Graph Convolution Network Based Analysis For Systematic Performances of Networked Evolutionary Minority Game

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Abstract

As a classical multi-agent game model, the evolutionary minority game(EMG) has wide applications in financial markets and distributed control systems. The factors that influence the systematic behavior and individual strategy preferences in the networked version of EMG (NEMG) have remained undetermined for a long time. We propose a Graph Convolution Network-based framework to learn a good strategy with a high payoff from the agent's own experience and information shared by its neighbors. Experiments performed on five classes of networks: star graph, Erdos-Renyi random graph, small-world graph, regular graph and complete graph illustrate the effectiveness of this framework and reveal some insights into NEMG settings.

1 Introduction

The minority game (MG) is a widely studied model for understanding competitive decision-making among adaptive agents in resource-limited environments. In its canonical form, a population of agents repeatedly chooses between two options, and those in the minority group receive a payoff. [13, 6] Despite its simplicity, the MG captures key features of complex adaptive systems such as financial markets [18], congestion dynamics [8], and distributed control [15]. The model highlights how global coordination can emerge from decentralized learning and how collective behavior depends on information availability, memory, and adaptation rules.

The evolutionary minority game (EMG) extends the classical MG by allowing agents' strategies or behavioral parameters to evolve over time [12]. Instead of relying on fixed strategy pools, agents adapt their decision rules through mechanisms such as reinforcement learning, mutation–selection dynamics, or strategy switching based on historical performance. This evolutionary framework enables the system to explore strategy spaces more flexibly and often leads to richer dynamical regimes, including self-organization around efficient equilibria and intermittency between coordinated [4, 9] and uncoordinated phases [1]. The EMG thus provides a bridge between individual learning and population-level evolution, offering deeper insight into how robust strategies emerge in heterogeneous, co-evolving agent populations.

Networked minority games (NMG) incorporate explicit interaction structures by embedding agents on a network [16]. In contrast to the mean-field assumption of the classical MG, NMG allows each agent to access only local information—such as neighbors' past actions or payoffs—while the global minority rule still determines rewards. The topology of the underlying network plays a crucial role in shaping both agent behavior and system-level outcomes [11]. Networked variants of the MG are therefore useful for modeling strategic dynamics in social, technological, or biological systems where

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information exchange is inherently local [5, 14]. Together, evolutionary and networked formulations provide a natural foundation for studying learning dynamics with competing and incomplete information features. However, how network topology affects the systematic and individual behaviors in NEMG is still not so clear.

As a natural generalization of Convolution Neural Network(CNN) [10], Graph Convolution Network(GCN) [19] is designed to learn representations from graph-structured data by explicitly incorporating relational information between nodes. GCNs iteratively aggregate and transform features from a node's local neighborhood, enabling the model capture both node attributes and topological structure. Thus, GCN appears to be a highly appropriate model for us to study the strategy suitable for NEMGs on different underlying networks. In this paper, we train GCNs with local and global market information on 6 classes of graphs to simulate NEMGs for comparison. Experiment results illustrate a surprising result that global information doesn't contribute to the improvement of strategies. Local information, however, can help agents benefit in certain network topologies.

2 Method

2.1 Problem Formulation

The traditional game involves N players (agents) on the nodes of a complex network forced to make a binary decision: 0 (eg. go to the bar, or take route A) or 1 (eg. do not go to the bar, or take route B). Note that N needs to be an odd number and the network all agents are on can be regarded as an information transport venue. We call this network the underlying network. At each round t, agent i selects a binary action $a_i^t \in \{0, 1\}$. Agent i receives a reward α at time t according to

$$r_i^{(t)} = 1[a_i^t = \arg\min_{a\in\{0,1\}} \sum_{j\in V} 1[a_j^t = a]$$

where $1[\cdot]$ is the indicator function. Each agent shares the same memory with the length m and employs a deterministic strategy mapping from the memory states to actions. The memory state at time t can be expressed as $M_i^{(t)} = (b_{t-m}, b_{t-m+1}, ..., b_{t-1})$ where b_i is the winning action at time t. A pure strategy S_i is a lookup table mapping each of the 2^m memory states to a recommended action. At the beginning, one initial strategy is assigned to all agents from the whole strategy pool randomly. The game evolves as the agents modify their behaviors (or strategies) based on their previous experiences with their own choices and their neighbors' choices. [9] After fixed rounds T_{evo} of game, each agent would change his strategy. In this paper, we apply a Fermi-rule based imitation process: Agent i compares its cumulative payoff π_i with a randomly chosen neighbor with a cumulative payoff π_j. The probability of agent i to adopt neighbor j's strategy is:

