

## Lesson 2: Elementary Signals

# About this presentation

Dr Chris Jobling ([c.p.jobling@swansea.ac.uk](mailto:c.p.jobling@swansea.ac.uk))

Digital Technium 123

Office Hours: Mondays 12:00 pm (noon).

You can view the notes for this presentation in HTML and PDF.

The source code of this presentation is available in Markdown format from GitHub: `elementary_signals.md`.

The GitHub repository EG-247 Resources also contains the source code for all the Matlab/Simulink examples.

# Review of Homework Problem from Lesson 1

Consider a signal

$$x = f(t) = \begin{cases} 0 & : t < -1 \\ t + 1 & : -1 \leq t \leq 1 \\ 0 & : t > 1 \end{cases}$$

Sketch this signal

Sketch the effect on this signal of applying the following basic signal operations

## Amplitude scaling

- ▶  $2f(t)$
- ▶  $0.5f(t)$

## Time scaling

- ▶  $f(2t)$
- ▶  $f(0.5t)$

## Mirroring

- ▶  $-f(t)$
- ▶  $f(-t)$
- ▶  $-f(-t)$

# Try this

## A combination of transformations

►  $-2f(-t + 2)$

Note that this involves *amplitude scaling*, *amplitude mirroring*, *time mirroring*, and a *time shift*. Each operation can be performed in sequence in any order.

Quiz: consider this circuit:

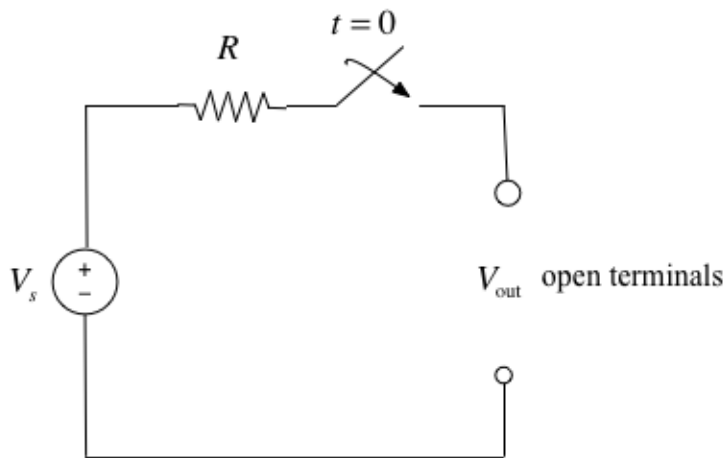


Figure 1: Circuit for quiz

# Elementary signals

# Unit Step Function

## Definition

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



## Sketch

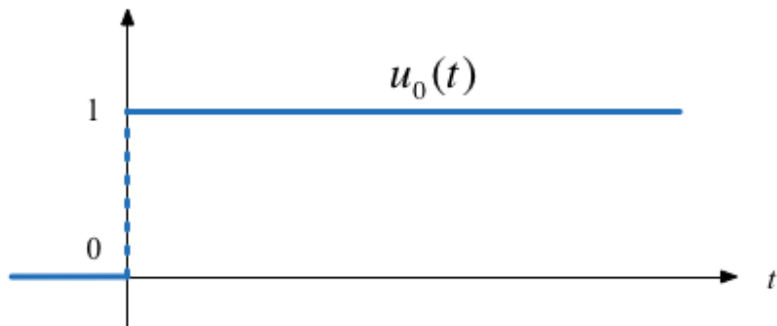


Figure 2: Unit step function

# Computing/Plotting in Matlab

In Matlab, we use the `heaviside` function (Named after Oliver Heaviside).

```
syms t  
ezplot(heaviside(t),[-1,1])
```

