State-dependent Riccati equation-based robust dive plane control of AUV with control constraints

通过非线性系统模型：

$$\dot{x} = \begin{bmatrix} A_1(x,p) & A_2(x,p) \\ A_3(x) & A_4(x) \end{bmatrix} x + \begin{bmatrix} B_1(p) \\ 0 \end{bmatrix} \delta_s + \begin{bmatrix} d_1 \\ 0 \end{bmatrix}$$
$$\triangleq A(x,p)x + B(p)\delta_s + d, \tag{7}$$

增广跟踪误差：

$$\dot{x}_{s1} = e = z - z_r = Cx - z_r. \tag{12}$$

The signal $x_{s1}(t)$ is the integral of the depth trajectory tracking error.

Define the augmented state vector as $x_{a1} = (x^{\mathrm{T}}, x_{s1})^{\mathrm{T}} \in \Omega_{a1} \subset R^5$, where Ω_{a1} is the open neighborhood of the origin. Then the composite system Eqs. (7) and (12) can be written as:

$$\dot{x}_{a1} = \begin{bmatrix} A(x,p) & 0_{4\times 1} \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_{s1} \end{bmatrix} + \begin{bmatrix} B(p) \\ 0 \end{bmatrix} \delta_s + \begin{bmatrix} d \\ -z_r \end{bmatrix}$$
$$\triangleq A_{a1}(x,p)x_{a1} + B_{a1}(p)\delta_s + Ev, \tag{13}$$

求解优化问题：

$$J_1 = \frac{1}{2}\int_0^{\infty} [x_{a1}^{\mathrm{T}} Q_1(x_{a1})x_{a1} + R_1\delta_s^2]\,\mathrm{d}t, \tag{14}$$

非线性微分方程很难求解，该文章采用了次优控制律设计，使用的是 SDRE 方法：

method (Cloutier et al., 1998). This control law is obtained by solving a simplified state-dependent Riccati equation given by

$$A_{na1}^{\mathrm{T}}(x)P_1 + P_1A_{na1}^{\mathrm{T}}(x) - P_1B_{na1}R_1^{-1}B_{na1}^{\mathrm{T}}P_1 + Q_1(x_{a1}) = 0, \tag{16}$$

该方法可以处理约束（饱和）问题，约束为：

For the design of a constrained control law, now we introduce a hard constraint on the fin angle given by

$$|\delta_s| \leqslant \delta_{sm}, \tag{27}$$

处理方法为将有约束控制问题转换为等效非线性调节器问题：

According to Cloutier et al. (1998), the design is accomplished by transforming the bounded control problem into an equivalent nonlinear regulator problem by introducing a slack variable x_{s2} that satisfies

$$\dot{x}_{s2} = u_{\mathrm{n}}, \tag{28}$$

where u_{n} is the new control input. The fin angle takes the form of a saturation sin function, given by

$$\delta_s = \mathrm{satsin}(\delta_{sm}, x_{s2}), \tag{29}$$

$$\mathrm{satsin}(\delta_{sm}, x_{s2}) = \begin{cases} \delta_{sm}\,\mathrm{sgn}(x_{s2}) & \text{for } |x_{s2}| > \dfrac{\pi}{2}, \\ \delta_{sm}\sin(x_{s2}) & \text{for } |x_{s2}| \leqslant \dfrac{\pi}{2}. \end{cases} \tag{30}$$

该方法的优点是通过求解优化问题得到控制律，可以处理约束，可以结合 MPC 使用；缺点是求解的不是最优解，动态性能无法保证；文章也只仿真了参考指令不变的情况，无法保证是否合适地形探测这种参考指令实时改变的情况。

Depth Control of Model-Free AUVs via Reinforcement Learning

该文章考虑到动力学模型未知以及AUV的纵摆运动和横摆运动之间的耦合，这些问题无法通过大多数**基于模型**或比例积分微分的控制器来有效解决。文章描述了在未知转移概率下的连续状态、连续动作马尔可夫决策过程AUV的深度控制问题。基于确定性策略梯度定理和神经网络近似，提出了一种**无模型强化学习**（RL）算法，从 AUV 的采样轨迹中学习状态反馈控制器。

状态和模型表示：

$\boldsymbol{\chi} = [z, \theta, w, q]^T$, including heave position z, heave velocity w, pitch orientation θ, and pitch angular velocity q.

