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Reconfigurable Fault Tolerant Flight Control based on Nonlinear Model Predictive Control

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Abstract— **Constrained Nonlinear Model Predictive Control (NMPC) is shown to have potentials for reconfigurable fault tolerant control of highly nonlinear, intrinsically unstable, high performance aircraft. Results on fault tolerance of NMPC autopilots were obtained for an F-16 fighter aircraft model, without the implementation of any prestabilizing controllers. It has been shown that NMPC has inherent fault detection capabilities due to its effective utilization of feedback and its internal model predictions. Actuator (control surface) faults, including extreme cases of total actuator failure are examined as test cases for the NMPC reconfigurable fault tolerant control scheme developed in this work. The NMPC autopilots implementation and simulations were done using the ACADO nonlinear optimization solver.**

I. INTRODUCTION

The increasing complexity of modern and future flight control systems demands advanced and reliable control techniques that will ensure safety and survivability. When faults occur, it is very important that system stability is maintained and an acceptable system performance is attained.

The reconfigurable fault-tolerant control systems review in [1] highlights important limitations on conventional approaches to solving reconfigurable control problems for constrained multivariable systems and systems with significant nonlinearities. In general, how to design fault-tolerant control systems which can work effectively in the entire range of nonlinear systems, and how to distinguish the changes induced by faults from that by operating condition variations is still a challenge.

In the past few decades it became apparent that predictive control methods display qualities that could be utilized in complex, nonlinear flight control applications [2]. Model predictive control, in general, has been identified as a method that offers good possibilities for reconfiguration and fault-tolerant control [1], [3]–[6]. This claim is simply due to model predictive control's ability of handling most of the challenges of reconfigurable control in a generic and systematic manner. The choice of NMPC over standard nonlinear control methods is therefore justified by the fact that they are not developed in order to handle constraints in a systematic way. In addition, many nonlinear methods depend on complicated design procedures that do not scale well to large systems [7].

Most fault tolerant control systems rely on or integrate fault detection and diagnosis (FDD) subsystems in order to accommodate component failures automatically. This work reveals further benefits of NMPC's limited reliance on external FDD systems and external reconfiguration mechanisms, compared to earlier indications in [3], which designed a model predictive controller for a linear stable aircraft model.

The work presented here deviates from the most common way of NMPC implementation. Predictive control in general is usually implemented on top of low-level controllers which take care of fast system dynamics and stabilization (if necessary) [2], [3]. In [8], NMPC is shown to provide better performance compared to a Linear Parameter Varying (LPV) control approach for the F-16 model used in the work presented here. The NMPC scheme in [8] was implemented on a prestabilized F-16 aircraft model, and it employs regulation of state perturbations to a set as its control objective. The present work, on the other hand, explores both fast inner-loop and relatively slow outer-loop NMPC autopilot functionalities for time varying references, and therefore demonstrates the powerful potentials of NMPC as capable of replacing the prestabilizing low-level controllers. This work focuses on the inherent fault tolerance of NMPC, and therefore reveals NMPC capabilities in the absence of a model reflecting severe actuator faults.

II. THE AIRCRAFT MODEL

In aircraft control design, it is a common practice to use decoupled equations of motion. The nonlinear equations of motion are based on formulations in [9].

The *longitudinal* equations are as follows:

$$\dot{h} = V_T \cos\alpha \sin(\gamma+\alpha) - V_T \sin\alpha \cos(\gamma+\alpha) \tag{1}$$
$$\dot{V}_T = (F_T \cos\alpha - D - \mathrm{m}g\sin\gamma)/\mathrm{m} \tag{2}$$
$$\dot{\gamma} = (F_T \sin\alpha + L - \mathrm{m}g\cos\gamma)/\mathrm{m}V_T \tag{3}$$
$$\dot{\alpha} = Q - \dot{\gamma} \tag{4}$$
$$\dot{Q} = \mathscr{M}/I_y \tag{5}$$

where m is the aircraft mass, $\mathscr{M}$ is the pitching moment, I_y is the moment of inertia about the *body* y-axis, and g is the gravitational acceleration. The drag force D and the lift force L are calculated from:

$$D = -X_T \cos\alpha - Z_T \sin\alpha$$
$$L = X_T \sin\alpha - Z_T \cos\alpha$$

where X_T and Z_T are the total longitudinal and vertical aerodynamic forces respectively. The longitudinal differential states and parameters are described in tables I and II.

