Evolutionary Minority Game

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Basic Minority Game[1]

- N agents choose to buy or sell assets
- Wins the game if it's one of the members of the minority group
- Finite amount of public information - memory stored in bit string (e.g. 3 most recent outcomes, 2^3 possible outcomes)
- Strategy space based on possible outcomes (e.g. 2^{2^3})
- Virtual score - strategy score, and real score - agent's wealth

[1] Sitabhra Sinha et al. *Econophysics: an introduction*. John Wiley & Sons, 2010.

Evolutionary Minority Game

- Darwinist Selection; Reset score;
- Version 1: the worst agent is replaced by a new one after some time-steps
- Version 2: all agents hold the same dynamic strategy (updated by outcomes), but an individual gene value (possibility to follow the strategy)
- Version 3: the poor agents of each round are replaced
- Version 4: a few poorest agents are replaced
- Mutation; Evolution threshold

Different Evolutionary Versions of Minority Game

Table 1: Comparison Table for Different Versions of EMG

Version	**Mutation**	**Threshold**	**Time to Eliminate**
Basic	None	None	None
EMG(V1)[2]	Best+Random	Dynamic	Fixed Time-steps
EMG(V2)[3]	Random	Fixed	Anytime
EMG(V3)	Best+Random	Fixed	Fixed Time-steps
EMG(V4)[4]	Best+Random	Dynamic	Fixed Time-steps

[2] Damien Challet and Y-C Zhang. "Emergence of cooperation and organization in an evolutionary game". In: *Physica A: Statistical Mechanics and its Applications* 246.3-4 (1997), pp. 407–418.

[3] TS Lo, PM Hui, and NF Johnson. "Theory of the evolutionary minority game". In: *Physical Review E* 62.3 (2000), p. 4393.

[4] Yi Li, Rick Riolo, and Robert Savit. "Evolution in minority games.(I). Games with a fixed strategy space". In: *Physica A: Statistical Mechanics and its Applications* 276.1-2 (2000), pp. 234–264.

Basic Parameters

Table 2: Basic Parameters

Parameter	Value
Rounds(ite)	4000
Rounds Before Recording(iteq)	200
Memory Size(m)	5
Sample Size(nsam)	10
Number of Agents(N)	Odd
Way of Updating Scores	Linear Score

