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Emergence of cooperation and organization in an evolutionary game

D. Challet, Y.-C. Zhang *

Institut de Physique Théorique, Pérolles, Université de Fribourg, 1700 Fribourg, Switzerland

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Abstract

A binary game is introduced and analysed. N players have to choose one of the two sides independently and those on the minority side win. Players use a finite set of ad hoc strategies to make their decision, based on the past record. The analysing power is limited and can adapt when necessary. Interesting cooperation and competition patterns of the society seem to arise and to be responsive to the payoff function.

Keywords: Evolution; Game; Emergence of organization

Most current economics theories are deductive in origin. One assumes that each participant knows what is best for him given that all other participants are equally intelligent in choosing their best actions. However, it is recently realised that in the real world the actual players do not have the perfect foresight and hindsight, most often their actions are based on trial-and-error inductive thinking, rather than the deductive rationale assuming that there are underlying first principles. Whether deductive or inductive thinking is more relevant is still under debate [1].

Evolutionary games have also been studied within the standard framework of game theory [2]. However, it has been recently pointed out that the approach traditionally used in economics is not convenient to generalise to include irrationality, and an alternative Langevin-type equation is proposed [3]. As physicists, we would like to view a game with a large number of players, i.e. a statistical system, we need to explore new approaches in which the emerging collective phenomena can be better appreciated. One recent approach using bounded rationality is particularly inspiring, put forward by B. Arthur in his *El Farol* bar problem [4]. Following a similar philosophy, in this work we propose and study a simple evolutionary game.

* Corresponding author.

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Table 1

Signal	Prediction
000	1
001	0
010	0
011	1
100	1
101	0
110	1
111	0

Let us consider a population of N (odd) players, each has some finite number of strategies S. At each time step, everybody has to choose to be in side A or side B. The payoff of the game is to declare that after everybody has chosen side independently, those who are in the minority side win. In the simpliest version, all winners collect a point. The players make decisions based on the common knowledge of the past record. We further limit the record to contain only yes and no, e.g. side A is the winning side or not, without the actual attendance number. Thus, the system's signal can be represented by a binary sequence, meaning A is the winning side (1) or not (0).

Let us assume that our players are quite limited in their analysing power, they can only retain last M bits of the system's signal and make their next decision basing only on these M bits. Each player has a finite set of strategies. A strategy is defined to be the next action (to be in A or B) given a specific signal's M bits. An example of a strategy is illustrated in Table 1 for $M = 3$.

There are 8 ($= 2^M$) bits we can assign to the right side, each configuration corresponds to a distinct strategy, this makes the total number of strategies to be $2^{2^M} = 256$. This is indeed a fast increasing number, for $M = 2, 3, 4, 5$ it is 16, 156, 65 536, $65\,536^2$. We randomly draw S strategies for each player, and some strategies maybe by chance-shared. However, for moderately large M, the chance of repetition of a single strategy is exceedingly small. Another special case is to have all 1's (or 0's) on the RHS of the table, corresponding to the fixed strategy of stay at one side no matter what happens.

Let us analyse the structure of this minority game to see what to expect. Consider the extreme case where only one player takes a side, all the others take the other side. The lucky player gets a reward point, nothing for the others. Equally extreme example is that when $(N-1)/2$ players at one side, $(N+1)/2$ at the other. From the society point of view, the second situation is preferable since the whole population gets $(N-1)/2$ points, whereas in the first example, only one point – a huge waste. Perfect coordination and timing would approach the second, disaster would be the first example. In general, we expect the population to behave between the above two extremes.

This binary game can be easily simulated for a large population of players. Initially, each player draws randomly one out of his S strategies and uses it to predict the next step; an artificial signal of M bits is also given. All the S strategies in a players's bag collect points depending if they would win or not given the M past bits, and the actual outcome of the next play. However, these points are only *virtual* points as they record the merit of a strategy as if it were used each time. The player uses the strategy having the highest accumulated points (capital) for his action; he gets a real point only if the strategy used happens to win in the next play.

In Fig. 1 we plot the actual number of attendance at side A, for a population of 1001 players, having various brain size (i.e. M bits). As one may expect, the temporal signal indeed fluctuates around the 50%. Whoever takes side A wins a point at a given time step when the signal is below 501. The precise number is not known to the players, they only know if a side is winning or not, after their bet is made. Note that large fluctuations imply large waste since still more players could have taken the winning side without harm done to the others. On the other hand, smaller fluctuations imply more efficient usage of available resources, in general this would require coordination and cooperation – which are not built-in explicitly. We see that the population having larger brains (i.e. M larger) cope with each other better: the fluctuation is indeed in decreasing order for ever increasingly "intelligent" players (i.e. $M = 6, 8, 10$). Remarkable is that each player is by definition selfish, not considerate to fellow players, yet somehow they manage to somewhat share the limited available resources.