See: `heaviside_function.m`

# Result

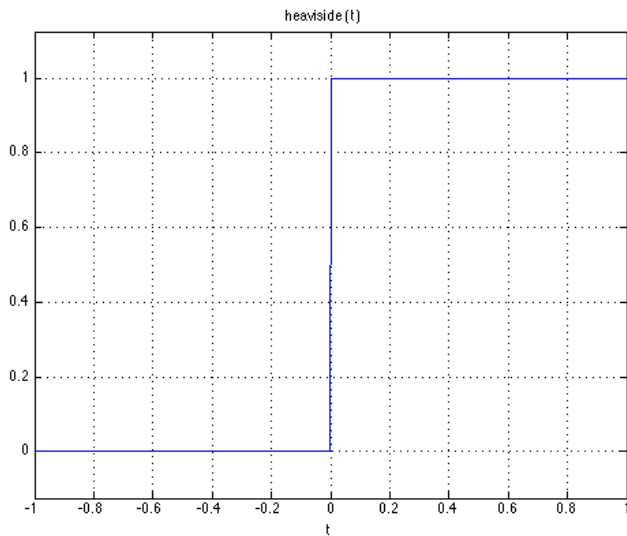
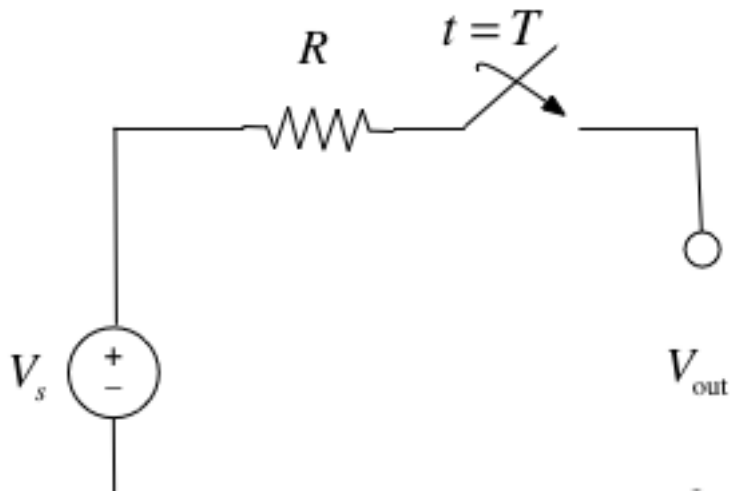


Figure 3: The Heaviside function (unit step)

## Circuit Revisited

Consider the network shown below, where the switch is closed at time  $t = T$ .



Express the output voltage  $v_{\text{out}}$  as a function of the unit step function, and sketch the appropriate waveform.

## Simple Signal Operations

# Amplitude Scaling

Sketch  $Au_0(t)$  and  $-Au_0(t)$

# Time Reversal

Sketch  $u_0(-t)$



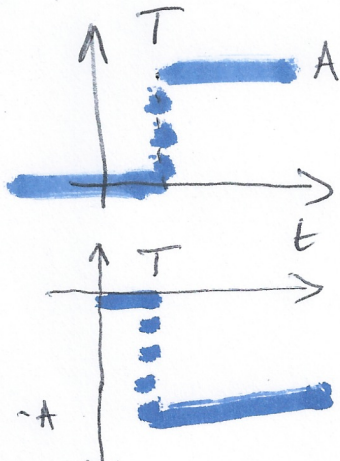
# Time Delay and Advance

Sketch  $u_0(t - T)$  and  $u_0(t + T)$

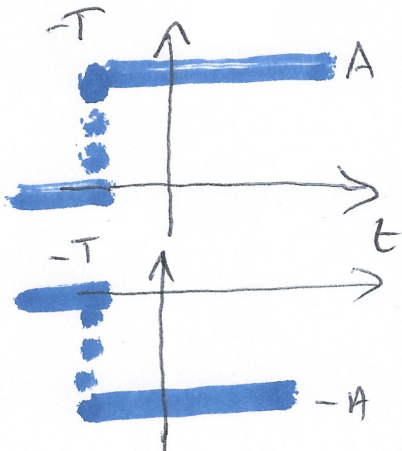
## Example 1

Which of these signals represents  $-Au_0(t + T)$ ?

