Introduction to control systems
TI0118

Homework III

Guidelines

You must solve each exercise using hand calculations and compare your results on Matlab/Octave. The report for the homework must be sent in a **single pdf document** by **July 10, 2017**. Note that **late submissions will not be considered**.

Your report must include:

- The relevant steps, the results and your comments on each exercise.
- The Matlab/Octave code developed for solving each exercise. This can be included as part of the single exercise or in a separate section of the report (for instance, as an appendix).
- The graphs required in each exercise.

Exercise 1 25 Points

Consider the negative feedback system shown in Figure 1. For this system, the transfer functions, $G_1(s), G_2(s)$ and $H(s)$, are given in equation 1.

$$G_1 = K,\ G_2(s) = \frac{s^2+5s+6}{(s+6)(s^2+2s+3)(s^2+9s+20)} \text{ and } H(s) = 1 \qquad (1)$$

It is required to:

1. Sketch the root locus and explain the main steps for plotting it.
2. Find the point and gain where the root locus crosses the $\zeta = 0.45$ line.
3. Find the point and gain where the locus crosses the $j\omega$-axis.
4. Find breakaway and break-in points on the real axis.
5. Find range of K within which the system is stable.

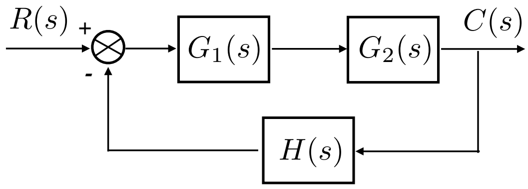
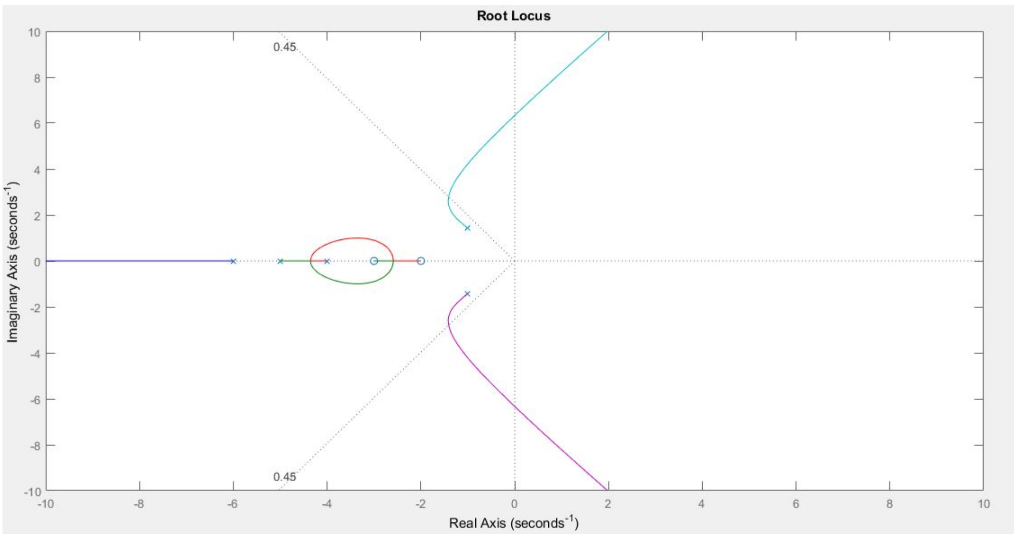


Figura 1: Negative feedback system.

Solution 1

1. Primeiramente, é necessário achar os polos e os zeros. Com isso, verificando seus angulos, achamos os trechos sobre o eixo real que correspondem ao espaço das raizes. Depois, procura-se as assintotas do sistema, calculado através de seus polos e zeros. Depois, devemos achar os pontos de break away e break-in. Para tal, utilizamos a relação $\frac{dK}{ds} = 0$, obtida através de $1 + G(s)H(s) = 0$.Por fim, achamos onde o *Root Locus* cruza o eixo imaginário. Para tal, achamos o máximo ganho(K) para qual o sistema se mantém marginalmente estável, usando qualquer método para tal (Tabela de Routh ou substituição de jw na função de transferência). Com isso, conseguimos ter um esboço do gráfico da função.
   Já no MATLAB, precisamos apenas modelar a função de transferência e aplicar a função *rlocus*.



