Self-healing Control Against Actuator Stuck Failures Under Constraints: Application to Unmanned Helicopters

Xin Qi, Didier Theilliol, Juntong Qi, Youmin Zhang and Jianda Han

Abstract This paper investigates the problem of actuator stuck failures under constraints. In order to guarantee the post-failure system stability and acceptable performance, self-healing control framework is proposed which includes self-healing management module, fault-tolerant controller, reference redesigner and anti-windup compensator. Because of the existence of actuator constraints, the post-failure system may be unstable and the reference may be unreachable. Hence, fault-tolerant controller with anti-windup compensator was used to guarantee stability which was proved by introducing H_∞ performance. Reachability of reference was analyzed by self-healing management module and a new reference could be calculated by reference redesigner. At last, the proposed self-healing framework was applied to a linear unmanned helicopter model for velocities and yaw tracking control.

Keywords Fault-tolerant systems · Actuators · Stuck · Saturation · Autonomous vehicles

容错控制器和抗饱和补偿器用于保证稳定性
自救管理模块用于分析可达性

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1 Introduction

Traditional control techniques cannot guarantee correct and safe operation of equipment in the event of malfunctions in actuators [10]. Abundant approaches have been proposed against actuator faults, using hardware redundancy and fault-tolerant control (FTC) techniques [17]. Generally, actuator malfunctions are divided into two categories: faults and failures. Actuator failures signify all efficiency is lost and actuator cannot respond control signal completely. One typical actuator failure is actuator stuck malfunction. fault(故障) 会导致 failure(失效)

Compared to actuator faults, few researches focused on actuator stuck failures. In [12], an adaptive state feedback method against actuator failures was proposed. Actuator stuck failures can be compensated by remaining fault-free actuators adaptively and the system can keep tracking the original reference. In [15], a stuck failure was modeled as a bounded input and its effect on the closed-loop system is described by a peak-to-peak gain. The FTC controller is designed based on H_∞. In [4], an iterative learning observer (ILO) for actuator stuck failures was presented which can provide both the estimates of the system states and information on the transient of failure compensation. The drawback of these approaches is lack of consideration the constraints of actuators. In this paper, a FTC method against actuator stuck failures under actuator constraints is investigated.

System inputs of stuck actuators have to be compensated by remaining fault-free actuators. Taking into account actuator constraints, the remaining actuators margin is degraded after stuck-failure compensation. Thus, post-failure system may not track the original reference without offset. For the sake of system stability and preventing failure deterioration, reference redesign is necessary. In [6], a new reference of post-failure system was generated according to system remaining performance. The distance between the new reference and the original one before failure is minimum. In [13], a model predictive control strategy was proposed to redesign the new reference on-line which is achieved by solving an optimization problem. The drawback of this method is that the new reference needs to be calculated in real-time. So it is impossible to obtain the steady-state reference at the beginning of failure occurrence. In [16], a control input management approach was investigated to compute a new steady-state reference which is based on the open-loop gain of post-failure system in steady-state case. The method is based on experience so that the new reference may not be optimal. The disadvantage of these methods is the shortage of reachability analysis of original references after actuator stuck occurrence. In this paper, a reference admissible set is computed for analyzing reachability.

Similar researches against actuator failures and constraints are limited such as [3] where flatness technique was used for quadrotors specially. However, actuator constraints and only actuator faults were considered and analysis of reference reachability was absent.

On the other hand, actuator constraints also affect dynamic performance of both fault-free and post-failure systems especially at the moment when controller is switched from fault-free one to fault-tolerant one. One way to improve dynamic performance is to consider anti-windup techniques. Two categories of anti-windup methods have been proposed. The first one is to take into account anti-windup technology when the system controller is designed [9]. The second one is to design a nominal controller without anti-windup first and then add into an anti-windup compensator. The second method is more popular. In [14], an anti-windup compensator design method was proposed by introducing sector theory. The difference between the controller output and saturated actuator output is assumed bounded and the compensator design problem is recast into a robust control paradigm. In [5], based on invariant theory, both anti-windup compensator and stability domain can be achieved by solving a group of linear matrix inequalities (LMIs). Besides anti-windup techniques, command governor was used as an added primal compensator to modify the reference inputs so as to avoid violation of the constraints [1].

