STOR 455 Extra Credit HW. Due 04/30 11:55pm. Submit through Sakai Assignments.

```
library(Stat2Data)
data("FranticFingers")
# Chapter 1 Exercises
# 3, 5, 23, 27, 31, 47
data("Sparrows")
head(Sparrows)
    Treatment Weight WingLength
##
## 1
      control 14.9
                           29.0
## 2
      control
                15.0
                           31.0
## 3
      control 14.3
                           25.0
## 4 control 17.0
                           29.0
      control 16.0
## 5
                           30.0
## 6
      control 16.2
                           31.5
mod_sparrows=lm(Weight~WingLength,data=Sparrows)
summary(mod_sparrows)
##
## Call:
## lm(formula = Weight ~ WingLength, data = Sparrows)
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -3.5440 -0.9935 0.0809 1.0559 3.4168
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.36549
                       0.95731
                                    1.426
                                             0.156
                                            <2e-16 ***
                          0.03472 13.463
## WingLength
               0.46740
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.4 on 114 degrees of freedom
## Multiple R-squared: 0.6139, Adjusted R-squared: 0.6105
## F-statistic: 181.3 on 1 and 114 DF, p-value: < 2.2e-16
#To extract the intercept
B1=summary(mod_sparrows)$coefficients[2,1]
B1
## [1] 0.467404
```

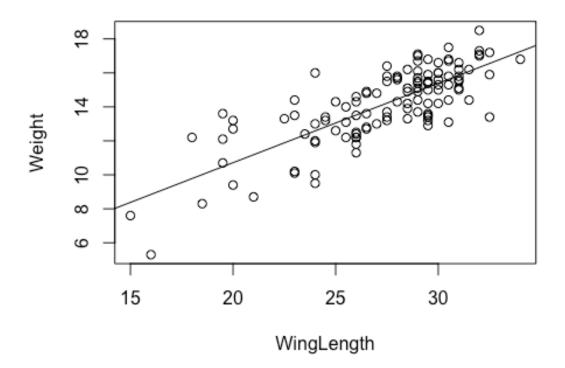
```
#the slope of the least squares regression line for predicting sparrow weight
from wing length is 0.467404

#5
B0=summary(mod_sparrows)$coefficients[1,1]
B0

## [1] 1.36549

#the intercept of the least squares regression line for predicting sparrow
weight from wing length is 1.36549

#23
#a
plot(Weight~WingLength,data=Sparrows)
abline(mod sparrows)
```

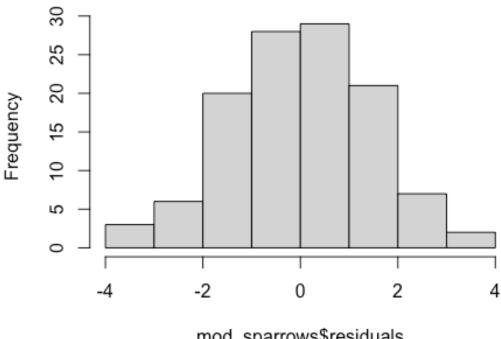


#The general fit is pretty good according to the plot and no obvious outliers or influential points.

#b

hist(mod_sparrows\$residuals)

