Lesson 2: Elementary Signals

About this presentation

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Office Hours: Mondays 12:00 pm (noon).

You can view the notes for this presentation in HTML and PDF.

The source code of this presentation is available in Markdown format from GitHub: elementary signals.md.

The GitHub repository EG-247 Resources also contains the source code for all the Matlab/Simulink examples.

Review of Homework Problem from Lesson 1

Consider a signal

$$x = f(t) = \begin{cases} 0 : t < -1 \\ t+1 : -1 \le t \le 1 \\ 0 : t > 1 \end{cases}$$

Sketch this signal

Sketch the effect on this signal of applying the following basic signal operations

Amplitude scaling

- 2*f*(*t*)
- 0.5f(t)

Time scaling

- f(2t)
- f(0.5t)

Mirroring

- -f(t)
- f(-t)
- -f(-t)

Time shifting - delay and advance

- f(t-1)
- f(t+1)

Try this

A combination of transformations

• -2f(-t+2)

Note that this involves amplitude scaling, amplitude mirroring, time mirroring, and a time shift. Each operation can be performed in sequence in any order.

Quiz: consider this circuit:

Elementary signals

Unit Step Function

Definition

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

Sketch

Computing/Plotting in Matlab

In Matlab, we use the heaviside function (Named after Oliver Heaviside).

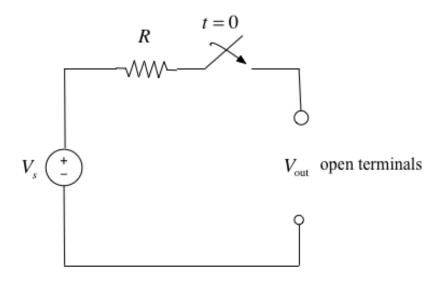


Figure 1: Circuit for quiz

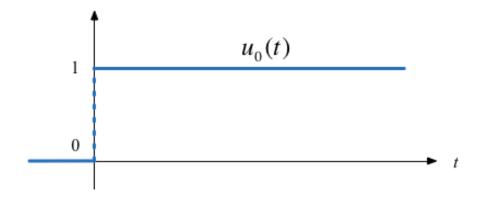


Figure 2: Unit step function

syms t
ezplot(heaviside(t),[-1,1])

See: $heaviside_function.m$

Result

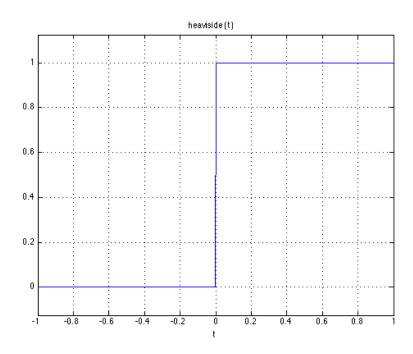


Figure 3: The Heaviside function (unit step)

Note that, so it can be plotted, Matlab defines the *heaviside function* slightly differently from the mathematically ideal unit step:

$$\text{heaviside}(t) = \begin{cases} 0 & t < 0 \\ 1/2 & t = 0 \\ 1 & t > 0 \end{cases}$$

Circuit Revisited

Consider the network shown below, where the switch is closed at time t=T.

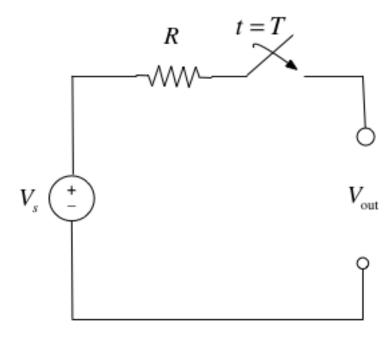


Figure 4: The circuit revisited

Express the output voltage $v_{\rm out}$ as a function of the unit step function, and sketch the appropriate waveform.

Simple Signal Operations

Amplitude Scaling

Sketch $Au_0(t)$ and $-Au_0(t)$

Time Reversal

Sketch $u_0(-t)$

Time Delay and Advance

Sketch $u_0(t-T)$ and $u_0(t+T)$

Example 1

Which of these signals represents $-Au_0(t+T)$?