The main contribution of this paper is to present a self-healing control framework against actuator stuck failures under actuator constraints. The self-healing framework includes self-healing management module, reconfigurable controller with anti-windup compensator, reference redesigner and fault diagnosis and identification (FDI) module, as shown in Fig. 1, where ref represents the original reference while ref_{new} is the redesigned new reference, f represents actuator faults/failures and ψ is difference between actuator output with/without saturation. The self-healing management module is used to analyze the reachability of the original reference by calculating reference admissible set. It allows to pre-evaluate the reachability of post-failure system before it is in motion. The reconfigurable controller is designed to guarantee the closed-loop stability and tracking performance while an anti-windup module is used to improve dynamic performance of the system when actuators are in saturation. The function of reference redesigner is to compute a new reachable reference which satisfies actuator constraints and conditions of the post-failure system. FDI module is used to detect, isolate and identify the stuck failures. Furthermore, with the information provided by FDI module, the proposed framework can be designed online. In the following discussion, stuck-failure magnitude is assumed to be provided by FDI module correctly with no time delay. The investigation of FDI methods is not included in this paper.

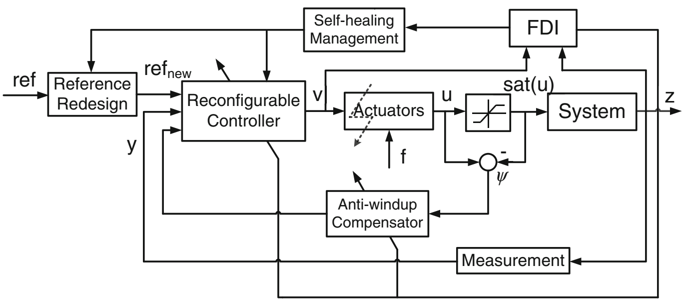

Fig. 1 The structure of self-healing framework

The paper is organized as follows: Problem statement is dedicated in Sect. 2. Section 3 is devoted to the development of the proposed approach. Fault-tolerant control method is proposed with related stability certification. Then, reachability of reference and reference redesign are investigated based on achieved admissible set of reference. In Sect. 4, a linear unmanned helicopter (UH) model is considered to illustrate the proposed method. Section 5 ends the paper.

2 Problem Statement

Consider an open-loop linear time-invariant (LTI) system which is stabilizable and detectable as following:

$$\begin{cases} \dot{x}(t) = A_p x(t) + B_p sat(u(t)) \\ z(t) = C_{p1} x(t) \\ y(t) = C_{p2} x(t) \end{cases} \tag{1}$$

where $x \in R^n$ is the system state vector. $u \in R^m$ is the system input vector neglecting actuator constraints. $z \in R^{p1}$ is the system controlled output vector, and $y \in R^{p2}$ is the system measurement output vector. $A_p,\ B_p,\ C_{p1},\ C_{p2}$ are constant matrices with appropriate dimensions. Furthermore, $B_p = [b_1\ b_2\ \ldots\ b_m]$, where b_i represents the ith column of matrix B_p. $sat(\cdot)$ represents a vector function defined by

$$sat(u_i(t)) = \begin{cases} u_i^{max} & if \quad u_i(t) > u_i^{max} \\ u_i(t) & if \quad u_i^{min} \leq u_i(t) \leq u_i^{max} \\ u_i^{min} & if \quad u_i(t) < u_i^{min} \end{cases} \tag{2}$$

for $i = 1, 2, ..., m$. u^{min}, u^{max} are actuator constraints, and $sat(u(t))$ is the control input vector with actuator constraints. Assume that $u^{min} = -u^{max} = \tilde{u}$.

Actuator stuck failures can be modeled as following:

$$u(t) = \Phi v(t) + (I - \Phi)\bar{u} \tag{3}$$

where $v(t)$ represents controller output vector, and $\bar{u}$ is a constant vector representing the magnitude of stuck failure. $\Phi = diag(\phi_1,\ \phi_2, \ldots,\ \phi_m)$, and $\phi_i = 1\ or\ 0$ where $\phi_i = 1$ represents that the ith actuator is fault-free and $\phi_i = 0$ represents that the ith actuator is lock-in-place because the actuator cannot respond the control signal completely.

Hence, the control input vector neglecting actuator constraints can be divided into two parts $u(t) = [u_0^T(t)\ \bar{u}_f^T]^T$ in stuck-failure case, where $u_0(t) \in R^{m_0}$ is the fault-free control input, $\bar{u}_f \in R^{m_f}$ is the stuck control input which is constant and $m_0 + m_f = m$.