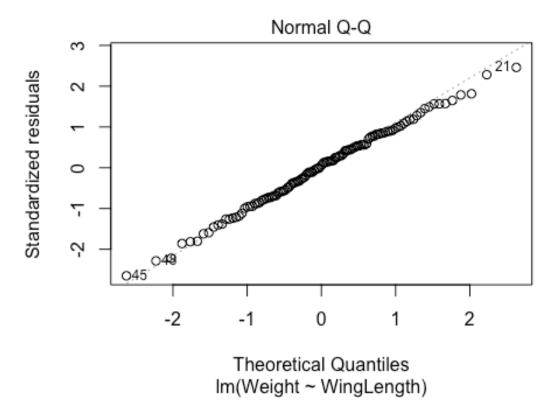
Histogram of mod_sparrows\$residuals



mod_sparrows\$residuals

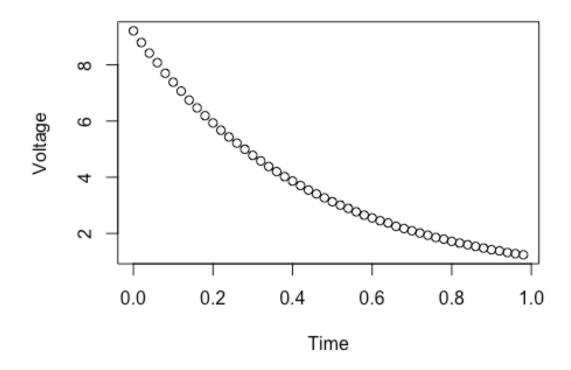
#no obvious outlier, the distribution look pretty normal and it's a bell shape with center at zero.

#c plot(mod_sparrows,2)

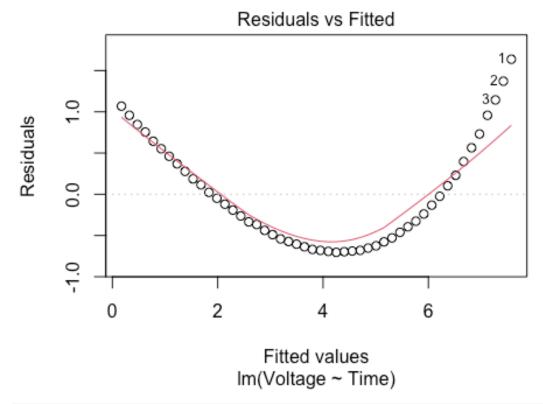


#Normal qqplot demonstrates good fit, with very small deviation on the right tail. Thus the normality condition is good.

```
#27
#a
data("Volts")
plot(Voltage~Time, data=Volts)
```

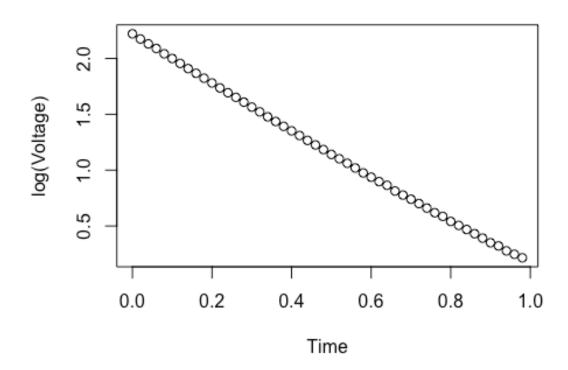


mod_voltage=lm(Voltage~Time,data=Volts)
#It is showing a negative nonlinear relationship between voltage and time. As
time increasing, voltage is decreasing at a decreasing rate.
#b
plot(mod_voltage,1)



#Residual vs. fitted plot is showing big problem with linearity since there is a clear curved pattern. Thus, it is not a good idea to fit a linear model to predict Voltage from Time

#c
plot(log(Voltage)~Time,data=Volts)

