# Estimation of Individual Failure Rates for Power System Components Based on Risk Functions

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***Abstract*—The failure rate is essential in power system reliability assessment and thus far, it has been commonly assumed as constant. This is a basic approach that delivers reasonable results. However, this approach neglects the heterogeneity in component populations, which has a negative impact on the accuracy of the failure rate. This paper proposes a method based on risk functions, which describes the risk behavior of condition measurements over time, to compute individual failure rates within populations. The method is applied to a population of 12 power transformers on transmission level. The computed individual failure rates depict the impact of maintenance and that power transformers with long operation times have a higher failure rate. Moreover, this paper presents a procedure based on the proposed approach to forecast failure rates. Finally, the individual failure rates are calculated over a specified prediction horizon and depicted with a 95% confidence interval.**

***Index Terms*—Asset management, condition monitoring, failure rate, failure rate modeling, power transformer diagnostics.**

## I. Introduction

THESE days, the power system experiences a huge change due to the integration of renewables, the Smart Grid revolution, and the integration of power system markets [1]. This change has also an impact on the system reliability which can result in higher outage times for customers and higher costs for utilities. Therefore, a major goal is the maximization of the system reliability while minimizing the annual costs to find the socio-economically optimal reliability level [2].

An essential strategy to achieve this goal is maintenance planning, scheduling, and optimization as a midterm asset management task [3]. Here, the failure rate is used as the reliability measure of power system components. A functional relationship between component failure rates and maintenance has been developed in [4] to propose a reliability-centred asset maintenance (RCAM) method. Identifying failure modes as crucial, [3] proposed a general and straightforward reliability-centred maintenance framework with an ensuing application in [5]. Reference [6] formulates a structured optimization problem for selecting the correct maintenance strategy in a RCAM context. Likewise, the failure rate is also used as the preferred reliability measure. Moreover, the failure rate is used as input parameter for a midterm transformer maintenance scheduler in [7] or in optimal maintenance planning for overhead lines [8].

All aforementioned methods need accurate component failure rates to gain the best results. Estimation of the failure rate requires failure data from life tests of $n$ identical components to obtain a particular lifetime distribution $F(t)$. However, the long lifetimes and tenuous equipment documentation cause incomplete failure data sets which lead to censoring and truncation when predicting the remaining life of, for example, power transformers [9]. In addition, critical power system components, particularly on transmission level, get replaced before actual failures occur. The accurate estimation of failure rates is therefore rarely possible.

These challenges have led to the use of average or experience failure rates in power system reliability calculations [10], [11]. Although the use of a constant failure rate is a basic approach, it has produced valuable results for power system planning [12]. However, two disadvantages of this approximation remain. Firstly, a constant failure rate cannot indicate the individual condition of the equipment and therefore might overrate or underrate the actual failure rate [11]. Furthermore, applying a component failure rate to all components of the same type, limits the accuracy of the subsequent maintenance optimization since all components are assumed to be equal which neglects the individual risk. Secondly, it is not possible to identify the impact of maintenance activities. Hence, there is the need to develop methods that can deal with the aforementioned challenges to estimate an individual failure probability for power system components.

Certain attempts have been made to combine the average failure rate with condition-monitoring data [11], [13], [14]. Failure rate modelling based on inspection data has been proposed in [11]. This approach presents a failure rate for the best, average, and worst condition score and by using a specific formula, calculates the failure rate as a function of the condition score. Whereas the average failure rate can be found in literature, the failure rate for the best and worst is more difficult to determine [11]. To do so, sufficient historical failure data is required. A Hidden Markov model approach is used in [13] to calculate the failure rate for power transformer based on the condition. However, this method is data intensive since the component needs to be in every predefined state before the transition rates between the states can be computed. Moreover, to calculate an individual failure rate, this method needs to be repeated for each

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component in a population which seems not practical. A method to calculate individual failure rates is presented [14] which addresses heterogeneity of components by using a relative risk approach and thereby overcomes the remaining difficulties of the methods described in [11], [13]. Even though accurate results are presented, the method solely considers single time condition measurements and assumes linearity between condition measurement and the failure rate. Particularly, the time independence of the method does not allow any individual failure rate prediction. In the proposed paper, we formulate a method, based on [14], for time-dependent condition monitoring data, addressing the non-linearity between the failure rate and the condition information, and use condition measurement information to predict an individual failure rate. The advantage of this method is that no actual failure data is required to calculate the individual failure rates which is especially beneficial for new components.

### A. Summary of Contributions

This paper presents the novel concept of individual failure rates while considering condition measurement data as time-dependent. The method utilizes available component information in utilities such as the population/baseline failure rate, failure statistics, and measurement uncertainty. Risk functions describe the relative risk of one component towards the population by using condition monitoring information. Applying existing methods to forecast stochastic or non-stochastic condition monitoring data, the individual failure rates can be predicted with a predefined prediction horizon.

### B. Article Structure

The following section describes the theoretical aspects of the proposed method. Afterwards in Section III, the method is applied to a population of transmission system power transformers to present its applicability. The case study results are presented in Section IV and further discussed in V. The last section concludes the work.

## II. Method

In this section, the proposed method is formulated to calculate individual failure rates based on the concept of risk functions. However, this method shall be seen as a general framework and not be limited to the three proposed risk functions since the functional space to describe risks is large.

Suppose a homogeneous population of $n$ non-identical power system components in operation without failure occurrence until time $t$. Thus far, the unspecified and arbitrary population baseline failure rate function

$$\lambda(t) = \lim_{\Delta t \longrightarrow 0} \frac{P(t < T \leq t + \Delta t \mid T > t)}{\Delta t} = \frac{f(t)}{R(t)} \qquad (1)$$

with $T$ as survival time, $f(t)$ the probability density function, and $R(t)$ the reliability function, is assumed to be the failure rate for each component. A covariate, which failure might depend upon, is an internal or external risk factor which has an impact on the failure rate of the component. This method solely considers internal covariates which are condition measurements such as the temperature or gas and oil results for power transformers. Given the basic and time dependent covariate vector $\boldsymbol{x}_i(t) = (x_{i_1}(t), x_{i_2}(t), \dots x_{i_d}(t))$ of $d$ covariates, the relative risk model, proposed in [15], is

$$\begin{aligned} \lambda(t; \boldsymbol{x}_i(t)) &= \lim_{\Delta t \longrightarrow 0} \frac{P(t < T \leq t + \Delta t \mid T > t, \boldsymbol{x}_i(t))}{\Delta t} \\ &= \lambda_0(t)\, r(t, \boldsymbol{x}_i(t)), \quad t > 0 \end{aligned} \qquad (2)$$

where $\lambda_0(t)$ is an arbitrary unspecific baseline failure rate function and $r(t, \boldsymbol{x}_i(t))$ the relative risk function which connects the basic covariates $\boldsymbol{x}_i(t)$ with the baseline failure rate function. When all covariates are zero, $r(t, \boldsymbol{x}_i(t)) = 1$ and then the failure rate for an individual is the baseline failure rate. The purpose of the paper is to define a relative risk function which describes the individual risk of a component but without using actual failures as in the regression approach in [15].