2. Usando a função *rlocusfind*, achamos os pontos $-1.4056 \pm 2.7923i$ com ganho $K = 61.6824$.

3. Usando a função *rlocusfind*, achamos os pontos $\pm 6.3316i$ com ganho

$K = 444.0072$.

4. $1 + KG(s)H(s) = 0 \longrightarrow K = \dfrac{-1}{G(s)}$

   As raízes do numerador e do denominador de $\dfrac{dK}{ds}$ identificam, respectivamente, os pontos break-in e o breakaway.

5. O sistema só será estável quando estiver na parte real à esquerda do eixo imaginário, ou seja, na parte negativo do eixo real. Sendo assim, como no limiar o $K = 444.0072$ (verificado no item 3), o intervalo de K que o sistema será estável é $0 < K < 444.0072$

```
%%Exercise 1
clear all;clc;

syms s;
num = [1 5 6];
f1 = [1 6];
f2 = [1 2 3];
f3 = [1 9 20];
den = conv(f1,conv(f2,f3));
G = tf(num,den);

rlocus(G)
% axis([-1.7 -1.2 2.6 3.2])
%Eixo para selecionar melhor o ponto zeta=0.45
% axis([-0.01 0.01 6.31 6.35])
%Eixo para selecionar melhor o ponto onde cruza o eixo
    imaginario
axis([-10 10 -10 10])
z=0.45; wn=0;
sgrid(z,wn)
[k,p] = rlocfind(G)
```

Exercise 2

25 Points

Consider the negative feedback system shown in Figure 1. For this system, the transfer functions, $G_1(s), G_2(s)$ and $H(s)$, are given in equation 2.

$$G_1 = 10,\ G_2(s) = \frac{s^2 + 3s + 2}{s^2(s^3 + 15s^2 + 74s + 120)} \text{ and } H(s) = \frac{s + 6}{s^2 + 17s + 72} \quad (2)$$

Do the following:

1. Define the system type.

2. Define the static error constants.

3. Find the steady-state error for the inputs $10u(t)$ and $10tu(t)$.

Solution 2

1. $G(s) = \dfrac{10s^4 + 200s^3 + 1250s^2 + 2500s + 1440}{s^7 + 32s^6 + 401s^5 + 2448s^4 + 7178s^3 + 7480s^2 - 2300s - 1320}$

   Logo, o sistema é do tipo 0.

2. $K_p = \lim_{s \to 0} G(s) = -\dfrac{12}{11} = -1.0909$
   $K_v = \lim_{s \to 0} sG(s) = 0$
   $K_a = \lim_{s \to 0} s^2 G(s) = 0$

3. $\text{SSError}_1 = \dfrac{10}{1 + K_p} = -110$

   $\text{SSError}_2 = \dfrac{10}{K_v} = \infty$

```
%% Exercise2
%2.1 e 2.2
clear all;clc;

G1 = tf(10);

f1 = [1 3 2];
f2 = [1 0 0];
f3 = [1 15 74 120];
G2 = tf(f1,conv(f2,f3));

f4 = [1 6];
f5 = [1 17 72];
H = tf(f4,f5);

T = feedback(G1*G2,H);
G = T /(1-T);
G = minreal(G)
Kp = dcgain(G)

error = 10 / (1 + Kp)
```

Exercise 3

25 Points

Consider the negative feedback system shown in Figure 1. For this system, the transfer functions, $G_2(s)$ and $H(s)$, are given in equation 3.

$$G_2(s) = \frac{1}{(s+1)(s-2)(s+3)} \text{ and } H(s) = 1 \tag{3}$$

Using the root locus, the goal is to design a controller that stabilises the process and provides a settling time of less than 2 seconds.

1. Among the following possible controllers, define which one satisfies the required conditions.

   (a) $G_1 = K$;

   (b) $G_1 = \dfrac{K(s+1)}{(s+a)}$, where $a = 10$;

   (c) $G_1 = \dfrac{K(s+1)(s+3)}{(s+a)^2}$, where $a = 15$.

