University of North Carolina at Charlotte
Department of Electrical and Computer Engineering

Laboratory Report 3

Network Analysis
Thevenin and Norton Equivalent Circuits
Time Constant of a RC Circuit

Laboratory Experiment Reports 8,9,10
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1 Objectives

1.1 Network Analysis

The objective is to analyze a circuit and measure the real values to validate the calculated values.

1.2 Thevenin and Norton Equivalent Circuits

The objective of the lab is to utilize the Thevenin and Norton methods to calculate the current and voltage across any one of several resistors in a circuit and verify the calculated values by measuring the values in the circuits.

1.3 Time Constant of a RC Circuit

The objective is to measure the time constant of an RC circuit in order to verify the calculated values.

2 Equipment List

2.1 Network Analysis

- Digital Multimeter
- DC Power Supply
- Resistors: 470Ω, $1K\Omega$ (2), $5.1k\Omega$, $10k\Omega$

2.2 Thevenin and Norton Equivalent Circuits

- Digital Multimeter
- DC Power Supply
- Resistors: $1.2k\Omega$, $3.3k\Omega$, $10k\Omega$

2.3 Time Constant of a RC Circuit

- Digital Multimeter
- DC Power Supply
- Resistor: $20k\Omega$
- Capacitor: $2,200\mu$F
- Alligator (Clips) Jumper

3 Relevant Theory/Background Information

3.1 Network Analysis

In terms of relevant theory there is a wide variety of methods to calculate the current, voltage, and resistance across a circuit or individual elements. The primary base for all of these methods are Kirchoff's Voltage and Current laws which show the sum of the currents or voltages across a circuit will equal zero ergo:

$$0 = \sum_{i=1}^{n} V_i$$

and:

$$0 = \sum_{i=1}^{n} I_i$$

With the understanding of Kirchoff's Laws the methods of mesh current analysis, node voltage analysis, and superposition. When analysing using mesh current analysis the first step is to break a circuit into meshes which are smaller pieces of a circuit and are functionally the most basic circuit possible. Then apply Kirchoff's Voltage Law where the voltage across each resistor will be accounted for with Ohm's Law, V = IR, with current in resistors which are located in two loops the currents in each loop must be subtracted. Finally simply solve for each unknown element in each equation. When analysing using Node Voltage Analysis the first step is to selecting a node as reference or ground. Then use Kirchoff's Current Law for each node and repeat the process described previously. When utilizing Superposition the first step is to replace all but one current or voltage source with either an open circuit or short circuit respectively. Then apply whatever analysis functions best and repeat the process until all sources have been used. Finally sum all of the current and voltage values to solve for each respectively as seen here:

$$V = \sum_{i=1}^{n} V_i$$

and here:

$$I = \sum_{i=1}^{n} I_i$$

3.2 Thevenin and Norton Equivalent Circuits

When doing circuit analysis there are occasions when the resistance, voltage, and current at specific terminals and there are two main theorems to conduct these calculations, Thevenin and Norton's theorem. Each theorem abides by Ohm's Law but it is focused on the the premise that $R_{Thevenin}$ is equal to R_{Norton} and in the the Ohm's Law equation for these theorems is $V_{oc} = I_{sc} * R_{thevenin}$ or R_{norton}. V_{oc} is equal to $V_{thevenin}$ and I_{sc} is equal to I_{norton}. The objective of both theorems is to simplify the circuit to an equivalent resistance , $R_{Thevenin}$ or R_{Norton}, and a source, $V_{thevenin}$ with $R_{Thevenin}$ in series or I_{norton} with R_{Norton} in parallel, with a load resistor attached at the terminals. To calculate the previously mentioned equivalent resistance the load resistor must be removed and "turn off" all the sources in the circuit if all are independent sources and then calculate the resistance. To find $V_{thevenin}$ simply calculate voltage across the resistor parallel to the open circuit using any method preferable to find the open circuit voltage. If given the short circuit current and the resistance $V_{thevenin}$ is simply the multiplication of these two values. In the case of Norton equivalent cirucits repeat the process to calculate the R_{Norton} then solve for I_{sc} by placing a short circuit across the terminals and calculate the current flowing through the short or solve for V_{oc} and solve for the current using R_{Norton}.

3.3 Time Constant of a RC Circuit

The voltage and current change in a capacitor is a natural exponential relationship in the case of charging for voltage the value is found by the expression:

$$V_c = V_S(1 - e^(- t/\tau))$$

wherein τ is equal to equivalent resistance times equivalent capacitence and t is the time passed since the switch which determines when voltage is passing through a capacitor. Once a steady state is reached at a factor of τ the change of voltage is no longer significant and that means the capacitor is holding its max charge which is calculated by Q = CV.

