Intro to ML Homework3

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March 4, 2020

1 Question 1

(a) My choice is:

c: a positive constant or 0

because there should be no regularization on w if all parameters w are free to be determined.

(b) My choice is:

$L1Norm$

because each w is of an absolute value function thus when whole $L(w)$ getting to smallest, w will rapidly get to 0 point

(c) My choice is:

$L2Norm$

because when $L2Norm$ getting smaller with $L(w)$, magnitude of w on average will shrink.

(d) My choice is:

$\sum_{j=2}^{d}(w_j - w_{j-1})^2$

because when this $R(w)$ getting smaller with $L(w)$, w_j and w_{j-1} will approach and overlap for most.

(e) My choice is:

$R(w) = \sum_{i=1}^{n} e^{-w_i}$

because when $R(w)$ is getting smaller with $L(w)$, gradient will let w_i go to positive side because negative w_i cause penalty on $L(w)$, however, I think we must set a relative large learning rate to help w_i cross -1 to 0 due to a relatively mild gradient/"slope" in this area.

2 Question 2

(1)Load data set

Question 2:

(1)load dataset

```
[1] import pandas as pd
    import numpy as np
    from sklearn.datasets import load_boston
    boston_dataset = load_boston()
    boston = pd.DataFrame(boston_dataset.data,
    columns=boston_dataset.feature_names)
    boston['MEDV'] = boston_dataset.target
    boston.head()
```

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

(2)Preprocess: normalize data

(2) Preprocess

```
[2] import sklearn
    boston_dataset.target = boston_dataset.target.reshape(boston_dataset.target.shape[0],1)
    sklearn.preprocessing.normalize(boston_dataset.data, norm='max', axis=0, copy=False, return_norm=False)
    sklearn.preprocessing.normalize(boston_dataset.target, norm='max', axis=0, copy=False, return_norm=False)

    #print(boston_dataset.target)
    boston = pd.DataFrame(boston_dataset.data,
    columns=boston_dataset.feature_names)
    boston['MEDV'] = boston_dataset.target
    boston.head()
```

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.000071	0.18	0.083273	0.0	0.617681	0.748861	0.652	0.337278	0.041667	0.416315	0.695455	1.000000	0.131156	0.480
1	0.000307	0.00	0.254867	0.0	0.538462	0.731321	0.789	0.409607	0.083333	0.340366	0.809091	1.000000	0.240716	0.432
2	0.000307	0.00	0.254867	0.0	0.538462	0.818337	0.611	0.409607	0.083333	0.340366	0.809091	0.989746	0.106136	0.694
3	0.000364	0.00	0.078587	0.0	0.525832	0.797039	0.458	0.499913	0.125000	0.312236	0.850000	0.994281	0.077430	0.668
4	0.000776	0.00	0.078587	0.0	0.525832	0.814009	0.542	0.499913	0.125000	0.312236	0.850000	1.000000	0.140374	0.724

(3)put a test on train_test_split() to split data set 80-20 train-test for one time.

(3)put a test on train_test_split() to split dataset 80-20 train-test for one time.

```
[3] from sklearn.model_selection import train_test_split
    X = boston_dataset.data
    y = boston_dataset.target

    print('Xshape: ', X.shape)
    print('yshape: ', y.shape)

    train_X, test_X, train_y, test_y = train_test_split(X, y, test_size=0.2, shuffle = True)
    print('train_Xshape: ', train_X.shape)
    print('train_yshape: ', train_y.shape)
    print('test_Xshape: ', test_X.shape)
    print('test_yshape: ', test_y.shape)
```

```
Xshape:  (506, 13)
yshape:  (506, 1)
train_Xshape:  (404, 13)
train_yshape:  (404, 1)
test_Xshape:  (102, 13)
test_yshape:  (102, 1)
```

(4) Because I am little bit confused about exactly way to implement 10-fold, I made it by two ways as below:
Method(a): do 10-Fold and ridge regression by repeating 10 times of splitting 80-20 train and test, each split will shuffle data set

