HW1Notebook

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1 Departamento de Engenharia de Teleinformática

2 Inteligência Computacional Aplicada - HW1

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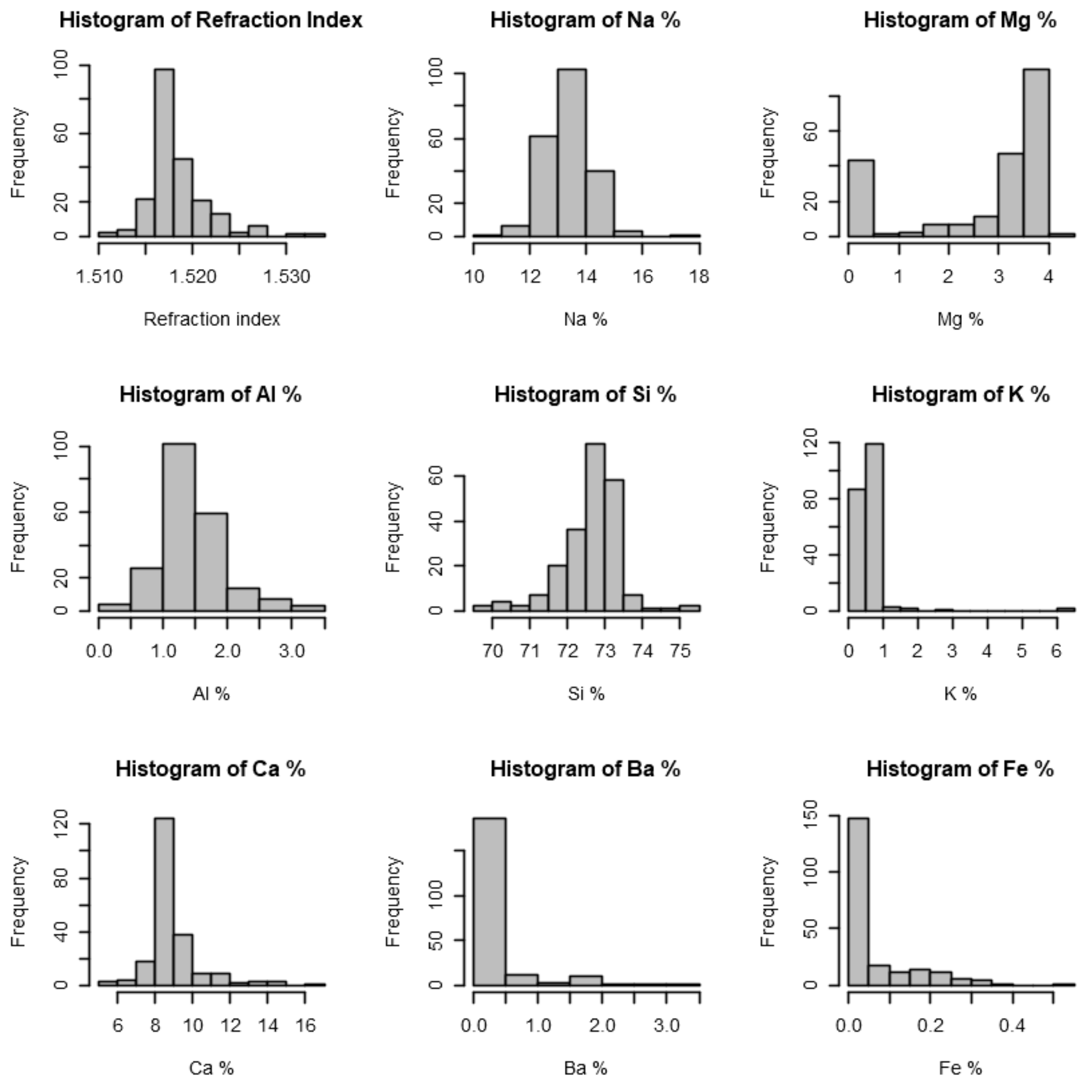
4.1 Questão 1 e 2

Nas primeiras duas questões, foi pedido que analisassemos os histogramas dos *predictors*, além de suas médias, desvio padrão e *skewness*, sendo na primeira englobando todas as amostras e na segunda dividindo-os por tipo. Começaremos, portanto, primeiro com o histograma por *predictors* de todas as amostras.

```
In [38]: library(mlbench);
         library("AppliedPredictiveModeling");
         library("e1071");
         library(corrplot);
         library(ggplot2);
         library(ggfortify);
         data(Glass);

         #Questao 1

         par(mfrow=c(3,3))

         RI.hist <- hist(Glass$RI, main = "Histogram of Refraction Index",
                         xlab = "Refraction index", col = "Gray")

         Na.hist <- hist(Glass$Na, main = "Histogram of Na %",
                         xlab = "Na %", col = "Gray")

         Mg.hist <- hist(Glass$Mg, main = "Histogram of Mg %",
                         xlab = "Mg %",col = "Gray")

         Al.hist <- hist(Glass$Al, main = "Histogram of Al %",
```

```
                xlab = "Al %", col = "Gray")

Si.hist <- hist(Glass$Si, main = "Histogram of Si %",
                xlab = "Si %",col = "Gray")

K.hist <- hist(Glass$K, main = "Histogram of K %",
               xlab = "K %",col = "Gray")

Ca.hist <- hist(Glass$Ca, main = "Histogram of Ca %",
                xlab = "Ca %",col = "Gray")

Ba.hist <- hist(Glass$Ba, main = "Histogram of Ba %",
                xlab = "Ba %",col = "Gray")

Fe.hist <- hist(Glass$Fe, main = "Histogram of Fe %",
                xlab = "Fe %", col = "Gray")

meanAll <- c(mean(Glass$RI), mean(Glass$Na), mean(Glass$Mg),
             mean(Glass$Al), mean(Glass$Si), mean(Glass$K),
             mean(Glass$Ca), mean(Glass$Ba), mean(Glass$Fe))
devAll <- c(sd(Glass$RI), sd(Glass$Na), sd(Glass$Mg),
            sd(Glass$Al), sd(Glass$Si), sd(Glass$K),
            sd(Glass$Ca), sd(Glass$Ba), sd(Glass$Fe))
skewAll <- c(skewness(Glass$RI), skewness(Glass$Na), skewness(Glass$Mg),
             skewness(Glass$Al), skewness(Glass$Si),
             skewness(Glass$K), skewness(Glass$Ca),
             skewness(Glass$Ba), skewness(Glass$Fe))
dataAll <- data.frame(meanAll, devAll, skewAll)
rownames(dataAll) <- c("RI","Na","Mg","Al","Si","K","Ca","Ba","Fe")
colnames(dataAll) <- c("Mean", "Standard Deviation", "Skewness")

dataAll
```

	Mean	Standard Deviation	Skewness
RI	1.51836542	0.003036864	1.6027151
Na	13.40785047	0.816603556	0.4478343
Mg	2.68453271	1.442407845	-1.1364523
Al	1.44490654	0.499269646	0.8946104
Si	72.65093458	0.774545795	-0.7202392
K	0.49705607	0.652191846	6.4600889
Ca	8.95696262	1.423153487	2.0184463
Ba	0.17504673	0.497219261	3.3686800
Fe	0.05700935	0.097438701	1.7298107