$$P_{i \longleftarrow j} = \frac{1}{1 + \exp(-\beta(\pi_j - \pi_i))}$$

where β is the selection pressure parameter controlling the strength of the fitness-based selection. With probability P_{mu}, the adopted strategy will undergo a mutation: a random subset of entries in the strategy lookup table are flipped. [7]

2.2 Network Topologies

There are 5 underlying networks we considered in this paper.

Star Graph: a tree with one internal node and k leaves where a tree is a graph in which there is a single path between any two nodes of the graph and leaves are degree one nodes in the tree. [3]

Regular Graph: a graph in which every vertex has the same degree, which means the same number of neighbors. [3]

Erdos-Renyi Random Graph: a graph that is constructed by connecting n labeled nodes randomly. Each edge is included in the graph with probability p, independently from every other edge. [3]

Small-world Graph: a graph characterized by a high clustering coefficient and low distances, which means that the typical distance L between two randomly chosen nodes (the number of steps required) grows proportionally to the logarithm of the number of nodes N in the network ($L \propto \log N$) while the global clustering coefficient is not small. [17, 2]

Complete Graph: a graph in which every pair of distinct vertices is connected by a unique edge. In this case, NEMG actually reduces to EMG. [3]

2.3 Graph Convolutional Networks for Strategy Learning

We employ Graph Convolutional Networks (GCNs) to learn agent strategies from behavioral data, investigating whether neural networks can capture emergent strategic patterns in networked games. We compare two architectural approaches that differ in their information scope.

Each graph convolutional layer can be expressed as below:

$$h_i^{l+1} = \sigma\left(\sum_{j \in N(i) \cup \{i\}} \frac{1}{\sqrt{d_i d_j}} W^{(l)} h_j^{(l)}\right)$$

where

- h_i^l: feature vector of node i at layer l;
- $N(i)$: neighbors of node i;
- d_i, d_j: degrees of node i and j;
- $W^{(l)}$: learnable weight matrix at layer l;
- σ: activation function, in our experiments we use $ReLU$;

Full Network GCN: The full network GCN processes the entire graph structure simultaneously. It consists of three graph convolutional layers followed by a fully connected output layer. We use the full network GCN to simulate an agent with global information in the game.

Ego Network GCN: The ego network model processes each agent's local subgraph independently. For the target agent i, we extract the ego network which is the induced network of $\{i\} \cup N(i)$, denoted by G_i. The architecture uses two convolutional layers followed by a fully connected output layer. One convolutional layer was omitted because less information is needed to handle.

3 Experiments

Both GCN models share the same setting in our experiments, not only the training settings, but also the game settings.

3.1 Settings

Input Feature: For each agent, we extract the most recent $h = 20$ binary actions, forming the memory vector as feature vectors x.

Target Label: The evolved strategies s_i after evolutionary dynamics serve as ground truth labels.

Loss Function: We use the binary cross-entropy loss.

Optimizer: We use Adam optimizer with learning rate $\alpha = 0.001$ trains for 200 epochs.

Game Settings: We set the memory length $m = 4$, selection pressure $\beta = 1.0$, mutation rate $p_{mut} = 0.01$ and the number of agents is $N = 301$. For bipartite graphs, additionally, we run experiments with 351 and 501 agents to see a systematic trend in the change of the attendance number.

3.2 Experiment Framework

We assess learned strategies through simulation-based evaluation. After the training of the full network GCN and ego network GCN, we binarize predicted strategies and initialize a new NMG with the same network topology. By assigning predicted strategies to agents and simulating 20000 rounds without evolution, we can figure out which strategy performs better by comparing the cumulative average payoff $\overline{P} = \frac{1}{N}\sum_{i=1}^{N} P_i(T)$ where $P_i(T)$ is the cumulative payoff of the agent i after T rounds of games. Note that the cumulative average payoff in fact relies on the attendance number A and can be expressed as $\overline{P} = \min\{\frac{A}{N}, \frac{N-A}{N}\}$, it is always no more than $\frac{N}{2}$. A basic strategy is also applied in the simulations as a comparison reference, which is exactly the evolved strategy setting in basic NEMG we introduced before.

