

# Fourier Transforms for Circuit and LTI Systems Analysis

# Scope and Background Reading

This session we will apply what we have learned about Fourier transforms to some typical circuit problems. After a short introduction, this session will be an examples class.

The material in this presentation and notes is based on Chapter 8 of Steven T. Karris, Signals and Systems: with Matlab Computation and Simulink Modelling, 5th Edition. from the **Required Reading List**. I also used Benoit Boulet, Fundamentals of Signals and Systems from the **Recommended Reading List**.

# Agenda

- ▶ The system function
- ▶ Examples

# The System Function

# System response from system impulse response

Recall that the convolution integral of a system with impulse response  $h(t)$  and input  $u(t)$  is

$$h(t) * u(t) = \int_{-\infty}^{\infty} h(t - \tau)u(\tau) d\tau.$$

We let

$$g(t) = h(t) * u(t)$$

Then by the time convolution property

$$h(t) * u(t) = g(t) \Leftrightarrow G(\omega) = H(\omega).U(\omega)$$

# The System Function

We call  $H(\omega)$  the *system function*.

We note that the system function  $H(\omega)$  and the impulse response  $h(t)$  form the Fourier transform pair

$$h(t) \Leftrightarrow H(\omega)$$

# Obtaining system response

If we know the impulse response  $h(t)$ , we can compute the system response  $g(t)$  of any input  $u(t)$  by multiplying the Fourier transforms of  $H(\omega)$  and  $U(\omega)$  to obtain  $G(\omega)$ . Then we take the inverse Fourier transform of  $G(\omega)$  to obtain the response  $g(t)$ .

1. Transform  $h(t) \rightarrow H(\omega)$
2. Transform  $u(t) \rightarrow U(\omega)$
3. Compute  $G(\omega) = H(\omega) \cdot U(\omega)$
4. Find  $\mathcal{F}^{-1}\{G(\omega)\} \rightarrow g(t)$

## Example 1

Karris example 8.8: for the linear network shown below, the impulse response is  $h(t) = 3e^{-2t}$ . Use the Fourier transform to compute the response  $y(t)$  when the input

$$u(t) = 2[u_0(t) - u_0(t - 3)].$$

Verify the result with Matlab.

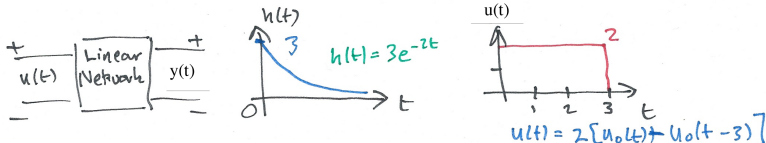


Figure 1: Example 1



# Matlab verification

See ft3\_ex1.m

Result:

```
y = 3*heaviside(t) - 3*heaviside(t - 3) + ...  
3*heaviside(t - 3)*exp(6 - 2*t) ...  
- 3*exp(-2*t)*heaviside(t)
```

Which after gathering terms gives

$$y(t) = 3(1 - 3e^{-2t})u_0(t) - 3(1 - 3e^{-2(t-3)})u_0(t - 3)$$

And here's a plot:

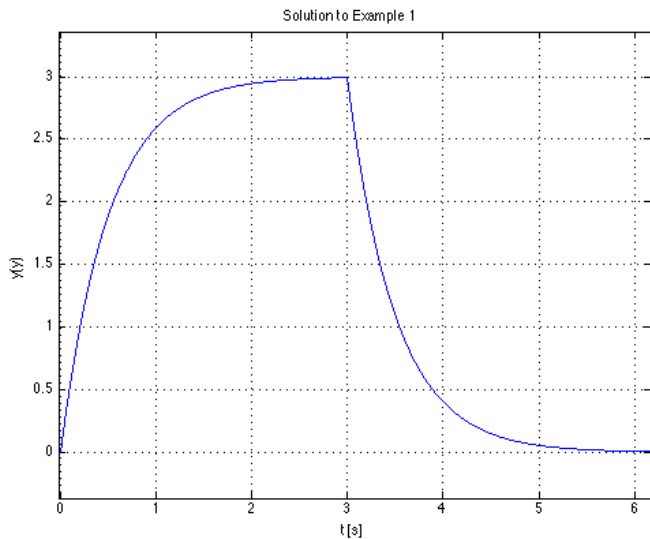


Figure 2: Solution for example 1

## Example 2

Karris example 8.9: for the circuit shown below, use the Fourier transform method, and the system function  $H(\omega)$  to compute  $V_L(t)$ . Assume  $i_L(0^-) = 0$ . Verify the result with Matlab.