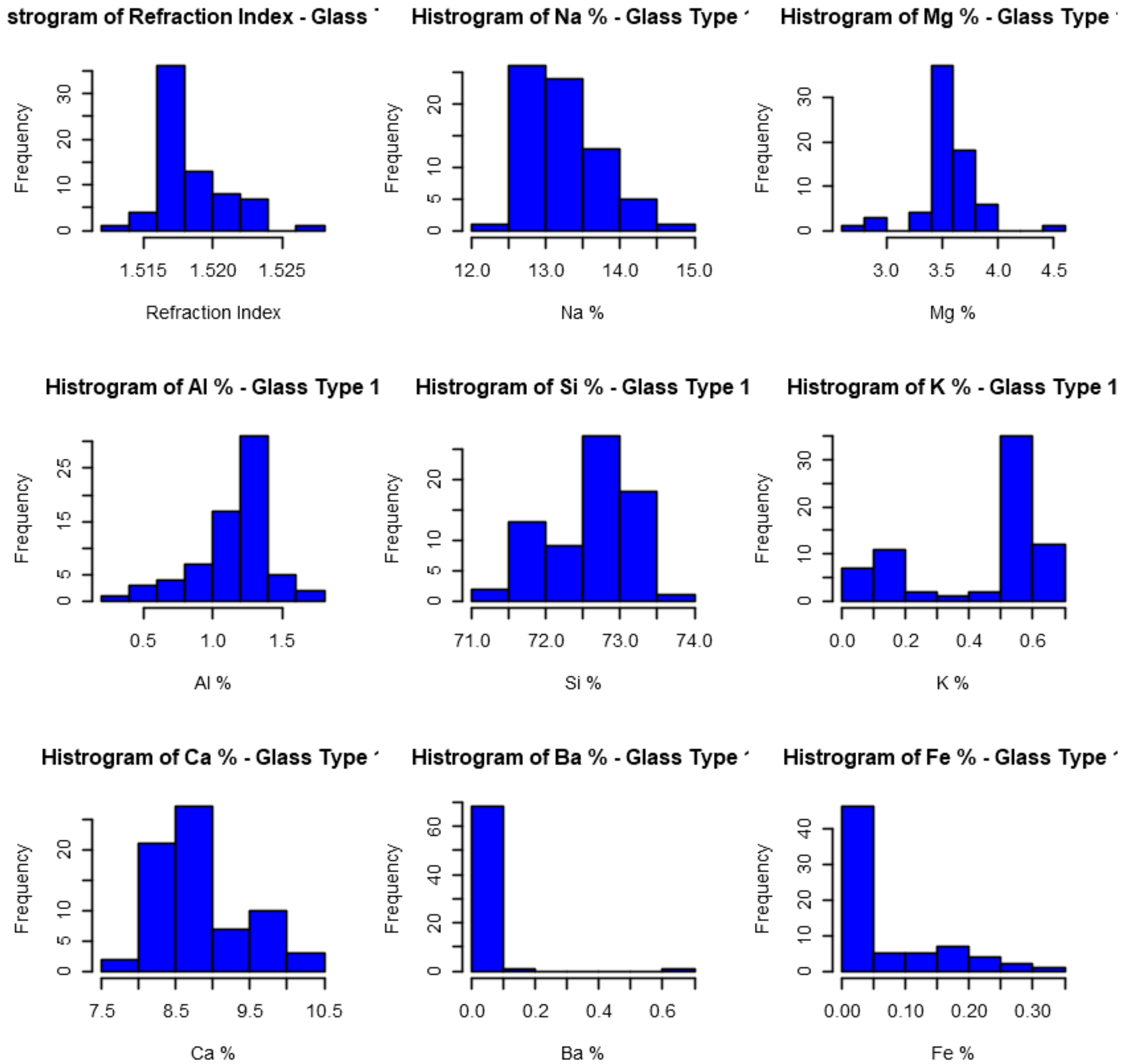
Agora, analisaremos os histogramas das amostras por Tipo, indo de 1 a 7 (ressaltamos que não existem amostras do tipo 4).

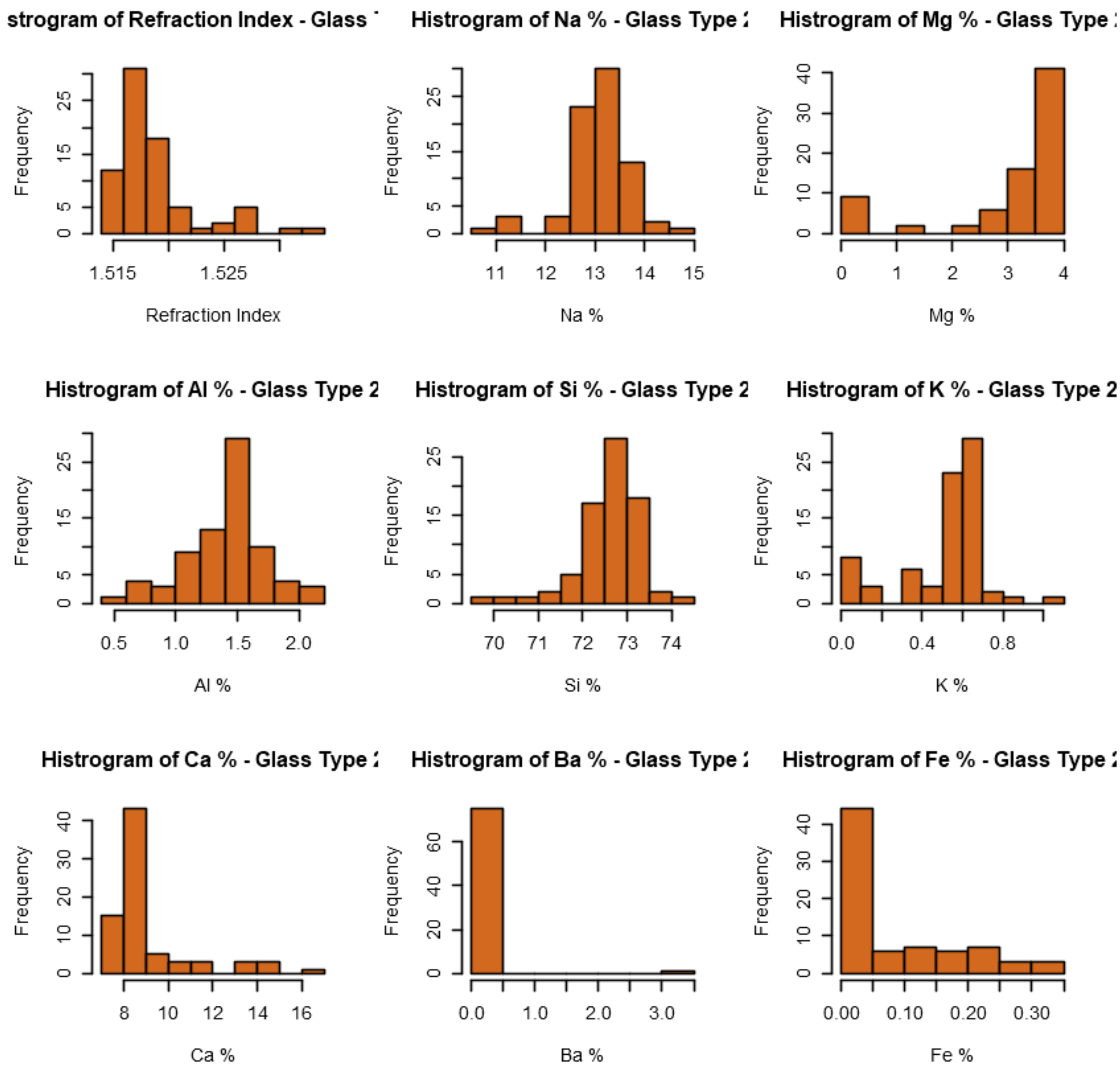
```
In [12]: #Questao 2

         par(mfrow=c(3,3))
         GlassByType <- split(Glass, Glass$Type)
         meanByType <- data.frame(RI = numeric(), Na = numeric(), Mg = numeric(),
                                  Al = numeric(), Si = numeric(), K = numeric(),
                                  Ca = numeric(), Ba = numeric(), Fe = numeric())
         devByType <- data.frame(RI = numeric(), Na = numeric(), Mg = numeric(),
                                 Al = numeric(), Si = numeric(), K = numeric(),
                                 Ca = numeric(), Ba = numeric(), Fe = numeric())
         skewByType <- data.frame(RI = numeric(), Na = numeric(), Mg = numeric(),
```

