



Survey paper

Reference and command governors for systems with constraints: A survey on theory and applications[☆]



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ABSTRACT

Reference and command governors are add-on control schemes which enforce state and control constraints on pre-stabilized systems by modifying, whenever necessary, the reference. This paper surveys the extensive literature concerning the development of such schemes for linear and nonlinear systems. The treatment of unmeasured disturbances and parametric uncertainties is also detailed. Generalizations, including extended command governors, feedforward reference governors, reduced order reference governors, parameter governors, networked reference governors, and decentralized/distributed reference governors, are discussed. Practical applications of these techniques are presented and surveyed as well. A comprehensive list of references is included. Connections with related approaches, including model predictive control and input shaping, are discussed. Opportunities and directions for future research are highlighted.

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1. Introduction

With the advances in control theory, many effective techniques have become available for the design of feedback control laws with the desired stability, performance and disturbance rejection properties. The interest in treating requirements that have the form of *pointwise-in-time state and control* constraints has also been growing, given their importance for industrial applications. Examples of constraints in real-world applications include actuator magnitude and rate limits, bounds imposed on process variables to ensure safe and efficient system operation, and collision/obstacle avoidance requirements.

A control engineer faced with the task of satisfying constraints has several choices. One route is to re-design the controller within the Model Predictive Control (MPC) framework

(Camacho & Bordons, 2007; Goodwin & de Doná, 2005; Kwon & Han, 2006; Maciejowski, 2002; Morari & Lee, 1999). Another route is to augment a well-designed nominal controller, that already achieves high performance for small signals, with constraint handling capability for larger signals and transients that have the potential to induce constraint violation. This second route is attractive to practitioners who may be interested in preserving an existing/legacy controller or are concerned with the computational effort, tuning complexity, stability, robustness, certification issues, and/or other requirements satisfactorily addressed by the existing controller. Anti-windup compensation (Åström & Wittenmark, 1997) and the augmentation of Lyapunov controllers with barrier functions (Tee, Ge, & Tay, 2009) are examples of this second approach, and so are the reference governors (RGs) and command governors (CGs).

As its name suggests, the reference governor (see Fig. 1) is an *add-on* scheme for enforcing pointwise-in-time state and control constraints by modifying the reference command to a well-designed (for small signals) closed-loop system. The reference governor plays the role of a pre-filter that, based on the current value of the *desired reference command* $r(t)$ and of the state (measurement or estimate) $x(t)$, generates a *modified reference command* $v(t)$ whenever propagating the reference command without modifications may lead to constraints violations.

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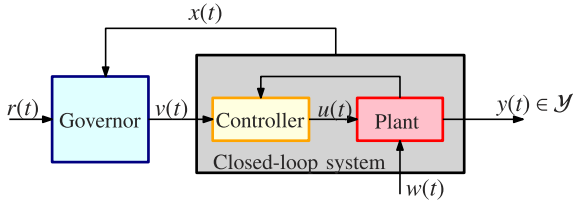


Fig. 1. Reference/command governor applied to closed-loop (Plant + Controller) system subject to constraints.

The use of low pass pre-filters to enforce constraints is a classical control technique, see e.g., the discussion in (Vahidi, Kolmanovsky, & Stefanopoulou, 2007), which usually results in a modified reference which is always different from the actual reference (other than asymptotically). Instead, the reference governor exploits state feedback, prediction, optimization, and set-invariance arguments (Blanchini, 1999; Blanchini & Miani, 2007) to ensure that modifications of the command are performed only when necessary to avoid compromising the system performance.

As an example, consider the application of a reference governor to the double integrator $x_1(t+1) = x_1(t) + 0.1x_2(t)$, $x_2(t+1) = x_2(t) + 0.1u(t)$, controlled through an LQ control law, $u(t) = 0.9170(v(t) - x_1(t)) - 1.6821x_2(t)$, and subject to constraints $|u(t)| \leq 0.1$, $|x_1(t)| \leq 1$, $|x_2(t)| \leq 0.1$, and $|v(t)| \leq 0.6$. The operation of this system is illustrated in Fig. 2, for $v(t) = r(t)$ and for $v(t)$ assigned by a reference governor. Note that, using a reference governor, the command and response are slowed down in order to keep constraints satisfied. However, the modification to the reference is much smaller than what would be with a low pass filter. In fact, with the reference governor, the constrained variables ride the constraint boundary, which is a behavior that is usually impossible to achieve through a simple low pass filter.

A number of governor schemes have been proposed in the literature. The range of potential options includes, among others, scalar and vector reference governors, command governors, extended command governors, incremental reference governors, feedforward reference governors, network reference governors, reduced order reference governors, distributed reference governors, parameter governors, and virtual state governors. While different in obtained properties and implementation aspects, the common intent of these governors is to preserve, whenever possible, the response of the closed loop system designed by conventional control techniques. Frequently (but not always), they achieve this by ensuring that the modified reference command is as close as possible to the original reference command subject to satisfying the constraints.

Reference governors were first proposed as continuous-time algorithms in Kerasouris, Athans, and Stein (1990). Techniques and analysis sharing a similar philosophy with the continuous-time reference governor of Kerasouris et al. (1990) have appeared in Blanchini and Miani (1997, 2000, 2001). The discrete-time framework (Gilbert, Kolmanovsky, & Tan, 1995, 1994) has also emerged later due to some advantages from an implementation standpoint. The static reference governor (Gilbert et al., 1994) used $v(t) = \kappa(t)r(t)$, where the parameter $\kappa(t)$, $0 \leq \kappa(t) \leq 1$, was maximized subject the condition $x(t+1) \in O_\infty$, where O_∞ is the maximal output admissible set (Gilbert & Tan, 1991) of all states that, with reference command equal to zero, do not lead to subsequent constraint violation. Because of the possibility of oscillations (Gilbert et al., 1994), the static reference governor was abandoned and replaced by a dynamic reference governor for which finite-time convergence for constant or nearly constant reference commands is ensured. Other formulations of reference and command governors have appeared in Bemporad, Casavola, and Mosca (1997), Bemporad and Mosca (1994, 1995, 1998), Casavola, Mosca, and Angeli

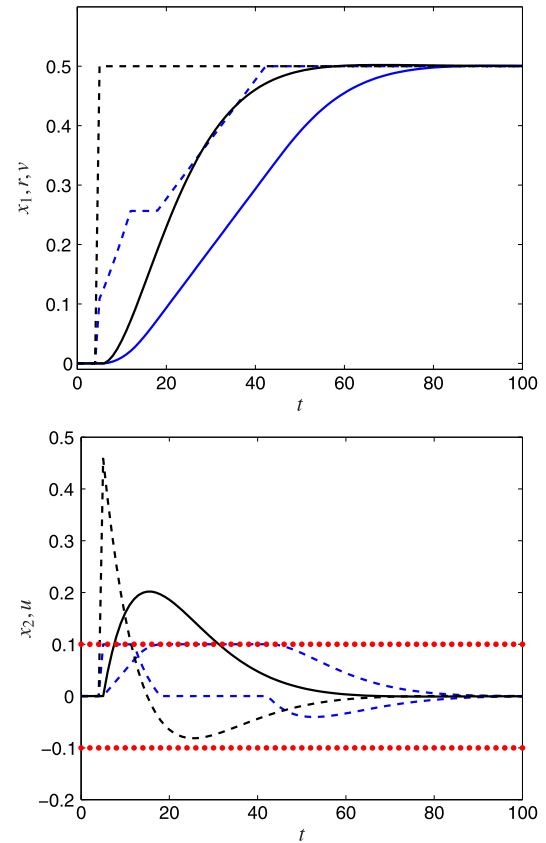


Fig. 2. Double integrator simulations. Above: Time histories of command (dash) and position responses (solid), without (black) and with (blue) reference governor. Below: Time histories of velocity (solid) and control input (dash) responses, without (black) and with (blue) reference governor; constraints in red (dot). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(2000) and Gilbert and Kolmanovsky (1999a), see also references therein. The developments included the treatment of linear systems with uncertainties and set-bounded disturbance inputs, the case of output feedback, and the implementation based on non-positively invariant sets. Extended command governors (Gilbert & Ong, 2011) provided a further generalization with the potential to achieve a larger constrained domain of attraction and faster response, at the price of increased computational complexity.

Several reference governors for nonlinear systems have also been developed, see e.g., Bemporad (1998b), Bemporad, Tarn, and Xi (1999), Borrelli, Falcone, Pekar, and Stewart (2009), Gilbert and Kolmanovsky (1999b, 2002), Miller, Kolmanovsky, Gilbert, and Washabaugh (2000), and references therein. Some of these approaches exploit on-line prediction through simulations or level sets of Lyapunov functions to guard against constraint violation. The parameter governor has been proposed in Kolmanovsky and Sun (2006) to adjust constant controller parameters or controller states based on prediction and optimization.

More recently, classical reference and command governor ideas have been extended in several directions related to the general area of cyber-physical systems (CPS), including the distributed control of large scale systems, modular control architectures, and network control systems.

The aim of this survey paper is to collect and systematize in a common framework the numerous contributions that at the current stage are dispersed in a number of different papers, to discuss the most recent results, to illustrate the potential for the impact on real world applications, and to provide a perspective on some open research directions. Accordingly, we will first

survey the most important results concerning linear (Section 2) and nonlinear (Section 3) reference governor schemes. Then, we will describe the most recent research directions (Section 4) and we will provide an overview of the use of reference governors in different applications (Section 5). Discussions on connections with other control schemes (such as MPC) and on directions for future research are included at the end of the paper (Section 6). A summary of the existing results, substantially more limited in scope and aim compared to this survey, has appeared as a tutorial paper (Kolmanovsky, Garone, & Di Cairano, 2014) at the 2014 American Control Conference.

Notation: \mathbb{R} and \mathbb{Z}_+ denote the real and non-negative integer numbers, respectively. Relational operators $<$, $>$, \leq , \geq are intended componentwise for vectors, while for matrices indicate (semi)definiteness. The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $\|x\| = \sqrt{x_1^2 + \dots + x_n^2}$ whereas $\|x\|_\Psi^2$, with $\Psi = \Psi^T > 0$, denotes the quadratic form $x^T \Psi x$. For given sets $X, Y \subset \mathbb{R}^n$, $X \sim Y := \{x : x + y \in X, \forall y \in Y\}$ is the *Pontryagin (or Minkowski) set difference* and $X \oplus Y := \{x + y : x \in X, y \in Y\}$ the *Minkowski set sum*. (a, b) denotes the stacked vector $(a, b) = [a^T \ b^T]^T$.

2. Reference governors for linear systems

In this section we survey the theory of reference and command governors for linear systems. In Section 2.1 an overview of the basic theory for nominal linear systems is provided assuming the state can be measured. Then we discuss how this theory can be extended to the case of linear systems subject to process disturbances and to the case of noisy output measurements (Section 2.2). Sections 2.3 and 2.4 address the computational aspects related to the design and online implementation of the presented schemes.

2.1. Nominal case—reference and command governors

Reference governors for linear systems are designed based on discrete-time models in the form,

$$\begin{aligned} x(t+1) &= Ax(t) + Bv(t), \\ y(t) &= Cx(t) + Dv(t), \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $v(t) \in \mathbb{R}^m$ is the input vector, and $y(t) \in \mathbb{R}^p$ is the output vector. Model (1) represents a closed-loop system, thereby reflecting the combined *closed-loop* dynamics of the plant with a stabilizing controller. Consequently, the closed-loop system is assumed to be asymptotically stable, i.e., the matrix A is Schur (all eigenvalues are strictly in the interior of the unit disk). Constraints are imposed on the output variables $y(t)$,

$$y(t) \in Y \quad \text{for all } t \in \mathbb{Z}_+, \quad (2)$$

where $Y \subset \mathbb{R}^p$ is a specified set. Note that, since (1) is a model of the closed-loop system, (2) can represent constraints on either state or control variables. For instance, a control constraint $|u_1(t)| \leq 1$ where the control is generated by a state feedback law, $u = Kx$, can be restated as $|y_1(t)| \leq 1$ with $y_1 = \hat{e}_1 Kx$, and $\hat{e}_1 = [1 \ 0 \ \dots \ 0]$.

In this survey, an assumption on the convexity of Y is made throughout, because of theoretical and computational simplifications which occur when the set Y is convex.

The common feature of most reference governor schemes proposed in the literature is that they compute at each time instant a command $v(t)$ such that, if it is constantly applied from the time instant t onward, the ensuing output will always satisfy the constraints. More formally, define the *maximal output admissible set* (a.k.a. maximal constraint admissible set) O_∞ (Gilbert & Tan, 1991) as the set of all states x and inputs, v , such that the predicted

response from the initial state x and with the input v kept constant satisfies the constraints (2), i.e.,

$$O_\infty = \{(v, x) : \hat{y}(k|v, x) \in Y, \forall k \in \mathbb{Z}_+\}, \quad (3)$$

where for system (1) the output prediction $\hat{y}(k|x, v)$ is defined as

$$\begin{aligned} \hat{y}(k|v, x) &= CA^k x + C \sum_{j=1}^k A^{j-1} Bv + Dv \\ &= CA^k x + C(I - A)^{-1}(I - A^k)Bv + Dv. \end{aligned} \quad (4)$$

On the basis of the currently available state $x(t)$, reference governor schemes compute $v(t)$ so that

$$(v(t), x(t)) \in P, \quad (5)$$

where $P \subseteq O_\infty \subset \mathbb{R}^m \times \mathbb{R}^n$. Although, in principle, $P = O_\infty$ is an acceptable choice, it is usually avoided for computational reasons. The most usual choice for P is $P = \tilde{O}_\infty$, where \tilde{O}_∞ is a slightly tightened version of O_∞ obtained by constraining the command v so that the associated steady state output $\bar{y}_v = (D + C(I - A)^{-1}B)v$ satisfies constraints with a nonzero (typically small) margin $\epsilon > 0$, i.e.,

$$\tilde{O}_\infty = O_\infty \cap O^\epsilon, \quad (6)$$

where

$$O^\epsilon = \{(v, x) : \bar{y}_v \in (1 - \epsilon)Y\}. \quad (7)$$

Clearly, \tilde{O}_∞ can be made arbitrary close to O_∞ by decreasing ϵ . It can be proven (Gilbert & Tan, 1991) that if A is Schur, (A, C) is observable, and Y is compact, then the set \tilde{O}_∞ is *finitely determined*, i.e. there exists a finite index k^* such that

$$\tilde{O}_\infty = \{(v, x) \mid \hat{y}(k|v, x) \in Y, k = 0, \dots, k^*\} \cap O^\epsilon. \quad (8)$$

Moreover, it is possible to prove (Gilbert & Tan, 1991) that \tilde{O}_∞ is *positively invariant*, which means that if $(v(t), x(t)) \in \tilde{O}_\infty$ and $v(t)$ is applied to the system at time t , then $(v(t), x(t+1)) \in \tilde{O}_\infty$. Furthermore, if Y is convex, also \tilde{O}_∞ is *convex*.

For the double integrator example (with simulations presented in Fig. 2), the \tilde{O}_∞ set is shown in Fig. 3, together with some sections for different values of v that illustrate the allowed values of states for the corresponding values of the modified reference command.

The *Scalar Reference Governor*, introduced in Bemporad and Mosca (1995), Gilbert et al. (1995, 1994) and Gilbert and Kolmanovsky (1999a), is based on the idea of computing at each time instant t and on the basis of the current state $x(t)$, a command $v(t)$ which is the best approximation of the desired set-point $r(t)$ along the line segment connecting $v(t-1)$ and $r(t)$ that ensures $(v(t), x(t)) \in \tilde{O}_\infty$. More formally, the scalar RG solves at each discrete time t , the following optimization problem,

$$\begin{aligned} \kappa(t) &= \max_{\kappa \in [0,1]} \kappa \\ \text{s.t. } & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & (v, x(t)) \in \tilde{O}_\infty, \end{aligned} \quad (9)$$

where $\kappa(t)$ is a scalar adjustable bandwidth parameter and $v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1))$ is the command to be applied to the system. If no danger of constraint violation exists, $\kappa(t) = 1$, and $v(t) = r(t)$ so that the RG does not interfere with the desired operation of the system. If $v(t) = r(t)$ would cause a constraint violation, the value of $\kappa(t)$ is decreased by the RG. In the extreme case, $\kappa(t) = 0$, $v(t) = v(t-1)$, which means that the RG momentarily isolates the system from further variations of the reference command to ensure safety, in terms of constraint enforcement.

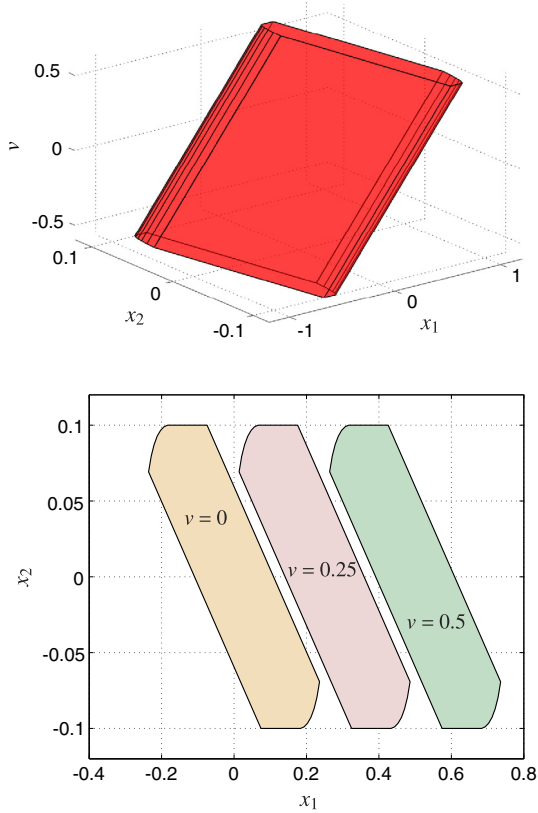


Fig. 3. \tilde{O}_∞ for the double integrator example. Above: entire \tilde{O}_∞ . Below: sections of \tilde{O}_∞ in the (x_1, x_2) -plane obtained for $v = 0, 0.25, 0.5$.

Due to the positive invariance of \tilde{O}_∞ , $v(t) = v(t - 1)$ always satisfies the constraints, which ensures *recursive feasibility* under the condition that at the time instant $t = 0$ a command $v(0)$ such that $(v(0), x(0)) \in \tilde{O}_\infty$ is known. Response properties of the scalar RG, including conditions for the *finite-time convergence* of $v(t)$ to $r(t)$, are detailed in Gilbert et al. (1995) and Gilbert and Kolmanovsky (1999a). Essentially, if $r(t)$ remains constant for $t \geq t_0$ (i.e., $r(t) = \bar{r}, \forall t \geq t_0$) and \bar{r} is strictly steady-state constraint admissible (i.e., if $\bar{r} \in O^\epsilon$), then $v(t)$ converges to \bar{r} in *finite-time*. If $r(t)$ is not strictly steady-state constraint admissible, then $v(t)$ converges in a finite time to a constant command $\bar{v} \in O^\epsilon$ that is the closest feasible approximation of \bar{r} along the line connecting $v(t_0)$ and \bar{r} . Under appropriate assumptions, similar finite-time convergence results can be proved for $r(t)$ varying in a vicinity of a constant value (Gilbert & Kolmanovsky, 1999a) and for $r(t)$ sufficiently slowly varying (Kalabić, 2015; Kalabić & Kolmanovsky, 2014). Finite-time convergence is a desirable property indicating that, after the transients caused by large changes in $r(t)$, the RG becomes inactive and the nominal closed-loop system performance is recovered.

For the double integrator example, Fig. 4 shows the state trajectories from Fig. 2 superimposed onto sections of \tilde{O}_∞ for different values of v .

One of the main strengths of the scalar RG is that only the scalar parameter $\kappa(t)$ is optimized on-line, and hence the computational effort is small. However, this is also its potential weakness. In fact, the command decision space is restricted from \mathbb{R}^m to \mathbb{R} , potentially limiting the control performance in the case of systems with $m > 1$. A first scheme proposed to overcome this limitation is the *Vector Reference Governor*. The vector RG uses a diagonal matrix $\mathbf{K}(t) \in \mathbb{R}^{m \times m}$ in place of a scalar $\kappa(t)$ to decouple the governing of different channels (Gilbert et al., 1995). A further generalization is the so-called *Command Governor* (CG) proposed in Bemporad et al. (1997)

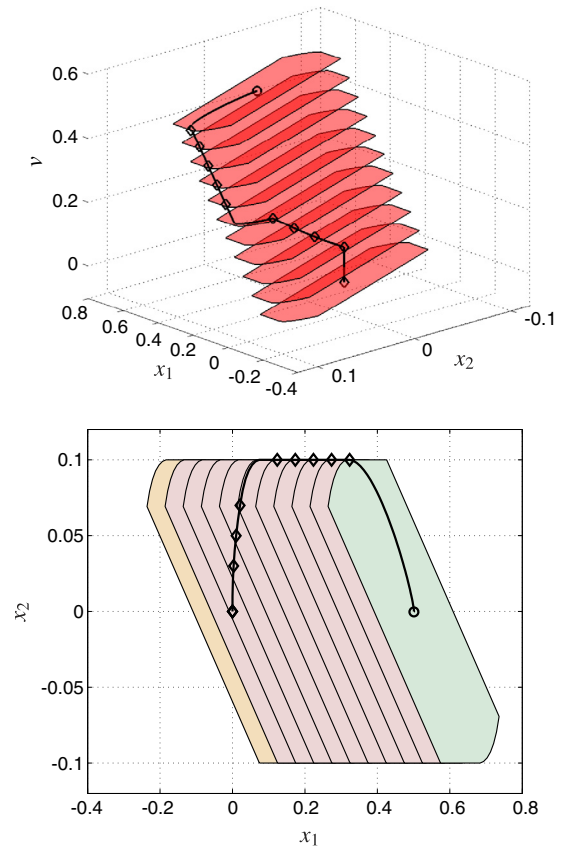


Fig. 4. Trajectory of the double integrator plotted on top of sections of \tilde{O}_∞ . The diamond markers indicate where the state trajectory first enters a shown section. Above: trajectories of states and command. Below: state trajectory projected onto the (x_1, x_2) -plane.

and Casavola et al. (2000). The CG directly uses $v(t) \in \mathbb{R}^m$ as an optimization variable, and solves at each time instant t the optimization problem

$$v(t) = \arg \min_v \|v - r(t)\|_Q^2 \quad (10)$$

$$\text{s.t. } (v, x(t)) \in \tilde{O}_\infty,$$

with $Q > 0$. Compared with the scalar RG, the CG can provide a faster response if $m \geq 2$. It also ensures recursive feasibility under the condition that an admissible command $v(0)$ such that $(x(0), v(0)) \in \tilde{O}_\infty$ exists at time $t = 0$. In contrast, in the scalar RG case a safe $v(0)$ needs to be known, and hence the constrained domain of attraction may be a function of the initialization procedure or requiring that the system is started in steady state. In addition, for a constant $r(t) = \bar{r}$, finite time convergence to $\bar{v} \in O^\epsilon$, which minimizes the norm $\|\bar{v} - \bar{r}\|_Q$, can be proved for the CG (Bemporad et al., 1997). The improved performance of the CG as compared to the scalar RG comes at the price of increased computational effort due to solving the optimization problem (10). Both the scalar RG and the CG have been intensively studied in the last three decades and have been proposed for a significant number of real world applications (see Section 5).

