# Intro to ML Homework6

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## 1 Question 1

```
[ ] import numpy as np
    X = np.array([[3,2,1],[2,4,5],[1,2,3],[0,2,5]])
    print(X)
```

```
[[3 2 1]
 [2 4 5]
 [1 2 3]
 [0 2 5]]
```

a. Find the sample mean

```
[ ] X_mean = np.mean(X, axis = 0)
    print(X_mean)
```

```
[1.5 2.5 3.5]
```

b. Zero-center the samples, and find the eigenvalues and eigenvectors of the data covariance matrix Q.

(1) Zero-center the samples

```
[ ] X_original = X.copy()
    X = X - X_mean
    print(X)
```

```
[[ 1.5 -0.5 -2.5]
 [ 0.5  1.5  1.5]
 [-0.5 -0.5 -0.5]
 [-1.5 -0.5  1.5]]
```

(2) Find the eigenvalues and eigenvectors of the data covariance matrix Q.

```
Q = 1/4 * np.matmul(X.T,X)  # covariance matrix Q
evalue,evector = np.linalg.eig(Q)
print(evalue)   # eigenvalues  #print(evector)  # eigenvectors(a row is a vector)
evector1 = np.array(evector[:,0])
evector2 = np.array(evector[:,1])
evector3 = np.array(evector[:,2])
evec = np.array([evector1,evector2,evector3])
print(evec)
```

```
[3.56166464 1.1733803  0.01495506]
[[-0.45056922  0.19247228  0.87174641]
 [-0.66677184 -0.72187235 -0.18524476]
 [-0.59363515  0.66472154 -0.45358856]]
```

First list is of eigenvalues, each row of the matrix above is an eigenvector.

c. Find the PCA coefficients corresponding to each of the samples in X.

```
[ ]  scores = np.matmul(X,evec.T)
     print(scores)
```

```
[[-2.95145599 -0.17610969 -0.0888421 ]
 [ 1.37104342 -1.69406159  0.0198819 ]
 [-0.30682473  0.78694448  0.19125108]
 [ 1.8872373   1.0832268  -0.12229089]]
```

PCA coefficients(scores) produced by eigenvector for each samples are shown above, values in each row are three scores of each sample.

d. Reconstruct the original samples from the top two principal components, and report the reconstruction error for each of the samples.

PCA reconstruction= PC scores * Eigenvectors$^T$ + Mean

```
[ ]  np.set_printoptions(precision=4)
     eigen12 = np.delete(evec, 2, 0)
     #print(eigen12)
     PC12 = np.delete(scores, 2, 1)
     #print(PC12)
     recons = np.matmul(PC12,eigen12)+ X_mean
     print(recons)
```

```
[[ 2.9473  2.0591  0.9597]
 [ 2.0118  3.9868  5.009 ]
 [ 1.1135  1.8729  3.0867]
 [-0.0726  2.0813  4.9445]]
```

Reconstruction error

```
[ ]  #print(recons-X_original)
     RE = np.linalg.norm((recons-X_original),axis=1)
     print(RE)
```

```
[0.0888 0.0199 0.1913 0.1223]
```

Reconstruction errors for each sample are shown above in the output list.

## 2 Question 2

**(a)**
Loss function of k-means is:

$$F(\{S_1, ..., S_k\}, \{\mu_1, ..., \mu_k\}) = \sum_{j=1}^{k} \sum_{x_i \in S_j} \|x_i - \mu_j\|^2$$

We want it in the form:

$$F(\eta, \mu) = \sum_{i=1}^{n} \sum_{j=1}^{k} \eta_{ij} \|x_i - \mu_j\|^2$$

So we want $\sum_{i=1}^{n} \eta_{ij}$ serves the function of $\sum_{x_i \in S_j}$, $\sum_{i=1}^{n} \sum_{j=1}^{k} \eta_{ij}$ bonds $x_i$ and corresponding $\mu_j$, so $\eta$ is a binary matrix of size of (n,k), for its element, for instance, if a $x_i$'s corresponding center is $\mu_j$, then $\eta_{ij}$ is 1, rest $\eta_i$ in the same row of the matrix is 0.