The control matrix B_p can be decomposed into $[B_{p0}\ B_{pf}]$ with $B_{p0} \in R^{n\times m_0}$ and $B_{pf} \in R^{n\times m_f}$ correspondingly.

Then system (1) in stuck-failure case can be described as:

$$\begin{cases} \dot{x}(t) = A_p x(t) + B_{p0} sat(u_0(t)) + B_{pf} sat(\bar{u}_f) \\ z(t) = C_{p1} x(t) \\ y(t) = C_{p2} x(t) \end{cases} \tag{4}$$

Compared with the fault-free system defined by (1), the post-failure system with stuck actuators has degraded input matrix and an additional constant item. The degraded input matrix will affect state and output controllability so that the following assumption is necessary:

Assumption The post-failure system $(Ap,\ B_{p0})$ is stabilizable.

The additional constant item will affect state value in steady case. Taking into account actuator constraints, the problem will be more troublesome. Actuator constraints affect the post-failure system in two ways:

- *Global stability may not be guaranteed.* Because of actuator constraints, actuator output is limited so that the controllable domain may not be guaranteed as the global space. Thus, regional stability is considered instead [8].
- *Reference may be unreachable.* For a set-point tracking problem, the following relationship is established in steady case:

$$ref = z(\infty) = \begin{cases} H(\infty) sat(u(\infty)) & Normal \\ H_0(\infty) sat(u_0(\infty)) + H_f(\infty)\bar{u}_f & Stuck \end{cases} \tag{5}$$

  where ref is reference, $H(\infty) = [H_0(\infty)\ H_f(\infty)]$ is open-loop gain in steady case, $H_0(\infty)$ and $H_f(\infty)$ are related to fault-free and post-failure actuators respectively. Because actuator allowance is consumed by stuck-failure compensation, the second equation might not be valid. In other words, the reference may be unreachable.

Hence, the proposed self-healing control framework should solve the following two problems:

Problem 1 Design a fault-tolerant controller to guarantee stability of the post-failure system with acceptable set-point tracking under actuator stuck failures and saturation.

Problem 2 Analyze reference reachability subject to closed-loop post-failure system and compute a new one if required.

3 Main Results

3.1 Fault-Tolerant Control Method

In addition to guarantee the post-failure system being stable, the task of fault-tolerant controller also includes compensating stuck failures with capability of tracking set-points with offset-free and anti-windup. The three functions are achieved by three additional items of the dynamic output feedback controller. Hence, the dynamic fault-tolerant controller against post-failure system (4) is proposed as following.

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c y(t) + E_c(sat(u_0(t)) - u_0(t)) + K_f \bar{u}_f + K_r ref \\ u_0(t) = C_c x_c(t) + D_c y(t) \end{cases} \tag{6}$$

where $x_c \in R^{n_c}$ is controller state vector which has the same dimension as the open-loop system $n_c = n$, $u_0 \in R^{m_0}$ is controller output vector, ref is reference vector, E_c is anti-windup compensator, K_f is stuck-failure compensator, and K_r is feedforward gain matrices. A_c, B_c, C_c, and D_c are constant feedback controller matrices. Note that, controller (6) can be divided into four parts: (1) Matrices A_c, B_c, C_c, and D_c consist a classical dynamic output feedback controller which can be designed according to (1) without considering actuator saturation [5]. The design methods of A_c, B_c, C_c, D_c are classical and out of research scope of this paper; (2) Anti-windup compensator E_c which can be designed based on a nominal closed-loop system; (3) Stuck-failure compensator K_f is used to reduce the impact of actuator stuck and (4) Feedforward controller K_r is used to guarantee tracking performance. The design method of matrices E_c, K_f and K_r will be introduced in the following.

Define extended state vector $\xi(t) = \left[x^T(t)\ x_c^T(t)\right]^T \in R^{n+n_c}$, exogenous input vector $\omega = \left[\bar{u}_f^T\ ref^T\right]^T$, and function $\psi(u_0(t)) = u_0(t) - sat(u_0(t))$. Then, the post-failure closed-loop system can be described by:

u0(t) 状态反馈

$$\begin{cases} \dot{\xi}(t) = A\xi(t) - (B_0 + RE_c)\psi(K\xi(t)) + D\omega \\ z(t) = C\xi(t) \end{cases} \tag{7}$$

where

$$A = \begin{bmatrix} A_p + B_{p0}D_cC_{p2} & B_{p0}C_c \\ B_cC_{p2} & A_c \end{bmatrix},\ R = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} B_{p0} \\ 0 \end{bmatrix},\ B_f = \begin{bmatrix} B_{pf} \\ K_f \end{bmatrix},\ L = \begin{bmatrix} 0 \\ K_r \end{bmatrix}$$

$$D = \begin{bmatrix} B_f & L \end{bmatrix},\ C = \begin{bmatrix} C_{p1} & 0 \end{bmatrix} \text{ and } K = \begin{bmatrix} D_cC_{p2} & C_c \end{bmatrix}$$

