Introduction to control systems
TI0118

Homework I

Guidelines

You must solve each exercise using hand calculations and compare your results on Matlab/Octave. The report for the homework must be sent in a **single pdf document**, which must include:

- The relevant steps, the results and your comments on each exercise.
- The Matlab/Octave code developed for solving each exercise. This can be included as part of the single exercise or in a separate section of the report (for instance, as an appendix).
- The graphs required in the exercises.

Exercise 1

15 Points

Find the inverse Laplace transform by hand calculations of the following:

1. $F_1(s) = \dfrac{s-10}{(s+2)(s+5)}$
2. $F_2(s) = \dfrac{100}{(s+1)(s^2+4s+13)}$
3. $F_3(s) = \dfrac{s+18}{s(s+3)^2}$

Verify your results using the Symbolic toolbox in Matlab/Octave.

Exercise 2

30 Points

For a system with the following transfer function:

$$G(s) = \frac{\alpha s + 10}{(s^2 + 12s + 32)}$$

1. For $\alpha = 1$, find and plot the unit step and impulse response.

2. For $\alpha = [-4,\ -2,\ -1,\ 0,\ 1,\ 2,\ 4]$, plot and compare the unit step and impulse response.

3. Discuss your results.

Exercise 3

35 Points

Two tanks have cross-sectional areas S_1 and S_2 [m^2], respectively, and are arranged as shown in Figure 1.

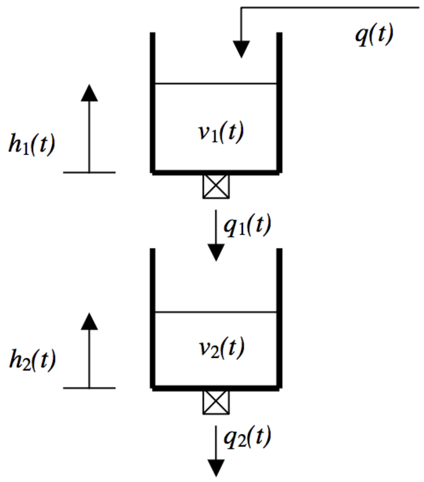

Figure 1: Two-tank system for Exercise 3.

The liquid heights on the two tanks are, respectively, $h_1(t)$ and $h_2(t)$ [m] and $v_1(t)$ and $v_2(t)$ are the liquid volumes in the tanks.

The first tank is fed by a flow-rate $q(t)$ [m^3/s] and has an output flow-rate given by $q_1(t) = K_1 h_1(t)$ [m^3/s]. The first tank feeds the second one whose output flow-rate is given by $q_2(t) = K_2 h_2(t)$ [m^3/s]. The mass conservation law for an incompressible fluid states that the derivatives of the liquid fluid $v(t)$ in the tank is given as

$$\frac{dv(t)}{dt} = q_{in}(t) - q_{out}(t)$$

1. Define a state space model for the system where $x_1(t) = v_1(t)$ and $x_2(t) = v_2(t)$ are the state variables, $u(t) = q(t)$ as input and $y(t) = h_2(t)$ as output.

2. Simulate the state space system, given the following values $S_1 = 2,\ S_2 = 4,\ K_1 = 3,\ K_2 = 4$ and the following inputs:

(a) A step with amplitude 2 [m^3/s] (Hint: Use `stepDataOptions` for modifying the step amplitude).

(b) A square signal with period 5 seconds, duration 30 seconds, and sampling every 0.1 second (Hint: Use `gensig` to generate the input signal).

3. Discuss your results.

Exercise 4

20 Points

Given a system described by the IO model in Equation 1:

$$4\ddot{y}(t) + 7\dot{y}(t) + 3y(t) = \ddot{u}(t) + 4\dot{u}(t) + 4u(t) \tag{1}$$

1. Express the system as transfer function $G(s) = U(s)/Y(s)$.
2. Find a state space representation of the system.
3. Verify and comment your results on Matlab/Octave.