**(b)**
Lloyd's algorithm can only decrease the value of $F$, because at each iteration:

(1)it re-arrange points to center which is closest to them, and then compute new centers as mean of each cluster,

(2)$F(\{S_1, ..., S_k\}, \{\mu_1, ..., \mu_k\}) = \sum_{j=1}^{k} \sum_{x_i \in S_j} \|x_i - \mu_j\|^2 = \sum_{j=1}^{k} |S_j| * Variance S_j$,

(3)set centers to mean points $\mu_j$ in $S_j$. This is equivalent to minimizing the pairwise squared deviations of points in the same cluster in the function above, thus $F$ is decreased.

**(c)**
The algorithm will terminate in no more than T iterations, where T is some finite number, because at each iteration of Lloyd's algorithm, it will reduce the value of $F$, and it can not iterate infinitely which means $F$ can be reduced infinitely.

For upper bound of T, considering there are n points and k centers, there could be $k^n$ combination of points and their corresponding centers, assume the worst case: only one point changes its corresponding center and algorithm has to try $k^n$ combination and find the best combination at last. So the upper bound of T is $O(k^n)$.

# 3 Question 3

Using the Senate Votes dataset demo'ed in Lecture 11, perform k-means clustering with k = 2 and show that you can learn (most of) the Senators' parties in a completely unsupervised manner. Which Senators did your algorithm make a mistake on, and why?

(1) Import dataset

```
[ ] import numpy as np
    import matplotlib
    import matplotlib.pyplot as plt
```

```
[ ] import pandas as pd

    url_votes = 'https://raw.githubusercontent.com/exemplary-citizen/PCA-and-Senate-Voting-Data/master/senator_pca_problem/senator_data_p
    senator_df =  pd.read_csv(url_votes + 'data_matrix.csv',error_bad_lines=False)
    af = pd.read_csv(url_votes + 'politician_labels.txt', header=None)
    af["affiliations"] = af[0].str.split().str[-1]
```

```
[ ] senator_df.head()
```

| | Unnamed: 0 | bill_type bill_name bill_ID | missing_votes | Arlen Specter (PA) | Barack H. Obama (IL) | BarbaraA Mikulski(MD) | BarbaraBoxer(CA) | Bill Nelson(FL) | BlancheL.Linc |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | Appropriations_Transit Security Amendment_3866 | 0.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | |
| 1 | 1 | Budget_Spending_and_Taxes_2007 Budget Resoluti... | 0.0 | 1.0 | -1.0 | -1.0 | -1.0 | -1.0 | |
| 2 | 2 | "Budget, Spending and Taxes_Debt Limit Increas... | 0.0 | 1.0 | -1.0 | -1.0 | -1.0 | -1.0 | |
| 3 | 3 | "Budget, Spending and Taxes_Education Funding ... | 0.0 | -1.0 | 1.0 | 1.0 | 1.0 | 1.0 | |
| 4 | 4 | Budget, Spending and Taxes_Reinstate Pay-As-Yo... | 0.0 | -1.0 | 1.0 | 1.0 | 1.0 | 1.0 | |

5 rows × 103 columns

```
[ ] af.head()
```

| | 0 | affiliations |
|---|---|---|
| 0 | Specter Red | Red |
| 1 | Obama Blue | Blue |
| 2 | Mikulski Blue | Blue |
| 3 | Boxer Blue | Blue |
| 4 | Nelson Blue | Blue |

```
[ ] X = np.array(senator_df.values[:, 3:].T, dtype='float64') #transpose to get senators as rows
```

## (2) Check shape of matrix, affiliations

Check shape of data sets

```
[ ] print(senator_df.shape)
    print(af.shape)
    #print(af)
    #print(af["affiliations"])
    print(X.shape)
    print(X)
```

```
(542, 103)
(100, 2)
(100, 542)
[[ 1.  1.  1. ...  1.  1.  1.]
 [ 1. -1. -1. ...  0.  0.  0.]
 [ 1. -1. -1. ...  1. -1. -1.]
 ...
 [-1.  1.  1. ...  1.  1.  1.]
 [-1.  1.  1. ...  0.  0.  0.]
 [-1.  1.  1. ...  1.  1.  1.]]
```

