sound is less distorted as we sample below the sampling frequency of 8 kHz.

Practical application - digital audio

Human beings can hear sounds with frequencies up to around 20 kHz so when recording music in the modern sound studio (or phone or PC for that matter) the audio signal is antialiased with a 22 kHz filter. The signal is then sampled at 44.1 kHz before being stored for later processing and/or playback.

DT Processing of CT Signals

DT Processing of CT Signals

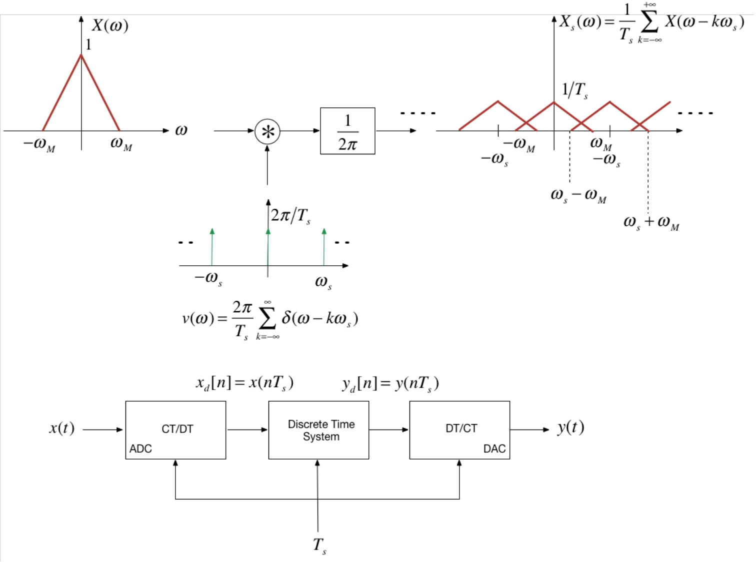

Figure 17:DT Processing of CT Signals

Sampling of DT Signals

- In modern signal processing and digital communications many of the operations that were once done in continuous time are now done entirely in discrete time.
- For example, we can implement sampling and modulation in discrete time.
- We can also up-sample (interpolate between samples) or downs-ample (reduce the number of samples in a discrete-time signal)

These topics are left to you for further study.

Summary

- Sampling of Continuous-Time Signals
- Signal Reconstruction
- Discrete-time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Systems

Next session

- The Z-Transform

Matlab Functions used

See notes.

Homework

You should take the scripts home and play with them.

Try increasing the sampling frequency: 8000 Hz, 11025 Hz, 22050 Hz, 44100 Hz, 48000 Hz, and 96000 Hz are supported by most PC sound cards.

Try increasing the bits per sample: 8, 16, 24 are available.

Lab Work

We explore sound generation and manipulation in the final lab session.