CITS2211: Assignment Three 2022

This assignment has 9 questions with a total value of 50 marks. Follow the instructions on LMS for submission. Due 5pm Friday 21 October 2022.

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1. Prove, using induction, that for any odd integer $n > 1$, the sum of all positive odd integers less than n is less than the sum of all positive even integers less than n.

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let n = odd integer > 1

let $P(n)$ = the sum of all positive odd integers less than n is less than the sum of all positive even integers less than n: $P(n): 1+3+5+..+(n-2) < 2+4+6+..+(n-1)$

let $n=3$, so LHS = 1, RHS = 2

$1 < 2$ $\therefore$ LHS < RHS, hence $P(3)$ is true.

Assuming $P(k)$ is true,

where k is an odd integer greater than 1.

so, $P(k): 1+3+5+...(k-2) < 2+4+6+...(k-1)$

LHS < RHS.

let $n = k+2$, LHS = sum of positive odd integers $< k+2$

$= 1+3+5+..+k = 1+3+5+..+(k-2)+k < 2+4+6+..(k-1)+k$ from above

$= 2+4+6+..+k$, and clearly $k < k+1$ so we can insert this:

LHS $= 1+3+5+..+k < 2+4+6+..+k < 2+4+6+..+(k+1)$

which is equal to the sum of all positive even integers less than $k+2$, hence $P(k+2)$ is true for all $P(k)$

Therefore, by principle of mathematical induction, $P(n)$ is true for all positive odd integers, n.

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2. Let X be the set $\{a, b, c, d, e\}$. Give answers to each of the following questions, justifying your answer in each case. 2

(a) How many functions are there which map from X to X?

(b) How many distinct total orders can be defined on X?

(c) For each function f in the set of functions from X to X, consider the relation that is the symmetric closure of the function f. Let us call the set of these symmetric closures Y. List at least two elements of Y.

(d) Suppose R is some partial order on X. What is the smallest possible cardinality R could have? What is the largest?

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a) every element of X has one value $f(x)$ under f.
for every $x \in X$, there are 5 possibilities to choose $f(x)$
hence there are 5^5 functions $= 3125$, $f: X \to X$

b) Since every element can only be mapped onto the same value once, total order can be represented as $(a \leq b \leq c \leq d \leq e)$ for each permutation of X, which results in a total order of $5!$, or 120.

c) Symmetric closure is obtained by adding (b,a) to R for each $(a,b) \in R$
$Y = X \cup X^{-1}$, and some elements of Y include: (a,b), (b,a), (c,d)

d) If we consider the partial order R as $R \subseteq X$, the least cardinality of partial order R is 5.
$\{(a,a), (b,b), (c,c) (d,d) (e,e)\}$
the greatest cardinality is 15
$\{(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (a,c), (a,d), (a,e), (b,c), (b,d), (b,e), (c,d), (c,e), (d,e)\}$

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3. State which of the following are equivalence relations and justify your answer carefully. 3

(a) The relation between each person and their current age in years: eg $(Anna, 20)$.

(b) The relation between any two sets of people, which says that the two sets differ by at most one person, i.e. they are the same set or there is just one person who is in one set but not in the other.

(c) The relation between any two sets of natural numbers, which says that the two sets differ by only a finite number of numbers.

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a) This is not an equivalence relation, since a name of a person does not relate to their own name, eg. (Anna, Anna) $\notin$ this relation, and hence it is not reflexive.

b) This is not an equivalence relation, since, if there are 3 sets of people, A, B, C, A has 2 people, B has 3 people and C has 4 people, these sets differ by 1 person, $\therefore$ A is related to B, and B relates to C, however A and C differ by 2, and are not related, hence it is not transitive.

c) This <u>is</u> an equivalence relation, since, by reflexive property, every set is related to itself, eg. A is a set and differs by 0, which is finite number.
By symmetric property, two sets A and B, A relates to B by a finite number, and B relates to A by a finite number.
By transitive property, three sets A, B, C, A relates to B by a finite number, B relates to C by a finite number, and A relates to C as they differ by a finite number.