(4)(a)do 10-Fold and ridge regression by repeating 10 times of splitting 80-20 train and test, each split will shuffle dataset

```
[4] from sklearn.model_selection import train_test_split
    from sklearn.model_selection import KFold
    from sklearn.linear_model import RidgeCV

    MAXRSQ = 0
    Rsq = []

    for iteration in range(10):
      train_X, test_X, train_y, test_y = train_test_split(X, y, test_size=0.2, shuffle = True)
      reg = RidgeCV(alphas=np.logspace(-6, 6, 13))
      reg.fit(train_X, train_y)
      #print("coef:",reg.coef_)
      #print("intercept:",reg.intercept_)
      R2 = reg.score(test_X, test_y, sample_weight=None)
      Rsq.append(R2)
      if Rsq[-1] > MAXRSQ:
        MAXRSQ = Rsq[-1]
    print("R^2s: ",Rsq)
    print("max R^2: ",MAXRSQ )
```

```
R^2s:  [0.7680858851610384, 0.7874675986247345, 0.5301104494185349, 0.7940209670900414, 0.753972605410243, 0.6979271656739077, 0.67
max R^2:  0.7940209670900414
```

Result: R^2s: [0.7680858851610384, 0.7874675986247345, 0.5301104494185349, 0.7940209670900414, 0.753972605410243, 0.6979271656739077, 0.6771072254856478, 0.786203696643711, 0.6760426283022105, 0.6737388696246557]
max R2: 0.7940209670900414

Method(b): do 10-Fold and ridge regression by performing KFold function from sklearn on 80% dataset (train set), and apply best model to 20% dataset (test set) at last.

KFold rom sklearn divide train set into k equal parts and use one parts as test set each time and iterate k times , at last choose best model.

(4)(b)do 10-Fold and ridge regression by perform KFold function ftome sklearn on 80% dataset (train set), and apply best model to 20% dataset (test set) at last.

```
KFold divide train set into k equal parts and use one parts as test set each time and iterate k times , at last choose best model.
```

```
from sklearn.model_selection import KFold
from sklearn.linear_model import RidgeCV

kf = KFold(n_splits=10)
kf.get_n_splits(train_X)
MAXRSQ = 0
Rsq = []

for train_index, test_index in kf.split(train_X):
  #print("TRAIN:", train_index, "TEST:", test_index)
  X_kftrain, X_kftest = train_X[train_index], train_X[test_index]
  y_kftrain, y_kftest = train_y[train_index], train_y[test_index]
  reg = RidgeCV(alphas=np.logspace(-6, 6, 13))
  reg.fit(X_kftrain, y_kftrain)
  #print("coef:",reg.coef_)
  #print("intercept:",reg.intercept_)
  R2 = reg.score(X_kftest, y_kftest, sample_weight=None)
  Rsq.append(R2)
  if Rsq[-1] > MAXRSQ:
    REG = reg
    MAXRSQ = Rsq[-1]
print("R^2: ",Rsq)
print("max R^2: ",MAXRSQ )
```

```
R^2:  [0.7066356164882359, 0.7978579718649058, 0.8817957851320075, 0.6285918652207365, 0.6816921028698321, 0.7551258611869247, 0.736
max R^2:  0.8817957851320075
```

Training result: R^2: [0.7066356164882359, 0.7978579718649058, 0.8817957851320075, 0.6285918652207365, 0.6816921028698321, 0.7551258611869247, 0.7362578047680395, 0.6882121578205618, 0.6116558913590023, 0.6655671564513179]
max R^2: 0.8817957851320075

apply best model gained from 4(b) to test set and report R2

apply best model gained from 4(b) to test set and report R^2

```
R2 = REG.score(test_X, test_y, sample_weight=None)
print('R^2 for test set: ',R2 )
```

```
R^2 for test set:  0.6763008535410039
```

Result for test: R^2 for test set: 0.6763008535410039

3 Question 3

$$
\begin{aligned}
L(w) &= \frac{1}{2} \parallel x - w \parallel_2^2 + \lambda \parallel w \parallel_1 \\
&= \frac{1}{2} \sum_{i=1}^{d} (x_i - w_i)^2 + \lambda \sum_{i=1}^{d} |w_i|
\end{aligned}
\tag{1}
$$

To get a minimum $L(w)$:

$$
\frac{\partial L(w)}{\partial w} = 0 \rightarrow \frac{\partial \left(\frac{1}{2} \sum\limits_{i=1}^{d} (x_i - w_i)^2 + \lambda \sum\limits_{i=1}^{d} |w_i| \right)}{\partial w} = 0
$$

Because $\sum\limits_{i=1}^{d} (x_i - w_i)^2$ and $\sum\limits_{i=1}^{d} |w_i|$ are larger or equal to zero, we discuss each situation for i independently:

$$
\frac{\partial \left(\frac{1}{2} (x_i - w_i)^2 + \lambda |w_i| \right)}{\partial w_i} = 0
$$

