

# Reconstruction of Large Scale Structure by Nonlinear **Lagrangian** Displacement Fields

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We investigate the ability of using nonlinear displacement field to reconstruct the primordial linear perturbations of the large scale structure. According to the linear Lagrangian perturbation theory, the first order primordial linear density field is given by the negative divergence of the displacement field. By running  $N$ -body simulations, we show that this **new** reconstruction algorithm recovers great amount of nonlinear information in the early universe. In reality, the displacement fields can be solved from only the final stage of the density fields. This demonstrates the potential to reconstruct baryonic acoustic oscillation (BAO) from current and future large scale structure surveys.

## I. INTRODUCTION

Our universe starts from primordial Gaussian perturbations at a very early stage, and from those fluctuations, the gravitational instability drives the formation of the large scale structure (LSS) distribution of matter [1, 2]. These structures grow linearly until the perturbations are large enough so that the first order perturbation theories are unable to analytically describe the LSS distributions [3]. As a result, the final nonlinear LSS distribution contains higher order statistics, and thus makes it more challenging to be interpreted into basic cosmological parameters. One such example is that, the baryonic acoustic oscillation (BAO) scale can be used as a “standard ruler” to constrain the cosmic expansion history and thus probes the dark energy properties [4], but nonlinear evolution smears the BAO features and lowers the measurement accuracy [5, 6]. There are various attempts to recover earlier stages of LSS, in which statistics are closer to Gaussian [7, 8]. Because Gaussian fields can be adequately described by two-point statistics, ideally after some recovery algorithms, more information can be extracted, more straightforwardly, by power spectra or two-point correlation functions [9, 10].

Standard BAO reconstruction algorithms smooth the nonlinear density field on linear scale ( $\sim 10$  Mpc/ $h$ ) and reverse the large scale bulk flows by a negative Zel’dovich linear displacement [11–13]. Here we propose a **new reconstruction method** that uses the nonlinear displacement field to recover the primordial density field. In the linear Lagrangian perturbation theory, the negative divergence of the the displacement field  $\Psi(\mathbf{q})$  respect to Lagrangian coordinates  $\mathbf{q}$  gives the linear density field [14]. Of course, the full displacement field  $\Psi(\mathbf{q})$  is non-observable, as it requires the initial distributions of matter, however there are many techniques to estimate the nonlinear displacement field from a final distribution of matter. For example, when a homogeneous initial matter distribution is assumed, there is a unique solution of curl-less displacement field to relate the initial and final distributions without shell-crossing. This solution can be solved by a metric transformation equation [15, 16]. In 1-dimensional (1D) case, the exact solution of [15] simplifies to an ordering of matter elements by Eulerian coordinates. Zhu *et al.* [19] apply this algorithm to the result of

an 1D simulation [3] and obtain an estimated displacement field  $\tilde{\Psi}(\mathbf{q})$ , and find that **this new method** well recovers the linear information and reconstructs BAO peak in correlation function. In 3D case, there are various techniques to obtain  $\tilde{\Psi}(\mathbf{q})$ . However one needs to carefully consider effects of curl, shell-crossing in 3D case.

Before these steps, we need to quantify the amount of linear information that can be recovered from the full nonlinear displacement field  $\Psi(\mathbf{q})$ , and further estimations  $\tilde{\Psi}(\mathbf{q})$  can be compared with this result. In this paper we run a LSS  $N$ -body simulation and track the motion of particles to obtain  $\Psi(\mathbf{q})$ . According to this we reconstruct the linear density field and compare to the primordial density field of the initial conditions. We describe the simulation and reconstruction algorithm in section II, and we show the results in section III. Discussion and conclusion are in section IV.

## II. METHOD

We show the LSS simulation and displacement field setups in section II A, and the reconstruction processes are presented in section II B.

