CITS2211: ASSIGNMENT TWO 2022

This assignment has 10 questions with a total value of 50 marks. Follow the instructions on LMS for submission. Due 5 pm Friday 30th September 2022.

Important:

- All work is to be done individually.
- You do not need to write on this question sheet but do put your name and student number on every page of your writing and do number your pages clearly in order. Your assignment will not be marked if you don't follow these instructions.

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1. Prove that if n is an integer and $n^3 + 5$ is odd, then n is even using

   (a) a proof by contrapositive (an indirect proof)

   (b) a proof by contradiction

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a) suppose n is not even

then $n = 2k - 1$

so, $n^3 = (2k-1)^3$

$\rightarrow n^3 + 5 = (2k-1)^3 + 5$

$\rightarrow n^3 + 5 = 8k^3 - 12k^2 + 6k + 4$

$\rightarrow n^3 + 5 = 2(4k^3 - 6k^2 + 3k + 2)$

$\therefore n^3 + 5$ is even

so, if $n^3 + 5$ is even, n is odd

b) given $n^3 + 5$ is odd, suppose n is not even

if $n = 2k - 1$

$\rightarrow n^3 = (2k-1)^3$, $n^3 + 5 = (2k-1)^3 + 5$

$\rightarrow n^3 + 5 = 2(4k^3 - 6k^2 + 3k + 2)$

$\therefore n^3 + 5$ is even,

which contradicts the fact $n^3 + 5$ is odd,

hence the assumption of n not being even is wrong, n is even.

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2. For each of these arguments determine whether the argument is correct or incorrect and explain why.

   (a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

   (b) Every computer science major takes discrete math- ematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

   (c) All parrots like fruit. My pet bird is not a parrot. There- fore, my pet bird does not like fruit.

   (d) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

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a) correct - by modus ponens and universal instantiation.

b) incorrect - by fallacy of affirming conclusion

c) incorrect - by fallacy of denying hypothesis

d) correct - by universal instantiation and modus tollens

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3. For each of the Venn diagrams below, write down a set theory expression which equals the shaded area of the diagram.

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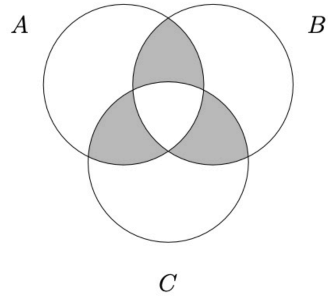
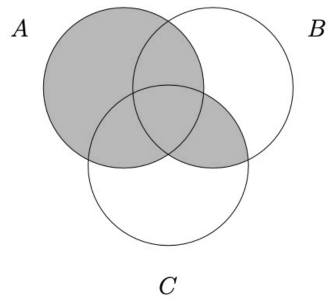
(i)



(ii)



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i) $((A \cap B) \cup (A \cap C) \cup (B \cap C)) - (A \cap B \cap C)$

ii) $(A \cup (B \cap C))$

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4. Consider two relations R, S on a set X (so that $R, S \subseteq X \times X$). Prove or disprove (by giving a counter example) the following statements:
   (a) If R and S are transitive, then so is $R \cup S$.
   (b) If R and S are transitive, then so is $R \cap S$.

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a) Let $X = \{1,2,3\}$
Let relation R on set X
$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$
Let relation S on set X
$S = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$
so, $R \cup S$ $\{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$
$\therefore (1,2) \in R$ and $(2,3) \in S$
since $(1,2) \in R \cup S$ and $(2,3) \in R \cup S$
however, $(1,3) \notin R \cup S$, $\therefore$ not transitive relation by counter example.

b) Let $X = \{a,b,c\}$
Let $(a,b),(b,c) \in R \cap S \Rightarrow (a,b),(b,c) \in R$
and $(a,b),(b,c) \in S \Rightarrow (a,c) \in R$ and $(a,c) \in S$
$\therefore (a,c) \in R \cap S$, hence $R \cap S$ is transitive.

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5. Consider the two sets R and S, where

$$R = \{\ \varnothing,\ \{\varnothing, 2, \{5, 7\}\}\ \}$$

and

$$S = \{\ \{\varnothing, 2, \{5, 7\}\},\ \varnothing,\ \{1, 2, 5, 7\}\ \}$$

Answer the following questions about them, giving a brief explanation.

(i) Which of the sets contain the empty set as an element?

(ii) Which of the sets have the empty set as a subset?

(iii) Which set has the greater number of members?

(iv) Is S a subset of itself?

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i) in Both R and S sets, $\varnothing$ is an element and an empty set.
In set R, two elements in which $\varnothing$ is an element.
In set S, three elements in which $\varnothing$ is an element.

ii) In set R, $\varnothing$ is an element, and subset of R.
In set S, $\varnothing$ is an element, and subset of S.
$\therefore$ Both sets R and S have the empty set $\varnothing$ as a subset.

iii) In set R, there are two element members however in set S, there are three element members, $\therefore$ S has greater number of members.

iv) Yes, since every set is a subset of itself.

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6. Consider the function $f(n) = n^2 + 1$, which has the integers, $\mathbb{Z}$, as its domain and codomain.

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(a) Which of the properties of being injective, surjective, or bijective does the function f have? Give a brief explanation of why.

(b) Consider the relation F (on $\mathbb{Z}$), where $F = \{(2, 5), (4, 17), (5, 26)\}$. Is F a subset of f? Explain why or why not.

(c) What would you need to add to F to obtain F', the transitive closure of F? Explain why.