(1)when $w_i > 0$:

$$
(x_i - w_i) * (-1) + \lambda = 0 \rightarrow w_i = x_i - \lambda \qquad (w_i > 0 \rightarrow x_i > \lambda)
$$

(2)when $w_i < 0$:

$$
(x_i - w_i) * (-1) - \lambda = 0 \rightarrow w_i = x_i + \lambda \qquad (w_i < 0 \rightarrow x_i < -\lambda)
$$

(3)otherwise:

$$
w_i = 0
$$

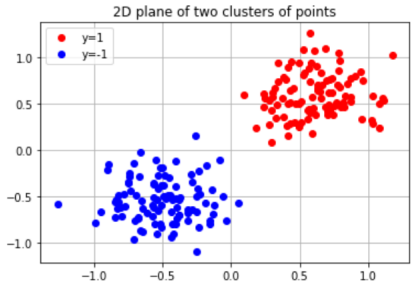
Thus question solved.

4 Question 4

(a)

```
[ ] import numpy as np
    import matplotlib.pyplot as plt
    meanx1 = (0.6, 0.6)
    covx1 = [[0.05, 0], [0, 0.05]]
    x1 = np.random.multivariate_normal(meanx1, covx1, 100)
    meanx2 = (-0.5, -0.5)
    covx2 = [[0.06, 0], [0, 0.06]]
    x2 = np.random.multivariate_normal(meanx2, covx2, 100)
    print("x1.shape:" ,x1.shape)
    print("x2.shape:" ,x2.shape)
    plt.figure(1)
    plt.title("2D plane of two clusters of points")
    plt.plot(x1[:,0],x1[:,1],'ro',label="y=1")
    plt.plot(x2[:,0],x2[:,1],'bo',label="y=-1")
    plt.legend()
    plt.grid()

    y1 = np.random.normal(loc=1.0, scale=0.0, size=100)
    y2 = np.random.normal(loc=-1.0, scale=0.0, size=100)
    y1 = y1.reshape(y1.shape[0],1)
    y2 = y2.reshape(y2.shape[0],1)
    print("y1.shape:" ,y1.shape)
    print("y2.shape:" ,y2.shape)
```

```
x1.shape: (100, 2)
x2.shape: (100, 2)
y1.shape: (100, 1)
y2.shape: (100, 1)
```



(b)because y_i equals to 1 or -1, I solve this question by two methods as below:

Method(1): According to what we learn, deduce a formula suits 1 & -1 situation. The formula by my deduction are:

$$L(w) = -\sum_{i=1}^{n}[(\frac{1+y_i}{2})log\frac{1}{1+e^{-z}} + (\frac{1-y_i}{2})log\frac{e^{-z}}{1+e^{-z}}]$$

$$w_{k+1} = w_k + \alpha\sum_{i=1}^{n}[\frac{1+y_i}{2} - \frac{1}{1+e^{-z}}]x_i$$

Method(2): use formula for 1 and 0 as we learnt in class, preprocess all -1 to 0.

define function for solving lgistic regression, it will shuffle data set before iteration.

Although it is set with epoch parameter but I think we don't know when it will exactly converges so we can't compare convergence rate between GD and SGD (question (c)), therefore I design it to stop and report iteration number if it converges judging by a threshold (if newest loss function value variation is less than 0.1% of that of last interation, this model's $L(w)$ variation/oscillation is very small thus this model is considered as convergence).

Set mode argument to 1 to use a formula for 1 & -1, set mode argument to 2 to use formula in class for 1 & 0.

