Fault-tolerant control for constrained unmanned marine vehicles based on model predictive control with integral sliding mode control

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***Abstract*—In this paper, a fault-tolerant control method for dynamic positioning of constrained unmanned marine vehicles is investigated. Due to the optimization essence, model predictive control can explicitly solve the practical constraints of the system, meanwhile, considering the robustness of integral sliding mode control, the effect of system uncertainties can be compensated. Hence, the proposed control scheme is based on the combination of model predictive control and integral sliding mode control. The inner integral sliding mode control loop is designed to compensate the thruster faults and reject the ocean disturbances. While the external model predictive control loop can ensure an optimal solution of the dynamic system with the constraints on state and input variables. Finally, the effectiveness of our proposed fault tolerant control scheme is demonstrated in the simulation.**

***Index Terms*—integral sliding mode, model predictive control, fault-tolerant control, constrained unmanned marine vehicles**

I. Introduction

Dynamic positioning of unmanned marine vehicles (UMVs) has been a long standing control problem that have attracted much attention in ocean engineering and control communities over recent years [1]. In the actual dynamic positioning applications, the UMVs system constraints on state variables [2] and control input [3] are inevitable for the sake of security, performance and implementation. Hence, it is of practical significance to consider these constraints in the UMVs controller design.

Model predictive control (MPC) represents an effective control method, and requires a model that describes the dynamic behavior of the objects to predict the future dynamics of the system and get the optimal solution online. Nowadays, it is widely used in complex dynamical systems, providing an optimal control strategy to solve these problems with constraints [4]. Moreover, as the increasing requirements for the safety of UMVs, the fault-tolerance capacity and robustness needs to be taken into account. Sliding mode control (SMC) is considered as an effective control method since it can overcome the uncertainties of the system and its robustness to disturbances. Integral sliding mode control (ISMC) improves the robustness compared with SMC by designing the initial state of the integrator, whose goal is to take the state on the sliding surface from the initial time. For this reason, ISMC method has been well developed in both theoretical research and practical application and it has been a relatively mature technology, and fortunately, we have obtained some results: A fault-tolerant control scheme based on ISM techniques for uncertain systems with signals quantization [5] is addressed. [6] and [7] proposed an fault-tolerant ISMC strategy and an adaptive sliding mode fault-tolerant control scheme for autonomous underwater vehicle (AUV), respectively.

While, in this paper, if the constraints are also considered, inspired by [8], it is an effective scheme to combine MPC with ISMC methods. In this way, not only the robustness of the system can be satisfied, but solve the problem of constraints on state and input variables. For all the above considerations, we proposed a fault-tolerant control strategy consists of two loops of the constrained UMVs system: an inner loop contains an ISM controller with the role of rejecting the external ocean disturbances and compensating for the thruster faults at a high rate; and an external loop includes a nominal MPC controller, which is designed to ensure the optimality at a lower rate subjected to the state and input constraints.

The main research contents and results are as follows:
(1) Different from [5], [9], [10], this paper considers the constraints of state and input variables of the UMVs system, then proposed a fault-tolerant control scheme combined MPC with ISMC.
(2) In this paper, the integral sliding mode control method is designed to compensate thruster faults and reject the ocean disturbances.

This paper is structured as following: Section II introduces the system description and rises the problems to be solved in this paper. Section III contains the design of ISM and MPC controllers, the proof of stability and feasibility. Section IV and Section V give the comparison of simulation results and conclusions of this paper, respectively.

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II. System Description and Problem Statement

A. UMVs system model

Six degrees of freedom for dynamic positioning of unmanned marine vehicles are included surge, sway, heave, roll, pitch and yaw, which uniquely determine the motion of the UMVs. For a normalized model of horizontal motion, the components of motion that have been studied includes surge, sway, and yaw motions. The position variables of the UMVs, $x_p(t)$, $y_p(t)$ and the yaw angle $\psi(t)$, are expressed in the body-fixed reference frame. Considering that the state constraints are unavoidable in a real application system, referring to [2], we suppose that the yaw angle $\psi(t)$ satisfies $\psi(t) \in (-\frac{\pi}{6}, \frac{\pi}{6})$ in this paper. And the attitude variables, surge velocity $\zeta(t)$, sway velocity $v(t)$ and yaw velocity $r(t)$ are represented in the earth-fixed reference frame. And the dynamical equation is described as

$$\dot{\eta}(t) = J(\psi(t))\, v(t), \tag{1}$$

where

$$J = \begin{bmatrix} cos(\psi(t)) & -sin(\psi(t)) & 0 \\ sin(\psi(t)) & cos(\psi(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$\eta(t) = [x_p(t), y_p(t), \psi(t)]^T$ and $v(t) = [\zeta(t), v(t), r(t)]^T$.