Check part of affiliations

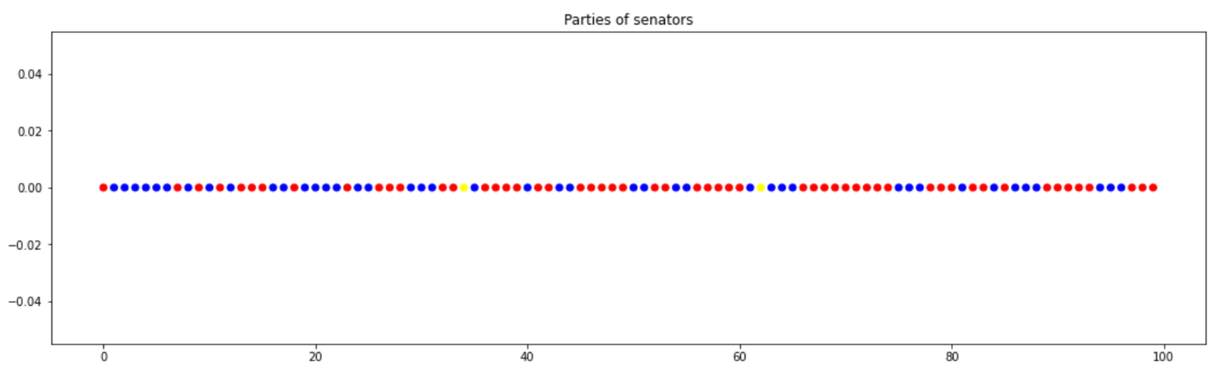
```
[ ] affiliations = af["affiliations"]
    print(affiliations[30:40])
```

```
30      Blue
31      Blue
32       Red
33       Red
34    Yellow
35      Blue
36       Red
37       Red
38       Red
39       Red
Name: affiliations, dtype: object
```

## (3) Visualize affiliations

Visualize affiliations

```
[50] affiliations = af["affiliations"]
     senator_num = range(100)
     plt.figure(figsize=(18,5))
     plt.scatter(senator_num, np.zeros_like(senator_num), c=affiliations)
     plt.title('Parties of senators')
     plt.show()
```



## (4)Perform K-means

Perform k-means clustering with k = 2 and show that you can learn (most of) the Senators' parties in a completely unsupervised manner.

```
[51] from sklearn.cluster import KMeans
     kmeans = KMeans(2)
     labels = kmeans.fit(X).predict(X)
     print(labels)
```

```
[0 1 1 1 1 1 1 0 1 0 1 0 1 0 0 0 1 1 0 1 1 0 1 0 1 1 0 0 0 1 1 1 0 0 1 1 0
 0 0 0 1 0 0 1 1 0 0 0 0 0 1 1 0 0 1 1 0 0 1 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0
 0 1 1 1 0 0 0 1 0 0 1 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0]
```