ps: Emailed professor to confirm feasibility of this, thanks.

```
def gradient_descent(X,y,learning_rate,epoch,mode):
  import time

  #preoprocess: make all -1 to 0 if mode = 2
  if mode == 2 :
    print("GD Result when use 1 & 0 formula learnt in class:")
    for n in range(X.shape[0]):
      if y[n] == -1 :
        y[n] = 0
  if mode == 1 :
    print("GD Result when use 1 & -1 formula from my deduction:")
    for n in range(X.shape[0]):
      if y[n] == 0 :
        y[n] = -1

  #preproess X and y: shuffle X,y pairs
  #combine X & y, shuffle and then split
  pairs = np.concatenate((X, y), axis=1)
  #print("pairs.shape: ",pairs.shape,"pairs:")
  np.random.shuffle(pairs)
  #print(pairs)
  X = pairs[:,0:pairs.shape[1]-1]
  y = pairs[:,pairs.shape[1]-1:pairs.shape[1]]
```

```
#print(X)
#print(y)

n = X.shape[0]
d = X.shape[1]
x0 = np.ones((n,1))
X = np.hstack((x0,X))# add a column of 1 to the left of x matching for w0
w = np.random.randn(d+1,1)
print("X.shape: ",X.shape," y.shape: ",y.shape," w.shape: ",w.shape)

cost = []
gd = []
t0 = time.time()
for iteration in range(epoch):
  Lw = 0
  gradient = np.zeros((d+1,1))
  for i in range(0,n):
    Xi = X[i].reshape(1,X[i].shape[0])
    fx = ( 1+np.exp((-1)*np.matmul(Xi,w)) )**(-1)# Sigmoid
    if mode == 1 : #use formula for 1 & -1
      gradient = gradient - ( (1+ y[i])/2 - fx ) * Xi.T
      Lw = Lw +  (-1)*(  (1+y[i])/2 * np.log( fx ) + (1-y[i])/2 * np.log( 1 - fx ) )
    if mode == 2 : #use formula for 1 & 0
      gradient = gradient - ( y[i] - fx ) * Xi.T
      Lw = Lw +  (-1)*(  y[i] * np.log( fx ) + (1-y[i]) * np.log( 1 - fx ) )
```

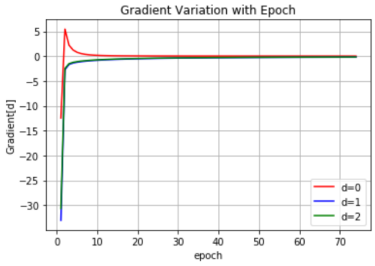
```
    w = w - learning_rate * gradient
    cost.append(Lw)
    gd.append(gradient)
    if len(cost) > 1 and abs(Lw - cost[-2]) / abs(cost[-2]) < 0.001:
      break
    # print("Lw:  ",Lw)
    # print("gradient: ",gradient)
  run_time = time.time() - t0
  print("converge at ",iteration+1,"th epoch")
  print("iteration time: ", run_time)

  gd = np.array(gd)
  cost = np.array(cost)
  cost = cost.reshape(cost.shape[0],1)
  iterations = range(1,iteration+2)
  print("time per iteration : ", run_time / epoch)
  plt.figure(2)
  plt.title("Gradient Variation with Epoch")
  plt.plot(iterations,gd[:,0],'r',label = "d=0")
  plt.plot(iterations,gd[:,1],'b',label = "d=1")
  plt.plot(iterations,gd[:,2],'g',label = "d=2")
  plt.grid()
  plt.legend()
  plt.xlabel("epoch")
  plt.ylabel("Gradient[d]")
```

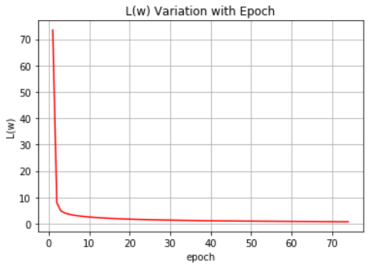
```
  plt.figure(3)
  plt.title("L(w) Variation with Epoch")
  plt.plot(iterations,cost,'r')
  plt.xlabel("epoch")
  plt.ylabel("L(w)")
  plt.grid()
  print("w: ")
  print(w)
  print("Last Gradient: ")
  print(gd[-1])
  print("Loss (Lw): ",cost[-1])
  return w
```

combine two cluster of points together and train the logistic regression model

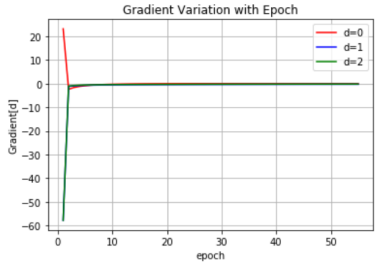
```
[122] X = np.concatenate((x1, x2), axis=0)
      #print("X.shape: ",X.shape,"X:")
      #print(X)
      y = np.concatenate((y1, y2), axis=0)
      #print("y.shape: ",y.shape,"y:")
      #print(y)
      Train_X, Test_X, Train_y, Test_y = train_test_split(X, y, test_size=0.2, shuffle = True)

      gdw = gradient_descent(Train_X,Train_y,0.1,200,mode = 1)
```