4 Experimental Data/Analysis

4.1 Network Analysis

Table 8-1: Resistors Values

Resistance	Measured ($K\Omega$)	Color Code ($K\Omega$)	Error (%)
R_1	$5.047k\Omega$	$5.1k\Omega$	1.0392 %
R_2	467.5Ω	470Ω	.5319%
R_3	$9.935k\Omega$	$10k\Omega$	.65%
R_4	$.999k\Omega$	$1k\Omega$	.1%
R_5	$.997k\Omega$	$1k\Omega$	.3%

Table 8-2: Mesh Currents

Current	Measured (mA)	Calculated (mA)	Error (%)
I_A	.806 mA	.813 mA	.861%
I_B	.043 mA	.0404 mA	5.581%
I_C	.910 mA	.913 mA	.329%

Table 8-3: Resistors Voltages

	Measured	Mesh Method	Nodal Analysis	Superposition	Simulation
V_{R1}	4.14 V	4.1463 V	4.43368 V	4.1466 V	4.147 V
V_{R2}	.021 V	.018988 V	.018 V	.01897 V	.01897 V
V_{R3}	9.123 V	9.13 V	9.128 V	9.158 V	9.128 V
V_{R4} .852 V		.854 V	.853 V	.853 V	.8534 V
V_{R5}	.873 V	.8721 V	.872 V	.873 V	.8724 V

Table 8-4: Resistors Current

	Measured	Mesh Method	Nodal Analysis	Superposition	Simulation
I_{R1}	.803 mA	.813 mA	.8131 mA	.81305 mA	.8131 mA
I_{R2}	.043 mA	.0406 mA	.0404 mA	.04037 mA	.04034 mA
I_{R3}	.911 mA	.913 mA	.9128 mA	.91276 mA	.9128 mA
I_{R4}	.825 mA	.8536 mA	.853 mA	.853 mA	.8534 mA
I_{R5}	.834 mA	.8271 mA	.872 mA	.873 mA	.8724 mA

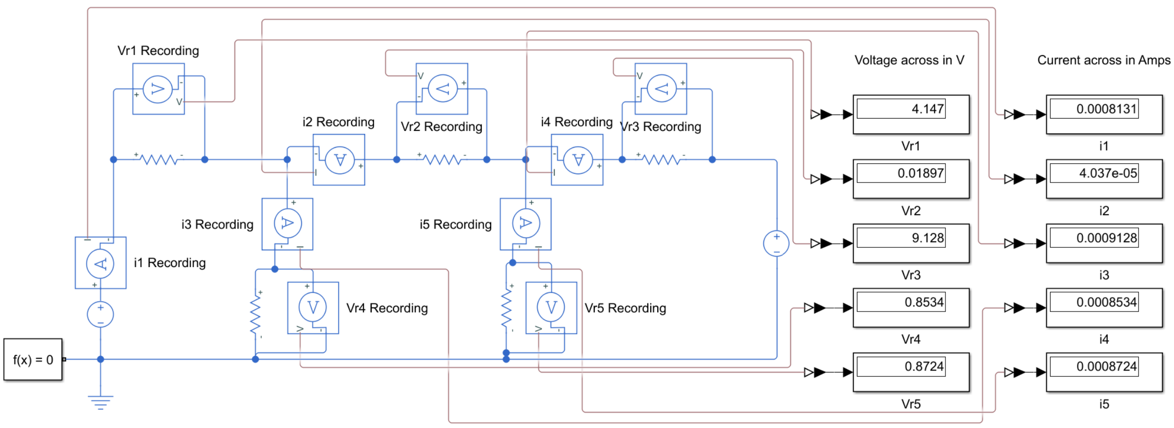

Figure 8-2: Simulated model of the circuit in Figure 8-1

4.2 Thevenin and Norton Equivalent Circuits

Table 9-1: Calculated Voltage and Current for Resistor R3

	Thevenin Equivalent	Norton Equivalent
I_{R3}	4.9 mA	4.9 mA
V_{R3}	16.17 V	16.17 V

Table 9-2: Measured Thevenin and Norton Equivalents

Thevenin Equivalent		Norton Equivalent	
v_{TH}	21.43 V	i_N	18.3 mA
R_{TH}	$1.056k\Omega$	R_N	$1.056k\Omega$

Table 9-3: Measured Voltage and Current for Resistor R3

	Figure 9-3	Thevenin Equivalent	Norton Equivalent
I_{R3}	4.961 mA	4.961 mA	4.596 mA
V_{R3}	16.19 V	16.18 V	15 V