①



②



## Example 2

What is represented by

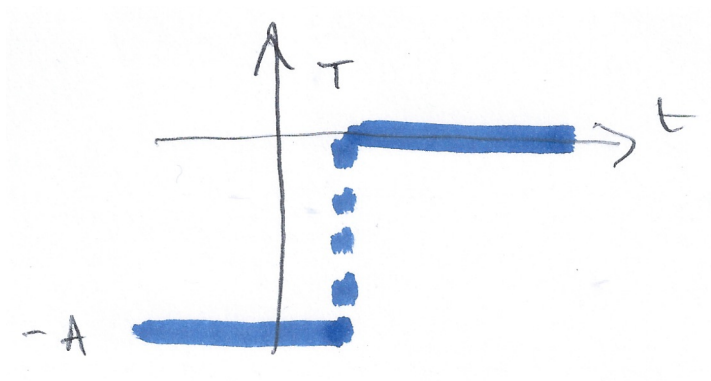


Figure 6: Example 2

1.  $-Au_0(t - T)$
2.  $-Au_0(-t + T)$
3.  $-Au_0(-t - T)$
4.  $-Au_0(t - T)$

# Synthesis of Signals from Unit Step

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

# Synthesize Rectangular Pulse

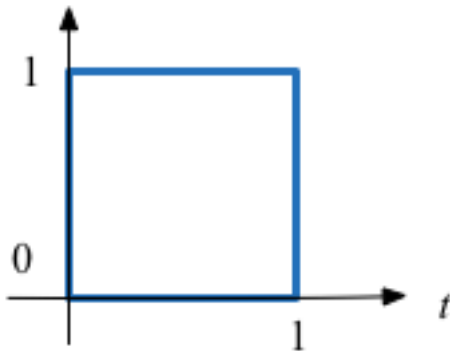


Figure 7: Rectangle function

## Synthesize Square Wave

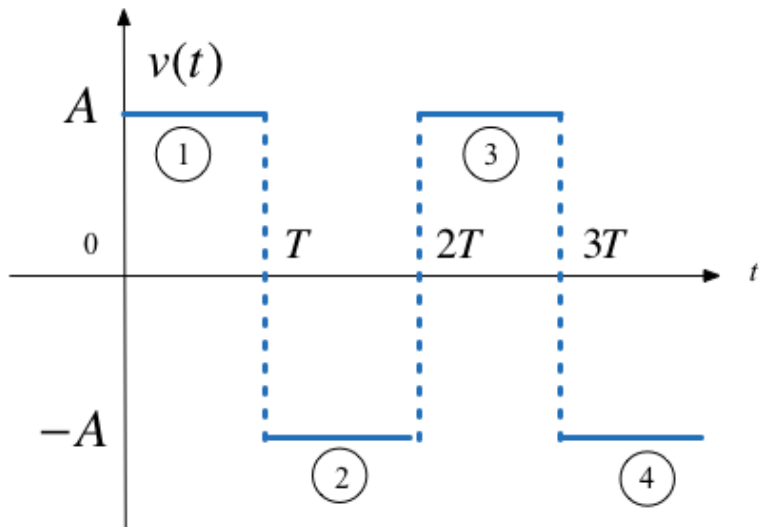


Figure 8: Square wave

## Synthesize Symmetric Rectangular Pulse

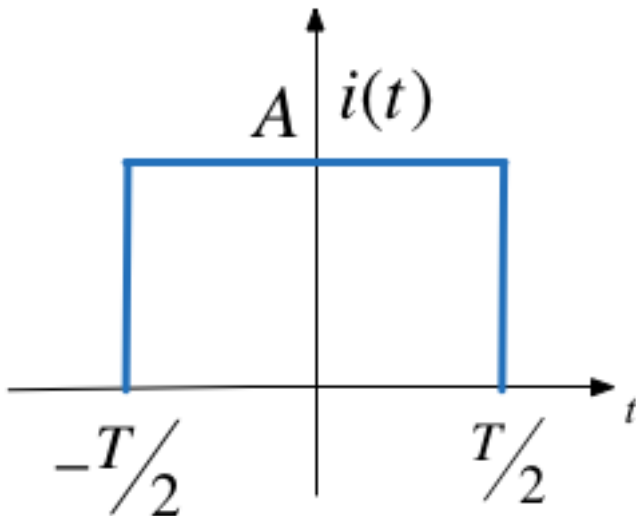


Figure 9: Symmetric rectangular pulse



## Synthesize Symmetric Triangular Pulse

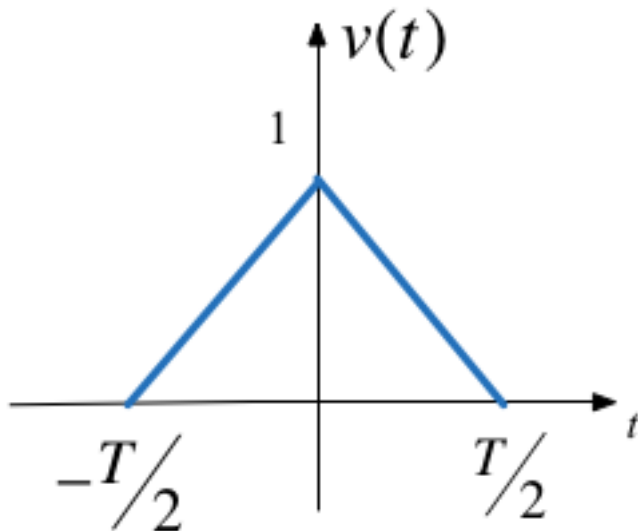
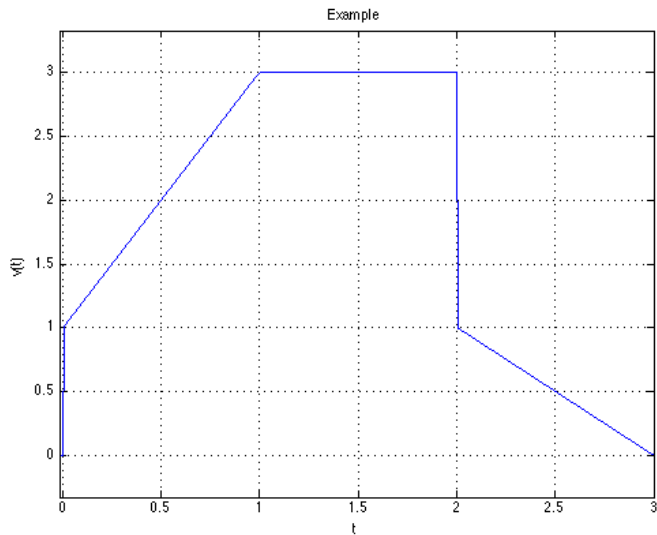