### A. Simulation

We use the open source cosmological simulation code CUBE [20]. Cosmological parameters are set as  $\Omega_m = 0.27$ ,  $\Omega_\Lambda = 0.73$ ,  $h_0 = 0.68$ ,  $n_s = 0.96$  and  $\sigma_8 = 0.83$ . Initial conditions are generated at redshift  $z = 50$  using Zel’dovich approximation, and using a CLASS transfer function.  $N_p = 512^3$   $N$ -body particles are evolved via their mutual gravitational interactions to  $z = 0$ , in a periodic box with  $L = 300$  Mpc/ $h$  per side. The code is set to use standard a particle-mesh (PM) algorithm [21] on a two-level mesh grids (details see [22]) and cloud-in-cell (CIC) is used in particle interpolations in force calculation and obtaining the density field  $\rho(\mathbf{x})$  in Eulerian coordinates  $\mathbf{x}$  at late stages. We use density contrast  $\delta \equiv \rho/\langle\rho\rangle - 1$  to describe the density fluctuations. The primordial linear density field  $\delta_L$  is given by the initial

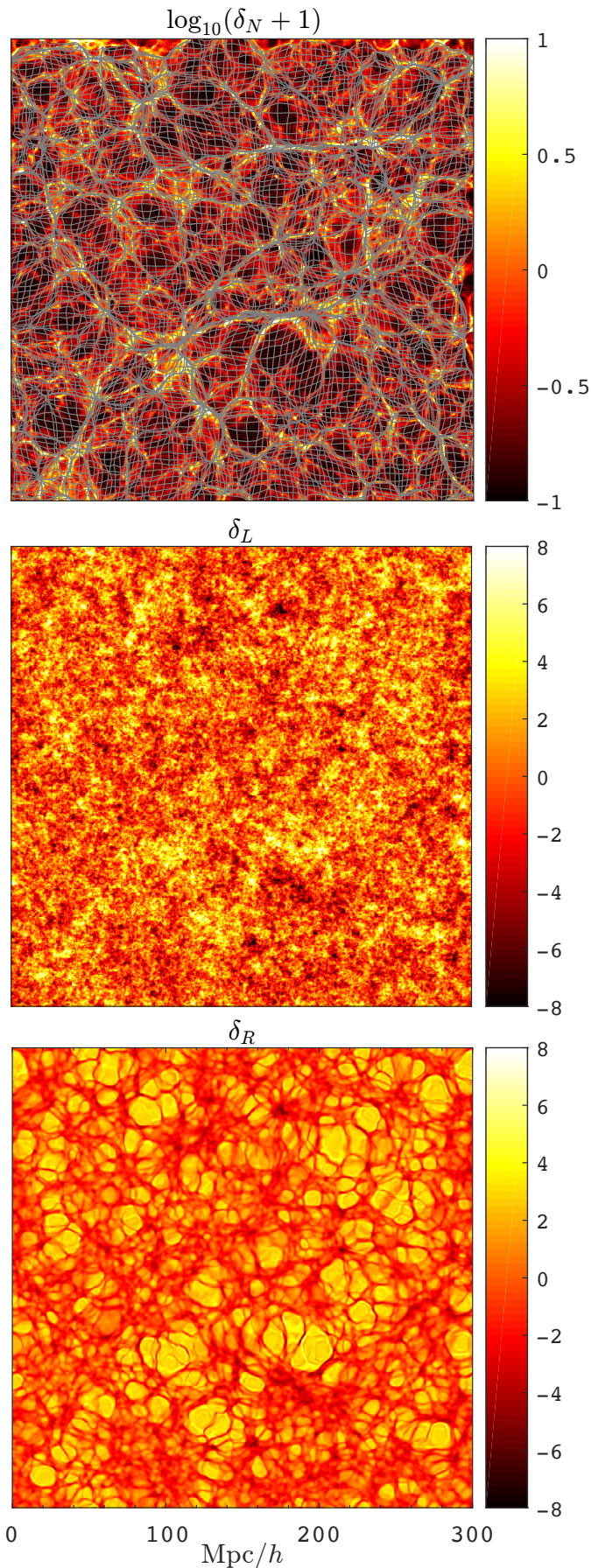
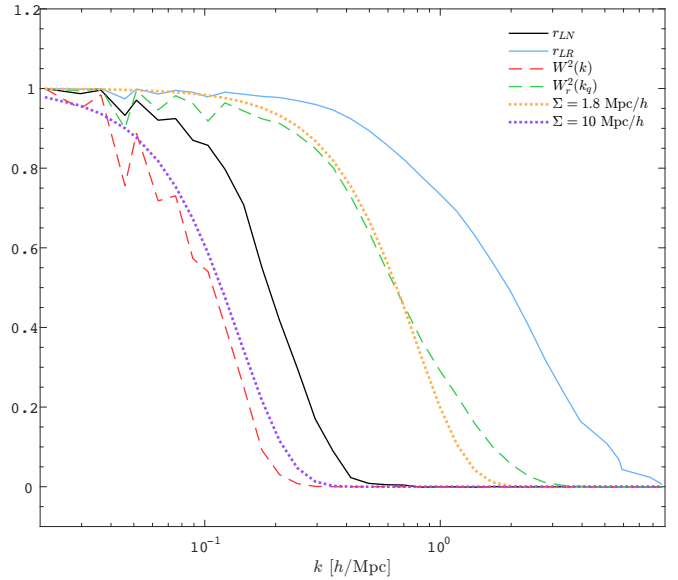