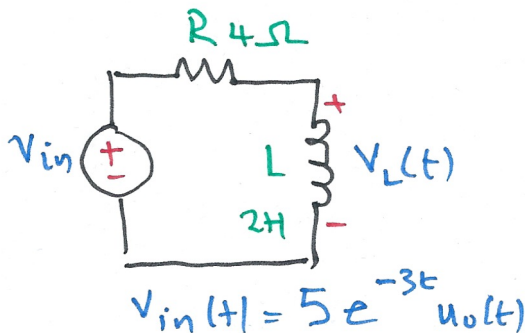


Figure 3:Example 2

# Matlab verification

See ft3\_ex2.m

Result:

$$v_{out} = -5 \exp(-3t) \operatorname{heaviside}(t) * (2 \exp(t) - 3)$$

Which after gathering terms gives

$$v_{out} = 5 \left( 3e^{-3t} - 2e^{-2t} \right) u_0(t)$$

And here's a plot:

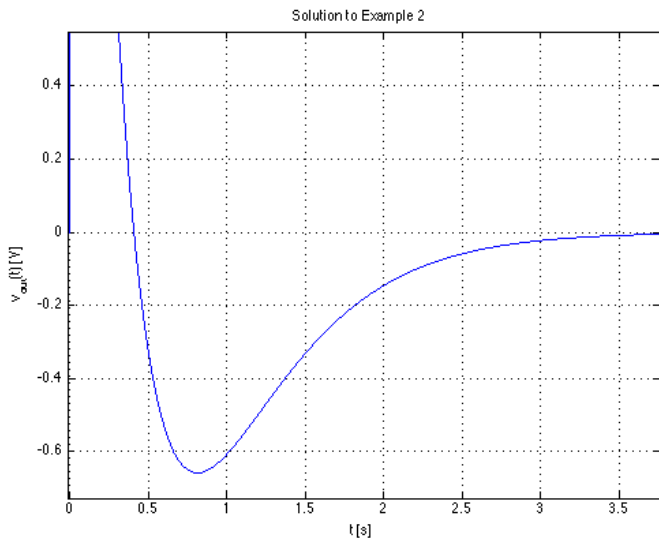


Figure 4: Solution for example 2

## Example 3

Karris example 8.10: for the linear network shown below, the input-output relationship is:

$$\frac{d}{dt}v_{\text{out}} + 4v_{\text{out}} = 10v_{\text{in}}$$

where  $v_{\text{in}} = 3e^{-2t}$ . Use the Fourier transform method, and the system function  $H(\omega)$  to compute the output  $v_{\text{out}}$ . Verify the result with Matlab.

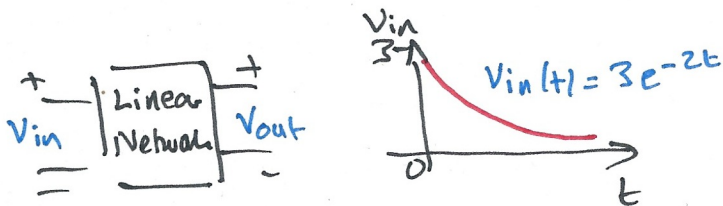


Figure 5:Example 3

# Matlab verification

See ft3\_ex3.m

Result:

$$15 \cdot \exp(-4 \cdot t) \cdot \text{heaviside}(t) \cdot (\exp(2 \cdot t) - 1)$$

Which after gathering terms gives

$$v_{\text{out}}(t) = 15 \left( e^{-2t} - e^{-4t} \right) u_0(t)$$

And here's a plot:

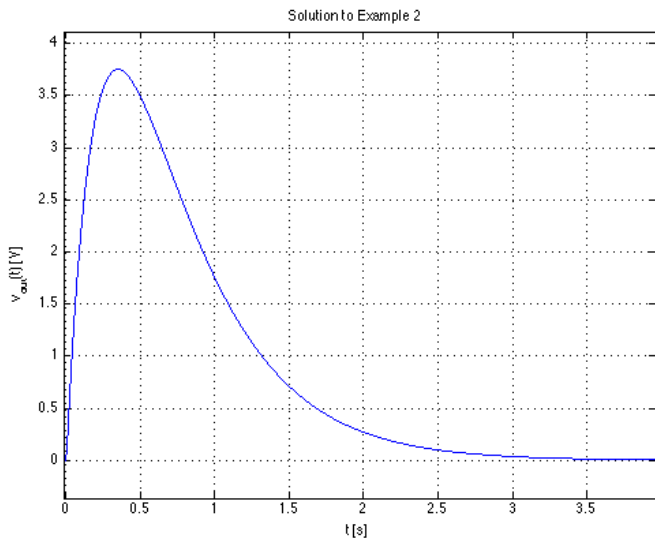


Figure 6: Solution of example 3



## Example 4

Karris example 8.11: the voltage across a  $1\ \Omega$  resistor is known to be  $V_R(t) = 3e^{-2t}u_0(t)$ . Compute the energy dissipated in the resistor for  $0 < t < \infty$ , and verify the result using Parseval's theorem. Verify the result with Matlab.

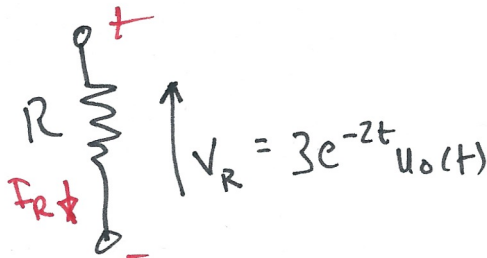


Figure 7:Example 4

Note from tables of integrals

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C.$$

# Matlab verification

See ft3\_ex4.m

Result:

$$Wr = (51607450253003931 \cdot \pi) / 72057594037927936 = 2.25$$

# Homework

Attempt the end of the chapter exercises 7-11 (Section 8.10) from Karris. Don't look at the answers until you have attempted the problems.

# Lab Work

We will verify the results and examine the frequency responses of selected examples from this session.