The dynamical equation of the AUV is defined as follows:

$$\dot{\boldsymbol{\chi}} = f(\boldsymbol{\chi}, \boldsymbol{u}, \boldsymbol{\xi}) \tag{1}$$

参考指令根据 x 坐标计算得出：

The purpose of the depth control is to control the AUV to track a desired depth with minimum energy consumption, where the desired depth trajectory z_r is given by

$$z_r = g(x). \tag{2}$$

文章考虑了恒定深度控制、曲线深度控制以及海底地形跟踪控制三种情景，使用马尔可夫决策过程（MDP）进行控制

Depth control of the INFANTE AUV using gain-scheduled reduced order output feedback

本文解决了在**没有完整状态信息**的情况下自主水下航行器（AUV）控制的问题。应用于在垂直平面上控制原型AUV。控制器设计采用的方法是非线性的增**益调度控制**。

本文总结了控制器设计步骤，值得参考。

在固定螺旋桨转速下的解耦建模：

Surge motion equation:

$$m\dot{u} = C_X u^2 + C_{X_{ww}} w^2 + C_{X_{qq}} q^2 + u^2 C_{X_{\delta_b \delta_b}} \delta_b^2 + u^2 C_{X_{\delta_s \delta_s}} \delta_s^2 + C_{X_{\dot{u}}} \dot{u} + T, \tag{9}$$

Heave motion equation:

$$m(\dot{w} - uq) = (W - B)\cos(\theta) + C_{Z_w} uw + C_{Z_q} uq + C_{Z_{\delta_b}} \delta_b + C_{Z_{\delta_s}} \delta_s + C_{Z_{\dot{w}}} \dot{w} + C_{Z_{\dot{q}}} \dot{q}, \tag{10}$$

$$\dot{z} = -u \sin(\theta) + w \cos(\theta), \tag{11}$$

Pitch motion equation:

$$I_y \dot{q} = z_{CB} B \sin(\theta) + C_{M_w} uw + C_{M_q} uq + C_{M_{\delta_b}} \delta_b + C_{M_{\delta_s}} \delta_s + C_{M_{\dot{w}}} \dot{w} + C_{M_{\dot{q}}} \dot{q}, \tag{12}$$

$$\dot{\theta} = q, \tag{13}$$

控制系统设计：

(i) *Linearizing* the plant about a finite number of representative operating points,
(ii) Designing *linear controllers* for the plant linearizations at each operating point,
(iii) *Interpolating* the parameters of the linear controllers of Step (ii) to achieve adequate performance of the linearized closed-loop systems at all points where the plant is expected to operate. The interpolation is performed according to an external scheduling variable (vehicle's forward speed), and the resulting family of linear controllers is referred to as a *gain-scheduled controller*,
(iv) *Implementing* the gain-scheduled controller on the original nonlinear plant.

在有代表性的操作点线性化，在这些点上设计线性控制器，然后进行插值。

不适用于容错控制。

Experimentally verified depth regulation for AUVs using constrained model predictive control

这篇文章将非线性模型进行线性化，通过将升降速度、俯仰角速度设为 0 ，以及将阻尼力（damping force） 设为一个中间值（合适的值）。得到线性化模型后用标准 MPC 计算控制律。

Robust depth control of a hybrid autonomous underwater vehicle withpropeller torque’s effect and model uncertainty

本文对混合动力AUV的深度跟踪控制器设计进行了研究，该研究存在模型不确定性和螺旋桨扭矩的影响。首先，六自由度（6-DOF）非线性运动方程，以及混合AUV的操作机制和具体特性。随后通过对6自由度AUV**模型进行解耦和线性化，提取了深度平面模型**。此外，一个构建了**非线性扰动观测器（NDO）来处理深度平面模型中的线性化误差和不确定分量**，然后根据反步设计深度跟踪控制器保证跟踪误差收敛到任意小邻域为零的技术。

3.1. NDO design

To estimate d, we define the new variable $F = d - Lq$, where $L > 0$ is the observer gain which needs to be selected later. Then the nonlinear disturbance observer for d is constructed as follows:

$$\begin{cases} \widehat{d} = \widehat{F} + Lq \\ \dot{\widehat{F}} = -L\left(f_{\dot{q}} + g_{\dot{q}} x_m + \widehat{d}\right) \end{cases} \tag{14}$$

where $\widehat{d}$ and $\widehat{F}$ are the estimations of d and F, respectively. The estimated error is defined:

$$\widetilde{d} = d - \widehat{d}$$

and its derivative is:

$$\dot{\widetilde{d}} = \dot{d} - \dot{\widehat{d}} = \dot{d} - \dot{\widehat{F}} - L\dot{q} = \dot{d} + L\left(f_{\dot{q}} + g_{\dot{q}} x_m + \widehat{d}\right) - L\left(f_{\dot{q}} + g_{\dot{q}} x_m + d\right) = \dot{d} - L\widetilde{d} \tag{15}$$

这篇文章的非线性扰动观测器思路值得参考，通过扰动观测器来补偿线性化时的误差。AUV的运动是一个较为缓慢的过程，并且能够长时间保持在稳定的状态，所以有足够的时间让观测器收敛。