

Evolutionary minority games with small-world interactions

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Abstract

In the Evolutionary Minority Game (EMG), agents compete for a limited resource and are rewarded if they correctly select the minority behaviour. At each time step, agents make their decision based on the aggregate history of past moves and an internal parameter — the probability that the individual follows the given strategy. In this study, the effects of strategic imitation among agents are examined. Here, I combine and extend previous work using local information transmission mechanisms to promote coordination in the population. Extensive numerical simulations using different network architectures, ranging from regular lattices to random networks, are used to investigate the population dynamics. The results suggest that agents sharing information in small-world networks can coordinate their behaviour more effectively than agents playing the standard EMG. However, both the network re-wiring probability and level of imitation significantly impact on performance.

1. Introduction

Agent-based models of complex adaptive systems provide valuable insights into the emergent properties and large-scale effects of locally interacting components (Holland, 1995). One model that combines the general properties of such systems is the Minority Game (MG) (Challet and Zhang, 1997) inspired by the El Farol bar-attendance problem (Arthur, 1994). In the MG, a population of agents with bounded rationality compete for a limited resource. The traditional game involves N players (agents) forced to make a binary decision: 0 (eg. go to the bar, or take route A) or 1 (eg. do not go to the bar, or take route B). The agents share a common “memory” or look-up table, containing the outcomes from the m most recent occurrences. The resulting 2^m possible histories are then used to generate strategies for making an appropriate binary choice. At each time step, an agent receives one point if their decision places them in the minority and loses a point otherwise. The game evolves as the agents modify their behaviours (or strategies) based on the previous experiences.

Johnson and co-workers (1999) introduced an extension to this basic game—the *Evolutionary Minority Game* (EMG)—where every agent employs the same strategy, based on the m most recent occurrences. However, the differentiating factor between the agents is a gene parameter p characterising the probability that the agent takes an action based on the prediction of the strategy. That is, the probability p is the chance that an agent decides to

follow the strategy's prediction, and $1 - p$ is the chance that the agent decides to act opposite to the current trend. If an agent's utility (number of successes) falls below some threshold d , their p -value is mutated. In this sense, each agent tries to learn from past mistakes and modifies their strategy in order to survive.

Typically, in the EMG, the agents do not have direct interactions. As such, the models are really mean-field descriptions. In contrast, in many real-world scenarios agents can combine local information, accessed via dialogues with their peers or local consultant, with public information in order to make decisions. Subsequently, some researchers have studied variations of both the MG and EMG with local interactions (Moelbert and De Los Rios, 2002; Chau et al., 2003; Quan et al., 2003; Burgos et al., 2004; Caridi and Ceva, 2004; Namatame and Sato, 2004). The results reported indicate that local communication within the agent population may improve the efficiency of the systems. It should be noted, however, that a detailed study of how coordination in the EMG is related to the underlying communication topology of agents playing the game is lacking.

In this study, local information transmission mechanisms are extended and combined with recent studies in complex network theory in order to investigate the population dynamics of the EMG. Here, the agents playing the EMG are mapped to the nodes of a small-world network (Watts and Stogatz, 1998). The fundamental rules of the EMG have not changed. However, when an agent's utility falls below the threshold d , the agent basically "starts again". That is, its utility is set to zero and the agent is forced to modify its p -value. The new gene is a mutated copy of the p -value of the local neighbour with the highest utility. The rationale behind this approach is based on the fact that network topology significantly influences the dynamical behaviour in ecological and social networks (Dorogovtsev and Mendes, 2003; Watts and Stogatz, 1998; Watts, 1999). It is to be expected that the local information available to the agents differs across the network. This in turn will allow for the formation of alternative clusters of like individuals and consequently a reduced fluctuation in the number of agents correctly selecting the minority group.

2. Background

2.1 EMG description

A simple example illustrating the basic functionality of the EMG was given by Johnson et al., (1999). Consider the following look-up table $(xyz)w$, containing the outcomes from the $m=3$ most recent occurrences. Here, the bit string (xyz) represents the corresponding sequence and the outcome w . An example memory would comprise (000)1, (001)0, (010)0, (011)1, (100)0, (101)1, (110)0, (111)1. In this scenario, a sequence of three 0s in the past was followed by a 1. Therefore, the look-up table available to all agents suggests that the agent selects 1. The agent follows the trend (prediction) with probability p , and makes the opposite prediction with probability $1 - p$. If the result from the next iteration of the game turned out to be 0, the look-up table entry (000)1 would be changed to (000)0. Agents who win (select the minority group) gain a point on their utility, whereas the others lose a point.