FIG. 1.

FIG. 2. Correlation functions  $r(\delta_L, \delta)$  and  $r(\delta_L, \delta_R)$  (solid lines). Their damping of the power spectra  $W^2$  (dashed lines), and their best fit BAO damping models (dotted lines).

stage and scaled to  $z = 0$  by the linear growth factor.

Two-point statistics of the density fields are quantified by the cross power spectrum  $P_{ij}(k) \equiv (2\pi)^{-3} \langle |\delta_i(k)| |\delta_j(k)| \rangle$ , where subscripts  $i, j$  may refer to linear ( $L$ ), nonlinear ( $N$ ), or reconstructed ( $R$ ) density fields. When  $i = j$  it reduces to the auto power spectrum  $P_{ii}(k)$  or  $P(k)$ . We usually plot the dimensionless power spectrum  $\Delta^2(k) \equiv k^3 P(k) / 2\pi^2$ . The blue solid and black dashed curves in Fig.1 show the power spectra of  $\delta$  and  $\delta_L$ . Their difference shows the nonlinear evolution of LSS on small scales. Their cross power drops to a very low value, indicating a loss of linear information in the nonlinear power spectrum.

## B. Reconstruction

In the simulation, we use particle-ID (PID) to record the initial (Lagrangian) location  $\mathbf{q}$  of particles, and the information is tracked until the  $z = 0$  and we can get the Lagrangian displacement vector  $\Psi \equiv \mathbf{x} - \mathbf{q}$  for every particle. Then these vectors are interpolated onto the initial Lagrangian coordinates  $\mathbf{q}$  of particles and we get the displacement field  $\Psi(\mathbf{q})$ . The raw reconstructed density field is given by the differential motion of matter elements,

$$\delta_R = -\nabla \cdot \Psi(\mathbf{q}). \quad (1)$$

Because the reconstruction processes are implemented on Lagrangian coordinates,  $\delta_R$  takes the coordinates of  $\mathbf{q}$  instead of  $\mathbf{x}$ . We just write  $\mathbf{q}$ 's Fourier wave number  $k_q$  as  $k$  to simplify the expression. In Fig.1 we show the power spectrum of  $\delta_R$  and its cross power with  $\delta_L$ . Despite of a lowered power of  $\delta_R$  compared to  $\delta$ , it has a much higher cross power with  $\delta_L$  compared to  $\delta_N$ , up to a relatively smaller scale (higher  $k$ ).