4 Results

4.1 Complete Graphs

We run experiments on complete graphs independently, since in this case, NEMG reduces to EMG. Hence, the complete graph NEMG can be regarded as a baseline to show the evolutionary dynamics of an NEMG/EMG system. We mainly focus on the attendance number, cumulative payoff distribution, and the payoff lines versus rounds of three representative agents: best agent, median agent and the worst agent.

The attendance number is defined as the number of agents who choose 1 in each round. As more agents join the game, the attendance number tends to show more and larger fluctuations, which implies that the system becomes less stable.

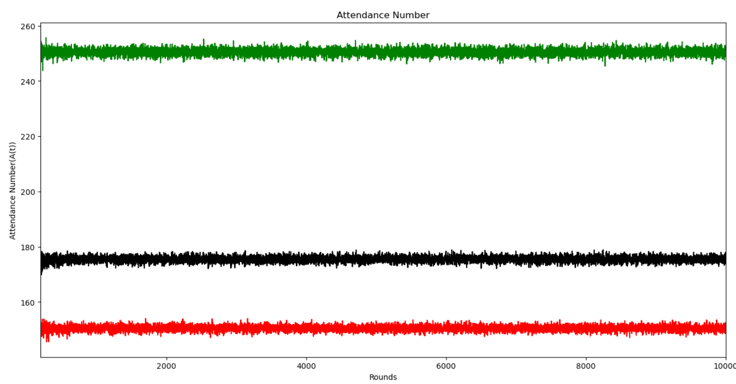

Figure 1: Attendance Number for Different Number of Agents

Though all agents get the same global market information, the worst agent still turns out to be significantly behind the others. This phenomenon reveals the importance of a good starting strategy in NEMG. Once you are left behind, it would be hard to catch up.

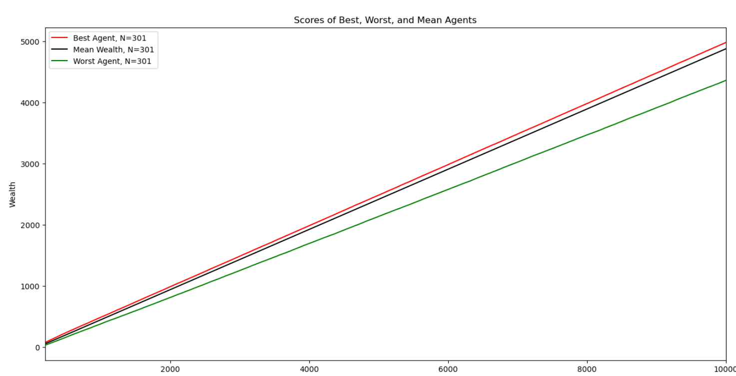

Figure 2: Payoff Curves For Representative Agents

The cumulative payoff distribution further supports our observation. The payoffs are centered around the line $payoff = 4875$, which is slightly smaller than half the total rounds. Global information helps agents to behave similarly but the initial strategies lead to the main payoff differences.

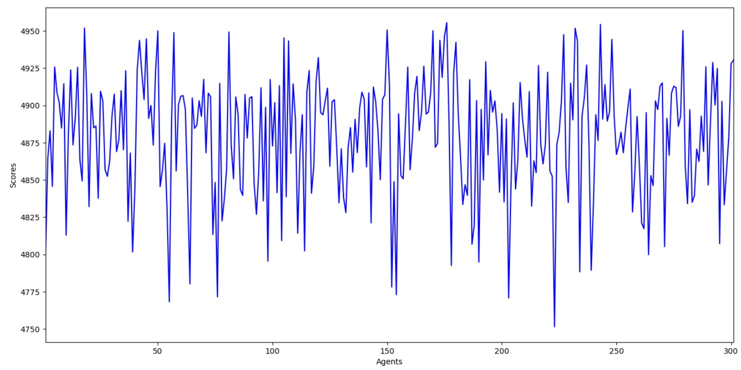

Figure 3: Payoff Distribution

4.2 Star Graph

For star graphs, both full network GCN and ego network GCN turn out to show significantly different patterns compared with the complete graph case.