Since the desired yaw angle is as small as possible for dynamic positioning system, obviously, we have $cos(\psi(t)) \approx 1, sin(\psi(t)) \approx 0, J(\psi(t)) \approx I$ [14]. So (1) becomes

$$\dot{\eta}(t) = v(t).$$

The body-fixed equation of motion for surge, sway and yaw is described as

$$M\dot{v} + N_0 v + G_0 \eta = S(t), \tag{2}$$

where M is the inertial matrix, satisfying $M = M^T > 0$, N_0 expresses damping, and G_0 denotes mooring forces. The control input vector of forces and moment $S(t) = [S_1(t), S_2(t), S_3(t)]^T$ is provided by the thrusters, which denotes the force in surge, sway and the moment in yaw [11], respectively. And in the UMVs system, it can be written as

$$S(t) = \Upsilon u(t). \tag{3}$$

For the sake of security, performance and implementation, the input variable $u(t)$ of a dynamic system are always constrained [3]. In the unmanned vessels control system, the thrust of the propeller is always limited. For example, according to [13] , the podded propulsion units are available in power ranges up to at least $25MW$. In this paper, we define the input variables satisfy $u \in U$, where U is a compact set with the origin as an interior point. And it is such that $|u| \leqslant u_{max}$, where u_{max} represents the range of the resultant thrust.

The UMVs considered in this article are equipped with six thrusters, including one azimuth thruster, two main thrusters and three tunnel thrusters [15]. The thruster configuration matrix Υ which can distributes the thrust force is defined as

$$\Upsilon = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & cos\varphi \\ 0 & 0 & 1 & 1 & 1 & sin\varphi \\ l_1 & -l_2 & -l_3 & -l_4 & l_5 & l_6 sin\varphi \end{bmatrix},$$

where $l_i (i = 1, \cdots, 6)$ represents the ith yaw moment arm, and due to the symmetry of the main propeller, we have $l_2 = l_1$, φ denotes an arbitrary angle.

Taking the control vector (3) and the external ocean disturbance $w(t)$ into the motion equation (2), we have

$$\dot{v} = -M^{-1}N_0 v(t) + M^{-1}K(u(t) + w(t)) - M^{-1}G_0\eta(t).$$

Note that $x_{ref} = \begin{bmatrix} \eta_{ref} \\ v_{ref} \end{bmatrix}$, our goal is to control the tracking error $e(t) = \begin{bmatrix} e_\eta(t) \\ e_v(t) \end{bmatrix} = \begin{bmatrix} \eta(t) - \eta_{ref} \\ v(t) - v_{ref} \end{bmatrix}$ as small as possible. Without loss of generality, we choose the origin as the reference state point x_{ref}. And then we have

$$\dot{e}(t) = Ae(t) + Bu(t) + Bw(t), \tag{4}$$

where $A = \begin{bmatrix} 0 & I \\ -M^{-1}G_0 & -M^{-1}N_0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1}K \end{bmatrix}$. Assume that, the state variables are limited to the constraint $e \in \chi$, where χ is a compact set with the origin as an interior point.

In this paper, the controlled output $z(t)$ is defined as

$$z(t) = Ce(t). \tag{5}$$

Based on (4) and (5), it yields

$$\begin{aligned} \dot{e}(t) &= Ae(t) + Bu^F(t) + Bw(t), \\ z(t) &= Ce(t). \end{aligned} \tag{6}$$