To quantify the linear information in the reconstructed density

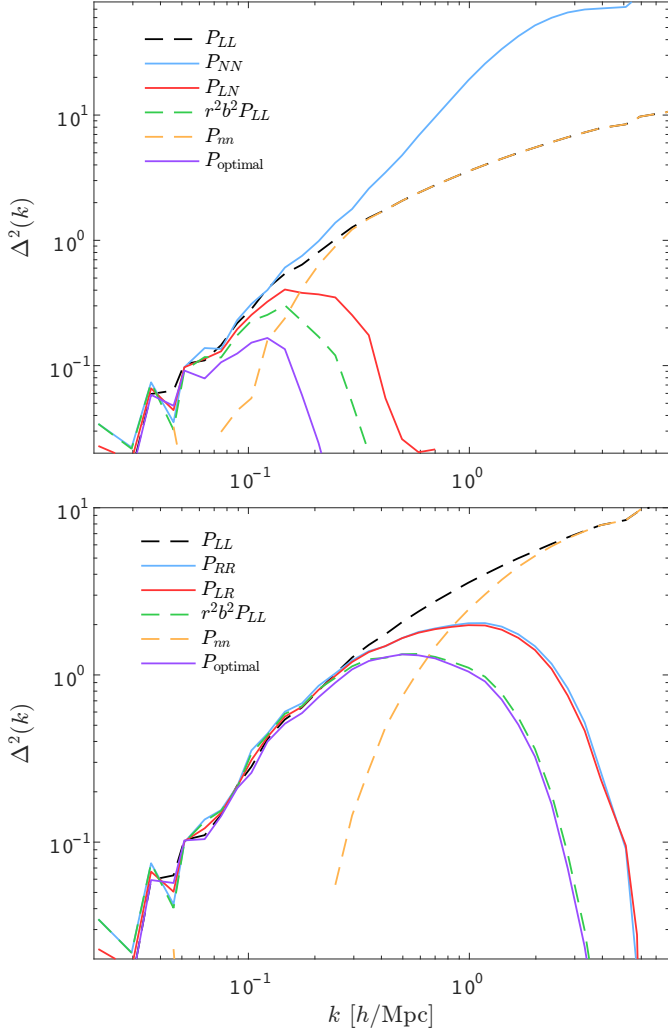


FIG. 3. Power spectra of the nonlinear density field and its filtered density field.

field, we decompose  $\delta_R$  in Fourier space as

$$\delta_R(k) = r'\delta_L + \delta_N, \quad (2)$$

where  $r'\delta_R$  is completely correlated with linear density  $\delta_L$ . Correlating equation (2) with  $\delta_L$  gives

$$P_{LR} = r'P_{LL} + P_{LN}, \quad (3)$$

where  $P_{ij} \equiv \langle \delta_i \delta_j \rangle$  denotes the cross power spectrum. Since  $\delta_N$  is uncorrelated with  $\delta_L$ ,  $P_{LN} = 0$ . With the definition of cross correlation coefficient  $r(\delta_L, \delta_R) \equiv P_{LR} / \sqrt{P_{LL}P_{RR}}$  and bias  $b^2 = P_{RR}/P_{LL}$ , we solve  $r' = P_{LR}/P_{LL} = rb$ . We plot the cross correlation coefficient  $r_{LN}$  and  $r_{LR}$  in Fig.2. Clearly,  $\delta_R$  contains much more linear information on smaller scales.