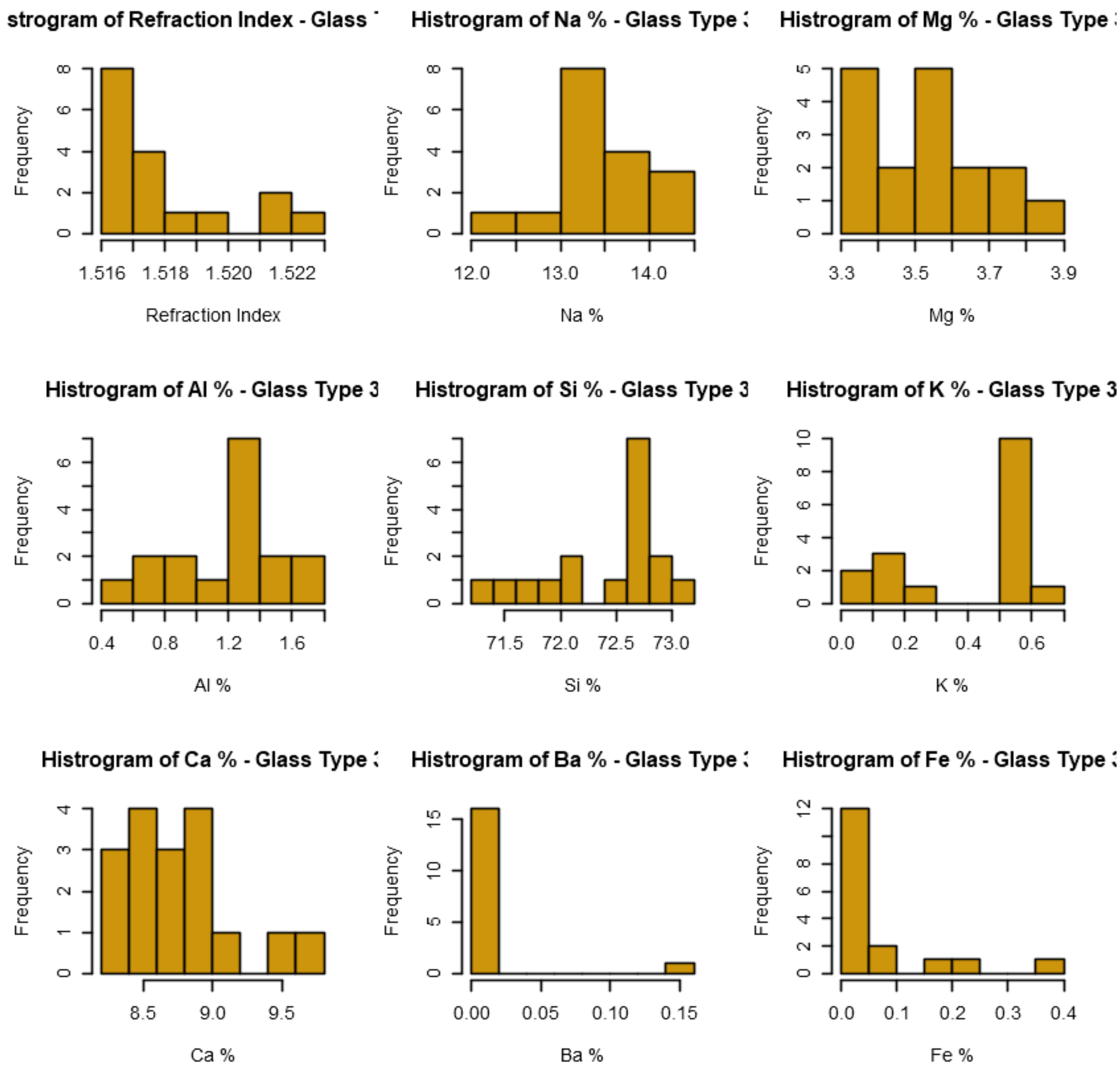
```r
                           Al = numeric(), Si = numeric(), K = numeric(),
                           Ca = numeric(), Ba = numeric(), Fe = numeric())

for (i in 1:3){

  hist(GlassByType[[i]]$RI,
       main = paste(c("Histrogram of Refraction Index - Glass Type",i), collapse = " ")
       xlab = "Refraction Index", col = colors()[26*i])

  hist(GlassByType[[i]]$Na,
       main = paste(c("Histrogram of Na % - Glass Type",i), collapse = " "),
       xlab = "Na %", col = colors()[26*i])

  hist(GlassByType[[i]]$Mg,
       main = paste(c("Histrogram of Mg % - Glass Type",i), collapse = " "),
       xlab = "Mg %", col = colors()[26*i])

  hist(GlassByType[[i]]$Al,
       main = paste(c("Histrogram of Al % - Glass Type",i), collapse = " "),
       xlab = "Al %", col = colors()[26*i])

  hist(GlassByType[[i]]$Si,
       main = paste(c("Histrogram of Si % - Glass Type",i), collapse = " "),
       xlab = "Si %", col = colors()[26*i])

  hist(GlassByType[[i]]$K,
       main = paste(c("Histrogram of K % - Glass Type",i), collapse = " "),
       xlab = "K %", col = colors()[26*i])

  hist(GlassByType[[i]]$Ca,
       main = paste(c("Histrogram of Ca % - Glass Type",i), collapse = " "),
       xlab = "Ca %", col = colors()[26*i])

  hist(GlassByType[[i]]$Ba,
       main = paste(c("Histrogram of Ba % - Glass Type",i), collapse = " "),
       xlab = "Ba %", col = colors()[26*i])

  hist(GlassByType[[i]]$Fe,
       main = paste(c("Histrogram of Fe % - Glass Type",i), collapse = " "),
       xlab = "Fe %", col = colors()[26*i])

meanByType <- rbind(meanByType, c(mean(GlassByType[[i]]$RI), mean(GlassByType[[i]]$Na),
                                  mean(GlassByType[[i]]$Mg), mean(GlassByType[[i]]$Al),
                                  mean(GlassByType[[i]]$Si), mean(GlassByType[[i]]$K),
                                  mean(GlassByType[[i]]$Ca), mean(GlassByType[[i]]$Ba),
                                  mean(GlassByType[[i]]$Fe)))
devByType <- rbind(devByType, c(sd(GlassByType[[i]]$RI), sd(GlassByType[[i]]$Na),
                                sd(GlassByType[[i]]$Mg), sd(GlassByType[[i]]$Al),
```

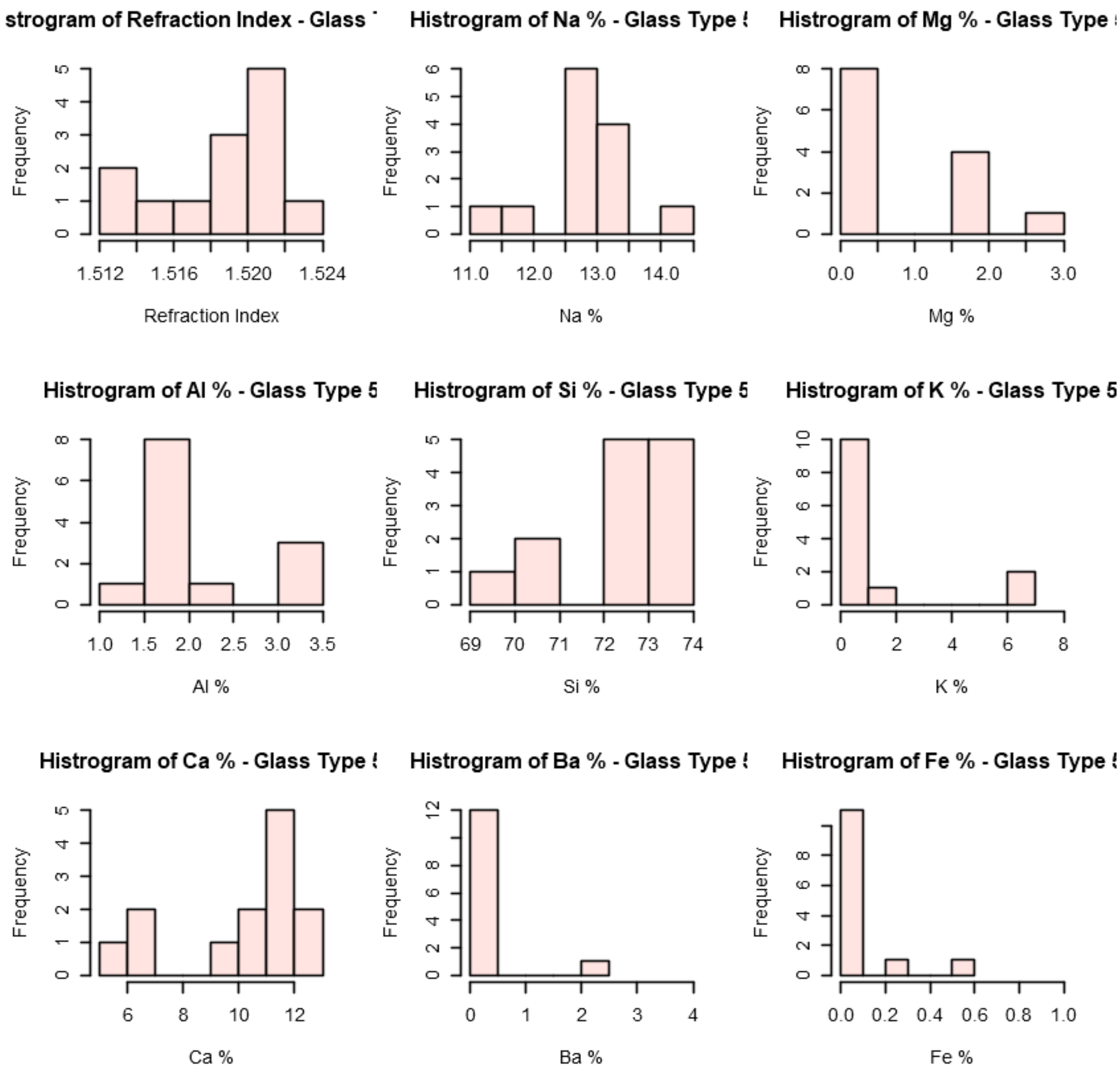
```r
                                   sd(GlassByType[[i]]$Si), sd(GlassByType[[i]]$K),
                                   sd(GlassByType[[i]]$Ca), sd(GlassByType[[i]]$Ba),
                                   sd(GlassByType[[i]]$Fe)))
skewByType <- rbind(skewByType, c(skewness(GlassByType[[i]]$RI),
                                    skewness(GlassByType[[i]]$Na),
                                    skewness(GlassByType[[i]]$Mg),
                                    skewness(GlassByType[[i]]$Al),
                                    skewness(GlassByType[[i]]$Si),
                                    skewness(GlassByType[[i]]$K),
                                    skewness(GlassByType[[i]]$Ca),
                                    skewness(GlassByType[[i]]$Ba),
                                    skewness(GlassByType[[i]]$Fe)))

}
for (i in 4:6){

  hist(GlassByType[[i]]$RI,
       main = paste(c("Histrogram of Refraction Index - Glass Type",i+1), collapse = "
       xlab = "Refraction Index", col = colors()[120*i])

  hist(GlassByType[[i]]$Na,
       main = paste(c("Histrogram of Na % - Glass Type",i+1), collapse = " "),
       xlab = "Na %", col = colors()[120*i])

  hist(GlassByType[[i]]$Mg,
       main = paste(c("Histrogram of Mg % - Glass Type",i+1), collapse = " "),
       xlab = "Mg %", col = colors()[120*i])

  hist(GlassByType[[i]]$Al,
       main = paste(c("Histrogram of Al % - Glass Type",i+1), collapse = " "),
       xlab = "Al %", col = colors()[120*i])

  hist(GlassByType[[i]]$Si,
       main = paste(c("Histrogram of Si % - Glass Type",i+1), collapse = " "),
       xlab = "Si %", col = colors()[120*i])

  hist(GlassByType[[i]]$K,
       main = paste(c("Histrogram of K % - Glass Type",i+1), collapse = " "),
       xlab = "K %", col = colors()[120*i], xlim = c(0,8))

  hist(GlassByType[[i]]$Ca,
       main = paste(c("Histrogram of Ca % - Glass Type",i+1), collapse = " "),
       xlab = "Ca %", col = colors()[120*i])

  hist(GlassByType[[i]]$Ba,
       main = paste(c("Histrogram of Ba % - Glass Type",i+1), collapse = " "),
       xlab = "Ba %", col = colors()[120*i], xlim = c(0,4))
```