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4. For each of the following languages over the alphabet $\Sigma = \{a, b, c\}$ specified by the regular expressions (i)–(iii), provide two strings in Σ^* that are members and two strings in Σ^* that are not members of the language (four strings each).

(a) $ab + a$

(b) $((bc)^* + b)a$

(c) $(a + ab + abc)^*(b + c)$

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a) i) members: aba, abba
not members: b, abc

ii) members: bcba, ba
not members: bb, aaa

iii) members: bc, abcbc
not members: a, bbb

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5. Assume the alphabet $\Sigma = \{0,1\}$. Give three different regular expressions (besides the one given) that specify the language described by this regular expression:

$$((11)^*1^*)^* + (11+1)^* + (0+\epsilon)^*$$

In each case, explain **why** your regular expression specifies the same language.

Note: For the purposes of this exercise only, changing the order of a union does not count as a different regular expression. Your examples also should not be more complicated than the original regular expression.

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Considering the rules of regular expressions:

$(r^* s^*)^* = (r+s)^*$ and $(r+s)^* = (r^* s)^* r^*$

let $r = 1$ and $s = 1$

so a) $((11)^* 1^*)^* \rightarrow (11+1)^*$

and b) $(11+1)^* \rightarrow (11^* 1)^* (11)^*$

given: $((11)^* 1^*)^* + (11+1)^* + (0+\epsilon)^*$

$((11)^* 1^*)^*$ reprasents any no. of 1s including 0 1s

$(11+1)^*$ reprasents any no. of 1s excluding 0. 1's

$(0+\epsilon)^*$ reprasents any no. of 0s including 0 0's

Hence, three alternate regular expressions are:

1. $(11+1)^* + (11+1)^* + (0+\epsilon)^*$ from a

2. $(11+1)^* + (11+1)^* + 0^*$ since $(0+\epsilon)^*$ generates the same set of strings as $(0)^*$

3. $(11+1)^* + 0^*$ due to the property $r^* + r^* = r^*$, where $r = (11+1)$

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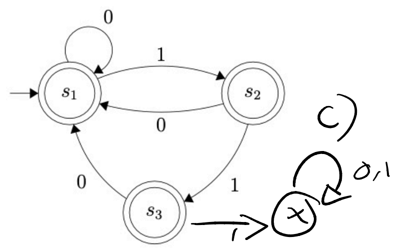
6. Given the following deterministic FSM M over the alphabet $\Sigma = \{0, 1\}$:



(a) Deterministic FSMs are comprised of a 5-tuple. For the deterministic FSM depicted by the above diagram, describe each of the items in the relevant tuple.

(b) Give an English language description of $L(M)$, the language recognised by M.

(c) Add an error state (labelled X) to the diagram, and draw all transitions to it.

(d) Describe how to derive an FSM that accepts the complement of $L(M)$ over the set Σ^*. (That is, an FSM that accepts the language $\Sigma^* - L(M)$.)

(e) Give a regular expression for the complement of $L(M)$.

a) 5 tuple is : $(\Sigma, Q, \delta, q, F)$

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so, $\Sigma = \{0,1\}$

$Q = \{s_1, s_2, s_3\}$

q = the starting state

$f = \{s_1, s_3\}$

$\delta =$

δ	0	1
s_1	s_1	s_2
s_2	s_1	s_3
s_3	s_1	ϕ

b) Set of all binary strings which does not include 111 as a substring

d) In order to derive the complement of $L(M)$, the states must be switched from final to non-final, and non-final to final states in M, resulting in M' which accepts the complement of $L(M)$.

e) $(0+1)^* \, 111 \, (0+1)^*$

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7. Consider the language of all binary strings that end with a different symbol than they start with. 7

(a) Write the production rules of a grammar for this language. Show some derivations to test that your grammar is correct.

(b) Design a pushdown automata (PDA) to recognise this language. Draw a diagrammatic state machine model showing all the moves of your PDA.