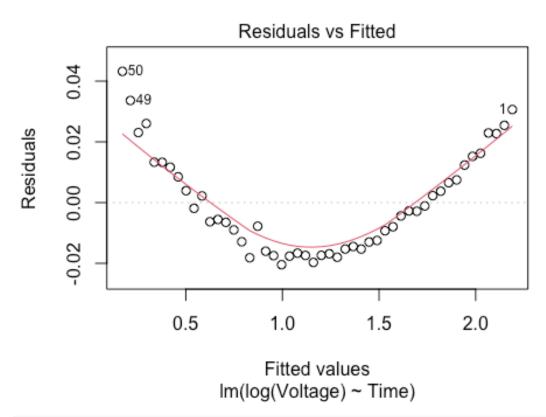


#The plot has a very linear pattern for logvoltage and time(linear negative slope) #d mod_voltage_2=lm(log(Voltage)~Time,data=Volts) summary(mod_voltage_2) ## ## Call: ## lm(formula = log(Voltage) ~ Time, data = Volts) ## ## Residuals: ## Min **1Q** Median 3Q Max ## -0.020448 -0.015084 -0.003621 0.012190 0.043212 ## ## Coefficients: Estimate Std. Error t value Pr(>|t|)## <2e-16 *** ## (Intercept) 2.189945 0.004637 472.3 ## Time -2.059065 0.008154 -252.5 <2e-16 *** ## ---0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 ## Signif. codes: ## Residual standard error: 0.01664 on 48 degrees of freedom

```
## Multiple R-squared: 0.9992, Adjusted R-squared: 0.9992
## F-statistic: 6.377e+04 on 1 and 48 DF, p-value: < 2.2e-16

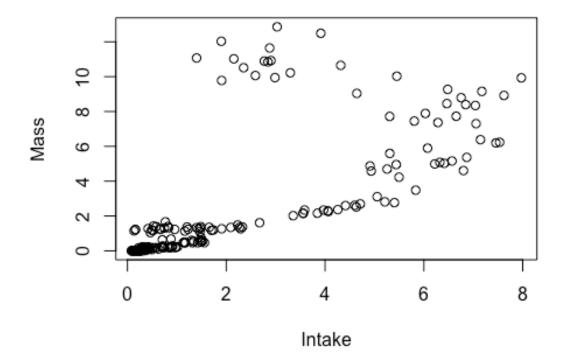
#Prediction equation: Log(Voltage)=2.18994-2.059065*Time

#e
plot(mod_voltage_2,1)</pre>
```



#Though there is still a curved pattern(nonlinear), but from the value of yaxis(residuals), the magnitude of residuals are much smaller than the
previous one(part b, before the transformation)
#31
#a

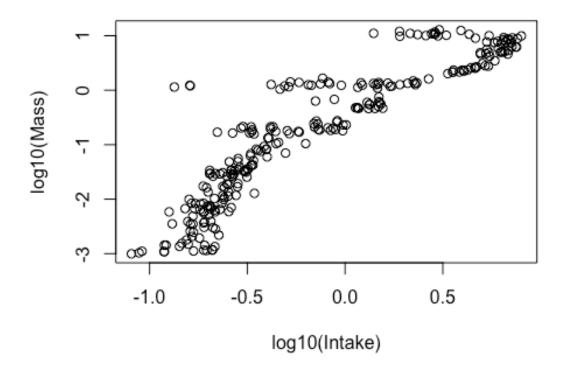
data("Caterpillars")
plot(Mass~Intake,data=Caterpillars)



#the plot is showing there is a nonlinear relationship between Mass and Intake

#b

plot(log10(Mass)~log10(Intake),data=Caterpillars)



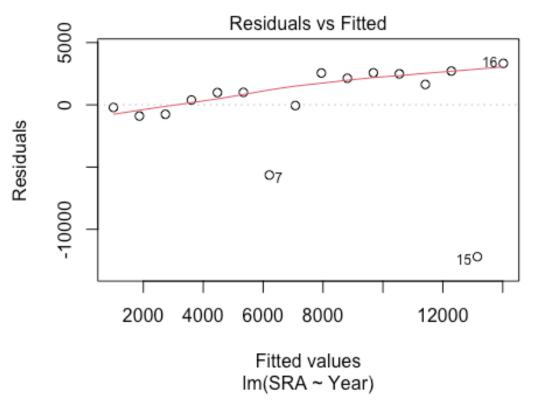
#The plot is showing a positive relationship between log10(Mass) and log10(Intake). But there are also some deviations and a slight curved pattern(nonlinear), so the relationship is increasing at a decreasing rate.

#*c*

#no,linear model should not be used to model either of the relationship in part a and b because both plots are demonstrating curved patterns, so the relationship is nonlinear between two variables which means we can't use linear model to model them. If comparing these two models, the second model's linearity condition is better than the first model but still not linear enough.