Attendance Number I

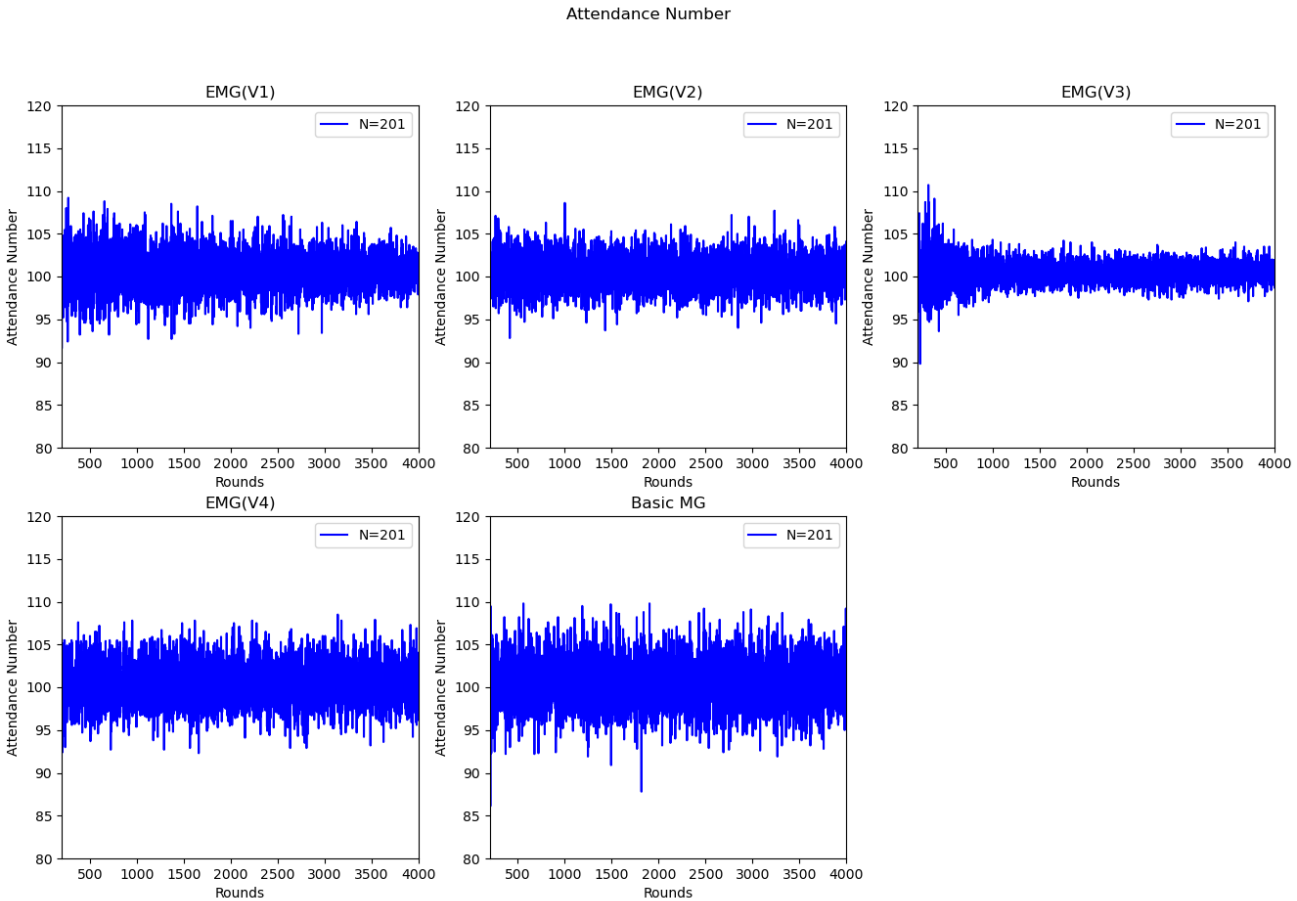

Figure 1: Attendance Number When $N = 201$

Attendance Number II

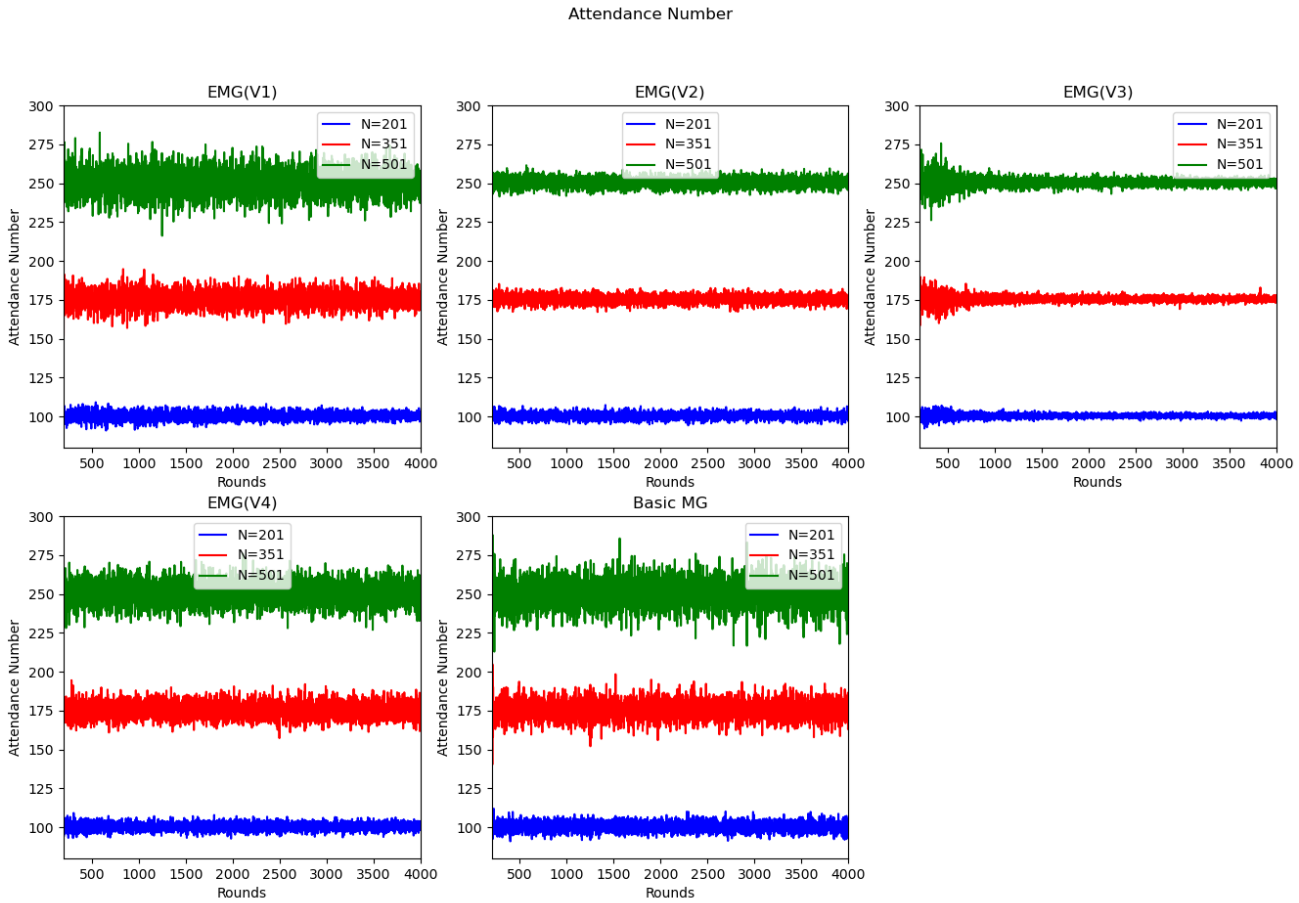

Figure 2: Attendance Number When $N = 201, 351$ and 501

Observations

- All versions of EMG appear to have smaller amplitudes of fluctuation. $\longrightarrow$ The presence of evolution in the model can control fluctuations
- Version 3 seems to be more close to Nash equilibrium
- Version 1 and 4 (dynamic threshold) vs Version 2 and 3 (fixed threshold)