In the standard version of the EMG, when the utility of an agent drops below a certain threshold a new p -value is assigned to the agent. The new value is a randomly generated value within a range R centred on the current gene value Johnson et al. (1999). A reflective boundary condition is imposed in the p -space so that $|R| \leq 2$. The magnitude of R determines the correlation between the original gene value and the new mutated value. The extreme values are represented by $R = 2$, where the genes values are uncorrelated, and $R = 0$, where the new gene value is a clone of the original value.

The most interesting feature in the EMG is that agents who behave in an extreme way perform better than the cautious ones. That is, there is a self-segregation of the population in the sense that the distribution of p -values tends to peak at $p \approx 0$ and $p \approx 1$ (Johnson et al., 1999; Lo et al., 2000; Hod and Nakar, 2002; Quan et al., 2003). It has also been concluded that the results are very robust across different values of N , m , d , and the initial distribution of p .

2.2 *Minority games with local information*

A number of modifications have been introduced to both the MG and EMG, designed to reflect the influence of local information transmission on an agent's decision-making behaviour. One way to incorporate local information into the decision-making process is to build a local lookup table. That is, as well as using the global common memory, a local common memory based on local neighbours can be used to guide selection. The local neighbours can be randomly drawn from the population, or more commonly be found as nearest neighbours on a regular lattice (Moelbert and De Los Rios, 2002; Caridi and Ceva 2004; Burgos et al., 2004). Additional points may be awarded/deducted depending upon whether the agent belongs to the local minority/majority.

An alternative approach incorporates strategic imitation by agents mapped onto a linear chain with periodic boundary conditions (Kalinowski et al., 2000; Quan et al., 2003). Here, agents mimic, with some level of mutation, the corresponding gene value of their nearest neighbours if their utility value is less than their neighbour's utility. The resultant self-segregation of the population leads to more winners per time step than the standard MG or EMG. In another interesting study emphasizing the role of spatial structure, Schweitzer and co-workers (2002) investigated the impact of local information generated by the agents with a finite lifetime and dissemination radius. Detailed analysis revealed that more "efficient" information exchange within the spatially distributed population provided a suitable way to stabilize a groups of agents minority/majority status and to reduce diversity and uncertainty in the decision making process.

3. Small-World connections in the EMG

The EMG model presented in this paper can be represented as a dynamic network of interconnected agents sending signals to other agents (in the local neighbourhood), with global feedback available to all agents based on aggregate measures. Here, the agents occupy nodes of alternative network architectures, ranging from regular lattices to random networks. Before describing the enhanced EMG model in detail, small-world networks are introduced.

3.1 *Small-World networks*

In recent years, a number of papers have appeared investigating the widespread presence of the so called "small-world" pattern of social, economic and technological networks (Watts and Stogatz, 1998; Watts, 1999). A small-world network is typically described as a transition from a regular to random network. In these networks, each link is re-wired with some probability λ . The effect of re-wiring is the substitution of some short-range connections with long-range connections (Fig 1). The regular lattice $\lambda = 0$ and the random graph $\lambda = 1$ represent the two extreme models. Regular networks are highly clustered with relatively high shortest average path lengths. On the other hand, random networks are rather homogeneous, that is, most of the nodes have approximately the same number of links. Random networks have

relatively short average path lengths and tend to have low clustering. For intermediate values, small-world networks have a large overlap of neighbourhoods (clustering), and yet only relative short paths connecting any two individuals in the network.

Formally, a small-world network can be modelled as a graph $G(N, E)$ where N is a finite set of *nodes* (vertices) and E a finite set of *edges* (links) such that each edge is associated with a pair of nodes i and j . G can be represented by simply giving the $N \times N$ adjacency (or connection) matrix whose entry a_{ij} is 1 if there is an edge joining node i to node j and is 0 otherwise. Typically, two measures are used to characterise the structural properties of the network: a global property—the *average path length* (L), and a local property—the *clustering coefficient* (C) (Watts and Stogatz, 1998).