Let us remark that the very simplest strategy by playing randomly is not included here, for generating random numbers more bits are needed. In a perfect timing, the average gain in the population would be 1/2 per play. Waste is proportional to fluctuation's amplitude, hence the average gain is always below 1/2 in reality. Since the game is symmetrical in A and B, one may be tempted to use the simple strategy to stay at A or B, hoping to get exactly 1/2 gain. Let us mention if this strategy indeed rewards 1/2 gain on average, many would imitate. Suppose that there is a group sitting at A no matter what signal is shown (this is included in the strategy space). The active players will soon recognise that they win less often choosing A than B. In fact, for them the game is no longer symmetrical and they will adopt accordingly so that the apparent advantage disappears for those sitting at one side fixed. This is similar to the arbitrage opportunities in finance: any obvious advantage will be arbitraged away – no easy "risk-free way" to make a living both for our players and those in the real world.

The advantage of the larger brain sizes over the smaller ones can be better appreciated inspecting Fig. 2 . Identical parameters ($N = 1001$, $S = 5$) for a mixed population having $M = 1, \ldots, 10$. We thus force unequally equiped players to play together. One may fear that the "poorly" brained players may get exploited by the more powerfully brained ones; indeed this is the case. We plot the average gain per time step after a long time. We see that within a sub-population (same M) there are better and worse performers. We have noticed that better players do not necessarily stay that way for a long time, but exceptions exist. For $M = 1$, there appears fewer points, since there are more degeneracies. As a group the more intelligent players gain more and the spread between the rich and the poor is smaller, even though the in-fighting among them is more intensified. Note that above a certain size ($M \approx 6$) the average performance

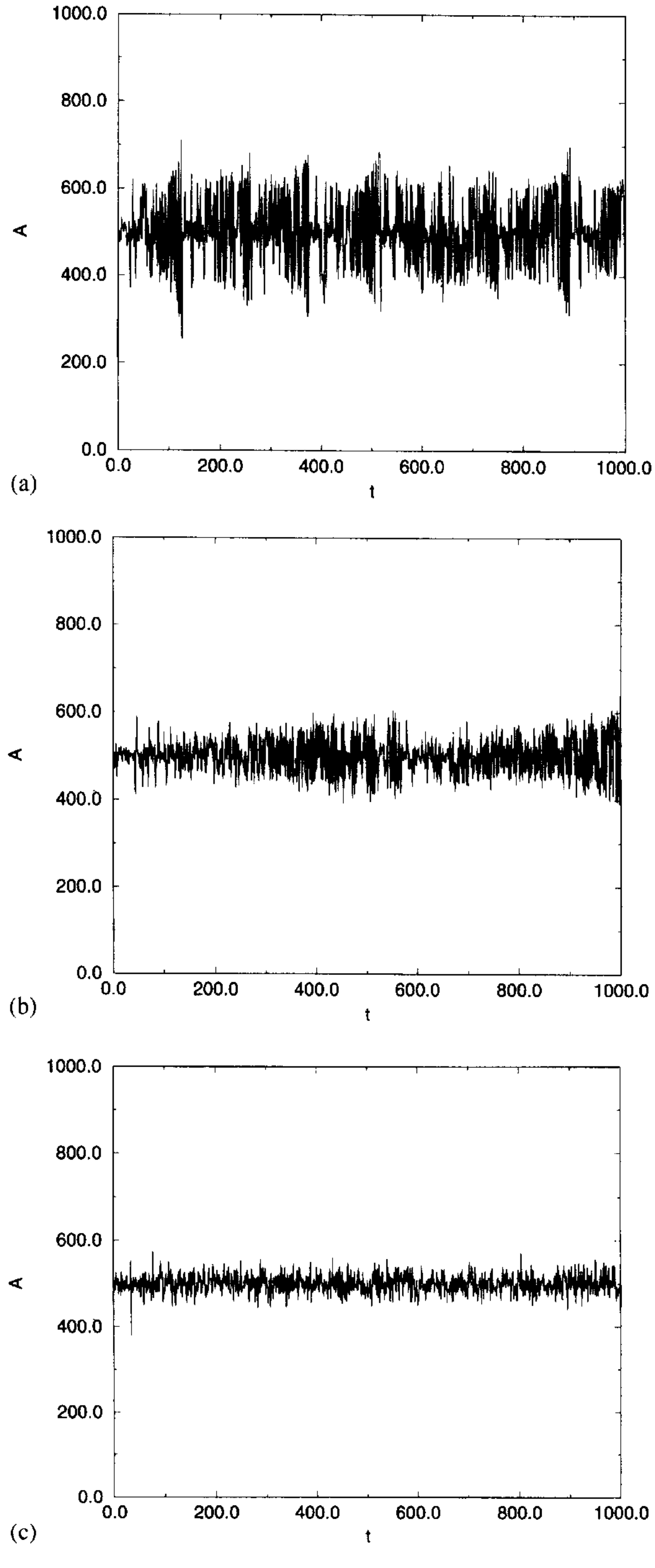

Fig. 1. Actual number of attendance at the side A against time, for a population of 1001 players, having brain size of (a) 6, (b) 8 and (c) 10 bits.