The baseline failure rate is observed from historical failure data of a comparable population $\eta$ with $m$ distinct failure types $j \in \{1, \dots, m\}$. A failure type-specific failure rate can be denoted as

$$\lambda_j(t; \boldsymbol{x}_i(t)) = \lim_{\Delta t \longrightarrow 0} \frac{P(t < T \leq t + \Delta t, j \mid T > t, \boldsymbol{x}_i(t))}{\Delta t} \qquad (3)$$

which is the instantaneous failure rate of failure type $j$ for the $i$-th individual with $i \in \{1, \dots, n\}$. Assuming that all failure types can occur but separately, we get

$$\lambda(t; \boldsymbol{x}_i(t)) = \sum_{j=1}^{m} \lambda_j(t; \boldsymbol{x}_i(t)) = \sum_{j=1}^{m} \lambda_{0j}(t) r_j(t; \boldsymbol{x}_i(t)) \qquad (4)$$

which is known to be the competing risk approach in survival analysis and further described in [16]. Henceforth, we describe and discuss the proposed method based on the form and idea of eq. (4).

Let the number of occurrences of each failure type $j$ be known from population $\eta$, the vector $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_m)$ denotes the proportion of each failure type $j$ with the property $\sum_{j=1}^{m} \alpha_j = 1$. Let $\sum_{j=1}^{m} \lambda_{0j}(t) = \lambda_0(t) \sum_{j=1}^{m} \alpha_j$, the competing risk approach can then be written as

$$\lambda(t; \boldsymbol{x}_i(t)) = \lambda_0(t) \sum_{j=1}^{m} \alpha_j r_j(t; \boldsymbol{x}_{i,j}(t)) \qquad (5)$$

Since this is not a regression model where the covariates are tested to assess their significance to failure type $j$, it must be assumed that each covariate of $k$ internal covariates related to $j$ in $\boldsymbol{x}_{i,j}(t) = (x_{i,j_1}(t), x_{i,j_2}(t), \dots x_{i,j_k}(t))$ with $p \in \{1, \dots, k\}$ and $x_{i,j_p}(t) \in \Re_{>0}$ is a valid failure indicator of $j$. The benefit of expressing the competing risk approach in the form of eq. (5) is that $\lambda_0(t)$ and $\boldsymbol{\alpha}$ are usually given by population data $\eta$. The relative risk function $r_j(t; \boldsymbol{x}_{i,j}(t))$ links the covariate data which is related to the $j$-th failure type to the baseline failure rate. It is assumed that each internal covariate in vector $\boldsymbol{x}_{i,j}(t)$ is a valid condition indicator of $j$ and has a measurement uncertainty vector $\boldsymbol{\rho} = (\rho_1, \rho_2, \dots \rho_m)$ related to it. The concept of the measurement uncertainty $\rho$ is introduced in [14] and describes the assurance of successfully measuring the internal covariates. This leads to the time dependent model

$$\lambda(t;\boldsymbol{x}_i(t)) = \lambda_0(t)\sum_{j=1}^{m}(\alpha_j\rho_j r_j(t;\boldsymbol{Z}_{i,j}(t)) + \alpha_j(1-\rho_j)) \quad (6)$$

where $\boldsymbol{Z}_{i,j_p}(t) = (Z_{i,j_1}(t), Z_{i,j_2}(t), \ldots Z_{i,j_k}(t))$ is a vector of time dependent covariates derived from $\boldsymbol{x}_{i,j}(t)$. This notation is useful since the basic covariates might be transformed such that $\boldsymbol{Z}_{i,j}(t) = g(\boldsymbol{x}_{i,j}(t))$ where $g$ depends on the internal covariate data being chosen. This transformation might be needed to describe the risk functions in II-A. The model in eq. (6) is presented in time-independent form in [14]. The term relative risk function for $r_j(t;\boldsymbol{x}_{i,j}(t))$ will be avoided hereafter and the general expression risk function is introduced to clearly distinguish between the regression model in eq. 3 and the proposed model in eq. (6).

### A. Risk Functions Based on Relative Risk

The differentiation between the population failure rate and the failure rate of the $i$-th individual is described by $r_j(t;\boldsymbol{Z}_{i,j}(t))$ in eq. (6). This is a relative risk function because if $r_j(t;\boldsymbol{Z}_{i,j}(t)) > 1$ the failure probability increases, when $r_j(t;\boldsymbol{Z}_{i,j}(t)) < 1$ it decreases, and with $r_j(t;\boldsymbol{Z}_{i,j}(t)) = 1$ the failure probability is unchanged. Thus, knowledge of how an individual differs within a population is required. As presented in [17], an individual's failure rate is dependent on the environment, the operational stress, and its current condition which has been impacted by static and dynamic factors over time. If the condition is an indicator for the failure probability of failure type $j$, we use the function $r_j(t;\boldsymbol{Z}_{i,j}(t))$ to adjust the population failure rate to gain an individual risk. This section presents different functional types of $r_j(t;\boldsymbol{Z}_{i,j}(t))$. Henceforth, it is assumed that there is only one failure type for the $i$-th individual, so that the notation for the risk function can be simplified to $r(t;\boldsymbol{Z}(t))$.