Further, the fault model which represents the following three types of faults: outage, stuck and loss of effectiveness is $u^F(t) = \rho u(t) + \sigma u_s(t)$, where ρ is a diagonal matrix representing the effectiveness of each thruster, which is contained in the sets $\Delta_{\rho^j} = \left\{\rho^j | \rho^j = diag\left\{\rho_1^j, \rho_2^j, \cdots, \rho_n^j\right\}, \rho_i^j \in [\underline{\rho}_i^j, \bar{\rho}_i^j]\right\}$, where $\underline{\rho}_i^j$ is the lower bound of ρ_1^j and $\bar{\rho}_i^j$ is the upper bound, and the elements satisfy $0 \leqslant \underline{\rho}_i^j \leqslant \rho_i^j \leqslant \bar{\rho}_i^j \leqslant 1$, where $i(i = 1, \cdots, m)$ represents the ith thruster, $j(j = 1, \cdots, L)$ represents the jth fault mode and L denotes the whole number of the fault modes. σ is a diagonal matrix, where the elements $\sigma_i^j = 0$ or $\sigma_i^j = 1$, and $u_s(t)$ represents the nonparametric bounded time-varying thruster stuck fault. Hence, the UMVs system (6) with thruster faults can be modeled as

$$\begin{aligned} \dot{e}(t) &= Ae(t) + B(\rho u(t) + \sigma u_s(t)) + Bw(t), \\ z(t) &= Ce(t). \end{aligned} \tag{7}$$

where $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{p \times n}$.

Remark 1. *Different from the previous literature [10], [16], this paper considers the state and input constrained problem of the UMVs system, and the limited range and sets are given.*

It is assumed that the system (7) in this paper is fully controllable and fully observable for the purpose of fault tolerance. In addition, the following assumptions are valid for the entire article, which are given in [17]–[19], respectively.

Assumption 1. *The nonparametric stuck faults and external disturbances are piecewise continuous bounded equations, i.e.,* *the existence of unknown positive constant $\bar{u}_s$ and $\bar{w}_s$ can ensure the following equations hold:*

$$||u_s(t)||_2 \leqslant \bar{u}_s, \quad ||w(t)||_2 \leqslant \bar{w}.$$

Assumption 2. *It satisfies $rank(B\rho) = rank(B) = l$ for all $\rho \in \Delta_{\rho^j}, j = (1, \cdots, L)$.*

Assumption 3. *When $m - l(1 \leqslant l \leqslant m-1)$ thrusters are outage or stuck, the remaining thrusters can still achieve the desired control objectives, and all the thrusters can fail simultaneously.*

B. Problem Statement

Given the UMVs system (7), the purpose of this work is to design a new fault-tolerant control system to realize DP of the constrained UMVs system with thruster faults and external ocean disturbances. The considered control strategy includes two key blocks: an ISM controller and an MPC controller.

III. MAIN RESULTS

The control variable $u(t)$ is designed as:

$$u(t) = u_1(t) + u_{mpc}(t), \tag{8}$$
$$u_1(t) = -\beta(t)\hat{\mu}_0 N^T sign(\alpha),$$
$$u_{mpc}(t) = u^*_{mpc0}(t_k), t \in [t_k, t_{k+1}),$$

where u_1 and u_{mpc} represent the ISM controller and the MPC controller, respectively. The role of the ISM controller is to achieve the fault-tolerant objective and reject the matched ocean disturbances of the UMVs, while the MPC controller aims to guarantee the optimality with the constraints of states and inputs. These will be described in the following two subsections.

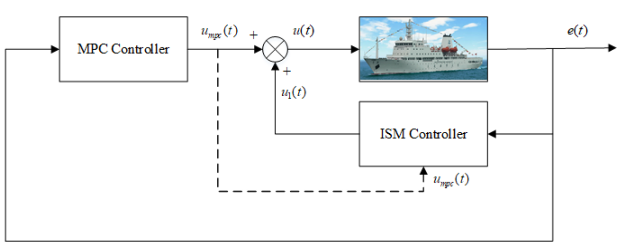

Fig. 1: Control scheme of the UMVs system

A. Integral Sliding Mode Controller

In this subsection, the ISM controller is designed to achieve the fault-tolerant objective and reject the ocean disturbances.

Assume that the input matrix B of the system can be factored into:

$$B = B_v N, \tag{9}$$

where $B_v \in R^{n\times l}$, $N \in R^{l\times m}$, and the rank of both of them is l, satisfying $l < m$.