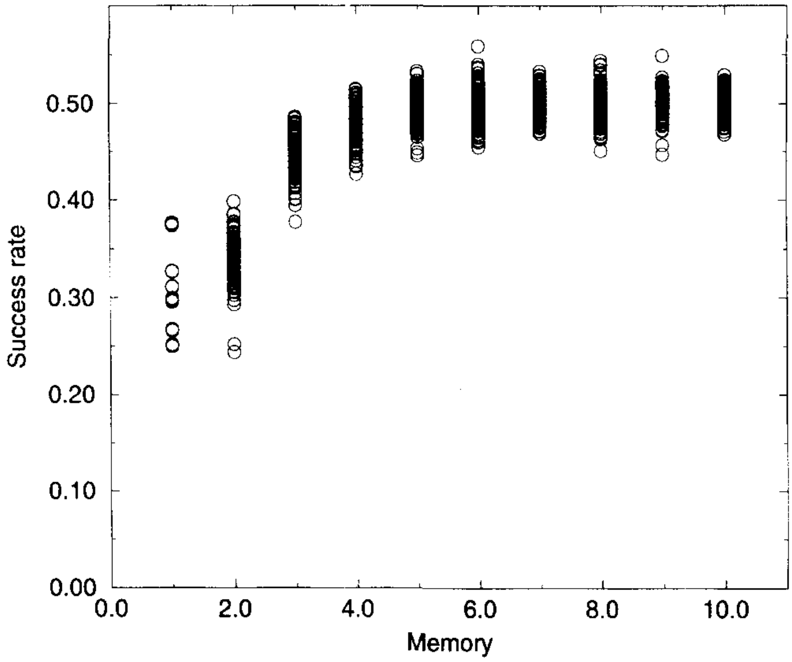

Fig. 2. Success rate of a mixed population players against their memory ($N = 1001$, $S = 5$).

of a population appears to saturate, further increasing the brain size does not seem to improve further. This is due to the simple structure of this version of the game, there is nothing more to gain. Recall that only most crude information is transmitted to the players, i.e. only yes and no, not the exact attendance number. More precise information would necessitate more analysing power, more complicated payoff functions and games also provides incentives to develop more sophisticated brains. However, in the present work, we stick to the binary functions and will report more complicated applications using neural networks elsewhere.

Of course, the game is symmetrical for A and B. This can be observed in Fig. 3 , where the histograph shows the attendance of A (hence B is the mirror image at the point $N = 501$). B. Arthur's *El Farol* problem uses 60% rule and does not give rise to new questions, and results appear to be similar.

One may argue that our payoff function is too simple, i.e. a step function without differentiating a "good" minority from a "bad" one. Let us consider the payoff function $N/x-2$, i.e. these many (nearest integer values) points awarded to every player choosing the minority side, the number of winning players being $x < N/2$. Clearly, this structure favours smaller minority. This is like in lottery you would like to be on the winning side, but even better you are alone there. The players thus face an extra type of competition, a winner would prefer less fellow winners in company. If, for instance, a player wins on a mediocre play, his winning strategies are hardly enhanced with respect to not winning at all. Globally, the population ($N = 1001$, $M = 4$) responds to having a histograph (Fig. 4) with two peaks. Although the jackpot (winning alone) is very appealing, this is very unlikely to happen since the fellow players are just as intelligent. The players need a sizeable gain to get motivation to win. There appears to