A substantial amount of research on RG and CG for linear systems has been devoted to the computational aspects, some of which will be detailed in Section 2.4. Concerning computational aspects, it is worth mentioning that, if needed, a way to reduce the computational burden is through the use of simpler subsets $P \subset \tilde{O}_\infty$ in place of \tilde{O}_∞ . Choices for P not being positively invariant are possible, but in this case the optimization problem may not admit a solution at some step. In these situations, simply applying $v(t) = v(t - 1)$ ensures that the constraints are still satisfied, since

$(v(t-1), x(t-1)) \in P \subset O_\infty$ and hence $(v(t-1), x(t)) \subset O_\infty$, even if $(v(t-1), x(t)) \notin P$ due to the non-invariance of P . The subset P can be obtained from \tilde{O}_∞ by the systematic elimination of *almost* redundant constraints and applying a pull-in procedure, see Gilbert and Kolmanovsky (1999a). This strategy can lead to a ten-fold reduction in the on-line computing effort at the price of a loss in performance, see e.g., Vahidi et al. (2007).

Several variants of the RG and CG schemes have been proposed in the literature, such as the *Prioritized Reference Governor*, see Kalabić, Chitalia, Buckland, and Kolmanovsky (2013). The Prioritized Reference Governor enforces hard constraints and it satisfies soft constraints in the order of priority. The soft constraints are relaxed by slack variables and the penalty on the slack variables is added to the cost with lower weights corresponding to lower priority constraints.

A scheme that may further improve the performance over the CG by using an even larger decision space is the *Extended Command Governor* (ECG) (Gilbert & Ong, 2011). The starting point of this approach is that the notion of prediction (4) can be seen as a special case of the more general prediction,

$$\hat{y}(k|\hat{v}(\cdot), x) = CA^k x + C \sum_{j=1}^k A^{j-1} B \hat{v}(j) + D \hat{v}(j), \quad (11)$$

where the command $\hat{v}(\cdot)$ is kept constant, i.e. $\hat{v}(\cdot) = v$. The idea of the ECG is to use, instead of a constant command, a command that is the sum of a constant part $\bar{\rho}$ and of a vanishing part $\hat{\mu}(\cdot)$,

$$\hat{v}(\cdot) = \bar{\rho} + \hat{\mu}(\cdot), \quad (12)$$

where the vanishing part $\hat{\mu}(\cdot)$ is the output of a fictitious autonomous system,

$$\chi(t+1) = A_\chi \chi(t), \quad (13)$$

$$\hat{\mu}(t) = C_\chi \chi(t), \quad (14)$$

and A_χ, C_χ are design parameters with A_χ being a Schur matrix. Since this system is fictitious, its initial state is an extra decision variable that can be used to improve the performance of the ECG.

The simplest way to use this idea consists in computing the maximal output admissible set \tilde{O}_∞ (using the very same procedures developed for the classical RG/CG schemes) for the extended system

$$\begin{bmatrix} x(t+1) \\ \chi(t+1) \end{bmatrix} = \begin{bmatrix} A & BC_\chi \\ 0 & A_\chi \end{bmatrix} \begin{bmatrix} x(t) \\ \chi(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \rho(t),$$

and then solve at each time step t the optimization problem,

$$\begin{aligned} (\bar{\rho}(t), \chi(t)) = \arg \min_{\bar{\rho}, \chi} & \frac{1}{2} \|\bar{\rho} - r(t)\|_Q^2 + \frac{1}{2} \|\chi\|_P^2 \\ \text{s.t.} & \left(\bar{\rho}, \begin{bmatrix} x(t) \\ \chi \end{bmatrix} \right) \in \tilde{O}_\infty \end{aligned} \quad (15)$$

where $P > 0$ satisfies the discrete-time Lyapunov inequality,

$$A_\chi^T P A_\chi - P < 0.$$

The command that is applied based on the solution of the optimization problem (15) is

$$v(t) = \bar{\rho}(t) + C_\chi \chi(t).$$

The ECG enjoys the same theoretical properties as the CG while providing a faster response (especially in the case of systems with rate limited actuators) and enjoying a larger domain of attraction. Note that if the dimension of the auxiliary system is $n_\chi = 0$ the ECG becomes a CG (Gilbert & Ong, 2011). For what concerns A_χ and C_χ , in principle any arbitrary choice can be made as long as A_χ is a Schur matrix. The first paper using this framework appears

to be Bemporad et al. (1997) where $A_\chi = \gamma I_m$ was chosen with $\gamma \in [0, 1)$. Another common choice for A_χ consists in the use of shift registers (Gilbert & Ong, 2011),

$$A_\chi = \begin{bmatrix} 0 & I_m & 0 & 0 & \cdots \\ 0 & 0 & I_m & 0 & \cdots \\ 0 & 0 & 0 & I_m & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad (16)$$

$$C_\chi = [I_m \quad 0 \quad 0 \quad 0 \quad \cdots],$$

where I_m is an $m \times m$ identity matrix. For the choice in (16), ECG can be re-formulated as a particular MPC scheme (Gilbert & Ong, 2011). Another approach (Kalabić, Kolmanovsky, Buckland, & Gilbert, 2011), motivated by Rossiter, Wang, and Valencia-Palomo (2010), uses the Laguerre sequences,

$$A_\chi = \begin{bmatrix} \alpha I_m & \beta I_m & -\alpha \beta I_m & \alpha^2 \beta I_m & \cdots \\ 0 & \alpha I_m & \beta I_m & -\alpha \beta I_m & \cdots \\ 0 & 0 & \alpha I_m & \beta I_m & \cdots \\ 0 & 0 & 0 & \alpha I_m & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad (17)$$

$$C_\chi = \sqrt{\beta} \times [I_m \quad -\alpha I_m \quad \alpha^2 I_m \quad -\alpha^3 I_m \quad \cdots \quad (-\alpha)^{N-1} I_m],$$

where $\beta = 1 - \alpha^2$, and $0 \leq \alpha \leq 1$ is a design parameter. Note that for the choice $\alpha = 1$, (17) reduces to (16).

2.2. Disturbance, noise and output feedback

An attractive feature of RG and CG schemes is that they can be easily modified to account for bounded unmeasured disturbances,

$$x(t+1) = Ax(t) + Bv(t) + B_w w(t),$$

$$y(t) = Cx(t) + Dv(t) + D_w w(t), \quad (18)$$

where $w(t)$ is a disturbance that is assumed to be unknown but bounded, i.e., $w(t) \in W$, $\forall t$ where W is a compact set containing the origin. The main difference with respect to the nominal case is that the *maximal output admissible set* is defined as follows Kolmanovsky and Gilbert (1998)

$$O_\infty = \{(v, x) : \hat{y}_d(k|v, x) \subseteq Y, \forall k \in \mathbb{Z}_+\}, \quad (19)$$

where $\hat{y}_d(k|x, v)$ is now a set-valued prediction of the output taking into account all possible realizations of the disturbance. Since (18) is linear, the set-valued output predictions (19) can be written as the Minkowski set-sum of a nominal prediction plus the set of all the possible effects of the disturbance on the output,

$$\hat{y}_d(k|v, x) = \hat{y}(k|v, x) \oplus \hat{\mathcal{Y}}_k, \quad (20)$$

where the nominal predictions $\hat{y}(k|v, x)$ are as in (4) and

$$\hat{\mathcal{Y}}_k = C \left(\bigoplus_{i=0}^{k-1} A^{k-i-1} B_w W \right) \oplus D_w W. \quad (21)$$

Using the sets $\hat{\mathcal{Y}}_k$ from (21) and the Pontryagin set difference, it is possible to rewrite the maximal output admissible set as

$$O_\infty = \{(v, x) : \hat{y}(k|v, x) \in Y \sim \hat{\mathcal{Y}}_k, \forall k \in \mathbb{Z}_+\}. \quad (22)$$

Since A is a Schur matrix and W is compact, with $0 \in W$, the monotonically increasing sequence $\hat{\mathcal{Y}}_{k+1} \supseteq \hat{\mathcal{Y}}_k$ converges to a finite set $\hat{\mathcal{Y}}_\infty = \lim_{k \rightarrow \infty} \hat{\mathcal{Y}}_k$. As a consequence, it is possible to prove (Kolmanovsky & Gilbert, 1995) that if Y is convex and compact, (A, C) is observable, and W is compact, then the set

$\tilde{O}_\infty = O_\infty \cap O^\epsilon$, with O^ϵ defined similarly to (7) and accounting for the effects of disturbances in steady-state is finitely determined, convex, compact, and positively invariant. This implies that all the strategies developed for disturbance-free linear systems can be extended to the case where bounded unmeasured disturbances are present, by simply computing \tilde{O}_∞ while accounting for the disturbances as in (22). Furthermore, the same properties (recursive feasibility and finite time convergence) are retained.

Note that the idea of pre-stabilizing a system and then using the boundedness and convergence of the resulting sequence \hat{y}_k to deal with bounded disturbances, introduced for RG in Gilbert and Kolmanovsky (1995) based on Bertsekas and Rhodes (1971), Blanchini (1990) and Kolmanovsky and Gilbert (1995), has subsequently become popular also in the robust MPC (see, e.g., Kerrigan, 2001; Langson, Chrysoschoos, Rakovic, & Mayne, 2004; Rakovic, 2005 and references therein).

Using similar arguments, it is also possible to treat the case when the state of the system is not measured, but only a possibly noisy output is measured. More precisely, suppose that for system (18) only the output,

$$z(t) = C_z x(t) + \xi(t), \quad (23)$$

is measured, where $\xi(t)$ is an unknown bounded measurement error satisfying $\xi(t) \in \mathcal{E}$, $\forall t \in \mathbb{Z}_+$, and \mathcal{E} is a bounded convex set containing the origin. The approach proposed in Angeli, Casavola, and Mosca (2001) consists in using the set-valued recursions introduced in Glover and Schweppe (1971) to build a *set-membership state estimator*. To do so, it is assumed that a set $\hat{\mathcal{X}}(0|0)$ which contains the true initial state, $x(0)$, is known. At each time instant t , the set of plausible state estimates, $\hat{\mathcal{X}}(t|t)$, can be computed as the intersection of the set of plausible state predictions and of the set of states compatible with the output observations,

$$\hat{\mathcal{X}}(t|t) = \hat{\mathcal{X}}(t|t-1) \cap \hat{\mathcal{X}}_z(t), \quad (24)$$

where the predictions are

$$\hat{\mathcal{X}}(t|t-1) = A\hat{\mathcal{X}}(t-1|t-1) \oplus \{B_v v(t-1)\} \oplus B_w W, \quad (25)$$

and the states compatible with the output measurements are in the set

$$\hat{\mathcal{X}}_z(t) = \{C_z^T (C_z C_z^T)^{-1} z(t)\} \oplus \{x \in \mathbb{R}^n : -C_z x \in \mathcal{E}\}. \quad (26)$$

Once $\hat{\mathcal{X}}(t|t)$ is computed, it is possible to rewrite it as the Minkowski set sum of a nominal state estimate, $\hat{x}(t) \in \hat{\mathcal{X}}(t|t)$, plus a set, $\hat{\mathcal{E}}(t)$, containing the origin, i.e., $\hat{\mathcal{X}}(t|t) = \{\hat{x}(t)\} \oplus \hat{\mathcal{E}}(t)$. A sequence of maximal output admissible sets can be defined as

$$O_\infty(t) = \{(v, \hat{x}) : \hat{y}(k|v, \hat{x}) \in Y \sim (\hat{y}_k \oplus CA^k \hat{\mathcal{E}}(t)), \forall k \in \mathbb{Z}_+\}. \quad (27)$$

It is possible to prove (Angeli et al., 2001) that the sequence of sets, $\tilde{O}_\infty(t) = O_\infty(t) \cap O^\epsilon$, is positively invariant and each set in the sequence is finitely determined and convex. As a consequence, all schemes developed for nominal linear systems can be applied in the case of output feedback, by using the estimate, \hat{x} , in place of the (unknown) state, x . The feasibility and convergence properties are also preserved. However, as t increases, the complexity of the representation of the set $\hat{\mathcal{X}}(t|t)$ increases, complicating the computations of $\tilde{O}_\infty(t)$. Also, $\tilde{O}_\infty(t)$ depends on $\hat{\mathcal{E}}(t)$ which in turn depends on a priori unknown measurements, $z(t)$. Hence, in general, $\tilde{O}_\infty(t)$ must be computed online, which may cause significant computational challenges. For these reasons, it may be convenient to use simpler outer approximations of $\hat{\mathcal{X}}(t|t)$, at the price of an increased conservativeness.

In Angeli et al. (2001), some sufficient conditions to ensure that these outer approximations still lead to a positively invariant,

tightened version of \tilde{O}_∞ have been highlighted. A very relevant case is when a Luenberger observer (Casavola, 2005; Hatanaka & Takaba, 2005b) is used to estimate the state. In this case, starting from an initial estimate $\hat{x}(0)$ and given an initial uncertain set $\hat{\mathcal{E}}(0)$, the set-valued observer becomes

$$\begin{aligned} \hat{x}(t+1) &= A\hat{x}(t) + B_v v(t) + L(C_z \hat{x}(t) - z(t)), \\ \hat{\mathcal{E}}(t+1) &= A\hat{\mathcal{E}}(t) \oplus B_w W \oplus L\mathcal{E}, \end{aligned} \quad (28)$$

where L is the Luenberger observer gain that must be chosen so that $A + LC_z$ is a Schur matrix. Since $A + LC_z$ is Schur, the uncertainty set $\hat{\mathcal{E}}(t)$ remains bounded and will eventually converge,

$$\begin{aligned} \lim_{t \rightarrow \infty} \hat{\mathcal{E}}(t) &= \lim_{t \rightarrow \infty} A^t \hat{\mathcal{E}}(0) \oplus \left(\bigoplus_{i=0}^{t-1} A^{t-i-1} (B_w W \oplus L\mathcal{E}) \right) \\ &= \bigoplus_{i=0}^{\infty} A^i (B_w W \oplus L\mathcal{E}) = \hat{\mathcal{E}}_\infty. \end{aligned} \quad (29)$$

Note that, in the special case $\hat{\mathcal{E}}(0) = \hat{\mathcal{E}}_\infty$, the uncertainty set is stationary, i.e. $\hat{\mathcal{E}}(t) = \hat{\mathcal{E}}_\infty$, $\forall t \in \mathbb{Z}_+$, and O_∞ can be defined as

$$O_\infty = \{(v, \hat{x}) : \hat{y}(k|v, \hat{x}) \in Y \sim (\hat{y}_k \oplus C\hat{\mathcal{E}}_\infty), \forall k \in \mathbb{Z}_+\}. \quad (30)$$

In this case, the effect of the observation uncertainty can be taken into account with a simple Pontryagin set difference, and then $\tilde{O}_\infty = O_\infty \cap O^\epsilon$ can be computed off-line. Furthermore, with some further loss of performance, this approach can be used whenever $\hat{\mathcal{E}}(0) \subseteq \hat{\mathcal{E}}_\infty$.

Finally, a more extreme scenario that is worth mentioning is the case where no measurement is available (Casavola, Garone, & Tedesco, 2014c). This is equivalent to the case when observer (29) is used with $L = 0$. Reference/command governors not making use of any measurements are known in the literature as *feedforward reference/command governors* (Garone, Tedesco, & Casavola, 2009, 2011). The fact that RG schemes can be built even in the absence of measurements of the state (although at the cost of a conservatism that grows with the size of the disturbance set W) is due to the fact that since A is Schur, the nominal system is open loop detectable (Casavola et al., 2014c).

2.3. Design aspects

For what concerns the design of RG schemes, the main aspect is the computation of \tilde{O}_∞ . In Gilbert and Tan (1991) results for the computation of the maximal output admissible set O_∞ were developed for autonomous asymptotically stable systems without reference. Such results do not apply to the case of governors, where the reference is modeled as a constant control signal, and the system can be viewed as an augmented system with a marginally stable mode. However, in Kolmanovsky and Gilbert (1998) it is shown that for every $\epsilon > 0$, \tilde{O}_∞ is finitely determined.

To compute \tilde{O}_∞ one must define

$$\tilde{O}_k := \{(v, x) : \hat{y}(i|v, x) \in Y \sim \hat{y}_k, i = 0, \dots, k\} \cap O^\epsilon,$$

and increase k until k^* , which is the lowest k such that $\tilde{O}_k = \tilde{O}_{k+1}$, or in other words the lowest k such that the new constraint $\hat{y}(k+1|v, x) \in Y \sim \hat{y}_{k+1}$ is redundant with respect to the previous ones (Casavola & Mosca, 1996; Kolmanovsky & Gilbert, 1995). From the computational viewpoint the two main difficulties to compute \tilde{O}_∞ are: (i) to verify that constraints are redundant, and (ii) to perform the Pontryagin set differences.

To verify that a single constraint $h(y) \leq 0$ is redundant with respect to a set \mathcal{H} , one must verify that there exists no $y \in \mathcal{H}$, such

that $h(y) > 0$. A possible way to do so is to solve the optimization problem,

$$\max_{y \in \mathcal{H}} h(y). \quad (31)$$

If the result of (31) is positive, the constraint is not redundant, otherwise it is redundant. Note that if $h(y)$ is convex, this maximization problem is, in general, non-convex. A special case where (31) results in a convex optimization problem is when $h(y)$ is affine.

Following from the definition, the Pontryagin set difference of a generic set $Y = \{y \mid h(y) \leq 0\}$ minus another set \mathcal{Y} can be written as $Y \sim \mathcal{Y} = \{y \mid h_{\mathcal{Y}}(y) \leq 0\}$ where the function $h_{\mathcal{Y}}(y)$ is defined as

$$h_{\mathcal{Y}}(y) = \max_{\hat{y} \in \mathcal{Y}} h(y + \hat{y}). \quad (32)$$

If Y is convex, the Pontryagin set difference results in a convex set. While in the general case the computation of $h_{\mathcal{Y}}(y)$ may be involved, whenever $h(y)$ is an affine function, i.e., of the form, $h(y) = h^T y - \bar{h}$, the computations become much simpler. In fact, (32) translates into

$$\begin{aligned} h_{\mathcal{Y}}(y) &= \max_{\hat{y} \in \mathcal{Y}} h^T [y + \hat{y}] - \bar{h} \\ &= h^T y - \bar{h} + \max_{\hat{y} \in \mathcal{Y}} h^T \hat{y} \end{aligned} \quad (33)$$

with $\max_{\hat{y} \in \mathcal{Y}} h^T \hat{y}$ being an optimization problem that is convex when \mathcal{Y} is convex, and is solved offline. Note that if \mathcal{Y} is a polytope, then the maximum in (33) is achieved at one of the vertices.

It is clear from the above discussion that, whenever possible, it is convenient to define (or to approximate) the set Y in the constraints $y \in Y$ as a polytope described by a set of linear inequalities in the form

$$Y = \{y \in \mathbb{R}^p \mid Hy \leq \bar{h}\}, \quad (34)$$

for which it is relatively easy to determine offline a minimal representations of \tilde{O}_{∞} (Gilbert & Tan, 1991; Kolmanovsky & Gilbert, 1998), as a minimal set of linear inequalities of the form

$$\tilde{O}_{\infty} = \{(v, x) : H_x x + H_v v \leq s\}, \quad (35)$$

since the above methods require only the solutions of linear programming problems. Similarly, if the disturbance set W is a polytope, the sets $\hat{\mathcal{Y}}_k$ in (21) are also polytopes, and again the computations to construct O_{∞} in (22) are usually manageable.

An alternative way to compute \tilde{O}_{∞} is through the method presented in Bemporad, Oliveri, Poggi, and Storace (2011) to compute polyhedral invariant sets under state constraints. It is worth to mention also that, beside computing \tilde{O}_{∞} explicitly offline, it is possible to compute it (and k^*) implicitly online using explicit predictions and terminal invariant sets, similarly to MPC schemes.

2.4. Computational and implementation aspects

One of the main advantages of RG and CG with respect to other more complex constrained control strategies, such as MPC, is the limited computational footprint of the online implementation. Next, we describe some of the most common algorithms for evaluating the RG and CG laws for linear systems, with particular focus on the case of constraints defined by polyhedra.