$\frac{\sigma^2}{N}$ Versus $\frac{2^m}{N}$

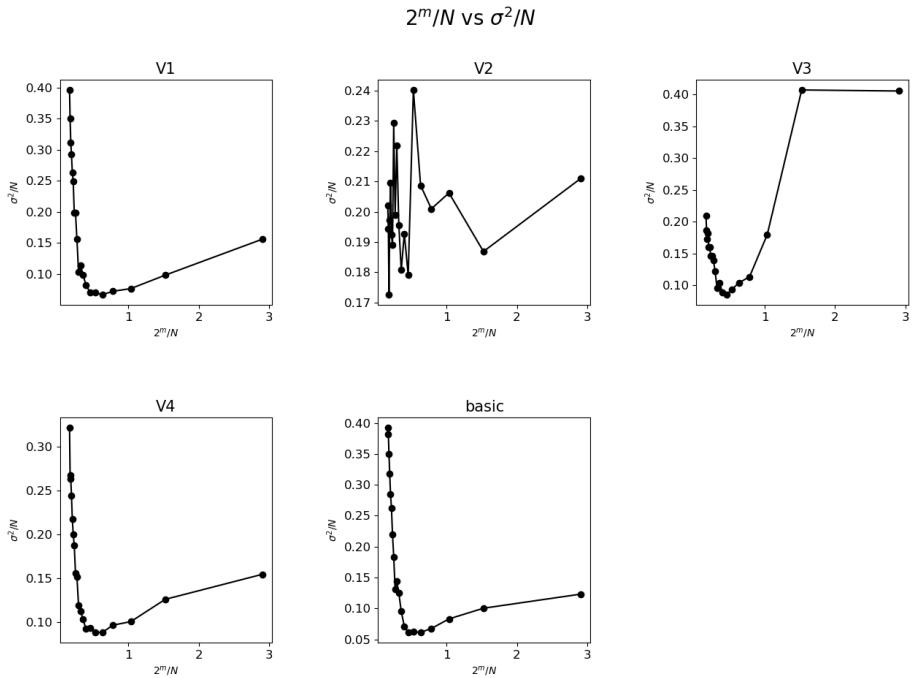

Figure 3: $\frac{\sigma^2}{N}$ Versus $\frac{2^m}{N}$

- σ^2: variance of the numbers of buyers, m=5 and different odd N

Wealth Curves

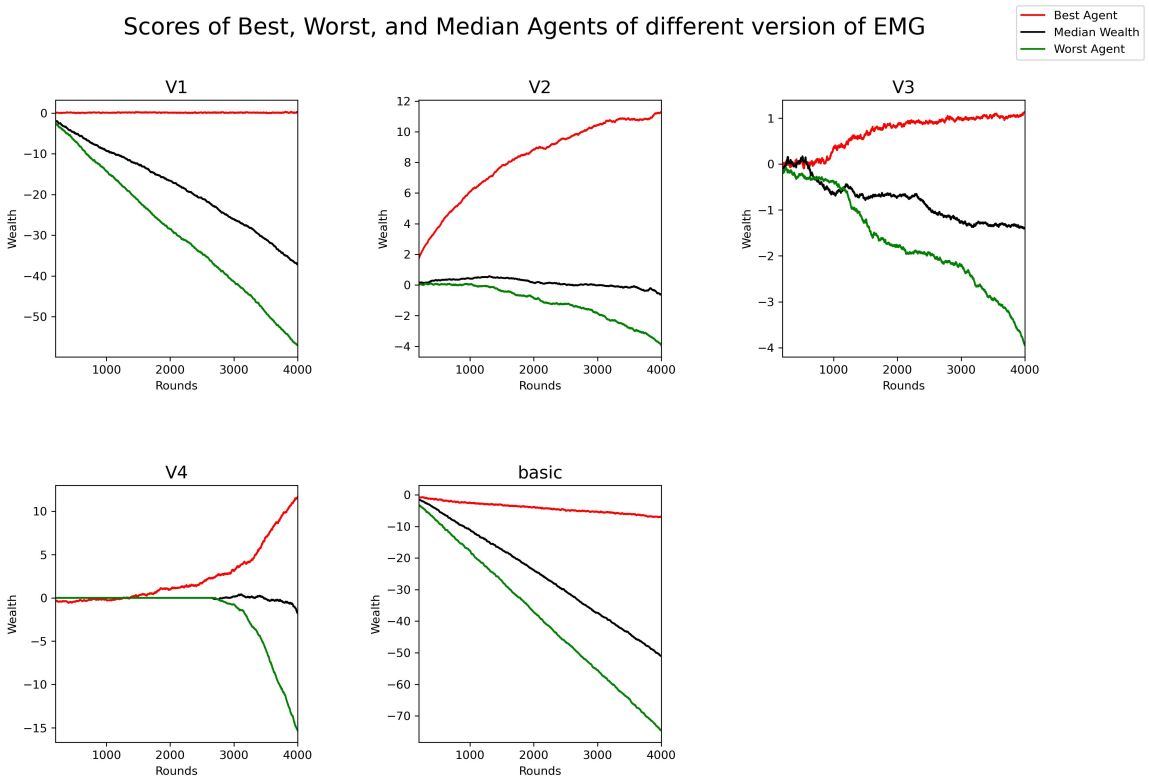

Figure 4: Wealth Curves of Representative Agents

- Gene Value Distribution ($P(p)$): The frequency distribution of gene values;
- Lifespan ($L(p)$): The average length of time a gene value p survives between modifications;
- Hamming Distance(D_h): The number of positions at which the corresponding symbols are different;
- Behavioral Distance: The normalized sum of the Hamming distances between each pair of agent i's and j's strategies;
- Average Behavioral Distance (D_b): the average behavioral distance of the i_{th} agent from all other agents playing the game.

