demo4

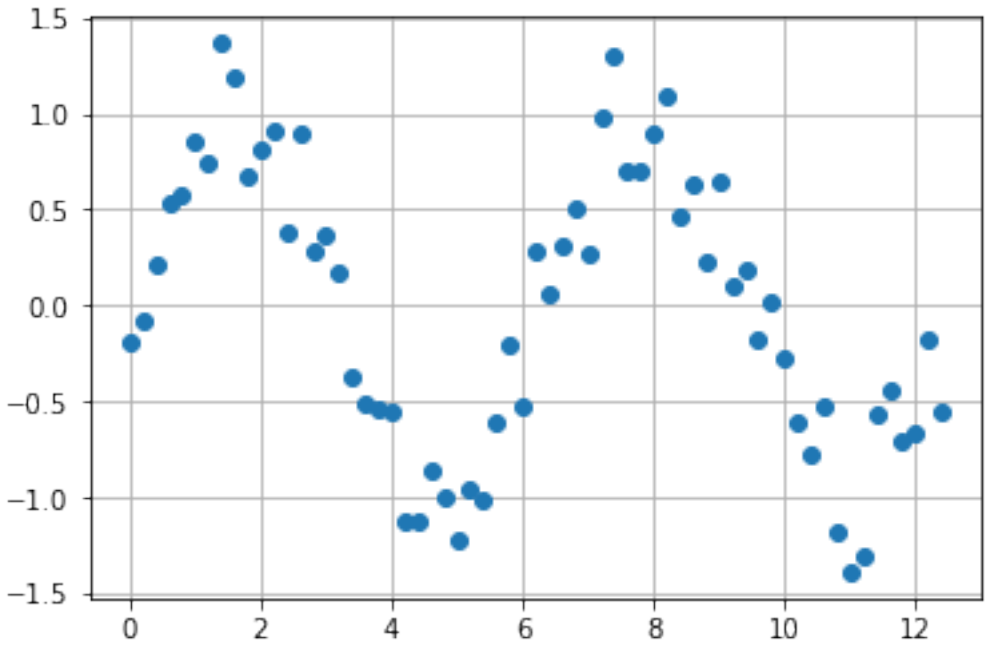
February 26, 2020

Let's demo the bias-variance issue discussed in class! We will cook up some synthetic data; in the next homework we will actually test similar ideas on real-world prediction problems.

[0]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import random

np.random.seed(1337)

def gen_data(x):
  b = 0.4
  t = np.sin(x)
  y = t + np.random.uniform(-b,b,len(x))
  return y, t

def get_rand_subsets(len_ratio,x,y,t_x):
  nsamples = int(np.round(len(x)*len_ratio))
  ind = random.sample(range(len(x)),nsamples)
  ind = np.sort(ind)
  x_subset = x[ind]
  t_subset = t[ind]
  y_subset = y[ind]
  return x_subset, y_subset, t_subset
```

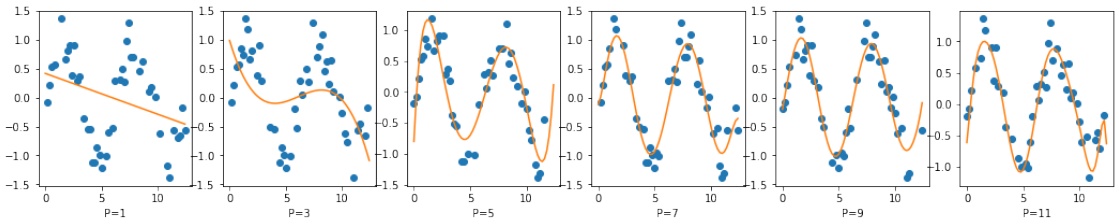
OK! Let's generate some noisy sinusoidal data, and try to fit various polynomials.

[58]:
```
x = np.arange(0,4*np.pi,0.2)
y_orig, t_orig = gen_data(x)

plt.plot(x,y_orig,'o')
plt.grid()
```



[59]:
```
p_order = [1,3,5,7,9,11]
num_trials = 100

f, ax = plt.subplots(1, len(p_order), figsize=(18, 3))
P=[]
biases=[]
variances=[]
for i, p_val in enumerate(p_order):
    y_pred = np.zeros(len(x))
    var = np.zeros(len(x))
    for j in range(num_trials):
        x_sub, y, t_sub = get_rand_subsets(0.7, x, y_orig, t_orig)
        p = np.poly1d(np.polyfit(x_sub, y, p_val))
        P.append(p)
        y_pred = y_pred + p(x)

    y_pred = y_pred/num_trials
    ax[i].plot(x_sub, y, 'o', x, y_pred, '-')
    ax[i].set_xlabel('P='+str(p_order[i]))
    bias = np.linalg.norm(y_pred-t_orig)
    biases.append(bias)

    for j in range(num_trials):
        p_t = P.pop(0)
        var = var + np.square(p_t(x)-y_pred)
```