Considering steady-state case of system (7), if all of the fault-free actuators are not saturated, which means that $\psi(u_0(t)) = 0$, the controlled output will be

$$\begin{aligned} z(\infty) = &- C_{p1}\left[A_p + B_{p0}(-C_c A_c^{-1} B_c + D_c)C_{p2}\right]^{-1} \\ &\left[(-B_{p0} C_c A_c^{-1} K_f + B_{pf})\bar{u}_f - B_{p0} C_c A_c^{-1} K_r ref\right] \end{aligned} \tag{8}$$

In order to compensate actuator stuck failures, the coefficient of $\bar{u}_f$ should be equal to zero. Thus, the stuck-failure compensator can be calculated by:

$$K_f = \left(C_{p1} M B_{p0} C_c A_c^{-1}\right)^{+} C_{p1} M B_{pf} \tag{9}$$

where $M = \left[A_p + B_{p0}(-C_c A_c^{-1} B_c + D_c)C_{p2}\right]^{-1}$ and $(\cdot)^{+}$ represents pseudo inverse.

For set-point tracking problem, $z(\infty) = ref$ should be satisfied. Thus, the tracking matrix can be computed by:

$$K_r = (C_{p1} M B_{p0} C_c A_c^{-1})^{+} \tag{10}$$

Remark Considering columns of matrix B_p, if $\alpha_1 b_{i_1} + \alpha_2 b_{i_2} + \cdots + \alpha_q b_{i_q} = 0,\ \alpha_i \neq 0$ is satisfied, the column rank of B_p will be less than m. In other words, assume $Rank(B_p) = Rank(B_{p0}) = q < m$, then $B_{pf} = B_{p0} Q$ where Q is a constant matrix. So the control inputs of stuck actuators can be compensated by the remaining actuators directly such as $u_0 = Q\bar{u}_f$. Most of the research works against actuator stuck failures are under the above condition [4, 12]. Obviously, these results will be useless when $Rank(B_p) < m$ is not satisfied such as $Rank(B_p) = m$. Compared to these method, the proposed fault-tolerant controller design method in this paper can work under both $Rank(B_p) < m$ and $Rank(B_p) = m$.

Before anti-windup compensator design and stability certification, the following lemma [5] is recalled which is required for the coming theorem.

Lemma 1 *Consider a matrix $G \in R^{m\times(n+n_c)}$ and define the following polyhedral set:*

$$\Omega = \left\{\xi(t) |\ |(K_i - G_i)\xi(t)| \leq \tilde{u}_i,\ i = 1, \ldots, m\right\} \tag{11}$$

where i represents the ith row of matrix K and G.

If $\xi(t) \in \Omega$, then the relation

$$\psi(K\xi(t))^T T\left[\psi(K\xi(t)) - G\xi(t)\right] \leq 0 \tag{12}$$

is verified for any positive-definite matrix $T \in R^{m\times m}$.

Clearly, Lemma 1 defines a set of system states related to actuator saturation and the relation based on the set is useful for the coming theorem.

Theorem 1 *Given* $\gamma > 0$ *and a symmetric positive-definite matrix* $R \in R^{(n+n_c)\times(n+n_c)}$, *if there exist a symmetric positive-definite matrix* $W \in R^{(n+n_c)\times(n+n_c)}$, *matrices* $Y \in R^{m\times(n+n_c)}$, $Z \in R^{n_c\times m}$, *and a diagonal positive-definite matrix* $S \in R^{m\times m}$ *satisfying*

$$\inf_{W,Y,Z,S} \quad \lambda$$

$$\begin{bmatrix} W & WK_i^T - Y_i^T \\ * & \tilde{u}_i^2 \end{bmatrix} \geq 0, \quad i = 1, \ldots, m \tag{13}$$

$$\begin{bmatrix} \frac{1}{\gamma}(WA^T+AW) & \frac{1}{\gamma}(B_0S+RZ-Y^T) & \frac{1}{\gamma}B_f & \frac{1}{\gamma}L & -WC^T \\ * & -\frac{2}{\gamma}S & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -\gamma I & I \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} \lambda R & I \\ * & W \end{bmatrix} \geq 0 \tag{15}$$

then the anti-windup compensator is $E_c = ZS^{-1}$, *and the stability domain is* $\varepsilon(P) = \left\{\xi(t)\ |\xi^T(t)P\xi(t) \leq 1\right\}$ *with* $P = W^{-1}$.