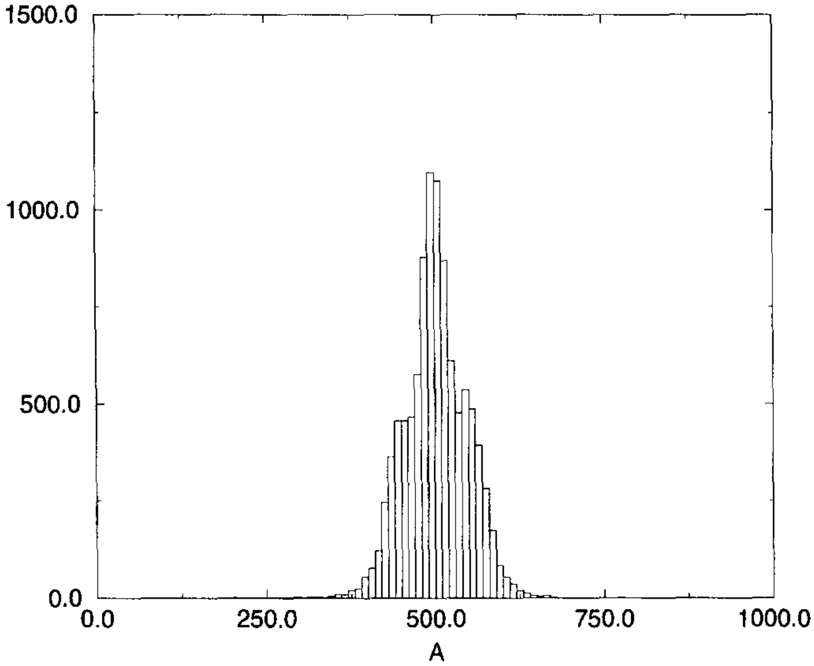

Fig. 3. Histograph of the attendance of A ($N = 1001$, $M = 8$, $S = 5$).

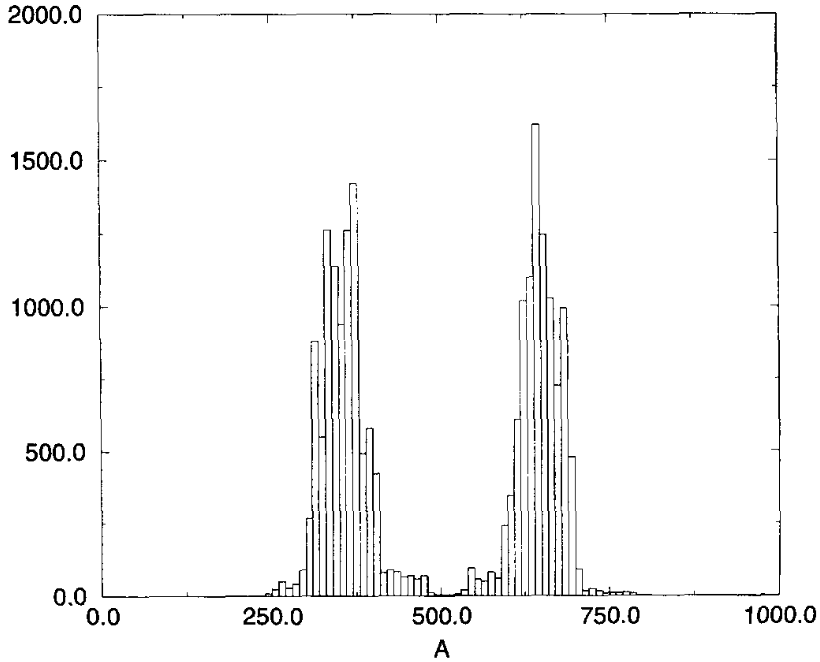

Fig. 4. Histograph of the attendance of A for a $N/x - 2$ payoff ($N = 1001$, $M = 4$, $S = 5$).

be a compromise that they effectively (not through any enforceable agreement) agree to show up on the minority side a smaller number of players. What is remarkable here is that entropy, i.e. the most likely configuration, does not favour the distribution in Fig. 4. The players manage to defy entropy; in other words, to get themselves organised to occupy less unlikely configurations.

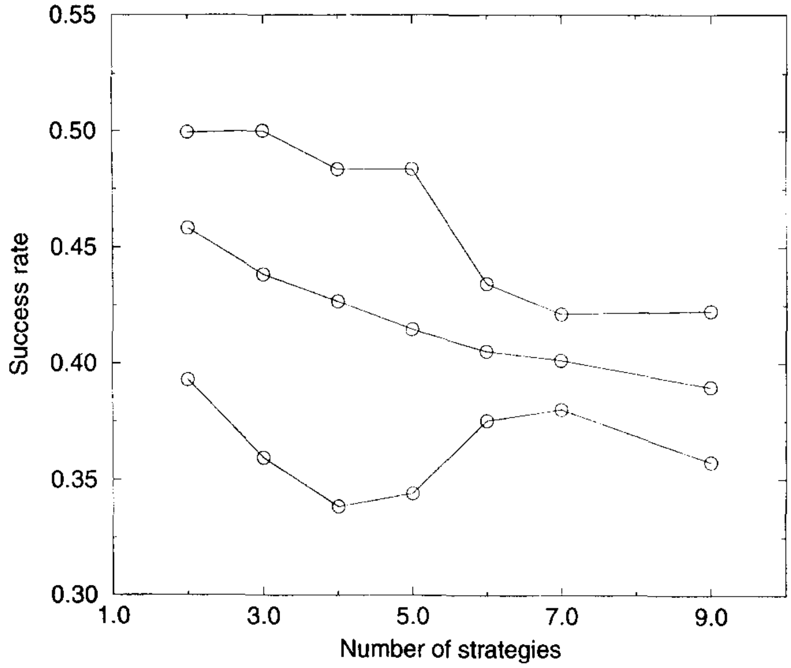

Fig. 5. Success rate against the number of strategies ($N = 1001$, $M = 5$).