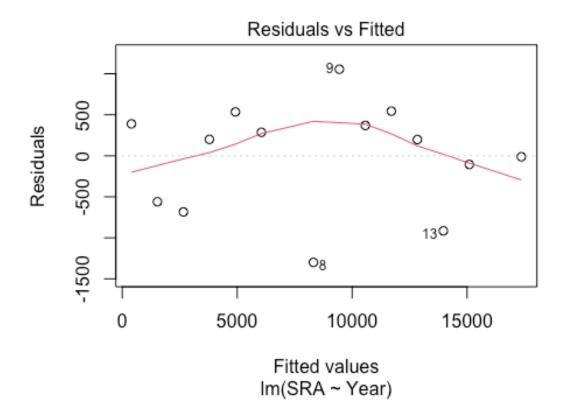
```
#47
#a
data("Retirement")
mod_retirement=lm(SRA~Year, data=Retirement)
summary(mod_retirement)
##
## Call:
## lm(formula = SRA ~ Year, data = Retirement)
##
## Residuals:
```

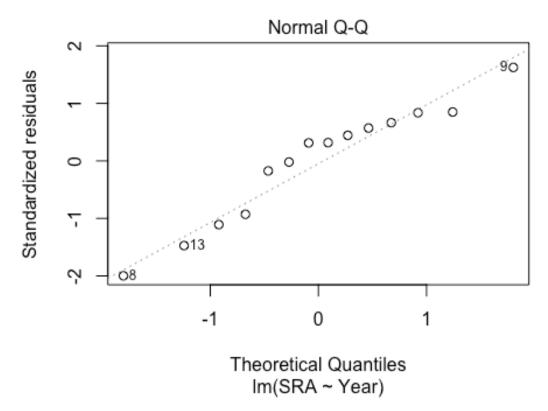
```
Min
                  1Q
                       Median
                                    30
                                            Max
                        990.2
## -12201.0
              -350.9
                                2503.6
                                         3328.7
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -1732400.2
                            439864.9
                                     -3.938
                                            0.00148 **
                                       3.956 0.00144 **
## Year
                    868.0
                               219.4
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4046 on 14 degrees of freedom
## Multiple R-squared: 0.5278, Adjusted R-squared: 0.494
## F-statistic: 15.65 on 1 and 14 DF, p-value: 0.001436
plot(mod_retirement,1)
```



```
resid(mod_retirement)[7]
## 7
## -5642.725
resid(mod_retirement)[15]
```

```
## -12200.96
#The fit is SRA=-1732400.2+868.0*Year. The residual for 7th
observation(Year=2003) is -5642.725, and the residual for 15th
obersvation(Year=2011) is -12200.96
rstandard(mod_retirement)[7]
##
## -1.445406
rstandard(mod retirement)[15]
##
          15
## -3.343753
#7th obervation(Year=2003) has a standardized residual of -1.445406 which is
within the magnitude of 2 bound, so it is not very significant outlier.
However, 15th observation(Year=2011) has a standardized residual of -3.343753
which is greater than the magnitude of 3 bound, thus it is very significant
and thus it is indeed an outlier.
#b
Retiremen_new=Retirement[c(1:6,8:14,16),]
mod_retirement2=lm(SRA~Year, data = Retiremen_new)
summary(mod retirement2)
##
## Call:
## lm(formula = SRA ~ Year, data = Retiremen_new)
##
## Residuals:
                1Q Median
##
      Min
                                30
                                       Max
## -1299.1 -446.8 198.8 384.4 1055.1
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.258e+06 7.891e+04 -28.62 2.06e-12 ***
               1.131e+03 3.937e+01 28.72 1.97e-12 ***
## Year
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 674.7 on 12 degrees of freedom
## Multiple R-squared: 0.9857, Adjusted R-squared: 0.9845
## F-statistic: 825.1 on 1 and 12 DF, p-value: 1.97e-12
plot(mod_retirement2,1:2)
```





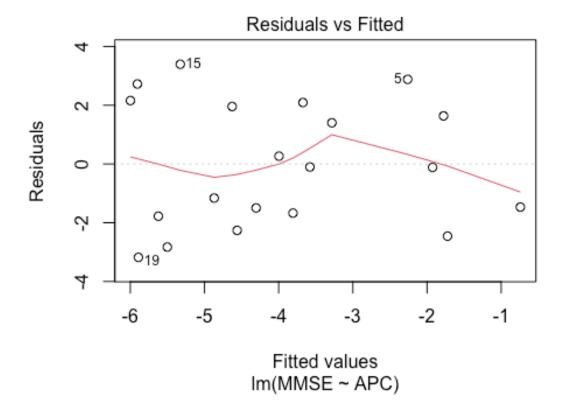
#The new model indeed provide a better fit for the annual contributions and the fit is SRA= -2257996.88 + 1130.89*Year. Though there is still a curved pattern, the residual is much smaller and we can't see any obvious outlier. Not only the linearity condition is better, the normal applot is also showing that normality condition is good.

