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Model Predictive Control for an AUV with Dynamic Path Planning

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Abstract: This paper investigates the model predictive control (MPC) for an autonomous underwater vehicle (AUV). We aim to develop a tracking control algorithm integrated with a dynamic path planning for the AUV. Considering that the effective range of onboard sensors cannot be large, we formulate the path planning problem into a receding horizon optimization framework with spline path templates. Once the local optimal path is constructed for the current time, it is viewed as a reference trajectory of the vehicle. In order to control the depth of AUV simultaneously and to have a friendly interaction with the dynamic path planning method, a nonlinear model predictive control (MPC) scheme is adopted. The simulation results demonstrate the effectiveness of the proposed tracking control algorithm.

Keywords: Autonomous Underwater Vehicle, Path Planning, Receding Horizon, Tracking, Nonlinear Model Predictive Control

1. INTRODUCTION

The core characteristic of AUVs is the ability of choosing appropriate actions in light of the objectives without interventions of human pilots [1]. Such high level of autonomy depends on an elaborately designed control system, while the highly nonlinear, time-varying AUV dynamics [2] challenges the designing process. The majority of AUV controllers design take a classic linearization regime followed by the extensively studied linear control techniques such as proportional-integral-derivative (PID)[2, 3]. However, AUVs are typically small multi-purpose underwater vehicles, operating over numerous working points, which complicates the application of linear control theory. Nonlinear effects from hydrodynamic damping and lift, added mass, Coriolis and centripetal forces may produce highly degraded performance even though AUVs are operating a little bit far from preset working points. Therefore, AUV applications using advanced control techniques are constantly drawing researchers' attention. In [3] Yuh summarizes a wide range of advanced controllers studied for AUVs including sliding mode control, nonlinear control, adaptive control, neural network control and fuzzy control.

Meanwhile, model predictive control (MPC), known as a receding horizon optimization based control strategy, has been extensively studied for more than forty years [14]. Due to its handling of nonlinearities and directly enforcing constraints, MPC finds an increasing number of applications across different disciplines[4-7]. With the constantly updating computer technology as well as the great breakthrough in nonlinear programming area [15], MPC appears an attractive solution in AUV control community. Kwiesielewicz [8] shows that an MPC controller could have better performance comparing to a PID controller and an adaptive controller; Caldwell [9] combines the sampling based motion planning and MPC with the kinematic model of AUVs; Medagoda [10] explores the MPC application to an AUV providing an estimate of system state in the presence of ocean current; Truong [11] develops an intermittent MPC controller for AUVs that can tolerate lower sampling frequency. Nevertheless, the study of MPC into AUV field is relatively immature, which offers both opportunities and challenges.

AUV control problems can generally be categorized into four different types [13]: vertical and horizontal plane control, pose control, trajectory tracking and path following control, and cooperative motion control of multiple vehicles. Conventionally, people design a controller exclusively for one purpose of them, e.g. [16, 17]. However, in view of the typical form of the objective function, MPC controller is capable of dealing with the scalarization of multi-objective optimization problems, which means it might be possible to tackle two or more control goals simultaneously. In this paper, we consider the tracking control problem of an AUV in the xy-plane. At the same time, the depth and orientation of the AUV are also taken into account within the MPC scheme. Before tracking, as a highly autonomous system, AUV itself is responsible for generating the reference signal. A modified path planning method is proposed for the generation of reference signals. Subsequently, the AUV control problem can be formulated into a nonlinear model predictive control scheme. The contributions of this paper are three-fold:

- A receding horizon optimization framework is proposed for the reference path planing.
- With an MPC scheme, in addition to the tracking task, the depth and orientation of the AUV can also be taken into consideration.
- By the receding horizon nature of both MPC and the proposed path planning method, a control algorithm smoothly incorporating tracking control with reference path generation is developed.

† Chao Shen is the presenter of this paper.

The remainder of this paper is organized as follows. In Section 2, we present the problem formulation. In Section 3, a receding horizon path planing method is proposed. Section 4.1 introduces the mathematical model of the AUV; formulation of the tracking control problem into an MPC scheme is discussed in Section 4.2; in Section 4.3, the control algorithm combining MPC and path planning for the AUV is depicted. Simulation results are demonstrated in Section 5; Section 6 concludes the paper.