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TABLE I: NMPC flight path control design parameters and prediction model

Differential states:	γ	flight path
	Q	pitch rate
	α	angle of attack
	V_T	total speed
	$\delta_{e1}, \delta_{e2}, \quad \dot{\delta}_{e1}, \dot{\delta}_{e2}$	elevator states
Controls:	$\delta c_{e1}, \delta c_{e2}$	elevator command
	F_T	thrust command
Model Parameter:	$\bar{q}$	dynamic pressure
Optimization/prediction parameter values:		
	$T_s = 0.05\,\mathrm{s}$	sample time
	$T_h = 1.0\,\mathrm{s}$	horizon (start: 0.0 s)
	$N = 20$	intervals in horizon
Diagonal weighting matrix values:		
	$\mathscr{Q}_p(1,1) = 100.0$	h error weight
	$\mathscr{Q}_p(2,2) = 0.010$	Q error weight
	$\mathscr{Q}_p(3,3) = 0.100$	α error weight
	$\mathscr{Q}_p(4,4) = 0.001$	V_T error weight
	$\mathscr{R}_p(1,1) = \mathscr{R}_p(2,2) = 0.001$	δ_{e1}, δ_{e2} error weights
	$\mathscr{R}_p(3,3) = 1.000$	F_T error weight
Objective function parameters:		
	$\{\gamma, Q, \alpha, V_T, \delta c_{e1}, \delta c_{e2}, F_T\}$	states and controls
	$\{r_\gamma, 0, 4^\circ, 700\,\mathrm{ft/s}, \delta_{e1_0}, \delta_{e2_0}, 2169\,\mathrm{lb}\}$	setpoints
Differential equation (prediction model):		
(2) - (5), and (10) for elevators		
	$[\gamma_0, Q_0, \alpha_0, V_{T0}, \delta_{e1_0}, \delta_{e2_0}, \dot{\delta}_{e1_0}, \dot{\delta}_{e2_0}], [\bar{q}_0]$	initial values (last measured values)
Constraints on states, parameters and controls:		
	$-10^\circ \leq \alpha \leq 45^\circ$	angle of attack
	$900\,\mathrm{ft/s} \leq V_T \leq 1000\,\mathrm{ft/s}$	total speed
	$-45^\circ \leq \gamma \leq 45^\circ$	flight path
	$-15^\circ \leq \delta c_e \leq 15^\circ$	elevator deflection
	$-60^\circ/\mathrm{s} \leq \dot{\delta} c_e \leq 60^\circ/\mathrm{s}$	elevator rate
	$1000\,\mathrm{lb} \leq F_T \leq 19000\,\mathrm{lb}$	thrust force
	$\bar{q} = \bar{q}_0$	dynamic pressure

The equations of motion for the *lateral* channel assume constant longitudinal states α_0, V_{T0}, Θ_0 (pitch angle):

$$\dot{\beta} = P \sin\alpha_0 - R\cos\alpha_0 + (C - F_{T0}\cos\alpha_0 \sin\beta + \mathrm{m}g_2)/\mathrm{m}V_{T0} \quad (6)$$

$$\dot{P} = \mathscr{L}\, I_z/(I_xI_z - I_{xz}^2) + \mathscr{N}\, I_{xz}/(I_xI_z - I_{xz}^2) \quad (7)$$

$$\dot{R} = \mathscr{L}\, I_{xz}/(I_xI_z - I_{xz}^2) + \mathscr{N}\, I_x/(I_xI_z - I_{xz}^2) \quad (8)$$

$$\dot{\Phi} = P + R\tan\Theta_0 \cos\Phi \quad (9)$$

where $\mathscr{L}$ and $\mathscr{N}$ are the rolling and yawing moments respectively. I_x and I_z are, respectively, the moments of inertia about the *body* x and z axes, and I_{xz} is the product of inertia. The gravity component g_2 takes the form:

$$g_2 = g(\cos\alpha_0 \sin\beta \sin\Theta_0 + \cos\beta \sin\Phi \cos\Theta_0 - \sin\alpha_0 \sin\beta \cos\Phi \cos\Theta_0)$$

and the cross-wind force C is calculated from:

$$C = -X_{T0}\cos\alpha_0 \sin\beta + Y_T \cos\beta - Z_{T0}\sin\alpha_0 \sin\beta$$

where Y_T is the total transverse force. The lateral differential states and parameters are described in tables III and IV.