2. Discuss and motivate your choices.

Solution 3

Para cada um dos itens, plotamos o *Root Locus* correspondente. Assim, temos:

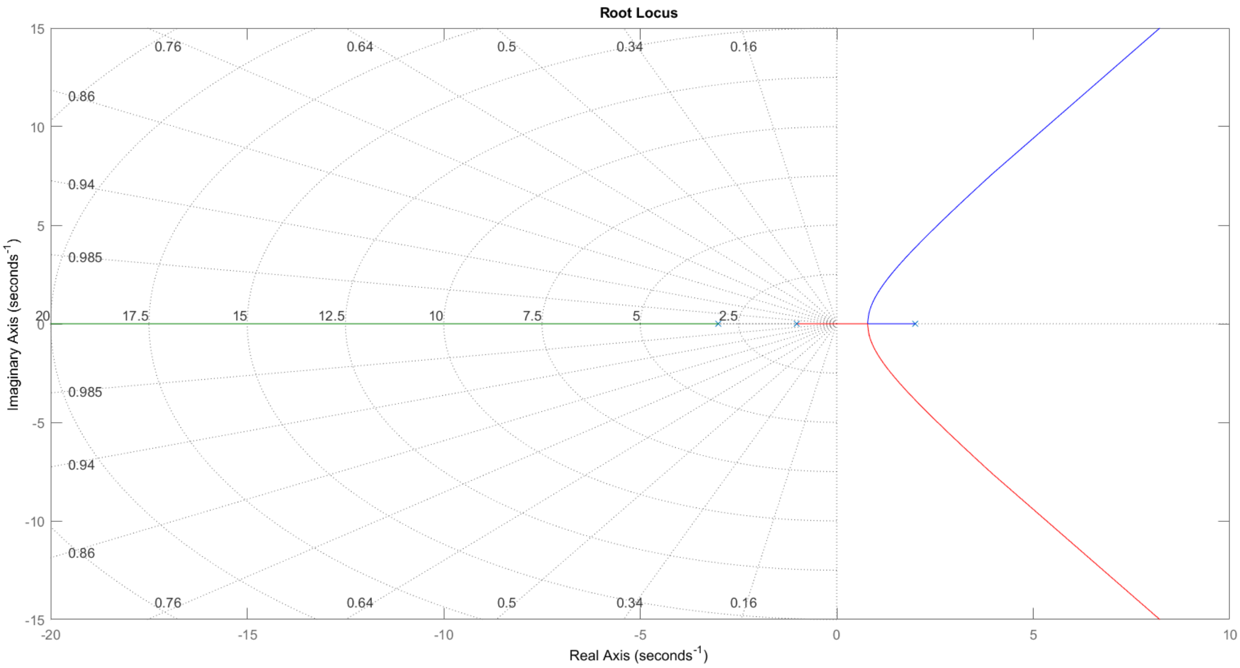


Figura 2: Root Locus a

Nesse primeiro *Root Locus*, verificamos que as raizes positivas estáveis estão todas sobre o eixo real. Desse modo, todas tem $\zeta = 1$, o que caracteriza um sistema criticamente amortecido. Tal sistema é ideal, e não existe no mundo real. Logo, é desconsiderado para nosso controlador.

