CITS2211: Assignment One 2022

This assignment has 10 questions with a total value of 50 marks. Follow the instructions on LMS for submission. Due 5 pm Friday 26th August 2022.

Ciarān Engelbrecht
23169641

1. Use a *truth table* to prove that following proposition is a contradiction. *Hint: This question specifically asks for a truth table. No marks for answering the question using other methods.*

$$\neg P \vee (\neg Q \wedge P) \leftrightarrow P \wedge Q$$

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P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg Q \wedge P$	$\neg P \vee (\neg Q \wedge P)$	$-P \vee (-Q \wedge P) \Leftrightarrow P \wedge Q$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	F
F	T	T	F	F	F	T	F
F	F	T	T	F	F	T	F

Therefore, since for all cases it is false it is a contradiction

Ciaran Engelbrecht
23169641

2. Use the logical equivalence laws from lectures to show that

$$(\neg Q \vee P) \rightarrow \neg(\neg P \wedge Q)$$

is a tautology. In each step name the equivalence law you are using.

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Prove: $(\neg Q \vee P) \rightarrow \neg(\neg P \wedge Q)$

RHS $= \neg(\neg P \wedge Q)$, using De Morgans law change $\neg(\neg P \wedge Q)$ to $\neg\neg P \vee \neg Q$

using double negation, $\neg\neg P \vee \neg Q$ to $P \vee \neg Q$

LHS $= (\neg Q \vee P)$, using commutativity $\neg Q \vee P$ to $P \vee \neg Q$.

Let $T = P \vee \neg Q$, we have $T \rightarrow T$

$T \rightarrow T$ is a tautology, as they are logically equivalent.

Ciaran Engelbrecht

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3. Let P and Q be the propositions:

P: Swimming at the New Jersey shore is allowed.
Q: Sharks have been spotted near the shore.

Express each of these compound propositions as an English sentence:

(a) $\neg Q$
(b) $\neg P \vee Q$
(c) $\neg P \to \neg Q$
(d) $\neg P \wedge (P \vee \neg Q)$
(e) $P \leftrightarrow \neg Q$

a) Sharks have not been spotted near the shore

b) Swimming at the New Jersey shore is not allowed or sharks have been spotted near the shore.

c) If swiming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

d) Swimming at the New Jersey shore is not allowed and swimming at the Jersey shore is allowed or sharks have not been spotted.

e) swimming at the New Jersey shore is allowed only if sharks have not been spotted by the shore.

Ciaran Engelbrecht
23169641

4. Let P, Q and R be the propositions:

P: Bears have been seen in the area.
Q: Hiking is safe on the trail.
R: Berries are ripe along the trail.

Write these propositions using P, Q and R and logical connectives:

(a) Berries are ripe along the trail, but bears have not been seen in the area.

(b) Bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

(c) It is not safe to hike on the trail, but bears have not been seen in the area and the berries along the trail are ripe.

(d) If berries are ripe along the trail, hiking is safe if and only if bears have not been seen in the area.

(e) Hiking is not safe on the trail whenever bears have been seen in the area and berries are ripe along the trail.

a) $R \wedge \neg P$

b) $\neg P \wedge Q \wedge R$

c) $\neg Q \wedge \neg P \wedge R$

d) $R \rightarrow (Q \leftrightarrow \neg P)$

e) $(P \wedge R) \rightarrow \neg Q$

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5. State whether each of the following propositions is a **tautology** or a **contradiction** or **contingent** (i.e. neither). You must give a (brief) reason to justify each of your answers.

(a) P

(b) $Q \to (P \vee Q)$

(c) $(P \vee \neg P) \to (P \wedge \neg P)$

(d) $((P \to Q) \wedge (Q \to R)) \leftrightarrow (P \to R)$

(e) $(R \vee P) \to (P \vee (Q \vee R))$

a) contingent: as it depends on truth or falsity of P, if P is true proposition is true

b) Tautology: if Q is true, then $(P \vee Q)$ is always true, if Q is false $(P \vee Q)$ is true.

c) contradiction: if P is false $(P \wedge \neg P)$ is false, if P is true $(P \wedge \neg P)$ is still false

d) contingent: if P, Q and R is true, the proposition is true, if P and R is true and Q is false, then the proposition is false.

e) tautology: if R is true, then $(P \vee (Q \vee R))$ is true. if P is true then $(P \vee (Q \vee R))$ is also true.