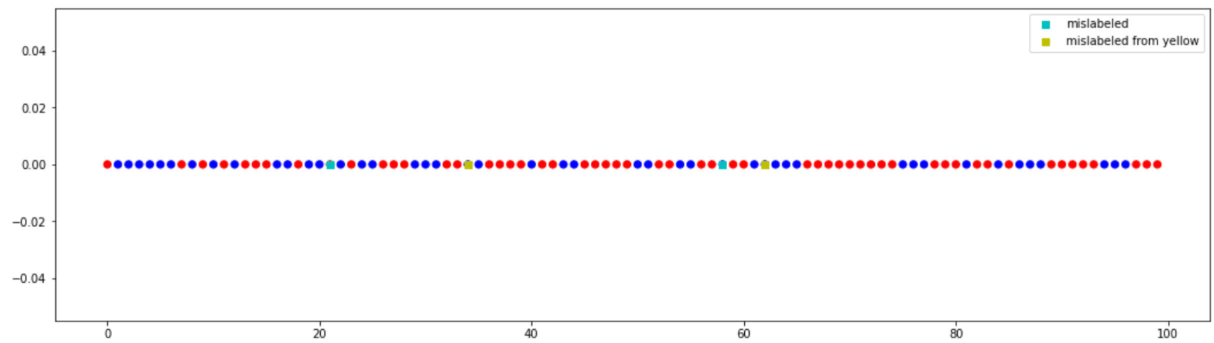
```
[54] af_kmeans = []
     mislabel = []
     yellowbabels = []
     for i in range(100):
       if labels[i] == 1:
         af_kmeans.append('Blue')
       elif labels[i] == 0:
         af_kmeans.append('Red')
       if af_kmeans[i] != affiliations[i]:
         mislabel.append(i)
       if affiliations[i] == 'Yellow':
         yellowbabels.append(i)
     print("Senators who are mis labelled: ",mislabel)
     print("Among them, these(Yellow) are neither of each label: ",yellowbabels)
     for i in mislabel:
       print(af.values[i][0])
```

```
Senators who are mis labelled:  [21, 34, 58, 62]
Among them, these(Yellow) are neither of each label:  [34, 62]
Nelson Blue
Jeffords Yellow
Chafee Red
Dayton Yellow
```

Senators who are mislabeled are shown above in the output.

## (5) Visualize the result of k-means

```
[58] plt.figure(figsize=(18,5))
     plt.scatter(senator_num, np.zeros_like(senator_num), c=af_kmeans, s=40, cmap='viridis');
     plt.plot(mislabel,np.zeros_like(mislabel),'cs',label='mislabeled')
     plt.plot(yellowbabels,np.zeros_like(yellowbabels),'ys',label='mislabeled from yellow')
     plt.legend()
     plt.show()
```



Square points are mislabeled senators whose original label is red/blue, yellow points are senators who are originally of yellow label. Which Senators did your algorithm make a mistake on, and why? **21,34,58,62.**

There are two senator originally labeled with 'Yellow', these will inevitably be wrong labeled due to only two clusters.And there are many potential drawbacks of k-means leading to rest two senators mislabeled: maybe these two are on the boundary of two clusters and closer to wrong cluster thus they are easily mislabeled, maybe distribution is not spherical.

# 4 Question 4

There is problem loading The Places Rated Almanac ratings from txt file provide so I find same csv file online.

**a.** Load the data and construct a table with 9 columns containing the numerical ratings.

import data

```
[2] import pandas as pd

    Ratings =  pd.read_csv('places.csv',error_bad_lines=False)
```

```
[ ] Ratings.head()
```

| | Place | Climate_and_Terrain | Housing | Health_Care_and_Environment | Crime | Transportation | Education | The_Arts | Recreation | Economics |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | Abilene_TX | 521 | 6200 | 237 | 923 | 4031 | 2757 | 996 | 1405 | 7633 |
| 1 | Akron_OH | 575 | 8138 | 1656 | 886 | 4883 | 2438 | 5564 | 2632 | 4350 |
| 2 | Albany_GA | 468 | 7339 | 618 | 970 | 2531 | 2560 | 237 | 859 | 5250 |
| 3 | Albany-Schenectady-Troy_NY | 476 | 7908 | 1431 | 610 | 6883 | 3399 | 4655 | 1617 | 5864 |
| 4 | Albuquerque_NM | 659 | 8393 | 1853 | 1483 | 6558 | 3026 | 4496 | 2612 | 5727 |

Code Text

Extract places and ratings

```
[3] import numpy as np

    place = np.array(Ratings.values[:,0])
    print(place[0])
    print(place.shape)

    ratings = np.array(Ratings.values[:,1:],dtype='float64')
    print(ratings[0,:])
    print(ratings.shape)
```

```
Abilene_TX
(329,)
[ 521. 6200.  237.  923. 4031. 2757.  996. 1405. 7633.]
(329, 9)
```

**b.** Replace each value in the matrix by its base-10 logarithm. (This pre-processing is done for convenience since the numerical range of the ratings is large.)

```
[4]  ratings_original = ratings
     ratings = np.log10(ratings)
     print(ratings[0,:])
     print(ratings.shape)
```

```
[2.71683772 3.79239169 2.37474835 2.9652017  3.6054128  3.44043677
 2.99825934 3.14767632 3.88269526]
(329, 9)
```