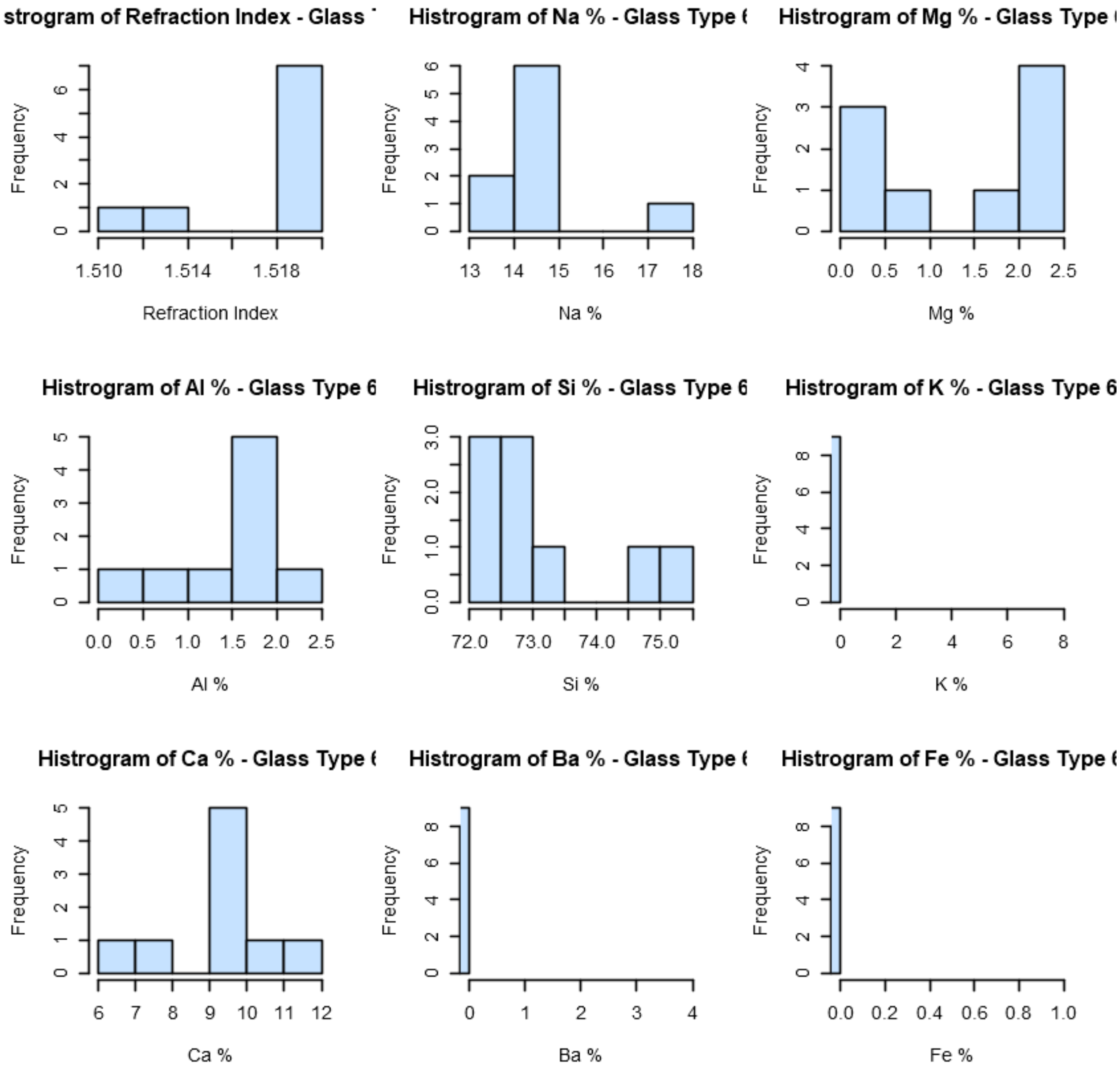
```r
  hist(GlassByType[[i]]$Fe,
       main = paste(c("Histrogram of Fe % - Glass Type",i+1), collapse = " "),
       xlab = "Fe %", col = colors()[120*i], xlim = c(0,1))

meanByType <- rbind(meanByType, c(mean(GlassByType[[i]]$RI), mean(GlassByType[[i]]$Na),
                                  mean(GlassByType[[i]]$Mg), mean(GlassByType[[i]]$Al),
                                  mean(GlassByType[[i]]$Ca), mean(GlassByType[[i]]$Ba),
                                  mean(GlassByType[[i]]$Fe)))
devByType <- rbind(devByType, c(sd(GlassByType[[i]]$RI), sd(GlassByType[[i]]$Na),
                                  sd(GlassByType[[i]]$Mg), sd(GlassByType[[i]]$Al),
                                  sd(GlassByType[[i]]$Si), sd(GlassByType[[i]]$K),
                                  sd(GlassByType[[i]]$Ca), sd(GlassByType[[i]]$Ba),
                                  sd(GlassByType[[i]]$Fe)))
skewByType <- rbind(skewByType, c(skewness(GlassByType[[i]]$RI),
                                  skewness(GlassByType[[i]]$Na),
                                  skewness(GlassByType[[i]]$Mg),
                                  skewness(GlassByType[[i]]$Al),
                                  skewness(GlassByType[[i]]$Si),
                                  skewness(GlassByType[[i]]$K),
                                  skewness(GlassByType[[i]]$Ca),
                                  skewness(GlassByType[[i]]$Ba),
                                  skewness(GlassByType[[i]]$Fe)))

}
rownames(meanByType) <- c("Tipo 1", "Tipo 2", "Tipo 3", "Tipo 5","Tipo 6","Tipo 7")
rownames(devByType) <-  c("Tipo 1", "Tipo 2", "Tipo 3", "Tipo 5","Tipo 6","Tipo 7")
rownames(skewByType) <- c("Tipo 1", "Tipo 2", "Tipo 3", "Tipo 5","Tipo 6","Tipo 7")

colnames(meanByType) <- c("RI","Na","Mg","Al","Si","K","Ca","Ba","Fe")
colnames(devByType) <- c("RI","Na","Mg","Al","Si","K","Ca","Ba","Fe")
colnames(skewByType) <- c("RI","Na","Mg","Al","Si","K","Ca","Ba","Fe")
```