Hamming Distance D_h

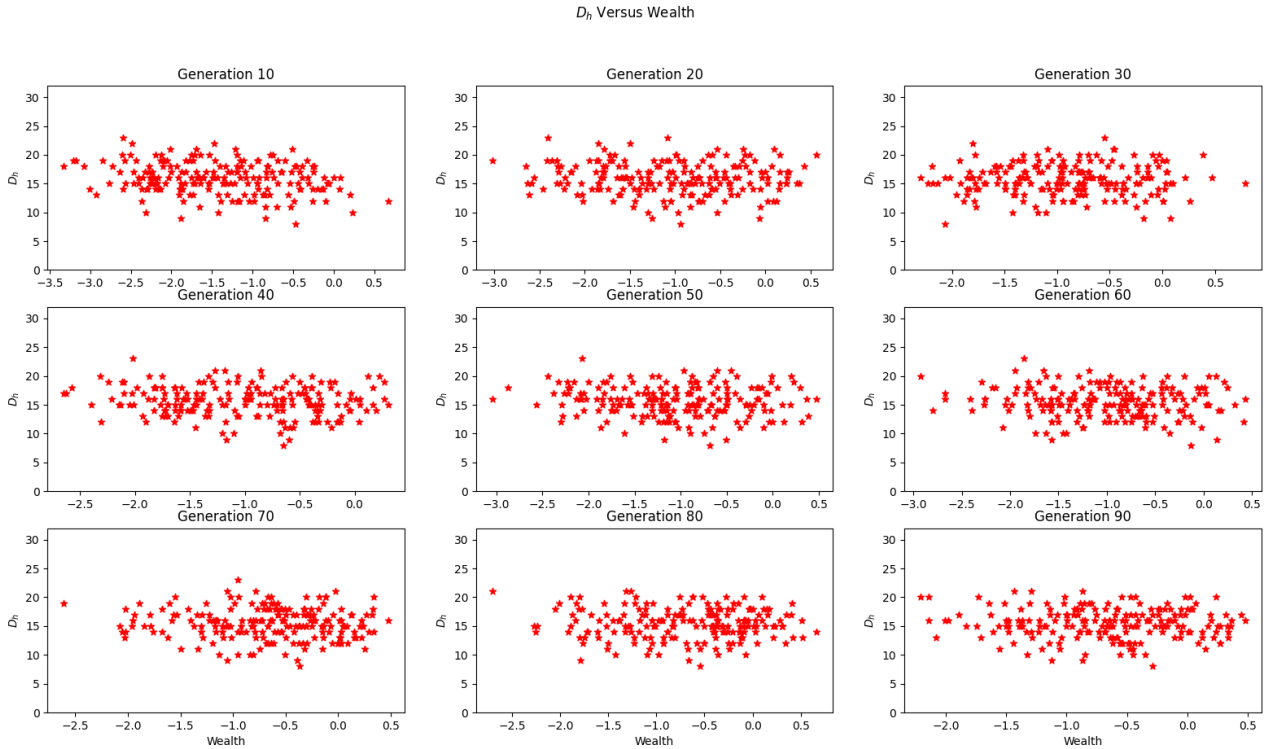

Figure 5: D_h Versus Wealth

Average Behavioral Distance D_b

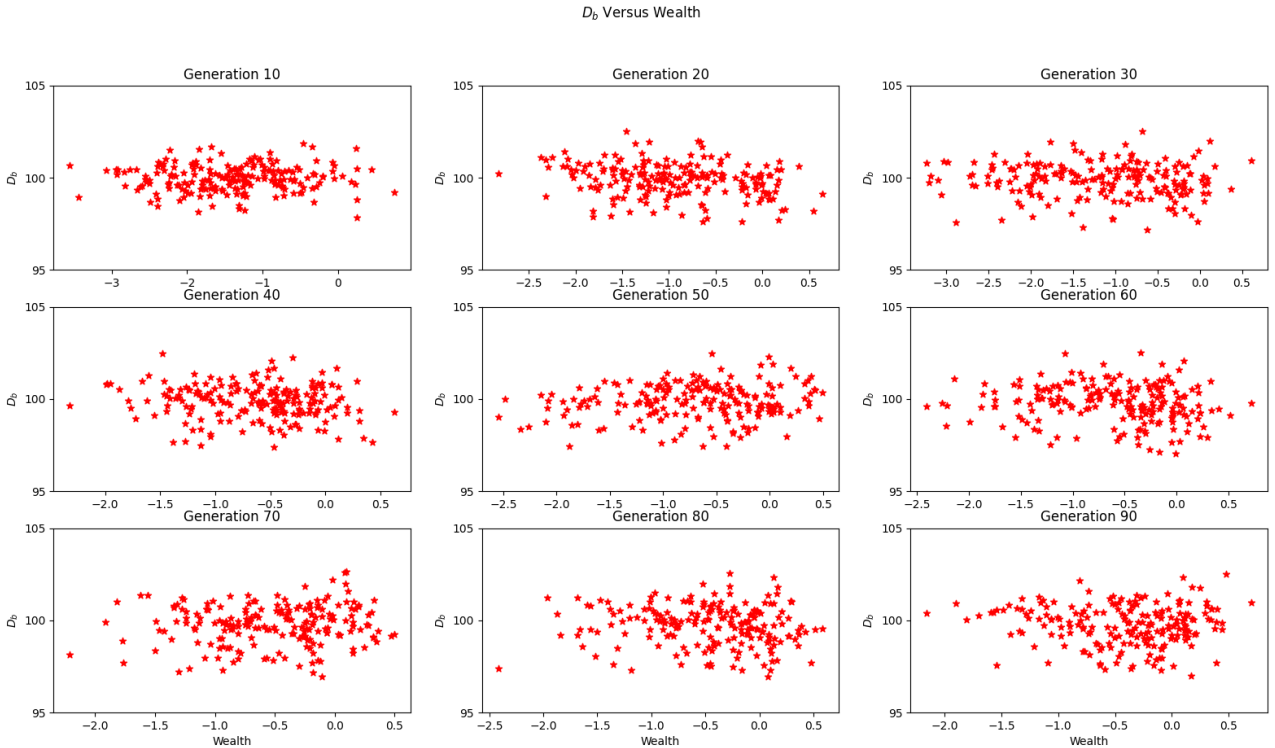

Figure 6: D_b Versus Wealth

Gene Value Distribution

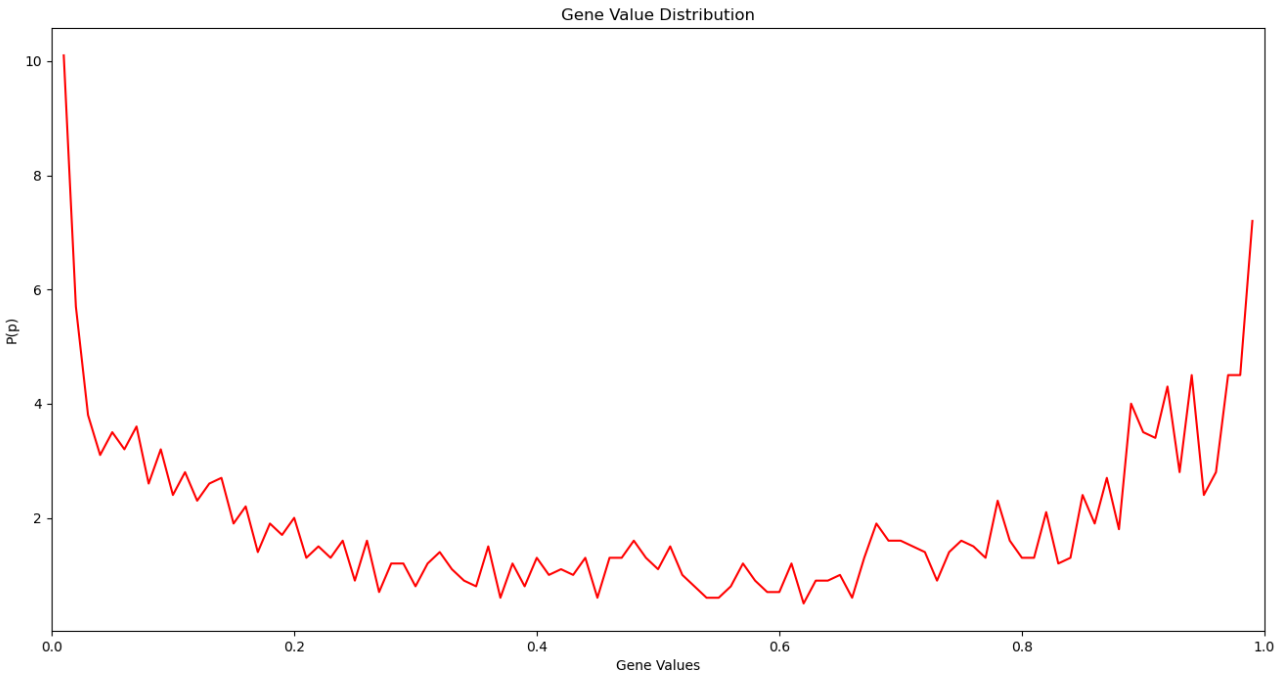