The risk function $r(t;\boldsymbol{Z}(t))$ is a relative risk function, thus a form of 'reference group' is required. We are interested in how the individual's probability of failure differs from the population or baseline failure rate. It is assumed that if an individual indicates a condition which is different from the average population condition, the failure rate would increase or decrease. Assume that $X_p(t) = \{x_p(u) : 0 \leq u < t\}$ is the covariate history of $p$ in form of a time series until time $t$ and recall that $\boldsymbol{Z}(t)$ is the vector of $k$ transformed time dependent internal covariates related to failure type $j$. Assuming the $i$-th individual has been operated under normal external conditions, the calculation of the arithmetic mean over time reflects this normal condition such that

$$\overline{X}_p(t) = \frac{1}{t}\int_0^t X_p(u)du \quad (7)$$

This leads to $\overline{\boldsymbol{X}}(t) = (\overline{X}_1(t), \ldots, \overline{X}_k(t))$ and $\boldsymbol{X}_{n\times k}$ is the matrix with all $\overline{\boldsymbol{X}}(t)$ for every individual. Now, we can calculate the average population condition based on the arithmetic mean of the $n$ individuals in the population. Thus, the vector

$$\overline{\boldsymbol{C}} = \frac{1}{n}\mathbf{1}\boldsymbol{X}_{n\times k} \quad (8)$$

describes the average population condition for each covariate $p$ with $\mathbf{1} = (1,\ldots,1)_{1\times n}$. Likewise, the variance is $\boldsymbol{V} = (Var(X_1(t)),\ldots,Var(X_k(t))$ and the average of the variance is

$$\overline{\boldsymbol{V}} = \frac{1}{n}\mathbf{1}\boldsymbol{V}_{n\times k} \quad (9)$$

and the average standard deviation is calculated with $\boldsymbol{\sigma} = \sqrt{\overline{\boldsymbol{V}}}$.

The arithmetic mean of all internal covariates in eq. 8 can be used as a reference measure and can be compared to the current measurements $\boldsymbol{Z}(t)$. Now, a risk function $S_p(t;Z_p(t)$ for the covariate $p$ can be calculated such that $\boldsymbol{S}(t;\boldsymbol{Z}(t)) = (S_1(t;Z_1(t)),\ldots S_k(t;Z_k(t)))$. When several covariates exists for failure type $j$ the risk functions must be combined. Thus, the overall risk function is

$$r(t;\boldsymbol{Z}(t)) = \boldsymbol{S}(t;\boldsymbol{Z}(t))\mathbf{w} \quad (10)$$

where $\mathbf{w} = (w_1,\ldots,w_k) \in \Re_{>0}$ with the property $\mathbf{w}\mathbf{1}_{k\times 1} = 1$. The choice of $w_p$ will be discussed in Section II-B.

*1) Linear Risk Function:* A straightforward approach is to assume that the relationship between the failure rate and a covariate is linear. This has been proposed in [14] where a positive and negative linear relationship is presented. To calculate a linear risk function, the basic covariate data needs to be transformed with

$$Z_p(t) = \begin{cases} 0 & \text{if } x_p(t) < 0 \\ x_p(t) & \text{if } x_p(t) \geq 0 \end{cases} \quad (11)$$

if there is a linear positive relationship and

$$Z_p(t) = \begin{cases} x_p(t) & \text{if } x_p(t) \leq x_{p,new}(t) \\ 0 & \text{if } x_p(t) > x_{p,new}(t) \end{cases} \quad (12)$$

if a linear negative relationship between covariate and failure rate exists. The vector $\boldsymbol{x}_{new}(t)$ denotes a reference value for covariate $p$ where the condition would be as new. Thus, we get the linear positive risk function for each covariate with

$$S_p(t;Z_p(t)) = Z_p(t)/\overline{C}_p \quad (13)$$

and the negative linear function is characterized as

$$S_p(t;Z_p(t)) = (x_{p,new}(t) - Z_p(t))/\big(x_{p,new}(t) - \overline{C}_p\big) \quad (14)$$

*2) Non-Linear Risk Function:* The linear risk function assumed a linear relationship between covariate and failure rate. However, this assumption might not be plausible for all covariate types. One indicator of the probability of failure is how a internal covariate changes over time. Particularly, a sudden change of the internal covariate might be an indicator of an increased failure rate, whereas a constant change over time might be tolerated. Thus, if the time series $X_p(t)$ of covariate $p$ has a moderate and constant slope, the failure rate is not affected. However, if the rate of change changes, it reflects an increased risk. This can be described by the second derivative of $X_p(t)$. The basic covariate needs to be standardized to become unit-free and thus

$$Z_p(t) = \frac{x_p(t) - \overline{C}_p}{\sigma_p} \tag{15}$$

Accordingly the risk function can be modelled as

$$S_p(t; Z_p(t)) = exp(\ddot{Z}_p(t)) \tag{16}$$

The exponential form is used in this function to satisfy the failure rate property $\lambda(t; x(t)) \geq 0$.

*3) Cumulative Risk Function:* Another risk indication is the cumulative deviation of the covariate $Z_p(t)$ from its expected value $\overline{C}_p$. This concept of a risk indicator is proposed in [18] where the deviation between the actual measurement and the expected value of a normal behaviour model is accumulated over time. In the proposed approach, the standardisation is necessary to provide a unit-free risk function and to better interpret the main effects while considering natural variations in the covariate over time. Thus, the covariate $x_p(t)$ is standardised with

$$Z_p(t) = \frac{x_p(t) - \overline{C}_p}{2\sigma_p} \tag{17}$$

where the common scale is two times the standard deviation to contain only significant deviations from the mean. The cumulative risk function is thus defined as

$$S_p(t; Z_p(t)) = \int_0^t |Z_p(u)|\, du \tag{18}$$

*4) Selection of Risk Function:* The proposed risk functions can be selected based on the following covariate risk behaviour.

1) *High deviation from population:* If the condition measurement has a high deviation from the population mean in a negative understanding, it shows an increased probability of failure. This straightforward case is best described by the positive or negative linear risk function.
2) *Abrupt change:* If the condition measurement suddenly changes from its approximately constant path, this might be an indication for an increase failure rate. Then, the non-linear function is able to capture this risk type.
3) *High volatility:* The rapid and severe changes of a condition measurement, the volatility, can describe an upcoming failure. In this case, the cumulative risk function is suitable to identify such risk.

Whereas the linear risk function is a rather general approach and applicable for a wide range of condition measurements, the selection of the non-linear and cumulative risk function requires process knowledge of the condition measurement and how it signals a possible failure.