All the actuators of the control surfaces are modeled as first order dynamics with time constants $T = 0.0495\,\mathrm{s}$ (which specify the bandwidths). That is,

$$\dot{\delta}(t) = T^{-1}(\delta_c(t) - \delta(t)) \quad (10)$$

TABLE II: NMPC altitude-hold (climb control) autopilot design parameters and prediction model

Differential state:	h	altitude
Controls:	γ_c	flight path command
Model Parameters:	α	angle of attack
	V_T	total speed
Optimization/prediction parameter values:		
	$T_s = 0.5\,\mathrm{s}$	sample time
	$T_h = 5.0\,\mathrm{s}$	horizon (start: 0.0 s)
	$N = 10$	intervals in horizon
Weighting matrix values:		
	$\mathscr{Q}_a(1,1) = 0.01$	h error weight
	$\mathscr{R}_a(1,1) = 0.10$	γ error weight
Objective function parameters:		
	$\{h, \gamma_c\}$	state and control
	$\{r_h, \gamma_0\}$	setpoints (γ_0: last measured value)
Differential equation (prediction model):		
	(1)	altitude dynamics
	$[h_0], [\alpha_0, V_{T0}]$	initial values (last measured values)
Constraints on states, parameters and controls:		
	$-833.3\,\mathrm{ft/s} \leq \dot{h} \leq 833.3\,\mathrm{ft/s}$	climb rate
	$-25^\circ \leq \gamma_c \leq 25^\circ$	flight path command
	$\alpha = \alpha_0$	angle of attack
	$V_T = V_{T0}$	total speed

where δ_c is the commanded input and δ is the actual actuator deflection. The control surfaces are considered as the main control effectors in this work. The engine thrust force was therefore kept constant in all the test simulations.

The nonlinear mathematical model uses aerodynamic data from NASA-Langley wind tunnel tests on a subscale model of an F-16 aircraft [10]. The F-16 aerodynamic look-up tables and the coefficient equations used to sum up the contributions to the forces $X_T(\alpha,\beta,Q,\delta_e)$, $Y_T(\alpha,\beta,P,R,\delta_a,\delta_r)$, $Z_T(\alpha,\beta,Q,\delta_e)$, and the moments $\mathscr{L}(\alpha,\beta,P,R,\delta_a,\delta_e,\delta_r)$, $\mathscr{M}(\alpha,\beta,Q,\delta_e)$, $\mathscr{N}(\alpha,\beta,P,R,\delta_r,\delta_a,\delta_e)$, are also given in [9], [10]. The original (*high fidelity*) data cover -20° to 90° angle of attack (α) range, and sideslip angle (β) range from -30° to 30°. For simplicity and reduction of required computational power, the *low fidelity* data found in [9] were used in the NMPC autopilot prediction model calculations. The reduced aerodynamic data cover a $-10^\circ \leq \alpha \leq 45^\circ$ range, and the β dependence is also approximated in some cases. However, the model constructed from the low fidelity data exhibits steady state flight trim conditions, and corresponding dynamic modes that are close to those of the full high fidelity data model [9].

III. THE NMPC PROBLEM

Model Predictive Control (MPC) is an advanced control methodology that uses a multivariable process model (*internal model*) to predict future system and control behavior. MPC using *nonlinear* process models is termed nonlinear MPC (NMPC). Typical reasons for using NMPC are that the process operates in several steady states with significantly different dynamics, or there are large disturbances that excite nonlinearities.

The formulation of the NMPC problem includes constraints on inputs (*manipulated variables*) and states (*con-*

TABLE III: NMPC turn coordination (roll-angle hold) autopilot parameters and prediction model