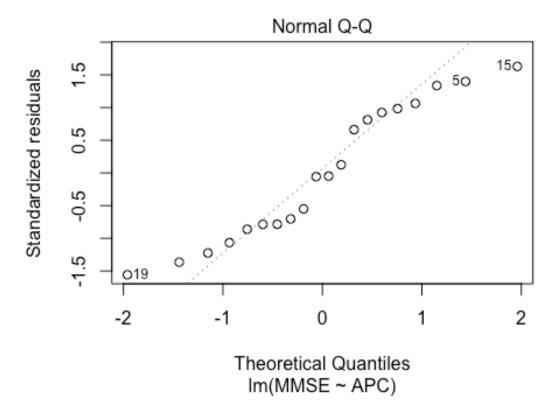
```
# Chapter 2 Exercises
# 15, 17, 23
#15
#a
data("Cereal")
mod_cereal=lm(Calories~Sugar,data=Cereal)
summary(mod_cereal)
##
## Call:
## lm(formula = Calories ~ Sugar, data = Cereal)
## Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                        Max
## -37.428
           -9.832
                     0.245
                              8.909 40.322
##
## Coefficients:
```

```
Estimate Std. Error t value Pr(>|t|)
                                            <2e-16 ***
## (Intercept) 87.4277
                           5.1627 16.935
## Sugar
                           0.7074
                                            0.0013 **
                2.4808
                                    3.507
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 19.27 on 34 degrees of freedom
## Multiple R-squared: 0.2656, Adjusted R-squared: 0.244
## F-statistic: 12.3 on 1 and 34 DF, p-value: 0.001296
#Null hypothesis: The coefficient for sugar is zero(slope(\theta)=0), Alternative
hypothesis: the coefficient for sugar is not zero(8 not equal to zero). From
the summary table, the t-statistic is 3.507 and the p-value is 0.0013 which
is smaller than 0.05, thus we reject the null hypothesis and conclude that
there is indeed a linear relationship between calories and sugar.
#h
confint(mod_cereal,level=.95)
##
                  2.5 %
                           97.5 %
## (Intercept) 76.935859 97.919521
## Sugar
               1.043205 3.918421
#The 95% confidence interval for the slope is (1.043205,3.918421), the slope
coefficient means that for each unit change in sugar, calories will increase
by 1.043205 to 3.918421
#17
#a
data("LewyDLBad")
mod_dementia=lm(MMSE~APC, data=LewyDLBad)
summary(mod_dementia)
##
## Call:
## lm(formula = MMSE ~ APC, data = LewyDLBad)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -3.1791 -1.6991 -0.1081 1.9911 3.3963
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.4359
                           0.6858 -3.552 0.00228 **
                                   3.182 0.00516 **
## APC
                1.3444
                           0.4225
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.184 on 18 degrees of freedom
## Multiple R-squared: 0.36, Adjusted R-squared: 0.3245
## F-statistic: 10.13 on 1 and 18 DF, p-value: 0.005161
```

#Null hypothesis: The coefficient for APC is $zero(slope(\theta)=0)$, Alternative hypothesis: the coefficient for APC is not $zero(\theta)$ not equal to $zero(\theta)$. From the summary table, the t statistic is 3.182 and the p-value is 0.00516 which is smaller than 0.05, thus we reject the null hypothesis and conclude that there is indeed a statistically significant linear relationship between MMSE and APC

#b
plot(mod_dementia,1:2)