### B. Determination of Covariate Significance

When multiple covariate data is available to indicate one particular failure type, the $k$ risk functions in $\boldsymbol{S}(t; \boldsymbol{Z}(t))$ must be combined with $\boldsymbol{w}$ to a single function $r(t; \boldsymbol{Z}(t))$ as shown in eq. (10). According to [14], the weight $w_p$ is calculated with

$$w_p = c_p / \sum_{p=1}^{k} c_p \tag{19}$$

where $c_p \in \Re_{>0}$ is the weight score of covariate $p$ and describes the significance as a failure indicator. Depending on the amount of historical covariate and failure data available, different approaches are possible to determine $c_p$. These are sorted from high to low data requirements:

1) *Cox (Proportional Hazards) Regression:* This model presented in eq. (2) requires failure, covariate, and survival time data to estimate the regression parameters which are an estimate of the covariate effect, given that the covariate is significant. Here, the covariate effect would describe the impact on the failure rate and can be used as $c_p$.
2) *Logistic Regression:* Similar but without the need of the survival time, the covariates can be regressed towards the failure data to estimate the covariate effect which can be employed as $c_p$.
3) *Weights based on expert knowledge:* Particularly, missing or incomplete data within the power system domain suggest that the previous regression techniques cannot be applied to estimate the weights $c_p$. Consequently, weights based on expert knowledge are commonly used [11].

### C. Prediction of Stochastic Covariate Behavior

The individual failure rates are estimated based on internal covariates which are stochastic or non-stochastic. Both can be forecasted with a certain confidence level with techniques such as time series analysis, state space models, and Markov Chains in the stochastic case or with a set of different functions in the non-stochastic case as described in [19]. Even though the assumptions on covariate behaviour and models should be case specific [19], this paper only discusses stochastic covariates modelled with univariate time series analysis. The autoregressive integrated moving average model (ARIMA(p,d,q)) is defined in general form as

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right) (1 - L)^d X_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \epsilon_t \tag{20}$$

with $L$ as the lag operator, $\epsilon_t \sim WN(0, \sigma^2)$, and $\phi$ and $\theta$ the autoregressive and moving average parameter, respectively. If $d = 0$ the model reduces to an ARMA(p,q) model. Using the Box-Jenkins approach [20], a univariate time series can be modelled as an ARIMA process with the following procedure: Data preparation, model identification and selection, parameter estimation, model validation, and forecasting. Although this procedure is straightforward, it requires experience and can take several iterations until a suitable model is selected. Thus, we refer to [20] for a more detailed description of the procedure.

This covariate forecast can be used in the proposed method to calculate the individual failure rates for the length of the predicted covariate paths with the following procedure. Firstly, the covariates must be modelled with the aforementioned Box-Jenkins approach or other techniques. To do so, a time series of the covariate process needs to be accessible and a reasonable forecast horizon must be chosen. Secondly, the time series is forecasted over the prediction horizon. Since the forecast might vary, Monte Carlo simulation is used to calculate up to $c$ sample paths. Thirdly, from these $c$ forecasted covariate paths an average path and the upper and lower confidence levels are computed.

TABLE I
TRANSMISSION POWER TRANSFORMER POPULATION CHARACTERISTICS

| **Number** | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Production year** | 1983 | 2003 | 1974 | 1977 | 1985 | 1981 | 1987 | 1993 | 1970 | 1995 | 1975 | 1985 |
| **Symbol in figures** | ○ | + | * | • | × | □ | ◇ | △ | ▽ | ▷ | ◁ | ★ |

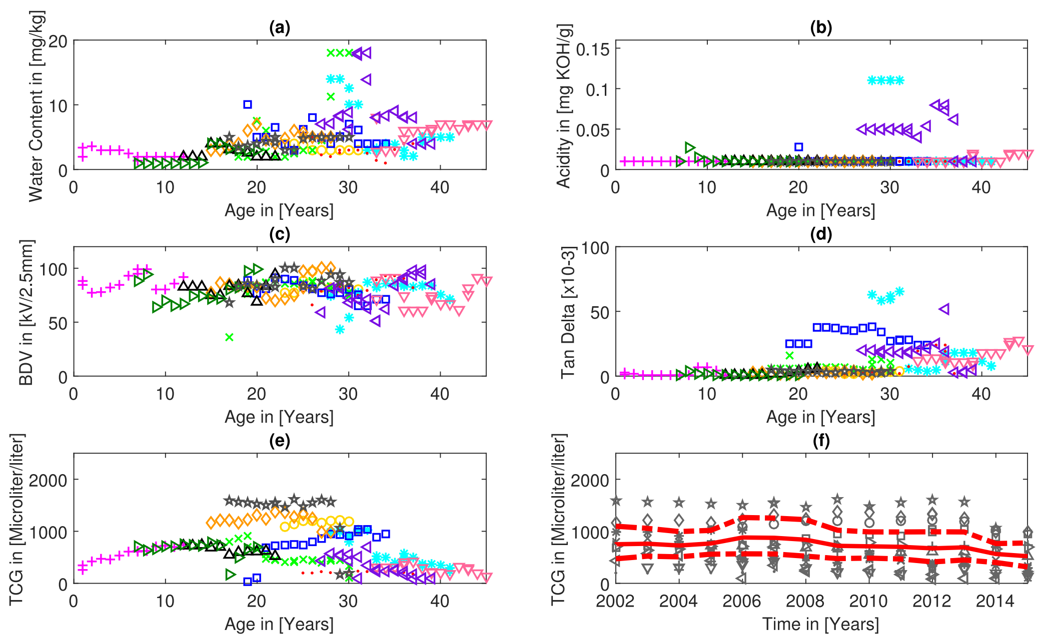



Fig. 1. Obtained covariates for the transmission power transformer population over the period 2002 to 2015 illustrated over component age.

Finally, these 3 covariate paths can be used with the proposed method to calculate the individual failure rate for the time over the prediction horizon.

## III. CASE STUDY: POWER TRANSFORMER POPULATION

A population of 12 power transformer on transmission level is employed to show the applicability of the method. The power transformer's characteristics such as production year, power rating, and symbol and colour code, used throughout this paper, are shown in Table I. The operation time varies between 12 and 45 years. Historical failure mode statistics of power transformer of the same population are not available. Hence, the failure mode results of the survey conducted in [21] are used with: 1 - dielectric 27.7%, 2 - electrical 27.7%, 3 - thermal 15.2%, 4 - physical chemistry 8.9%, 5 - mechanical 17.0%, and 6–3.9% unknown. We can write this in vector form such that $\boldsymbol{\alpha} = (0.277, 0.277, 0.152, 0.089, 0.17, 0.039)$. These failure mode results are particularly suitable since they include a high share of power transformers on transmission level.