We first focus on the attendance number. The evolved strategy turns out to show erratic behavior, which makes it unpredictable. Both the full network GCN strategy and the ego network GCN strategy control the fluctuations very well and remain almost the same at a low level, which implies that 1 is always the winning choice. This reveals a surprising phenomenon that no matter agents learn their strategies by GCN using global or local information, they finally obtain an equilibrium where everyone shares equal payoff from the perspective of some period. The system is not only predictable, but also stable.

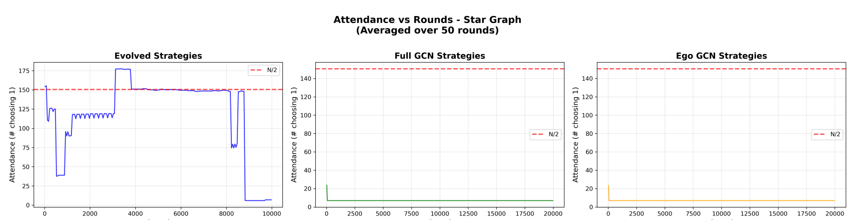

Figure 4: Attendance Number For Star Graph

The cumulative average payoff curves support our former observation. Two GCN strategies obtain a stable average payoff after several rounds of adjustments. As a comparison baseline, the traditional evolved strategy shows an obvious decreasing pattern. As a centralized network, a star network has a unique center. While evolving, the centering agent is actually the only agent that masters the global information in this game. For an arbitrary leaf agent, the experience of another leaf agent does not contribute to the improvement of his understanding of the market, because his neighboring agent, the centering one, already incorporates strategies of all leaf agents into his own. This accounts for the similar pattern and dynamics that two network GCN strategies show.

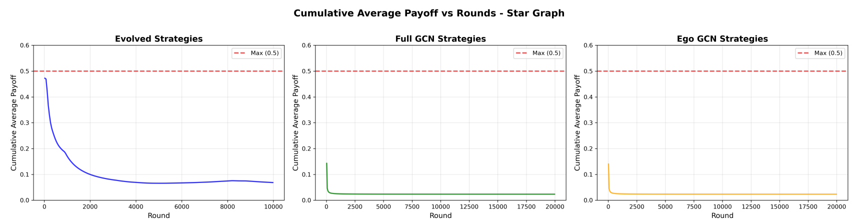

Figure 5: Cumulative Payoff Curve For Star Graph

4.3 Erdos-Renyi Graph

In the experiments of the Erdos-Renyi graph, we set $p = 0.015$.

Unlike the evolved strategy that shows fluctuations around the half the total number line, both the full and ego network GCN strategies turn out to have periodic transitions. Putting two network GCN strategies together, the ego network GCN strategy controls the fluctuations better, leading to a more stable overall system. This reveals a fact that global information brings nothing additional but noise to the strategy determination in this case.

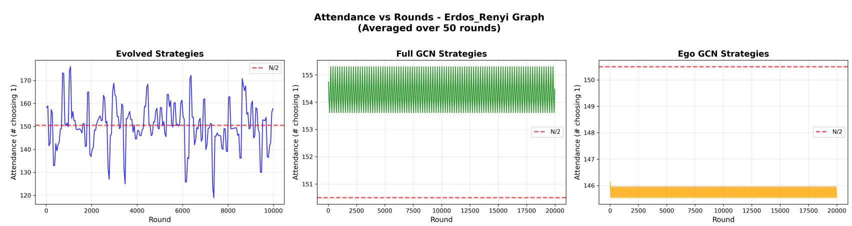

Figure 6: Attendance Number For Erdos Renyi Graph

Cumulative average payoff curves provide more evidence. The ego network GCN strategy not only gets the highest payoff within three settings of strategies, but remains a stable systematic performance as well. Local information is already enough for agents to refine a well-performing strategy while the market information flow in the market is uncertain and inconsistent.