**c.** Perform PCA on the data. Remember to center the data points first by computing the mean data vector and subtracting it from every point. With the centered data matrix, do an SVD and compute the principal components.

```
[5]  from sklearn.decomposition import PCA

     X = ratings
     X_mean = np.mean(X, axis = 0)
     X_zc = X - X_mean
     pca = PCA(n_components=2)
     pca.fit(X_zc)
     e_vectors = pca.components_ # get eignvectors
```

**d.** Write down the first two principal components v1 and v2. Provide a qualitative interpretation of the components. Which among the nine factors do they appear to correlate the most with?

```
[6]  print('eignvectors:')
     print(e_vectors)
```

```
eignvectors:
[[ 0.03507288  0.09335159  0.40776448  0.10044536  0.15009714  0.03215319
   0.87434057  0.15899622  0.01949418]
 [ 0.0088782   0.00923057 -0.85853187  0.22042372  0.05920111 -0.06058858
   0.30380632  0.33399255  0.0561011 ]]
```

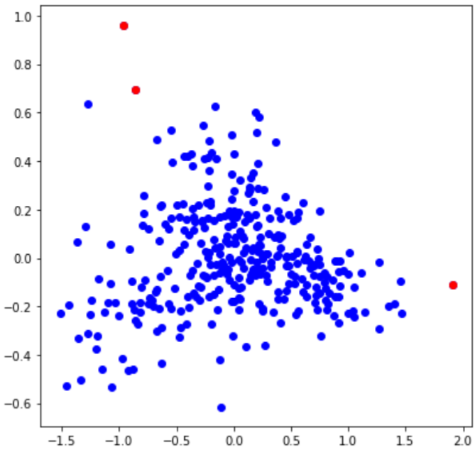
Principal components(eigenvectors) are shown above, these vectors point the direction data varies the most.

For first component, 3rd and 7th factors contribute more for it, we can say they are correlated, also we can see 7th factor is much larger than rest, it is dominant, in fact, we could state that based on the correlation of 0.87434057 that this principal component is primarily a measure of the Arts. 3rd factor is Health Care and Environment, 7th factor is The Arts.

For second component, 3rd factor contributes the most for it. It seems 3rd factor dominate the vector and have small correlation with other unimportant factors because -0.85853187 is much larger than others. 3rd factor is Health Care and Environment.

**e.** Project the data points onto the first two principal components. (That is, compute the highest 2 scores of each of the data points.) Plot the scores as a 2D scatter plot. Which cities correspond to outliers in this scatter plot?

```
import matplotlib.pyplot as plt
scores = np.matmul(X_zc,e_vectors.T)
s_mean = np.mean(scores,axis=0)
s_std = np.std(scores,axis=0)
outliers = []
for i in range(scores.shape[0]):
  if (scores[i][0] > s_mean[0]+3*s_std[0] or scores[i][0] < s_mean[0]-3*s_std[0]) or (scores[i][1] > s_mean[1]+3*s_std[1] or scores[i
    outliers.append(i)
plt.figure(figsize=(6.5,6.5))
plt.scatter(scores[:,0],scores[:,1],c='b')
for i in outliers:
  plt.plot(scores[i][0],scores[i][1],'ro')
```




For outliers, it is decided by $3\sigma$ threshold from Gaussian Distribution. Red points above are outliers.

Print outliers:

```
for i in outliers:
  print(place[i])
```

```
Brownsville-Harlington_TX
Midland_TX
New_York_NY
```

Cites which are outliers are shown above.

**f.** Repeat Steps 2-5, but with a slightly different data matrix – instead of computing the base-10 logarithm, use the z-scores. (The z-score is calculated by computing the mean u and standard deviation d for each feature, and normalizing each entry x by (x-u)/d ). How do your answers change?