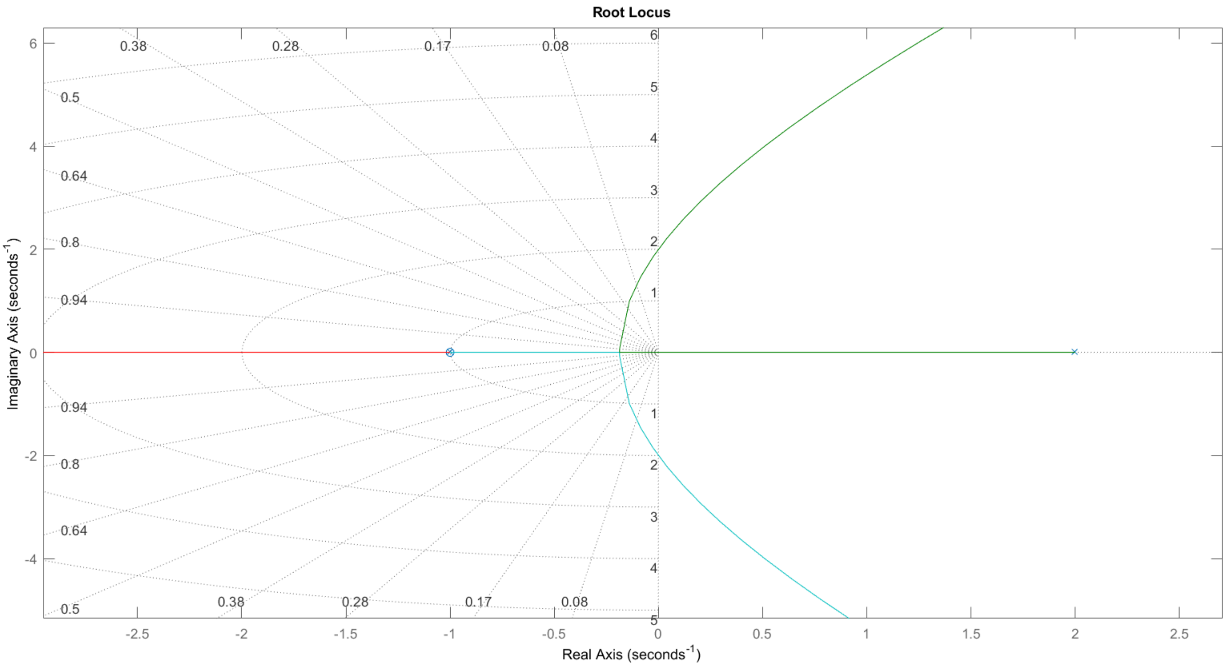


Figura 3: Root Locus b

Já no segundo *Root Locus*, temos raizes no plano esquerdo ao eixo imaginário com $\zeta \neq 1$, que caracteriza sistemas amortecidos. Sabemos que $\zeta = \cos\theta$, com isso $0 < \zeta < 1$. Logo, como $T_s = \dfrac{4}{\zeta w_n}$, temos que $T_s < 2$ segundos, somente se $w_n > 2$. Contudo, analisando os pontos no gráfico, encontramos que o maior w_n estável é menor ou igual a 2, em uma situação que o ζ é quase 0. Com isso, ainda não é o controlador ideal para nosso problema, pois seu T_s ainda é maior que 2.