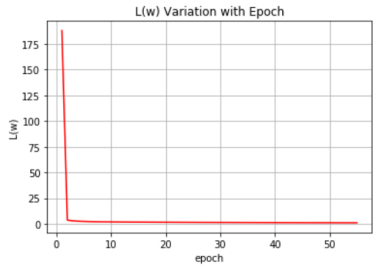
```
GD Result when use 1 & -1 formula from my deduction:
X.shape:  (160, 3)  y.shape:  (160, 1)  w.shape:  (3, 1)
converge at  74 th epoch
iteration time:  0.3808877468109131
time per iteration :  0.0019044387340545654
w:
[[-0.45542826]
 [ 6.58578279]
 [ 7.37148804]]
Last Gradient:
[[ 0.01659029]
 [-0.19584131]
 [-0.17044524]]
Loss (Lw):  [0.76413498]
```






```
GD Result when use 1 & 0 formula learnt in class:
X.shape:  (160, 3)  y.shape:  (160, 1)  w.shape:  (3, 1)
converge at  55 th epoch
iteration time:  0.2302231788635254
time per iteration :  0.001151115894317627
w:
[[-0.73405886]
 [ 7.15831395]
 [ 6.88834324]]
Last Gradient:
[[ 0.02941278]
 [-0.24599076]
 [-0.1599582 ]]
Loss (Lw):  [0.98526853]
```





For training I split 80-20 train and test data sets. As we can see from outputs above, my two methods using two different formula can converge.

(c)

define a function to generate indices of mini batch

```
[ ]  def mini_batch(full_size,mini_size):
       import numpy as np
       fullbatch = []
       for n in range(full_size):
         fullbatch.append(n)
       #print("full batch: ",fullbatch)
       np.random.shuffle(fullbatch)
       minibatch = fullbatch[0:100]
       #print(len(minibatch))
       #print("mini batch: ",minibatch)
       return minibatch
     #mini_batch(200,100)
```

According to function above to define a function for mini_batch gradient descent

```
[117] def mini_batch_gd(X,y,learning_rate,epoch,mode,batch_size = X.shape[0]):
        import time

        if batch_size > X.shape[0] :
          print("batch size > sample amount")
          return 0
        #preoprocess: make all -1 to 0 if mode = 2
        if mode == 2 :
          print("SGD Result when use 1 & 0 formula learnt in class:")
          for n in range(X.shape[0]):
            if y[n] == -1 :
              y[n] = 0
        if mode == 1 :
          print("SGD Result when use 1 & -1 formula from my deduction:")
          for n in range(X.shape[0]):
            if y[n] == 0 :
              y[n] = -1

        #preproess X and y: shuffle X,y pairs #first combine X & y, shuffle and then split
        pairs = np.concatenate((X, y), axis=1)
        #print("pairs.shape: ",pairs.shape,"pairs:")
        np.random.shuffle(pairs)
        #print(pairs)
        X = pairs[:,0:pairs.shape[1]-1]
        y = pairs[:,pairs.shape[1]-1:pairs.shape[1]]
```

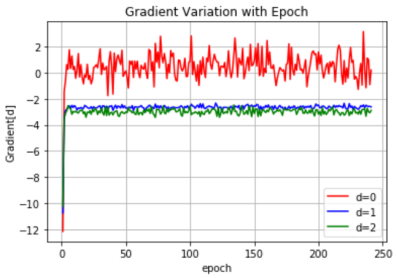
```
        n = X.shape[0]
        d = X.shape[1]
        x0 = np.ones((n,1))
        X = np.hstack((x0,X))# add a column of 1 to the left of x matching for w0
        w = np.random.randn(d+1,1)
        print("X.shape: ",X.shape," y.shape: ",y.shape," w.shape: ",w.shape)

        cost = []
        gd = []
        t0 = time.time()
        for k in range(epoch):
          Lw = 0
          gradient = np.zeros((d+1,1))
          minibatch = mini_batch(n,batch_size)
          for i in range(0,batch_size):
            Xi = X[minibatch[i]].reshape(1,X[minibatch[i]].shape[0])
            fx = ( 1+np.exp((-1)*np.matmul(Xi,w)) )**(-1)# Sigmoid
            if mode == 1 : #use formula for 1 & -1
              gradient = gradient - ( (1+ y[minibatch[i]])/2 - fx ) * Xi.T
              Lw = Lw +  (-1)*(  (1+y[minibatch[i]])/2 * np.log( fx ) + (1-y[minibatch[i]])/2 * np.log( 1 - fx ) )
            if mode == 2 : #use formula for 1 & 0
              gradient = gradient - ( y[minibatch[i]] - fx ) * Xi.T
              Lw = Lw +  (-1)*(  y[minibatch[i]] * np.log( fx ) + (1-y[minibatch[i]]) * np.log( 1 - fx ) )
          learning_rate = learning_rate / (k+1)
          w = w - learning_rate * gradient
          cost.append(Lw)
```