The notations in this paper are explained as follows. $\mathbf{E} > \mathbf{0}$ represents that the matrix $\mathbf{E}$ is positive definite; the identity matrix is denoted as $\mathbf{I_n}$; given a vector $\boldsymbol{\lambda}$, $\| \boldsymbol{\lambda} \|$ denotes the Euclidian norm while $\| \boldsymbol{\lambda} \|_P$ denotes the weighted norm $\sqrt{\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{P}\boldsymbol{\lambda}}$; the column operation $[\boldsymbol{\lambda}_1^{\mathrm{T}}, ...\boldsymbol{\lambda}_n^{\mathrm{T}}]^{\mathrm{T}}$ is denoted by $col(\boldsymbol{\lambda}_1, ..., \boldsymbol{\lambda}_n)$. Furthermore, for a function $h(x)$, the time derivative is denoted as $\dot{h}$ while the derivative with respect to x is denoted as h'. The diagonal operation is abbreviated by $\mathrm{diag}(\cdot)$.

2. PROBLEM STATEMENT

The work condition of AUVs is limited in space or cluttered with obstacles, e.g., exploring an underwater shipwreck or source seeking applications [21]. If this is the case, path planning or reference trajectory generation should be incorporated into the control design. As shown in Fig.1, a reference path (dashed line) is initially to be calculated and then the AUV is controlled to track the generated path.

Definitions of the reference path may be diverse for various applications. Basically, the reference path is at least continuous with some objectives optimized, such as piecewise linear path with minimum fuel consumption. The higher order of smoothness may be expected. That is, the resulting trajectory of path planner is continuous with its derivatives. In many industrial applications, a feasible trajectory with smooth 2nd order derivative is required, since it represents that the acceleration of the control object is continuous.

At current stage, most AUVs are torpedo-shaped [2, 3] hence in an underactuated situation, e.g. *Sirius* [18]. Therefore, some researchers model the planar kinematic equation of AUVs with nonholonomic constraints [9, 21]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} cos\psi & 0 \\ sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} \tag{1}$$

where x and y are the position and ψ is the yaw angle, while u and r are the tangential and angular velocities of the AUV. Based upon (1), the open-loop control inputs u_{oc} and r_{oc} are calculated if there suppose no disturbances and initial errors

$$\begin{aligned} u_{oc} &= \sqrt{\dot{x}_r^2 + \dot{y}_r^2} \\ r_{oc} &= (\dot{x}_r\ddot{y}_r - \dot{y}_r\ddot{x}_r)/(\dot{x}_r^2 + \dot{y}_r^2) = u_{oc} \cdot \kappa \end{aligned} \tag{2}$$

where x_r and y_r are coordinates of the reference path, and κ represents the curvature. Like mobile robots, the existence of the minimum turning radius, or equivalently a maximum angular velocity prevents the AUV from tracking harsh trajectories in terms of too large path curvature. Seen from (2), to make the tracking easier for the underactuated AUV, a reference path with least curvature may be practical and helpful.

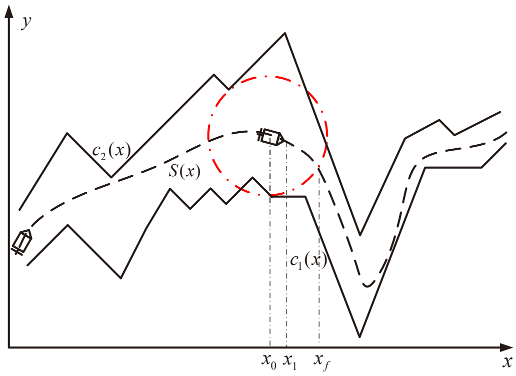

Fig. 1 *Illustration of the AUV Control Problem.*

However, due to large signal decaying rate in water, the effective sensing range onboard is indispensably limited (as the dashed circle shown in Fig.1). Consequently, the reference path has to be planned without global environment information. After the reference path being generated, the control system is responsible for steering the AUV to track it probably in the presence of constraints on states and/ or control inputs of the vehicle. Furthermore, when the AUV drives a little bit forward, new measured data of local environment information comes in, a new round of path planning and tracking should be performed. Therefore, in our application, the path planning and tracking control are inherently integrated.