#From the residual vs. fitted plot, the linearity condition is pretty good and constant variance is decent too. However, from the normal qaplot, there is a slight curved pattern and two short tails, but the overall, the normal applot is not bad thus the normality condition is somewhat satisfied

```
#c
summary(mod_dementia)
##
## Call:
## lm(formula = MMSE ~ APC, data = LewyDLBad)
##
## Residuals:
                1Q Median
       Min
                                3Q
                                       Max
## -3.1791 -1.6991 -0.1081 1.9911
                                    3.3963
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -2.4359
                            0.6858
                                    -3.552 0.00228 **
## APC
                 1.3444
                                     3.182 0.00516 **
                            0.4225
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Residual standard error: 2.184 on 18 degrees of freedom
                         0.36, Adjusted R-squared:
## Multiple R-squared:
## F-statistic: 10.13 on 1 and 18 DF, p-value: 0.005161
#from the summary table, estimated slope=1.3444 and standard error=0.4225
#d
confint(mod dementia, level=.9)
                               95 %
## (Intercept) -3.6251781 -1.246683
## APC
                0.6117672 2.076951
#The 90% confidence interval for the slope is (0.6117672,2.076951), the slope
coefficient means with 90% confidence the MMSE increase between 0.6117672 and
2.076951 with one unit increase in APC
#My interval does not contain zero, the interval again shows that there is a
statistically significant linear relationship between MMSE and APC which is
what we conclude for part (a)
#23
#a
data("USstamps")
USstamps new=USstamps[c(5:25),]
USstamps_new
##
     Year Price
## 5 1958
## 6 1963
               5
## 7 1968
               6
## 8 1971
               8
## 9 1974
              10
## 10 1975
              13
## 11 1978
              15
## 12 1981
              18
## 13 1981
              20
## 14 1985
              22
## 15 1988
              25
## 16 1991
              29
## 17 1995
              32
## 18 1999
              33
## 19 2001
              34
## 20 2002
              37
## 21 2006
              39
## 22 2007
              41
## 23 2008
              42
## 24 2009
              44
## 25 2012
              45
mod stamp=lm(Price~Year, data = USstamps new)
summary(mod_stamp)
```

```
##
## Call:
## lm(formula = Price ~ Year, data = USstamps_new)
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -2.9232 -0.9478 0.1195 1.1899 4.5325
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
                                              <2e-16 ***
## (Intercept) -1.647e+03 4.686e+01 -35.15
## Year
               8.410e-01 2.357e-02
                                      35.68
                                              <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.737 on 19 degrees of freedom
## Multiple R-squared: 0.9853, Adjusted R-squared: 0.9845
## F-statistic: 1273 on 1 and 19 DF, p-value: < 2.2e-16
#98.53% of variation in postal rates is explained by Year.
#b
summary(mod_stamp)
##
## Call:
## lm(formula = Price ~ Year, data = USstamps new)
##
## Residuals:
               10 Median
##
      Min
                               3Q
                                      Max
## -2.9232 -0.9478 0.1195 1.1899 4.5325
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                                              <2e-16 ***
## (Intercept) -1.647e+03 4.686e+01 -35.15
## Year
                                              <2e-16 ***
               8.410e-01 2.357e-02
                                      35.68
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.737 on 19 degrees of freedom
## Multiple R-squared: 0.9853, Adjusted R-squared: 0.9845
## F-statistic: 1273 on 1 and 19 DF, p-value: < 2.2e-16
#Null hypothesis: The coefficient for Year is zero(slope(\theta)=0), Alternative
hypothesis: the coefficient for Year is not zero(8 not equal to zero). From
the summary table, the t statistic is 35.68 and the p-value is almost zero,
thus we reject the null hypothesis and conclude that there is indeed a
statistically significant linear relationship between Price and Year.
```