Internal covariates in form of oil and gas analysis have been gathered in yearly intervals from all power transformers in the period 2002 to 2015. More specifically, the following basic covariates $x_{i,j}(t)$ have been chosen for this analysis: $x_{i,2_1}(t)$ is the breakdown voltage (BDV), $x_{i,2_2}(t)$ is the dissipation factor or tan delta, $x_{i,4_1}(t)$ is the water content, $x_{i,4_2}(t)$ is acidity in the oil, and $x_{i,4_3}(t)$ the total combustible gases (TCG) which are hydrogen ($H_2$), methane ($CH_4$), acetylene ($C_2H_2$), ethylene ($C_2H_4$), ethane ($C_2H_6$), and carbon monoxide ($CO$). These covariates are depicted over the power transformer age in Fig. 1. The covariates BDV and dissipation factor are indicators for electrical failure mode $j = 2$ and the water content, acidity, and TCG are indicators for the failure mode physical chemistry $j = 4$ which is described and illustrated in [22]. For all measurements, a confidence level of $\rho_j = 95\%$ is assumed as probability of successfully measuring the target value which results in $\boldsymbol{\rho} = 0.95\mathbf{1}_{1\times k}$. A detailed description of how to set $\rho_j$ is given in [14]. Since multiple measurements are related to one failure mode, the significance of the covariates needs to be set in form of weights. The missing failure data of this population and statistical results in general about the significance of the covariates require that the weights are based on expert knowledge, in this case [23]. An overview of the covariates, related information, and calculation of the weights is shown in Table II. Moreover, the baseline failure rate is assumed to be exponentially distributed and thus constant with $\lambda_0(t) = 0.02$.

All covariates are within acceptable ranges except the TCG levels of power transformer T1, T7, and T12 which are higher than normal according to [24]. However, these TCG levels do

TABLE II
INFORMATION ABOUT USED COVARIATES

| Measurement | Critical limit | Standard | Failure mode | Confidence level $\rho_j$ | Assigned score $c_p$ based on [23] | Weights $w_p$ |
|---|---|---|---|---|---|---|
| $x_{i,2_1}(t)$ - Oil Breakdown Voltage (BDV) | 23 [kV/2.5mm] | IEC 60156/95 | electrical | 95% | $3/(3+3) = 0.5$ | 0.5 |
| $x_{i,2_2}(t)$ - Dissipation Factor (DF) | 0.01 | IEC 60247 | electrical | 95% | $3/(3+3) = 0.5$ | 0.5 |
| $x_{i,4_1}(t)$ - Water Content | 35 [mg/kg] | IEC 60814 | physical chemistry | 95% | $4 * 6/5 = 4.8$ | 0.3 |
| $x_{i,4_2}(t)$ - Acidity | 0.2 [mg KOH/g] | IEC 62020-1 | physical chemistry | 95% | $1 * 6/5 = 1.2$ | 0.075 |
| $x_{i,4_3}(t)$ - Total Combustible Gases (TCG) | 1920 [$\mu L/L$] | IEC 60567 | physical chemistry | 95% | 10 | 0.625 |

not indicate trends or the necessity of immediate actions, but suggest further continuous monitoring. Eventually, all population information are gathered to apply the method previously discussed and the results are presented in the next section.

## IV. Results

This section presents the calculated and the forecasted individual failure rates of the transformer population described in the previous Section III.

### A. Individual Failure Rates Based on Covariate Information

The general method to calculate the individual failure rates can be divided into five steps. These steps are:

1) *Gather and set population parameters:* The necessary population characteristics and covariate information need to be gathered or set as in Section III.
2) *Define common time intervals for covariates:* The covariate information is, in this particular case, not gathered at the exact same point in time for each component. Thus, an expected value for the normal population condition is calculated for each year because the covariate information is gathered once a year for every power transformer. Moreover, the measurement data needs to be resampled due to unevenly spread measurement data. This has been done with linear interpolation.
3) Calculate the expected value for covariate $j_p$ for each time interval.
4) *Choose a risk function for covariate $j_p$:* A risk function should be chosen for each covariate depending on the expected risk behaviour. As discussed later in this paper, an extensive analysis might be required to calculate results for several risk functions.
5) Calculate the individual failure rate for each component $i$ at time $t$.

An example of the results of the expected value for the normal population condition of TCG is shown in Fig. 1-f.). Fig. 1-f.) depicts the covariates sorted over the years 2002 to 2015 and the expected value and confidence intervals of the normal population condition for each year. The expected value of the normal condition ranges always on the boundary of condition 1 and 2, according to [24], which is 720 [Microliter/liter].

The individual failure rates have been calculated twice to show the impact of using different risk functions. Firstly, a positive linear risk function has been chosen for measurements 1, 2, and 4 and a negative linear risk function for measurement 3 because a decreasing BDV indicates a declining condition and increased probability of failure. For measurement 5, the TCG, a non-linear risk function has been chosen because a sudden increase of the TCG is associated with a higher probability of failure. The results of the 12 power transformers are depicted in Fig. 2a and 2b. Secondly, the cumulative risk function is assigned to all covariates and the results are depicted in Fig. 2c and 2d.

### B. Prediction of Individual Failure Rates

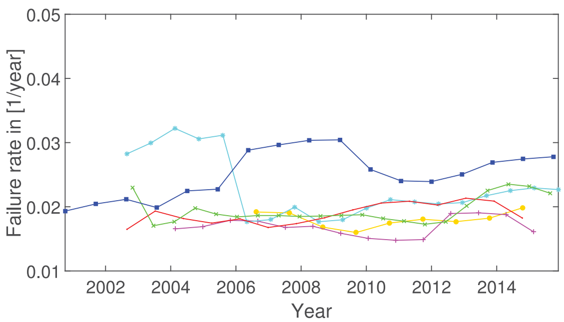
The possibility of predicting the internal covariate behaviour in combination with the proposed method allows us to forecast the individual failure rates for each transformer. Thus, the Box-Jenkins procedure has been applied to model the univariate time series of the covariates. Owing to the fact that each covariate has to be modelled separately, only the covariates of power transformers T6 and T9 are selected to show the general principle. The time series of the covariates have been modelled as ARIMAX and ARMA processes depending on the stationarity of the time series. The validated models are used to forecast the covariate behaviour for a time interval of 5 years. Assuming that the minimum mean square error of the processes is normally distributed, Monte Carlo simulation can be used to calculate the forecast responses. The forecast responses are calculated with a 5 year horizon and 1000 simulation paths. From these 1000 possible forecasts, a mean forecast with 95% prediction intervals can be calculated. The measured and forecasted time series for the covariate water content of transformer T6 and T9 is shown in Fig. 3. It is recommend to have at least 50 or 100 observations to effectively fit a Box-Jenkins model [25] which is not possible in this case study. However, to generally illustrate the proposed method, the amount of covariate data is sufficient but hence the exact model parameters are not presented in this paper.