3. LEAST CURVATURE OPTIMAL PATH PLANNING

Inspired by [22] and [12], for the path planning part, we take advantage of spline functions as the path template. Spline functions [19] own many desirable properties that make them suitable and very common for the path planning problem in engineering practice. A spline can be conveniently selected to provide enough order of smoothness and is parameterized to facilitate the optimal path computation.

A spline function represented in b-form is defined as

$$S(x) = \sum_{i=1}^{n} \alpha_i S_{i,k}(x) \tag{3}$$

where n is the number of knots, α_i are the control parameters and $S_{i,k}(x)$ are kth order basis splines defined recursively,

$$S_{i,1}(x) = \begin{cases} 1, & x_i \le x < x_{i+1} \\ 0, & otherwise \end{cases}$$

and

$$S_{i,k}(x) = \frac{(x - x_i)S_{i,k-1}(x)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - x)S_{i+1,k-1}(x)}{x_{i+k} - x_{i+1}}$$

For a spline $S(x)$, the measure of smoothness, which is also the objective function in the optimal path planning problem, is chosen as:

$$F(x)=\int_{x_0}^{x_f}\frac{\kappa'(x)^2}{\sqrt{1+S'(x)^2}}dx$$

where $\kappa(x)$ represents the curvature of the spline

$$\kappa(x)=\frac{S''(x)}{(1+S'(x)^2)^{3/2}}$$

For simplicity, the unknown environment in xy-plane is assumed as two polygonal chains which are piecewise linear functions (see Fig.1). Then the path planning problem can be formulated as an optimization problem:

Problem $\mathbf{A_0}$. Given upper and lower bounds of the environment $c_1(x)\le c_2(x)$, with proper numbers of knots n, order o and multiplicity l to computer spline function $S(x)$ such that

$$\begin{array}{ll} \min. & F(x) \\ \text{s.t.} & c_1(x)\le S(x)\le c_2(x) \end{array}$$

Here, we should notice that the resulting spline is always chosen with at least order 3 to guarantee the continuity of the acceleration of the AUV. At the very beginning, we may impose an additional constraint that the spline starts at the initial position of the vehicle, i.e. $S(x_0)=y_0$.

Remark 1. It is worth noting that the number of control parameters is equivalent to the number of knots n, thus the selection of knots appears a significant issue here. And this issue has been well discussed in [20].

In this paper, the larger number of knots makes the description of the environment more accurate, but it also makes the calculation of optimal path $S(x)$ more time-consuming. In view of the limited computational resource and memory size on chip, usually we require as small number of knots as possible without losing main characteristics of the environment. A possible way is to choose the local extrema of the environment. Accordingly, the environment is approximated by polygonal chains which connecting these extrema sequentially. A safe margin concept can be used [12].

Remark 2. Although we simplify the unknown xy-plane environment as two bound polygonal chains, any further complicated environment can be tackled by transforming it to constraints in the optimization problem or equivalently feasible searching regions for an appropriate nonlinear programming (NLP) algorithm. NLP algorithms (such as SQP method and interior-point method [15]) usually require a connected feasible region. However, complicated environment may be characterized as several disjoint feasible sub-regions. This will result in more than one time NLP to calculate the optimal path.

Because the sensing range cannot be large, especially for the underwater case where the decaying rate of any (usually acoustic) signal is thousands of times larger than in air, a receding horizon way is adopted here to approximate the global optimum for the path planing problem. That is, for example, with 5-meter sensing range, an optimal path is generated based on the current 5-meter data, however, when the AUV moves 1 meter forward, a new optimal path will be generated based on the new 5-meter data. Therefore, the continuity problem between neighbor splines should be taken into consideration. We impose the continuities of the 1st and 2nd derivative of the optimal path and the path itself as constraints in the optimization formulation:

Problem $\mathbf{A_1}$. Given the newest measured environment information $c_1(x)\le c_2(x)$ (only defined on $\Omega=[x_0,x_f]$) and the last calculated optimal path $S_0(x)$, with proper numbers of knots n, order o and multiplicity l to computer spline function $S(x)$ such that

$$\begin{array}{ll} \min. & F(x) \\ \text{s.t.} & c_1(x)\le S(x)\le c_2(x) \\ & \| S(x_0)-S_0(x_1^0)\|\le \epsilon_1 \\ & \| S'(x_0)-S_0'(x_1^0)\|\le \epsilon_2 \\ & \| S''(x_0)-S_0''(x_1^0)\|\le \epsilon_3 \end{array}$$

where x_1^0 is the second knot (see Fig.1) of $S_0(x)$ and ϵ_i are the tolerance of discontinuity.