Differential states:	β	sideslip
	Φ	roll
	P	roll rate
	R	yaw rate
	$\delta_{a1},\,\delta_{a2},\quad \dot{\delta}_{a1},\,\dot{\delta}_{a2}$	aileron states
	$\delta_r,\,\dot{\delta}_r$	rudder states
Controls:	$\delta c_{a1},\,\delta c_{a2}$	aileron command
	δc_r	rudder command
Model Parameters:	$\bar{q}$	dynamic pressure
	α	angle of attack
	V_T	total speed
	Θ	pitch
	F_T	thrust force
	$X_T,\quad Z_T$	total force in X, Z
Optimization/prediction parameter values:		
	$T_s = 0.05\,\mathrm{s}$	sample time
	$T_h = 1.0\,\mathrm{s}$	horizon (start: 0.0 s)
	$N = 20$	intervals in horizon
Diagonal weighting matrix values:		
	$\mathscr{Q}_t(1,1) = 1.00$	β error weight
	$\mathscr{Q}_t(2,2) = 10.0$	Φ error weight
	$\mathscr{Q}_t(3,3) = 1.00$	P error weight
	$\mathscr{Q}_t(4,4) = 1.00$	R error weight
	$\mathscr{R}_t(1,1) = \mathscr{R}_t(2,2) = 0.05$	$\delta_{a1},\,\delta_{a2}$ error weights
	$\mathscr{R}_t(3,3) = 0.10$	δc_r error weight
Objective function parameters:		
	$\{\beta,\,\Phi,\,P,\,R,\,\delta c_{a1},\,\delta c_{a2},\,\delta c_r\}$	states and controls
	$\{0,\,r_\Phi,\,0,\,0,\,\delta_{a1_0},\,\delta_{a2_0},\,\delta_{r0}\}$	setpoints
Differential equation (prediction model):		
	(6) - (9), and (10) for ailerons and rudder	initial values:
	$[\beta_0,\Phi_0,P_0,R_0,\delta_{a1_0},\dot{\delta}_{a2_0},\dot{\delta}_{a1_0},\dot{\delta}_{a2_0},\delta_{r0},\dot{\delta}_{r0}]$	for states
	$[\bar{q}_0,\,\alpha_0,\,V_{T0},\,\Theta_0,\,F_{T0},\,X_{T0},\,Z_{T0}]$	for parameters (last measured values)
Constraints on states, parameters and controls:		
	$-308^\circ/\mathrm{s} \le P \le 308^\circ/\mathrm{s}$	roll rate
	$-15^\circ \le \delta c_a \le 15^\circ$	aileron deflection
	$-80^\circ/\mathrm{s} \le \dot{\delta} c_a \le 80^\circ/\mathrm{s}$	aileron rate
	$-15^\circ \le \delta c_r \le 15^\circ$	rudder deflection
	$-120^\circ/\mathrm{s} \le \dot{\delta} c_r \le 120^\circ/\mathrm{s}$	rudder rate
	$\bar{q} = \bar{q}_0$	dynamic pressure
	$\alpha = \alpha_0,\quad V_T = V_{T0}$	angle of attack, speed
	$\Theta = \Theta_0,\quad F_T = F_{T0}$	pitch, thrust force
	$X_T = X_{T0},\quad Z_T = Z_{T0}$	total force in X, Z

trolled variables). The formulated problem is then solved by using mathematical programming to optimize predicted future performance at each sampling interval.

Since NMPC relies on its internal model for predictions, it will require a fault model of the system if the severity of the damage is such that the dynamics of the system changes.

A. Formulation of the NMPC problem

A very general and simple formulation of the nonlinear model predictive problem has the control objective of minimizing a *cost function* J, which takes the form:

$$J(x[0,T],u[0,T]) = \int_0^T \ell(x(t),u(t),t)\,\mathrm{d}t \tag{11}$$

where $\ell(x(t),u(t),t)$ is a non-negative function, termed the *stage* cost, and $T > 0$ is the *horizon*. The stage cost $\ell(.)$ is chosen as an l_2 type cost function [7]:

$$\ell(x(t),u(t),t) = \|x(t)-r_x(t)\|^2_{\mathscr{Q}} + \|u(t)-r_u(t)\|^2_{\mathscr{R}}$$

TABLE IV: NMPC heading-hold autopilot parameters and prediction model

Differential state:	Ψ	heading
Controls:	Φ_c	roll command
Model Parameter:	V_T	total speed
Optimization/prediction parameter values:		
	$T_s = 0.5\,\mathrm{s}$	sample time
	$T_h = 5.0\,\mathrm{s}$	horizon (start: 0.0 s)
	$N = 10$	intervals in horizon
Weighting matrix values:		
	$\mathscr{Q}_h(1,1) = 10.10$	Ψ error weight
	$\mathscr{R}_h(1,1) = 0.100$	Φ error weight
Objective function parameters:		
	$\{\Psi,\,\Phi_c\}$	state and control
	$\{r_\Psi,\,\Phi_0\}$	setpoints
Differential equation (prediction model):		
	(15)	bank-to-turn
	$[\Psi_0],\,[V_{T0}]$	initial values (last measured values)
Constraints on states, parameters and controls:		
	$-65^\circ \le \phi_c \le 65^\circ$	roll command
	$V_T = V_{T0}$	total speed