While their formulations may be slightly different, the governors all seek the closest admissible command to the current reference, given the current state. This amounts to seeking $v(t)$ as the projection of the reference $r(t)$ onto the section of \tilde{O}_{∞} taken at

the current state $x(t)$. In the case of Euclidean norm, the projection problem is

$$\begin{aligned} v(t) &= \arg \min_v \frac{1}{2} \|r(t) - v\|_Q^2 \\ \text{s.t. } (v, x(t)) &\in \tilde{O}_{\infty}, \end{aligned} \quad (36)$$

where $Q > 0$. If \tilde{O}_{∞} is convex, (36) is a convex optimization problem.

Different governors allow for different projections, by either limiting the directions of the projection, e.g., for the scalar RG (9) when $m > 1$, or by lifting the problem to higher dimensions by introducing auxiliary variables, e.g., the ECG (15).

In the scalar RG (9) only one optimization variable is considered. If \tilde{O}_{∞} is convex, this results in a simple line-search problem. In fact, since $v = v(t-1)$ is admissible, the solution is in the line segment connecting $v(t-1)$ and $r(t)$. In this case, a bisection search can be applied, which ensures that the error between the feasible solution v^{ℓ} at iterate ℓ and the optimal solution \bar{v} satisfies $\|v^{\ell} - \bar{v}\| \leq 2^{-\ell} \|r - v(t-1)\|$. Thus, for a scalar RG a simple algorithm with linear convergence rate of $1/2$ is available. Each step of the search requires only computing the admissibility of a candidate solution v^{ℓ} , that is, checking whether the condition $(v, x(t)) \in \tilde{O}_{\infty}$ is satisfied. An even simpler algorithm can be used in the case \tilde{O}_{∞} is polyhedral as in (35). In fact, in this case the problem reduces to finding the largest κ such that

$$H_x x(t) + H_v [v(t-1) + \kappa(r(t) - v(t-1))] \leq s,$$

which amounts to solving a series of linear inequalities in one variable that can be solved in closed form (Gilbert & Kolmanovsky, 1999a) as follows

$$\kappa(t) = \min \left\{ \min_{j \in J^+} \left\{ \frac{s_j - h_{x,j}^T x(t) - h_{v,j}^T v(t-1)}{h_{v,j}^T (r(t) - v(t-1))} \right\}, 1 \right\},$$

where $h_{x,j}^T$ and $h_{v,j}^T$ are the j th row of H_x and H_v , respectively, s_j is the j th element of s in (35), and $J^+ \subseteq \{1, 2, \dots, n_c\}$ is the set of indices such that $h_{v,j}^T (r(t) - v(t-1)) > 0$. Note that in this case the computations to be performed online amount to a fixed number of simple operations that scales linearly with respect to the number of constraints in \tilde{O}_{∞} and with respect to the dimension of the system. Efficient computational strategies using similar ideas for a scalar RG with affine, quadratic, and general convex constraints have been presented in Nicotra, Garone, and Kolmanovsky (in press).

More complex governors for linear systems, such as the CG, require the solution of Euclidean-norm projection problems with multiple optimization variables. When \tilde{O}_{∞} is polyhedral (35) such problem can be transformed into the parametric quadratic programming (pQP) problem,

$$\begin{aligned} v(t) &= \arg \min_v \frac{1}{2} v^T Q v - r(t)^T Q v \\ \text{s.t. } H_v v &\leq s - H_x x(t), \end{aligned} \quad (37)$$

where $Q > 0$, and the current state $x(t)$ and the current reference $r(t)$ are the parameters, that may change at every time step. Several classes of algorithms exist for solving (37). The interior point and active set methods (Boyd & Vandenberghe, 2004; Wright & Nocedal, 1999) typically exhibit faster convergence and reach the optimum in few iterations, however, each iteration may be fairly complicated, as it requires the solution of a linear system obtained from the KKT (Karush–Kuhn–Tucker) optimality conditions. Considering that in RG and CG (37) has only few variables, and that one of the attractive feature of governors is the reduced complexity of the computations, interior point and active set methods may be excessively complicated. Instead,

first order methods (Nesterov, 2004) perform computationally simple iterations, although many of them may be required to reach the optimum. Thus, their code is much simpler than in the cases of interior point and active set methods, which allows for faster implementation and verification, and implementability even on platforms with limited computational capabilities such as in automotive systems or in factory automation. These benefits may offset the drawbacks associated with the slower convergence when compared to active set and interior point methods. In recent years, several low complexity first order methods tailored to real-time optimization-based control have been developed, mostly motivated by MPC. In Richter, Jones, and Morari (2012) a first order method based on Nesterov's fast gradient algorithm (Nesterov, 1983) has been developed, which is very effective when only simple constraints, e.g., box constraints, are considered. However, when the constraints are non-simple, e.g., general polytopes like (35), the computational burden of the iteration becomes high. In Kögel and Findeisen (2011), Lagrangian methods were developed, where at each iteration an unconstrained Lagrangian-based QP is solved and the Lagrange multipliers are adjusted. An alternative approach for solving the QP (37) is to solve its dual problem, which is the non-negative QP

$$\eta^* = \arg \min_{\eta} \frac{1}{2} \eta^T H_v Q^{-1} H_v^T \eta + (s - H_v r(t) - H_x x(t))^T \eta \quad (38)$$

s.t. $\eta \geq 0$,

and then recover the solution of (37) as $v(t) = r(t) - QH_v^T \eta^*$, where η^* is the optimizer of (38).

The advantage of solving (38) in place of (37) is that the constraints in the former problem are simple and easily handled non-negativity constraints, while the disadvantages are that the problem is not strongly convex but only convex, i.e., $H_v Q^{-1} H_v^T \geq 0$, and that the number of variables in (38) is equal to the number of constraints in (37), which, for governors, tends to be significantly larger than the number of variables in (37). However, since, in general, the QPs for governor applications tend to be of small dimensions, even in terms of the absolute number of constraints, and only few constraints are active at any time due to the few variables, dual first order methods may be quite appealing for governor implementation. Among these, in Patrinos and Bemporad (2014) a dual fast gradient method has been proposed. In Patrinos, Guiggiani, and Bemporad (2015) a dual projected gradient method which allows for fixed point implementation was introduced, and in Di Cairano, Brand, and Bortoff (2013) a dual multiplicative update method was developed. Recently, a number of algorithms based on the alternating direction method of multipliers (ADMM) (Boyd, Parikh, Chu, Peleato, & Eckstein, 2011) for solving QPs in the context of MPC have been developed, see Ghadimi, Teixeira, Shames, and Johansson (2015), Giselsson and Boyd (2014), and Raghunathan and Di Cairano (2014a,b). The methods in Ghadimi et al. (2015) and Giselsson and Boyd (2014) require the QP to be strongly convex. In such a case, the governor problem includes general polyhedral constraints, resulting in computationally demanding iterations due to the operation of projecting on a polyhedron. Instead, the methods in Raghunathan and Di Cairano (2014a,b) do not require strong convexity, so that the reference governor problem can be re-formulated by introducing a vector of non-negative slack variables, one per constraint, to convert inequalities into equality constraints. With such a transformation, the problem is no-longer strongly convex but has only equality and simple (i.e., non-negativity) constraints, thus resulting in computationally inexpensive ADMM iterations because the projection operation is much simpler to evaluate. Thus, the methods in Raghunathan and Di Cairano (2014a,b) provide algorithms with linear convergence for advanced governors, such as CG, with simple code and iterations.

Yet another alternative is to compute the solution of (37) explicitly offline as a function of the parameters r, x by using multiparametric programming (Bemporad, 2015; Bemporad, Morari, Dua, & Pistikopoulos, 2002). It was shown in Bemporad et al. (2002) and Kogiso and Hirata (2003, 2009) that for QPs, the parametric solution amounts to a piecewise affine function of the parameters. For the governor, the parametric solution results in

$$v = \begin{cases} F_1^{PWA} x + E_1^{PWA} r + G_1^{PWA} & \text{if } H_1^{PWA} x + M_1^{PWA} r \leq K_1^{PWA} x \\ \vdots & \\ F_{\zeta}^{PWA} x + E_{\zeta}^{PWA} r + G_{\zeta}^{PWA} & \text{if } H_{\zeta}^{PWA} x + M_{\zeta}^{PWA} r \leq K_{\zeta}^{PWA} x \end{cases} \quad (39)$$

being a piecewise affine function defined over ζ polyhedral regions that partition the state-reference space into $\zeta \in \mathbb{Z}_+$ regions. For a governor, there is always a region $\bar{i} \in \mathbb{Z}_+$ $\bar{i} \leq \zeta$ that corresponds to \bar{O}_{∞} , and for such region, $H_{\bar{i}}^{PWA} = H_x$, $M_{\bar{i}}^{PWA} = H_v$, $K_{\bar{i}}^{PWA} = s$, and $F_{\bar{i}}^{PWA} = 0$, $E_{\bar{i}}^{PWA} = I$, $G_{\bar{i}}^{PWA} = 0$, so that there, $v = r$. Once the parametric solution is computed offline, the online computations reduce to first identifying the current active region, i.e., the (unique) region to which x, r belong, through the evaluation of the inequalities in (39), and then to evaluating corresponding affine function. Given the simplicity of operations and the sequential nature of the algorithm, it is relatively simple to bound the worst case number of operations at each control cycle. The disadvantages of the explicit implementation are similar to the ones in MPC: the reconfigurability to model and constraints changes is lost and the computational footprint for large problems can exceed that of the online optimizer-based solution. We note that the offline elimination of almost redundant constraints (Gilbert & Kolmanovsky, 1999a) can facilitate this explicit implementation.

3. Reference governors for nonlinear systems

In the case of nonlinear systems, the design of the RG can be based on model linearizations or can be performed based on the nonlinear system model. Several approaches along these lines are surveyed below.

3.1. Implementing linear model-based reference/command governor on a nonlinear system

As demonstrated in Section 2, effective RG/CG design techniques are available when the system model is linear, as in (1) or (18). However, many systems in real world applications are nonlinear. A “practical” way to deal with nonlinear systems consists in designing a governor based on an approximated linear model of the plant. In this case, however, the difference between the responses of the nonlinear system to which the governor is applied and of the linearized model on which the design is based, may need to be compensated for.

One approach to achieve such compensation is by treating the difference between the linearized and the nonlinear models as a set-bounded additive disturbance. Heuristic techniques proposed in Vahidi et al. (2007) treat such a disturbance as a constant over the prediction horizon. They proceed by extending the linear system model (1) as follows

$$\begin{aligned} x(t+1) &= Ax(t) + Bv(t), \\ w(t+1) &= w(t), \\ y(t) &= Cx(t) + Dv(t) + w(t), \end{aligned} \quad (40)$$

where the extra state $w(t)$ is a fictitious state added to approximately compensate the difference between the linear and the nonlinear system. At this point, the maximal output admissible

set \tilde{O}_∞ is computed for (40) and, at the time instant t , the RG/CG computes v so that

$$\left(v, \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \right) \in \tilde{O}_\infty,$$

where $w(t)$ can be computed in different ways, depending on the application. For instance, $w(t)$ may be set equal to the difference between the deviation of the current output of the nonlinear system from the nominal value (corresponding to the linearization point) and the output predicted according to the linear system model based on the state at the previous step, i.e., $\hat{y}(t|t-1) = C(Ax(t-1) + Bv(t-1)) + Dv(t-1)$. While this approach has been very useful in some practical applications (see e.g., Kalabić, Kolmanovsky, Buckland et al., 2011; Vahidi et al., 2007), it does not guarantee strong properties such as recursive feasibility.

Another approach to use linear theory in a nonlinear context is by exploiting feedback linearization (Khalil, 2001). Feedback linearization renders the state dynamics linear and hence the prediction of the state evolution becomes computationally straightforward. While this general approach is attractive, note that, after a feedback linearizing transformation, constraints, even if originally convex, can become non-convex. However, in some special cases, the resulting set \tilde{O}_∞ can be a convex set so that the linear RG theory can be directly applied. In the case \tilde{O}_∞ is non-convex, a possible sub-optimal approach is to approximate it through convex regions, mixed-logical-dynamic (MLD) constraints of *if-then-else* type, and concave constraints, see Kalabić (2015) and Kalabić, Kolmanovsky, and Gilbert (2011) for details.

3.2. Embedding a nonlinear system into a family of linear systems

A different approach to the reference management of nonlinear systems consists in embedding a nonlinear system into a family of linear systems.

Ref. Angeli, Casavola, and Mosca (1999) proposes a CG design based on embedding the nonlinear system model into a family of Linear Time Varying polytopic uncertain models. A similar idea is used in Chisci, Falugi, and Zappa (2005), where the nonlinear system is embedded into a Linear Parameter Varying (LPV) system that is controlled through an RG and a gain-scheduled tracking algorithm.

Nonlinear systems can also be treated by embedding a nonlinear model into a family of switched/hybrid linear models. A variety of schemes have been developed that exploit this kind of embedding in the RG design, see Famularo, Franzè, Furfaro, and Mattei (2011, 2015) and Franzè, Mattei, Ollio, Scordamaglia, and Tedesco (2014) for specific examples. Strategies for the RG design based on piecewise affine models have been proposed in Borrelli et al. (2009).

3.3. Reference governor design based on nonlinear models

Several techniques are available for designing RGs based directly on nonlinear models. The most common approaches and their extensions are discussed next.

In a typical setup, nonlinear RG schemes deal with a pre-compensated system described by a nonlinear discrete-time system model in the form,

$$x(t+1) = f(x(t), v(t), w(t)), \quad (41)$$

and subject to constraints,

$$y(t) = h(x(t), v(t)) \leq 0. \quad (42)$$

In (41), $w(t)$ is a disturbance that is assumed to be unknown but bounded, i.e., $w(t) \in W$, for all t where W is a compact set

containing the origin. The equilibrium associated with a constant reference $v(t) = v$, if no disturbance is present, is denoted by \bar{x}_v , i.e., $\bar{x}_v = f(\bar{x}_v, v, 0)$. It is typically assumed that for any v , the equilibrium \bar{x}_v is asymptotically stable if $w = 0$ or Input-to-State (ISS) stable (Jiang & Wang, 2001) with respect to w .

It appears that the first RG specifically designed for (disturbance-free) nonlinear systems has been proposed in Bemporad (1998b). This scheme is similar to the scalar RG for linear systems and finds at each time instant “the best” admissible reference along the line segment connecting the previously applied reference $v(t-1)$ and the desired $r(t)$ such that the predicted state response does not violate constraints:

$$\begin{aligned} \kappa(t) = \max_{\kappa \in [0,1]} \kappa \\ \text{s.t. } v = v(t-1) + \kappa(r(t) - v(t-1)), \\ \hat{x}(t+k+1|t) = f(\hat{x}(t+k|t), v), \\ h(\hat{x}(t+k|t), v) \leq 0, \quad \forall k \geq 0, \end{aligned}$$

where $\hat{x}(t+k|t)$ is the predicted trajectory propagating from $\hat{x}(t|t) = x(t)$, which is computed by simulating the nonlinear model. Once again, the reference to be applied is $v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1))$.

Similarly to the linear case, restricting the set of equilibria to the strictly steady-state admissible ones, and assuming that for any strictly steady-state admissible v the condition $h(x, v) \leq 0$ defines a compact set in x , the above infinite horizon problem can be rewritten as a finite horizon problem

$$\begin{aligned} \kappa(t) = \max_{\kappa \in [0,1]} \kappa \\ \text{s.t. } v = v(t-1) + \kappa(r(t) - v(t-1)), \\ \hat{x}(t+k+1|t) = f(\hat{x}(t+k|t), v), \\ h(\hat{x}(t+k|t), v) \leq 0, \quad k = 0, \dots, k^*, \\ h(\bar{x}_v, v) \leq -\varepsilon, \end{aligned} \quad (43)$$

where $\varepsilon > 0$ is sufficiently small and $k^* \in \mathbb{Z}_+$ is sufficiently large to satisfy the finite determination-like property in Bemporad (1998b), i.e., if the constraints are satisfied up to k^* , they will be satisfied after k^* . Note that the online optimization problem in (43) is a scalar optimization problem in the parameter $\kappa \in [0, 1]$. While, in general, it is non-convex, it is still relatively simple and can be solved by bisections or grid search, while checking the feasibility of $\kappa = 1$ first.

Based on arguments in Bemporad (1998b), it is possible to prove that the strategy (43) ensures *recursive feasibility* if a feasible $v(0)$ is known at time $t = 0$. For a constant reference $r(t) = \bar{r}$, $t \geq 0$, asymptotic convergence of $v(t)$ to \bar{r} can be proved if \bar{x}_v is a continuous function of v and if any reference contained in the segment between $v(0)$ and \bar{r} is strictly steady-state admissible, i.e., $h(\bar{x}_{v(0)+\kappa(\bar{r}-v(0))}, v(0) + \kappa(\bar{r} - v(0))) \leq -\varepsilon, \forall \kappa \in [0, 1]$. These results can be strengthened to finite-time convergence under the assumptions and small modifications presented in Gilbert and Kolmanovsky (2002). Furthermore, as shown in Garone and Nicotra (2016) and Nicotra, Naldi, and Garone (2014), the approach can be extended to the case of non-scalar v if a continuous curve of strictly steady-state admissible references connecting $v(0)$ and \bar{r} is known. In this case, the RG can operate by optimizing v along this curve rather than along the line segment between $v(0)$ and \bar{r} , while guaranteeing the usual recursive feasibility and convergence properties. Such a strategy is useful, for instance, in obstacle avoidance scenarios.

The main difficulty in the implementation of this method concerns the determination of k^* . If a Lyapunov function $V(x, v)$ is known, the invariance properties of the level sets of the Lyapunov function can be used for the implicit computation of k^* online. The idea is to stop further predictions at the first time instant k such that the condition $V(\hat{x}(t+k|t), v) \leq V_{\min}$ is satisfied, where V_{\min}

satisfies

$$\begin{cases} V(x, v) \leq V_{\min}, \\ h(\bar{x}_v, v) \leq -\varepsilon \end{cases} \Rightarrow h(x, v) \leq 0.$$

Such a value of V_{\min} can be pre-computed off-line by solving the following optimization problem,

$$\begin{aligned} V_{\min} = \min_{v, x} & V(x, v) \\ \text{s.t.} & h(\bar{x}_v, v) \leq -\varepsilon, \\ & h(x, v) \geq 0. \end{aligned} \quad (44)$$

Note that V is not required to be a strict Lyapunov function (i.e., only invariance of the level sets is required). Methods to estimate k^* offline can be devised using similar invariance ideas.

A different approach to nonlinear RG design consists in explicitly making use of the invariance of the level sets of Lyapunov functions (Gilbert & Kolmanovsky, 1999b; Miller et al., 2000) to build a RG scheme. Specifically, given a Lyapunov function $V(x, v)$, suppose it is possible to find a bound $\Gamma(v)$ such that

$$V(x, v) \leq \Gamma(v) \Rightarrow h(x, v) \leq 0. \quad (45)$$

Then a Lyapunov function-based RG is based on the solution of the following optimization problem,

$$\begin{aligned} \kappa(t) = \max_{\kappa \in [0, 1]} & \kappa \\ \text{s.t.} & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & V(x(t), v) \leq \Gamma(v), \\ & h(\bar{x}_v, v) \leq -\varepsilon. \end{aligned} \quad (46)$$

This scheme shares the very same properties of recursive feasibility and convergence with (43). In terms of online implementation, (46) is less computationally demanding than (43) as it does not involve any explicit prediction. Moreover, it has inherent robustness properties in that it is able to deal with any system such that the set $\{x | V(x, v) - \Gamma(v) \leq 0\}$ is invariant and constraint admissible. On the other hand, (46) is typically more conservative and the response of $v(t)$ may be slow. Note that changing the constraints from (42) into $V(x(t), v) \leq \Gamma(v)$ through (45) may be viewed as a form of constraint re-modeling where the output constraints are replaced by a constraint on the value of the Lyapunov function.

From the design viewpoint, assuming a Lyapunov function V is available, one of the main issues in implementing (46) concerns the off-line determination of $\Gamma(v)$. Note first that $\Gamma(v) = V_{\min}$ with V_{\min} as in (44) is always a feasible (although potentially a conservative) value for $\Gamma(v)$. A notable case where the computation of $\Gamma(v)$ is easy is presented in Garone and Nicotra (2016) where it is shown that, in the case of a linear constraint in the form $h_x^T x + h_v^T v \leq h$, and for a Lyapunov function that is lower-bounded by a quadratic form, i.e., $V(x, v) \geq (x - v)^T P (x - v)$, for $P = P^T > 0$, a feasible $\Gamma(v)$ can be computed in closed form as

$$\Gamma(v) = \frac{(h_x^T \bar{x}_v + h_v^T v + h)^2}{h_x^T P^{-1} h_x}. \quad (47)$$

Note that $\Gamma(v)$ as in (47) is optimal in the case $V(x, v) = (x - v)^T P (x - v)$. In the case of multiple constraints, $\Gamma(v)$ can be computed as the minimum of the bound computed for each constraint. Other special cases for which the computation of $\Gamma(v)$ is easy are reported in Nicotra and Garone (2015a).

Finally, note that the approach based on (46) can be used even in the cases where disturbances are present, provided $V(x, v)$ is an ISS-Lyapunov function. In this case, there exists a value V_{ISS} such that $V(x, v) > V_{ISS}$ implies $V(f(x, v, w), v) - V(x, v) < 0$, $\forall w \in W$. The method (46) can be used for the disturbed system under the only conditions that ε is sufficiently large to ensure that $V_{\min} > V_{ISS}$, with V_{\min} defined as in (44).

In Gilbert and Kolmanovsky (2002), a generalization of both prediction-based and Lyapunov function-based nonlinear reference governors applicable to constrained systems with disturbances is presented. The basis of this method is to define a continuous function $S(x, v)$ so that, for any $(x(t), v)$ such that $S(x(t), v) \leq 0$, if v is kept constant from t onward, the trajectory $\hat{x}(t + k|t)$, $k \geq 0$ is

- *safe*: constraints are never violated, i.e. $h(\hat{x}(t + k|t), v) \leq 0$, $\forall k \geq 0$;
- *strongly returnable*: there exists a finite integer k^* , which may depend on $x(t)$, such that $S(\hat{x}(t + k^*|t), v) < 0$, which means that after a finite time k^* the trajectory returns to the interior of the set $\{(x, v) : S(x, v) \leq 0\}$.