4.3 Time Constant of a RC Circuit

Table 10-1: Data Table for RC Time Constant

Time (min:sec)	Current (mA)			Resistor Voltage (V)	Capacitor Voltage (V)
	Trial 1	Trial 2	Average		
0:00	1.745 mA	1.745 mA	1.745 mA	34.9 V	0 V
0:15	1.242 mA	1.24 mA	1.241 mA	24.82 V	10.08 V
0:30	.901 mA	.900 mA	.9005 mA	18.01 V	16.86 V
0:45 τ	.662 mA	.666 mA	.664 mA	13.28 V	21.62 V
1:00	.490 mA	.500 mA	.495 mA	9.9 V	25 V
1:15	.365 mA	.373 mA	.369 mA	7.38 V	27.52 V
1:30	.277 mA	.274 mA	.2755 mA	5.51 V	29.39 V
1:45	.212 mA	.207 mA	.2095 mA	4.19 V	30.71 V
2:00	.165 mA	.158 mA	.1615 mA	3.23 V	31.67 V
2:15	.125 mA	.112 mA	.1185 mA	2.37 V	32.53 V
2:30	.100 mA	.092 mA	.096 mA	1.92 V	32.98 V
2:45	.076 mA	.072 mA	.074 mA	1.48 V	33.42 V
3:00	.062 mA	.057 mA	.0595 mA	1.19 V	33.71 V
3:15	.050 mA	.045 mA	.0475 mA	.95 V	33.95 V
3:30	.041 mA	.036 mA	.0385 mA	.77 V	34.13 V
3:45	.034 mA	.029 mA	.0315 mA	.63 V	34.27 V
4:00	.028 mA	.024 mA	.026 mA	.52 V	34.38 V
4:15	.023 mA	.020 mA	.0215 mA	.43 V	34.47 V
4:30	.020 mA	.016 mA	.018 mA	.36 V	34.54 V
4:45 5τ	.017 mA	.014 mA	.0155 mA	.31 V	34.59 V
5:00	.015 mA	.012 mA	.0135 mA	.27 V	34.63 V
5:15	.013 mA	.010 mA	.0115 mA	.23 V	34.67 V
5:30	.011 mA	.009 mA	.010 mA	.2 V	34.7 V
5:45	.010 mA	.008 mA	.009 mA	.18 V	34.72 V
6:00	.009 mA	.007 mA	.008 mA	.16 V	34.74 V

5 Conclusions

5.1 Network Analysis

Each method of calculation will introduce error due to its assumption of ideal conditions. Furthermore, each almost all methods of calculations will give similar value but will vary in precision.

5.2 Thevenin and Norton Equivalent Circuits

Thevenin and Norton equivalent circuits are way to reduce circuits to a simpler form that makes finding current though, voltage in, and resistance in a circuit. The Norton Current and Thevenin Voltage will be the values across the load resistor. Norton Resistance equals Thevenin Resistance and the Thevenin Voltage is equal to the Thevenin Resistance times the Norton Current.

5.3 Time Constant of a RC Circuit

As a capacitor is charged current will decrease until the capacitor is fully charged thus making an open circuit. Furthermore, the voltage will be collected into the capacitor as the voltage across the resistor goes down.

6 Post Lab

6.1 Network Analysis

1. From the data obtained in this experiment, calculations and simulations; discuss on the validity of Mesh Analysis, Nodal Analysis, and superposition.
   Depending on the precision required to measure the voltage and current will result in each method being more or less accurate. Those which require more precision in the measure of voltage will mean that the the node method is more accurate whereas in the case of current the loop method is more acurrate.

6.2 Thevenin and Norton Equivalent Circuits

1. From the data obtained in this experiment and prelab calculations simulations; discuss on

   (a) Observations regarding the current through resistor R3.
   The current through resistor R3 is equal to 4.961 mA which is equivalent to the Thevenin equivalent current based of the Ohm's Law calculations and is higher than the Norton equivalent current, 4.596; however, this is down to fluctuation due to the shorting of the power supply.

   (b) Observations regarding the voltage across resistor R3.
   The same can be said of the voltage across R3 for similar reasons as observed in the current across the same resistor and due to Ohm's Law with resistance is constant the current fluctuates meaning the voltage will fluctuate.

   (c) Observations regarding the Thevenin equivalent.
   The Thevenin equivalent is larger than the voltage across the load resistor this is down to the fact the current is concentrated in the circuit with out being divided across the two resistors proportionally thus meaning a higher value for voltage with resistance remaining constant.

   (d) Observations regarding the Norton equivalent.
   The Norton equivalent is also much higher than the current across R3 because the resistance in a short circuit is far smaller or non existant when compared the resistance across the resistor.

6.3 Time Constant of a RC Circuit

1. From the data obtained in this experiment in a single graph;

   (a) Plot V_R vs. time

   (b) Plot V_c vs. time

2. After the graphs have been completed, do the following:

   (a) Describe the capacitor voltage behavior from 0 through 5τ, in terms of initial and final voltage magnitude, linearity and rate of change.

   (b) Describe the resistor voltage behavior from 0 through 5τ, in terms of initial and final voltage magnitude, linearity and rate of change.

   (c) To how many volts has V_c charged in one time constant?

   (d) To what % has the capacitor charged to at this point?

   (e) Using the equation in the introduction section of this experiment, show the calculation of V_C for a time equal to one time constant.

(f) How many volts are across the resistor at the end of one time constant? What % is this of the total possible voltage change?

(g) From the graphs read the values of capacitor and resistor voltage at 4 minutes.

(h) Calculate the values of capacitor and resistor voltage at 4 minutes.

(i) From the results of g and h above, what do you conclude about the accuracy of your graphs?