Figure 10: Symmetric triangular pulse

# Homework

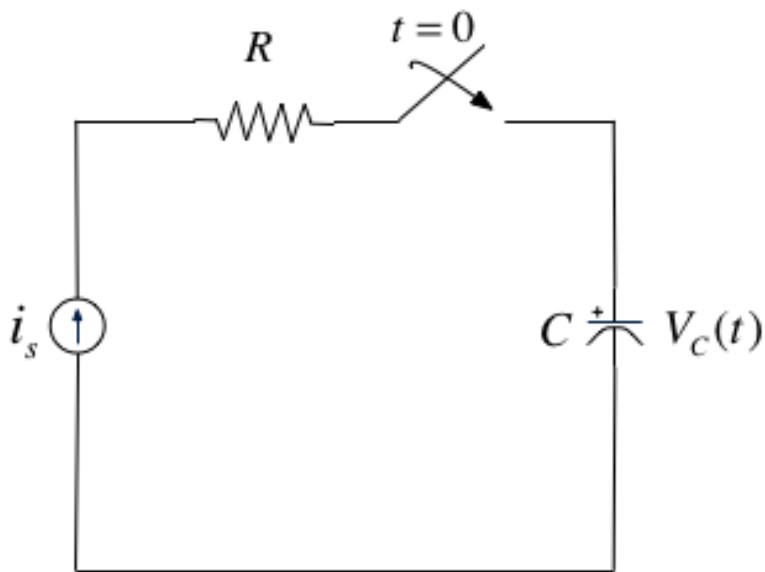
Show that the waveform shown below



can be represented by the function

$$v(t) = (2t+1)u_0(t) - 2(t-1)u_0(t-1) - tu_0(t-2) + (t-3)u_0(t-3)$$

## The Ramp Function



In the circuit shown in the previous slide  $i_s$  is a constant current source and the switch is closed at time  $t = 0$ . Show that the voltage across the capacitor can be represented as

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.

# The unit ramp function

The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^t u_0(\tau) d\tau$$

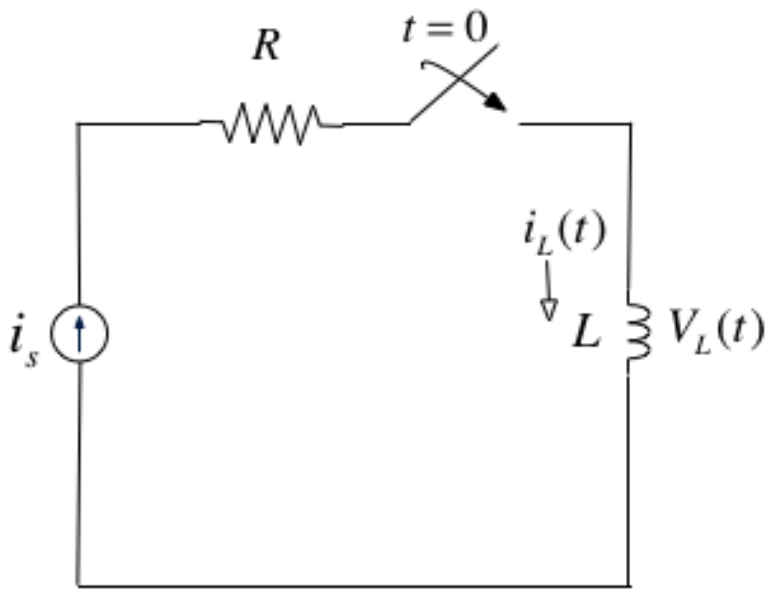
so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt} u_1(t)$$

## The Dirac Delta Function



In the circuit shown on the previous slide, the switch is closed at time  $t = 0$  and  $i_L(t) = 0$  for  $t < 0$ . Express the inductor current  $i_L(t)$  in terms of the unit step function and hence derive an expression for  $v_L(t)$ .