One may enquire what happens if the players are provided with a bigger "idea bag" with more alternative strategies. In Fig. 5 we show the results for various populations ($N = 1001$, $M = 5$) with $S = 2, 3, \ldots, 9$. We see that, in general, with increasing number of alternatives the players tend to perform worse. What happens is that the players tend to switch strategies often and are more likely to get "confused", i.e. some outperforming strategy may distract the player's attention, after being chosen turns out to be underperforming. We recognise this has also to do with the observation time, currently a player switches immediately if another strategy has one virtual point more than that in use. If a higher threshold is set, then the hinderance by increasing the number of alternatives can be in part avoided. In the neural network version of our game, just one network (with adjustable weights) is given to a player. Let us recall that in a recent study, Borkar et al. [5] have proven that in an evolutionary game players tend to specialise in a single strategy, even though alternatives exist.

In Fig. 6 we plot the switching rate against the success rate for various populations. The general tendency that the oftener one switches, less successfull one would end up. The phase space seems to be highly fragmented and many substructures appear, this having to do with the binary nature of our game.

It is also instructive to follow the performance record. In Fig. 7, we select 3 top players, 3 bottom players and 3 randomly chosen players. They are chosen at the last time step and we trace back their past record. Their capital gains are scaled such that the average gain (over the population) appears in an almost horizontal line. We see that the general tendency for best and worst players are rather consistent even though setbacks for the best and bursts for the worst do occur. Notice that the gap between the

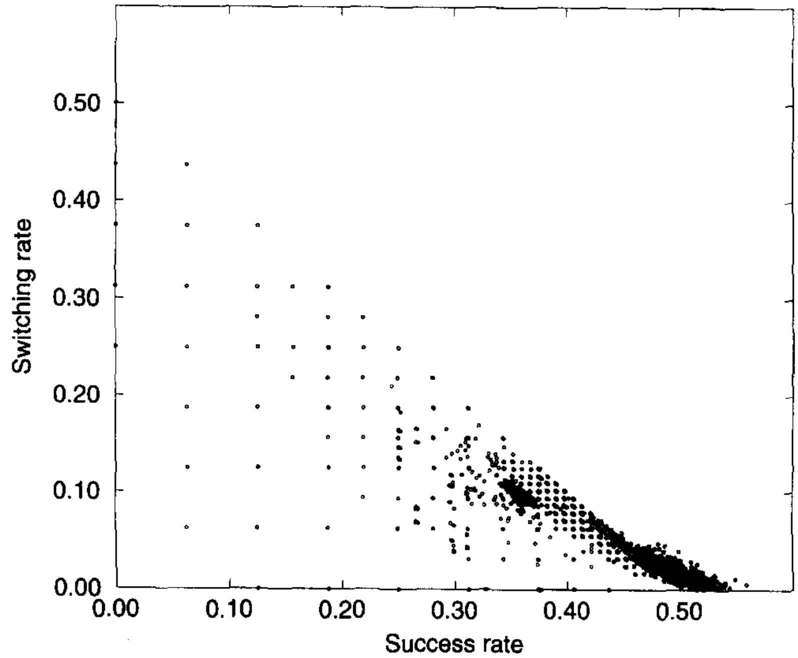

Fig. 6. Switching rate against the success rate for various populations.