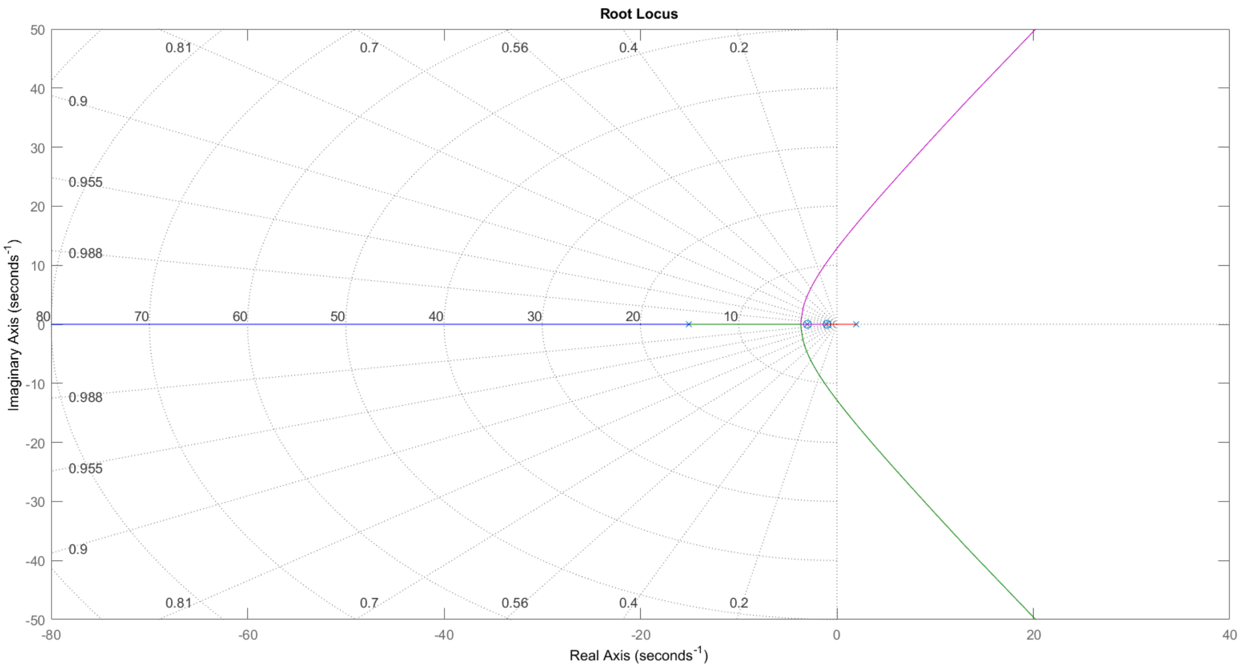


Figura 4: Root Locus b

No último caso, também temos raizes no plano esquerdo ao eixo imaginário com $\zeta \neq 1$. Contudo, percebemos que tais raizes estão mais distantes do eixo imaginário, de modo que alcançam frequências w_n mais elevadas. Verificando os dados do gráfico, observamos que $3.67 < w_n < 12.8$. Com isso, existem vários ganhos que satisfazendo o tempo de estabilização desejado para o nosso controlador, sendo esse intervalo aproximadamente $728 < K < 1930$.

```
%% Exercise3
clear all;clc;

f1 = [ 1 1]
f2 = [ 1 -2]
f3 = [ 1 3]
den = conv(f1,conv(f2,f3))
G1=tf(1,den)

figure(1);
rlocus(G1)
grid on;

T2 = tf([1 1],[1 10]);
G2 = G1 * T2
figure(2);
rlocus(G2)
```

```
grid on;

T3 = tf(conv([1 1],[1 3]),[1 30 225]);
G3 = G1 * T3
figure(3);
rlocus(G3)
grid on;
```

Exercise 4

25 Points

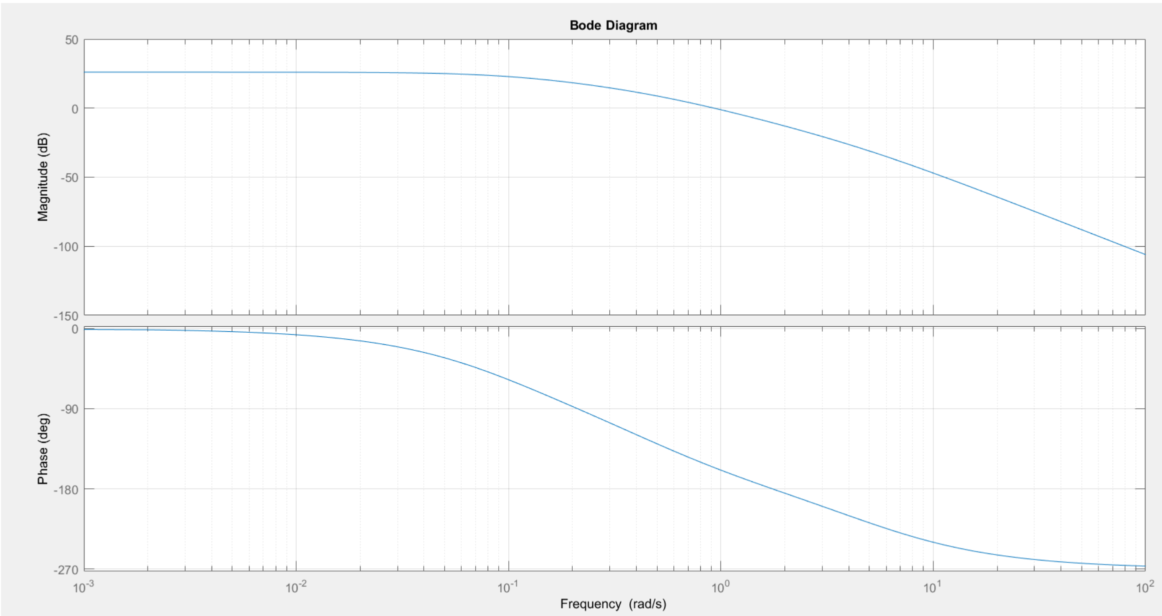
Consider the negative feedback system shown in Figure 1. For this system, the transfer functions, $G_1(s), G_2(s)$ and $H(s)$, are given in equation 4.