Given such a function S , $\kappa(t)$ can be chosen at each time instant t by solving the following scalar optimization problem,

$$\begin{aligned} \kappa(t) = \max_{\kappa \in [0, 1]} & \kappa \\ \text{s.t.} & S(x(t), v(t-1) + \kappa(t)(r(t) - v(t-1))) \leq 0, \\ & h(\bar{x}_v, v(t-1) + \kappa(t)(r(t) - v(t-1))) \leq -\varepsilon. \end{aligned} \quad (48)$$

If no feasible solution to (48) exists, $\kappa(t) = 0$ is used. Note that the prediction-based RG (43) may be viewed as a particular case of (48) with $S(x, v) = \max_{0 \leq k \leq k^*, i=1, \dots, n_y} \{h_i(\hat{x}(t+k|t))\}$ where n_y is the dimension of the output vector. The Lyapunov function-based RG (46) may also be viewed as a particular case of (48) with $S(x, v) = V(x, v) - \Gamma(v)$. For such a generalized nonlinear RG, recursive feasibility and finite-time convergence properties hold under suitable assumptions, see Gilbert and Kolmanovsky (2002).

Again, as in the previous scheme, the main difficulty of the method is to construct the function $S(x, v)$. However it should be remarked that the fact that sublevel sets of $S(x, v)$ must be only strongly returnable rather than positive invariant (as in the case of a Lyapunov function) may simplify the computation of S . In fact, for any S and for any v such that $\bar{x}_v \in \{x : S(x, v) < 0\}$, strong returnability is automatically ensured due to the fact that \bar{x}_v is asymptotically stable (and thus attractive).

To give an example of how S can be computed, consider the case of a system for which it is possible to find a function $\delta(\epsilon)$ such that for any v , $\|x - \bar{x}_v\| \leq \epsilon$, implies $\|h(\bar{x}_v, v) - h(\hat{x}(t + k|t), v)\| \leq \delta(\epsilon)$, $\forall k \geq 0$, then S can be built as $S(x, v) = h(\bar{x}_v, v) - \delta(\|x - \bar{x}_v\|)$. Note that although possibly quite conservative, this type of a set does not need to be based explicitly on the detailed system dynamic model and, in principle, could be constructed from intrinsic properties of the system (e.g., energy conservation principles, physics laws, etc.), or experimentally, using estimation/learning procedures.

Another approach to construct S in the disturbance-free case uses off-line simulations of the closed-loop system and generates S as a classifier so that $S(x(t), \bar{v}) \leq 0$ distinguishes safe pairs of states and constant reference commands from the unsafe ones. Machine learning techniques (Mitchell, 1997; Ong, Sathiyar Keerthi, Gilbert, & Zhang, 2004) can be used to compute such a classifier from simulation data. Note that combining/aggregating several of such classifiers, $S_j(x, v)$, $j = 1, \dots, J$, i.e., using $S(x, v) = \max_{j=1, \dots, J} S_j(x, v)$, is possible. This “union of classifiers” approach has the potential to significantly simplify the machine learning task by aggregating several locally valid classifiers (Gilbert & Kolmanovsky, 2002). In the case with disturbances, the offline computations are considerably more involved as the classification of pairs of states and constant reference commands into safe and unsafe pairs involves evaluating multiple scenarios or solving optimal control problems with respect to $w(\cdot)$.

3.4. Explicit reference governor

A different approach to the reference management of constrained nonlinear systems is the *Explicit Reference Governor* (ERG) recently proposed in Garone and Nicotra (2016) and Nicotra and Garone (2015b). This scheme represents an alternative to optimization-based solutions and, similarly to the first reference governor proposed in Kpasouris et al. (1990) for linear systems, is a continuous-time scheme. More precisely, the ERG manages the reference of pre-compensated continuous-time systems in the form

$$\dot{x} = f(x, v), \quad (49)$$

subject to constraint (42). The main idea is to ensure constraints enforcement by continuously manipulating the reference v so that the current state $x(t)$ always belongs to a safe invariant set centered at the steady-state $\bar{x}_{v(t)}$. To build the scheme, first a Lyapunov function $V(x, v)$ and a function $\Gamma(v)$ as in (46) are computed. Then, the derivative of $v(t)$ is manipulated accordingly to the following static state-feedback control law

$$\dot{v} = k_r [\Gamma(v) - V(x, v)] \frac{r - v}{\max\{\|r - v\|, \varepsilon\}} \quad (50)$$

where $k_r > 0$ and $\varepsilon > 0$ are arbitrary scalars. It is possible to prove (Garone & Nicotra, 2016) that, if at time $t = 0$ an admissible $v(0)$ is known and if the reference is steady-state admissible, feasibility and convergence are ensured under the same conditions as for the other nonlinear schemes presented in Section 3.3. Variants of (50) ensuring convergence for any r are presented in Nicotra and Garone (2015b). Further extensions of the method can be found in Garone and Nicotra (2016), Nicotra (2016), Nicotra, Bartulovic, Garone, and Sinopoli (2015), and Nicotra and Garone (2015a, 2016). In terms of performance, the ERG is usually more conservative than the optimization-based schemes. However, it is, in general, simpler to implement and less computationally demanding.

3.5. Parameter governor

Parameter governors (Kolmanovsky & Sun, 2004, 2005, 2006) are schemes inspired by the reference governor philosophy that adjust parameters, $\theta(t) \in \Theta$, of a nominal control law so as to optimize over a finite horizon the predicted system response subject to constraints. Parameters are assumed to remain constant over the prediction horizon. The cost minimized at each time instant is of the form

$$J(t) = \|\theta\|_{\psi_\theta}^2 + \sum_{k=0}^T \Omega(\hat{x}(t+k|t), \theta(t), r(t)), \quad (51)$$

and penalizes the system response as well as the parameter deviations. The assumption of constant parameters over the prediction horizon reduces computational and implementation effort, and simplifies the analysis. In fact, Θ may consist of a finite number of elements and so the evaluation of (51) and of the constraints reduces to a finite set of simulations.

Specific parameter governor schemes considered in Kolmanovsky and Sun (2006) include the feedforward¹ governor and the gain governor. For these schemes terminal set conditions need not to be imposed to ensure stability, provided that the horizon is

chosen sufficiently long and in agreement with the appropriate assumptions. In the feedforward governor approach of Kolmanovsky and Sun (2006), a disturbance-free system is considered with an integrator included as a part of the overall system,

$$\begin{aligned} x(t+1) &= f(x(t), u(t)), \\ x_i(t+1) &= x_i(t) + z(t) - r, \\ z(t) &= h_z(x(t)), \end{aligned} \quad (52)$$

where z is an output which is supposed to track the reference, r . The control law includes an integral action and an adjustable feedforward offset $\theta(t)$,

$$u(t) = u_e(r) - \epsilon x_i(t) + \bar{u}_{fb}(x(t), r) + \theta(t), \quad (53)$$

where $\epsilon > 0$ is a small gain, and $x_e(r)$, $u_e(r)$ denote the equilibrium values of state and control variables corresponding to the given r , respectively. Due to the use of integral action, if θ is constant, then as $t \rightarrow \infty$ it follows, under suitable assumptions, that $z(t) \rightarrow r$, $x_p(t) \rightarrow x_e(r)$, $u(t) \rightarrow u_e(r)$. The small gain integral control leads to dynamics decomposition into slow and fast modes (Desoer & Lin, 1985). The fast dynamics can be made to avoid constraint violations by changing $\theta(t)$; consequently, the constraints will be satisfied if the slow manifold is well within the constraint admissible region. The feedforward governor of Kolmanovsky and Sun (2006) can thus handle large reference changes and recover a large set of initial states. Note that the cost (51) is modified to include the penalty on the integral states.

3.6. Other developments

Other approaches to the design of RGs and related schemes for nonlinear systems have been presented in the literature. Developments are reported in Do, Choi, and Kyung (2011) (and earlier in Do and Choi (2005), for linear systems) concerning RGs for nonlinear systems subject to input constraints only. In Hatanaka and Takaba (2005a) an output feedback RG for nonlinear systems is presented. Ref. Angeli and Mosca (1999) presents the conditions for a CG for nonlinear systems where $\|r - v\|^2$ is minimized by solving at each time instant a (possibly non-convex) optimization problem. In Angeli et al. (1999) a similar CG for nonlinear systems is considered and compared with the RG based on embedding the nonlinear model into a family of Linear Time Varying polytopic uncertain systems. In the incremental RG approach of Tsourapas, Sun, and Stefanopoulou (2009) in order to reduce the computational effort, the solution of (43) is distributed over time by checking the feasibility of a single value of $v(t)$ that differs from $v(t-1)$ by a fixed and “small enough” step size. In Kolmanovsky, Kalabić, and Gilbert (2012) a predictor–corrector form of Newton’s method, with one iteration/update per time step, is proposed to be applied to the parametric root-finding problem in the command governor case. Landing RGs have been proposed for systems with terminal constraints, e.g., for reaching a desired position with a small velocity in mechanical systems (Kahveci & Kolmanovsky, 2010b; Kolmanovsky & Gilbert, 2001; Kolmanovsky & Haghgooie, 2004). In Sun and Kolmanovsky (2005), the robust RG approach is introduced in the context of a fuel cell application. There, the case when disturbances $w(t)$ are constant parameters is treated, and a Taylor series approximation (first order with a quadratic bound on omitted terms) is employed in the implementation to predict constraint violation.

4. Recently proposed reference governor schemes

Several special RG schemes and design procedures have recently been proposed. They include reduced order reference governors, network reference governors for systems controlled through

¹ Although it has a similar name, the (parameter) feedforward governor presented in this section should not be confused with the (sensorless) feedforward command governor presented in Section 2.2.

communication networks, virtual state governors, reference governors for fault handling, and reference governors for decentralized systems.

4.1. Reduced order reference governors

Reduced order reference and extended command governors (Kalabić, 2015; Kalabić, Kolmanovsky, Buckland, & Gilbert, 2012; Kalabić, Kolmanovsky, & Gilbert, 2014) enforce constraints based on a reduced order model. These governors are typically applied to systems decomposable into slow and fast subsystems, with the reduced order model representing the dynamics of slow states, while the contributions of fast states and observer errors are accounted for implicitly.

To illustrate the design approach, consider a transformation (e.g., by diagonalization) of a system, assumed to be disturbance-free for simplicity, into a “slow” subsystem and a “fast” subsystem, the outputs of which are additively combined, i.e.,

$$\begin{aligned} x_s(t+1) &= A_s x_s(t) + B_s v(t), \\ x_f(t+1) &= A_f x_f(t) + B_f v(t), \\ y(t) &= C_s x_s(t) + C_f x_f(t) + D v(t) \in Y. \end{aligned} \quad (54)$$

By decomposing the state of the fast subsystem into a sum of the steady state part, $\bar{x}_{f,v}(t) = (I - A_f)^{-1} B_f v(t)$, and transient deviation, $e_f(t)$, i.e., $x_f(t) = e_f(t) + \bar{x}_{f,v}(t)$, system (54) can be rewritten as

$$\begin{aligned} x_s(t+1) &= A_s x_s(t) + B_s v(t), \\ e_f(t+1) &= A_f e_f(t) + B_\Delta \Delta v(t), \\ y(t) &= C_s x_s(t) + C_f e_f(t) + D_e v(t) \in Y, \end{aligned} \quad (55)$$

where $\Delta v(t) = v(t) - v(t-1)$, $B_\Delta = -(I - A_f)^{-1} B_f$, and $D_e = D + C_f(I - A_f)^{-1} B_f$. The main idea of the approach is that, under suitable assumptions on the sampling time of the system, it is possible to ensure the existence of a compact set E_x that can be made positively invariant for the fast subsystem (i.e. such that $e_f(t+1) \in E_x$ if $e_f(t) \in E_x$) by using an additional constraint on the variation of the reference, i.e. $\Delta v(t) \in \Delta \mathcal{V}$. At this point, using this auxiliary constraint, and under the assumption $e_f(0) \in E_x$, the reduced order governor enforces the constraint on the reduced system treating the contribution of the fast system as a bounded disturbance, i.e. $y(t) \in Y \sim C_f E_x$. Under suitable assumptions, constraint enforcement, recursive feasibility and desirable response properties, such as finite-time convergence to steady-state admissible constant commands, can be guaranteed.

In Kalabić et al. (2012) this approach is demonstrated for an infinite-dimensional system (free-free beam) and constraints are successfully enforced based on a finite-dimensional reduced order model that reflects the dynamics of the first several modes. The developments in Li, Kalabić, and Kolmanovsky (2014) imply that if the reduced order model can be made second order, an efficient and fast implementation of the RG computations is possible.

The general principle of maintaining specified error states in an appropriately defined invariant set by constraining the rate of change of the command, Δv , and treating the effect of these error states as bounded disturbances is similar to Garone et al. (2009) and Garone et al. (2011). This principle underlies several other proposed RG designs discussed in this survey article.

4.2. Network reference governor handling variable time-delays

Several approaches for networked control systems, where the governor and the plant are connected via a non-ideal communication network, have been initially proposed in Bemporad (1998a), Casavola, Mosca, and Papini (2006), Casavola, Papini, and Franzè (2006), Kogiso and Hirata (2006), and more recently in Di Cairano,

Kalabić, and Kolmanovsky (2015) and Di Cairano and Kolmanovsky (2010, 2012). Refs. Bemporad (1998a), Casavola, Mosca et al. (2006) and Casavola, Papini et al. (2006) focus on cases when the communication between the plant and the CG is synchronized, which results in the communication network introducing delays that are multiple of the sampling period. Such delays, which can represent also packet drops when they become infinitely long, are dealt with by different types of redundancy strategies, based either on the best case assumptions, when resynchronization strategies are applied to recover in case of command delay/loss, or on the worst case assumptions, when commands that are robust with respect to any possible sequence of data losses are selected.

Refs. Di Cairano, Kalabić et al. (2015) and Di Cairano and Kolmanovsky (2010, 2012) focus on the case when the plant and the RG/CG are not synchronized by a common clock, thus resulting in asynchronous communication and the delay being real valued. In such a case, modifications to the O_∞ set construction are exploited for robustly dealing with the network induced delay. In Di Cairano, Kalabić et al. (2015) and Di Cairano and Kolmanovsky (2010, 2012), the command $v(t)$ is transmitted through a communication channel that has variable real-valued time delay, $\delta(t) \in [0, \bar{\delta}]$. When the delay $\delta(t)$ is smaller than the RG update period, T_s , the effect of the delay on the state is shown in Di Cairano and Kolmanovsky (2010) to satisfy the relation,

$$x(t+1) = Ax(t) + Bv(t) + R(\delta(t))\Delta v(t), \quad (56)$$

where $\Delta v(t) = v(t+1) - v(t)$, $R(\delta) = -\int_0^\delta e^{A_c(T_s-\tau)} B_c d\tau$, and (A_c, B_c) are the matrices of the continuous-time system model. Consequently, the effective disturbance introduced by the delay in (56) is proportional to the change in the command, $\Delta v(t)$. Thus, the network RG manipulates the reference change to ensure that the delay-induced disturbance does not cause a constraint violation. With some slightly conservative approximation, the network RG reduces to solving a QP. The case of longer-than-sampling period time delay is dealt with by augmenting the design with a simple acceptance/rejection logic located at the plant site, which provides guarantees in terms of constraint satisfaction and convergence in probability of the command to the reference.

While originally developed for network control systems, the network RG can prove useful in any application when the time delay is time varying. Several extensions of these results are developed in Di Cairano and Kolmanovsky (2010, 2012), including delays in both command and measurement channels, the output feedback case, and longer (random, possibly unbounded) delay for which a simple command acceptance/rejection logic is implemented at the plant side. A further extension developed in Di Cairano and Kolmanovsky (2012) considers slowly-varying delays with known bound on the rate of change.

4.3. Virtual state governor for integrating existing controllers

The Virtual State Governor (VSG) (Di Cairano & Kolmanovsky, 2011) is a strategy for modular control system design that aims at integrating multiple actuators, each equipped with an assigned non-modifiable feedback control law, while enforcing constraints and minimizing the use of those actuators that are “expensive” to operate. This problem is of interest in automotive applications, for instance in cornering control (Di Cairano, Tseng, Bernardini, & Bemporad, 2013), engine control (Di Cairano, Doering, Kolmanovsky, & Hrovat, 2014; Di Cairano, Yanakiev, Bemporad, Kolmanovsky, & Hrovat, 2012), energy management in hybrid powertrains (Di Cairano, Liang, Kolmanovsky, Kuang, & Phillips, 2011), and in aerospace applications such as spacecraft attitude control (Di Cairano & Kolmanovsky, 2011).

Given a constrained plant with two actuator groups, each with a pre-assigned controller $u_1(t) = K_1(t)x(t)$, $u_2(t) = K_2x(t)$,

the virtual state governor produces the “virtual states” $x_1, x_2 \in \mathbb{R}^n$, $(x_1(t) \ x_2(t)) = g(x)$, where $g(x)$ is a nonlinear mapping to be determined, that are provided to the pre-assigned controllers in place of the system state x . Thus, the virtual state governor modulates the effect of the controllers by modifying the state from which the feedback is computed. The virtual states are obtained by decomposing the actual state in a way that minimizes the usage of the expensive actuator while ensuring constraint satisfaction. Such a decomposition is computed by solving a QP based on the maximal output admissible sets for the plant in closed loop with each single controller and a Lyapunov function of the loop involving the expensive actuator.

4.4. Governors and fault tolerant control

A further recent use of reference and command governor schemes is in the development of fault-tolerant systems. The main idea behind these schemes is that in the presence of constraints, after the occurrence of a fault (e.g. the loss of an actuator), a system is not able to achieve the same nominal performance that it had before the fault occurrence. Consequently, it may not be enough to reconfigure only the feedback control law, but also the control objectives (i.e., the reference) should be modified. Reference and command governor schemes represent a quite natural solution to this problem.

The first contributions on the subject appear to be Casavola, Franzè, and Sorbara (2007a,b) where the reference-offset governor was introduced. The reference-offset governor consists in the joint use of a command governor and a parameter governor, where the latter is the feedforward governor (Kolmanovsky & Sun, 2006). In this approach, system (1) is extended with a fictitious input $\theta(t)$,

$$x(t+1) = Ax(t) + Bv(t) + B_\theta\theta(t),$$

the corresponding \tilde{O}_∞ , is computed and the problem

$$(v(t), \theta(t)) = \arg \min_{v, \theta} \|v - r(t)\|_Q^2 + \|\theta\|_{Q_\theta}^2, \\ \text{s.t. } (v, \theta, x(t)) \in \tilde{O}_\infty,$$

with $Q > 0$ and $Q_\theta > 0$ is solved. The idea behind this scheme is that, after a fault event, the parameter $\theta(t)$ can be used to add an offset to the nominal system that automatically counteracts some of the effects of the fault. A similar approach has been proposed in De Almeida (2011). A number of papers exploiting this idea have appeared. In Casavola, Famularo, Franzè, and Garone (2009) and Franzè and Tedesco (2011) the reference-offset governor was used within a distributed one-master-to-many-slaves framework and was augmented with the capability to manage communication latencies between the master and the slaves. In Boussaid, Aubrun, and Abdelkrim (2010) the reference-offset governor was applied in a scheme where a fault detection and identification module informs the governor of the changes of the system model due to the fault. This idea has been extended in Boussaid, Aubrun, and Abdelkrim (2011a,b); Boussaid, Aubrun, Jiang, and Abdelkrim (2014) and Weber, Boussaid, Khelassi, Theilliol, and Aubrun (2012) where both the reference governor and the stabilizing feedback control are reconfigured on the basis of the identified fault. The idea of changing the model and the constraints after a fault has been formalized in Franzè, Furfaro, Mattei, and Scordamaglia (2013) and Franzè, Furfaro, Mattei, and Scordamaglia (2015) within the framework of the hybrid command governor introduced in Famularo et al. (2011, 2015).

4.5. Decentralized command governors for large scale and multi-agent systems

Several recent research efforts have been devoted to the development of *Decentralized Command Governors* applied to systems

consisting of N dynamically coupled subsystems, which are subject to local and global constraints. The initial solutions made use of the feedforward command governor approach developed in Garone et al. (2009) that allows to reformulate the decentralized reference management problem as a static problem of determining at each decision step local references $v_i(t)$, $i = 1, \dots, N$, such that the aggregated vector $v(t) = (v_1(t), \dots, v_N(t))$ belongs to the static set O^ϵ and such that the variation of $v(t)$ within two update times is constrained in the set of admissible variations $\Delta \mathcal{V}$, i.e., $v(t+1) - v(t) \in \Delta \mathcal{V}$. Following this philosophy, both sequential and parallel approaches have been proposed. Sequential approaches (Casavola, Garone, & Tedesco, 2011a) are schemes where only one agent at the time is allowed to modify its command. Parallel approaches (Casavola, Garone, & Tedesco, 2011b) are schemes where all the agents are allowed to move the command at the same time and each agent makes a local decision assuming that the other agents will make the worst case choices. This second approach has proven to work well when the aggregated command v is far from the boundary of O^ϵ , but unsatisfactorily when v is close to its boundary. For this reason, hybrid approaches switching between parallel and sequential modes have been proposed in Garone, Tedesco, and Casavola (2010) and Tedesco, Casavola, and Garone (2012b). An interesting aspect of these schemes is that, although ensuring constraints satisfaction, they may experience problems with convergence to “good approximations” of the desired reference signals $r_i(t)$, $i = 1, \dots, N$. In fact, as shown in Casavola et al. (2011a), these schemes may experience convergence of the commands $v_i(t)$, $i = 1, \dots, N$ to Nash equilibria that are not Pareto-optimal. In Casavola, Garone, and Tedesco (2011c, 2014b) and Tedesco, Casavola, and Garone (2012c) this phenomenon has been carefully investigated and some algorithms to check the existence of these anomalies and to eliminate them are provided. Following the same idea of feedforward distributed command governor strategies, decentralized schemes making use of state measurements have been presented in Tedesco, Casavola, and Garone (2012a); Tedesco, Raimondo, and Casavola (2014), and Casavola, Garone, and Tedesco (2014a, 2016). Finally, it is worth mentioning decentralized the command governor schemes (Casavola, Garone, & Tedesco, 2014d; Soleymani, Garone, & Dorigo, 2015) that have been developed for the case of independent systems subject to coupling constraints. In this case, using colorability theory, it is possible to partition the agents into sets of agents that are not directly coupled by a constraint. Following this idea, a sequential scheme has been proposed where all agents belonging to the same set can move their reference at the same time independently one from each other, while the remaining agents keep the previous command.