The forecasted covariate paths can now be used to calculate the individual failure rates for power transformers T6 and T9. Here, the linear and cumulative risk functions have been chosen and the results simulated in Fig. 4a and 4b, respectively. The nonlinear risk function describes a sudden change in the covariate path which might be due to a specific event. Thus, this function type is unsuitable for this type of forecast.

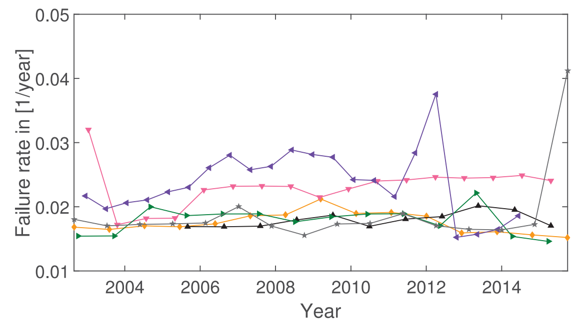
## V. Discussion

### A. Case Study

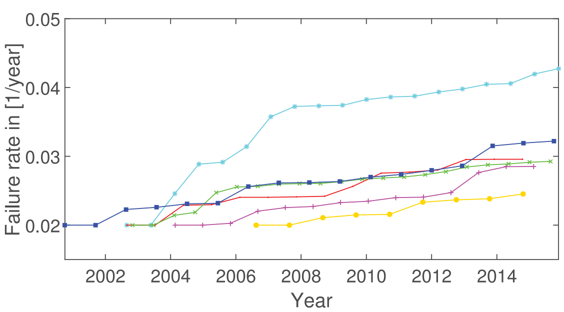
After careful assessment of the obtained covariates, it can be concluded that none of the 12 power transformer is in a critical condition or any action is required. Hence, no unusual high failure rates should be observed from the calculated results in Fig. 2. Nevertheless, two relevant observations can be drawn



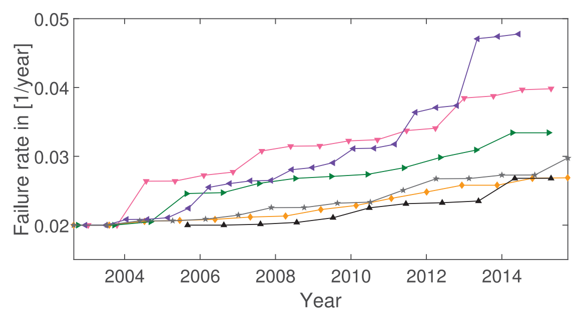

(a) Individual failure rates for transformers 1-6 with a positive linear risk function for measurement 1, 2, and 4, a negative linear for measurement 3, and the non linear risk function for measurement 5.




(b) Individual failure rates for transformers 7-12 with a positive linear risk function for measurement 1, 2, and 4, a negative linear for measurement 3, and the non linear risk function for measurement 5.

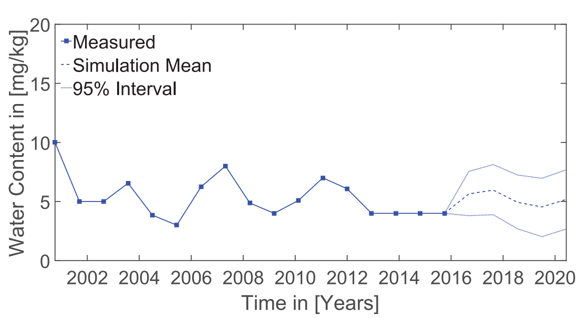



(c) Individual failure rates for transformers 1-6 calculated with the cumulative risk function for all measurements.

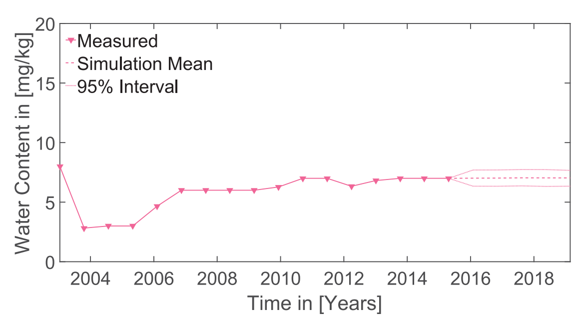



(d) Individual failure rates for transformers 7-12 calculated with the cumulative risk function for all measurements.

Fig. 2. Estimated individual failure rates with suggested risk functions over the time period 2002 to 2015.

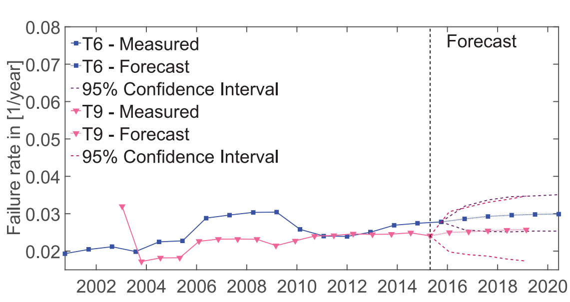



(a) Water content forecast with ARMA(2,2) process.

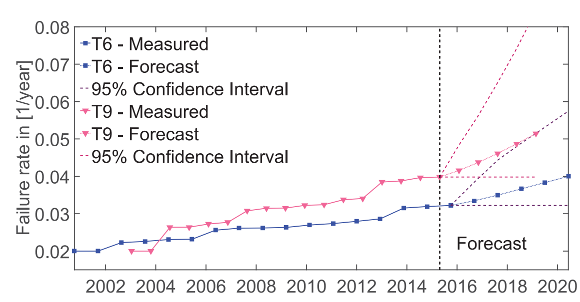



(b) Water content forecast with ARIMA(1,1,1) process.

Fig. 3. Five year forecast of the water content covariates for transformer T6 and T9.




(a) Forecasted individual failure rates for transformers 6 with baseline failure rate of 0.02 and a positive linear risk function for measurement 1, 2, 4, and 5 a negative linear for measurement 3.




(b) Forecasted individual failure rates for transformers 9 with baseline failure rate of 0.02 and the cumulative risk function for all measurements.