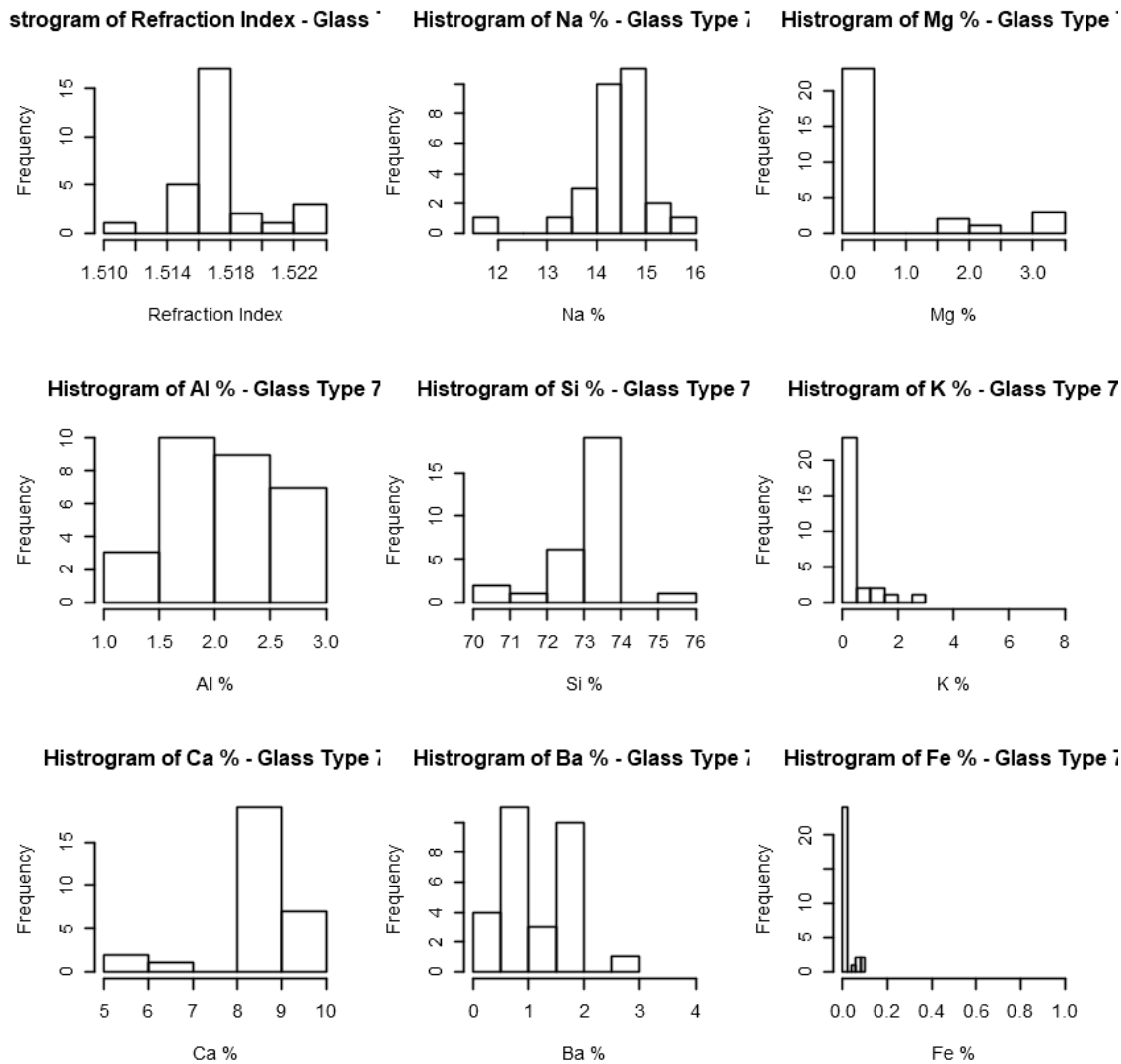

strogram of Refraction Index - Glass
Frequency
0 10 20 30
1.515 1.520 1.525
Refraction Index
Histrogram of Na % - Glass Type
Frequency
0 5 10 20
12.0 13.0 14.0 15.0
Na %
Histrogram of Mg % - Glass Type
Frequency
0 10 20 30
3.0 3.5 4.0 4.5
Mg %
Histrogram of Al % - Glass Type 1
Frequency
0 5 15 25
0.5 1.0 1.5
Al %
Histrogram of Si % - Glass Type 1
Frequency
0 5 10 20
71.0 72.0 73.0 74.0
Si %
Histrogram of K % - Glass Type 1
Frequency
0 10 20 30
0.0 0.2 0.4 0.6
K %
Histrogram of Ca % - Glass Type
Frequency
0 5 10 20
7.5 8.5 9.5 10.5
Ca %
Histrogram of Ba % - Glass Type
Frequency
0 20 40 60
0.0 0.2 0.4 0.6
Ba %
Histrogram of Fe % - Glass Type
Frequency
0 10 20 30 40
0.00 0.10 0.20 0.30
Fe %


strogram of Refraction Index - Glass
Frequency
Refraction Index
Histrogram of Na % - Glass Type
Na %
Histrogram of Mg % - Glass Type
Mg %
Histrogram of Al % - Glass Type 2
Al %
Histrogram of Si % - Glass Type 2
Si %
Histrogram of K % - Glass Type 2
K %
Histrogram of Ca % - Glass Type
Ca %
Histrogram of Ba % - Glass Type
Ba %
Histrogram of Fe % - Glass Type
Fe %


strogram of Refraction Index - Glass
Frequency
0 2 4 6 8
1.516 1.518 1.520 1.522
Refraction Index
Histrogram of Na % - Glass Type
Frequency
0 2 4 6 8
12.0 13.0 14.0
Na %
Histrogram of Mg % - Glass Type
Frequency
0 1 2 3 4 5
3.3 3.5 3.7 3.9
Mg %
Histrogram of Al % - Glass Type 3
Frequency
0 2 4 6
0.4 0.8 1.2 1.6
Al %
Histrogram of Si % - Glass Type 3
Frequency
0 2 4 6
71.5 72.0 72.5 73.0
Si %
Histrogram of K % - Glass Type 3
Frequency
0 2 4 6 8 10
0.0 0.2 0.4 0.6
K %
Histrogram of Ca % - Glass Type
Frequency
0 1 2 3 4
8.5 9.0 9.5
Ca %
Histrogram of Ba % - Glass Type
Frequency
0 5 10 15
0.00 0.05 0.10 0.15
Ba %
Histrogram of Fe % - Glass Type
Frequency
0 2 4 6 8 12
0.0 0.1 0.2 0.3 0.4
Fe %


strogram of Refraction Index - Glass
Frequency
Refraction Index
Histrogram of Na % - Glass Type
Frequency
Na %
Histrogram of Mg % - Glass Type
Frequency
Mg %
Histrogram of Al % - Glass Type 5
Frequency
Al %
Histrogram of Si % - Glass Type 5
Frequency
Si %
Histrogram of K % - Glass Type 5
Frequency
K %
Histrogram of Ca % - Glass Type
Frequency
Ca %
Histrogram of Ba % - Glass Type
Frequency
Ba %
Histrogram of Fe % - Glass Type
Frequency
Fe %


strogram of Refraction Index - Glass
Frequency
Refraction Index
Histrogram of Na % - Glass Type
Frequency
Na %
Histrogram of Mg % - Glass Type
Frequency
Mg %
Histrogram of Al % - Glass Type 6
Frequency
Al %
Histrogram of Si % - Glass Type 6
Frequency
Si %
Histrogram of K % - Glass Type 6
Frequency
K %
Histrogram of Ca % - Glass Type
Frequency
Ca %
Histrogram of Ba % - Glass Type
Frequency
Ba %
Histrogram of Fe % - Glass Type
Frequency
Fe %



4.1.1 Mean by type

```
In [6]: meanByType
```

	RI	Na	Mg	Al	Si	K	Ca	Ba	Fe
Tipo 1	1.518718	13.24229	3.5524286	1.163857	72.61914	0.4474286	8.797286	0.012714286	0.05700000
Tipo 2	1.518619	13.11171	3.0021053	1.408158	72.59803	0.5210526	9.073684	0.050263158	0.07973684
Tipo 3	1.517964	13.43706	3.5435294	1.201176	72.40471	0.4064706	8.782941	0.008823529	0.05705882
Tipo 5	1.518928	12.82769	0.7738462	2.033846	72.36615	1.4700000	10.123846	0.187692308	0.06076923
Tipo 6	1.517456	14.64667	1.3055556	1.366667	73.20667	0.0000000	9.356667	0.000000000	0.00000000
Tipo 7	1.517116	14.44207	0.5382759	2.122759	72.96586	0.3251724	8.491379	1.040000000	0.01344828

4.1.2 Standard Deviation by type

```
In [13]: devByType
```

	RI	Na	Mg	Al	Si	K	Ca	Ba	Fe
Tipo 1	0.002268097	0.4993015	0.2470430	0.2731581	0.5694842	0.2148790	0.5748066	0.08383769	0.08907496
Tipo 2	0.003802126	0.6641594	1.2156615	0.3183403	0.7245726	0.2137262	1.9216353	0.36234044	0.10643275
Tipo 3	0.001916360	0.5068871	0.1627859	0.3474889	0.5122758	0.2298897	0.3801112	0.03638034	0.10786361
Tipo 5	0.003345355	0.7770366	0.9991458	0.6939205	1.2823191	2.1386951	2.1837908	0.60825096	0.15558821
Tipo 6	0.003115783	1.0840203	1.0971339	0.5718610	1.0794675	0.0000000	1.4499483	0.00000000	0.00000000
Tipo 7	0.002545069	0.6863588	1.1176828	0.4427261	0.9402337	0.6684931	0.9735052	0.66534094	0.02979404