Example 2

What is represented by

- 1. $-Au_0(t-T)$
- 2. $-Au_0(-t+T)$
- 3. $-Au_0(-t-T)$
- 4. $-Au_0(t-T)$

Synthesis of Signals from Unit Step

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

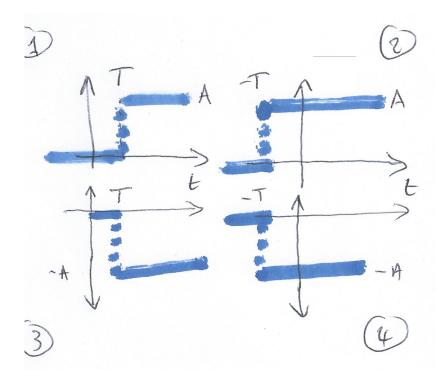


Figure 5: Example 1

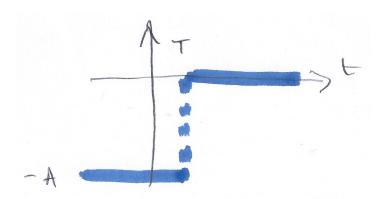


Figure 6: Example 2

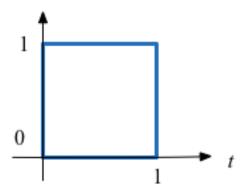


Figure 7: Rectangle function

Synthesize Rectangular Pulse

Synthesize Square Wave

Synthesize Symmetric Rectangular Pulse

Synthesize Symmetric Triangular Pulse

Homework

Show that the waveform shown below

can be represented by the function

$$v(t) = (2t+1)u_0(t) - 2(t-1)u_0(t-1) - tu_0(t-2) + (t-3)u_0(t-3)$$

The Ramp Function

In the circuit shown in the previous slide i_s is a constant current source and the switch is closed at time t=0. Show that the voltage across the capacitor can be represented as

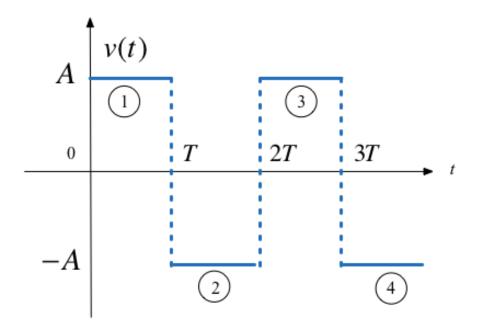


Figure 8: Square wave

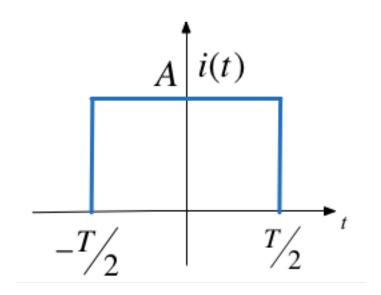


Figure 9: Symmetric rectangular pulse

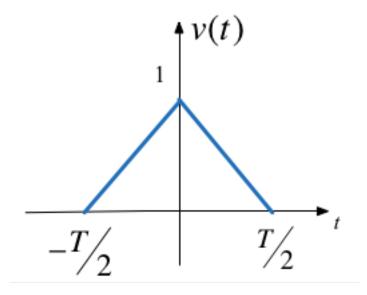


Figure 10: Symmetric triangular pulse

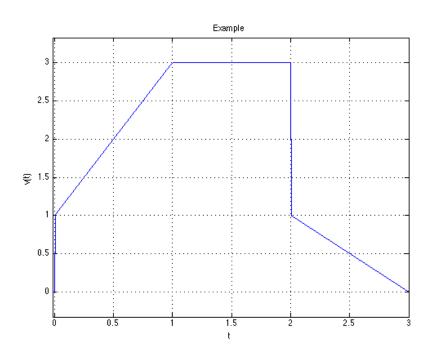


Figure 11: Homework

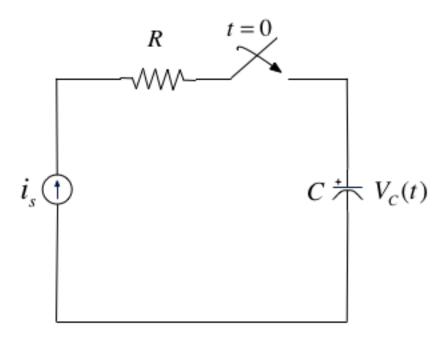


Figure 12: RC circuit

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.

The unit ramp function

The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^t u_0(\tau) d\tau$$

so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \ge 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt}u_1(t)$$

Note

Higher order functions of t can be generated by the repeated integration of the unit step function.