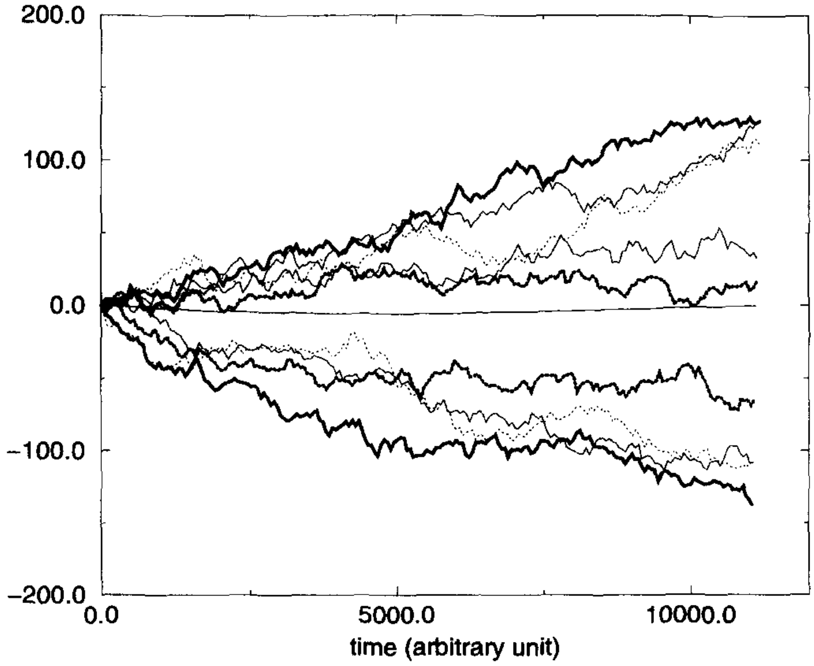

Fig. 7. Performance record of the 3 best, the 3 worst and 3 randomly chosen players ($N = 1001$, $M = 10$, $S = 5$).

rich and the poor appears to increase linearly with time, though reversion is possible but the poor players in general are doomed to stay poor.

Another result enhances this conclusion: one may blame bad players for their bad strategies. In order to check whether there are really good and bad strategies, we plot the virtual gains of all the strategies in the population. In Fig. 8 we see three different

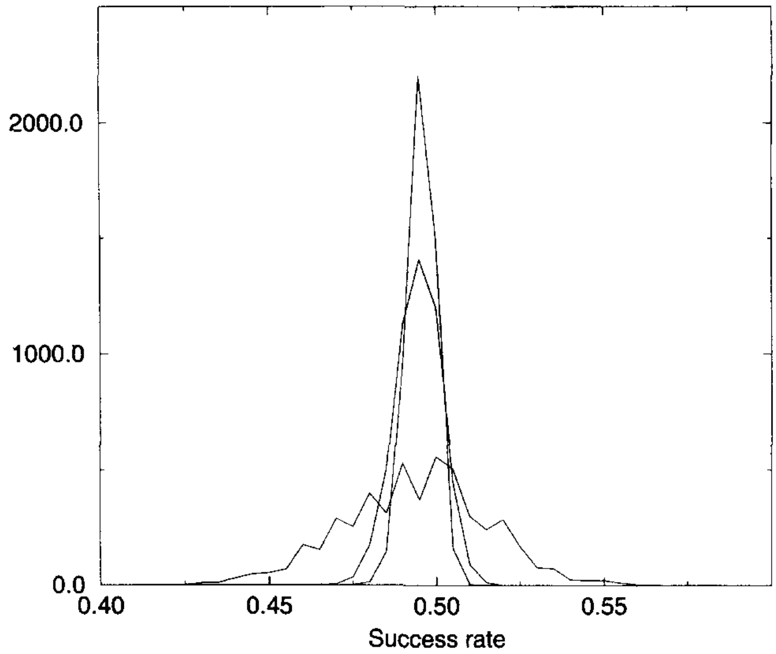

Fig. 8. Different distributions of the average value of all the strategies with increasing iterations numbers (1000, 5000 and 10 000), showing that all strategies are equivalent in the $t \to \infty$ limit.

distributions of the average (time) gains. The longer the time the more concentrate is the distribution, indicating that the relative values of the strategies are about the same. Indeed, it can be analytically shown that all the strategies are equivalent to each other, since our game is symmetrical in A and B. So the bad players are bad because they have used the strategies inopportunely and are unlucky, also their specific composition is to blame. Note that a player is only distinguished from others by this composition, if two players have the same composition, they are clone sisters. In that case, initial conditions can still set them apart and they may know different fortunes only in the beginning.

The above discussion calls for a genetic approach in which the poor players are regulary weeded out from the game and new players are introduced to replace the eliminated ones. Let us consider our minority game generalised to include the Darwinist selection: the worst player is replaced by a new one after a finite time steps, the new player is a clone of the best player, i.e. it inherits all the strategies but with corresponding virtual capitals reset to zero. This is analogous to a new born baby, though having all the predispositions from the parents, it does not inherit their knowledge.