# The delta function

The *unit impulse* or the *delta function*, denoted as  $\delta(t)$ , is the derivative of the unit step.

This function is tricky because  $u_0(t)$  is discontinuous at  $t = 0$  but it must have the properties

$$\int_{-\infty}^t \delta(\tau) d\tau = u_0(t)$$

and

$\delta(t) = 0$  for all  $t \neq 0$ .

## Sketch of the delta function



Figure 14: The delta function

## Important properties of the delta function

# Sampling Property

The *sampling property* of the delta function states that

$$f(t)\delta(t-a) = f(a)\delta(t-a)$$

or, when  $a = 0$ ,

$$f(t)\delta(t) = f(0)\delta(t)$$

Multiplication of any function  $f(t)$  by the delta function  $\delta(t)$  results in sampling the function at the time instants for which the delta function is not zero.

The study of discrete-time (sampled) systems is based on this property.

*You should work through the proof for yourself.*

# Sifting Property

The *sifting property* of the delta function states that

$$\int_{-\infty}^{\infty} f(t)\delta(t - \alpha)dt = f(\alpha)$$

That is, if multiply any function  $f(t)$  by  $\delta(t - \alpha)$ , and integrate from  $-\infty$  to  $+\infty$ , we will get the value of  $f(t)$  evaluated at  $t = \alpha$ .

*You should also work through the proof for yourself.*

# Higher Order Delta Functions

the  $n$ th-order *delta function* is defined as the  $n$ th derivative of  $u_0(t)$ , that is

$$\delta^n(t) = \frac{d^n}{dt^n}[u_0(t)]$$

The function  $\delta'(t)$  is called the *doublet*,  $\delta''(t)$  is called the *triplet* and so on.

By a procedure similar to the derivation of the sampling property we can show that

$$f(t)\delta'(t-a) = f(a)\delta'(t-a) - f'(t)\delta(t-a)$$

Also, derivation of the sifting property can be extended to show that

$$\int_{-\infty}^{\infty} f(t) \delta^n(t - \alpha) dt = (-1)^n \left. \frac{d^n}{dt^n} [f(t)] \right|_{t=\alpha}$$



# Examples

## Example 3

Evaluate the following expressions

1.

$$3t^4\delta(t-1)$$

2.

$$\int_{-\infty}^{\infty} t\delta(t-2)dt$$

3.

$$t^2\delta'(t-3)$$

## Example 4

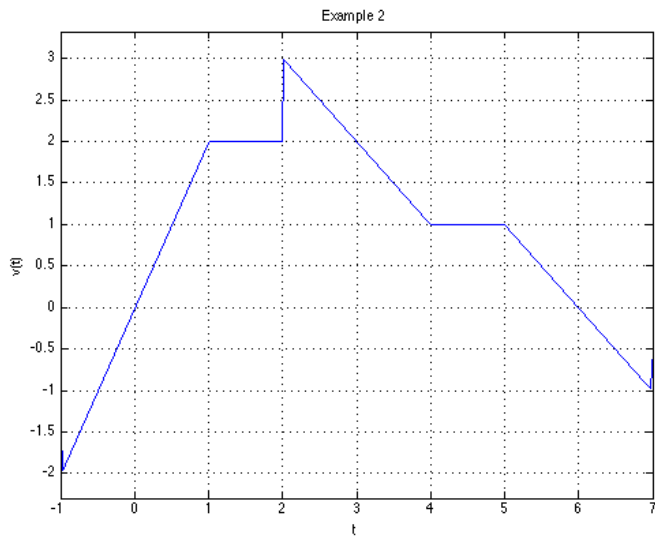


Figure 15: Example 4

1. Express the voltage waveform  $v(t)$  shown above as sum of unit step functions for the time interval  $-1 < t < 7$  s
2. Using the result of part (1), compute the derivative of  $v(t)$  and sketch it's waveform.

# Self-study

Do the end of the chapter exercises (Section 1.7) from the textbook. Don't look at the answers until you have attempted the problems.

# Lab Work

In the lab, a week on Friday, we will solve Example 2 using Matlab/Simulink following the procedure given between pages 1-17 and 1-22 of the textbook. We will also explore the heaviside and dirac functions.