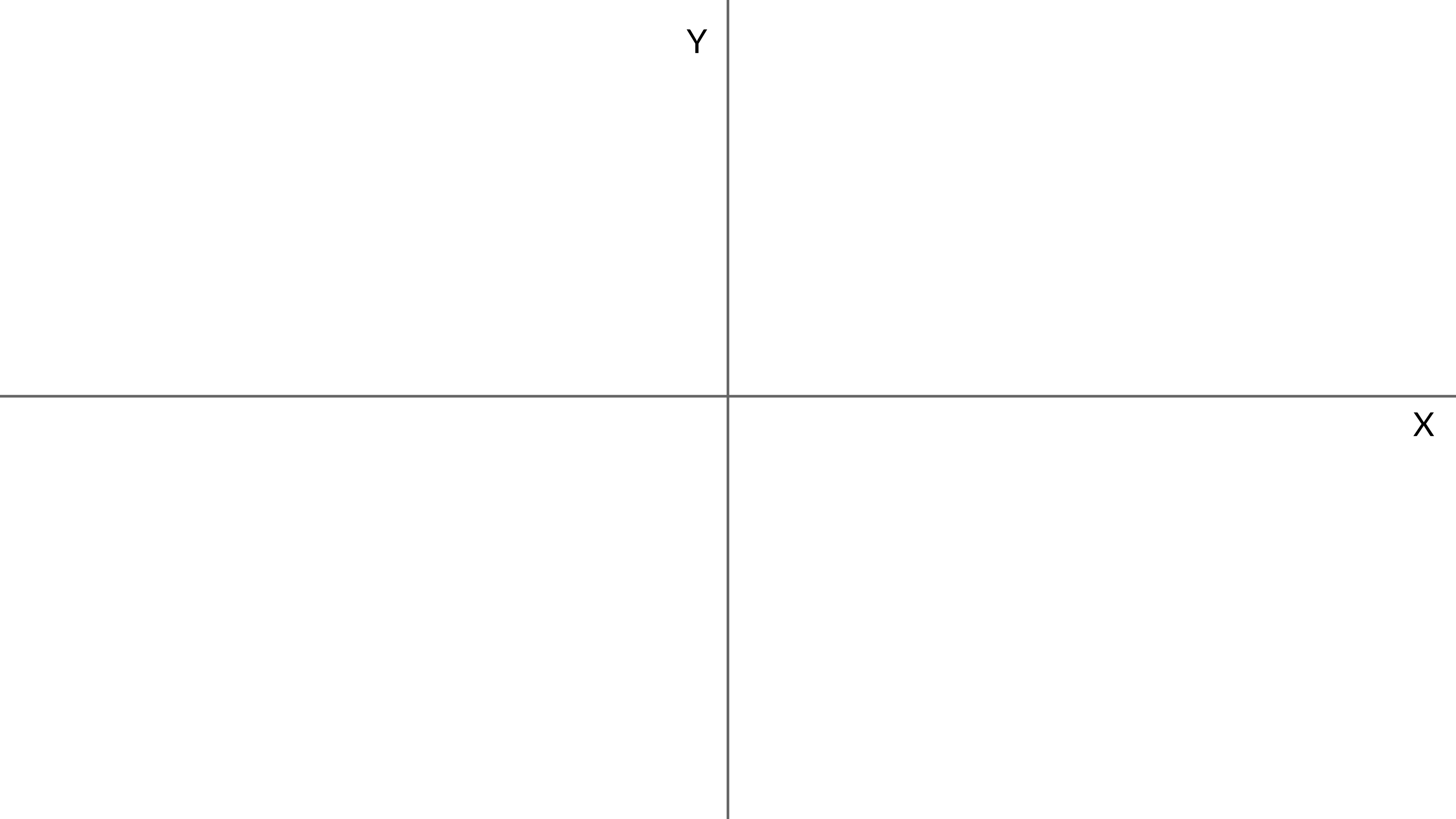
Grupo de Ciencia Computacional HIMFG