Proof If relations in (13) are valid, $\varepsilon(P) \subset \Omega$ will be satisfied with $G = YP$ [2]. Thus, $\forall \xi(t) \in \varepsilon(P)$, $\psi(K\xi(t))$ satisfies sector condition (12) [5]. Taking into account H_∞ performance $\|ref - C\xi(t)\|_2 \leq \gamma\|\omega\|_2$, it can be written as

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$$J = \int_0^T \left[(ref - C\xi(t))^T(ref - C\xi(t)) - \gamma^2\omega^T\omega\right] dt < 0$$

Considering zero initial condition and Lyapunov function $V(\xi) = \xi(t)^T P\xi(t)$,

$$J = \int_0^T J_1 dt - V(\xi(T))$$

充分条件 where $J_1 = (ref - C\xi(t))^T(ref - C\xi(t)) - \gamma^2\omega^T\omega + \frac{d}{dt}V(\xi(t))$. Clearly, if $J_1 < 0$ is satisfied, H_∞ performance will be guaranteed. In the following analysis, symbol t will be ignored and $\psi(K\xi(t))$ will be replaced by ψ for simplicity. According to Lemma 1,

$$
\begin{aligned}
J_1 &\le (ref - C\xi)^T(ref - C\xi) - \gamma\omega^T\omega + \dot{V}(\xi) - 2\psi(K\xi)^T T\,[\psi(K\xi) - G\xi(t)] \\
&= \begin{bmatrix} \xi \\ -\psi \\ \bar{u}_f \\ ref \end{bmatrix}^T \left(\begin{bmatrix} -C^T \\ 0 \\ 0 \\ I \end{bmatrix} [\,-C\ 0\ 0\ I\,] \right. \\
&\quad \left. + \begin{bmatrix} A^TP+PA & P(B_0+RE_c)+G^TT & PB_f & PH \\ * & -2T & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \right) \begin{bmatrix} \xi \\ -\psi \\ \bar{u}_f \\ ref \end{bmatrix} \\
&\le 0
\end{aligned}
$$

Considering Schur complement, the following inequality is achieved:

$$
\begin{bmatrix} A^TP+PA & P(B_0+RE_c)+G^TT & PB_f & PH & -C^T \\ * & -2T & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\gamma^2 I & I \\ * & * & * & * & -I \end{bmatrix} \le 0
$$

Then pre- and post-multiplying the above inequality by $diag\left[\gamma^{-1/2}P^{-1}\ \gamma^{-1/2}T^{-1}\ \gamma^{-1/2}I\ \gamma^{-1/2}I\ \gamma^{1/2}I\right]$ and considering $W = P^{-1}$, $S = T^{-1}$, $Y = GP^{-1}$, and $Z = E_C T^{-1}$, relation (14) is achieved. If relation (14) is valid, $\dot{V}(\xi) \le 0$ will be satisfied. Thus, $\forall \xi(t) \in \varepsilon(P)$, $\varepsilon(P)$ is a positively invariant and contractive region. In other words, $\varepsilon(P)$ is the stability domain for system (7).

Finally, the objectives to be optimized $inf\ \lambda$ and relation (15) are used to enlarge the domain [7].

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Thus, a solution of Problem 1 has been found.

3.2 Self-healing Management

The main target of self-healing management is to analyze the remaining capability of post-failure system and select suitable strategy to guarantee system stabilization with acceptable performance. In order to guarantee post-failure system stability, system states in steady case should be inside stability domain achieved by Theorem 1 such as

$$
\xi(\infty)^T P \xi(\infty) \le 1 \tag{16}
$$

Note that, $\xi(\infty) \in \varepsilon(P) \subset \Omega$ can guarantee $|(K-G)\xi(\infty)| \le \tilde{u}$ but not $|K\xi(\infty)| \le \tilde{u}$. In other words, actuators may be saturated. In order to guarantee offset-free tracking performance, control input should never be saturated in steady-state case. Thus, system inputs of post-failure system should satisfy $|u_0(\infty)| = |K\xi(\infty)| \le \tilde{u}$ where $u_0(\infty)$ can be described as function of $\bar{u}_f$, and ref such as:

$$
u_0(\infty) = M_1\bar{u}_f + M_2 ref \tag{17}
$$

where

$$M_1 = -C_c A_c^{-1} K_f$$
$$M_2 = \left[(-C_c A_c^{-1} B_c + D_c) C_{p2} M B_{p0} - I\right] C_c A_c^{-1} K_r$$

Because M_1, M_2, and $\bar{u}_f$ can be obtained based on information of system and FDI module, the system inputs in steady case can be achieved under reference ref.