Figure 7: Gene Value Distribution

Lifespan

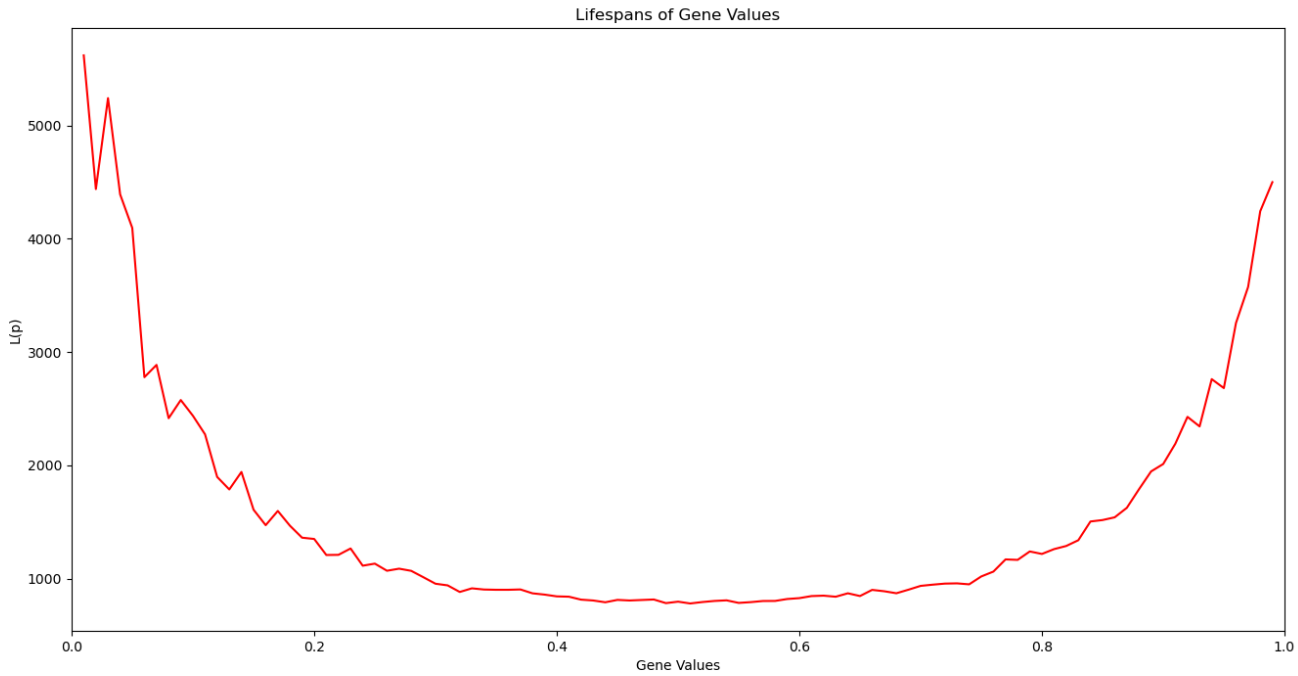

Figure 8: Lifespan

Hamming Distance D_h

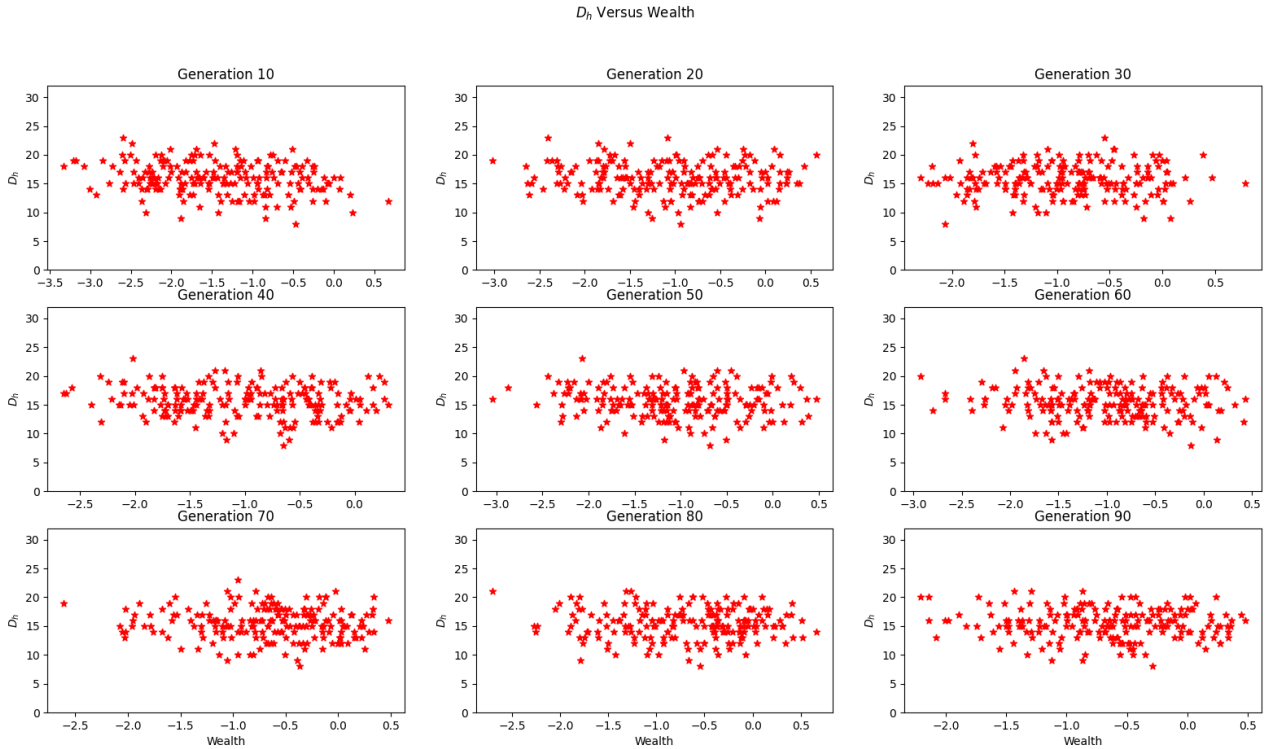

Figure 9: D_h Versus Wealth

Average Behavioral Distance D_b

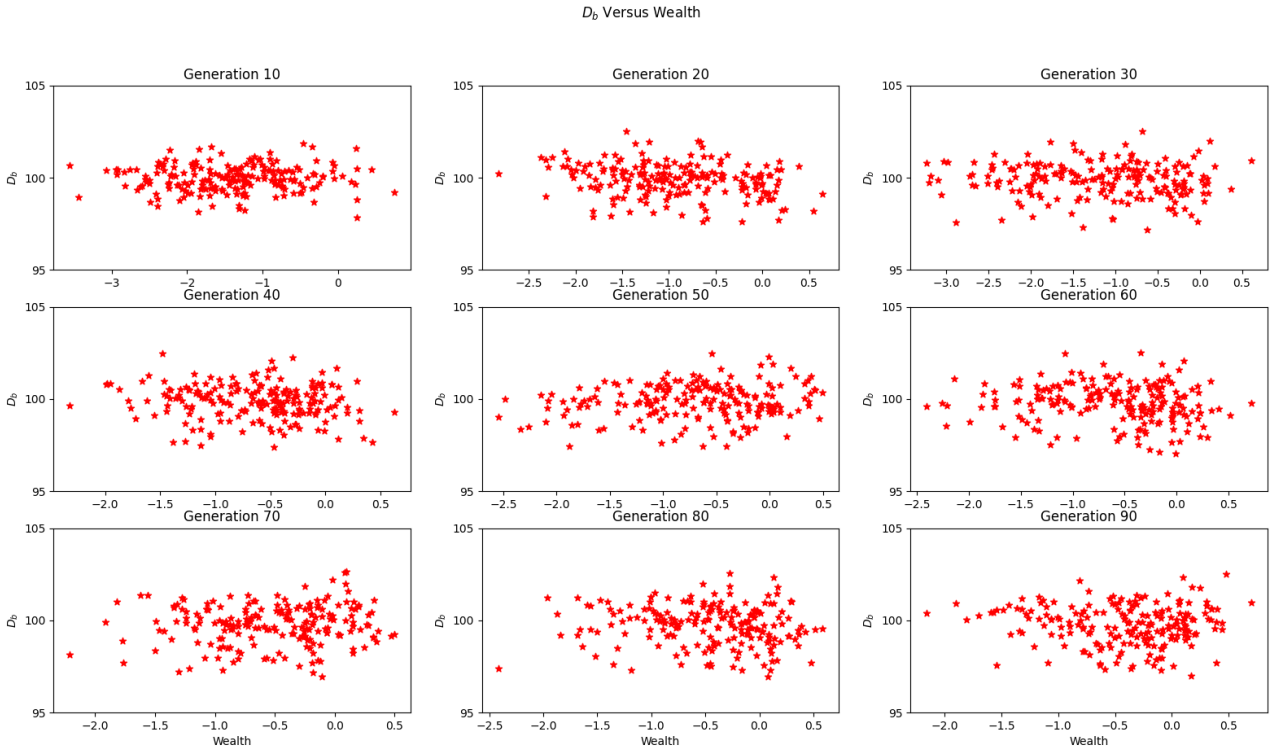

Figure 10: D_b Versus Wealth

In this report, we focus on the comparison of different versions of evolutionary minority games and the basic minority game.

- For overall systematic performances, evolutionary populations can significantly control the fluctuation and help agents gain more. Fixed thresholds appear to have better effects.
- For agents' behaviors, agents' preferences of strategies highly depend on the elimination rule in the EMG.

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Thanks!