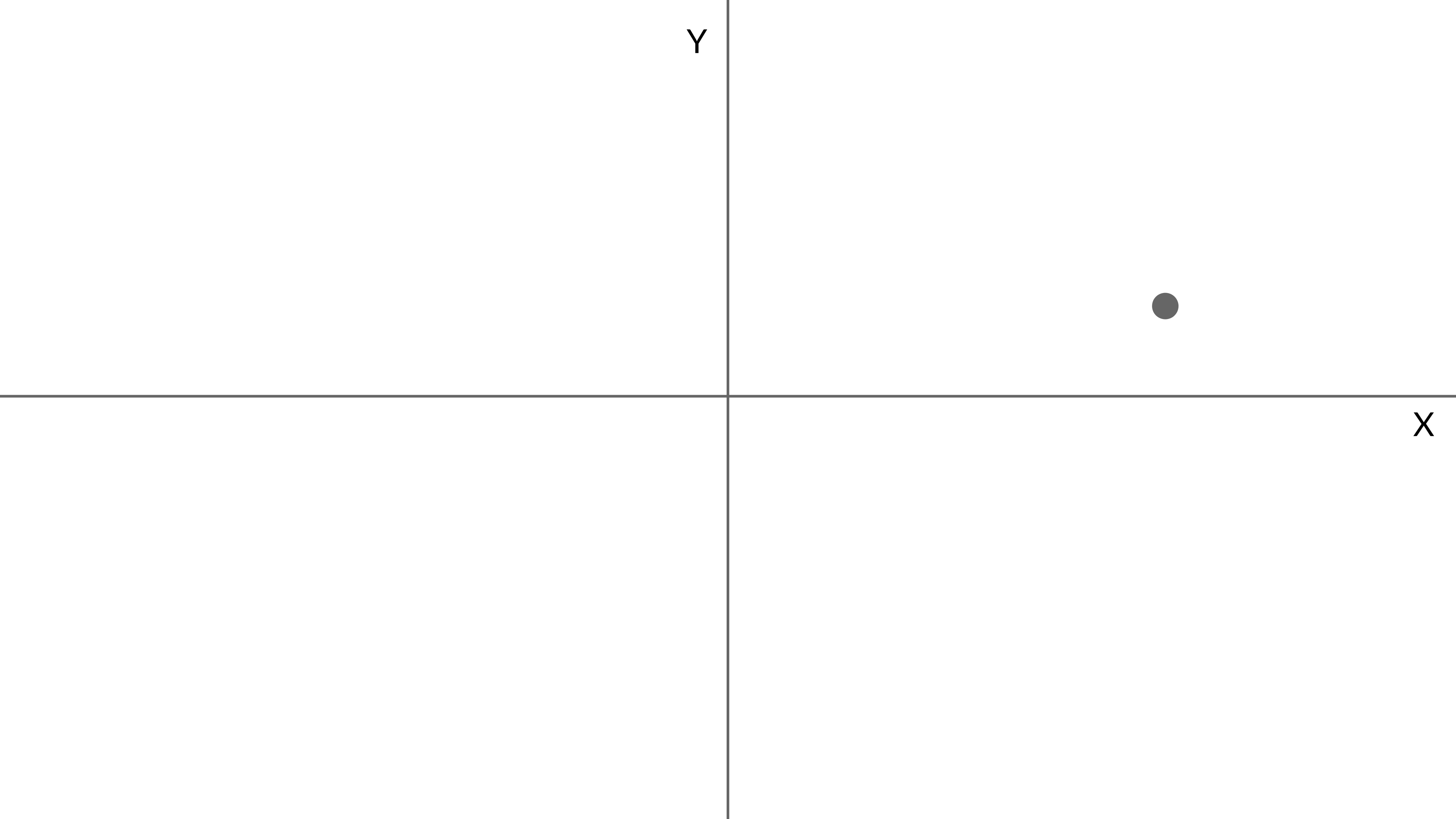
Análisis de Componentes Principales

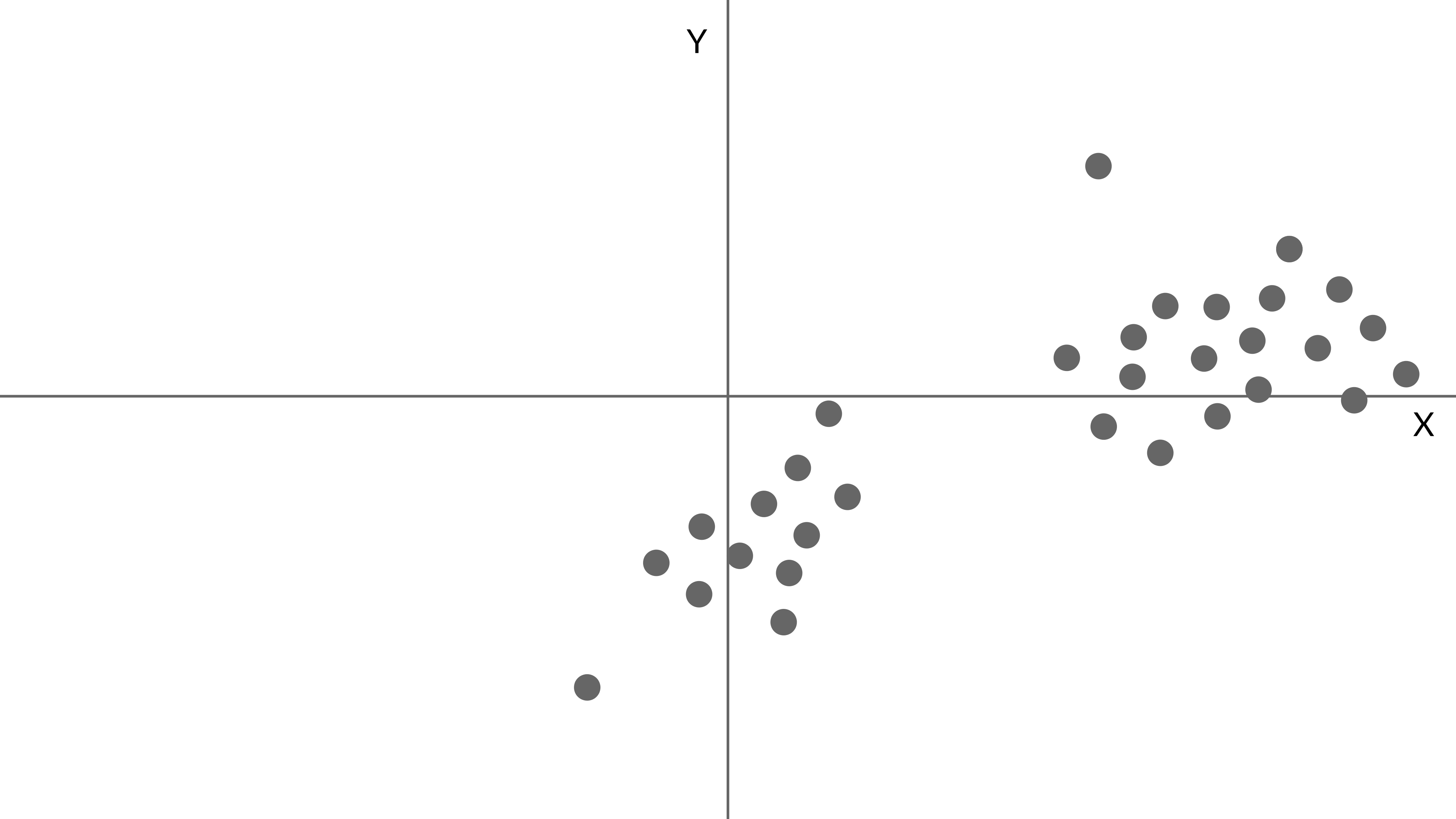
12 Julio 2024 || Diego Prada Gracia @ Grupo de Ciencia Computacional HIMFG