4.1.3 Skewness by type

```
In [14]: skewByType
```

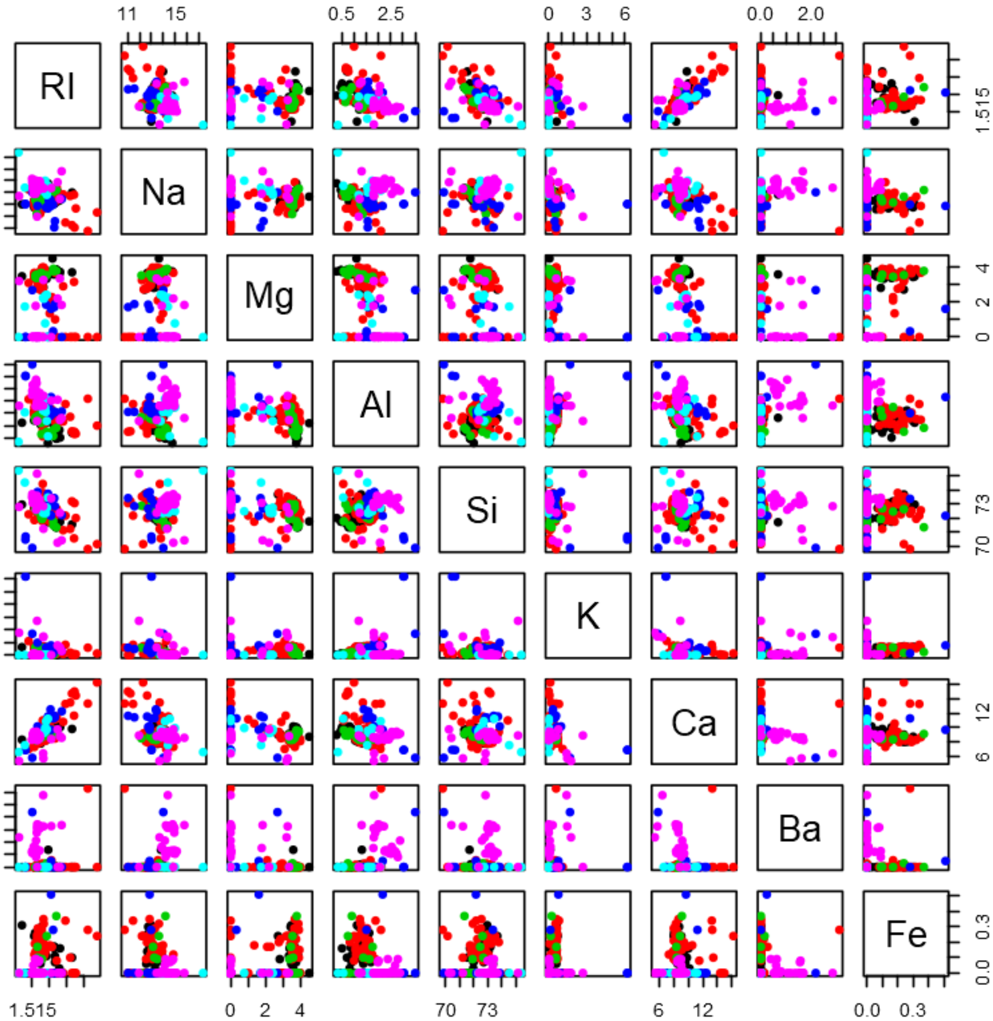
	RI	Na	Mg	Al	Si	K	Ca	Ba	Fe
Tipo 1	0.7437286	0.7538231	-0.6767263	-1.0800373	-0.5542691	-0.8997727	0.6863776	7.5619933	1.3041178
Tipo 2	2.0576334	-1.0495492	-1.7735903	-0.3715251	-1.3759015	-0.9699909	2.0816494	8.2385374	0.9489729
Tipo 3	0.9707950	-0.4619438	0.6021565	-0.3325460	-0.6913984	-0.6377215	0.7853627	3.4240324	1.6919393
Tipo 5	-0.5692461	-0.9348808	0.5860668	0.9982672	-0.6439219	1.5919612	-0.7898330	2.7606112	2.0163265
Tipo 6	-1.1310170	1.6786808	-0.2194845	-0.6702752	0.9974867	NaN	-0.5485900	NaN	NaN
Tipo 7	0.9765233	-1.4473029	1.6320747	-0.2928373	-1.2115324	2.1395094	-1.9355386	0.4505834	1.7805965

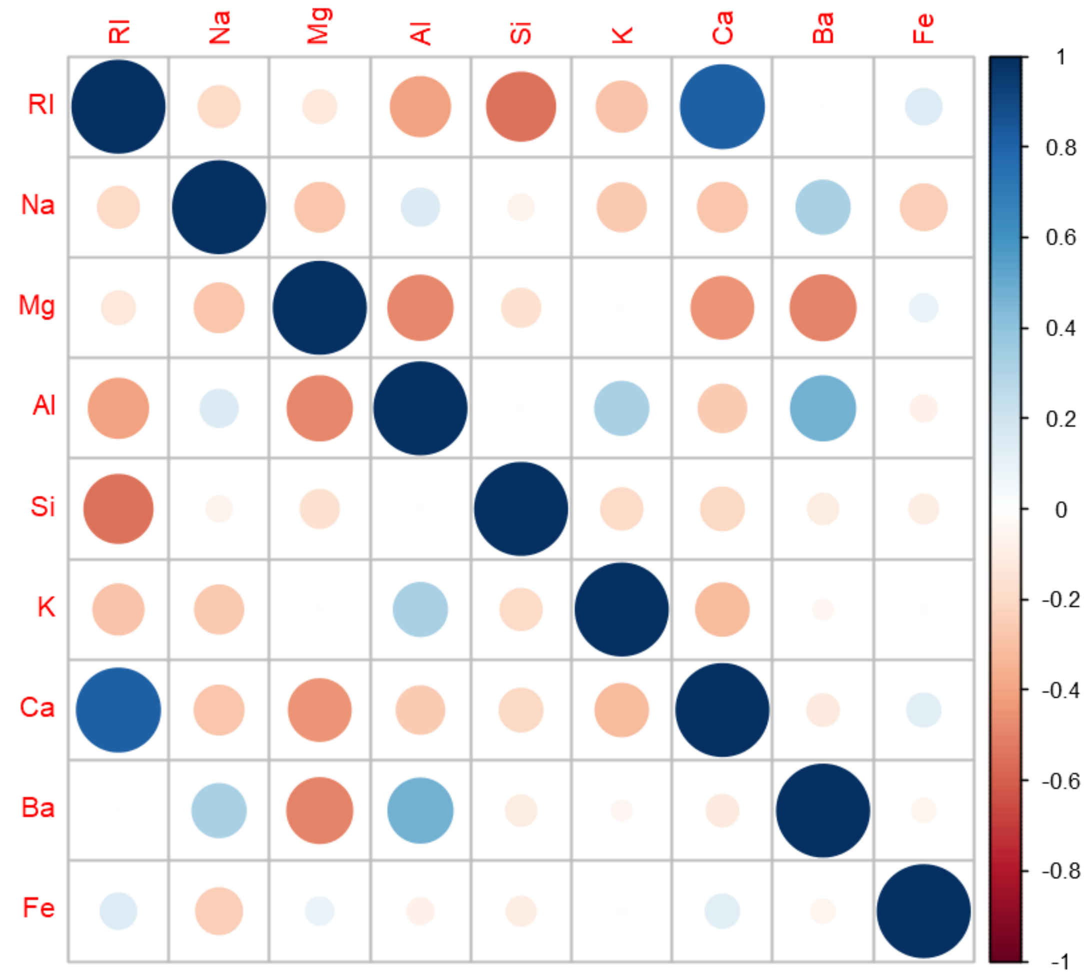
Analisando os histogramas, percebemos que não existe nenhum *predictor* com poder discriminativo. Ou seja, nenhum *predictor* sozinho consegue distinguir os tipos. Percebemos isso pois não há nenhuma relação distintiva dos histogramas. Nenhum deles aumentar linearmente, por exemplo.