Ciaran Engelbrecht
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6. Fill in the blanks in the following proof that

$$\exists x.(P(x) \vee (R(x) \wedge Q(x))) \wedge \forall x.(\neg P(x) \wedge R(x)) \rightarrow \exists x.Q(x)$$

1. $\exists x.(P(x) \vee (R(x) \wedge Q(x)))$	premise
2. $\forall x.(\neg P(x) \wedge R(x))$	premise
3. $P(a) \vee (R(a) \wedge Q(a))$	1, exist elimination
4. $\neg P(a) \wedge R(a)$	2, forall elimination
5. $\neg P(a)$	4, conjuction elimination
6. $\neg P(a) \rightarrow (R(a) \wedge Q(a))$	3, implication law
7. $R(a) \wedge Q(a)$	5,6 modus ponens
8. $Q(a)$	7, conjunction elimination
9. $\exists x.Q(x)$	8, exist introduction

Ciaran Engelbrecht
23169641

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7. Consider the following (incomplete) proof that $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$, for all $n \geq 1$ — i.e., that

$$2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 2$$

PROOF

Let $P(n)$ be the predicate: $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$. We will prove that this equality holds for all $n \geq 1$.

Base case

When $n = 1$, the left hand side in the equality is $\sum_{i=1}^{n} 2^i = 2^1 = 2$.

And the right-hand side is

(a)

RHS $= 2^{n+1} - 2$, for $n=1$, RHS $= 2^2 - 2 = 2$

LHS $= 2$ & RHS $= 2$

$\therefore$ RHS $=$ LHS

which is equal to the left-hand side.

Therefore, P(n) holds true (b) for the base case.

Inductive step: prove that P(n) is true for all $n \geq 1$ (c)

Assume that P(K) is true for all $k \geq 1$ (d).

We will show that the equality $P(k+1)$ holds, given this assumption.

Our predicates are:

$$P(k): \quad \sum_{i=1}^{k} 2^i = 2^1 + 2^2 + \ldots + 2^k \qquad = 2^{k+1} - 2$$

$$P(k+1): \quad \sum_{i=1}^{k+1} 2^i = 2^1 + 2^2 + \ldots + 2^k + 2^{k+1} \quad = 2^{(k+1)+1} - 2 = 2^{k+2} - 2$$

Therefore, the left-hand side of $P(k+1)$ is

$$\begin{aligned}
&(2^1 + 2^2 + \ldots + 2^k) \ + 2^{k+1} \\
\equiv &(2^{k+1} - 2) \quad + 2^{k+1} \\
\equiv &2^{k+1} - 2 + 2^{k+1} \\
\equiv &2(2^{k+1}) - 2 \\
\equiv &2^{k+2} - 2
\end{aligned}$$

which equals the right-hand side of $P(k+1)$.

Therefore, we've proved that $P(k+1)$ holds, given the assumption that **(d)**. Since we've also proved $P(1)$ holds, then by the principle of induction, $P(n)$ holds for all $n \geq 1$. □

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8. Prove by induction that for all positive natural numbers n, we have $2^n > n$.

Let $P(n)$ be the proposition, $2^n > n$ for all positive natural numbers.

For $P(1)$:
for the base case $n=1$, $2^1 > 1$,
since $2 > 1$

Inductive assumption is that $2^n > n$ where $n = k$ and $k \geq 1$.

$2^{k+1} = 2 \cdot 2^k > 2 \cdot k > (k+1)$ since $2k > k+1$
for all natural numbers k

$\therefore 2^{k+1} > (k+1)$

$\therefore$ by way of induction we have $2^n > n$ for all positive natural numbers, n.

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9. Express the following colloquial English statements using predicate logic, where the domain of discourse is all people.
Hint: First identify the predicates in each statement.
Use the constants $a =$ "Anna" and $b =$ "Ben".

(a) Somebody is Ben's neighbour.

(b) Anna is not anyone's neighbour.

(c) Some people are neighbours.

a) $\exists x.\ N(x,b)$
where $N(x,y)$ means
x is a neighbour of y.

b) $\neg\exists x.\ N(a,x)$
where $N(x,y)$ is as above

c) $\exists x.\ \exists y\ N(x,y)$

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10. Translate each of the English sentences (a)-(d) below into predicate logic formulas, with all *athletes* as the domain of discourse.

The only **predicates** you may use are

- $O(x)$ meaning that "x is an Olympic athlete", and
- $H(x)$ meaning that "x is training hard", and
- $M(x, y)$ meaning that "x has won more Olympic medals than y", and
- equality and inequality. (i.e. $=$ and $\neq$)

However, you may define whatever variables and constants you wish, and you are also allowed to use all the symbols from the alphabet of predicate logic, i.e. all connectives, quantifiers, brackets etc.

If you need to make any assumptions, state what they are.

(a) Some hard training athletes are not Olympic athletes.

(b) All Olympic athletes train hard.

(c) Some athletes have won more Olympic medals than everyone else.

(d) Michael Phelps is the only athlete, who has won more Olympic medals than Larisa Latynina.

a) $\exists x . H(x) \wedge \neg O(x)$

b) $\forall x . O(x) \rightarrow H(x)$

c) $\exists x . \forall y . M(x, y)$

d) P = Michael Phelps

L = Larisa Latynina

$\forall x . (M(x, L) \rightarrow M(P, L))$

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