4. MODEL PREDICTIVE TRACKING CONTROL

4.1. AUV Modeling

A simplified 4 degree of freedom (DOF) model of AUVs is adopted in this paper [2]. The dynamics of AUV can be expressed as

$$\mathbf{M}\dot{\mathbf{v}}+\mathbf{C}(\mathbf{v})\mathbf{v}+\mathbf{D}(\mathbf{v})\mathbf{v}+\mathbf{g}(\boldsymbol{\eta})=\boldsymbol{\tau} \tag{4}$$

where $\mathbf{v}=[u,v,w,r]^{\mathrm{T}}$ denotes the body-fixed linear and angular velocity vector and $\boldsymbol{\eta}=[x,y,z,\psi]^{\mathrm{T}}$ denotes the earth-fixed position and and orientation vector. $\mathbf{M}$ represents the inertia matrix including added mass, $\mathbf{C}(\mathbf{v})$ is the state dependent matrix of Coriolis and centripetal terms, $\mathbf{D}(\mathbf{v})$ is the hydrodynamic damping and lift matrix, $\mathbf{g}(\boldsymbol{\eta})$ represents the vector of gravitational forces and moments, and $\boldsymbol{\tau}$ is the vector of thrust forces and moments.

Meanwhile, the kinematic equation of the AUV motion with 4 DOF can be expressed as

$$\dot{\boldsymbol{\eta}}=\begin{bmatrix}\cos\psi & -\sin\psi & 0 & 0\\ \sin\psi & \cos\psi & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ r\end{bmatrix}=\mathbf{J}(\boldsymbol{\eta})\mathbf{v} \tag{5}$$

Note that, although the AUV may be torpedo-shaped, unlike ground vehicles, these underactuated underwater vehicles can still have sway velocity v due to inertia in water. For this reason, we do not impose nonholonomic constraint like (1) on the AUV kinematic model.

Defining the system state $\mathbf{x}=col(\mathbf{v},\boldsymbol{\eta})$ and the general control input $\bar{\mathbf{u}}=\boldsymbol{\tau}$, from (4) and (5), we build the general AUV model for system control

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{M^{-1}}(\bar{\mathbf{u}} - \mathbf{Cv} - \mathbf{Dv} - \mathbf{g}) \\ \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \end{bmatrix} \triangleq \bar{f}(\mathbf{x}, \bar{\mathbf{u}}) \tag{6}$$

For different AUVs, the propulsion system will be different, so the detailed expression of $\bar{\mathbf{u}}$ will be different accordingly. In this work, we use the $Sirius$ AUV model [18] with differential thrust, and the general control input $\bar{\mathbf{u}}$ can be written as

$$\bar{\mathbf{u}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ d/2 & -d/2 & 0 \end{bmatrix} \mathbf{u} \triangleq \mathbf{Bu} \tag{7}$$

where d denotes the distance between two differential thrusters, $\mathbf{u} = [u_1, u_2, u_3]^{\mathrm{T}}$ representing the thrusts from port, starboard and vertical thrusters is the real control input. Substituting (7) into (6), we have

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{M^{-1}}(\mathbf{Bu} - \mathbf{Cv} - \mathbf{Dv} - \mathbf{g}) \\ \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \end{bmatrix} \triangleq f(\mathbf{x}, \mathbf{u}) \tag{8}$$

4.2. MPC Scheme for Tracking of AUVs

After generating the reference trajectory, the main control objective is to track the reference trajectory as close as possible. MPC scheme is adopted. The reference kinematic state $\boldsymbol{\eta}_r(k)$ is defined with a specified time law

$$\begin{aligned} x_r(k) &= \beta k T \\ y_r(k) &= S(x_r(k)) \end{aligned} \tag{9}$$

where $\beta > 0$, T is the sampling period and $S(\cdot)$ denotes the reference spline. Also, the orientation of the AUV should also be considered because we need to keep the surge velocity u positive. Considering the underactuated system dynamics, define the reference yaw angle ψ_r as

$$\psi_r(k) = \mathrm{atan2}\left(\frac{y_r(k+1) - y_r(k)}{x_r(k+1) - x_r(k)}\right) \tag{10}$$

where atan2 is the four-quadrant inverse tangent operator.