(1) compute z

```
[30] np.set_printoptions(precision=4,suppress=True,floatmode = 'fixed')
     r_mean = np.mean(ratings_original,axis=0)
     r_std = np.std(ratings_original,axis=0)
     z = ratings_original
     print(ratings_original.shape)
     for i in range(329):
       for j in range(9):
         z[i][j] = (z[i][j] - r_mean[j]) / r_std[j]
     print(z)
```

```
(329, 9)
[[-0.1470 -0.9013 -0.9473 ... -0.4649 -0.5466  1.9464]
 [ 0.3007 -0.0876  0.4696 ...  0.5206  0.9744 -1.0855]
 [-0.5864 -0.4231 -0.5669 ... -0.6286 -1.2235 -0.2543]
 ...
 [ 0.0105  0.0103 -0.4720 ... -0.4593 -1.2446 -0.5351]
 [ 0.2592 -0.5566 -0.0886 ... -0.0763 -0.6433 -1.5066]
 [ 0.5742 -0.1980 -0.9723 ... -0.6534 -1.1504 -0.7678]]
```

data points are already centered

(2) Write down the first two principal components v1 and v2. Provide a qualitative interpretation of the components.

```
[31] from sklearn.decomposition import PCA

     pca = PCA(n_components=2)
     pca.fit(z)
     e_vectors = pca.components_ # get eignvectors
     print('eignvectors:')
     print(e_vectors)
```

```
eignvectors:
[[ 0.2064  0.3565  0.4602  0.2813  0.3512  0.2753  0.4631  0.3279  0.1354]
 [ 0.2178  0.2506 -0.2995  0.3553 -0.1796 -0.4834 -0.1948  0.3845  0.4713]]
```

Principal components(eigenvectors) are shown above, these vectors point the direction data varies the most.

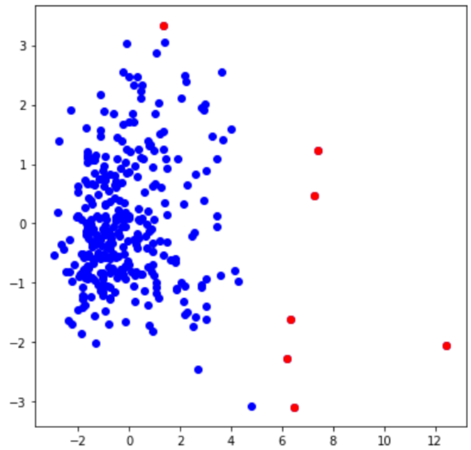
For first component, factor values are small and close to each others, we can relatively see that 3rd and 7th factors contribute more for it, we can say they are correlated. 3rd factor is Health Care and Environment, 7th factor is The Arts.

For second component, factor values are small and close to each others, we can relatively see that 6th and 9th factors contribute more for it, we can say they are correlated. 6th factor is Education, 9th factor is Economics.

(3) Project the data points onto the first two principal components. (That is, compute the highest 2 scores of each of the data points.) Plot the scores as a 2D scatter plot. Which cities correspond to outliers in this scatter plot?

```
[35] import matplotlib.pyplot as plt
     scores = np.matmul(z,e_vectors.T)
     s_mean = np.mean(scores,axis=0)
     s_std = np.std(scores,axis=0)
     outliers = []
     for i in range(scores.shape[0]):
       if (scores[i][0] > s_mean[0]+3*s_std[0] or scores[i][0] < s_mean[0]-3*s_std[0]) or (scores[i][1] > s_mean[1]+3*s_std[1] or scores[i
         outliers.append(i)
     print(outliers)
     plt.figure(figsize=(6.5,6.5))
     plt.scatter(scores[:,0],scores[:,1],c='b')
     for i in outliers:
       plt.plot(scores[i][0],scores[i][1],'ro')
```

```
[42, 64, 167, 178, 212, 269, 313]
```




For outliers, it is decided by $3\sigma$ threshold from Gaussian Distribution. Red points above are outliers.

Print outliers

```
[39] for i in outliers:
       print(place[i])
```

```
Boston_MA
Chicago_IL
Las_Vegas_NV
Los_Angeles_Long_Beach_CA
New_York_NY
San_Francisco_CA
Washington_DC-MD-VA
```

Cites which are outliers are shown above.