4.2 Questão 3

Nessa questão, foi pedido um *scatter plot* dos *predictors*, além de uma analise deles e da matriz de correlação. Começamos mostrando o *scatter plot*.

```
In [17]: #Questao 3

         pairs(Glass[1:9], col = Glass$Type, pch=20)

         par(mfrow=c(1,1))
         correlation <- cor(Glass[1:9])
         corrplot(correlation)
```


RI
Na
Mg
Al
Si
K
Ca
Ba
Fe
11
15
0.5
2.5
0
3
6
0.0
2.0
1.515
0
2
4
70
73
6
12
0.0
0.3



Analisando o *scatter plot*, conseguimos identificar alguns graficos com aparentes relações lineares, como o do RI x Ca e o do RI x Si. Com a matriz de correlação, confirmamos nossas suspeitas ao visualisarmos que os dois elementos com maiores modulo são o do RI x Ca e do RI x Si. Isso indica que existe uma relação linear entre esses *predictors*, sendo a do RI x Ca proporcinal e a do RI x Si inversamente proporcional.

4.3 Questao 4

Por último, foi pedido que façamos uma PCA dos *predictors*. Pelos dados passadas, analisando a matriz de correlação, podemos adiantar que os dois principais componentes provavelmente serão o do RI e do Ca. Dessa forma, os dados serão ortogonalizados nesses dois *predictors* Agora, iremos para o codigo. Primeiro, vamos verificar que a função **prcomp** fornece valores coerentes. Para isso, compararemos os valores de PCs fornecidos por ela, com os autovalores e autovetores fornecidos pela matriz de correlação.

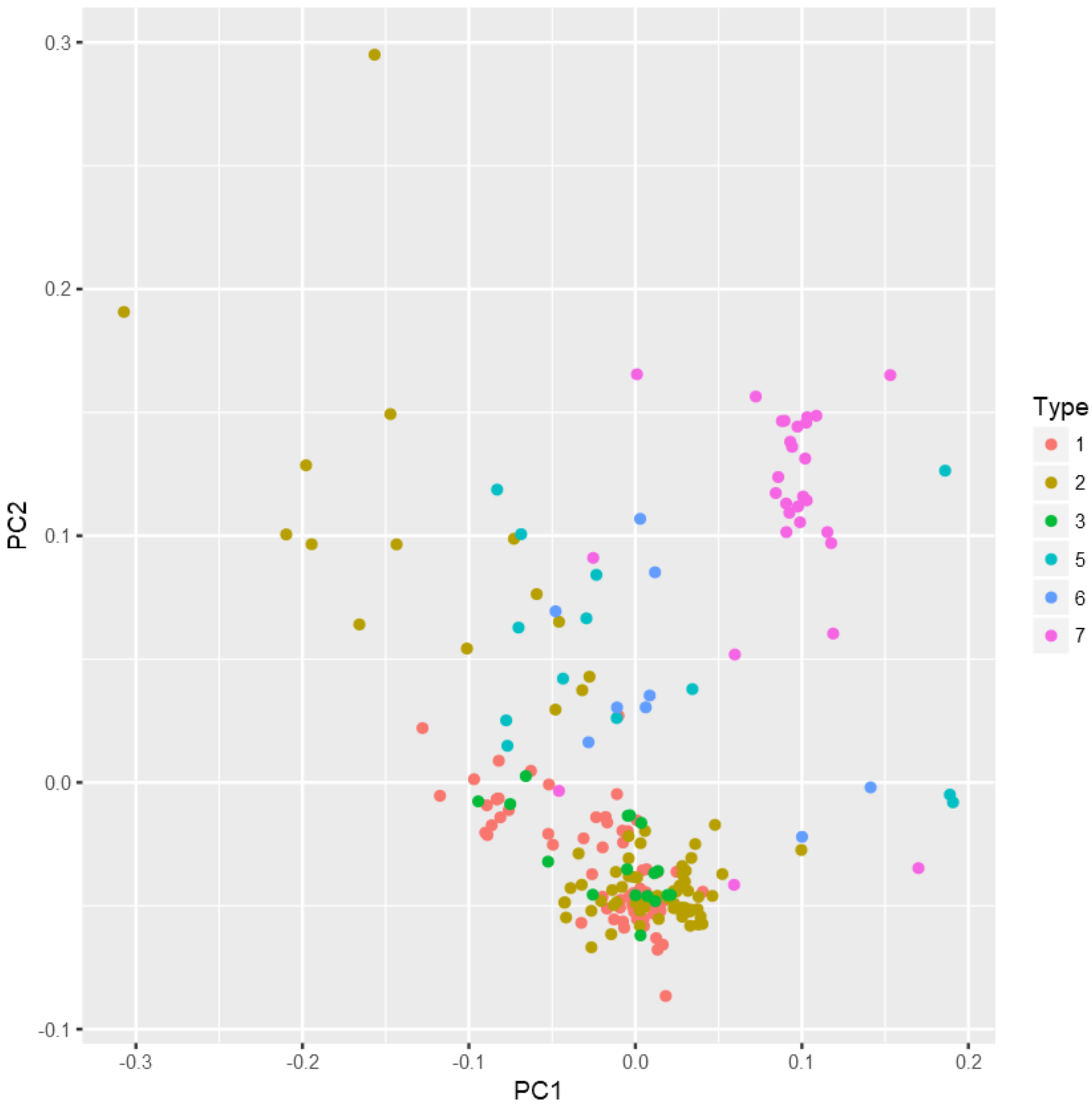
```
In [26]: #Questao 4

         pcaObject <- prcomp(Glass[1:9], center = TRUE, scale. = TRUE)

         pcaObject$rotation
         eigen(correlation)$vectors
```

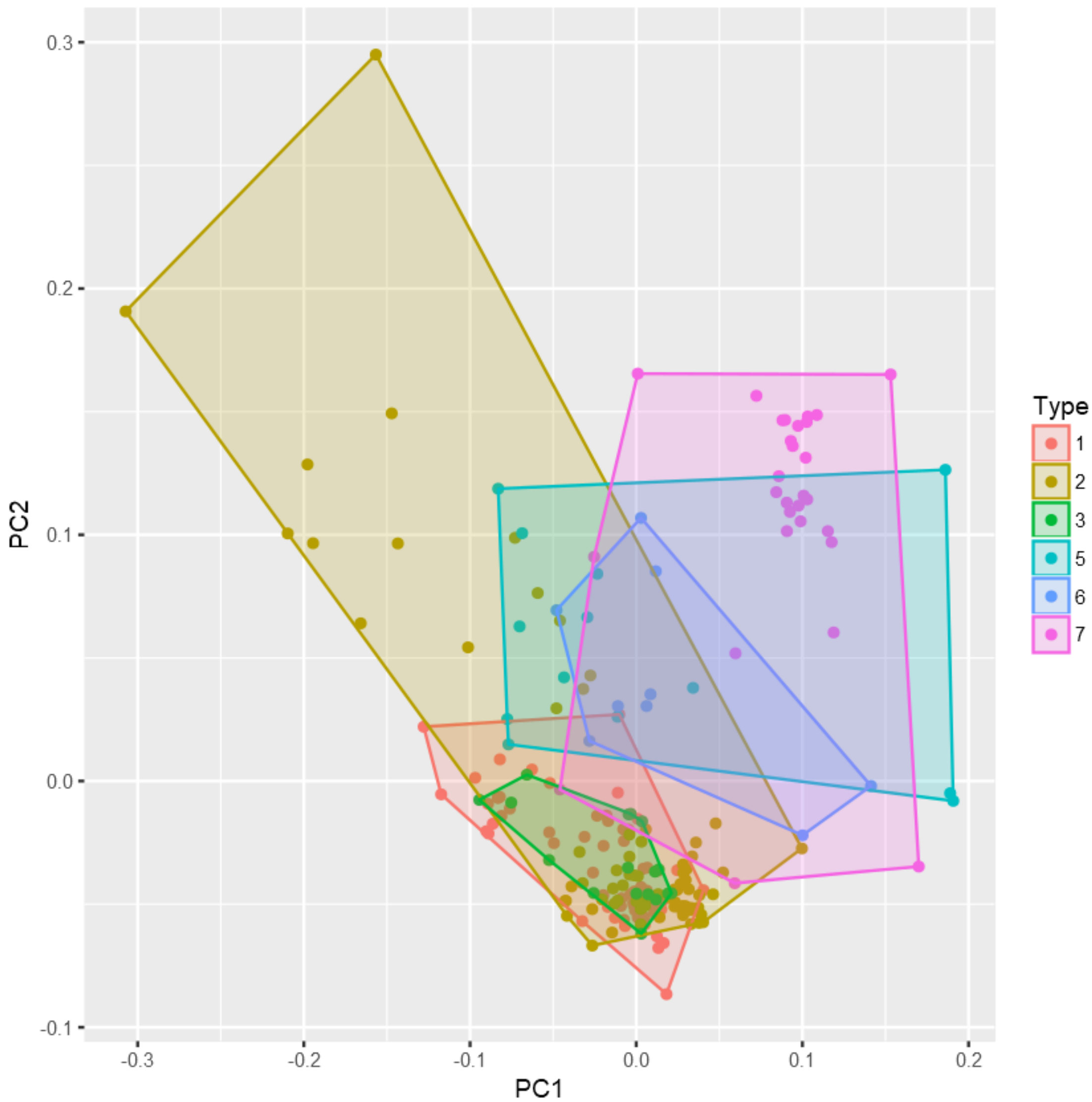
	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
RI	-0.5451766	0.28568318	-0.0869108293	-0.14738099	0.073542700	-0.11528772	-0.08186724	-0.75221590	-0.02573194
Na	0.2581256	0.27035007	0.3849196197	-0.49124204	-0.153683304	0.55811757	-0.14858006	-0.12769315	0.31193718
Mg	-0.1108810	-0.59355826	-0.0084179590	-0.37878577	-0.123509124	-0.30818598	0.20604537	-0.07689061	0.57727335
Al	0.4287086	0.29521154	-0.3292371183	0.13750592	-0.014108879	0.01885731	0.69923557	-0.27444105	0.19222686
Si	0.2288364	-0.15509891	0.4587088382	0.65253771	-0.008500117	-0.08609797	-0.21606658	-0.37992298	0.29807321
K	0.2193440	-0.15397013	-0.6625741197	0.03853544	0.307039842	0.24363237	-0.50412141	-0.10981168	0.26050863
Ca	-0.4923061	0.34537980	0.0009847321	0.27644322	0.188187742	0.14866937	0.09913463	0.39870468	0.57932321
Ba	0.2503751	0.48470218	-0.0740547309	-0.13317545	-0.251334261	-0.65721884	-0.35178255	0.14493235	0.19822820
Fe	-0.1858415	-0.06203879	-0.2844505524	0.23049202	-0.873264047	0.24304431	-0.07372136	-0.01627141	0.01466944