The integral sliding surface is designed as follows:

$$\alpha\left(e\left(t\right)\right) = Ge\left(t\right) - Ge\left(t_0\right) - G\int_{t_0}^{t}\left(Ae\left(\xi\right) + Bu_{mpc}\left(\xi\right)\right)d\xi, \tag{10}$$

where G is a projection matrix satisfying $GB_v = I_l$, and we design it as $G = B_v^+$ by assumption 1. In the following subsections, let's substitute α for $\alpha(e(t))$, for the sake of simplicity.

The ISM control law can be designed by

$$u_1 = -\beta(t)\hat{\mu}_0 N^T sign(\alpha), \tag{11}$$

where $\hat{\mu}_0$ is the estimate of $\mu_0 = \frac{1}{\mu}$, and μ is an unknown positive constant satisfying $\mu \in (0, 1)$. Further,

$$\beta(t) = \frac{1}{\lambda}(\sum_{i=1}^{m}||N_i||\hat{\sigma}_i\hat{\bar{u}}_{si} + \sum_{i=1}^{m}||N_i||\hat{\bar{w}}_i + \epsilon), \tag{12}$$

where λ is the smallest eigenvalue of matrix NN^T, N_i and A_i are the ith column of matrix N and matrix A, u_{mpci} represents the ith line of matrix u_{mpc}, ϵ is a positive scalar, and μ, σ_i, $\bar{u}_{si}$ and $\bar{w}_i$ are obtained by the adaptive laws:

$$\begin{aligned} &\dot{\hat{\mu}}_0(t) = \gamma_1||\alpha||\lambda\beta(t), \dot{\hat{\sigma}}_i(t) = -\gamma_{2i}||\alpha||\ ||N_i||\hat{\bar{u}}_{si}(t), \\ &\dot{\hat{\bar{u}}}_{si}(t) = \gamma_{3i}||\alpha||\ ||N_i||, \dot{\hat{\bar{w}}}_i(t) = \gamma_{4i}||\alpha||\ ||N_i||, \end{aligned} \tag{13}$$

where γ_1, γ_{2i}, γ_{3i} and γ_{4i} are the adaptive gains which are obtained from practical applications and $i = 1, \cdots, m$. Based on the above, it is clear that $u_{mpci}, \hat{\sigma}_i, \hat{\bar{u}}_{si}, \hat{\bar{w}} \in L_\infty$ such that $\beta(t) \in L_\infty$, which means that there exists an upper bound $\bar{\beta}(t)$ satisfying $\lim_{t\to\infty}\beta(t) = \bar{\beta}(t)$. Hence, by (11), the maximum thrust provided by the ISM controller satisfies $u_1 \leqslant u_{1max}$.

Define the adaptive estimate errors as $\tilde{\mu}_0(t) = \hat{\mu}_0(t) - \mu_0(t)$, $\tilde{\sigma}_i(t) = \hat{\sigma}_i(t) - \sigma_i$, $\tilde{\bar{u}}_{si}(t) = \hat{\bar{u}}_{si}(t) - \bar{u}_{si}$, $\tilde{\bar{w}}_i(t) = \hat{\bar{w}}_i(t) - \bar{w}_i$, further, the error systems becomes $\dot{\tilde{\mu}}_0(t) = \dot{\hat{\mu}}_0(t)$, $\dot{\tilde{\sigma}}_i(t) = \dot{\hat{\sigma}}_i(t)$, $\dot{\tilde{\bar{u}}}_{si}(t) = \dot{\hat{\bar{u}}}_{si}(t)$, $\dot{\tilde{\bar{w}}}_i(t) = \dot{\hat{\bar{w}}}_i(t)$ because the derivatives of μ_0, σ_i, u_{si} and w_i are zero.

Theorem 1. *Consider the UMVs system (7) subjected to the thruster faults, suppose Assumptions 1-3 are valid. Then the trajectory of the system can be driven into the integral sliding surface (10) by employing the control laws (11)-(12) and adaptive laws (13).*

Proof. Choose the following Lyapunov function to analyse the reachability:

$$\begin{aligned} &V(\tilde{\mu}_0, \tilde{\bar{u}}_s, \tilde{\sigma}, \tilde{\bar{w}}) \\ &= V_1(\alpha) + \frac{1}{2\gamma_1}\mu\tilde{\mu}_0^2 + \sum_{i=1}^{m}\frac{\tilde{\sigma}_i^2}{2\gamma_{2i}} + \sum_{i=1}^{m}\frac{\sigma_i\tilde{\bar{u}}_{si}^2}{2\gamma_{3i}} + \sum_{i=1}^{m}\frac{\tilde{\bar{w}}_i^2}{2\gamma_{4i}}, \end{aligned} \tag{14}$$

where $V_1(\alpha) = \frac{1}{2}\alpha^T\alpha$.