Remark 3. For the reference depth state z_r, we can define it with other specific requirements. For example, usually AUVs are equipped with sonar system to detect the seafloor, therefore the reference depth z_r may be determined with the seafloor information that AUV maintains a certain distance from the seafloor or we can simply keep z_r constant, e.g. 20 m.

With the discretized AUV model $\mathbf{x}(k+1) = f_d(\mathbf{x}(k), \mathbf{u}(k))$ we construct the cost function $V(\mathbf{x})$ as follows,

$$V(\mathbf{x}(k)) = \sum_{i=H_w}^{H_p} \left\| \hat{\boldsymbol{\eta}}(k+i|k) - \boldsymbol{\eta}_r(k+i) \right\|^2_{\mathbf{Q_i}} + \sum_{i=0}^{H_u-1} \left\| \Delta\hat{\mathbf{u}}(k+i|k) \right\|^2_{\mathbf{R_i}} \tag{11}$$

where $\hat{\boldsymbol{\eta}}(k+i|k)$ represents the kinematic state estimate i steps forward that made at time instant k, and $\Delta\hat{\mathbf{u}}(k+i|k) = \hat{\mathbf{u}}(k+i|k) - \hat{\mathbf{u}}(k+i-1|k)$ is the difference of input estimates between neighbor time instants. The prediction horizon is H_p. It is not necessary to punish the deviations of $\hat{\boldsymbol{\eta}}$ from $\boldsymbol{\eta}_r$ immediately, so we may allow a slight delay of tracking if $H_w > 1$. The control horizon always satisfies $H_u \leq H_p$ and $\hat{\mathbf{u}}(k+i|k) = \hat{\mathbf{u}}(k+H_u-1|k)$ for all $i \geq H_u$. $\mathbf{Q_i} > \mathbf{0}$, $\mathbf{R_i} > \mathbf{0}$ are corresponding weighting matrices. Now we define the optimal control problem to be solved at each sampling time instant:

Problem $\mathbf{P_k}$. Given the current system state $\mathbf{x(k)}$, the reference (kinematic) state $\boldsymbol{\eta}_r$ and weighting matrices $\mathbf{Q}_i$, $\mathbf{R}_i$, compute the optimal control sequence $\mathbf{u}^* \triangleq col(\mathbf{u}^*_0, ..., \mathbf{u}^*_{H_p-1}) = col(\hat{\mathbf{u}}(i|k), ..., \hat{\mathbf{u}}(i+H_p-1|k))$ by solving the following optimization problem:

$$\begin{aligned} \min. \quad & V(\mathbf{x}(k)) \\ \text{s.t.} \quad & \hat{\mathbf{x}}(i+1|k) = f_d(\hat{\mathbf{x}}(i|k), \hat{\mathbf{u}}(i|k)) \\ & \hat{\mathbf{x}}(i|k) \in X \\ & \hat{\mathbf{u}}(i|k) \in U, for\ i = k, k+1, ..., k+H_u \\ & \hat{\mathbf{u}}(j+1|k) = \hat{\mathbf{u}}(j|k), for\ j \geq k+H_u \\ & \hat{\mathbf{x}}(k+H_p) \in X_f \subset X \end{aligned} \tag{12}$$

Here, $\hat{\mathbf{x}}(k+i|k)$ denotes the full state estimate. X is always closed, polyhedral that constrains the state within a certain range; U is a compact set containing all the allowable control inputs. X_f is called the terminal constraint which can be designed to guarantee the stability [14].

4.3. Tracking Control Algorithm with Dynamic Path Planning

Because of MPC's sampled data nature, the tracking control can be integrated with the proposed path planning method. Let D_i denote the current sensor data. The dimension of sampled optimal path $S(x)$ between the first and second knot is $\mathcal{M}$. The prediction horizon $H_p = \mathcal{N} < \mathcal{M}$. Based on **Problem $\mathbf{A_0}$, $\mathbf{A_1}$** and **$\mathbf{P_k}$**, the tracking control algorithm can be summarized as **Algorithm 1**.

5. SIMULATION RESULTS

5.1. Parameter Selection

For the path planning, it is assumed that the vehicle can sense the environment up to 5 meters ahead; we uniformly choose 6 knots, $o = 4$, $l = 4$ and update the optimal path with current 5-meter data whenever the vehicle moves 1 meter forward. $\epsilon_1 = \epsilon_2 = \epsilon_3 = 10^{-2}$ (m).