Fig. 4. Five year forecast of individual failure rates for transformer T6 and T9.

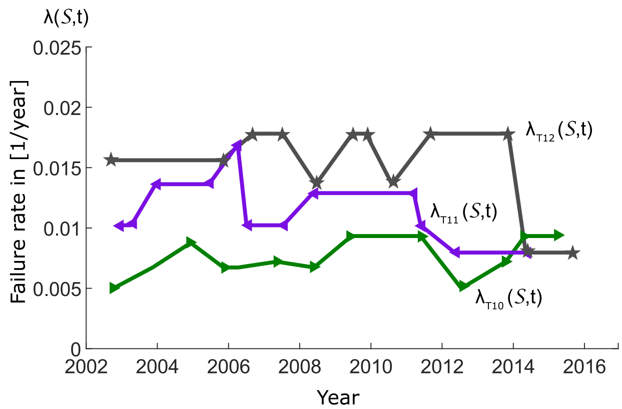



Fig. 5. Computed failure rates for power transformer T10, T11, and T12 based on the method in [11] and presented in [26]. The failure rate $\lambda(\mathcal{S}, t)$ is a function of the condition score $\mathcal{S}$ and time $t$.

from the results. Firstly, the maintenance impact becomes visible in, for example, Fig. 2b where the IFR of T11 first increases from 2011 to 2012 and suddenly decreases because maintenance was performed on this transformer. Secondly, from Fig. 2b and 2c, where the cumulative risk function has been applied, it can be noted that the IFRs of T3, T9, and T11 are higher compared to the rest which is reasonable since all of them are already operated for over 40 years. This indicates that the choice of the risk function has an impact on the results and consequently affects decision making and the outcome of subsequent calculations. For example, the linear and non-linear risk functions are better suited for maintenance decision making whereas the cumulative risk function reflects a long term view and thus is better for reinvestment decision making.

Applying the method developed in [11] to the available data, of power transformer T10, T11, and T12, for example, results in the failure rates presented in Fig. 5. To do so, a condition score $\mathcal{S}$ is calculated and transformed into a failure rate based on [11]. This has been described in detail in [26]. Comparing the results to the failure rate of T10, T11, and T12 in Fig. 2b to Fig. 5, the results are similar but an offset might be noted.

Fig. 4 depicts the forecasted IFRs with a horizon of 5 years. Applying the linear risk functions for the forecast in Fig. 4a, the predicted IFRs are rather constant but T9 has the wider confidence interval. In Fig. 4b where the cumulative risk functions have been applied, the mean of the IFRs is increasing from 0.033 to 0.040 and 0.041 to 0.051 for T6 and T9, respectively. Again, the confidence interval for T9 is wider than the confidence interval for T6 which can be explained by the greater variations in the covariate paths of T9. The illustration of the predicted IFRs with a mean and confidence interval has the advantage that we can interpret the predicted IFRs with some uncertainty. Moreover, the predicted IFR path for further applications can be chosen based on the risk behaviour of the decision maker. Here, the decision maker can be risk-averse by choosing the upper 95% confidence interval of the IFR, risk-neutral by using the mean of the predicted IFR, or risk-seeking by choosing the lower 95% confidence interval path. Depending on the choice, the results of subsequent calculations can be significantly different.

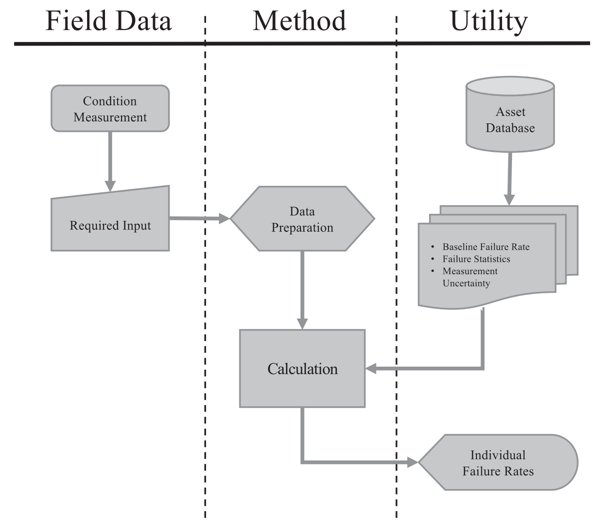



Fig. 6. Illustration of required field and utility data in connection to the IFR method.

## B. Theoretical Limitations

The validation of IFRs remains a challenge due to the limited amount of failure data in combination with extensive condition monitoring data required. A validation of IFRs with single time measurements has been conducted in [26] which showed a sensitivity of 77%. Here, the sensitivity describes the ability to correctly estimate failures based on the gathered information of condition measurements. However, conducting a similar validation for the time-dependent case requires to record all condition monitoring over a long interval combined with actual failures. Moreover, the fusion of multiple covariates to one failure mode and an extended set of risk functions would increase the applicability of the method. Particularly, setting the weights of the covariates is in practise highly dependent on expert knowledge. Thus, choosing the weights might be biased. The set of risk functions should be extended to fit more risk behaviour but also should be further assessed and validated.

## C. Practical Aspects

In an asset management context, the IFRs are input data to improve system reliability calculations and maintenance planning, scheduling, and optimization but can also be used as decision-making tool with caution [14]. Applied to maintenance and replacement decision-making, the IFRs, generally as all failure rates, should be correctly interpreted. A threshold might be determined to support the decision process. In comparison to other decision-making tools such as Health Indices, the IFRs do not provide a linguistic condition classification and the operator needs to interpret and understand the IFRs which prevent immature decisions. Generally, the concept of IFRs gives additional insights to risk management of power system components but is particularly of advantage for new components where little or no failure data exists. The method utilizes the available data in utilities such as an average or constant failure rate of different component types [11], related failure statistics [21], or the measurement uncertainty given in standards as presented in Table II.

This is illustrated in Fig. 6. In maintenance optimization the failure rate is used as an input parameter to the optimization algorithm. Traditionally, the average failure rate has been used and is thus equal for all components of the same type. This limits the results of the optimization algorithm. The IFRs are representing the individual probabilities of failure for each component and consequently the maintenance optimization algorithm computes better results.