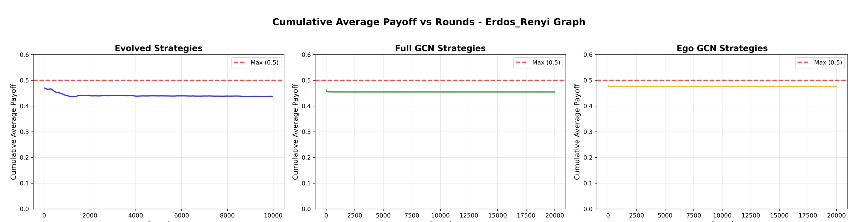

Figure 7: Cumulative Payoff Curve For Erdos Renyi Graph

4.4 Small World Graph

Figure 8 illustrates the attendance dynamics under three strategy learning mechanisms on a small-world network. The evolved strategies exhibit substantial temporal fluctuations around the theoretical equilibrium $N/2$, indicating persistent coordination instability despite long-term averaging. This is the same as the results on other networks.

In contrast, the full and ego network GCN strategies rapidly converge to a stable attendance level. While taking the fluctuations into consideration, however, unlike our former observations on the star graph and Erdos-Renyi graph, the ego network GCN strategy turns out to have larger fluctuations. This phenomenon is comprehensive. In a small-world network, it is generally accurate to say that the local structure is not a full representation of the global structure. A high clustering coefficient implies that nodes are embedded in tightly knit clusters that resemble lattices, while a short average path length gives rise to random-graph-like global connectivity. In this case, local information can be significantly different from global information, making the capacity to control the fluctuations worse.

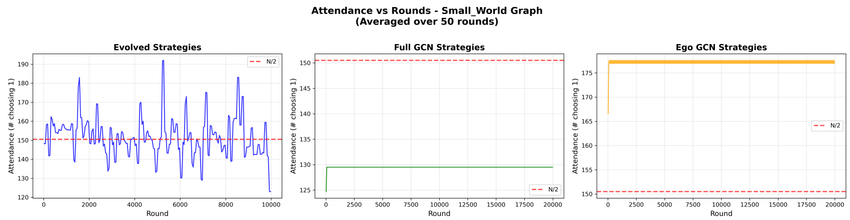

Figure 8: Attendance Number For Small World Graph

Cumulative average payoff provides another point of view. Random-graph-like global connectivity brings in many noises, which dominates the extra information and completely covers the positive effect of more agents' strategies, hence leading to the consequence that the ego network GCN strategy outperforms the full version. Though fluctuations seem to be a little large in the figure for the attendance number, the payoff curve illustrates that the performance of the ego network GCN strategy still remains stable and efficient.

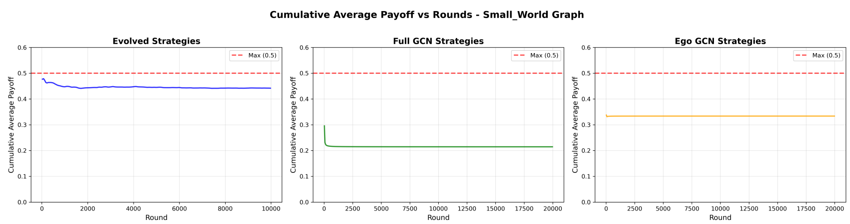

Figure 9: Cumulative Payoff Curve For Small World Graph

4.5 Regular Graph

In the experiments on regular graphs, we set the degree of all the nodes 4, to make the network more similar to a real-world social network.

Figure10 reports the attendance dynamics on a regular network under three learning mechanisms. Using the full network GCN strategy, the system rapidly converges to a stable attendance level. In the regular graph, where local neighborhoods are structurally identical, access to global information appears to amplify correlated decision-making across the population, leading to herding and a suboptimal steady state.

Though requiring a longer state to converge, the ego network GCN strategy achieves a surprisingly good outcome. All nodes share the same degree in the regular network, making all local neighborhoods exactly identical. Thus, all agents get the same amount of information while competing with each other. Due to the randomness of the initial strategies, no one would outperform significantly in the game, making the global information useless in strategy decisions. This partially explains the outstanding performance of the ego GCN strategy.