$$G_1 = 20,\ G_2(s) = \frac{1}{(1+10s)(1+2s)(1+0.2s)} \text{ and } H(s) = 1 \tag{4}$$

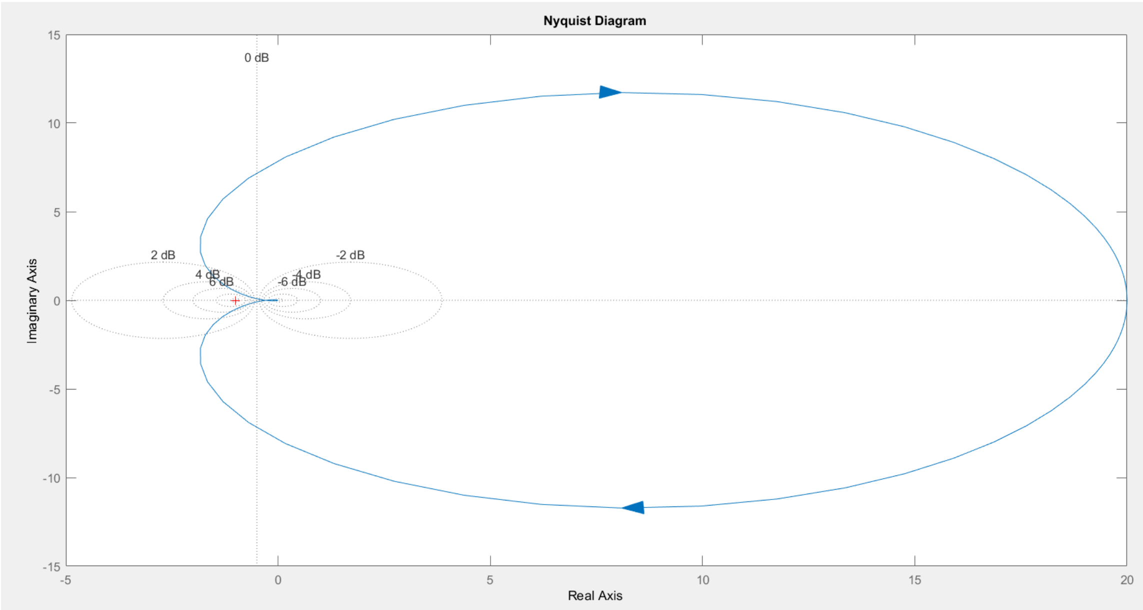
Do the following:

1. Sketch the Bode asymptotic magnitude and asymptotic phase plots.
2. Sketch the Nyquist diagram for the system.
3. Comment the plots.

Solution 4



1.



2.

```
%% Exercise4
clear all;clc;

G1 = tf(20)

f1 = [ 10 1 ]
f2 = [ 2 1 ]
f3 = [ 0.2 1 ]

G2 = tf(1,conv(f1,conv(f2,f3)))

G = G1*G2

figure(1);
bode(G)
grid on;

figure(2);
nyquist(G)
grid on;
```

Exercise 5 — 20 Points (Extra)

Consider the two-tank system given in HW1.

The liquid inflow to the first tank, $u(t) = q_0(t)$, can be controlled using a valve. The upper tank's outflow, $q_1(t)$, equals the lower tank's inflow. The

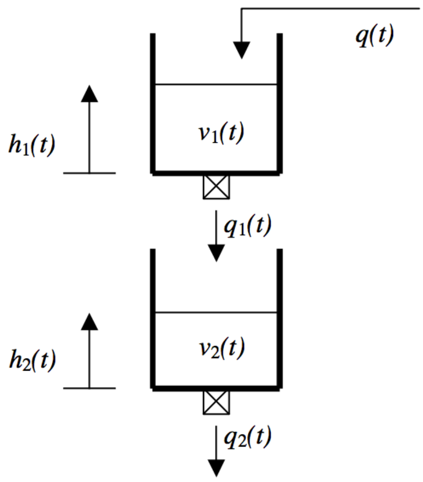


Figura 5: Two-tank system for Exercise 5.

outflow of the lower tank is $q_2(t)$. The objective of the design is to control the liquid level, $y(t) = h_2(t)$, in the lower tank.