## VI. Conclusion

In this paper, the concept of individual failure rates has been further developed by considering time-dependence of internal covariates and introducing new risk functions. In addition, the IFR can be forecasted by predicting the covariate behaviour. Based on the relative risk approach, an expected value of the normal condition can be calculated as a reference value and the actual risk behaviour of each individual component can be estimated. The results illustrate that by applying different risk functions IFRs can be estimated and predicted. Although, the studied power transformer population is in a very good condition, the impact of maintenance becomes visible and by using the cumulative risk function, it can be noted that the power transformer with the longest operation time have higher IFRs. The IFRs can subsequently be used in different methods such as component availability calculations, reliability block diagrams, Markov models, interruption indices as well as in maintenance and investment decision making depending on the chosen risk function.

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## References

[1] I. J. Pérez-Arriaga, *Regulation of the Power Sector*. Berlin, Germany: Springer Science & Business Media, 2014.

[2] K. Alvehag and K. Awodele, “Impact of reward and penalty scheme on the incentives for distribution system reliability,” *IEEE Trans. Power Syst.*, vol. 29, no. 1, pp. 386–394, Jan. 2014.

[3] P. Dehghanian, M. Fotuhi-Firuzabad, F. Aminifar, and R. Billinton, “A comprehensive scheme for reliability centered maintenance in power distribution systems—Part I: Methodology,” *IEEE Trans. Power Del.*, vol. 28, no. 2, pp. 761–770, Apr. 2013.

[4] L. Bertling, R. Allan, and R. Eriksson, “A reliability-centered asset maintenance method for assessing the impact of maintenance in power distribution systems,” *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 75–82, Feb. 2005.

[5] P. Dehghanian, M. Fotuhi-Firuzabad, F. Aminifar, and R. Billinton, “A comprehensive scheme for reliability-centered maintenance in power distribution systems—Part II: Numerical analysis,” *IEEE Trans. Power Del.*, vol. 28, no. 2, pp. 771–778, Apr. 2013.

[6] E. Shayesteh and P. Hilber, “Reliability-centered asset management using component reliability importance,” in *Proc. Int. Conf. Probabilistic Methods Appl. to Power Syst.*, Oct. 2016, pp. 1–6.

[7] A. Abiri-Jahromi, M. Parvania, F. Bouffard, and M. Fotuhi-Firuzabad, “A two-stage framework for power transformer asset maintenance management—Part I: Models and formulations,” in *Proc. IEEE Power Energy Soc. General Meeting*, Jul. 2013, p. 1.

[8] A. Moradkhani, M. R. Haghifam, and M. Mohammadzadeh, “Bayesian estimation of overhead lines failure rate in electrical distribution systems,” *Int. J. Elect. Power Energy Syst.*, vol. 56, pp. 220–227, 2014.

[9] Y. Hong, W. Q. Meeker, and J. D. McCalley, “Prediction of remaining life of power transformers based on left truncated and right censored lifetime data,” *Ann. Appl. Statist.*, vol. 3, no. 2, pp. 857–879, 2009.

[10] J. Pylvanainen, P. Verho, J. Jarvinen, S. Kunttu, and J. Sarsama, “Advanced failure rate and distribution network reliability modelling as part of network planning software,” in *Proc. 18th Int. Conf. Exhib. Electricity Distrib.*, 2005, pp. 1–5.

[11] R. Brown, G. Frimpong, and H. Willis, “Failure rate modeling using equipment inspection data,” *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 782–787, May 2004.

[12] R. E. Brown and M. Marshall, “Budget constrained planning to optimize power system reliability,” *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 887–892, May 2000.

[13] J. Pathak, Y. Jiang, V. Honavar, and J. McCalley, “Condition data aggregation with application to failure rate calculation of power transformers,” in *Proc. 39th Annu. Hawaii Int. Conf. Syst. Sci.*, Kauia, HI, USA, 2006, pp. 1–10.

[14] J. H. Jürgensen, L. Nordström, and P. Hilber, “Individual failure rates for transformers within a population based on diagnostic measures,” *Electric Power Syst. Res.*, vol. 141, pp. 354–362, 2016.

[15] D. R. Cox, “Regression models and life-tables,” *J. Roy. Statist. Soc. Ser. B Methodol.*, vol. 32, no. 2, pp. 187–220, 1972.

[16] J. D. Kalbfleisch and R. L. Prentice, *The Statistical Analysis of Failure Time Data*, vol. 360. Hoboken, NJ, USA: Wiley, 2011.

[17] J. H. Jürgensen, L. Nordström, and P. Hilber, “A review and discussion of failure rate heterogeneity in power system reliability assessment,” in *Proc. Int. Conf. Probabilistic Methods Appl. to Power Syst.*, Oct. 2016, pp. 1–8.

[18] R. J. de Andrade Vieira and M. A. Sanz-Bobi, “Failure risk indicators for a maintenance model based on observable life of industrial components with an application to wind turbines,” *IEEE Trans. Rel.*, vol. 62, no. 3, pp. 569–582, Sep. 2013.

[19] P.-J. an Vlok, “Dynamic residual life estimation of industrial equipment based on failure intensity proportions,” Ph.D. dissertation, Dep. Ind. Eng., Univ. Pretoria, Pretoria, South Africa, 2001.

[20] G. E. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time Series Analysis: Forecasting and Control*. Hoboken, NJ, USA: Wiley, 2015.

[21] S. Tenbohlen, F. Vahidi, P. Müller, J. Gebauer, and M. Krüger, “Zuverlässigkeitsbewertung von leistungstransformatoren,” CIGRE Working Group A 2.37 Transformer Reliability Survey, 2012.

[22] G. Pukel, H. Muhr, and W. Lick, “Transformer diagnostics: Common used and new methods,” in *Proc. Int. Conf. Condition Monit. Diagnosis*, 2006, pp. 1–4.

[23] A. Jahromi, R. Piercy, S. Cress, J. Service, and W. Fan, “An approach to power transformer asset management using health index,” *IEEE Elect. Insul. Mag.*, vol. 25, no. 2, pp. 20–34, Mar. 2009.

[24] *IEEE Guide for the Interpretation of Gases Generated in Oil-Immersed Transformers*, IEEE Standard C57.104-2008 (Revision of IEEE Standard C57.104-1991), Feb. 2009.

[25] C. Croarkin, P. Tobias, and C. Zey, *Engineering Statistics Handbook*. Gaithersburg, MD, USA: iTL, NIST, 2002.

[26] J. H. Jürgensen, “Individual failure rate modelling and exploratory failure data analysis for power system components,” Ph.D. dissertation, Dept. Electromagn. Eng., KTH, Zurich, Switzerland, 2018.