Thus, reference admissible set can be defined as:

$$\Upsilon = \left\{ ref \,|\, \xi(\infty) \in \varepsilon(P), |u_0(\infty)| \le \tilde{u} \right\} \tag{18}$$

If $ref \in \Upsilon$ is not valid, the original reference ref is recognised unreachable for the post-failure system. Hence, a new reference is required instead of the original one and Problem 2 is solved.

3.3 Reference Redesign

The target of reference redesign is to find a new optimal reference to guarantee the post-failure system stable with acceptable performance. If the new reference ref_{new} is expected to be as close as possible to the original one ref, the following optimization problem can be defined.

$$\min_{ref_{new}} \|N(ref_{new} - ref)\|_2$$

subject to:

$$ref_{new} \in \Upsilon$$

where $N \in R^{p1\times p1}$ is a diagonal weighting matrix. Thus, new optimal reference can be achieved and reachability is guaranteed.

4 Application to Unmanned Helicopters

A linear model of Unmanned helicopter, Fig. 2, including swashplate configuration and rotor speed control [11] is considered in this paper.

4.1 Unmanned Helicopter Model

The state vector is $x = [u\ v\ w\ \varphi\ \theta\ \psi\ p\ q\ r\ a_{1s}\ b_{1s}]^T$ where u, v, w are triaxial velocities, $\varphi\ \theta\ \psi$ are attitudes, $p\ q\ r$ are triaxial angular velocities, and $a_{1s}\ b_{1s}$ are flapping angle of main rotor. The control input vector is $u = [\theta_{M1}\ \theta_{M2}\ \theta_{M3}\ \theta_T\ \Omega]^T$,

Fig. 2 The unmanned helicopter

where the first four variables are output positions of servos for main rotor and tail rotor and the last one is rotor speed. The UH output vector is $z = [u\ v\ w\ \psi]^T$. Note, that actuator constraints are normalized in this paper, such as $\tilde{u} = [1\ 1\ 1\ 1\ 1]$, by multiplying an index matrix behind matrix B_p of [11] and the eigenvalues of the system matrix A_p are [0 −28.9183 10.0359 −5.9322+9.4855i −5.9322−9.4855i 1.5746 −1.2855 − 0.0085 + 0.3257i − 0.0085 − 0.3257i − 0.0001 − 0.0128] which means it is an open-loop unstable system.

4.2 Simulation Results

Assume the first actuator is stuck at 10 s with $\bar{u}_f = 0.3$ so that $B_{pf} = [b_1]$, and $B_{p0} = [b_2\ b_3\ b_4\ b_5]$. The stuck-failure compensator K_f and tracking matrix K_r can be calculated by (9) and (10). Anti-windup compensator E_c and related stability domain $\varepsilon(P)$ are achieved by the proposed Theorem 1. Stuck-failure information is assumed to be provided by FDI module without time delay.

Stabilization problem is illustrated firstly. Assume the initial sates of u, v, w are $[2\ 2\ -1]$ and the others are zero. Simulation results of control inputs and system outputs are shown in Figs. 3 and 4 respectively. Dash lines represent inputs and outputs of post-failure system with fault-free controller. As shown in Fig. 3, after the first actuator being stuck, the other four fault-free control inputs oscillate between the upper and lower bounds of constraints. Clearly, stability of the post-failure system cannot be guaranteed and outputs are out of order as shown in Fig. 4. In other words, after actuator stuck failures occurrence, the post-failure system will be out of order if controller is not reconfigured. Post-failure system with fault-tolerant controller is represented by solid lines. As shown in the figures, system stability is guaranteed and actuator outputs are not saturated. On the other hand, due to the existence of