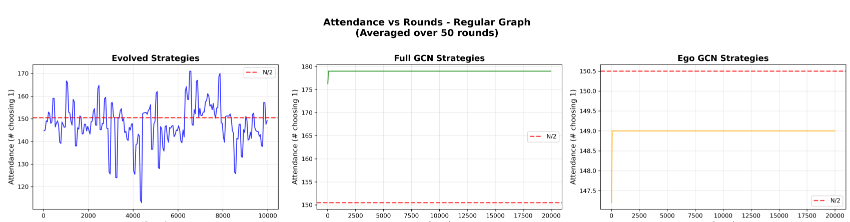

Figure 10: Attendance Number For Regular Graph

Cumulative average payoff curves reveal exactly the same pattern. Note that both the full and ego network GCN strategies achieve a relatively high payoff among all the networks we tested, indicating that equal access to information contributes to the improvement of the overall market performance. This teaches us a vital lesson in the mechanism design: the design of fairness has a positive effect on both systematic payoff and individual benefits. The ego network GCN strategy curve is highly close to the theoretical optimal value indicated by the red dashed line. This suggests that in a regular network, localized information supports stable and near-optimal collective outcomes.

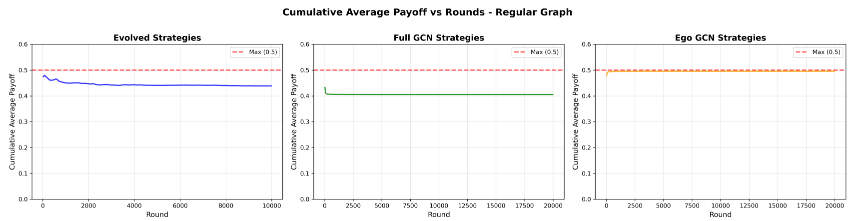

Figure 11: Cumulative Payoff Curve For Regular Graph

5 Conclusion

In this paper, we briefly analyzed the impact that network topologies have on the systematic performance of the Networked Evolutionary Minority Game(NEMG) by evaluating and comparing the dynamics of the strategy learnt by training two Graph Convolution Networks using global and local information respectively on six different classes of graphs. Generally speaking, both agents and the market can benefit from equal access to market information. In this case, localized information exchanges efficiently avoid the disturbance of irrelevant and redundant information, hence resulting in strategies with better performance. Agents who are significantly prior to the others block the propagation of the information, making the competition in an NEMG unbalanced, and eventually harming the whole market. The key lesson to take away here is that fairness guarantees win-win cooperation in an NEMG.

6 Future Work

It would be a good option to investigate the feasibility of learning suboptimal strategies (e.g., ego-network-based approaches) that leverage localized information to approximate globally optimal solutions (e.g., full-network strategies) under the realistic constraint of limited local information access in real-world markets. Additionally, exploring the benefits of maintaining a dynamic strategy pool with multiple candidate policies to enhance action flexibility during evolution is warranted, with a focus on determining the optimal pool size for ensuring expected payoffs and its dependence on the topological properties of the agent's network position (e.g., node centrality, degree distribution, or clustering coefficients). Furthermore, generalizing these findings across diverse network structures and market environments to develop adaptive mechanisms for autonomous adjustment of strategy pools and local information utilization based on real-time dynamics remains an open and promising direction.

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This work was completed independently by the author, including the key idea, experiment design, experiment implementations, and paper writing.

This work was inspired by a previous work of the author in which variants of the evolutionary mechanism in an Evolutionary Minority Game(EMG) were investigated. Based on it, we studied the networked version by applying machine learning techniques to form this work. For your reference, detailed information about the previous work such as codes and report can be found in the Github repository: https://github.com/Chwgraph/Evolutionary-Minority-Game.

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How This Project Relates to the Course

In the course, we studied the Convolutional Neural Network(CNN) as a useful architecture to handle image data. The convolutional layer is the key structure we use. Graph Convolutional Networks (GCNs) are a class of deep learning models designed to process graph-structured data, which consists of nodes and edges. They extend convolutional neural networks (CNNs) to non-Euclidean data by leveraging graph topology to propagate and aggregate node information. Hence, it can be considered a natural generalization of CNN. Despite the graph convolutional layer, the use of the fully connected layer, the activation functions, and dropout settings are exactly the same.