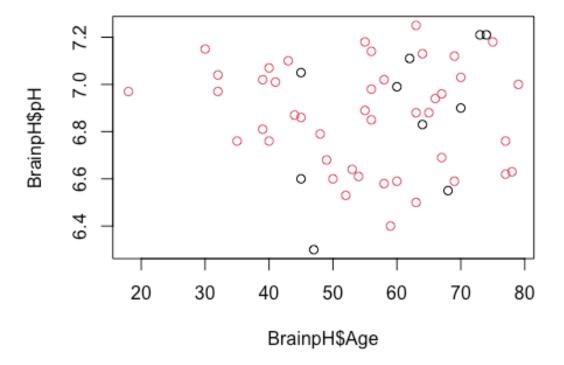
```
#c
anova(mod stamp)
## Analysis of Variance Table
##
## Response: Price
##
            Df Sum Sq Mean Sq F value
                                          Pr(>F)
             1 3841.2 3841.2 1273.1 < 2.2e-16 ***
## Year
## Residuals 19
                 57.3
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#Year has a F-value of 1273.1 and its p-value is almost zero, thus again, we
can conclude that year indeed has a significant impact on price.
# Chapter 3 Exercises
# 21, 23, 29, 31, 35
#21
#a
data("MathEnrollment")
MathEnrollment new=MathEnrollment[c(1:2,4:11),]
mod math=lm(Spring~Fall+AYear, data=MathEnrollment new)
summary(mod_math)
##
## Call:
## lm(formula = Spring ~ Fall + AYear, data = MathEnrollment_new)
## Residuals:
##
                       Median
        Min
                  1Q
                                    3Q
                                            Max
## -16.1945 -9.3982
                       0.3212
                                5.8503 18.2036
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -1.172e+04 2.686e+03 -4.361 0.00331 **
              -1.007e+00 2.041e-01 -4.933 0.00169 **
## Fall
               6.107e+00 1.337e+00
                                      4.566 0.00258 **
## AYear
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 13.37 on 7 degrees of freedom
## Multiple R-squared: 0.871, Adjusted R-squared:
## F-statistic: 23.64 on 2 and 7 DF, p-value: 0.0007704
#from the summary table, Multiple R-squared= 0.871, thus 87.1% variability in
spring enrollment is explained by the multiple regression model based on fall
enrollment and academic year
#From the summary table, the size of the typical error for this multiple
```

```
regression model is 13.37
nullmodel<- lm(Spring~1, data=MathEnrollment new)</pre>
anova(nullmodel,mod_math)
## Analysis of Variance Table
## Model 1: Spring ~ 1
## Model 2: Spring ~ Fall + AYear
              RSS Df Sum of Sq
    Res.Df
                                         Pr(>F)
## 1
         9 9697.6
                        8446.9 23.638 0.0007704 ***
## 2
         7 1250.7 2
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#Null hypothesis: The regression coefficients(betas) of Fall and AYear are
both zeros. Alternative hypothesis: At least one of the regression
coefficients(betas) of Fall and AYear is not zero. From the anova table, F-
value is 23.638 and p-value 0.0007704 which is extremely small, thus we
reject the null hypothesis and conclude that there is indeed a significant
relationship between response and at least one of the predictor
#d
summary(mod_math)
##
## Call:
## lm(formula = Spring ~ Fall + AYear, data = MathEnrollment_new)
##
## Residuals:
##
        Min
                  1Q
                      Median
                                    3Q
                                           Max
## -16.1945 -9.3982
                      0.3212
                                5.8503 18.2036
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -1.172e+04 2.686e+03 -4.361 0.00331 **
              -1.007e+00 2.041e-01 -4.933 0.00169 **
## Fall
## AYear
               6.107e+00 1.337e+00 4.566 0.00258 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 13.37 on 7 degrees of freedom
## Multiple R-squared: 0.871, Adjusted R-squared:
## F-statistic: 23.64 on 2 and 7 DF, p-value: 0.0007704
#For Fall variable: Null hypothesis: The coefficient for Fall is
zero(slope(\theta)=0), Alternative hypothesis: the coefficient for Fall is not
zero(β not equal to zero). From the summary table, the t statistic is -4.933
and the p-value is 0.00169, thus we reject the null hypothesis and conclude
that there is indeed a statistically significant linear relationship between
```

```
Spring and Fall.

#For AYear variable: Null hypothesis: The coefficient for AYear is
zero(slope(6)=0), Alternative hypothesis: the coefficient for AYear is not
zero(6 not equal to zero). From the summary table, the t statistic is 4.566
and the p-value is 0.00258, thus we reject the null hypothesis and conclude
that there is indeed a statistically significant linear relationship between