4.6. Other developments

Several other approaches and results have appeared in recent years. They include approaches in Taguchi and Ohta (2007) that combine reference governors and controller switching to improve performance. A related strategy is also used in Kolmanovsky and Sun (2006). In McDonough and Kolmanovsky (2015) a reference governor with the added capability of resetting internal closed-loop system states to avoid constraint violation is developed. With this approach the constrained domain of attraction, i.e., the set of states recoverable without constraint violation, is enlarged. This scheme is referred to as the Controller State and Reference Governor (CSRG). Refs. Franzè and Lucia (2016) and Franzè, Lucia, and Tedesco (2015) are proposing governors able to deal with switching systems. Refs. Kalabić (2015), Kalabić, Gupta, Di Cairano, Bloch, and Kolmanovsky (2014) and Kalabić and Kolmanovsky (2014, 2015) exploit contractive, rather than just invariant, sets and passivity to handle systems with time-varying reference commands and time-varying constraints and to treat the

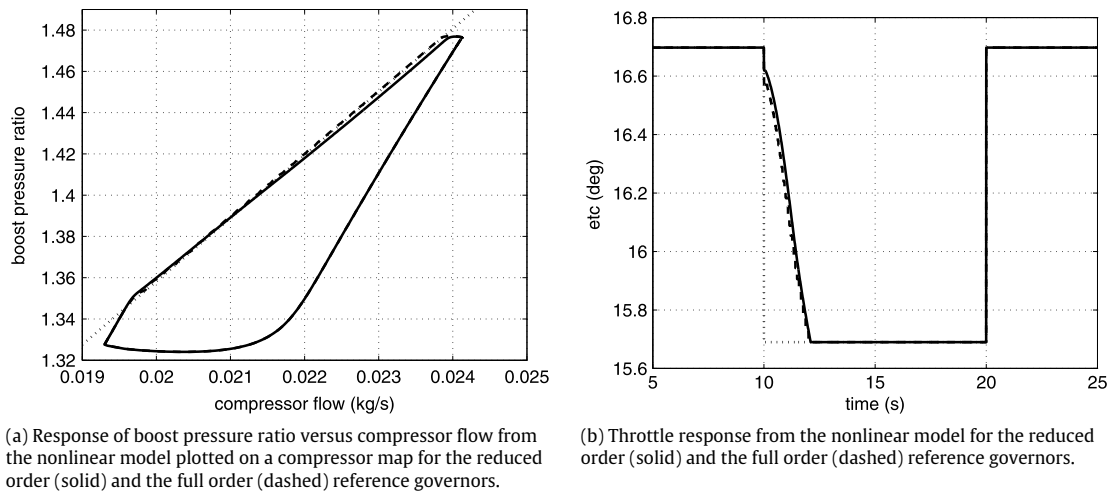


Fig. 5. Reference governor application to turbocharged gasoline engine.

unconventional reference governor placement inside rather than outside of a control loop. The papers (Oh, Kim, & Cho, 2008; Oh & Peng, 2001) propose the so called adaptive reference governor, in which $\hat{v}(t + k|t)$ is parameterized in terms of future values of the disturbance and a semi-definite programming problem is solved to achieve a less conservative design. The papers (Schatz, Yucelen, Johnson, & Holzapfel, 2013; Yucelen & Johnson, 2012) propose another command modification scheme to improve transient performance of adaptive controllers and to enforce constraints. A scheme to maintain Lyapunov function decay in presence of saturation constraints by modifying the reference is proposed in Kang and Grizzle (1999). In Kalabić, Gupta et al. (2014), a reference governor for constrained control on manifolds, and in particular on the special orthogonal group of dimension 3, $SO(3)$, has been proposed.

5. Applications

The key advantage of the governors is their capability of guaranteeing the enforcement of constraints with limited computational effort. Thus, the areas where these have been found most success are those involving applications with (i) relatively fast dynamics, (ii) computational platforms with limited capabilities, and (iii) the need to operate the system in the entire operating range, i.e., including close to the constraints. These areas include especially automotive, aerospace, and precision mechatronics and factory automation.

5.1. Automotive applications

Due to constant pressure to reduce costs and satisfy increasingly more stringent regulations, (i)–(iii) above are of significant relevance in automotive applications. As a consequence, several RG applications have been reported in the automotive domain.

Refs. Banker et al. (2015), Kalabić (2015), Kalabić, Buckland, Cooper, Wait, and Kolmanovsky (2016) and Kalabić, Kolmanovsky, Buckland et al. (2011) address compressor surge constraint handling in turbocharged gasoline engines. These engines are downsized and provide improved fuel efficiency, yet have tight operating constraints. Reference governor techniques are applied to modify the Electronic Throttle Command (ETC) and wastegate commands in Kalabić, Kolmanovsky, Buckland et al. (2011), and air charge command in Kalabić (2015) and Kalabić et al. (2016). Experimental results of RG implementation in a vehicle are reported in Kalabić (2015) and Kalabić et al. (2016). To further

reduce the computational footprint of the algorithm, in Kalabić et al. (2012) the reduced order RG is applied, taking advantage of the different time scales in the engine dynamics. The validation results based on the nonlinear model and the observer for unmeasured states, see Fig. 5, are presented in Kalabić et al. (2012).

Other effective applications or governors for powertrain control include constraint handling in diesel engines (Nakada, Martin, Milton, Iemura, & Ohata, 2014a; Nakada, Milton, Martin, Iemura, & Ohata, 2013; Nakada, Martin, Milton, Iemura, & Ohata, 2014b), HCCI engines (Jade, Hellström, Larimore, Stefanopoulou, & Jiang, 2014), and free piston engines (Zaseck, Brusstar, & Kolmanovsky, in press, 2014). Both RGs and CGs have been applied to engine speed control (Albertoni et al., 2003; Kolmanovsky, Gilbert, & Cook, 1997).

For non-conventional powertrains, significant developments were pursued for fuel cells (Gaynor, Mueller, Jabbari, & Brouwer, 2008; Laghrouche, Matraji, Ahmed, Jemei, & Wack, 2013; Sun & Kolmanovsky, 2005; Vahidi et al., 2007), where the main constraints are on keeping the oxygen over hydrogen ratio sufficiently high to prevent oxygen starvation, on avoiding compressor surge and choke regions, and on avoiding compressor voltage saturation. The model can be up to 10th order, and in Vahidi et al. (2007) the procedure described in Section 3.1 is exploited for compensating the mismatch between the linearized model and the actual nonlinear system. Parameter uncertainties in temperature and humidity are handled using the robust RG applied to the nonlinear fuel cell system model in Sun and Kolmanovsky (2005). The RG in Vahidi et al. (2007) was implemented in the production microcontroller obtaining computation time of 1.3ms for an update rate of 10ms and requiring 4kB of ROM occupancy. For hybrid electric and full electric vehicles, RGs were applied to handling constraints in electric batteries in Moura, Chaturvedi, and Krstic (2013) and Smith (2010).

For chassis and vehicle dynamics control, Kolmanovsky, Gilbert, and Tseng (2009) applies the RG and the ECG to vehicle roll control to modify the steering angle and operating the brakes so that constraints on the load transfer ratio, which represents a measure of roll instability, are enforced. Both a scalar RG and an ECG are shown to prevent rollover, the latter having higher computational burden, but much larger domain of recoverable states. As illustrated by Fig. 6(a)–(b) ECG is robust to the mismatch between the model and actual system dynamics. Further validation results of these approaches based on nonlinear vehicle models are presented in Bencatel, Girard, and Kolmanovsky (submitted for publication).

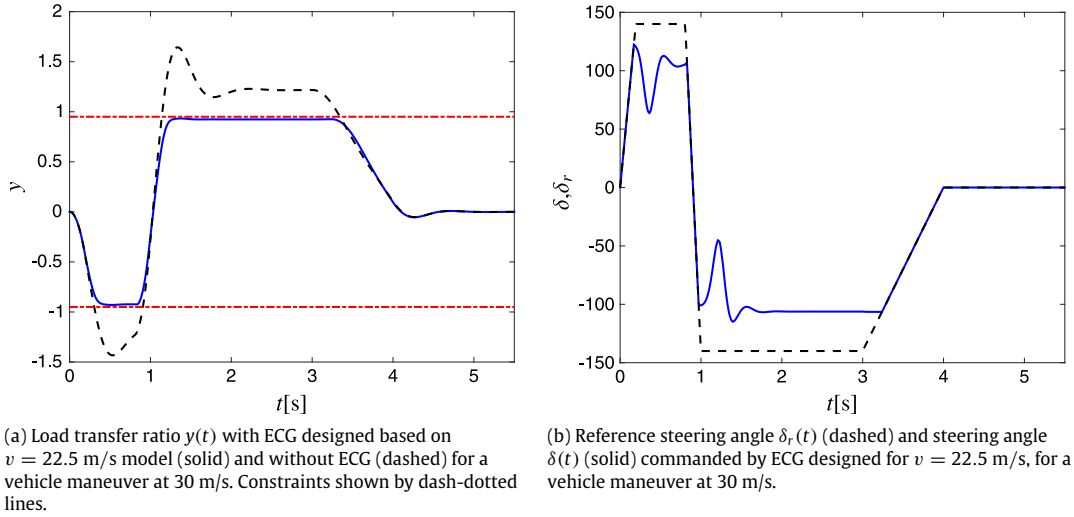


Fig. 6. Extended command governor for vehicle rollover prevention.

Other applications in active and passive vehicle safety include vehicle cornering and yaw stability control in [Borrelli et al. \(2009\)](#), where reference governor is developed based on a piecewise affine system model, control of steer-by wire systems ([Di Cairano & Zafeiropoulos, 2015](#); [Zafeiropoulos & Di Cairano, 2014](#)) where a command governor with a particular cost function is used, and belt restraint systems ([Van Der Laan, Heemels, Luitjen, Veldpaus, & Steinbuch, 2010](#)).

5.2. Precision mechatronics and manufacturing

Precision mechatronics deals with challenges that are similar to the ones in the automotive domain, i.e., fast dynamics, platforms with low computational power, and tight operating constraints. In precision mechatronics, reference governors have been exploited for various applications of electromagnetic actuators ([Kahveci & Kolmanovsky, 2010b](#); [Kalabić, Kolmanovsky, & Gilbert, 2011](#); [Kolmanovsky & Gilbert, 2001](#); [Kolmanovsky & Haghighi, 2004](#); [Miller et al., 2000](#)). In particular, the limited coil current leads to a force constraint expressed by a concave nonlinear function, while the soft landing constraint is of MLD type ([Di Cairano, Bemporad, Kolmanovsky, & Hrovat, 2007](#)), and can be handled using the techniques in [Kalabić, Kolmanovsky, and Gilbert \(2011\)](#). Additional applications of reference governors to mechatronic systems include cable robots ([Oh & Agrawal, 2005](#)), disk drives ([Guo, Ohta, Taguchi, & Masubuchi, 2012](#)), rotary cranes ([Iwasa, Terashima, Jian, & Noda, 2010](#)), and electrostatically actuated membrane mirrors ([Kolmanovsky, Miller, Washabaugh, & Gilbert, 1998](#)).

Recently, reference governors have been applied to motion control of dual-stage processing machines for precision manufacturing ([Di Cairano, Goldsmith, & Bortoff, 2015](#); [Haghighat, Di Cairano, Konobrytskyi, & Bortoff, 2014](#)). Dual-stage machines feature for each processing axis a fast stage, with large bandwidth and high accelerations but limited processing range, and a slow stage, with small bandwidth and limited accelerations, but large range. The two stages need to be coordinated to follow as fast as possible a given pattern, which results in a workpiece being properly processed with the worktool, e.g., a drill, a cutter, or a mill, mounted on the machine. Due to the stage dynamics time-scale separation, the problem reduces to controlling the slow stage such that the distance between its position (p_s) and the point to be currently processed (v) is smaller than the fast stage range (s_f), i.e., the reference dependent constraints $\|p_s(t) - v(t)\|_\infty \leq s_f$ must be

enforced. Since the pattern cannot be changed, the reference governor selects how many points are to be processed in the next sampling period, thus choosing $v = r_i$, where $\{r_i\}_i$ are the points to be processed. This effectively operates a time-scaling of an ideal reference trajectory to ensure constraint satisfaction. While it can be used alone, in [Di Cairano, Goldsmith et al. \(2015\)](#) the reference governor is used in combination with a Model Predictive Controller (MPC), which also minimizes secondary objectives such as acceleration-induced vibrations and/or energy consumption. From the previously predicted terminal state $\hat{x}(N + t - 1|t - 1)$, the reference governor extends the previous reference trajectory by a new point $\hat{v}(N + t|t)$, ensuring recursive feasibility of the MPC that enforces the terminal set constraint $(\hat{v}(N + t|t), \hat{x}(N + t|t)) \in O_\infty$, thus eventually ensuring the correct processing of the entire pattern. Due to the reference dependent constraints, the reference governor is necessary to ensure recursive feasibility, and it also provides a feasible solution that can be used to guarantee real-time computation for processors with limited computational capabilities. The trajectory obtained from the combination of reference governor and MPC with only one step prediction horizon for a pattern designed by a real CAD-CAM system is shown in [Fig. 7](#). Thanks to the reference governor, despite the very short MPC horizon, all the constraints are enforced.

5.3. Aerospace systems

Constraints are important considerations for aerospace systems such as fighter aircraft and missiles that are open-loop unstable and may become closed-loop unstable if large commands cause the actuator magnitude or rate saturation. Furthermore, maintaining aircraft operation within its flight envelope and preventing aircraft loss of control ([Wilborn & Foster, 2004](#)) is fundamentally a constrained control problem. Note that also structural/maneuver load alleviation in flexible aircraft and helicopters are problems that can be mitigated by using constrained control.

Several applications of RGs and CGs for constraint enforcement in aerospace systems have been investigated. The original work by [Kapasouris](#) on continuous-time RG ([Kapasouris, Athans, & Stein, 1988a,b](#)) used linearized models of fighter jet aircraft to demonstrate the benefits of continuous-time RG in handling actuator saturation. The article ([Pachter & Miller, 1998](#)) (see also the thesis [Miller, 1997](#)) considers several schemes including the discrete-time RG in the context of pilot command tracking control in aircraft with saturating actuators. The paper ([Famularo, Martino,](#)

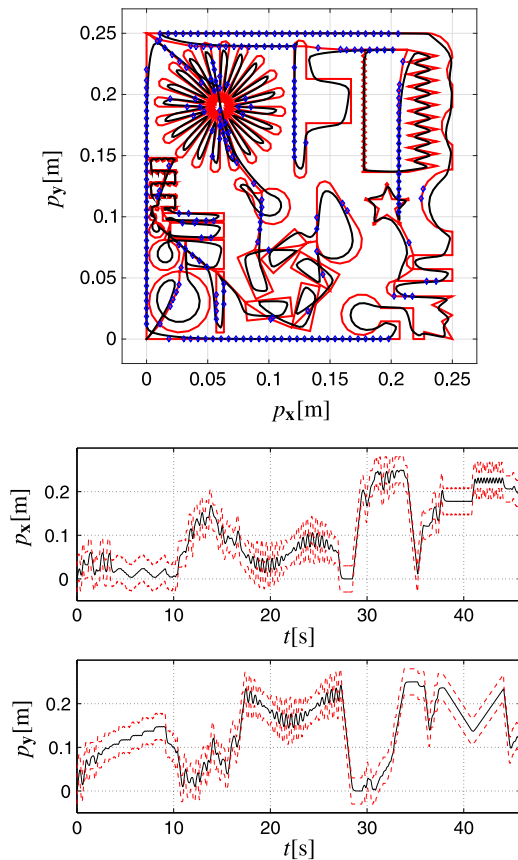


Fig. 7. Dual-stage machine motion control by reference governor and MPC. Above: spatial pattern (red), trajectory (black), points where reference governor reduces processing speed to ensure feasibility (blue diamonds). Below: trajectory (black solid) and constraints due processing point and fast stage range (red dash), i.e., $v(t) \pm s_f$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

& Mattei, 2008) (see also the conference paper Martino, 2008) investigates a CG strategy and presents simulation results for fighter aircraft addressing magnitude and rate constraints on actuators (rudder, tailerons, canards), roll rate, side-slip angle, and angle of attack. This paper also considers longitudinal constraint enforcement for small civil aircraft with actuator constraints on magnitude of aileron, rudder, and elevator deflection and constraint on the angle of attack, and concludes that “insertion of a command governor as an auxiliary device in the feedback loop involves a significant enhancement of the reliability and performance of both aircraft”. Simulation results of the hybrid CG on a P92 commercial aircraft model with constraints on aileron, rudder, and elevation magnitude and rates are reported in Franzè, Mattei, Ollio et al. (2014) where the reconfiguration in case of faults is also considered. Another application of hybrid CG to aircraft flight control is presented in Franzè, Mattei, and Scordamaglia (2014). The paper (De Almeida, 2011) exploits reference command filtering to ensure that feasible commands are provided to an MPC-based aircraft controller. In Weber et al. (2012) RGs, control allocation and failure mode handling are combined. In Ye, Chen, and Wu (2015) RGs are applied to flight envelope protection in high angle of attack maneuvers and a reference increment model is exploited to automate the computations. In Di Donato, Balachandran, McDonough, Atkins, and Kolmanovsky (in press), the controller state and command governor is incorporated into a larger scheme which mitigates

the aircraft loss of control. In Dillsafer, Kalabić, Kolmanovsky, and Cesnik (2013), the applications of scalar RG and ECG are considered to enforce limits on root curvature for a Very Flexible Aircraft model following a commanded flight path angle trajectory. In Kahveci and Kolmanovsky (2009), the RG is applied to enforce constraints in a glider UAV.

Applications of governors to missiles have been studied in Casavola and Mosca (2004) and Rodriguez and Wang (1996). The former paper develops a continuous-time RG (based on the ideas of Kapsouris et al., 1988b; Kapsouris et al., 1990) as well as the error governor (Kapsouris et al., 1988a) for a bank-to-turn (BTT) missile with saturating actuators. The latter paper considers a CG application to a bank-to-turn missile steering problem and demonstrates that missile stability can be maintained by avoiding elevator angle saturation. The design relies on output measurements and uses an observer for estimating the state.

For helicopters, Carlsson and Sunesson (2014) investigate various RG designs. Ref. Hirata and Minemura (2004) experimentally tests RGs and demonstrates the ability to enforce constraints on main and tail rotor swash plate angles.

Reference governors have been also employed for the obstacle avoidance of a single (Lucia, Szaier, & Franzè, 2014; Nicotra, Naldi, & Garone, 2016) or of multiple (Nicotra et al., 2015) quadrotors, and for the constraints management of tethered quadrotors (Nicotra et al., 2014; Nicotra, Naldi, & Garone, submitted for publication), kites (Eeckhout, Nicotra, Naldi, & Garone, 2014), and of a quadrotor manipulating an object (Nguyen & Garone, 2016).

The applications of governors to hypersonic vehicles have been studied in Petersen, Baldwin, and Kolmanovsky (2013) and Zinnecker, Serrani, Bolender, and Doman (2009). In Zinnecker et al. (2009), a reference governor strategy is employed to avoid input saturation and maintain stability with an adaptive controller. In Petersen et al. (2013), an ECG is used in the inner (flight control) loop to enforce constraints on the angle-of-attack, pitch rate, elevator deflection, elevator deflection rate, and on elastic deformation constraints. This ECG is integrated with an MPC-based controller in the guidance outer loop.

Supervisory switching schemes that exploit banks of RGs are considered in Bacconi (2005) and Bacconi, Mosca, and Casavola (2007) for spacecraft relative motion control with force and torque saturation limits and box constraints on positions and orientations. The paper (Kalabić, Kolmanovsky, & Gilbert, 2011) demonstrates that nonlinear quadratic constraints on Line-of-Sight approach cone and thrust magnitude in relative motion problems can be handled by the scalar RG. The ECG is applied to spacecraft relative motion problems in Petersen, Jaunzemis, Baldwin, Holzinger, and Kolmanovsky (2014) and its performance is compared with an MPC-based controller. The ECG is also a part of the proposed scheme in Dunham, Petersen, and Kolmanovsky (2016) for landing on an asteroid with uncertain gravity field. The applications of network RG to spacecraft constrained relative motion control in presence of communication delays and to spacecraft orientation with flexible appendage is considered in Di Cairano, Kalabić et al. (2015) and Di Cairano and Kolmanovsky (2010, 2012). The parameter governor is applied to coordinated control of a spacecraft formation in Frey et al. (2016).

In Kalabić, Gupta et al. (2014), a nonlinear RG strategy for constrained control on manifolds is introduced and applied to global spacecraft attitude control on $SO(3)$. A reference governor-like strategy for spacecraft attitude control that exploits a virtual net of equilibria on $SO(3)$ and a graph search for constraint admissible transitions is proposed in Weiss, Leve, Baldwin, Forbes, and Kolmanovsky (2014). An example illustrating the use of the virtual state governor to coordinate reaction wheels and thrusters for spacecraft attitude control is considered in Di Cairano and Kolmanovsky (2015).