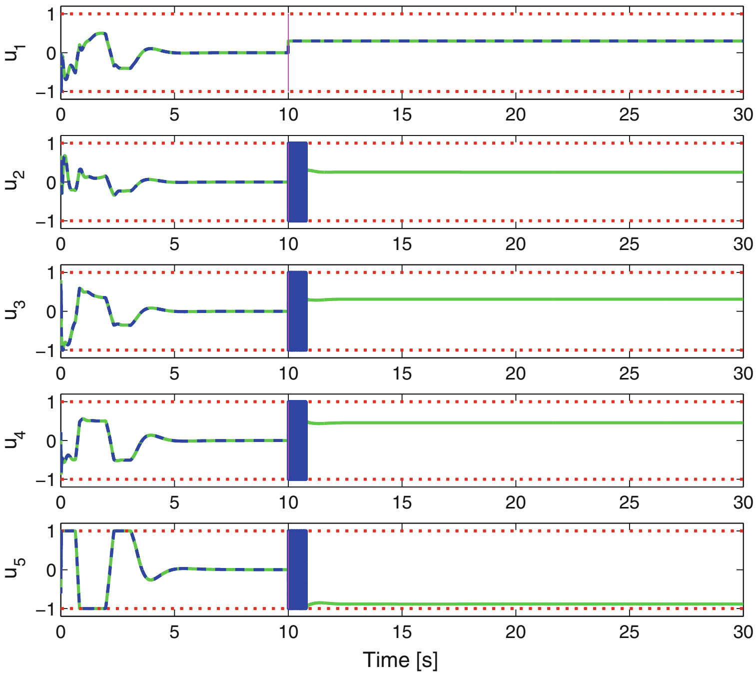

Fig. 3 Control inputs of stabilization problem

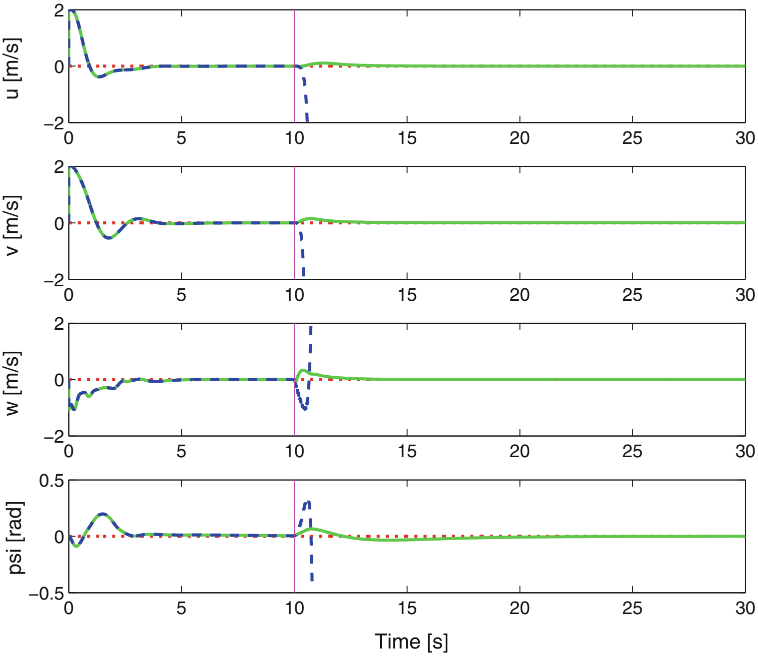

Fig. 4 System outputs of stabilization problem

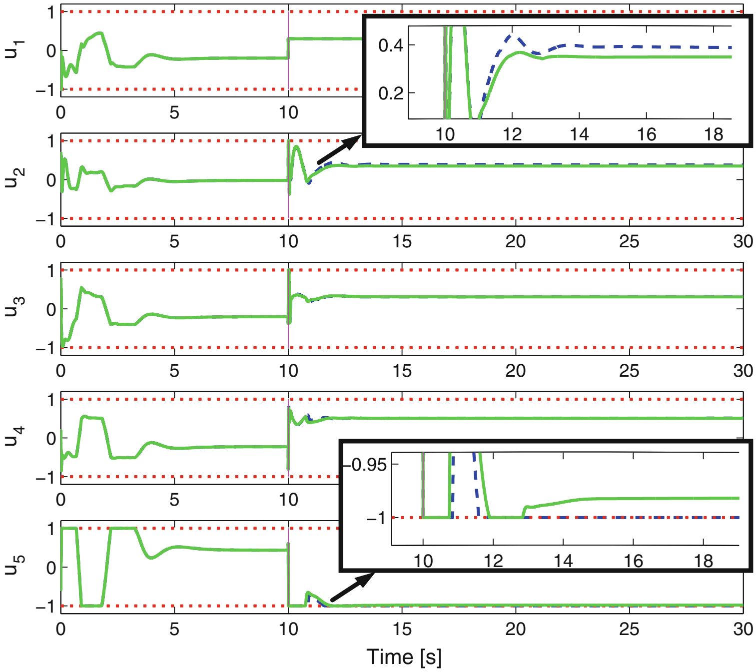

Fig. 5 Control inputs of set-point tracking

anti-windup compensator, actuator saturation is allowed during dynamic process as shown in Fig. 3.