For the AUV, the parameters of $Sirius$ is chosen based on [10]. The mass of $Sirius$ $m = 150.0$ (kg); the buoyancy $b = 150.2$ (kg); The distance between the differential thrusters $d = 0.4$ (m); the origin of body-fixed reference frame (BRF) is coincident with the center of gravity, while the center of buoyancy represented in BRF is $(x_B, y_B, z_B) = (0, 0, -0.04)$. The moment of inertia with respect to z axis in BRF $I_z = 179.0$ (kg$\cdot m^2$). The

Algorithm 1 Integrated Tracking Control Algorithm

$k_p = 0$.
Let $\mathbf{x}_0$ denote the initial state of the AUV.
procedure
  Given sensor data D_{k_p}, calculate the optimal path by solving $\mathbf{Problem\ A_0}$. Let $S_{k_p}(x)$ be the solution.
  $k = 0$.
  Given $S_{k_p}(x)$, $\mathbf{x}(k) = \mathbf{x}_0$, calculate $\mathcal{R} = [\boldsymbol{\eta}_r(k),...,\boldsymbol{\eta}_r(k+\mathcal{N})]$ by (9-10) and **Remark 3**; calculate $\mathbf{u}^* = col(\mathbf{u}_0^*, ..., \mathbf{u}_{H_p-1}^*)$ by solving $\mathbf{Problem\ P_k}$.
  for $k \leq \mathcal{M} - \mathcal{N}$ **do**
    $k = k + 1$.
    $\mathbf{x}(k) = f_d(\mathbf{x}(k-1), \mathbf{u}_0^*)$, calculate $\mathcal{R}$ and $\mathbf{u}^*$.
  end for
  $k_p = k_p + 1$.
  Collect another set of sensor data D_{k_p}, calculate optimal path by solving $\mathbf{Problem\ A_1}$. Let $S_{k_p}(x)$ be the solution.
  for $k \leq \mathcal{M}$ **do**
    $k = k + 1$.
    $\mathbf{x}(k) = f_d(\mathbf{x}(k-1), \mathbf{u}_0^*)$, calculate $\mathcal{R}$ using both $S_{k_p-1}(x)$ and $S_{k_p}(x)$; calculate $\mathbf{u}^*$.
  end for
end procedure
$\mathbf{x}_0 = \mathbf{x}(k)$, repeat **procedure.**

hydrodynamic coefficients $X_{\dot{u}} = -350$, $Y_{\dot{v}} = -350$, $Z_{\dot{w}} = -159.1$, $N_{\dot{r}} = -15.9$, $D_{\dot{x}} = 500$, $D_{\dot{y}} = 800$, $D_{\dot{z}} = 500$, $D_{\dot{\psi}} = 75.3$. And we define $M_x \triangleq m - X_{\dot{u}}$, $M_y \triangleq m - Y_{\dot{v}}$, $W \triangleq mg$, $B \triangleq bg$ where g is the gravitational acceleration. The maximum force that each thruster can provide is 3000 (N).

The inertia matrix $\mathbf{M}$=diag$(m - X_{\dot{u}}, m - Y_{\dot{v}}, m - Z_{\dot{w}}, I_z - N_{\dot{r}})$; the Coriolis and centripetal matrix

$$\mathbf{C}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & -M_y v \\ 0 & 0 & 0 & M_x u \\ 0 & 0 & 0 & 0 \\ M_y v & -M_x u & 0 & 0 \end{bmatrix}$$

The hydrodynamic damping matrix $\mathbf{D}(\mathbf{v})$=diag$(D_{\dot{x}}|u|, D_{\dot{y}}|v|, D_{\dot{z}}|w|, D_{\dot{\psi}}|r|)$. $\mathbf{g}(\boldsymbol{\eta}) = [0, 0, B - W, 0]^{\mathrm{T}}$ is the restoring forces and moments due to gravity.