Take the derivative of $V_1(\alpha)$, we have

$$\begin{aligned} &\dot{V}_1(\alpha) \\ &= \alpha^T N\left(\rho - I\right)u_{mpc} + \alpha^T N\rho u_1 + \alpha^T N\sigma u_s + \alpha^T Nw. \end{aligned}$$

Owing to the fact $||\rho|| < I$, the following inequality holds:

$$\alpha^T N(\rho - I)u_{mpc} \leqslant 0.$$

Then, according to Assumption 1 and applying Lemma 1 and Lemma 2 which are shown and proved in [5], we can obtain

$$\dot{V}_1(\alpha) \leqslant \sum_{i=1}^{m}||\alpha||\ ||N_i||\hat{\sigma}_i\hat{\bar{u}}_{si} - \sum_{i=1}^{m}||\alpha||\ ||N_i||\tilde{\sigma}_i\hat{\bar{u}}_{si}$$

$$- \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \sigma_i \tilde{\hat{u}}_{si} + \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \hat{\tilde{w}}_i$$
$$- \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \tilde{\tilde{w}}_i - \beta(t) \hat{\mu}_0 \mu \lambda ||\alpha||. \quad (15)$$

If β is chosen as (12), by applying the adaptive estimate errors and taking (15) into (14), $\dot{V}(t)$ becomes

$$\begin{aligned} \dot{V}(\tilde{\mu}_0, \tilde{\hat{u}}_s, \tilde{\sigma}, \tilde{w}) \leqslant & \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \hat{\sigma}_i \hat{\hat{u}}_{si} - \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \tilde{\sigma}_i \hat{\hat{u}}_{si} \\ & - \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \sigma_i \tilde{\hat{u}}_{si} + \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \hat{\hat{w}}_i \\ & - \sum_{i=1}^{m} ||\alpha|| \, ||N_i|| \tilde{\tilde{w}}_i - \beta(t) \tilde{\mu}_0 \mu \lambda ||\alpha|| \\ & - \beta(t) \lambda ||\alpha|| + \frac{1}{\gamma_1} \mu \tilde{\mu}_0 \dot{\hat{\mu}}_0 + \sum_{i=1}^{m} \frac{\tilde{\sigma}_i \dot{\tilde{\sigma}}_i}{\gamma_{2i}} \\ & + \sum_{i=1}^{m} \frac{\sigma_i \tilde{\hat{u}}_{si} \dot{\tilde{\hat{u}}}_{si}}{\gamma_{3i}} + \sum_{i=1}^{m} \frac{\tilde{\tilde{w}}_i \dot{\tilde{\tilde{w}}}_i}{\gamma_{4i}}. \end{aligned} \quad (16)$$

By applying the adaptive laws (13), we have

$$-\beta(t) \tilde{\mu}_0 \mu \lambda ||\alpha|| + \frac{1}{\lambda} \mu \tilde{\mu}_0 \dot{\hat{\mu}}_0 = 0.$$

Then, taking (12) and adaptive laws (13) into (16), the inequality is converted to

$$\dot{V}(\tilde{\rho}, \tilde{\hat{u}}_s, \tilde{\sigma}, \tilde{\bar{w}}) \leqslant -\epsilon ||\alpha|| \leqslant 0,$$

which follows that $V(\tilde{\rho}, \tilde{\hat{u}}_s, \tilde{\sigma}, \tilde{\bar{w}})$ is nonincreasing of time. Since $\dot{V}$ is not identical to zero in the trajectory of state for any non-zero solution of the equation (7), we complete the proof. □

Compute the derivative of the integral sliding surface (10),

$$\dot{\alpha} = G\dot{e} - GAe - GBu_{mpc}. \quad (17)$$

Substituting (7) into (17), we have

$$\dot{\alpha} = GB\left(\rho - I\right) u_{mpc} + GB\rho u_1 + GB\sigma u_s + GBw. \quad (18)$$