Spring and AYear.

#23
#a
data("BrainpH")
plot(BrainpH$Age,BrainpH$pH,col=BrainpH$Sex)
```



```
#There does not seem to have a linear relationship among the variables

#b

mod_brainph=lm(pH~Age,data = BrainpH)
summary(mod_brainph)

##

## Call:
## lm(formula = pH ~ Age, data = BrainpH)
##

## Residuals:
```

```
Min 10
                      Median
                                   30
                                           Max
## -0.56976 -0.21781 0.02032 0.16801 0.38649
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 6.8881113 0.1321194
                                      52.13
                                              <2e-16 ***
              -0.0003905 0.0022944
                                      -0.17
                                               0.866
## Age
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.235 on 52 degrees of freedom
## Multiple R-squared: 0.0005566, Adjusted R-squared: -0.01866
## F-statistic: 0.02896 on 1 and 52 DF, p-value: 0.8655
#Null hypothesis: The coefficient for Age is zero(slope(\theta)=0), Alternative
hypothesis: the coefficient for Age is not zero(8 not equal to zero). From
the summary table, the t statistic is -0.17 and the p-value is 0.866, thus we
fail to reject the null hypothesis and conclude that there is not a
statistically significant linear relationship between pH and Age.
#c
mod brainph2=lm(pH~Age+Sex,data = BrainpH)
summary(mod_brainph2)
##
## Call:
## lm(formula = pH ~ Age + Sex, data = BrainpH)
##
## Residuals:
                 10
##
       Min
                      Median
                                   30
                                           Max
## -0.58126 -0.21456 0.02306 0.16722 0.38942
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                                              <2e-16 ***
## (Intercept) 6.9025758 0.1613711 42.775
## Age
               -0.0004535 0.0023499 -0.193
                                               0.848
## SexM
               -0.0134258 0.0843155 -0.159
                                               0.874
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2372 on 51 degrees of freedom
## Multiple R-squared: 0.001053, Adjusted R-squared: -0.03812
## F-statistic: 0.02689 on 2 and 51 DF, p-value: 0.9735
b1=summary(mod_brainph2)$coefficients[2,1]
b1
## [1] -0.0004535487
#slope for age is -0.0004535487
slope <- b1
```

```
BrainpHfemale=subset(BrainpH,Sex=="F")
intercept female <- mean(BrainpHfemale$pH) - slope * mean(BrainpHfemale$Age)</pre>
intercept_female
## [1] 6.902576
#the best fit line for female is pH=6.902576-0.0004535487*Age
BrainpHmale=subset(BrainpH,Sex=="M")
intercept_male <- mean(BrainpHmale$pH) - slope * mean(BrainpHmale$Age)</pre>
intercept male
## [1] 6.88915
#the best fit line for female is pH=6.88915-0.0004535487*Age
# Chapter 4 Exercises
# 13, 15
#13
data("NCbirths")
White=117.87
White
## [1] 117.87
#the coefficients for fitted model: the 1st coefficient which is the baseline
group and the corresponding mother's race is white, and the predicted birth
weights(in ounces) of the babies is 117.87 ounces if the mother's race is
white.
Black=White-7.31*1
Black
## [1] 110.56
#-7.31 means that if the mother is Black, then the predicted birth weights(in
ounces) of the babies will decrease by 7.31. Thus, the predicted birth
weights(in ounces) of the babies is 117.87-7.31 = 110.56 ounces if the
mother's race is black.
Hispanic=White+0.65*1
Hispanic
## [1] 118.52
#0.65 means that if the mother is Hispanic, then the predicted birth
weights(in ounces) of the babies will increase by 0.65. Thus, the predicted
birth weights(in ounces) of the babies is 117.87+0.65 = 118.52 ounces if the
mother's race is hispanic.
Other=White-0.73*1
Other.
## [1] 117.14
#-0.73 means that if the mother is Other race, then the predicted birth
weights(in ounces) of the babies will decrease by 0.73. Thus, the predicted
```

```
birth weights(in ounces) of the babies is 117.87-0.73 = 117.14 ounces if the
mother's race is Other race.

# Chapter 5 Exercises
# 27, 37, 45

# Chapter 8 Exercises
# 3, 51, 55

# Chapter 9 Exercises
# 21, 33
```