```
    var = var/num_trials
    variance = np.linalg.norm(var)
    variances.append(variance)
```



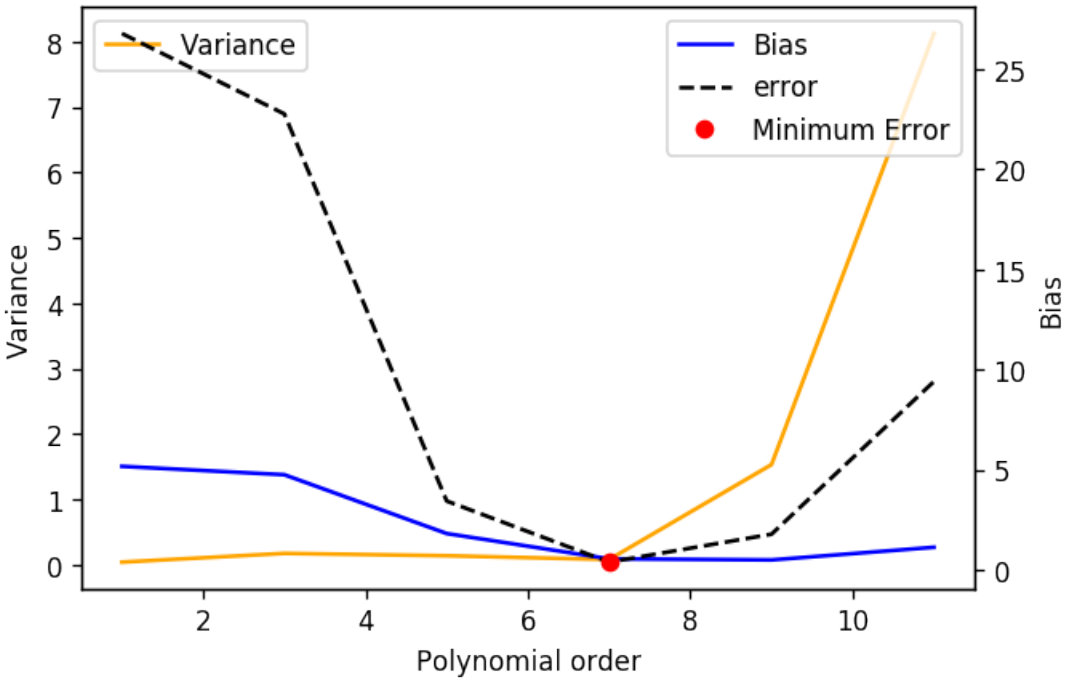
OK, how about the bias and variance?

[60]:
```
print("bias: ", biases)
print("var: ",variances)
```

```
bias:  [5.170999560339094, 4.752891242873352, 1.8160926840600309,
0.553650135786544, 0.49853881167521485, 1.1298515624854413]
var:  [0.04260942397570906, 0.1764327764977568, 0.1403822516124777,
0.08246808406672715, 1.5346404332614842, 8.129345001743484]
```

Seems to be working! Note that with higher model order, the fit (bias) is better but the variance is high across different trials. Let's plot the bias-variance curve.

[72]:
```
fig, ax1 = plt.subplots(dpi=120)

ax1.set_xlabel('Polynomial order')
ax1.set_ylabel('Variance')
ax1.plot(p_order,variances,color='orange',label = 'Variance')

ax2 = ax1.twinx()

ax2.set_ylabel('Bias')
ax2.plot(p_order, biases,color='blue',label = 'Bias')

error = np.square(biases)+variances
min_ind = np.argmin(error)
plt.plot(p_order, error, 'k--', label = 'error')
plt.plot(p_order[min_ind], error[min_ind], 'ro', label= 'Minimum Error')
leg = ax2.legend(loc='upper right')
leg = ax1.legend(loc='upper left')
plt.show()
```


Variance
Bias
error
Minimum Error
Variance
Bias
Polynomial order
0
1
2
3
4
5
6
7
8
0
5
10
15
20
25
2
4
6
8
10