According to the full-rank decomposition (9), $G = B_v^+$, it is easy to obtain $GB = N$. Substituting it into the above formula (18), we have

$$\dot{\alpha} = N\left(\rho - I\right) u_{mpc} + N\rho u_1 + N\sigma u_s + Nw.$$

Since the definition of the sliding surface $\alpha = \dot{\alpha} = 0$, the equivalent control u_{1eq} can be obtained as

$$u_{1eq} = -(N\rho)^+ \left(N(\rho - I)u_{mpc} + N\sigma u_s + Nw\right), \quad (19)$$

where $(N\rho)^+$ is the Moore-Penrose inverse of $(N\rho)$, which satisfies $(N\rho)(N\rho)^+ = I_l$. Taking (19) back into (7), the sliding dynamics are given by

$$\begin{aligned} \dot{e} &= Ae + B\rho[-(N\rho)^+(N(\rho - I)u_{mpc} + N\sigma u_s + Nw)] \\ &\quad + B\rho u_{mpc} + B\sigma u_s + Bw \\ &= Ae + Bu_{mpc}. \end{aligned}$$

So the equivalent dynamics system can be obtained as

$$\dot{e} = Ae + Bu_{mpc}. \quad (20)$$

Algorithm 1: MPC algorithm

1: Set simulation time t_s, prediction horizon N, matrix Q and scalar weight R;
2: Discretize (20) to obtain the discret system (21);
3: Input the objective function (22);
4: Measure the current state $e(t)$;
5: Solve the MPC problem $\mathbb{P}$ (23)-(27) to get the optimal solution;
6: Apply control $u_{mpc}(t) = u^*_{mpc0}(t_k), t \in [t_k, t_{k+1})$;
7: Repeat from step 4 at next sampling time instant.

B. Model Predictive Controller

In this subsection, the design of the MPC controller can be achieved in the current dynamical system (20) without the ocean disturbances, since it has been rejected by ISM controller.

After discretization, the system (20) becomes

$$e(t_{k+1}) = A_d e(t_k) + B_d u_{mpc}(t_k). \quad (21)$$

At any sampling time t_k , the cost function to be minimized in regard to the control sequence $u_{mpc}(t_k) = [u_{mpc0}(t_k), u_{mpc1}(t_k), \cdots, u_{mpcN-1}(t_k)]$ is designed as

$$\begin{aligned} J(e(t_k), u_{mpc}(t_k)) = & \sum_{j=0}^{N-1} \left\| e(t_{k+j}) \right\|_Q^2 + \sum_{j=0}^{N-1} \left\| u_{mpc}(t_{k+j}) \right\|_R^2 \\ & + \left\| e(t_{k+N}) \right\|_P^2, \end{aligned} \quad (22)$$

where $N \geqslant 1$ is the prediction horizon. Besides the inequality constraints on state and input variables, the cost function (22) is also restricted by the nominal system $e(t_{k+1}) = A_d e(t_k) + B_d u_{mpc}(t_k)$. Then the optimization problem ($\mathbb{P}$) can be recast as follows

$$\mathbb{P}: \quad \min_{u} \quad J(e(t_k), u_{mpc}(t_k)) \quad (23)$$

$$\begin{aligned} \text{s.t.} \quad & e(t_{k+j}) \in \chi, && (24) \\ & e(t_{k+N}) \in \chi_f, && (25) \\ & \left\| u_{mpc}(t_{k+j}) \right\| \leqslant u_{max} - u_{1max}, && (26) \\ & e(t_{k+1}) = A_d e(t_k) + B_d u_{mpc}(t_k). && (27) \end{aligned}$$

where $j = 1, \cdots, N-1$, and χ_f is the terminal set containing the origin as an interior point and satisfying

$$\chi_f = \left\{ e \mid \left\| x - x_{ref} \right\|_P^2 \leq \varepsilon \right\}, \ \chi_f \subseteq \chi. \quad (28)$$

