Transfer Functions

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Office Hours: 12:00-13:00 Mondays

You can view the notes for this presentation in HTML and PDF.

The source code of this presentation is available in Markdown format from GitHub: transfer_functions.md.

The GitHub repository EG-247 Resources also contains the source code for all the Matlab/Simulink examples and the Laboratory Exercises.

This Week

This week's sessions are based on Chapter 4 **Circuit Analysis with Laplace Transforms** from Steven T. Karris Signals and Systems: with MATLAB Computing and Simulink Modelling (5th Edition) [You need University Login to access]

Last Time

Last time we showed how Laplace and Inverse-Laplace Transforms could be used to model circuits and to analyse their responses to simple signals. We also introduced the idea of *complex impedence* and *complex admittance*.

This Time

This time we will examine the idea of a *Transfer Function* which opens up signals and systems analysis to a whole range of additional possibilities including

- block diagram modelling
- numerical simulation
- numerical analysis and the
- fundamental ideas of *Signal Processing*.

Agenda

- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulatiom Using Simulink Transfer Function Block

Transfer Functions for Circuits

When doing circuit analysis with components defined in the complex frequency domain, the ratio of the output voltage $V_{\text{out}}(s)$ ro the input voltage $V_{\text{in}}(s)$ *under zero initial conditions* is of great interest. This ratio is known as the *voltage transfer function* denoted $G_v(s)$:

$$G_v(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

Similarly, the ratio of the output current $I_{\text{out}}(s)$ to the input current $I_{\text{in}}(s)$ *under zero initial conditions*, is called the *cuurent transfer function* denoted $G_i(s)$:

$$G_i(s) = \frac{I_{\text{out}}(s)}{I_{\text{in}}(s)}$$

In practice, the current transfer function is rarely used, so we will use the voltage transfer function denoted:

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

Example 6

Derive an expression for the transfer function $G(s)$ for the circuit below. In this circuit R_g represents the internal resisteance of the applied (voltage) source v_s, and R_L represents the resistance of the load that consists of R_L, L and C.

Sketch of Solution

- Replace $v_s(t)$, R_g, R_L, L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g, R_L, sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\text{out}}(s)$ as a function of $V_s(s)$
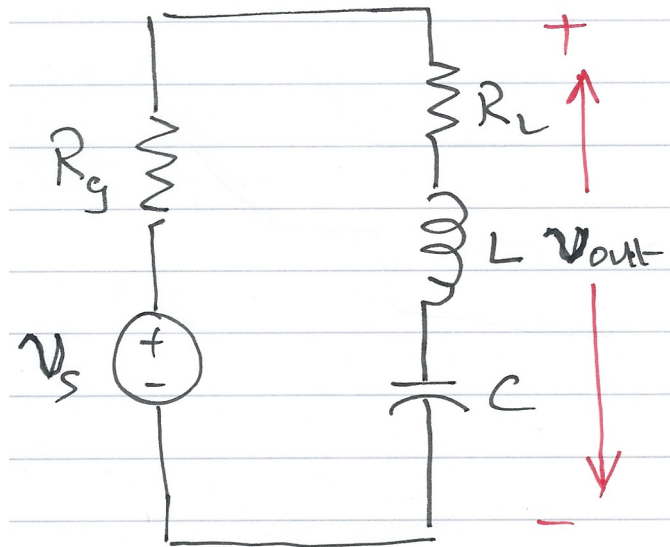- Form $G(s)$ by writing down the ratio $V_{\text{out}}(s)/V_s(s)$

Figure 1: Circuit for Example 6

Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants R_1, R_2, R_3, C_1 and C_2. Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude $|G(s)| = |V_{\text{out}}(s)/V_{\text{in}}(s)|$ versus radian frequency ω.