A reference governor approach to guarantee the satisfaction of critical altitude, tether tension, and angle of attack constraints in the presence of realistic setpoint variations and wind disturbance inputs for the Altaeros tethered, lighter-than-air wind energy flying turbine system is demonstrated in Kalabić, Vermillion, and Kolmanovsky (2013).

The control systems of aircraft gas turbine engines must handle numerous constraints such as surge avoidance, over-speed and over-temperature limits, combustion lean blowout limit, actuator magnitude and rate limits, etc. The conventional scalar RGs and ECGs have been applied to these problems in Kolmanovsky, Jaw, Merrill, and TranVan (2012) and Kolmanovsky and Merrill (2014). Several decentralized RG designs for the distributed constraint enforcement in gas turbine engines have been developed in Kalabić and Kolmanovsky (2013) and Kalabić (2015), motivated by the ongoing distributed engine control initiatives (DECWG, 2016). The application of prioritized and reduced order RGs is considered in Tian and Kolmanovsky (2014).

5.4. Power networks

In recent years, the use of reference and command governors has been proposed in several papers to augment the existing load/frequency control and voltage regulation of multi-area power distribution systems. Typically, existing schemes consist of local regulators (usually P and PI controllers) that reject small imbalances around the nominal equilibria. Reference and command governors appear to be promising solutions to enhance these control systems with the capability of managing larger transients (e.g. due to fault occurrence, sudden load changes, or changes in the distributed generation), while taking explicitly into account the constraints and the inter-area connections.

For what concerns load frequency regulation, several contributions have been presented for both medium voltage and high voltage grids (Casavola et al., 2009, 2007a; Casavola, Franzè, Tedesco, & Garone, 2011; Tedesco & Casavola, 2014, 2015). These papers emphasize the capability of CGs and reference offset-governors to change the frequency of the various areas, to manage constraints, and to counteract the effect of possible faults or load/distributed production anomalies. In Franzè and Tedesco (2011) the developed scheme is also able to take into account possible communication imperfections.

For what concerns voltage regulation, the first paper where a CG was proposed for the voltage regulation in medium and low voltage distribution networks is probably (Casavola, Franzè, & Patton, 2008). Other papers include Casavola, Franzè, Menniti, and Sorrentino (2011), and Tedesco and Casavola (2013) that explore the use of a distributed CG in a power grid with distributed generation.

5.5. Other applications

Several other applications of governors have been presented. A non-exhaustive list includes the power management system of an all-electric ship (Seenumani, Peng, & Sun, 2011), chemical process control (Kogiso, Noda, & Nishitani, 2006), water channel networks (Li & De Schutter, 2012), wind turbines (Boussaid et al., 2011a), tokamak reactors for thermonuclear fusion (Mattei, Labate, & Famularo, 2013), formations of mobile robots (Nguyen, Hatanaka, Garone, & Fujita, 2016; Soleymani et al., 2015), and lab-scale demonstrators such as inverted pendula (Casavola, Mosca, & Papini, 2004; Garone & Nicotra, 2016) and four-tanks systems (Casavola, Famularo, Franzè, & Furfaro, 2010).

6. Connections with other design techniques and future research topics

In this section we discuss the connections between reference/command governors and related control techniques. We also touch upon potential topics for future research.

6.1. Connections with model predictive control

As predictive control schemes for constrained reference tracking, governors have many common features with Model Predictive Control (MPC) and, in fact, can be designed within the MPC framework (Rossiter & Kouvaritakis, 1998). Note also that MPC controllers are often applied to manipulate set-points to lower level controllers which further diminishes the distinction between these schemes. In this sense, governors can be viewed as MPC controllers with control horizon of 1 (or longer in extended command governor case), quasi-infinite constraint horizon, special cost function and applied to manipulate references. At the same time, they are special schemes with unique motivation and several unique properties, results, and simplifications (such as finite-time convergence for constant reference commands or design procedures based on the elimination of almost redundant constraint) that are not easily available to more general MPC controllers.

Reference handling in MPC (Aghaei, Sheikholeslam, Farina, & Scattolini, 2013; Ferramosca, Limón, Alvarado, Alamo, & Camacho, 2009; Limón & Alamo, 2015; Limón, Alvarado, Alamo, & Camacho, 2008), tube MPC (Limon, Alvarado, Alamo, & Camacho, 2010) and reduced complexity MPC approaches in Kouvaritakis, Cannon, and Rossiter (2002); Kouvaritakis, Rossiter, and Schuurmans (2000) (arguably) incorporate features similar to reference/command governors. Several control algorithms that integrate MPC with reference governing for setpoint manipulation have been recently proposed with the objective of avoiding constraint violations in transients due to setpoint changes, and enlarging the domain of attraction. Some of these strategies are based on a cascade of an RG with an MPC (De Almeida, 2011; Di Cairano, Goldsmith et al., 2015; Falugi, 2015), while others, are based on a single algorithm that performs concurrently the setpoint optimization and the control of the plant in a so-called artificial setpoint-augmented MPC (Ferramosca et al., 2009; Limón et al., 2008; Mayne & Falugi, 2015) that can be seen as the merging of MPC and RG in a single control scheme. Strategies that allow to shape the reference without online state feedback, and then applying MPC for tracking with guarantees that a pre-assigned bound on the tracking error is enforced have been also proposed in Di Cairano and Borrelli (2013, 2016). Other relevant schemes include Ferramosca, Limon, González, Odloak, and Camacho (2010), Limon, Ferramosca, Alvarado, Alamo, and Camacho (2009) and Limon et al. (2016).

Parameter governors proposed in Kolmanovsky and Sun (2006) have similarities with the parameterized nonlinear MPC in Alami (2006).

The benefits of incorporating reference governing into MPC design to guarantee constraint feasibility and enlarge the constrained domain of attraction has been demonstrated in several applications such as spacecraft control (Guiggiani, Kolmanovsky, Patrinos, & Bemporad, 2015; Weiss, Baldwin, Erwin, & Kolmanovsky, 2015), and factory automation (Di Cairano, Goldsmith et al., 2015). It is finally worth mentioning that a number of “special” reference governor schemes that incorporate some aspects of MPC have been proposed such as extended command governor schemes (Gilbert & Ong, 2011), and schemes mixing the use of cost function based on the reference and on the output error (Sugie & Yamamoto, 2001).

6.2. Connections with input shaping

The input shaping techniques have been proposed to minimize residual vibrations in flexible structures, see e.g. Pao (1999), Singer and Seering (1990), Singhose (2009), and Singhose and Pao (1997). Similar to reference/command governors, input shapers modify the input to the system, however, they typically are not designed to enforce state and control constraints. The feedforward and reduced order reference governors may be suitable for problems where input shaping has traditionally been used and there are constraints. However, their properties in such applications remain to be further studied.

6.3. Future research topics

While the subject of reference governors has been researched for over twenty years and, as discussed above, connects naturally with model predictive control and input shaping, a variety of other research directions can be identified.

The nonlinear RG results (see e.g., Gilbert & Kolmanovsky, 2002; Sun & Kolmanovsky, 2005 and references therein) extend to the case when the system has uncertain set-bounded constant parameters. At the same time, a non-conservative application of RGs to systems with uncertain parameters being estimated online remains an area to be further explored. Special assumptions appear to be necessary in this case to guarantee recursive feasibility and other standard RG properties. Combining reference and/or command governors with direct adaptive controllers also remains to be studied. Resetting adaptive parameters in direct adaptive controllers can yield schemes similar to the controller state and reference governors in McDonough and Kolmanovsky (2015). In this context, the main challenge appears to be able to perform a fast and non-conservative prediction of possible constraint violation for an uncertain system.

From the practical standpoint, the infeasibility treatment, i.e., what policy to adopt to recover from a situation in which the online optimization problem does not admit a solution and/or the constraints are violated, deserves further attention. Note that the infeasibility typically occurs as a result of the mismatch between the assumed model and the actual system response. The usual strategy in the scalar or vector RG case is to continue applying the previous reference. However, this could lead to governor “hang up”, where it stops updating the reference. While this may be a reasonable approach to handle occasional infeasibility, it is more problematic if the “hang up” occurs for prolonged periods of time. The ECG possesses more degrees of freedom to find feasible solutions through optimization. However, where RG is infeasible, ECG may produce discontinuous jumps in the reference that may also not be desirable. Treating constraints as soft avoids the infeasibility but, if the standard quadratic penalty on slacks is used as in Kalabić, Chitalia et al. (2013), it may lead to undesirable constraint violations even when they can be avoided. The latter issue may be addressed by using exact penalization techniques (Fletcher, 2013).

The treatment of the case when constraints are time-varying or reconfigured dynamically is of interest for many applications. While there have been successful treatments for specific examples, see e.g., Kolmanovsky et al. (1997), Petersen et al. (2013) and Dunham et al. (2016), the theory remains largely to be developed. For initial results in this direction, see Kalabić (2015) and Kalabić and Kolmanovsky (2014).

Traditionally, reference and command governors modify the set-points to closed-loop systems. While the theory assumes that the set-points are given, in real systems the set-points may be adjusted by a human operator in response to external conditions. This dependence of set-points on external conditions can create

a feedback loop encompassing the governor and nominal closed-loop system. A closely related situation occurs when an RG augments control signals at the *actuator command* level. This implementation of RGs is appealing for certain applications as the design and calibration of the nominal controller can be changed without the need to re-design the governor (Kalabić et al., 2016; Kalabić, Kolmanovsky, & Gilbert, 2011). Theoretical properties of RGs and CGs in the loop remain to be studied. For initial results in this direction, see Kalabić and Kolmanovsky (2015).

Error governors are schemes related to reference governors, however, they act on the tracking error at the controller input and not on the reference command. They are also primarily intended for handling control input constraints and not output constraints. See Gilbert and Tan (1991), Kapsouris et al. (1988a), Tan (1991) and Tharayil and Alleyne (2002). As compared to RGs, error governors have received relatively little attention; obtaining similar convergence guarantees for them as they are available for RGs/CGs has been elusive. It is interesting that the error governor can be applied with relative ease to direct adaptive controllers (Kahveci & Kolmanovsky, 2010a).

Similar to current developments in MPC (Mesbah, in press), the treatment of uncertainties and disturbances in the design of RGs deserves further attention. Currently, robust (set-bounded or ISS Lyapunov function based) treatment of the disturbances appears to be the mainstream approach. Stochastic and scenario-based approaches with respect to reference and/or disturbance modeling may be pursued to reduce conservatism, and to take advantage of stochastic disturbance models available in many applications.

Several RG ideas have been thus far developed for linear models and could be generalized to more general nonlinear models. This, for instance, concerns the reduced order RGs for which such a generalization may be tractable.

In the design of RGs, the nominal closed-loop systems models are frequently formulated in discrete-time. Further consideration of these models formulated in continuous-time is of interest to address the effect of sampling, inter-sample constraint violations and to be able to treat systems with input and state time delays. Such extensions may be tractable taking advantage of the ongoing research on set invariance characterization for these classes of systems in the literature.

Further research into the development of real-time optimization algorithms for governors, that take advantage of the specific features of the governor law evaluation, e.g., few variables, large constraints-variables ratio, minimum norm projection form, etc., appears to be worthwhile. This also includes the possible preference for using QP solvers (Bemporad, 2015) rather than LP solvers in the governor design procedures in Section 2.3. The detailed analysis on what algorithms are best for the design and real-time execution of the different governors deserves further investigation, benchmarking, and formal proofs.

Finally, continuing research into a synergistic combination of an upper level reference/command governor and lower level MPC-based controller appears to be worthwhile. This concerns, in particular, the possibility of slowing down reference command changes to enable the optimizer for the inner loop MPC controller to maintain good accuracy while relying only on a few iterations to maintain the feasibility of the real-time implementations.

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References

- Aghaei, Shahram, Sheikholeslam, Farid, Farina, Marcello, & Scattolini, Riccardo (2013). An mpc-based reference governor approach for offset-free control of constrained linear systems. *International Journal of Control*, 86(9), 1534–1539.
- Alamir, Mazen (2006). *Lecture notes in control and information science, Stabilization of nonlinear systems using receding-horizon control schemes: a parametrized approach for fast systems*. Springer.
- Albertoni, Leonardo, Balluchi, Andrea, Casavola, Alessandro, Gambelli, Claudio, Mosca, Edoardo, & Sangiovanni-Vincentelli, Alberto L. (2003). Hybrid command governors for idle speed control in gasoline direct injection engines. In *Proc. American control conference*. Vol. 1 (pp. 773–778). IEEE.
- Angeli, David, Casavola, Alessandro, & Mosca, Edoardo (1999). Command governors for constrained nonlinear systems: direct nonlinear vs. linearization-based strategies. *International Journal of Robust and Nonlinear Control*, 9(10), 677–699.
- Angeli, David, Casavola, Alessandro, & Mosca, Edoardo (2001). On feasible set-membership state estimators in constrained command governor control. *Automatica*, 37(1), 151–156.
- Angeli, David, & Mosca, Edoardo (1999). Command governors for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 44(4), 816–820.
- Aström, Karl J., & Wittenmark, Björn (1997). *Computer-controlled systems*. Baccini, Fabio (2005). *Spacecraft attitude dynamics and control*. (Ph.D. thesis), Florence university.
- Bacconi, Fabio, Mosca, Edoardo, & Casavola, Alessandro (2007). Hybrid constrained formation flying control of micro-satellites. *IET Control Theory & Applications*, 1(2), 513–521.
- Banker, Adam Nathan, Buckland, Julia Helen, Ulrey, Joseph Norman, Kalabic, Uros Vojko, Gerhart, Matthew John, Pallett, Tobias John, Kolmanovsky, Ilya, & Wait, Suzanne Kay (2015). Methods and systems for torque control, November 3. US Patent 9,174,637.
- Bemporad, Alberto (1998a). Predictive control of teleoperated constrained systems with unbounded communication delays. In *Proc. 37th IEEE conf. on decision and control*. Vol. 2 (pp. 2133–2138). IEEE.
- Bemporad, Alberto (1998b). Reference governor for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 43(3), 415–419.
- Bemporad, Alberto (2015). A multiparametric quadratic programming algorithm with polyhedral computations based on nonnegative least squares. *IEEE Transactions on Automatic Control*.
- Bemporad, Alberto, Casavola, Alessandro, & Mosca, Edoardo (1997). Nonlinear control of constrained linear systems via predictive reference management. *IEEE Transactions on Automatic Control*, 42(3), 340–349.
- Bemporad, Alberto, Morari, Manfred, Dua, Vivek, & Pistikopoulos, Efstratios N. (2002). The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1), 3–20.
- Bemporad, Alberto, & Mosca, Edoardo (1994). Constraint fulfilment in feedback control via predictive reference management. In *Proc. IEEE conf. control applications* (pp. 1909–1914). IEEE.
- Bemporad, Alberto, & Mosca, Edoardo (1995). Nonlinear predictive reference filtering for constrained tracking. In *Proc. European control conf.* (pp. 1720–1725).
- Bemporad, Alberto, & Mosca, Edoardo (1998). Fulfilling hard constraints in uncertain linear systems by reference managing. *Automatica*, 34(4), 451–461.
- Bemporad, Alberto, Oliveri, Alberto, Poggi, Tomaso, & Storace, Marco (2011). Ultra-fast stabilizing model predictive control via canonical piecewise affine approximations. *IEEE Transactions on Automatic Control*, 56(12), 2883–2897.
- Bemporad, Alberto, Tarn, Tzyh-Jong, & Xi, Ning (1999). Predictive path parameterization for constrained robot control. *IEEE Transactions on Control Systems Technology*, 7(6), 648–656.
- Bencatel, Ricardo, Girard, Anouck, & Kolmanovsky, Ilya (2016). Reference governor strategies for vehicle rollover avoidance. *IEEE Transactions on Control Systems Technology*, submitted for publication. <https://arxiv.org/abs/1608.02266>.
- Bertsekas, Dimitri P., & Rhodes, Ian B. (1971). On the minimax reachability of target sets and target tubes. *Automatica*, 7(2), 233–247.
- Blanchini, F. (1990). Control synthesis for discrete time systems with control and state bounds in the presence of disturbances. *Journal of Optimization Theory and Applications*, 65(1), 29–40.
- Blanchini, Franco (1999). Survey paper: Set invariance in control. *Automatica*, 35(11), 1747–1767.
- Blanchini, Franco, & Miani, Stefano (1997). Constant signal tracking for state constrained dynamic systems. In *Proc. 36th IEEE conf. on decision and control*. Vol. 3 (pp. 2870–2875). IEEE.
- Blanchini, Franco, & Miani, Stefano (2000). Any domain of attraction for a linear constrained system is a tracking domain of attraction. *SIAM Journal on Control and Optimization*, 38(3), 971–994.
- Blanchini, Franco, & Miani, Stefano (2001). Set based constant reference tracking for continuous-time constrained systems. *Nonlinear Dynamics and Systems Theory*, 1(2), 121–131.
- Blanchini, Franco, & Miani, Stefano (2007). *Set-theoretic methods in control*. Springer Science & Business Media.
- Borrelli, Francesco, Falcone, Paolo, Pekar, Jaroslav, & Stewart, Greg (2009). Reference governor for constrained piecewise affine systems. *Journal of Process Control*, 19(8), 1229–1237.
- Boussaid, Boumedyen, Aubrun, Christophe, & Abdelkrim, Naceur (2010). Fault adaptation based on reference governor. In *Proc. conf. control and fault-tolerant systems* (pp. 257–262). IEEE.
- Boussaid, Boumedyen, Aubrun, Christophe, & Abdelkrim, Mohamed Naceur (2011a). Set-point reconfiguration approach for the ftc of wind turbines. In *Proc. IFAC world congress* (pp. 12395–12400).
- Boussaid, Boumedyen, Aubrun, Christophe, & Abdelkrim, Mohamed Naceur (2011b). Two-level active fault tolerant control approach. In *Proc. 8th int. multi-conference on systems, signals and devices* (pp. 1–6). IEEE.
- Boussaid, Boumedyen, Aubrun, Christophe, Jiang, Jin, & Abdelkrim, Mohamed Naceur (2014). Ftc approach with actuator saturation avoidance based on reference management. *International Journal of Robust and Nonlinear Control*, 24(17), 2724–2740.
- Boyd, Stephen, Parikh, Neal, Chu, Eric, Peleato, Borja, & Eckstein, Jonathan (2011). Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3(1), 1–122.
- Boyd, Stephen, & Vandenberghe, Lieven (2004). *Convex optimization*. Cambridge University Press.
- Camacho, Eduardo F., & Bordons, Carlos (2007). *Model predictive control*. In *Advanced textbooks in control and signal processing*.
- Carlsson, Victor, & Sunesson, Oskar (2014). *Reference governor for flight envelope protection in an autonomous helicopter using model predictive control*. Technical report. Linköpings Universitet, LiTH-ISY-EX-14/4780-SE.
- Casavola, Alessandro (2005). *The command governor approach for dummies*. Technical report, DEIS Technical Report 07-2005.
- Casavola, Alessandro, Famularo, Domenico, Franzè, Giuseppe, & Furfaro, Angelo (2010). A fault-tolerant real-time supervisory scheme for an interconnected four-tank system. In *Proc. American control conference* (pp. 6210–6215). IEEE.
- Casavola, Alessandro, Famularo, Domenico, Franzè, Giuseppe, & Garone, Emanuele (2009). Set-points reconfiguration in networked multi-area electrical power systems. *International Journal of Adaptive Control and Signal Processing*, 23(8), 808–832.
- Casavola, Alessandro, Franzè, Giuseppe, Menniti, Daniele, & Sorrentino, Nicola (2011). Voltage regulation in distribution networks in the presence of distributed generation: A voltage set-point reconfiguration approach. *Electric Power Systems Research*, 81(1), 25–34.
- Casavola, Alessandro, Franzè, Giuseppe, & Patton, Ron (2008). Fault tolerance enhancement in distribution power grids: a voltage set-point reconfiguration approach. In *Proc. IFAC world congress* (pp. 13563–13568).
- Casavola, Alessandro, Franzè, Giuseppe, & Sorbara, Michela (2007a). Reference-offset governor approach for the supervision of constrained networked dynamical systems. In *Proc. European control conf.* (pp. 7–14). IEEE.
- Casavola, Alessandro, Franzè, Giuseppe, & Sorbara, Michela (2007b). A supervisory control strategy for fault tolerance enhancement in networked power systems. In *Proc. American control conference* (pp. 4762–4767). IEEE.
- Casavola, Alessandro, Franzè, Giuseppe, Tedesco, Francesco, & Garone, Emanuele (2011). Distributed coordination-by-constraint strategies in networked multi-area power systems. In *IEEE int. symposium on industrial electronics* (pp. 1697–1702). IEEE.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2011a). Distributed coordination-by-constraint strategies for multi-agent networked systems. In *Proc. 50th IEEE conf. on decision and control* (pp. 6888–6893). IEEE.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2011b). Distributed reference management strategies for a networked water distribution system. In *Proc. IFAC world congress* (pp. 8951–8956).
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2011c). A liveness analysis of a distributed constrained coordination strategy for multi-agent linear systems. In *Proc. 50th IEEE conf. on decision and control* (pp. 8133–8138). IEEE.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2014a). The distributed command governor approach in a nutshell. In *Distributed model predictive control made easy* (pp. 259–274). Springer.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2014b). A distributed multi-agent command governor strategy for the coordination of networked interconnected systems. *IEEE Transactions on Automatic Control*, 59(8), 2099–2112.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2014c). Improved feed-forward command governor strategies for constrained discrete-time linear systems. *IEEE Transactions on Automatic Control*, 59(1), 216–223.
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2014d). Scalability and performance improvement of distributed sequential command governor strategies via graph colorability theory. In *Proc. IFAC world congress* (pp. 9400–9405).
- Casavola, Alessandro, Garone, Emanuele, & Tedesco, Francesco (2016). Distributed supervisory strategies for multi-agent networked systems. In *Complex systems* (pp. 411–427). Springer.
- Casavola, Alessandro, & Mosca, Edoardo (1996). Reference governor for constrained uncertain linear systems subject to bounded input disturbances. In *Proc. 35th IEEE conf. on decision and control*. Vol. 3 (pp. 3531–3536). IEEE.
- Casavola, Alessandro, & Mosca, Edoardo (2004). Bank-to-turn missile autopilot design via observer-based command governor approach. *Journal of Guidance, Control, and Dynamics*, 27(4), 705–710.
- Casavola, Alessandro, Mosca, Edoardo, & Angeli, David (2000). Robust command governors for constrained linear systems. *IEEE Transactions on Automatic Control*, 45(11), 2071–2077.
- Casavola, Alessandro, Mosca, Edoardo, & Papini, Maurizio (2004). Control under constraints: an application of the command governor approach to an inverted pendulum. *IEEE Transactions on Control Systems Technology*, 12(1), 193–204.
- Casavola, Alessandro, Mosca, Edoardo, & Papini, Maurizio (2006). Predictive teleoperation of constrained dynamic systems via internet-like channels. *IEEE Transactions on Control Systems Technology*, 14(4), 681–694.