creative commons

PCA en 2D

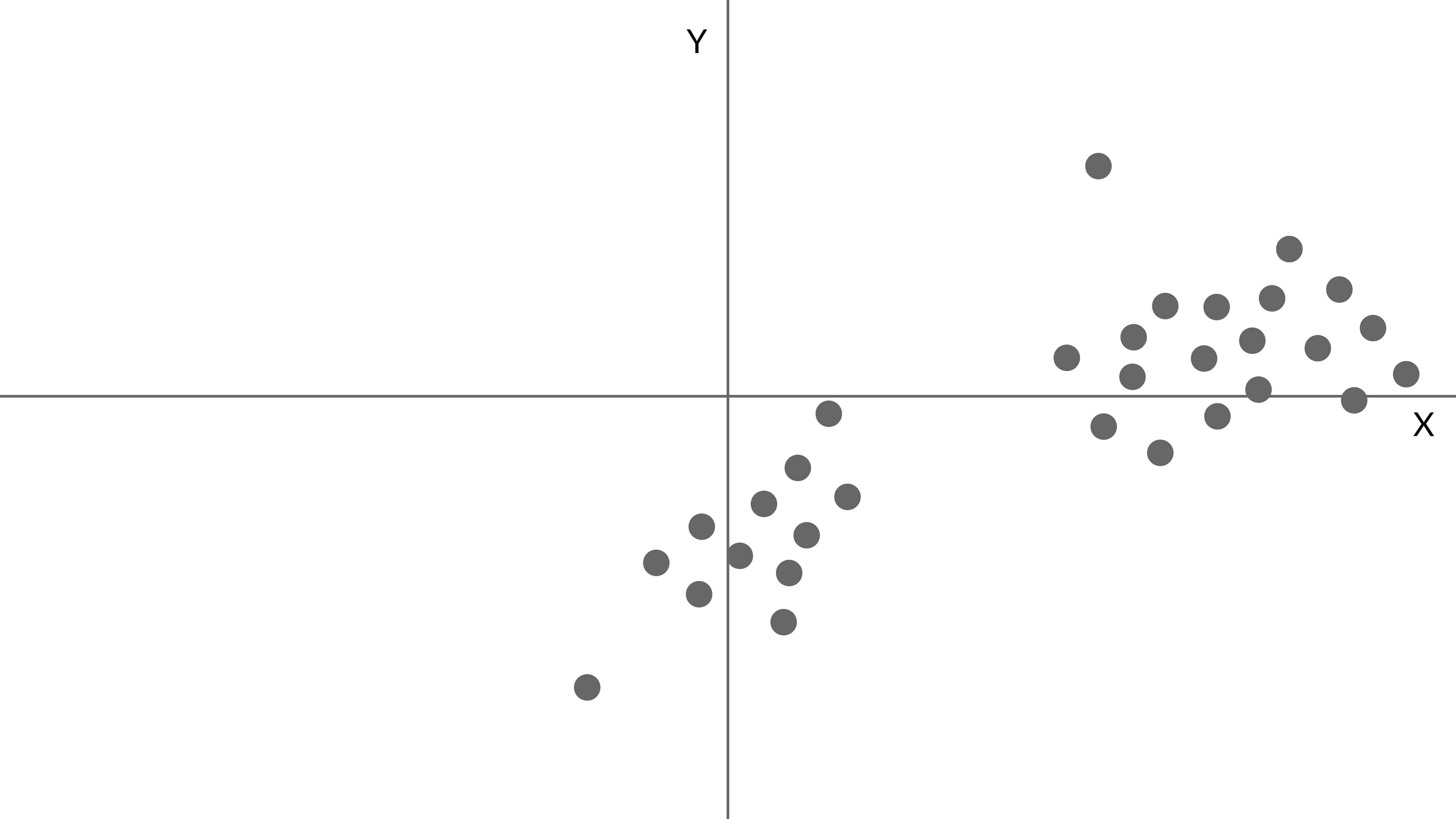

Y
X

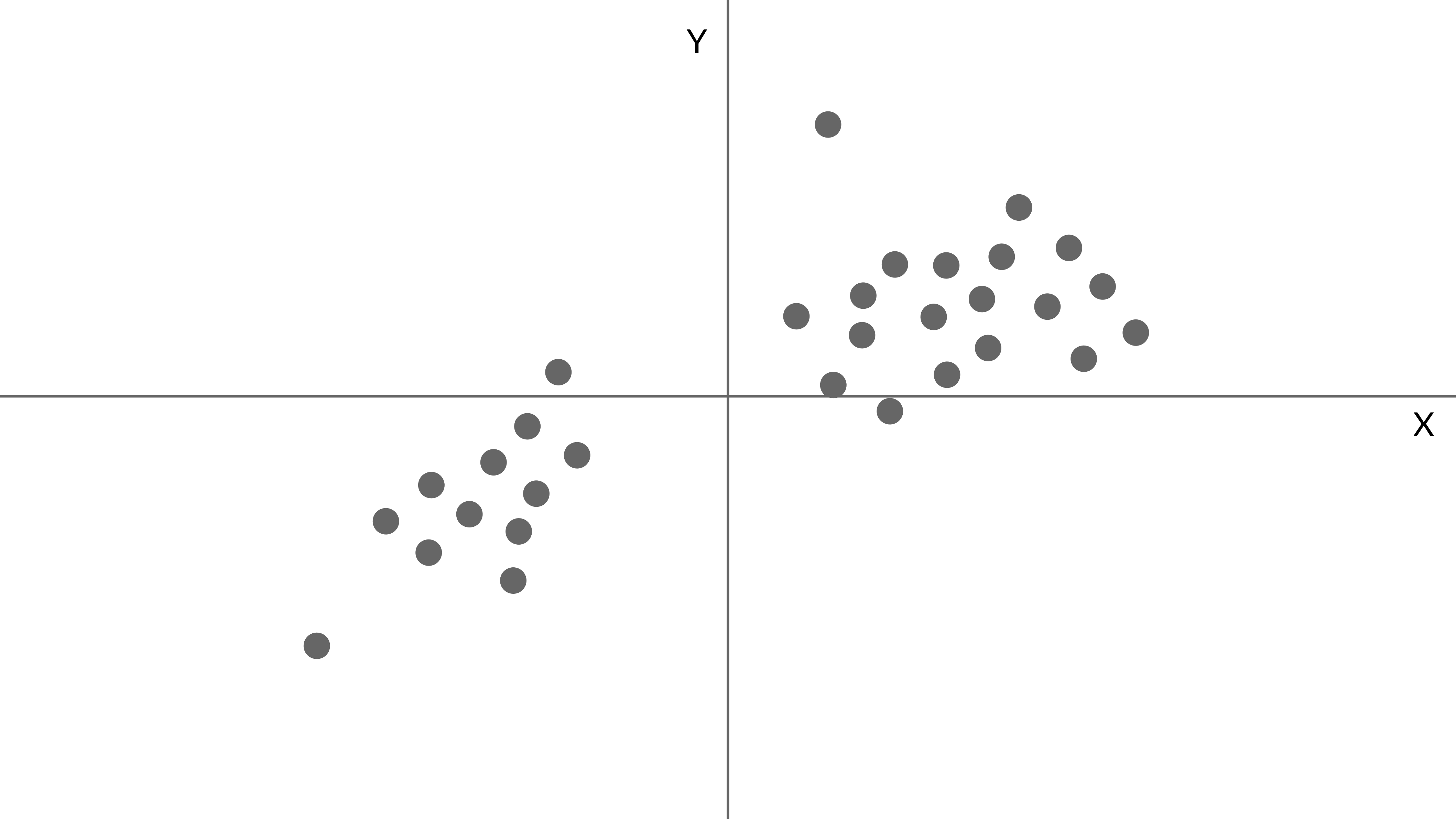

Y
X


Y
X

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle yx \rangle - \langle y \rangle \langle x \rangle \\ \langle xy \rangle - \langle x \rangle \langle y \rangle & \langle y^2 \rangle - \langle y \rangle^2 \end{pmatrix}$$


Y
X


Y
X

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle yx \rangle - \langle y \rangle \langle x \rangle \\ \langle xy \rangle - \langle x \rangle \langle y \rangle & \langle y^2 \rangle - \langle y \rangle^2 \end{pmatrix}$$