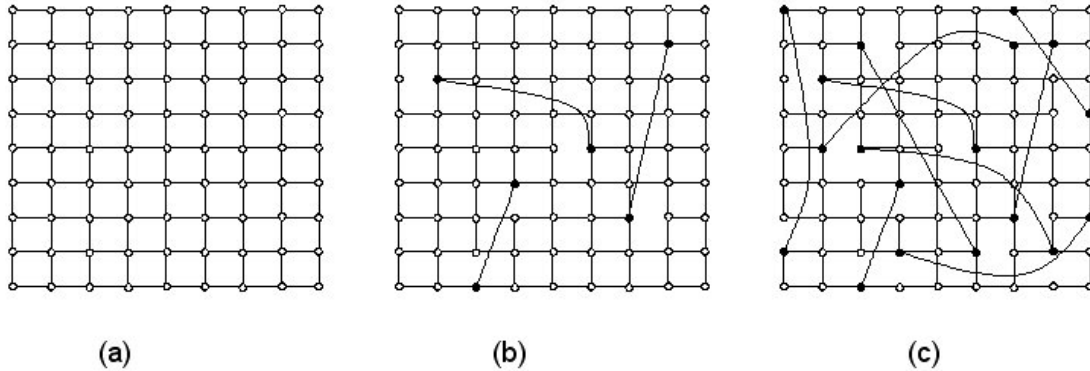


Figure 1. Small-world networks. (a) $\lambda = 0$, a regular lattice. (b) $\lambda = 0.1$, some of the links have been re-wired resulting in a small-world network. (c) $\lambda = 0.2$, additional re-wiring has occurred. As λ approaches 1, a transition to a random network will occur.

L measures the average separation between two nodes in the graph. The distance d_{ij} between two nodes, labelled i and j respectively, is defined as the number of edges along the shortest path connecting them. Thus $L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$. Short connecting paths suggest that contagious behaviour can spread more easily across the network.

C is the probability that two nearest neighbours of a node are also nearest neighbours of each other. C_i of node i is then defined as the ratio between the number E_i of edges that actually exist among these k_i nodes and the total possible number $k_i(k_i - 1)/2$ that is, $C_i = 2E_i / k_i(k_i - 1)$. The clustering coefficient C of the whole network is the average of C_i over all i . Clearly, $C < 1$ and $C = 1$ if and only if the network is globally coupled, which means that every node in the network connects to every other node. Generally speaking, high clustering implies that interaction in the network resembles interaction in a closed group.

3.2 The model

As is customary in the evolutionary-learning literature, the agents play the game over a fixed number of iterations. The agents follow the basic rules of the EMG (as described in Section 2). The specifics of the model are:

- The agents are mapped to the nodes of alternative small-world architectures.
- Global, aggregate performance is broadcast to all agents.
- All agents are initialised with a random gene value ($0 < p < 1$) and a zero utility value.
- Agents transmit their current utility and p -value to all other agents in the local neighbourhood.

- When an agent modifies their p -value, the new value is a randomly generated value within a range R , centred on the current p -value of the best neighbour (Quan et al., 2003). In the case of a tie between local neighbours, one of them is chosen at random to be the “best” neighbour.

For a given λ value, it is possible to associate a neighbourhood to each agent and subsequently provide a means by which an agent can modify their current decision processes (especially if they are performing poorly). The spatial constraints imposed by the underlying topology combined with the selection pressure (the utility threshold value d) and p -value correlation (R) provide appropriate mechanisms to guide the population trajectory. Here, the agent community is driven by “pushes and pulls” of individual decisions in certain directions. The better the agent is at predicting the minority group at time t , the greater its utility value. Consequently, the critical success factor for the agent population is based on the ability of individual agents to “follow the trend” as long as it is valid.

4. Simulations and results

Extensive computational simulations were carried out to investigate the population dynamics of the games played. All experiments were performed on a network consisting of $N = 31 \times 31$ nodes. The small-world networks were generated by systematically varying the value of λ , from 0 to 1, starting from a 2-D regular lattice base. For each network architecture, the value of R was varied across the range $R = 0$ to $R = 2$. The common “memory” or look-up table bit string length was set to $m = 3$ and the utility threshold value was set to $d = 4$ (Johnson et al., 1999). Synchronous updating of the model was used.

4.1 Attendance fluctuations

The main goal of this study was to determine the average behaviour of the population based on elementary rules. The EMG is a negative-sum game and as such, the average gain of the agents reaches its maximum when the number of agents in the minority group is as close as possible to $N/2$. Consequently, the standard deviation of the difference in the number of agents in the minority/majority groups is the most important statistic considered. Typically, this time averaged variation in the size of the groups is reported in the form of the *reduced variance* σ^2/N (Challet and Zhang, 1997). Fluctuations in this value indicate how the level of cooperation across the population varies.