To keep a certain diversity, we introduce mutation possibility in cloning. We allow one of the strategies of the best player to be replaced by a new one. Since strategies are not just recycled among the players any more, the whole strategy phase space is available for selection. We expect this population is capable of "learning" since self-destructive, obviously bad players are weeded out with time, fighting is among so-to-speak the best players. Indeed, in Fig. 9 we observe that the learning has emerged in time. Fluctuations are reduced and saturated, this implies the average gain for everybody is improved but never reaches the ideal limit. What would happen if no mutation

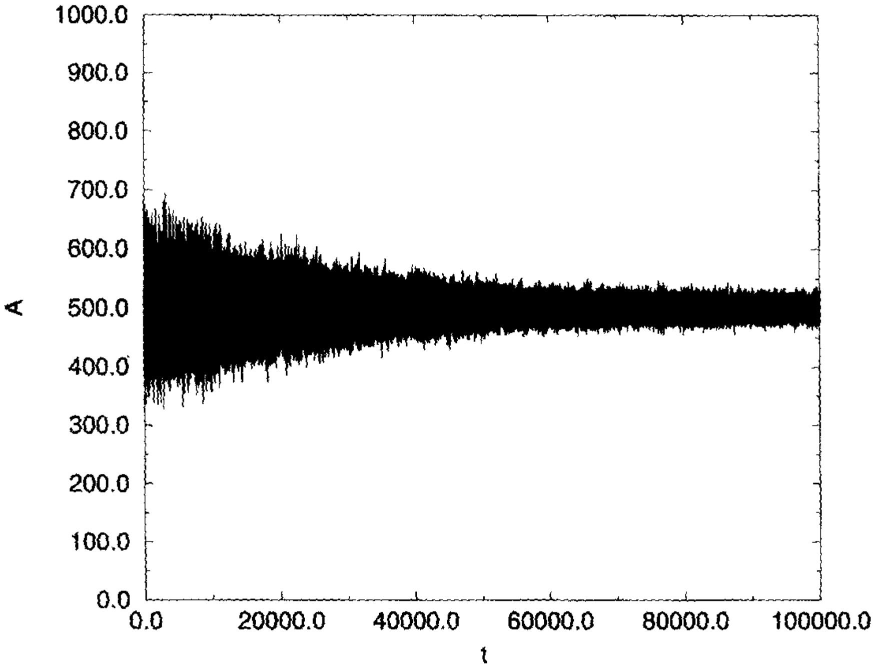

Fig. 9. Temporal attendance of A for the genetic approach showing a learning process.

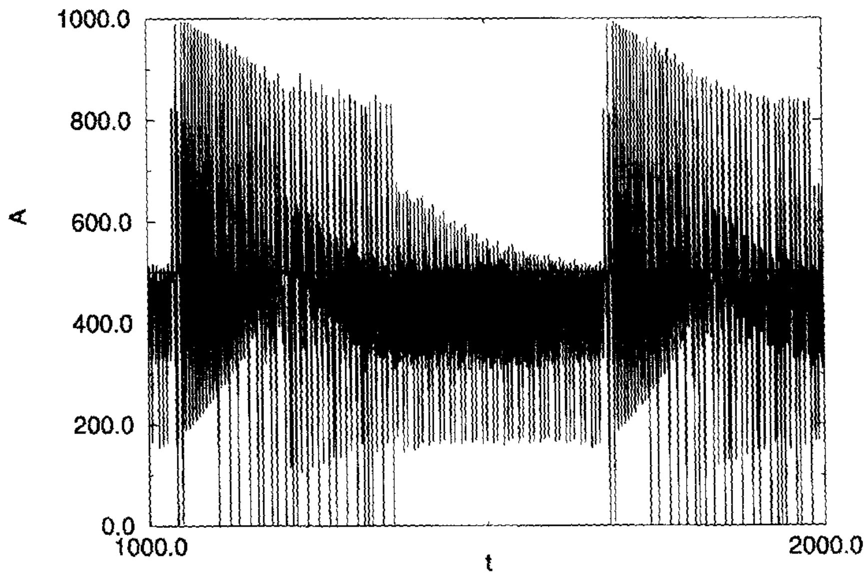

Fig. 10. Temporal attendance of A of an "pure" population.

is allowed and cloning is perfect? Eventually, population is full of the clone copies of the best player, each may still differ in their decision since the virtual capitals in their idea-bag can be different. In Fig. 10 we plot the performance of such a "pure" population; there appears tremendous waste and all strange things go loose. Indeed, the results from inbreeding look rather incestous.