In this article, we introduce $\kappa_f(e)$ as the auxiliary control law to design the terminal penalty and set. The $\kappa_f(e)$ is defined as

$$\kappa_f(e(t_k)) = K_{LQ} e(t_k).$$

In (28), ε is a positive real value which yields

$$e(t_k) \in \chi_f, \quad (29)$$

$$\|\kappa_f(e(t_k))\| \leqslant u_{max} - u_{1max}.$$

In addition, Q and R are positive definite matrix and scalar weight, respectively, and P denotes the positive definite terminal state weight, which can be obtained by the Riccati equation:

$$(A_d - B_d K_{LQ})^T P (A_d - B_d K_{LQ}) - P = -Q - K_{LQ}^T R K_{LQ},$$

where K_{LQ} is the control gain of an infinite horizon Linear-Quadratic (LQ) controller, which has the same cost function with (22). Then, the applied piecewise-constant control law can be obtained by the Receding Horizon strategy

$$u_{mpc}(t) = u^*_{mpc0}(t_k), t \in [t_k, t_{k+1}),$$

where $t_{k+1} - t_k$ is the sampling time, and $u^*_{mpc0}(t_k)$ is the first value of the optimal control sequence at t_k, which can be obtained by solving the optimization problem $\mathbb{P}$.

Theorem 2. *For the discrete system (21), controlled by (8), there exists a feasible solution obtained by solving $\mathbb{P}$ subjected to the nominal system (27) and the constraints on state and input variables (24)-(26), then the optimal control problem is recursively feasible and the controlled system is asymptotically stable.*

Proof. In order to proof Theorem 2, there are two steps have to be followed, i.e., the recursive feasibility and the stability of the system.

Step 1 (Feasibility) Recursive feasibility means that given the optimal solution at time t_k, there always exists a solution at time t_{k+1} which also satisfies the whole constraints. At time t_k, consider the optimal solution $u^*_{mpc}(t_k) = [u^*_{mpc0}(t_k), u^*_{mpc1}(t_k), \cdots, u^*_{mpcN-1}(t_k)]$, only the first value of the optimal sequence is applied by the Receding Horizon principle. The control sequence $\tilde{u}_{mpc}(t_{k+1}) = \begin{cases} u^*_{mpc}(t_k) \\ \kappa_f(e(t_{k+N})) \end{cases}$ satisfies the constrains (24)-(26) at time t_{k+1}. According to the Receding Horizon principle, it is not difficult to get $\tilde{u}_{mpc}(t_{k+1}) = u^*_{mpc}(t_k)$ satisfies the constraints (24) and (26). Since $e(t_{k+N}) \in \chi_f$, it also holds that $\|\kappa_f(e(t_{k+N}))\| \leqslant u_{max} - u_{1max}$ and $e(t_{k+N}) \in \chi$, so that (24) and (26) are fulfilled when $j = N$. Finally, from (29), we have $e(t_{k+N+1}) \in \chi_f$, which implies that the control law satisfies all the constraints when $j = N$. Therefore, the optimization problem $\mathbb{P}$ is recursively feasible.

Step 2 (Stability) For the discrete system, we choose the function $\tilde{J}(e(t_{k+1}))$ as the candidate Lyapunov function and $J^*(e(t_k))$ as the optimal function to prove the stability. Then, we have

$$\begin{aligned}\tilde{J}(e(t_{k+1})) =& \sum_{j=1}^{N} (\|e(t_{k+j})\|_Q^2 + \|u^*_{mpc}(t_{k+j})\|_R^2) \\ &+ \|e(t_{k+N+1})\|_P^2 \\ =& \sum_{j=1}^{\infty} (\|e(t_{k+j})\|_Q^2 + \|u^*_{mpc}(t_{k+j})\|_R^2) \\ =& J^*(e(t_k)) - (\|e(t_k)\|_Q^2 + \|u_{mpc}(t_k)\|_R^2).\end{aligned}$$

Since the function is suboptimal at time t_{k+1}, it holds the optimal value

$$\begin{aligned}J^*(e(t_{k+1})) \leqslant& \tilde{J}(e(t_{k+1})) \\ =& J^*(e(t_k)) - (\|e(t_k)\|_Q^2 + \|u_{mpc}(t_k)\|_R^2).\end{aligned}$$