In Fig 2, a plot of λ vs $\log R$ vs σ^2/N is presented. In all simulations, each set of data was recorded from 100 independent trials. In each trial, 5000 iterations after initialisation past before any measurements were made. The average values over 2000 steps after the equilibrium were then calculated. An inspection of Fig 2 clearly illustrates that the value of the reduced variance decreases as the value of R increases from very small values until $R \approx 0.5$, after which the value of σ^2/N increases. Interestingly, as λ is varied from 0 to approximately 0.3 the value of σ^2/N decreases. Beyond this point, the value of σ^2/N increases to a maximum at $\lambda = 1$.

Table 1 compares the value of σ^2/N for three different network models and the standard EMG. Once again, the statistics are reported at equilibrium. Here, the global efficiency calculations were carried out using a fixed p -value correlation ($R = 0.5$). There is a significant improvement in the value of σ^2/N for the small-world network reported compared to the standard EMG.

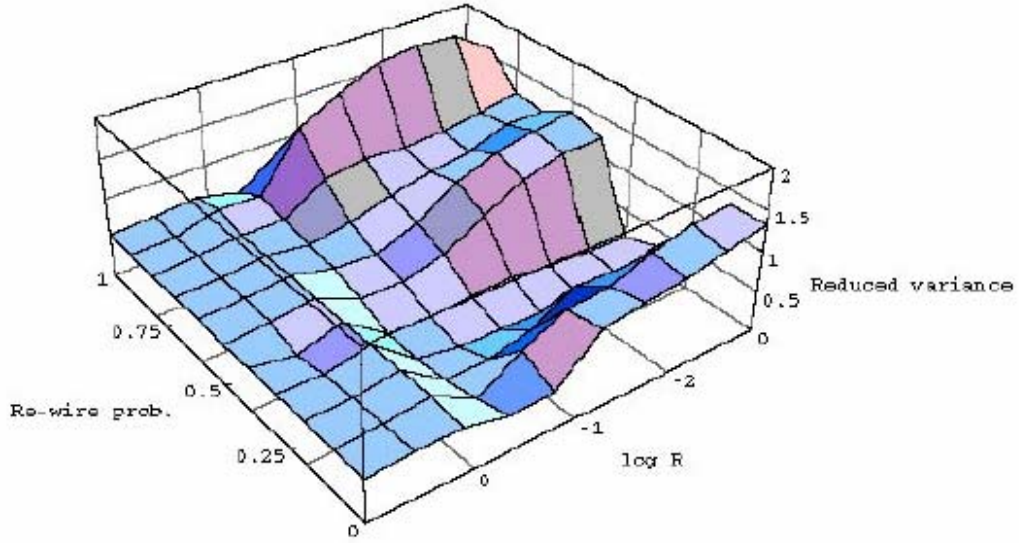


Figure 2. Network re-wire probability (λ) vs p -value correlation ($\log R$) vs Reduced variance (σ^2/N).

4.2 p -value probability density function

The self-organisation of the system is typically illustrated using a plot of the probability density function, $P(p)$ vs p . At equilibrium in the standard EMG, the function has a U-shape with two symmetric peaks at $p = 0$ and $p = 1$, indicating that the population have segregated into two groups making opposite choices (Johnson et al., 1999). Fig 3 plots $P(p)$ vs p for three different network models and the standard EMG. Each point represents an average value of 10 runs and 100000 time steps per run. In all models, the steady-state distribution of $P(p)$ is approximately symmetrical about $p = 0.5$, with peaks around $p = 0$ and $p = 1$. However, there is not a smooth transition at extreme λ values (especially at $p \approx 0$), indicating that the partition is not unique within differing local neighbourhoods.