According to equation (2), the auto power spectrum is decomposed as

$$P_{RR} = r^2b^2P_{LL} + P_{NN}, \quad (4)$$

and  $P_{NN} = (1 - r^2)P_{RR}$ . Then we construct a Wiener filter to filter

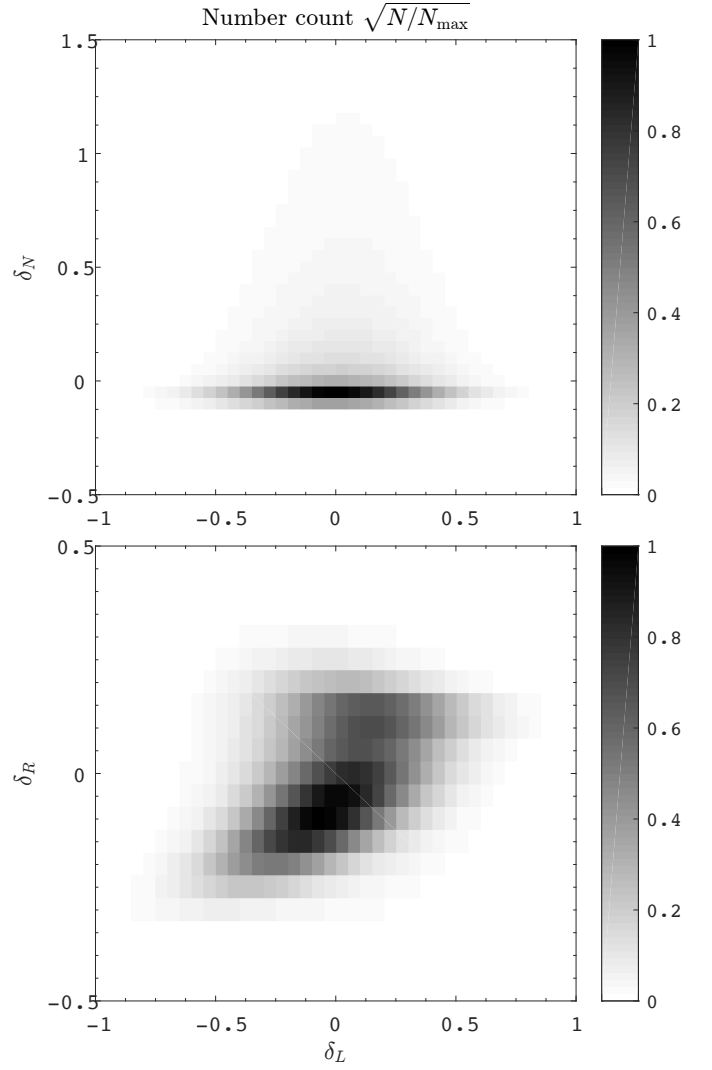


FIG. 4. Power spectra of the reconstructed density field and its filtered density field.

out the uncorrelated part in  $\delta_R$ :

$$W(k) = \frac{r^2b^2P_{LL}}{r^2b^2P_{LL} + P_{NN}} = r^2. \quad (5)$$

The Wiener filters are also plot in Fig.2. The estimated linear density, or the optimal reconstructed density is given by

$$\tilde{\delta}_R = Wb^{-1}\delta_R. \quad (6)$$

The full filters  $Wb^{-1}$  are also plotted in Fig.2. The optimal reconstructed power spectrum is given by

$$\tilde{P} = W^2b^{-2}P_{RR} = W^2P_{LL} + W^2b^{-2}P_{NN}. \quad (7)$$

The  $W^2$  describes the damping of the linear power spectrum.

### III. RESULTS

Fig.2 shows the cross correlation functions, damping factors  $W^2(k)$  for the optimal filtered nonlinear and reconstructed density fields. We can see that the cross correlation  $r_{LR}$  is recovered to nearly an order of magnitude smaller scales compared to  $r_{LN}$ . We also fit the Gaussian BAO damping model  $\mathcal{D}(k) = \exp(-k^2\Sigma^2/2)$  and give  $\Sigma = 1.8 \text{ Mpc}/h$  and  $\Sigma = 10 \text{ Mpc}/h$  for reconstructed and nonlinear fields. We repeat the analysis with various box sizes, and the results are consistent.

In Fig.3 and Fig.4 we show the power spectrum decomposition by equation (4) for both  $\delta_N$  (with subscripts “ $_R$ ” replaced by “ $_N$ ”) and  $\delta_R$ . In Fig.3 we can see that, on scales  $k > 0.2 \text{ h}/\text{Mpc}$ ,  $P_{NN}$  term dominates the power spectrum due to a low correlation between  $\delta_L$  and  $\delta_N$ . The optimal filtered power spectrum damps quickly on small scales. In comparison, in Fig.4,  $P_{NN}$  starts to dominate on scales  $k > 1 \text{ h}/\text{Mpc}$ . As a result, the optimal filtered reconstructed power spectrum (by equation (7)) is much higher.

### IV. DISCUSSION AND CONCLUSION

We extract the actual displacement field of matter elements in cosmological  $N$ -body simulations, and use this displacement field in Lagrangian coordinates to reconstruct the primordial lin-

ear perturbations. The result shows a prominent improvement from  $r_{LN}$  to  $r_{LR}$  in Fig.2 – recovering the lost linear information on nearly an order of magnitude smaller scales. This is achieved by implementing differential movement information of matter elements on Lagrangian coordinates, rather than on Eulerian coordinates. This result illustrates the feasibility of using estimated displacement field  $\tilde{\Psi}(\mathbf{q})$  to reconstruct primordial linear density field. A straightforward example of a estimation of  $\tilde{\Psi}(\mathbf{q})$  is given by [15, 16]. More complicated algorithms can be found in [cite], where shell-crossing effects are also considered. In reality, one needs to consider all aspects including vorticity, shell-crossing, bias, noise and data complexities. The impact of these factors can be quantitatively compared with the impact of different estimation methods, and with the exact solution by  $N$ -body simulations.

[17, 18] [methods of Peebles, david Goldberg? least action ]

Is this reconstruction method optimal? Is there any other better methods?

A proper name of this method?

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