Sketch of Solution

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3
- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$
- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace s by $j\omega$.
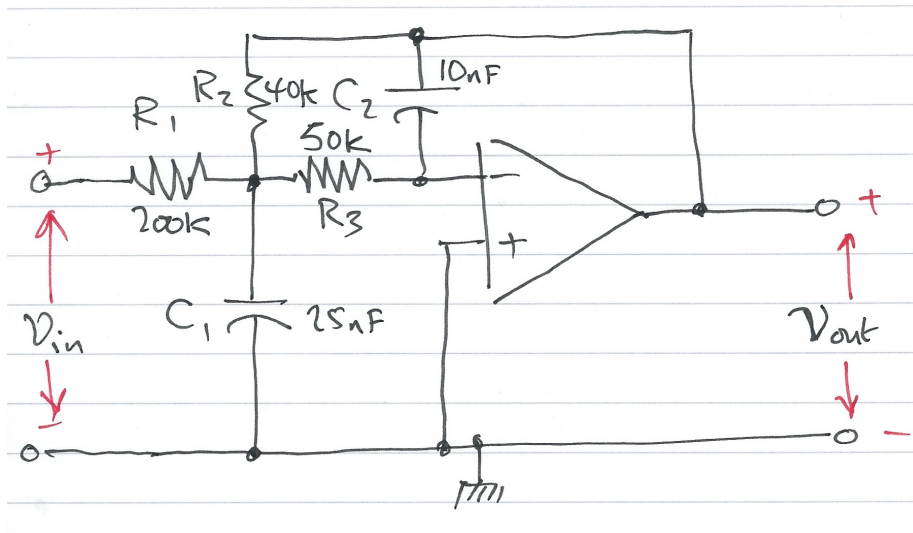- Plot $|G(j\omega)|$ on log-linear "paper"

3

Figure 2: Circuit for Example 7

The Matlab Bit

See attached script: solution7.m.

Using Transfer Functions in Matlab for System Analysis

Please use the file tf_matlab.m to explore the Transfer Function features provide by Matlab. Use the *publish* option to generate a nicely formatted document.

Using Transfer Functions in Simulink for System Simulation

The Simulink transfer function (**Transfer Fcn**) block shown above implements a transfer function representing a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

that it is not specific nor restricted to circuit analysis. It can, however be used in modelling and simulation studies.
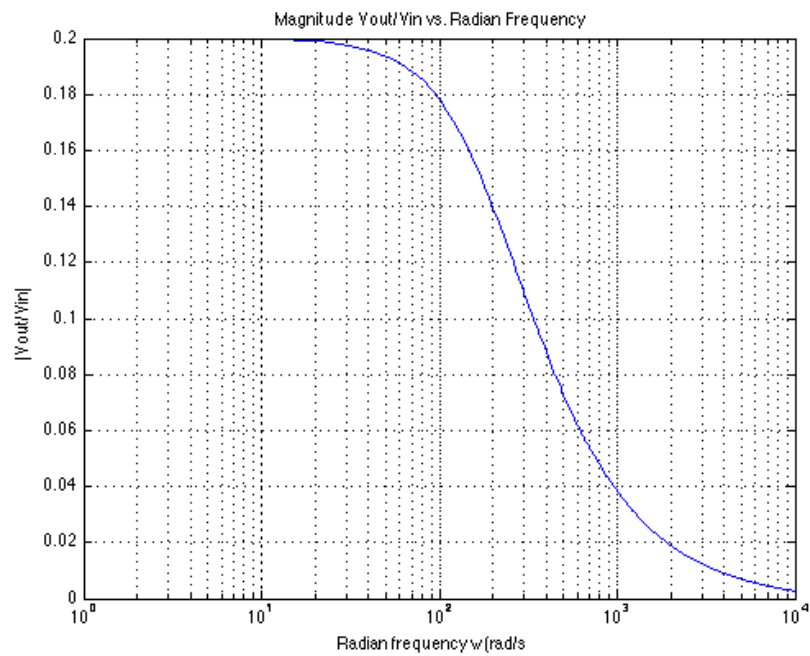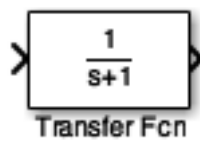
Figure 3: Solution to Example 7

Figure 4: Using Transfer Functions in Simulink for System Simulation

5

Example

Recast Example 7 as a Matlab problem using the LTI Transfer Function block. For simplicity use parameters $R_1 = R_2 = R_3 = 1\ \Omega$, and $C_1 = C_2 = 1$ F. Calculate the step response using the LTI functions.

The Matlab solution: example8.m

Verify the result with Simulink.

Homework

Do the end of the chapter exercises (Questions 5-7 from Section 4.7) from the textbook. Don't look at the answers until you have attempted the problems.

Lab Work

In the lab, a week on Friday, we will get you to play with using Matlab to solve circuit problems, build transfer functions, use transfer functions with the system analysis tools and the Simulink simulation tools. At the end of this exercise, you should be able to do Question 9 from the End of Chapter Problems Section 4.7).