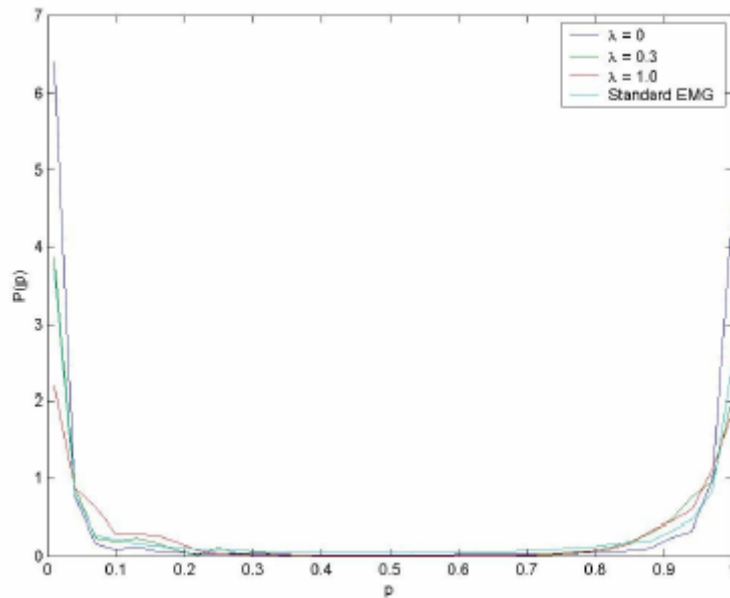


Figure 3. The strategy distribution $P(p)$ for different networks. Here, $d = 4$ and $R = 0.5$ with reflective boundary conditions.

5. Discussion

In the EMG, agents with limited information and rationality compete for a finite resource and are rewarded when they select the minority group. Agents have their own internal mechanism/strategy used to make a decision. Typically, individual agents react to the decisions of other agents, which often results in volatile aggregate behaviour that is far from efficient. Consequently, it is possible to describe the system dynamics at different levels: the *microscopic level*, where the decisions of the individual agents occur, and the *macroscopic level*, where a certain collective behaviour emerges.

In this study, enhancements to the standard EMG have been proposed that combine and extend previous local information transmission techniques and complex network theory. An important consideration when designing the model was to take into account mechanisms by which an agent decides their individual actions. Particular emphasis has been placed on the mode of interaction between agents. Agents share their decision-making strategies with their local neighbours as well as information regarding how successful their strategies have been. Poorly performing neighbours imitate (with some level of noise) another agent who has been more successful in their decision-making. Coordination is achieved via an adaptive learning mechanism where an agent has an opportunity to modify its behaviour.

Table 1. σ^2/N performance comparison using different EMG models.

Model , C L Mean SD					
Model	λ	C	L	Mean	SD
Regular	0.0	0.105	14.5	0.44	0.031
Small-world	0.3	0.054	7.7	0.20	0.035
Random	1.0	0.022	6.1	0.54	0.190
Standard EMG	-	-	-	0.61	0.040

The major goal of this work was to identify the way in which the aggregate or global behaviour emerges from the individual characteristics of the agents. The underlying hypothesis examined was that small-world architectures would provide a platform for more efficient coordination in the EMG. To meet the specific objectives, a number of simulation models were used to determine how different network structures and imitation rates affected the population dynamics. Detailed analysis of fluctuations in the number of agents selecting the minority group (Fig 2) revealed that optimal performance of the system was sensitive to the initial conditions.

The trend in system performance based on the value of R — the p -value correlation — follows a similar pattern to the work described by Quan et al., (2003), however, the magnitude of the value of R is significantly larger here. This is not unexpected given the increased population size. Agents do not have a unique “best behaviour” that should be played in all circumstances. If they did, all agents would lose due to symmetry reasons. Therefore, agents have no choice but to adapt a strategy in response to the population trends. Given the fact that R represents the “level of imitation” it is reasonable to expect that if R is very small, a poorly performing agent will almost be identical to the best nearest neighbour. It is necessary to add a small mutation to a copied strategy in order to explore unoccupied areas of the p -value space. As R increases (to some critical value) greater diversity will be introduced into the

population. However, if there is too much diversity in agent behaviour as a whole, self-organised/coordinated behaviour may prove to be difficult.

System performance was also found to be sensitive the value of λ — the network re-wire probability. The significance of the λ value may be attributed to the way in which the neighbourhoods overlap in the given network structure. Here, agents take on a new p -value in the vicinity of a better neighbours instead of in the vicinity of their own values as in the EMG. The active agent will only imitate the best local neighbour when its utility has fallen below the threshold and the local neighbour has accumulated wealth greater than the current agent. This in turn suggests that it is possible for a dominant individual to build a “cluster” of clones, albeit with slight variation. In this instance, it is very difficult for neighbours to increase their utility sufficiently to compete on an equal footing with the dominant neighbour. However, it is not unreasonable to expect diversity in behaviour to exist between different highly clustered groups of agents. For higher values of λ the impact of a distant dominant individuals decrease. Clearly, different values of the average path length and clustering coefficient of alternative networks can lead to the establishment and persistence of different types of populations. The simulation results suggest that a re-wiring level of $\lambda \approx 0.3$ leads to the smallest fluctuations in the size of the minority/majority groups. In related work, Caridi and Ceva (2004) have reported a similar improvement in coordination when a limited number of agents were allowed to interact (share decision-making strategies).