For the MPC parameters, we select $\beta = 1$, sampling period $T = 0.1sec$, prediction horizon $H_p = 8$, $H_u = H_p$, $H_w = 1$. The reference state is chosen according to (9), (10), and **Remark 3**, $\mathbf{Q}_i = \mathbf{Q} = \mathrm{diag}(10, 10, 0.5, 1)$, $\mathbf{R}_i = \mathbf{R} = 0.08\ \mathbf{I_3}$. Since we do not impose any constraint on the system state, the initial guess $\mathbf{u}_0$ for the NLP procedure can be determined as follows,

- For the very beginning, $\mathbf{u}_0 = \mathbf{0}_{3H_p \times 1}$.
- Given the optimal solution of the previous time instant $\mathbf{u}^* = col(\mathbf{u}_0^*, ..., \mathbf{u}_{H_p-1}^*)$, $\mathbf{u}_0 = col(\mathbf{u}_1^*, ..., \mathbf{u}_{H_p-1}^*, \mathbf{0}_{3\times 1})$

5.2. Tracking Performance

The xy-plane tracking result is shown in Fig.2. It can

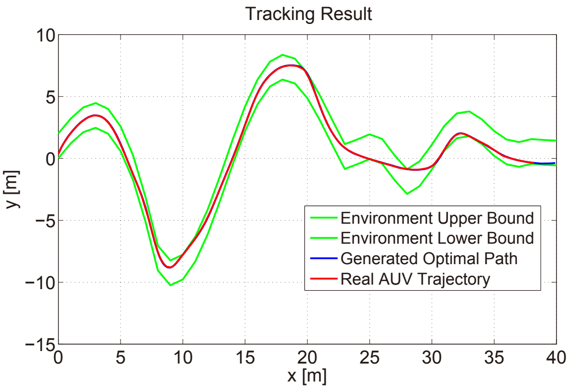

Fig. 2 *xy-plane tracking result.*

be observed that the generated reference path is smooth in light of curvature that facilitates the tracking of AUV. The MPC controller can steer the AUV to closely track the reference path.

From the cost function (11), we see the smoothness of the control input is also taken into consideration. Fig.3 depicts the thrusts during the simulation. All the thrusts are within the interval of $[-3000(N), 3000(N)]$, and the change of each thrust is far from too severe.

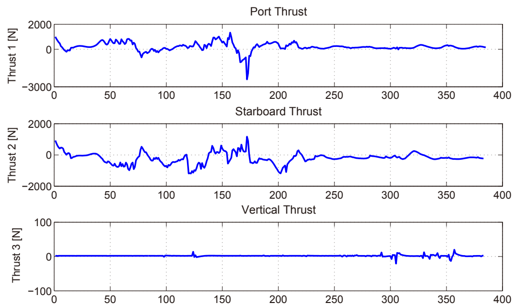

Fig. 3 *AUV system control inputs.*

At the same time, other kinematic states of AUV are also well controlled. From Fig.4, we notice that since we give some weights on the depth and orientation control, the MPC controller can simultaneously take care of these states, and these states are kept within relatively small ranges of variation around their desired working points. Moreover, Eqn.(7) implies that the vertical motion control is not coupled with the tracking control in xy-plane, and therefore the depth control performance is satisfactory as expected.

6. CONCLUSIONS

In this paper, an integrated scheme consisting of MPC and the dynamic path planning optimization has been investigated for the AUV. Based on the receding horizon optimization framework, a path planning method was proposed for the tracking control. Sequentially, a control procedure inherently combining MPC and the proposed path planning method was developed. Simulation results

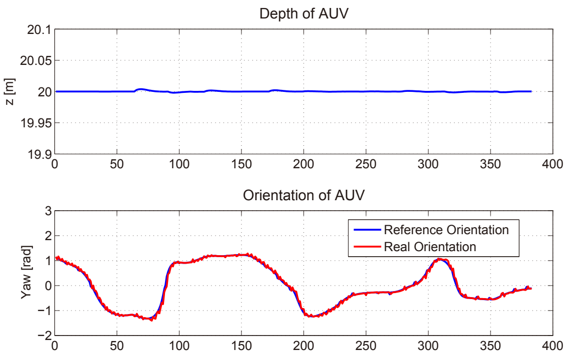

Fig. 4 *Other states of the AUV System.*

using a 4 DOF *Sirius* AUV model revealed the effectiveness of the proposed control algorithm.

There exist some application issues, e.g., the model mismatch, parameter uncertainty, influence of ocean current and the efficiency of NLP algorithms. These practical yet challenging problems will be tackled in our future work.

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