Problem 1

$$p(w|D)->w=(X^TX+\alpha*I)-1X^Tt$$

Solution I.

$$p(w \mid x,t,\alpha,\beta) \propto p(t \mid x,w,\beta)p(w \mid \alpha)$$

$$=> log(p(w \mid x,t,\alpha,\beta) \propto log(p(t \mid x,w,\beta)p(w \mid \alpha))$$

we have:

$$p(t \mid x,w,\beta) = \prod_{n=1}^{N} N(t_n \mid y(x_n,w),\beta^{-1})$$

$$p(t \mid x,w,\beta) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\beta^{-1}}} \times e^{\dfrac{-(t-y(x_n,w))^2}{2\beta^{-2}}}$$

$$log(p(t \mid x,w,\beta)) = \frac{-\beta}{2}\Sigma_{n=1}^{N}(t-y(x_n,w))^2 + noise$$

And:

$$p(\mathbf{w} \mid \alpha) = N(w \mid 0,\alpha^{-1}I) = \frac{1}{(2\pi)^{D/2} \mid \Sigma \mid^{1/2}} e^{\dfrac{-(w-0)^T\Sigma^{-1}(w-0)}{2}}$$

$$log(p(\mathbf{w} \mid \alpha)) = \frac{-1}{2}w^Tw + noise$$

So:

$$log(p(w \mid x,t,\alpha,\beta) \propto \frac{-\beta}{2}\Sigma_{n=1}^{N}(t-y(x_n,w))^2 + \frac{-1}{2}w^Tw$$

we find that the maximum of the posterior is given by the minimum of:

$$\frac{\beta}{2}\Sigma_{n=1}^{N}(t-y(x_n,w))^2 + \frac{1}{2}w^Tw$$

or we minimize:

$$Q = \mid\mid X\mathbf{w} - \mathbf{t} \mid\mid_2^2 + \lambda\mathbf{w}^{\mathbf{T}}\mathbf{w}$$

$$\bigtriangledown Q_w = 2X^T(X\mathbf{w}-\mathbf{t}) + 2\lambda\mathbf{w}$$

$$\rightarrow \mathbf{w} = (X^TX+\lambda I)^{-1}X^T\mathbf{t}$$