For future reference, you should determine $u_2(t)$, $u_3(t)$ and $u_n(t)$ for yourself and make a note of the general rule:

$$u_{n-1} = \frac{1}{n} \frac{d}{dt} u_n(t)$$

Details are given in equations 1.26-1.29 in the textbook.

The Dirac Delta Function

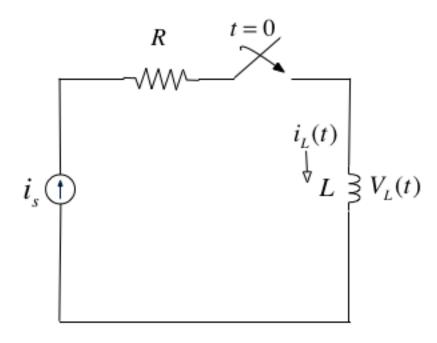


Figure 13: RL circuit

In the circuit shown on the previous slide, the switch is closed at time t = 0 and $i_L(t) = 0$ for t < 0. Express the inductor current $i_L(t)$ in terms of the unit step function and hence derive an expression for $v_L(t)$.

Note

To solve this problem we need to invent a function that represents the derivative of the unit step function. This function is called $\delta(t)$ or the *dirac delta* function (named after Paul Dirac).

The delta function

The unit impulse or the delta function, denoted as $\delta(t)$, is the derivative of the unit step.

This function is tricky because $u_0(t)$ is discontinuous at t=0 but it must have the properties

$$\int_{-\infty}^{t} \delta(\tau) d\tau = u_0(t)$$

and

 $\delta(t) = 0$ for all $t \neq 0$.

Sketch of the delta function



Figure 14: The delta function

Important properties of the delta function

Sampling Property

The sampling property of the delta function states that

$$f(t)\delta(t-a) = f(a)\delta(t-a)$$

or, when a = 0,

$$f(t)\delta(t) = f(0)\delta(t)$$

Multiplication of any function f(t) by the delta function $\delta(t)$ results in sampling the function at the time instants for which the delta function is not zero.

The study of descrete-time (sampled) systems is based on this property.

You should work through the proof for youself.

Sifting Property

The sifting property of the delta function states that

$$\int_{-\infty}^{\infty} f(t)\delta(t-\alpha)dt = f(\alpha)$$

That is, if multiply any function f(t) by $\delta(t-\alpha)$, and integrate from $-\infty$ to $+\infty$, we will get the value of f(t) evaluated at $t=\alpha$.

You should also work through the proof for yourself.

Higher Order Delta Fuctions

the nth-order delta function is defined as the nth derivative of $u_0(t)$, that is

$$\delta^n(t) = \frac{d^n}{dt^n} [u_0(t)]$$

The function $\delta'(t)$ is called the *doublet*, $\delta''(t)$ is called the *triplet* and so on.

By a procedure similar to the derivation of the sampling property we can show that

$$f(t)\delta'(t-a) = f(a)\delta'(t-a) - f'(t)\delta(t-a)$$

Also, derivation of the sifting property can be extended to show that

$$\int_{-\infty}^{\infty} f(t)\delta^{n}(t-\alpha)dt = (-1)^{n} \frac{d^{n}}{dt^{n}} [f(t)] \bigg|_{t=\alpha}$$

Examples

Example 3

Evaluate the following expressions

$$3t^4\delta(t-1)$$

2.
$$\int_{-\infty}^{\infty} t \delta(t-2) dt$$

3.
$$t^2\delta'(t-3)$$

Example 4

- 1. Express the voltage waveform v(t) shown above as sum of unit step functions for the time interval -1 < t < 7 s
- 2. Using the result of part (1), compute the derivative of v(t) and sketch it's waveform.

Self-study

Do the end of the chapter exercises (Section 1.7) from the textbook. Don't look at the answers until you have attempted the problems.

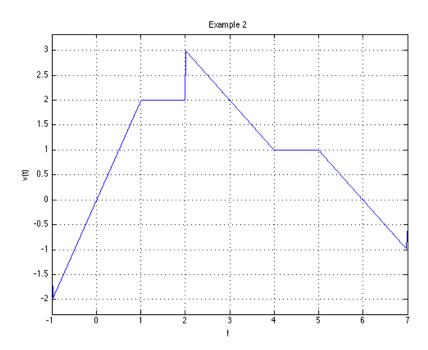


Figure 15: Example 4

Lab Work

In the lab, a week on Friday, we will solve Example 2 using Matlab/Simulink following the procedure given between pages 1-17 and 1-22 of the textbook. We will also explore the heaviside and dirac functions.