1. Given the cross-sectional areas $S_1 = 2$ and $S_2 = 4$ [m^2] and the coefficients $K_1 = 3$, $K_2 = 4$ for the upper and lower tanks, verify that the open loop transfer function for this system can be written as:

$$\frac{Y(s)}{Q_0(s)} = \frac{a_1/a_3}{s^2 + (a_1 + a_2)s + a_1 a_2}$$

   - where $a_1 = K_1/S_1,\ a_2 = K_2/S_2$ and $a_3 = S_2$.

2. The system is operating at 10% overshoot. Design a controller to reduce by half the system's settling time.

3. Discuss and justify your choices.

4. Verify your design through Matlab/Octave simulations.

Solution 5

1.

2. PD$= s + 6.5538$

3. Usando o MATLAB, calculamos os novos polos dominantes de acordo com o Overshoot(e consequentemente o ζ) dado na questão. Com eles, podemos projetar o sistema dado acima.

4.

```matlab
%% Exercise 5
clear all;clc;

S1 = 2 ; S2 = 4; K1 = 3; K2 = 4;
a1 = K1/S1 ; a2 = K2/S2 ; a3 = S2;
numg = [ a1/a3 ]; deng = [ 1 a1+a2 a1*a2];
G = tf( numg, deng)

overshoot = 10;
z=-log(overshoot/100)/sqrt(pi^2+[log(overshoot/100)]^2);
% Plot uncompensated root locus
figure(1);
rlocus(G)
axis([-10 10 -10 10])
% Overlay desired percent OS line
sgrid(z,0)
title(['Uncompensated Root Locus with ', num2str(overshoot),'% OS Line'])

%Generate gain, K, and closed-loop poles, p, for point selected ...
%interactively on the root locus
[K,p]=rlocfind(G)

Tss0 = 4/abs(real(p(1)));

% Simulate uncompensated closed-loop
T=feedback(K*G,1) % Find uncompensated T(s)
[y,t]=step(T); % Step response of uncompensated system

Tssf = Tss0/2;
% Calculate natural frequency
wn=4/(Tssf*z);
% Calculate desired dominant pole location
desired_pole=(-z*wn)+(wn*sqrt(1-z^2)*i);
% Calculate angular contribution to desired pole without PD compensator
angle_at_desired_pole=(180/pi)*...
angle(polyval(numg,desired_pole)/polyval(deng,desired_pole));
% Calculate required angular contribution from PD compensator
PD_angle=180-angle_at_desired_pole;
% Calculate PD zero location
zc=((imag(desired_pole)/tan(PD_angle*pi/180))-real(desired_pole));

% PD Compensator
numc=[1 zc]; % Calculate numerator of Gc(s)
denc=[0 1]; % Calculate denominator of Gc(s)
Gc=tf(numc,denc) % Create and display Gc(s)
Ge=G*Gc % Cascade G(s) and Gc(s)
```

```
% Plot compensated root locus
figure(2);
rlocus(Ge)
axis([-10 10 -10 10])
sgrid(z,0) % Overlay desired % OS line
title(['PD␣Compensated␣Root␣Locus␣with␣' , num2str
    (overshoot),'%␣OS␣Line'])

[K,p]=rlocfind(Ge)
Tssprat =4/abs(real(p(1)))

% PD compensated system
Tc=feedback(K*Ge,1) % PD compensated T(s).
[yc,tc]=step(Tc,t); % Step response for PD
    compensated system.

% Compare
figure(3);
plot(tc,yc,'LineWidth', 1.5); hold on; axis tight
plot(t,y, 'LineWidth', 1.5, 'Color','r'); grid on
legend('Compensated', 'Uncompensated')
title('System␣response␣comparison')
set(gca,'FontSize',12)
hold off
```