where the properties of the weight matrices $\mathscr{Q} \ge 0$ and $\mathscr{R} \ge 0$ are essential for performance, and in some cases also stabilty. The plant to be controlled has a state vector x (with setpoint r_x), an input vector u (with setpoint r_u), and a nonlinear behavior governed by the vector differential equation

$$\dot{x} = f(x,u) \tag{12}$$

In addition to the equality constraints introduced by the nonlinear plant model (12), general inequality constraints jointly on states and inputs can be defined as

$$g(x(t),u(t),t) \le 0 \tag{13}$$

and finally, the control input vector is constrained to some set $u(t) \in U$, where

$$U = \{u_{min} \le u(t) \le u_{max}\} \tag{14}$$

and u_{min}, u_{max} are, respectively, the minimum and maximum limits of $u(t)$.

In general, stability is not guaranteed for the simple NMPC formulation above, since the horizon is finite. Several approaches have therefore been proposed, introducing modifications such as *terminal* sets and constraints to ensure stability [11], [12]. However, stability enforcing terminal constraints may be relaxed, or even skipped completely, since they tend to be conservative and often not needed when the NMPC is carefully designed [7]. In this work, the NMPC autopilot design approach, the choice of horizons, and other design parameters resulted in the generation of reliable control trajectories, without the need of modifying the NMPC formulation. See [13] for further details, and [8] for a stability guaranteeing modification for the F-16 model.

Another key aspect of NMPC is the understanding of how design parameters such as the horizon, the weight matrices, and the time constant of the reference trajectory are tuned. The reference trajectory defines an ideal trajectory from the plant's current output to the desired set point, and therefore defines an important aspect of the closed-loop behavior of the NMPC scheme used in this work.

IV. NMPC AUTOPILOTS

A. Longitudinal Motion Control

An altitude-hold autopilot which relies on a flight path controller was implemented for maneuvers in the longitudinal channel. In a classical autopilot design sense, the altitude control is seen as the (slower) outer loop while the flight path control is the (faster) inner loop. The measured states of the flight path NMPC problem were introduced as time-varying parameters in the altitude NMPC problem formulation. Design details are shown in tables I and II. The design structure allows individual sampling times and horizons to be employed, reflecting the relative responses and sampling rates required for each autopilot dynamics.

B. Lateral Motion Control

The NMPC lateral autopilots include a Heading-hold controller, which performs coordinated turns using a roll-angle hold controller. The measured states of the turn-coordination (roll-angle hold) autopilot were introduced as time-varying parameters in the heading-hold NMPC problem formulation.

The following nonlinear bank-to-turn equation was used to specify the desired behavior of the aircraft during a turn maneuver.

$$\dot{\Psi} = g\sin(\Phi_c)/V_T \tag{15}$$

It states that a roll angle Φ_c different from zero will induce a heading rate $\dot{\Psi}$, which turns the aircraft. Design details are shown in tables III and IV, and the complete design process can be found in [13].

V. SOLVING THE NMPC PROBLEM

Many existing approaches to solving the NMPC problem are based on Sequential Quadratic Programming (SQP) methods, which make quadratic approximations to the objective function and linear approximations to the constraints, and iteratively solve a Quadratic Program (QP) to find the search direction leading to the optimal solution [7], [14].

The optimization package used in solving the NMPC problems formulated in this work is the ACADO Toolkit. Section V-A highlights the key features of ACADO and the optimizer parameters used in this work.

A. NMPC optimization solver: ACADO for MATLAB

ACADO Toolkit is an algorithm collection for automatic control and dynamic optimization, implemented as self-contained C++ code. The NMPC autopilots were implemented in two parts. The first consisting of the optimization problem, composed in MATLAB, and the second being the NMPC prediction model implemented as C-code.

The autopilot optimization problems were solved using ACADO's multiple-shooting SQP-type method combined with Runge-Kutta integrator (of order 4/5) for the state integration. Multiple-shooting is a discretization-type parameter option in ACADO. The KKT tolerance (Karush-Kuhn-Tucker first order (necessary) optimality conditions), which is used for the convergence criterion of the SQP algorithm, was set to 10^{-6} (default value in ACADO). The default value of the maximum number of iterations (1000) was also maintained for all simulations and tests. The maximum number of iterations offers a means of specifying a trade off between result accuracy and computational time. Smaller values will reduce the execution time, but the result could be far from optimal. Details about the ACADO Toolkit can be found in [15], [16].