Further, we have

$$J^*(e(t_{k+1})) \leqslant J^*(e(t_k)),$$

which implies the stability of the system. □

IV. Simulation Results

This section demonstrates the availability of the adaptive fault-tolerant control based on MPC with ISM for the UMVs system. We use a typical floating production ship [12], [15] with the length $L = 200.6m$ and the mass $m = 73097.15kg$. The values of matrices M, N_0 and G_0 are given in [15]. In this paper, we adopt the disturbance $w(t) = [0.35sin(1.2t),\ -0.5t,\ 0.15,\ 2M_1(s)N_1(t) + I_1 Ax_{ref},\ -cos(3t)exp^{-0.3t} + I_2 Ax_{ref},\ 2M_2(s)N_2(t) + I_3 Ax_{ref}]^T$ to better simulate the marine environment, where the selection of noise matrix and parameters are given in [16]. Without loss of generality, the origin is assumed to be the equilibrium of interest, therefore, the reference state x_{ref} is chosen as the origin. The range of the yaw angle [2] satisfies $\psi(t) \in (-\frac{\pi}{6}, \frac{\pi}{6})$. In this simulation, we consider the thrusters can work normally until $t = 20s$, and it is assumed that the third aft tunnel thruster I [15] is stuck at $0.5 + 0.1sin(t)$ from $t = 20s$. Moreover, the disturbances start from $t = 20s$ to $t = 25s$. The sampling time of the MPC block is set as $T = 0.1s$, positive definite matrix Q is defined as $Q = diag(100, 100)$, and the scalar weight $R = 0.1$.

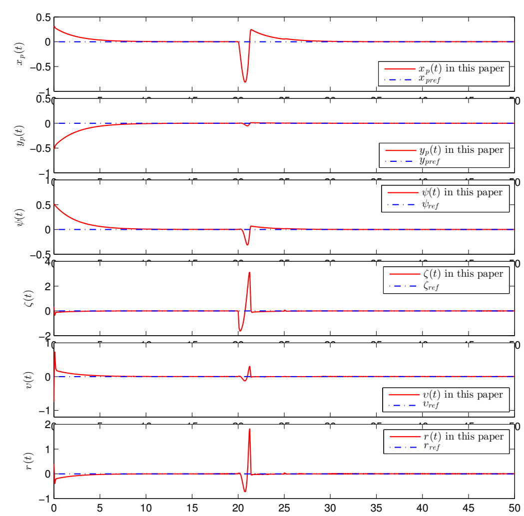

Fig. 2: Time responses of states using the proposed control scheme

Fig.2 shows the time responses of states of the UMVs system using the proposed control scheme. The red lines represents the states, and the blue line denotes the reference

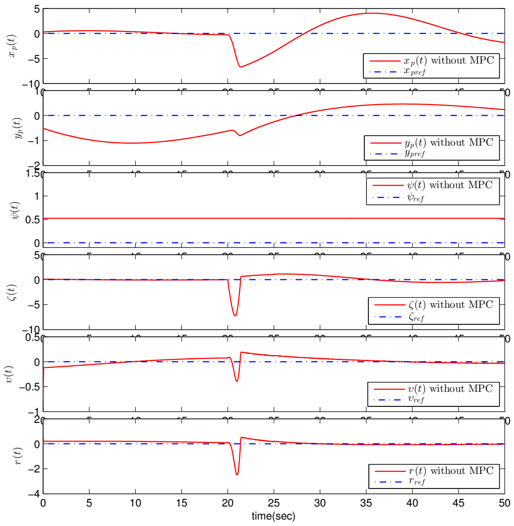

Fig. 3: Time responses of states without MPC controller

value. It can be easily seen that the curves from $t = 20s$ produce huge fluctuations caused by the thruster faults and disturbance. Fig.3 shows the states comparison of the UMVs system. Compared with Fig.2, the states ultimately can not be stabilized and the system is unstable under the constraints without MPC controller. Namely, the stability of the UMVs system with constraints could be guaranteed by the proposed control method.

V. Conclusion

Based on the combination of integral sliding mode control and model predictive control strategy, a fault-tolerant control method for dynamic positioning of unmanned marine vehicles is proposed to solve the problems of thruster faults, external disturbances and the constraints of states and inputs. Since the robustness of ISMC and the optimality of MPC which can deal with constraint problems, the proposal can reach the goals of this paper. Finally, the effectiveness of the control strategy is shown by the simulation results.

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