In Table 1, the performance of the standard EMG is compared with three different network models. Given the trends outlined in Fig 2, a small-world network with $\lambda \approx 0.3$ was compared with both regular and random networks as well as the standard EMG. There is a statistical significant improvement of the performance when $\lambda \approx 0.3$. The plots in Fig 3 follow the same basic U-shape as described in the original EMG studies. However, the inclusion of spatially distributed local information appears to degrade the “smooth” curve often depicted, indicating that there is still a significant probability for intermediate values of p . This result is to be expected given the dependence of the model on the value of λ for a given R . The clustered nature of the small-world model promotes a degree of “collaboration” between local neighbourhoods that may not be shared with other distant clusters.

6. Conclusion

In this paper, I have discussed an extension to the EMG which preserves the basic parameters of the standard game. Here, a framework for modelling individual interactions and decision-making was introduced based on small-world networks. The simulation results suggest that system efficiency depends both on the level of interactions between agents as well the mode of learning adopted by the agents. The population dynamics displayed were driven not only by the imitation of a neighbour’s successful strategy, but also by the positive reinforcement of an agents’ own successful strategies. The value of the reduced variance of the number of agents successfully selecting the minority group was directly related to both the connectivity of the network model, and the correlation between the gene value copied from previously successful agents and the new randomly generated gene value. A small degree of disorder introduced via “long-range connections” within the spatially distributed population leads to improvements in system efficiency.

References

- Arthur, W. B. (1994). Bounded Rationality and Inductive Behavior (the El Farol Problem). *American Economic Review*, 84:406-411.

- Burgos, E., Ceva, H., and Perazzo, R.P.J. (2004). The evolutionary minority game with local coordination. *Physica A*, 337:635-644.
- Caridi, I. and Ceva, H. (2004). The Minority Game with interactions. *Physica A*, 339:574-582.
- Challet, D. and Zhang, Y.C. (1997). Emergence of cooperation and organisation in an evolutionary game. *Physica A*, 246:407.
- Chau, H.F., Chow, F.K. and Ho, K.H. (2003). Minority Game with Peer Pressure. *arXiv:cond-mat/0307556v1*.
- Dorogovtsev S.N. and Mendes, J.F.F. (2003). *Evolution of Networks: From Biological Nets to the Internet and WWW*. Oxford University Press, Oxford.
- Holland, J. (1995). *Hidden Order: How adaptation build complexity*. Addison-Wesley, Reading.
- Hod, S. and Nakar, E. (2002). Self-segregation versus clustering in the evolutionary minority game. *Physical Review Letters*, 88(23):238702.
- Johnson, N.F., Hui, P.M., Jonson, R. and Lo, T.S. (1999). Self-Organized Segregation within an Evolving Population. *Physical Review Letters*, 82(16):3360.
- Kalinowski, T., Schuklz, H.-J. and Briese, M. (2000). Cooperation in the Minority Game with local information. *Physica A*, 277:502-508.
- Li, Y. and Savit, R. (2004). Toward a theory of local resource competition: the minority game with private information. *Physica A*, 335:217-239.
- Lo, T.S., Hui, P.M. and Johnson, N.F. (2000). Theory of the evolutionary minority game. *Physical Review E*, 62:4393.
- Moelbert, S. and De Los Rios, P. (2002). The Local Minority Game. *Physica A*, 302:217-227.
- Namatame, A. and Sato, H. (2004). Localized Minority Games and Emergence of Efficient Dynamic Order. *Lecture Notes in Economics and Mathematical Systems. Vol 550*. Springer. pp 71-86.
- Quan, H.-J., Wang, B.-H., Hui, P.M. and Luo, X.-S. (2003). Self-segregation and enhanced cooperation in an evolving population through local information transmission. *Physica A*, 321:300-308.
- Schweitzer, F., Zimmermann, J. and Muhlenbein, H. (2002). Coordination of decisions in a spatial agent model. *Physica A*, 303:189-216.
- Watts, D.J. (1999). *Small Worlds: The Dynamics of Networks between Order and Randomness*. Princeton University Press.
- Watts, D. and Strogatz, S.H. (1998). Collective dynamics of “small-world” networks. *Nature*, 393:440-441.