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a) if $n=1$ and $n=-1$, $f(n)=n^2+1$
$\rightarrow f(1)=1^2+1=2$ $\rightarrow f(-1)=-1^2+1=2$
we obtain the same value, hence not injective
if $n^2+1=0 \rightarrow n^2=-1, n=i \notin \mathbb{Z}$, hence this function is not surjective.
since this function is neither injective nor surgective, it is not bijective

b) given $F=\{(2,5),(4,17),(5,26)\}$ and $f(n)=n^2+1$
for $n=2$, $f(2)=2^2+1=5$
for $n=4$, $f(4)=4^2+1=17$
for $n=5$, $f(5)=5^2+1=26$,
therefore F is a subset of f.

c)
$$\begin{tikzcd} 2 \arrow[r] & 5 \arrow[d] & 4 \arrow[d] \\ & 26 & 17 \end{tikzcd}$$
$\therefore F'=\{(2,2),(2,5),(2,26),(4,4),(4,17),(5,26)\}$

F is not transitive, as it contains $(2,5)$ and $(5,26)$ but not $(2,26)$, we could add $(2,26)$.

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7. Suppose that $t_0 = 5$ and for each $n \geq 0$, $t_{n+1} = 2t_n + 1$. Use induction to show that $t_n \geq 20$ for all natural numbers $n \geq 2$.

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1. check $n=2$

$t_1 = 2t_0 + 1$

$t_1 = 2(5)+1 = 11$

$t_2 = 2t_1 + 1 = 2(11)+1 = 23$

$23 \geqslant 20 \therefore t_2 \geqslant 20$, so true for $n=2$

2. for $n=k$, $t_k \geqslant 20$

3. for $n = k+1$

$t_{k+1} = 2t_k + 1$ since $t_k \geqslant 20$

$\geqslant 20$ $\rightarrow 2t_k \geqslant 20$

$\therefore$ $\rightarrow 2t_k + 1 \geqslant 20$

$t_{k+1} \geqslant 20$

$\therefore$ true for $n = k+1$

and hence it is true for all $n \geqslant 2$

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8. Use induction to show that $n^2 + 3n - 5$ is greater than $4n$ for all natural numbers $n > 2$.

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for all $n>2$, $(n^2+3n-5) > 4n$

let $n=3$ for $(n^2+3n-5) > 4n$

LHS $= 3^2+3(3)-5 = 13$

RHS $= 4(3) = 12$

LHS > RHS, thus true

let $n=4$ for $(n^2+3n-5) > 4n$

LHS $= 4^2+3(4)-5 = 23$

RHS $= 4(4) = 16$

LHS > RHS, thus claim is true

assume for claim(k) true for $k \geq 2$

that is $(k^2+3k-5) > 4k$ → prove k+1 is true

→ $((k+1)^2+3(k+1)-5) > 4(k+1)$ LHS $= k^2+5k-1$

RHS $= 4k+4$

$\therefore ((k+1)^2+3(k+1)-5) > 4(k+1)$ LHS > RHS

and k+1 is true, and by principle of mathematical induction claim $n^2+3n-5 > 4n$ for all $n>2$.

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9. For each of the following, state whether it is possible to have a function meeting the criteria given, explain why or why not, and if it is possible, give an example.

(a) A function $f : \mathbb{Z} \to \mathbb{N}_{\geq 0}$ which is not surjective.

(b) A function $f : \mathbb{Z} \to \mathbb{N}_{\geq 0}$ which is injective.

(c) A function $f : \mathbb{N}_{\geq 0} \to \mathbb{Q}$ which is bijective.

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a) $f : \mathbb{Z} \to \mathbb{N}_{\geq 0}$ /

n	$f(n$
-1	1
0	0
1	1
2	2

consider

$f(n) = |n|$

not surjective and not one-to-one $\therefore$ possible.

b) $f : \mathbb{Z} \to \mathbb{N}_{\geq 0}$ /, consider $f(n) = \begin{cases} -2n & , n \leq 0 \\ 2n-1 & , n > 0 \end{cases}$

n	$f(n)$
-2	4
-1	2
0	0
1	1
2	3

negative numbers of n map to distinct even numbers and positive numbers of n map to distinct odd numbers.

$\therefore$ injective and possible

c) $f : \mathbb{N}_{\geq 0} \to \mathbb{Q}$

Cantor's diagonal argument proves this is possible.

for $1 \to \frac{1}{1}$, $2 \to \frac{2}{1}$, $3 \to \frac{1}{2}$, $4 \to \frac{3}{1}$, $5 \to \frac{1}{3}$...

which is bijective.

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10. For each of the following functions, state whether it is injective, surjective, and/or bijective, and why.

(a) The function $f(n) = n + 1$, mapping from integers to integers.

(b) The function $q(\phi)$, with codomain $\mathbb{N}_{\geq 0}$, which maps any formula of predicate logic to the number of symbols in that formula.

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$f(n)=n+1$ from $\mathbb{Z} \to \mathbb{Z}$

a) let $f(n_1) = f(n_2)$

$\to$ $n_1+1 = n_2+1$ $\Rightarrow$ $n_1 = n_2$

$\therefore$ f is injective

For every $n \in \mathbb{Z}$, $(n-1) \in \mathbb{Z}$

$\to$ $f(n-1) = (n-1)+1 = n$

hence f is surjective

$\therefore$ since f is injective and surjective it is bijective.

b) consider $q(\phi_1) = q(\phi_2)$, this may not be truly equal since two ϕ which may have the same number of symbols :

$\exists x \exists y P(x,y)$ and $\forall x \forall y P(x,y)$

$\therefore$ it is not injective.

For all natural numbers, $n \in \mathbb{N}_{\geq 0}$ there is alway predicate logic with n number of symbols $\therefore$ surjective.

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