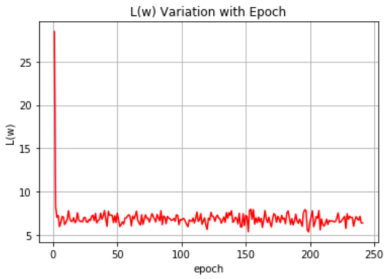
```
        if len(cost) > 1 and abs(Lw - cost[-2]) / abs(cost[-2]) < 0.001:
          break
        # print("Lw:  ",Lw)
        # print("gradient: ",gradient)
      run_time = time.time() - t0
      print("converge at ",k+1,"th epoch")
      print("iteration time: ", run_time)
      print("time per iteration : ", run_time / epoch)
      gd = np.array(gd)
      cost = np.array(cost)
      cost = cost.reshape(cost.shape[0],1)
      iterations = range(1,k+2)

      plt.figure(2)
      plt.title("Gradient Variation with Epoch")
      plt.plot(iterations,gd[:,0],'r',label = "d=0")
      plt.plot(iterations,gd[:,1],'b',label = "d=1")
      plt.plot(iterations,gd[:,2],'g',label = "d=2")
      plt.grid()
      plt.legend()
      plt.xlabel("epoch")
      plt.ylabel("Gradient[d]")
```

```
      plt.figure(3)
      plt.title("L(w) Variation with Epoch")
      plt.plot(iterations,cost,'r')
      plt.xlabel("epoch")
      plt.ylabel("L(w)")
      plt.grid()
      print("w: ")
      print(w)
      print("Last Gradient: ")
      print(gd[-1])
      print("Loss (Lw): ",cost[-1])
      return w
```

```
[166] sgdw = mini_batch_gd(Train_X,Train_y,0.1,400,2,50)
```

```
SGD Result when use 1 & 0 formula learnt in class:
X.shape:  (160, 3)  y.shape:  (160, 1)  w.shape:  (3, 1)
converge at  241 th epoch
iteration time:  0.3034689426422119
time per iteration :  0.0007586723566055298
w:
[[-0.0960937 ]
 [ 2.45887669]
 [ 1.39480995]]
Last Gradient:
[[ 0.18888454]
 [-2.6224503 ]
 [-2.8895921 ]]
Loss (Lw):  [6.35308716]
```





To make good comparison, I learning rate of both GD and SGD is 0.1, and they both apply 1 & 0 formula from class.

SGD output:
SGD Result when use 1 & 0 formula learnt in class:
X.shape: (160, 3) y.shape: (160, 1) w.shape: (3, 1)
converge at 241 th epoch
iteration time: 0.3034689426422119
time per iteration : 0.0007586723566055298

Also we can compare output with the **output of GD:**
GD Result when use 1 & 0 formula learnt in class:
X.shape: (160, 3) y.shape: (160, 1) w.shape: (3, 1)
converge at 55 th epoch
iteration time: 0.2302231788635254
time per iteration : 0.001151115894317627

Thus we can demonstrate that SGD exhibits a slower rate of convergence than GD, but is faster per-iteration, and does not suffer in terms of final quality(can be seen from (d) below).

(d):

For visulization I plot two classification line respectively for GD and SGD according to property of sigmoid function. Test points are also add to the graph, but we can see that final quality is still good.

(d)Visualize correctness of models

```
plt.figure()
plt.title("2D plane of two clusters of points with GD model dividing line")
plt.plot(x1[:,0],x1[:,1],'ro',label="y=1")
plt.plot(x2[:,0],x2[:,1],'bo',label="y=-1")
#plt.plot(x2[:,0],x2[:,1],'bo',label="Wrong Predition")
linex1 = [-1 , 1]
#when w0 + w1*x1 + w2*x2 = 0 (as input of sigmoid), probability outcome after sigmoid = 0.5 , when > 0.5 , judge as 1
linex2 = (0.0 - gdw[0] - gdw[1]*linex1) / gdw[2]
plt.plot(linex1,linex2,label = "GD")
linex4 = (0.0 - sgdw[0] - sgdw[1]*linex1) / sgdw[2]
plt.plot(linex1,linex4,label = "SGD")
# linex3 = (0.5 - gdw[0] - gdw[1]*linex1) / gdw[2]
# plt.plot(linex1,linex3,label = "model0.5")
plt.legend()
plt.grid()
```