- Casavola, Alessandro, Papini, Maurizio, & Franzè, Giuseppe (2006). Supervision of networked dynamical systems under coordination constraints. *IEEE Transactions on Automatic Control*, 51(3), 421–437.
- Chisci, Luigi, Falugi, Paola, & Zappa, Giovanni (2005). Predictive tracking control of constrained nonlinear systems. *IET Control Theory & Applications*, 152(3), 309–316.
- De Almeida, Fabio A. (2011). Reference management for fault-tolerant model predictive control. *Journal of Guidance, Control, and Dynamics*, 34(1), 44–56.
- DECWG (2016). Distributed control systems. <http://www.decwg.org/pages/DCS.html> (Accessed: 2016-08-06).
- Desoer, Charles, & Lin, Ching-An (1985). Tracking and disturbance rejection of mimo nonlinear systems with pi controller. *IEEE Transactions on Automatic Control*, 30(9), 861–867.
- Di Cairano, Stefano, Bemporad, Alberto, Kolmanovsky, Ilya V., & Hrovat, Davor (2007). Model predictive control of magnetically actuated mass spring dampers for automotive applications. *International Journal of Control*, 80(11), 1701–1716.
- Di Cairano, Stefano, & Borrelli, Francesco (2013). Constrained tracking with guaranteed error bounds. In *Proc. 52nd IEEE conf. on decision and control* (pp. 3800–3805). IEEE.
- Di Cairano, Stefano, & Borrelli, Francesco (2016). Reference tracking with guaranteed error bound for constrained linear systems. *IEEE Transactions on Automatic Control*, 61(8), 2245–2250.
- Di Cairano, Stefano, Brand, Matthew, & Bortoff, Scott A. (2013). Projection-free parallel quadratic programming for linear model predictive control. *International Journal of Control*, 86(8), 1367–1385.
- Di Cairano, Stefano, Doering, Jeff, Kolmanovsky, Ilya V., & Hrovat, Davor (2014). Model predictive control of engine speed during vehicle deceleration. *IEEE Transactions on Control Systems Technology*, 22(6), 2205–2217.
- Di Cairano, Stefano, Goldsmith, Abraham, & Bortoff, Scott (2015). MPC and spatial governor for multistage processing machines in precision manufacturing. In *IFAC nonlinear model predictive control conference*.
- Di Cairano, Stefano, Kalabić, Uroš V., & Kolmanovsky, Ilya V. (2015). Reference governor for network control systems subject to variable time-delay. *Automatica*, 62, 77–86.
- Di Cairano, Stefano, & Kolmanovsky, Ilya V. (2010). Rate limited reference governor for network controlled systems. In *Proc. American control conference* (pp. 3704–3709). IEEE.
- Di Cairano, Stefano, & Kolmanovsky, Ilya V. (2011). Constrained actuator coordination by virtual state governing. In *Proc. 50th IEEE conf. on decision and control* (pp. 5491–5496). IEEE.
- Di Cairano, Stefano, & Kolmanovsky, Ilya V. (2012). Further developments and applications of network reference governor for constrained systems. In *Proc. American control conference* (pp. 3907–3912). IEEE.
- Di Cairano, Stefano, & Kolmanovsky, Ilya V. (2015). Coordinating controllers for constrained linear systems by virtual state governors. *IEEE Transactions on Automatic Control*, 60(8), 2177–2182.
- Di Cairano, Stefano, Liang, Wei, Kolmanovsky, Ilya V., Kuang, Ming L., & Phillips, Anthony M. (2011). Engine power smoothing energy management strategy for a series hybrid electric vehicle. In *Proc. American control conference* (pp. 2101–2106). IEEE.
- Di Cairano, Stefano, Tseng, H. Eric, Bernardini, Daniele, & Bemporad, Alberto (2013). Vehicle yaw stability control by coordinated active front steering and differential braking in the tire sideslip angles domain. *IEEE Transactions on Control Systems Technology*, 21(4), 1236–1248.
- Di Cairano, Stefano, Yanakiev, Diana, Bemporad, Alberto, Kolmanovsky, Ilya V., & Hrovat, Davor (2012). Model predictive idle speed control: Design, analysis, and experimental evaluation. *IEEE Transactions on Control Systems Technology*, 20(1), 84–97.
- Di Cairano, Stefano, & Zafeiropoulos, Spyridon (2015). System and method for semi-autonomous driving of vehicles. US Patent 9,199,667.
- Di Donato, Pedro, Balachandran, Swee, McDonough, Kevin, Atkins, Ella, & Kolmanovsky, Ilya (2016). Envelope-aware flight management for loss of control prevention given rudder jam. *Journal of Guidance, Control, and Dynamics*, <http://dx.doi.org/10.2514/1.G000252>, in press.
- Dillsaver, Matthew J., Kalabić, Uroš V., Kolmanovsky, Ilya V., & Cesnik, Carlos E. S. (2013). Constrained control of very flexible aircraft using reference and extended command governors. In *Proc. American control conference* (pp. 1608–1613). IEEE.
- Do, Hyun Min, & Choi, Jin Young (2005). Design of reference governor for linear systems with input constraints. In *Proc. IFAC world congress* (pp. 554–554).
- Do, Hyun Min, Choi, Jin Young, & Kyung, Jin Ho (2011). Design of reference governor for a class of nonlinear systems with input constraints. In *Proc. 11th int. conference on control, automation and systems* (pp. 912–917). IEEE.
- Dunham, William, Petersen, Christopher, & Kolmanovsky, Ilya (2016). Constrained control for soft landing on an asteroid with gravity model uncertainty. In *Proc. American control conference* (pp. 5842–5847).
- Eeckhout, Sophie, Nicotra, Marco, Naldi, Roberto, & Garone, Emanuele (2014). Nonlinear control of an actuated tethered airfoil. In *Proc. 22nd mediterranean conf. control and automation* (pp. 1412–1417). IEEE.
- Falugi, Paola (2015). Model predictive control for tracking randomly varying references. *International Journal of Control*, 88(4), 745–753.
- Famularo, Domenico, Franzè, Giuseppe, Furfaro, Angelo, & Mattei, Massimiliano (2011). A hybrid real-time supervisory scheme for nonlinear systems. In *Proc. American control conference* (pp. 305–310). IEEE.
- Famularo, Domenico, Franzè, Giuseppe, Furfaro, Angelo, & Mattei, Massimiliano (2015). A hybrid real-time command governor supervisory scheme for constrained control systems. *IEEE Transactions on Control Systems Technology*, 23(3), 924–936.
- Famularo, Domenico, Martino, Davide, & Mattei, Massimiliano (2008). Constrained control strategies to improve safety and comfort on aircraft. *Journal of Guidance, Control, and Dynamics*, 31(6), 1782–1792.
- Ferramosca, Antonio, Limón, Daniel, Alvarado, Ignacio, Alamo, Teodoro, & Camacho, Eduardo F. (2009). MPC for tracking with optimal closed-loop performance. *Automatica*, 45(8), 1975–1978.
- Ferramosca, A., Limón, D., González, A. H., Odloak, D., & Camacho, E. F. (2010). MPC for tracking zone regions. *Journal of Process Control*, 20(4), 506–516.
- Fletcher, Roger (2013). *Practical methods of optimization*. John Wiley & Sons.
- Franzè, Giuseppe, Furfaro, Angelo, Mattei, Massimiliano, & Scordamaglia, Valerio (2013). An hybrid command governor supervisory scheme for flight control systems subject to unpredictable anomalies. In *Conference on control and fault-tolerant systems* (pp. 31–36). IEEE.
- Franzè, Giuseppe, Furfaro, Angelo, Mattei, Massimiliano, & Scordamaglia, Valerio (2015). A safe supervisory flight control scheme in the presence of constraints and anomalies. *International Journal of Applied Mathematics and Computer Science*, 25(1), 39–51.
- Franzè, Giuseppe, & Lucia, Walter (2016). A set-theoretic based control architecture for constrained switching systems. In *Proc. American control conference*. IEEE.
- Franzè, Giuseppe, Lucia, Walter, & Tedesco, Francesco (2015). A dwell-time based command governor approach for constrained switched systems. In *Proc. American control conference* (pp. 1077–1082). IEEE.
- Franzè, Giuseppe, Mattei, Massimiliano, Ollio, Luciano, Scordamaglia, Valerio, & Tedesco, Francesco (2014). A reconfigurable aircraft control scheme based on an hybrid command governor supervisory approach. In *Proc. American control conference* (pp. 1273–1278). IEEE.
- Franzè, Giuseppe, Mattei, Massimiliano, & Scordamaglia, Valerio (2014). Constrained predictive strategized for flight control systems. In *Congress of the int. council of the aeronautical sciences*. ICAS (pp. 1–10).
- Franzè, Giuseppe, & Tedesco, Francesco (2011). Constrained load/frequency control problems in networked multi-area power systems. *Journal of the Franklin Institute*, 348(5), 832–852.
- Frey, Gregory, Petersen, Christopher, Leve, Frederick, Garone, Emanuele, Kolmanovsky, Ilya, & Girard, Anouck (2016). Time shift governor for coordinated control of two spacecraft formation. In *Proc. IFAC symposium nonlinear control system design* (pp. 302–307).
- Garone, Emanuele, & Nicotra, Marco (2016). Explicit reference governor for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 61(5), 1379–1384.
- Garone, Emanuele, Tedesco, Francesco, & Casavola, Alessandro (2009). Distributed coordination-by-constraint strategies for networked control systems. In *Proc. distributed estimation and control in networked systems*.
- Garone, Emanuele, Tedesco, Francesco, & Casavola, Alessandro (2010). Distributed coordination strategies for interconnected multi-agent systems. In *Proc. of 8th IFAC symposium on nonlinear control systems*.
- Garone, Emanuele, Tedesco, Francesco, & Casavola, Alessandro (2011). Sensorless supervision of linear dynamical systems: The feed-forward command governor approach. *Automatica*, 47(7), 1294–1303.
- Gaynor, Robert, Mueller, Fabian, Jabbari, Faryar, & Brouwer, Jacob (2008). On control concepts to prevent fuel starvation in solid oxide fuel cells. *Journal of Power Sources*, 180(1), 330–342.
- Ghadimi, Euhanna, Teixeira, André, Shames, Iman, & Johansson, Mikael (2015). Optimal parameter selection for the alternating direction method of multipliers (ADMM): quadratic problems. *IEEE Transactions on Automatic Control*, 60(3), 644–658.
- Gilbert, Elmer, & Kolmanovsky, Ilya (1995). Discrete-time reference governors for systems with state and control constraints and disturbance inputs. In *Proceedings of the 34th IEEE conference on decision and control* (pp. 1189–1194).
- Gilbert, Elmer G., & Kolmanovsky, Ilya V. (1999a). Fast reference governors for systems with state and control constraints and disturbance inputs. *International Journal of Robust and Nonlinear Control*, 9(15), 1117–1141.
- Gilbert, Elmer G., & Kolmanovsky, Ilya V. (1999b). Set-point control of nonlinear systems with state and control constraints: A lyapunov-function, reference-governor approach. In *Proc. 38th IEEE conf. on decision and control*. Vol. 3 (pp. 2507–2512). IEEE.
- Gilbert, Elmer G., & Kolmanovsky, Ilya V. (2002). Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor. *Automatica*, 38(12), 2063–2073.
- Gilbert, Elmer G., Kolmanovsky, Ilya V., & Tan, Kok Tin (1995). Discrete-time reference governors and the nonlinear control of systems with state and control constraints. *International Journal of Robust and Nonlinear Control*, 5(5), 487–504.
- Gilbert, Elmer G., Kolmanovsky, Ilya V., & Tan, Kok Tin (1994). Nonlinear control of discrete-time linear systems with state and control constraints: A reference governor with global convergence properties. In *Proc. 33rd IEEE conf. on decision and control*. Vol. 1 (pp. 144–149). IEEE.
- Gilbert, Elmer G., & Ong, Chong-Jin (2011). Constrained linear systems with hard constraints and disturbances: An extended command governor with large domain of attraction. *Automatica*, 47(2), 334–340.
- Gilbert, Elmer G., & Tan, Kok Tin (1991). Linear systems with state and control constraints: The theory and application of maximal output admissible sets. *IEEE Transactions on Automatic Control*, 36(9), 1008–1020.
- Giselsson, Pontus, & Boyd, Stephen (2014). Metric selection in Douglas-Rachford splitting and ADMM. arXiv preprint arXiv:1410.8479.
- Glover, J. Duncan, & Schweppe, Fred C. (1971). Control of linear dynamic systems with set constrained disturbances. *IEEE Transactions on Automatic Control*, 16(5), 411–423.

- Goodwin, Graham C., & de Doná, José A. (2005). *Constrained control and estimation: an optimisation approach*. Springer Science & Business Media.
- Guiggiani, Alberto, Kolmanovsky, Ilya V., Patrinos, Panagiotis, & Bemporad, Alberto (2015). Constrained model predictive control of spacecraft attitude with reaction wheels desaturation. In *Proc. European control conf.* (pp. 1382–1387). IEEE.
- Guo, Hao, Ohta, Yuzo, Taguchi, Takaaki, & Masubuchi, Izumi (2012). Long seek control of hard disk drives using reference governor: An improved result. In *Proc. conf. industrial electronics and applications* (pp. 1951–1956). IEEE.
- Haghighat, Sohrab, Di Cairano, Stefano, Konobrytskyi, Dmytro, & Bortoff, Scott (2014). Coordinated control of a dual-stage positioning system using constrained model predictive control. In *ASME dynamic systems and control conf. American Society of Mechanical Engineers*.
- Hatanaka, Takeshi, & Takaba, Kiyotsugu (2005a). Output feedback reference governor for nonlinear systems. In *Proc. 44th IEEE conf. on decision and control* (pp. 7558–7563). IEEE.
- Hatanaka, Takeshi, & Takaba, Kiyotsugu (2005b). Reference management using state observers. *International Journal of Control*, 78(8), 566–576.
- Hirata, Kenji, & Minemura, Hiroshi (2004). Experimental evaluations of reference governor control schemes with applications to the constrained control of rc helicopters. In *Proc. IEEE conf. control applications* (pp. 855–859).
- Iwasa, Takahiro, Terashima, Kazuhiko, Jian, Nyioh Yong, & Noda, Yoshiyuki (2010). Operator assistance system of rotary crane by gain-scheduled h controller with reference governor. In *Proc. IEEE conf. control applications* (pp. 1325–1330). IEEE.
- Jade, Shyam, Hellström, Erik, Larimore, Jacob, Stefanopoulou, Anna G., & Jiang, Li (2014). Reference governor for load control in a multicylinder recompression hcci engine. *IEEE Transactions on Control Systems Technology*, 22(4), 1408–1421.
- Jiang, Zhong-Ping, & Wang, Yuan (2001). Input-to-state stability for discrete-time nonlinear systems. *Automatica*, 37(6), 857–869.
- Kahveci, Nazli E., & Kolmanovsky, Ilya V. (2009). Constrained control of UAVs using adaptive anti-windup compensation and reference governors. SAE Technical Paper.
- Kahveci, Nazli E., & Kolmanovsky, Ilya V. (2010a). Constrained control using error governors with online parameter estimation. In *Proc. 49th IEEE conf. on decision and control* (pp. 5186–5191). IEEE.
- Kahveci, Nazli E., & Kolmanovsky, Ilya V. (2010b). Control design for electromagnetic actuators based on backstepping and landing reference governor. In *5th IFAC symposium on mechatronic systems* (pp. 393–398).
- Kalabić, Uroš (2015). *Reference governors: Theoretical extensions and practical applications*. (Ph.D. thesis), University of Michigan.
- Kalabić, Uroš, Buckland, J., Cooper, S., Wait, S., & Kolmanovsky, Ilya V. (2016). Reference governors for enforcing compressor surge constraint. *IEEE Transactions on Control Systems Technology*, 24(5), 1729–1739.
- Kalabić, Uroš, Chitalia, Yash, Buckland, Julia, & Kolmanovsky, Ilya V. (2013). Prioritization schemes for reference and command governors. In *Proc. European control conf.* (pp. 2734–2739). IEEE.
- Kalabić, Uroš, Gupta, Rajesh, Di Cairano, Stefano, Bloch, Anthony, & Kolmanovsky, Ilya V. (2014). Constrained spacecraft attitude control on so (3) using reference governors and nonlinear model predictive control. In *Proc. American control conference* (pp. 5586–5593). IEEE.
- Kalabić, Uroš, & Kolmanovsky, Ilya V. (2013). Decentralized constraint enforcement using reference governors. In *Proc. 52nd IEEE conf. on decision and control* (pp. 6415–6421). IEEE.
- Kalabić, Uroš, & Kolmanovsky, Ilya V. (2014). Reference and command governors for systems with slowly time-varying references and time-dependent constraints. In *Proc. 53rd IEEE conf. on decision and control* (pp. 6701–6706). IEEE.
- Kalabić, Uroš, & Kolmanovsky, Ilya V. (2015). Inner loop reference governors: Placing reference governors inside the control loop by using passivity. In *Proc. 54th IEEE conf. on decision and control*. IEEE.
- Kalabić, Uroš, Kolmanovsky, Ilya V., Buckland, Julia, & Gilbert, Elmer G. (2011). Reference and extended command governors for control of turbocharged gasoline engines based on linear models. In *Proc. IEEE conf. control applications* (pp. 319–325). IEEE.
- Kalabić, Uroš, Kolmanovsky, Ilya V., Buckland, Julia, & Gilbert, Elmer G. (2012). Reduced order reference governor. In *Proc. 51st IEEE conf. on decision and control* (pp. 3245–3251). IEEE.
- Kalabić, Uroš, Kolmanovsky, Ilya V., & Gilbert, Elmer G. (2011). Reference governors for linear systems with nonlinear constraints. In *Proc. 50th IEEE conf. on decision and control* (pp. 2680–2686). IEEE.
- Kalabić, Uroš, Vermillion, Chris, & Kolmanovsky, Ilya V. (2013). Reference governor design for computationally efficient attitude and tether tension constraint enforcement on a lighter-than-air wind energy system. In *Proc. European control conf.* (pp. 1004–1010). IEEE.
- Kalabić, Uroš V., Kolmanovsky, Ilya V., & Gilbert, Elmer G. (2014). Reduced order extended command governor. *Automatica*, 50(5), 1466–1472.
- Kang, Jun-Mo, & Grizzle, Jessy (1999). Nonlinear control for joint air and fuel management in a si engine. In *Proceedings of American control conference*. Vol. 1 (pp. 17–23).
- Kapasouris, Petros, Athans, Michael, & Stein, Gunter (1988a). Design of feedback control systems for stable plants with saturating actuators. In *Proc. 27th IEEE conf. on decision and control* (pp. 469–479). IEEE.
- Kapasouris, Petros, Athans, Michael, & Stein, Gunter (1988b). *Design of feedback control systems for unstable plants with saturating actuators*. Technical report. Massachusetts Institute of Technology, IIDS-P-1838.
- Kapasouris, Petros, Athans, Michael, & Stein, Gunter (1990). Design of feedback control systems for unstable plants with saturating actuators. In *Proc. IFAC symposium on nonlinear control system design* (pp. 302–307).
- Kerrigan, Eric Colin (2001). *Robust constraint satisfaction: Invariant sets and predictive control*. (Ph.D. thesis), University of Cambridge.
- Khalil, Hassan (2001). *Nonlinear systems* (3rd ed.). Prentice Hall.
- Kögel, Markus, & Findeisen, Rolf (2011). Fast predictive control of linear systems combining nesterov's gradient method and the method of multipliers. In *Proc. 50th IEEE conf. on decision and control* (pp. 501–506). IEEE.
- Kogiso, Kiminao, & Hirata, Kenji (2003). A reference governor in a piecewise state affine function. In *Proc. 42th IEEE conf. on decision and control*. Vol. 2 (pp. 1747–1752). IEEE.
- Kogiso, Kiminao, & Hirata, Kenji (2006). Experimental validations of a remote control technique for constrained linear systems using reference governor plus switching control strategy. In *Int. joint conference SICE-ICASE* (pp. 5668–5672). IEEE.
- Kogiso, Kiminao, & Hirata, Kenji (2009). Reference governor for constrained systems with time-varying references. *Robotics and Autonomous Systems*, 57(3), 289–295.
- Kogiso, Kiminao, Noda, Masaru, & Nishitani, Hirokazu (2006). Constrained control for chemical processes using reference governor. *Computer Aided Chemical Engineering*, 21, 1437–1442.
- Kolmanovsky, Ilya V., Garone, Emanuele, & Di Cairano, Stefano (2014). Reference and command governors: A tutorial on their theory and automotive applications. In *American Control Conference, June* (pp. 226–241).
- Kolmanovsky, Ilya V., & Gilbert, Elmer G. (1995). Maximal output admissible sets for discrete-time systems with disturbance inputs. In *Proc. American control conference*. IEEE.
- Kolmanovsky, Ilya V., & Gilbert, Elmer G. (1998). Theory and computation of disturbance invariant sets for discrete-time linear systems. *Mathematical Problems in Engineering*, 4(4), 317–367.
- Kolmanovsky, Ilya V., & Gilbert, Elmer G. (2001). Landing reference governor. In *Proc. American control conference* (pp. 374–375). IEEE.
- Kolmanovsky, Ilya V., Gilbert, Elmer G., & Cook, J. A. (1997). Reference governors for supplemental torque source control in turbocharged diesel engines. In *Proc. American control conference*. Vol. 1 (pp. 652–656). IEEE.
- Kolmanovsky, Ilya V., Gilbert, Elmer G., & Tseng, H. E. (2009). Constrained control of vehicle steering. In *Proc. IEEE conf. control applications* (pp. 576–581). IEEE.
- Kolmanovsky, Ilya V., & Haghighi, Mohammad (2004). Control algorithm for soft-landing in electromechanical actuators, US Patent 6,693,787.
- Kolmanovsky, Ilya V., Jaw, Link C., Merrill, Walt, & TranVan, H. (2012). Robust control and limit protection in aircraft gas turbine engines. In *Proc. IEEE conf. control applications* (pp. 812–819). IEEE.
- Kolmanovsky, Ilya V., Kalabić, Uroš V., & Gilbert, Elmer G. (2012). Developments in constrained control using reference governors. In *IFAC nonlinear model predictive control conference* (pp. 282–290).
- Kolmanovsky, Ilya V., & Merrill, Walt (2014). Limit protection in gas turbine engines based on reference and extended command governors. In *Proc. of 50th AIAA/ASME/SAE/ASEE joint propulsion and energy forum and exposition*. AIAA/ASME/SAE/ASEE.
- Kolmanovsky, Ilya V., Miller, Robert H., Washabaugh, Peter D., & Gilbert, Elmer G. (1998). Nonlinear control of electrostatically shaped membrane with state and control constraints. In *Proc. American control conference* (pp. 2927–2931). IEEE.
- Kolmanovsky, Ilya V., & Sun, Jing (2004). Parameter governors for discrete-time nonlinear systems with pointwise-in-time state and control constraints. In *Proc. American control conference*. Vol. 4 (pp. 3075–3081). IEEE.
- Kolmanovsky, Ilya V., & Sun, Jing (2005). Approaches to computationally efficient implementation of gain governors for nonlinear systems with pointwise-in-time constraints. In *Proc. 44th IEEE conf. on decision and control* (pp. 7564–7569). IEEE.
- Kolmanovsky, Ilya V., & Sun, Jing (2006). A multi-mode switching-based command tracking in network controlled systems with pointwise-in-time constraints and disturbance inputs. In *World cong. intelligent control and automation* (pp. 199–204). IEEE.
- Kolmanovsky, Ilya V., & Sun, Jing (2006). Parameter governors for discrete-time nonlinear systems with pointwise-in-time state and control constraints. *Automatica*, 42(5), 841–848.
- Kouvaritakis, Basil, Cannon, Mark, & Rossiter, J. A. (2002). Who needs QP for linear MPC anyway? *Automatica*, 38(5), 879–884.
- Kouvaritakis, Basil, Rossiter, J. A., & Schuurmans, Jan (2000). Efficient robust predictive control. *IEEE Transactions on Automatic Control*, 45(8), 1545–1549.
- Kwon, Wook Hyun, & Han, Soo Hee (2006). *Receding horizon control: model predictive control for state models*. Springer Science & Business Media.
- Laghrouche, Salah, Matraji, Imad, Ahmed, Fayed, Shakil, Jemei, Samir, & Wack, Maxime (2013). Load governor based on constrained extremum seeking for pem fuel cell oxygen starvation and compressor surge protection. *International Journal of Hydrogen Energy*, 38(33), 14314–14322.
- Langson, W., Chrysoschoos, I., Rakovic, S. V., & Mayne, D. Q. (2004). Robust model predictive control using tubes. *Automatica*, 40, 125–133.
- Li, Yuhua, & De Schutter, Bart (2012). A robust feasibility problem for the design of a reference governor. In *Proc. American control conference* (pp. 6745–6750). IEEE.
- Li, Qinghua, Kalabić, Uroš V., & Kolmanovsky, Ilya V. (2014). Fast reference governors for second-order linear systems with constraints and an input time-delay. *Automatica*, 50(2), 641–645.
- Limon, D., Alvarado, I., Alamo, T., & Camacho, E. F. (2010). Robust tube-based mpc for tracking of constrained linear systems with additive disturbances. *Journal of Process Control*, 20(3), 248–260.
- Limon, D., Ferramosca, A., Alvarado, I., Alamo, T., & Camacho, E. F. (2009). MPC for tracking of constrained nonlinear systems. In *Nonlinear model predictive control* (pp. 315–323). Berlin, Heidelberg: Springer.