As a last experiment we start the population very "simple-minded", say $M = 2$. We allow in the cloning process mentioned above an additional feature that a bit of memory can be added or subtracted for the cloned new player, with a small probability. We

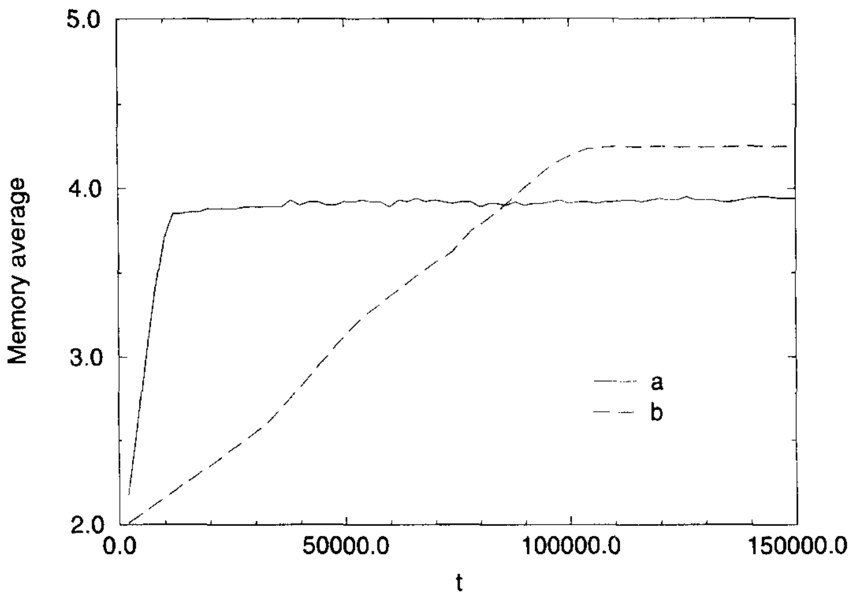

Fig. 11. Temporal record of the memory average starting from $M = 2$ population for $N = 101$ (a) and $N = 1001$ (b) ($S = 5$).

want to be sure that the rules are such that this structural mutation is strictly neutral, i.e. does not favour bigger brains over the smaller ones; we leave this to the invisible hand of evolution to decide. Indeed, something remarkable takes place: in Fig. 11 we plot the average brain size in the population started with $M = 2$, for population of $N = 101$ and $N = 1001$. The temporal record shows that there is an "arm race" among the players. We know by now that the more brain power leads to advantage, so in the evolution of survival-of-the-fittest the players develop bigger brains to cope with ever-aggressive fellow players. However, such an evolution appears to saturate and the "arm race" to settle at a given level. The saturation values are not universal, having to do with the time intervals of reproduction. In general, the larger brains need longer time to learn. Larger population ($N = 1001$) needs more powerful brains to sustain the apparent equilibrium than the smaller population ($N = 1001$), also the learning rate (the slope in Fig. 11) is smaller. We mention *en passant* that population's brain sizes do not concentrate on one value, only the average value is plotted. Some players manage to make do quite happily with a relatively small brain.

To conclude, what can we learn from these simple numerical experiments? First of all, the economical behaviour in the real-world seems to call for a general approach to systematically study the evolutionary nature of games. There are few most relevant questions to address: (1) Given each agent's selfishness what is his cooperative and cognitive skills in the course of competition? (2) What is the emerging collective behaviour that is the society's performance without an enforceable authority? (3) How can our *visible* hand modify the rules of the game (payoff functions) such that the global response may appear more cooperative? (4) How does evolution puts its *invisible* hand to work? Clearly, our study is far from answering all these. What we have presented in this work is not just an oversimplied model, but a general approach to ask the right questions. This approach, as the reader can readily convince himself, is very open to all sorts of variations. It is easy to include other situation-motivated payoff functions and game structures, there are qualitatively new questions to be asked when more realistic games are studied. It is a theoretical physicist's dream to have an Ising-type model, though oversimplified, and yet to capture some essential points of the real world. Our minority game may be indeed the simplest of the kind.

Our model is by design without fundamentals and insider information. Players are forced to fight each other. With the Darwinism included, everyone has to keep improving in order to survive – the *red-queen* effect. Unlike some examples in standard game theory, there is no commonly accepted optimal strategy (analogous to physical systems without obvious ground states). A rational approach is helpless here. Yet the emerging society appears to have a certain organisation. Even though the players care only their own gain, cooperation and timing does seem to spontaneously arise. Note that our learning mechanism is different from the traditional neural network studies, where a *pre-assigned* task like a pattern is given and performance is measured on how precisely the original is restored. Here the task is self-appointed and no ending is defined.

We may even speak of the emergence of intelligence. If the analysing power of the players can adapt to the increasingly challenging task (survival amongst ever-aggressive fellow players and larger number of players), the population seems to evolve to more equipped, larger brains appear to dominate and available resources are better explored, i.e. less fluctuation and waste in the attendance number. This is not unsimilar to the study of the prebiotic evolution: in the promodial soup only very simple organisms exist. Evolution allows these organisms to add one new feature (and reduce an existing one) from time to time. More complex organisms cope with the survival task better, on average, and more and more refined organisms spontaneously appear out of the monotonous soup [6].

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