For set-point tracking problem, the original reference for fault-free system is $ref = [1\ 1\ 1\ 0]^T$. Simulation results are shown in Figs. 5 and 6. Post-failure system with fault-free controller is represented by dash lines. As shown in the figures, the post-failure system is stable at last. However, because the tracked reference is the original one and, as analyzed before, it is not inside the admissible set of reference, the fifth actuator is saturated and the outputs cannot track the reference. Thus, analyzing reference reachability before system motion is necessary.

Based on self-healing management, the remaining control inputs of post-failure system in steady-state case can be obtained by (17) such as $u_0(\infty) = [0.3974\ \ 0.3079\ \ 0.5308\ \ -1.0301]$ which is outside the actuator constraints $\tilde{u}$. At the same time, $\xi(\infty)^T P \xi(\infty) = 2.4032 > 1$. In other words, the reference is unreachable for the post-failure system because of $ref \notin \Upsilon$. Thus, new reference is required. Based on the proposed reference redesign method, the optimal new reference is $ref_{new} = [0.61\ 0.67\ 0.4\ 0.05]^T$ with $N = [1\ 1\ 0.1\ 1]$. Post-failure system with self-healing control framework, including both fault-tolerant controller and reference redesign, is drawn by solid lines. As shown in Fig. 5, all of fault-free actuators are not saturated in steady case and Fig. 6 shows that system outputs can track the new reference without offset.

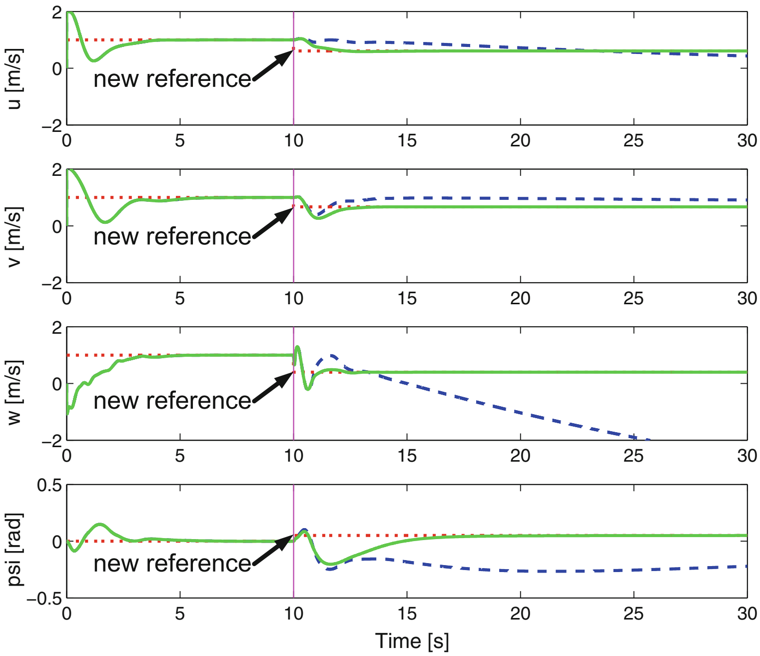

Fig. 6 System outputs of set-point tracking

5 Conclusions

Self-healing control framework against actuator stuck failures under constraints is proposed in this paper to guarantee the post-failure system stability and achieve acceptable performance. The proposed framework mainly includes fault-tolerant controller, reference redesign module, self-healing management module and anti-windup compensator. According to related information, self-healing management module can analyze reference reachability before system motion and reference redesign module can calculate an optimal new one if it is required. At last, the self-healing control method is illustrated by an unmanned helicopter model with velocities and yaw set-point tracking. Based on the proposed framework, stability and tracking performance of the post-failure system can be guaranteed. However, the fault-tolerant control, self-healing management and reference redesign methods are interconnected but the three parts are considered separately in the proposed framework. In the future, the three parts will be integrated together to reduce conservative.

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