- Limon, Daniel, & Alamo, Teodoro (2015). Tracking model predictive control. *Encyclopedia of Systems and Control*, 1475–1484.
- Limón, Daniel, Alvarado, Ignacio, Alamo, Teodoro, & Camacho, Eduardo F. (2008). MPC for tracking piecewise constant references for constrained linear systems. *Automatica*, 44(9), 2382–2387.
- Limon, Daniel, Pereira, Mario, Muoz de la Pea, David, Alamo, Teodoro, Jones, Colin N., & Zeilinger, Melanie N. (2016). MPC for tracking periodic references. *IEEE Transactions on Automatic Control*, 61(4), 1123–1128.
- Lucia, Walter, Szaier, Mario, & Franzè, Giuseppe (2014). An obstacle avoidance and motion planning command governor based scheme: the qball-x4 quadrotor case of study. In *Proc. 54th IEEE conf. on decision and control* (pp. 6135–6140). IEEE.
- Maciejowski, Jan Marian (2002). *Predictive control with constraints*. Pearson education.
- Martino, Davide (2008). Flight control with amplitude and rate constraints: A command governor approach. In *Proc. American control conference* (pp. 1788–1793). IEEE.
- Mattei, Massimiliano, Labate, Carmelo Vincenzo, & Famularo, Domenico (2013). A constrained control strategy for the shape control in thermonuclear fusion tokamaks. *Automatica*, 49(1), 169–177.
- Mayne, David, & Falugi, Paola (2015). Generalized stabilizing conditions for model predictive control. *Journal of Optimization Theory and Applications*, 1–16.
- McDonough, Kevin, & Kolmanovsky, Ilya V. (2015). Controller state and reference governors for discrete-time linear systems with pointwise-in-time state and control constraints. In *Proc. American control conference* (pp. 3607–3612). IEEE.
- Mesbah, Ali (2016). Stochastic model predictive control: An overview and perspectives for future research. *IEEE Control Systems Magazine*, in press.
- Miller, Russel B. (1997). *Manual tracking flight control with amplitude and rate constrained dynamic actuators..* (Ph.D. thesis), Air Force Institute of Technology.
- Miller, Robert H., Kolmanovsky, Ilya V., Gilbert, Elmer G., & Washabaugh, Peter D. (2000). Control of constrained nonlinear systems: A case study. *IEEE Control Systems Magazine*, 20(1), 23–32.
- Mitchell, Tom M. (1997). *Machine learning*. McGraw Hill.
- Morari, Manfred, & Lee, Jay H. (1999). Model predictive control: past, present and future. *Computers & Chemical Engineering*, 23(4), 667–682.
- Moura, Scott Jason, Chaturvedi, N. A., & Krstic, Miroslav (2013). Constraint management in Li-ion batteries: A modified reference governor approach. In *Proc. American control conference* (pp. 5332–5337). IEEE.
- Nakada, Hayato, Martin, Peter, Milton, Gareth, Iemura, Akiyuki, & Ohata, Akira (2014a). An application study of online reference governor to boost pressure control for automotive diesel engines. *Transactions of the Society of Instrument and Control Engineers*, 50, 197–202.
- Nakada, Hayato, Milton, Gareth, Martin, Peter, Iemura, Akiyuki, & Ohata, Akira (2013). Application of reference governor using soft constraints and steepest descent method to diesel engine aftertreatment temperature control. *SAE International Journal of Engines*, 6(2013-01-0350), 257–266.
- Nakada, Hidemoto, Martin, Patrick, Milton, Gareth, Iemura, Akiyuki, & Ohata, Akira (2014b). An application study of online reference governor to boost pressure control for automotive diesel engines. In *Proc. American control conference* (pp. 3135–3140). IEEE.
- Nesterov, Yurii (1983). A method of solving a convex programming problem with convergence rate $O(1/k^2)$. *Soviet Mathematics Doklady*, 27(2), 372–376.
- Nesterov, Yurii (2004). *Introductory lectures on convex optimization*, Vol. 87. Springer Science & Business Media.
- Nguyen, Tam, & Garone, Emanuele (2016). Control of a uav and ugv cooperating to manipulate an object. In *Proc. American control conference* (pp. 1347–1352). IEEE.
- Nguyen, Tam, Hatanaka, Takeshi, Garone, Emanuele, & Fujita, Masayuki (2016). A passivity-based approach for constrained mobile robotic networks. In *Proc. 55th IEEE conf. on decision and control*. IEEE.
- Nicotra, Marco (2016). *Constrained control of nonlinear systems: the explicit reference governor and its application to unmanned aerial vehicles*. (Ph.D. thesis), Université Libre de Bruxelles.
- Nicotra, Marco, Garone, Emanuele, & Kolmanovsky, Ilya (2016). A fast reference governor for linear systems subject to convex constraints. *Journal of Guidance, Control, and Dynamics* (in press).
- Nicotra, Marco, Naldi, Roberto, & Garone, Emanuele (2014). Taut cable control of a tethered uav. In *Proc. IFAC world congress*.
- Nicotra, Marco, Naldi, Roberto, & Garone, Emanuele (2016). Nonlinear control of a tethered uav: the taut cable case. *Automatica*, (submitted for publication).
- Nicotra, Marco M., Bartulovic, Mihovil, Garone, Emanuele, & Sinopoli, Bruno (2015). A distributed explicit reference governor for constrained control of multiple uavs. In *5th IFAC workshop on distributed estimation and control in networked systems*.
- Nicotra, Marco M., & Garone, Emanuele (2015a). Control of Euler–Lagrange systems subject to constraints: An explicit reference governor approach. In *Proc. 54th IEEE conf. on decision and control*. IEEE.
- Nicotra, Marco M., & Garone, Emanuele (2015b). Explicit reference governor for continuous time nonlinear systems subject to convex constraints. In *Proc. American control conference* (pp. 4561–4566). IEEE.
- Nicotra, Marco M., & Garone, Emanuele (2016). An explicit reference governor for the robust constrained control of nonlinear systems. In *Proc. 55th IEEE conf. on decision and control*. IEEE.
- Nicotra, Marco M., Naldi, Roberto, & Garone, Emanuele (2016). A robust explicit reference governor for constrained control of unmanned aerial vehicles. In *Proc. American control conference* (pp. 6284–6289).
- Oh, Jae-Hyuk, Kim, Hwa Soo, & Cho, Young Man (2008). Adaptive reference governor for constrained linear systems. *Journal of Mechanical Science and Technology*, 22(1), 61–69.
- Oh, S., & Agrawal, Sunil Kumar (2005). A reference governor-based controller for a cable robot under input constraints. *IEEE Transactions on Control Systems Technology*, 13(4), 639–645.
- Oh, Jae-Hyuk, & Peng, Pel-Yuan (2001). Sdp relaxation based adaptive reference governor for constrained linear systems. In *Proc. AIAA guidance, navigation, and control conference*. IEEE.
- Ong, Chong Jin, Sathya Keerthi, S., Gilbert, Elmer G., & Zhang, Z. H. (2004). Stability regions for constrained nonlinear systems and their functional characterization via support-vector-machine learning. *Automatica*, 40(11), 1955–1964.
- Pachter, Meir, & Miller, Russel B. (1998). Manual flight control with saturating actuators. *IEEE Control Systems Magazine*, 18(1), 10–20.
- Pao, Lucy Y. (1999). Multi-input shaping design for vibration reduction. *Automatica*, 35(1), 81–89.
- Patrinos, Panagiotis, & Bemporad, Alberto (2014). An accelerated dual gradient-projection algorithm for embedded linear model predictive control. *IEEE Transactions on Automatic Control*, 59(1), 18–33.
- Patrinos, Panagiotis, Guiggiani, Alberto, & Bemporad, Alberto (2015). A dual gradient-projection algorithm for model predictive control in fixed-point arithmetic. *Automatica*, 55, 226–235.
- Petersen, Christopher, Baldwin, Morgan, & Kolmanovsky, Ilya V. (2013). Model predictive control guidance with extended command governor inner-loop flight control for hypersonic vehicles. In *Proc. AIAA guidance, navigation, and control conference*. AIAA.
- Petersen, Christopher, Jaunzemis, Andris, Baldwin, Morgan, Holzinger, M.J., & Kolmanovsky, Ilya V. (2014). Model predictive control and extended command governor for improving robustness of relative motion guidance and control. In *Proc. AAS/AIAA space flight mechanics meeting*.
- Raghunathan, Arvind U., & Di Cairano, Stefano (2014a). ADMM for convex quadratic programs: Local convergence and infeasibility detection. *ArXiv Preprint arXiv:1411.7288*.
- Raghunathan, Arvind U., & Di Cairano, Stefano (2014b). Infeasibility detection in alternating direction method of multipliers for convex quadratic programs. In *Proc. 53rd IEEE conf. on decision and control* (pp. 5819–5824). IEEE.
- Rakovic, Sasa (2005). *Robust control of constrained discrete time systems: Characterization and implementation*. (Ph.D. thesis), Imperial College.
- Richter, Stefan, Jones, Colin Neil, & Morari, Manfred (2012). Computational complexity certification for real-time mpc with input constraints based on the fast gradient method. *IEEE Transactions on Automatic Control*, 57(6), 1391–1403.
- Rodriguez, Armando A., & Wang, Yanzhuo (1996). Performance enhancement methods for unstable bank-to-turn (btt) missiles with saturating actuators. *International Journal of Control*, 63(4), 641–678.
- Rossiter, J. Anthony, & Kouvaritakis, Basil (1998). Reference governors and predictive control. In *Proc. American control conference*, Vol. 6 (pp. 3692–3693). IEEE.
- Rossiter, J. Anthony, Wang, L., & Valencia-Palomo, G. (2010). Efficient algorithms for trading off feasibility and performance in predictive control. *International Journal of Control*, 83(4), 789–797.
- Schatz, Simon P., Yucelen, Tansel, Johnson, Eric N., & Holzapfel, Florian (2013). Constraint enforcement methods for command governor based adaptive control of uncertain dynamical systems. In *Proc. AIAA guidance, navigation, and control conference*.
- Seenumani, Gayathri, Peng, Huei, & Sun, Jing (2011). A reference governor-based hierarchical control for failure mode power management of hybrid power systems for all-electric ships. *Journal of Power Sources*, 196(3), 1599–1607.
- Singer, Neil C., & Seering, Warren P. (1990). Preshaping command inputs to reduce system vibration. *Journal of Dynamic Systems, Measurement, and Control*, 112(1), 76–82.
- Singhose, William (2009). Command shaping for flexible systems: A review of the first 50 years. *International Journal of Precision Engineering and Manufacturing*, 10(4), 153–168.
- Singhose, William, & Pao, L. (1997). A comparison of input shaping and time-optimal flexible-body control. *Control Engineering Practice*, 5(4), 459–467.
- Smith, Kandler (2010). Electrochemical control of lithium-ion batteries. *IEEE Control Systems Magazine*, 30(2), 18–25.
- Soleymani, Touraj, Garone, Emanuele, & Dorigo, Marco (2015). Distributed constrained connectivity control for proximity networks based on a receding horizon scheme. In *Proc. American control conference* (pp. 1369–1374). IEEE.
- Sugie, Toshiharu, & Yamamoto, Hiroyuki (2001). Reference management for closed loop systems with state and control constraints. In *Proc. American control conference vol. 2* (pp. 1426–1431). IEEE.
- Sun, Jing, & Kolmanovsky, Ilya V. (2005). Load governor for fuel cell oxygen starvation protection: A robust nonlinear reference governor approach. *IEEE Transactions on Control Systems Technology*, 13(6), 911–920.
- Taguchi, Takaaki, & Ohta, Yuzo (2007). A dual mode reference governor for discrete time systems with state and control constraints. In *22nd IEEE int. symposium on intelligent control*.
- Tan, Kok Tin (1991). *Maximal output admissible sets and the nonlinear control of linear discrete-time systems with state and control constraints*. (Ph.D. thesis), University of Michigan.
- Tedesco, Francesco, & Casavola, Alessandro (2013). A distributed command governor approach for voltage regulation in medium voltage power grids with distributed generation. In *Proc. American control conference* (pp. 3492–3497). IEEE.

- Tedesco, Francesco, & Casavola, Alessandro (2014). Fault-tolerant distributed load/frequency supervisory strategies for networked multi-area microgrids. *International Journal of Robust and Nonlinear Control*, 24(8–9), 1380–1402.
- Tedesco, Francesco, & Casavola, Alessandro (2015). Fault-tolerant distributed load/frequency coordination strategies for multi-area power microgrids. In *9th IFAC symposium on fault detection, supervision and safety for technical processes*.
- Tedesco, Francesco, Casavola, Alessandro, & Garone, Emanuele (2012a). A distributed parallel command governor strategy for the coordination of multi-agent networked systems. In *IFAC nonlinear model predictive control conference, Vol. 4* (pp. 478–483).
- Tedesco, Francesco, Casavola, Alessandro, & Garone, Emanuele (2012b). A parallel distributed coordination-by-constraint strategy for multi-agent networked systems. In *Proc. 51st IEEE conf. on decision and control* (pp. 284–289). IEEE.
- Tedesco, Francesco, Casavola, Alessandro, & Garone, Emanuele (2012c). Distributed command governor strategies for constrained coordination of multi-agent networked systems. In *Proc. American control conference* (pp. 6005–6010). IEEE.
- Tedesco, Francesco, Raimondo, Davide M., & Casavola, Alessandro (2014). Collision avoidance command governor for multi-vehicle unmanned systems. *International Journal of Robust and Nonlinear Control*, 24(16), 2309–2330.
- Tee, Keng Peng, Ge, Shuzhi Sam, & Tay, Eng Hock (2009). Barrier Lyapunov functions for the control of output-constrained nonlinear systems. *Automatica*, 45(4), 918–927.
- Tharayil, Marina, & Alleyne, Andrew (2002). Active supersonic flow control using hysteresis compensation and error governor. In *Proc. IFAC world congress*.
- Tian, Ye, & Kolmanovsky, Ilya V. (2014). Reduced order and prioritized reference governors for limit protection in aircraft gas turbine engines. In *Proc. AIAA guidance, navigation, and control conference*.
- Tsourapas, Vasilis, Sun, Jing, & Stefanopoulou, Anna (2009). Incremental step reference governor for load conditioning of hybrid fuel cell and gas turbine power plants. *IEEE Transactions on Control Systems Technology*, 17(4), 756–767.
- Vahidi, Ardan, Kolmanovsky, Ilya V., & Stefanopoulou, Anna (2007). Constraint handling in a fuel cell system: A fast reference governor approach. *IEEE Transactions on Control Systems Technology*, 15(1), 86–98.
- Van Der Laan, E. P., Heemels, W. P. M. H., Luitjen, H., Veldpaus, F. E., & Steinbuch, Maarten (2010). Reference governors for controlled belt restraint systems. *Vehicle System Dynamics*, 48(7), 831–850.
- Weber, Philippe, Boussaid, Boumedyen, Khelassi, Ahmed, Theilliol, Didier, & Aubrun, Christophe (2012). Reconfigurable control design with integration of a reference governor and reliability indicators. *International Journal of Applied Mathematics and Computer Science*, 22(1), 139–148.
- Weiss, Avishai, Baldwin, Morgan, Erwin, R. S., & Kolmanovsky, Ilya V. (2015). Model predictive control for spacecraft rendezvous and docking: Strategies for handling constraints and case studies. *IEEE Transactions on Control Systems Technology*, 23(4), 1638–1647.
- Weiss, Avishai, Leve, Frederick, Baldwin, Morgan, Forbes, James Richard, & Kolmanovsky, Ilya V. (2014). Spacecraft constrained attitude control using positively invariant constraint admissible sets on $SO(3) \times \mathbb{R}^3$. In *Proc. American control conference* (pp. 4955–4960). IEEE.
- Wilborn, James E., & Foster, John V. (2004). Defining commercial transport loss-of-control: A quantitative approach. In *AIAA atmospheric flight mechanics conference and exhibit*.
- Wright, Stephen J., & Nocedal, Jorge (1999). *Numerical optimization, Vol. 2*. New York: Springer.
- Ye, Hui, Chen, Mou, & Wu, Qingxian (2015). Flight envelope protection control based on reference governor method in high angle of attack maneuver. *Mathematical Problems in Engineering*.
- Yucelen, Tansel, & Johnson, E.N. (2012). Command governor-based adaptive control. In *Proc. AIAA guidance, navigation, and control conference* (p. 4618).
- Zafeiropoulos, Spyridon, & Di Cairano, Stefano (2014). Governor-based control for rack-wheel coordination in mechanically decoupled steering systems. In *Proc. 53rd IEEE conf. on decision and control* (pp. 4089–4094). IEEE.
- Zaseck, Kevin, Brusstar, Matthew, & Kolmanovsky, Ilya (2016). Stability, control, and constraint enforcement of piston motion in a hydraulic free-piston engine. *IEEE Transactions on Control Systems Technology*, (in press).
- Zaseck, Kevin, Brusstar, Matthew, & Kolmanovsky, Ilya V. (2014). Constraint enforcement of piston motion in a free-piston engine. In *Proc. American control conference* (pp. 1487–1492). IEEE.
- Zinnecker, Alicia, Serrani, Andrea, Bolender, Michael A., & Doman, David B. (2009). Combined reference governor and anti-windup design for constrained hypersonic vehicles models. In *Proc. AIAA guidance, navigation, and control conference*.



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