VI. TEST CASES AND RESULTS

This section focuses on using control surface (or actuator) fault simulations to primarily verify the fault tolerant capabilities of NMPC as a reconfigurable controller.

The simulations were initialized at trim conditions where $h_0 = 20000\,\text{ft}$, $V_{T0} = 500\,\text{ft/s}$, $\alpha_0 = 5.53°$, $F_{T0} = 2168.71\,\text{lb}$, $\delta_{e1_0} = \delta_{e2_0} = -2.77°$, and $\delta_{a1_0} = \delta_{a2_0} = \delta_{r0} = 0°$. The test maneuvers began with a 20° heading change demand to the heading and turn coordination autopilots, while the altitude autopilot was commanded to maintain 20000 ft. The altitude set point was changed to 20200 ft after 20 s, and the heading demand and was stepped down to 10° after 30 s. The main objective is to stabilize the aircraft and perform turn and climb maneuvers as well as attaining straight level flight in the presence of control surface faults.

The results for three test cases are presented. Case *1* assumes the existence of an FDD system that updates the NMPC autopilot with new control surface position limits or rate limits when faults occur (according to the FDD scheme in [17]), but it does not implement any external reconfiguration mechanism. Case *2* assumes no FDD system exists, and therefore simulates the case where the control system relies on only the inherent fault detection and reconfigurable capabilities of NMPC (enabled through the effective use of measured actuator position feedback). Case *3* assumes the existence of an FDD system and, in addition, employs the following simple reconfiguration mechanism.

1) *for each actuator*, scan for locked/jammed actuator by checking for changes in lower and upper limits (worst case $\delta_{min} = \delta_{max}$)
2) if an actuator is locked, decrease the weight of all the 'healthy' actuators in the same channel (longitudinal or lateral), by a predefined factor.
3) finally, reduce the weight on any secondary objectives that share the same remaining 'healthy' actuators (that are capable of achieving similar effects as the jammed actuator), by a predefined factor.

The tests were performed while the left aileron was locked at −15°. The results of the three test cases are compared in figure 1. Apart from differences noticed in the rudder input actions (figure 1e) and their accompanying sideslip trajectories (in figure 1c), the trajectories of Case *1* and Case *2* are close. In Case *2*, the NMPC turn coordination controller quickly realizes through measured actuator position feedback (implemented as initial values) that the left aileron is not responding to its commands. In other words, the controller perceives a situation similar to when it has driven the aileron to its limit, and starts to move other control surfaces. Not using any information about the failure situation from an FDD system, the results of Case *2* can be considered as good, compared to those of Case *1* and Case *3*.

It can be emphasized that in Case *1*, the locked aileron fault was reported to the controller (by setting $\delta_{a1_{min}} = \delta_{a1_{max}} = -15°$) and, in addition, the weights (error penalties) were adjusted in Case *3*. The weights on keeping set-points for sideslip ($\mathscr{Q}_t(1,1)$), roll rate ($\mathscr{Q}_t(3,3)$), and yaw rate ($\mathscr{Q}_t(4,4)$) were reduced by a factor of 10, to further prioritize the use of roll angle to achieve the desired heading. The same reduction was made on the weight for the right aileron ($\mathscr{R}_t(2,2)$), with the intention of making it more active.

Another interesting result was obtained when the weight on set-point error for heading was reduced from 10 to 2.5. The intention was to relax the objective of fast control from a guidance system point of view. The results, compared to the fault-free aileron test, are shown in figure 2. The performance improvement indicates that some reduction in the overall performance objective that reflects the fault situation may be useful in the event of an extreme control surface failure in order to achieve graceful degradation of performance.

Test cases that examine free-play (float-type) failures and significant changes in actuator position or rate limits are documented in [13]. Test results show that these failures have very little effect on NMPC performance, since actuator constraints are handled effectively as part of NMPC's primary objectives. The free-floating type failure is also rather mild compared to the locked actuator or hard-over failure tests, since no significant disturbance is introduced by the 'floating' control surface.

Simulations of the complete test setup, including all four autopilots and the fully coupled nonlinear F-16 model (plant), were done on a single core of a computer with Intel Core i7-3720QM 2.6 GHz processor and 8 GB RAM. Average computational times of 0.004 s, 0.006 s, 0.093 s, and 0.044 s were recorded for the heading-hold, altitude-hold, turn coordination, and flight path control autopilots respectively.