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a)

Production rules:

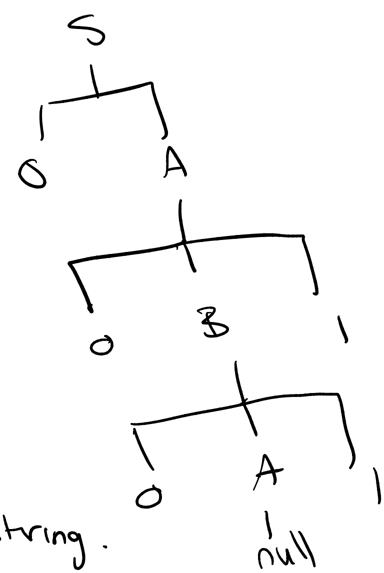
S, 0A, 0B1, 00B11, 0A1, 00A11...

S → 0A

A → 0B1

B → 0A1

, this is a sequence of 0s and 1s, starting with 0, ending in 1.



b)

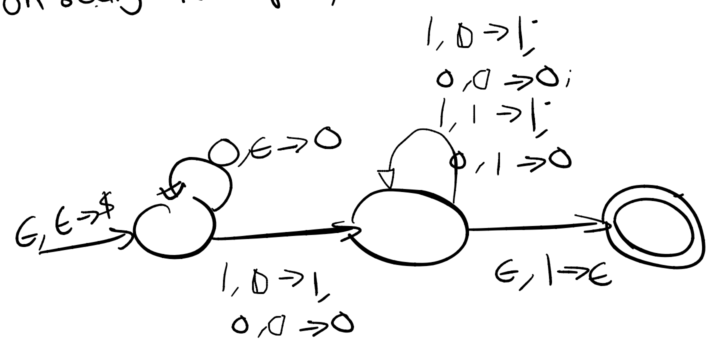
ε, ε → $, given stack is empty push $,

given input is 0, push 0 onto stack.

given input is either 1 or 0, push onto stack.

given input is 1, push 1 onto stack.

on seeing no input, and 1 on top, accept string.



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8. Give a context free grammar that generates the language $\{a\#b \mid a^R$ (the reverse of a) is a substring of b for $a, b \in \{0,1\}^*\}$. For example, the string 0110#001101 is in this language.

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using the example string, this language follows the rule that a string begins with a, followed by #, followed by something, followed by a^R (the reverse of a) followed by something.

Hence; $a\#(0\cup1)^* a^R (0\cup1)^*$

let $A = a\#(0\cup1)^* a^R$ and $B = (0\cup1)^*$

then $S \Rightarrow AB$

$A \rightarrow 0A0 \mid 1A1 \mid \#B$

$B \rightarrow 0B \mid 1B \mid \epsilon$

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9. Take the language $\{x^n 0 y^{4-|x|} | x, y$ are strings over the alphabet $\Sigma = \{1\}, 0 \le n \le 4\}$. Can this language be recognised by a Finite State Machine or only a Push Down Automata? Explain why.

If it can only be recognised by a PDA, draw one that recognises it. Otherwise, draw a FSM that recognises it.

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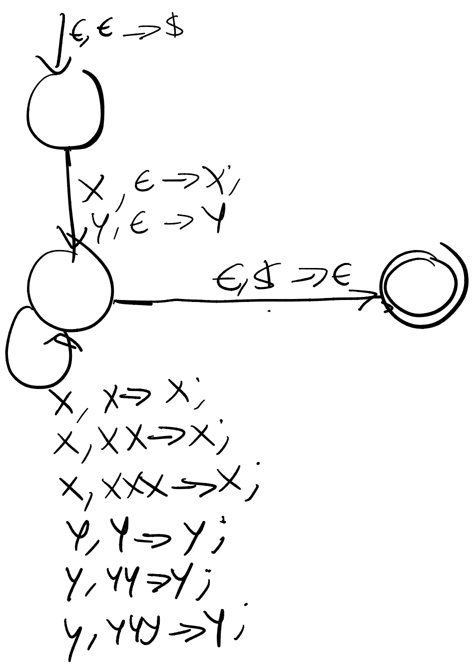
This language can only be recognised by a PDA, not a FSM, as it's not regular.
This language contains strings of different lengths, requiring infinite states to track all the strings.
Wheras a PDA can keep track of the number of 0s and 1s using the stack, recognising all strings regardless.



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