0.5451766	-0.28568318	-0.0869108293	0.14738099	0.073542700	-0.11528772	0.08186724	0.75221590	0.02573194
-0.2581256	-0.27035007	0.3849196197	0.49124204	-0.153683304	0.55811757	0.14858006	0.12769315	-0.31193718
0.1108810	0.59355826	-0.0084179590	0.37878577	-0.123509124	-0.30818598	-0.20604537	0.07689061	-0.57727335
-0.4287086	-0.29521154	-0.3292371183	-0.13750592	-0.014108879	0.01885731	-0.69923557	0.27444105	-0.19222686
-0.2288364	0.15509891	0.4587088382	-0.65253771	-0.008500117	-0.08609797	0.21606658	0.37992298	-0.29807321
-0.2193440	0.15397013	-0.6625741197	-0.03853544	0.307039842	0.24363237	0.50412141	0.10981168	-0.26050863
0.4923061	-0.34537980	0.0009847321	-0.27644322	0.188187742	0.14866937	-0.09913463	-0.39870468	-0.57932321
-0.2503751	-0.48470218	-0.0740547309	0.13317545	-0.251334261	-0.65721884	0.35178255	-0.14493235	-0.19822820
0.1858415	0.06203879	-0.2844505524	-0.23049202	-0.873264047	0.24304431	0.07372136	0.01627141	-0.01466944

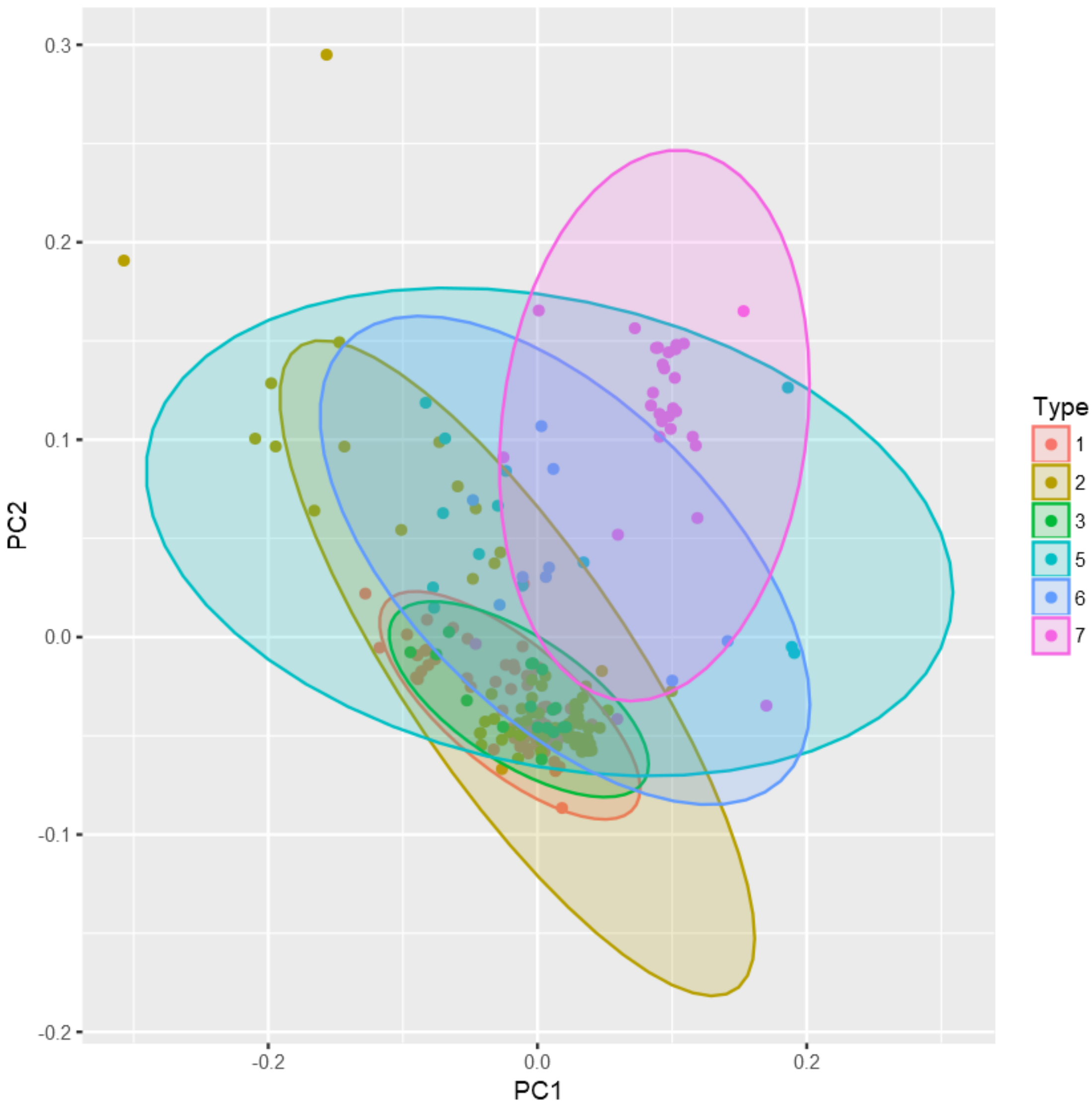
Com isso, verificamos que os autovetores estão iguais, comprovando a eficacia do **prcomp** e de seus parâmetros. Agora, iremos plotar os dados ortogonalizados em PC1 e PC2. Nos plots abaixos, utilizamos a biblioteca **ggfortify** e **ggplot2** para melhor visualização e distribuição dos dados.

```
In [44]: par(mfrow=c(1,1))
         autoplot(pcaObject, data = Glass, colour = 'Type')
         autoplot(pcaObject, data = Glass, colour = 'Type', frame = TRUE)
         autoplot(pcaObject, data = Glass, colour = 'Type', frame = TRUE, frame.type = 'norm')
```


0.3
0.2
0.1
0.0
-0.1
PC2
-0.3
-0.2
-0.1
0.0
0.1
0.2
PC1
Type
1
2
3
5
6
7



PC2
PC1
0.3
0.2
0.1
0.0
-0.1
-0.3
-0.2
-0.1
0.0
0.1
0.2
Type
1
2
3
5
6
7



Observando o PCA, vemos que os dados do tipo 7 foram os que ficaram mais clusterizados(*clustered*), desse modo, ficando razoavelmente simples sua identificação. Contudo, ao agruparmos os dados por frames, para melhor visualização, vemos que ainda existe muita sobreposição por todos os tipos. No primeiro gráfico agrupado, vemos um sistema que considera os pontos mais longíquos e envolve todos os pontos desse tipo, o que não é uma aproximação desejada, tendo em vista que pega todos os *outliers*. É fácil perceber isso quando vemos o agrupamento do tipo 2, que envolve quase todos. O segunda agrupamento, por elipses, é mais coerente, pois desconsidera alguns *outliers*, porém, ainda também não é o ideal, pois tipos com poucos dados, como o tipo 2, acabam envolvendo uma área muito grande, enquanto não deveria ser tanto.

Percebemos pelos gráficos que as fronteiras das classes projetadas nos PCAs não são lineares, e que são de difícil separação apenas observando o gráfico. Além disso, identificamos também que os tipos 1,2,3 são os mais sobrepostos, sendo os de mais dificil separação.