VII. CONCLUSIONS

This work illustrates the elegance of combining multivariable, nonlinear, and constrained controller behavior into a well structured and powerful NMPC framework in order to implement reconfigurable fault tolerant control.

Fault tolerance of NMPC has been shown, through autopilot implementation results, to be achievable through an effective combination of fault detection capabilities and reconfiguration. Observations made indicate a clear tendency of NMPC to effectively attempt to accomplish the set objectives, even with no explicit identification of the fault.

It can be further argued that in cases where fault detection is not available or the available FDD system is not reliable, fault tolerant graceful degradation of performance is achievable when the NMPC utilizes its inherent robustness.

Real-time implementation is known as a challenging aspect of NMPC. However, using fast NMPC solvers and/or hardware with adequate computational capacity, real-time implementation of the NMPC autopilots is possible. Moreover, the proposed autopilot formulations allow implementation in separate tasks on a multi-core computer or on separate processors.

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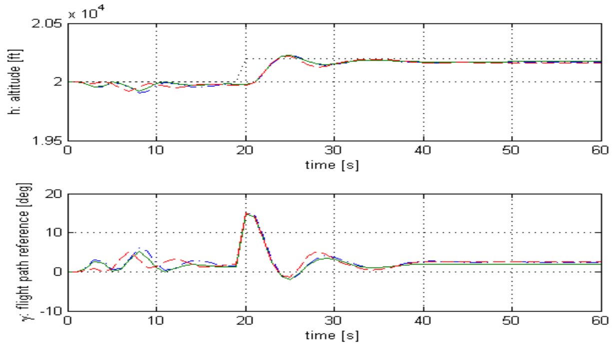
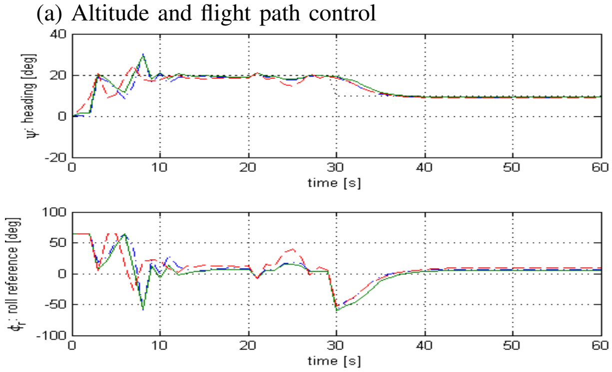
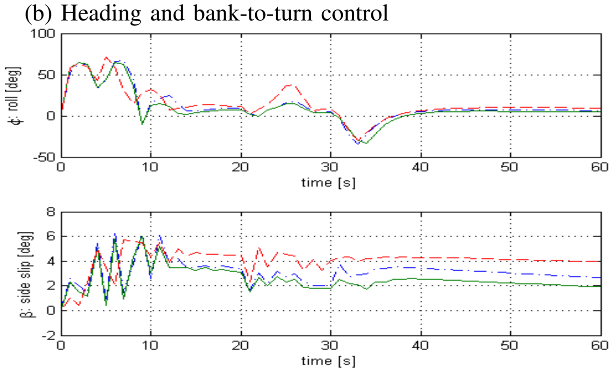


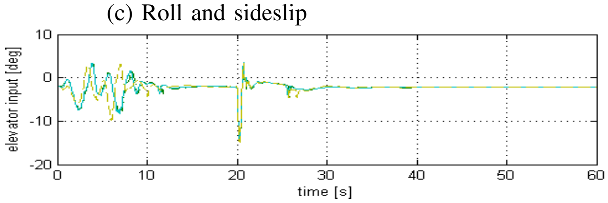
(a) Altitude and flight path control



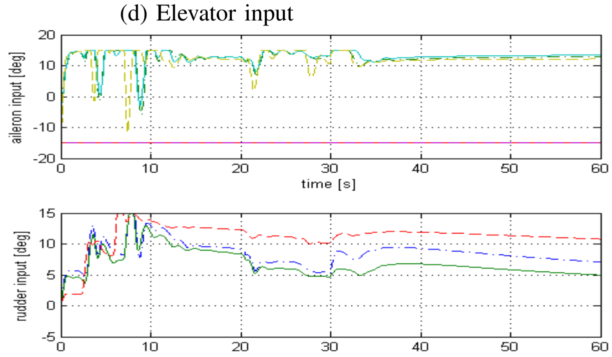
(b) Heading and bank-to-turn control



(c) Roll and sideslip

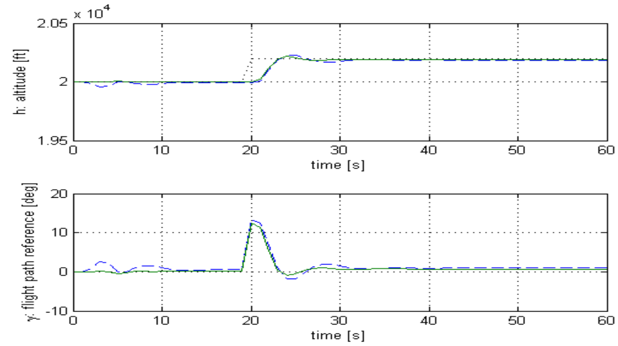


(d) Elevator input

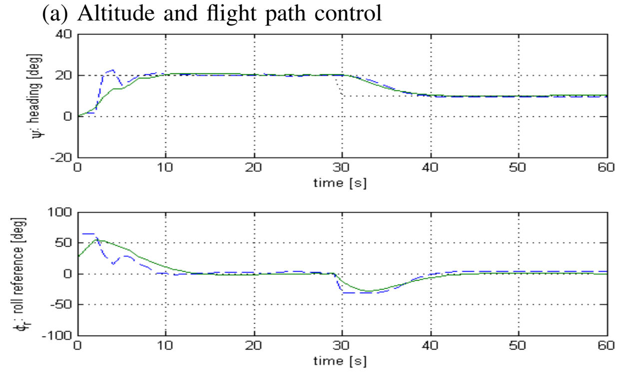


(e) Aileron and rudder inputs

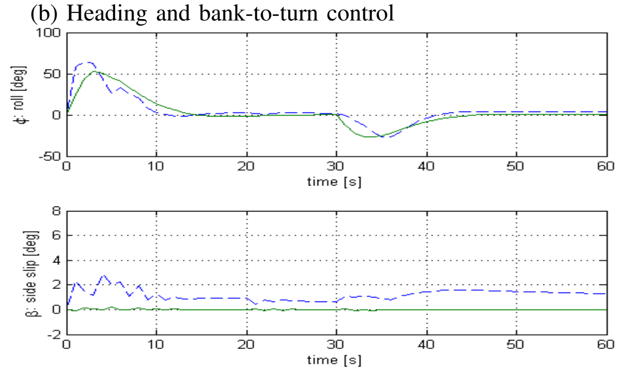
Fig. 1: NMPC fault tolerance: F-16 left aileron (δ_{a1}) locked at -15°. Case *1* (—·), Case *2* (—), Case *3* (– –).



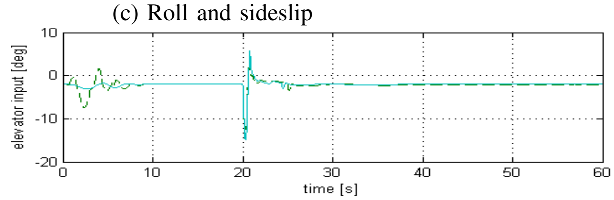
(a) Altitude and flight path control



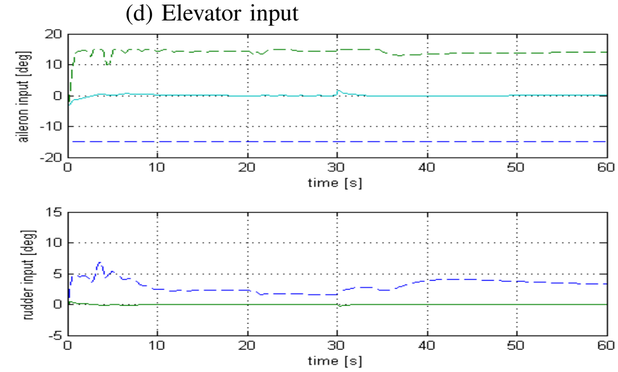
(b) Heading and bank-to-turn control



(c) Roll and sideslip



(d) Elevator input



(e) Aileron and rudder inputs

Fig. 2: Relaxed heading control objective. Left aileron (δ_{a1}) locked at -15° (Case *2* – –). Fault-free aileron test (—).