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix}$$

Valores propios y vectores propios

$$\lambda \vec{v} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \vec{v}$$

Valores propios y vectores propios

$$\begin{pmatrix} \lambda v_x \\ \lambda v_y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 v_1^x \\ \lambda_1 v_1^y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_1^x \\ v_1^y \end{pmatrix}$$

$$\begin{pmatrix} \lambda_2 v_1^x \\ \lambda_2 v_2^y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_2^x \\ v_2^y \end{pmatrix}$$

$$\lambda_1 > \lambda_2$$

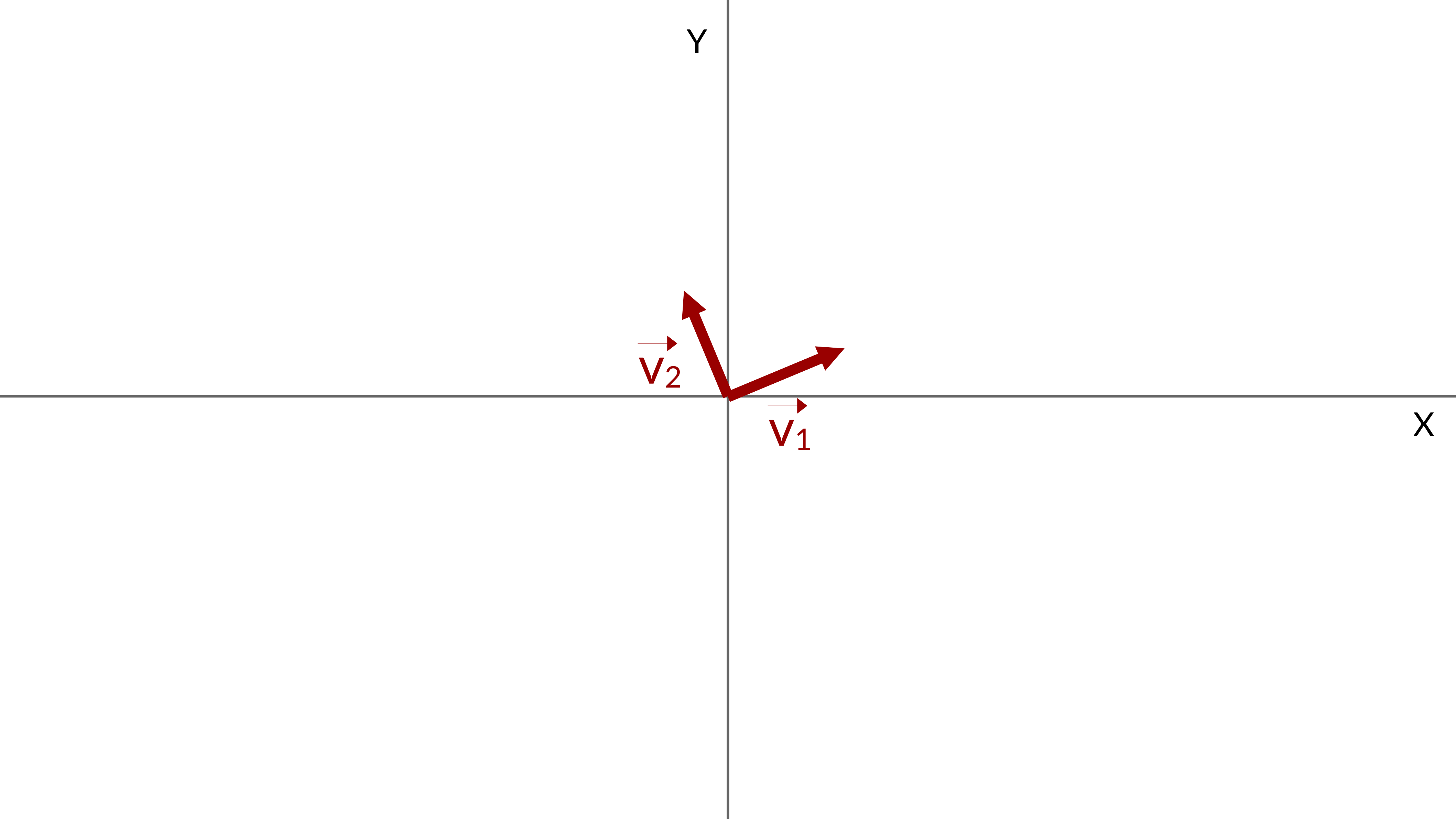
$$1 = v_1^x + v_1^y$$

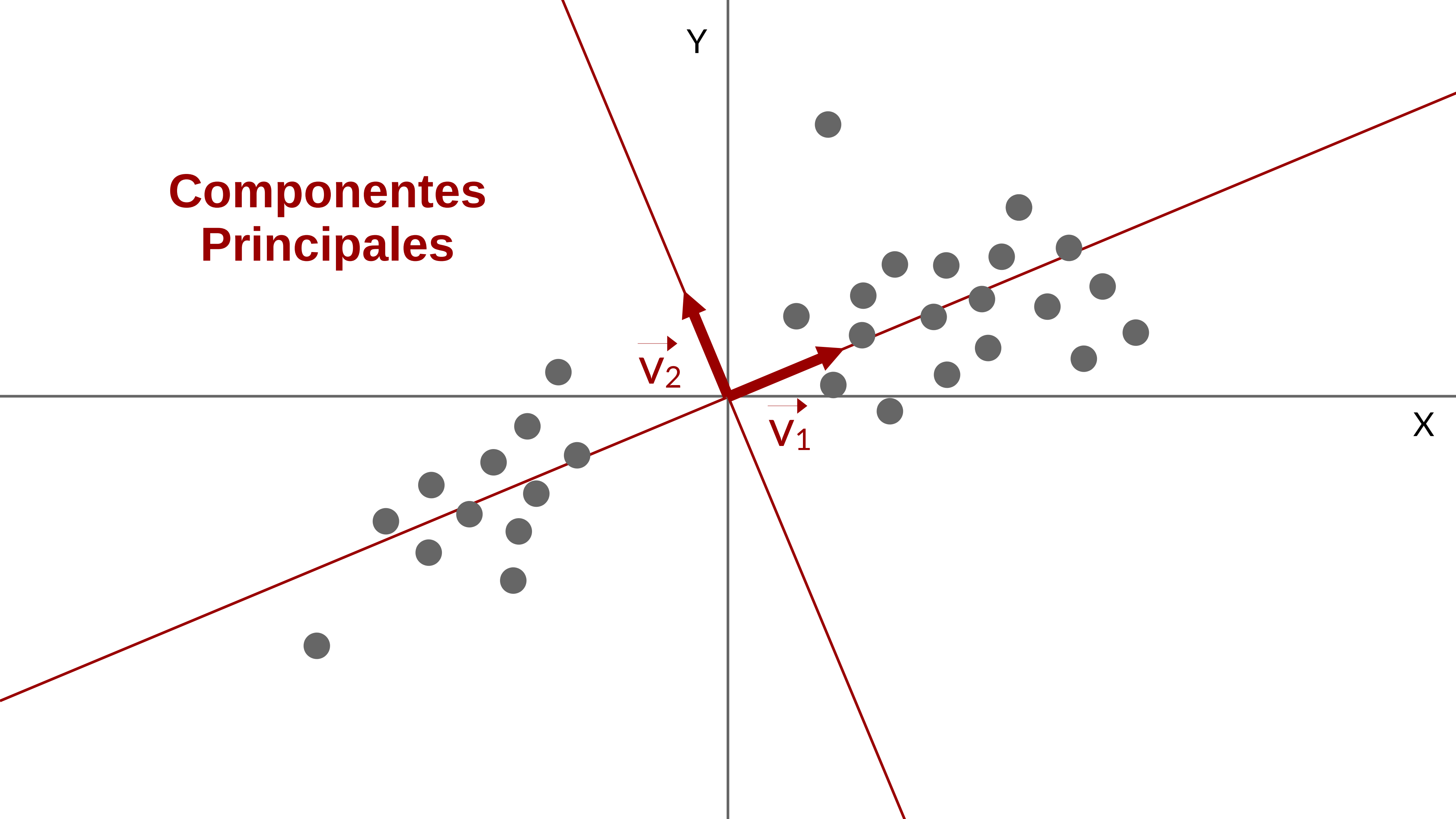
$$1 = v_2^x + v_2^y$$

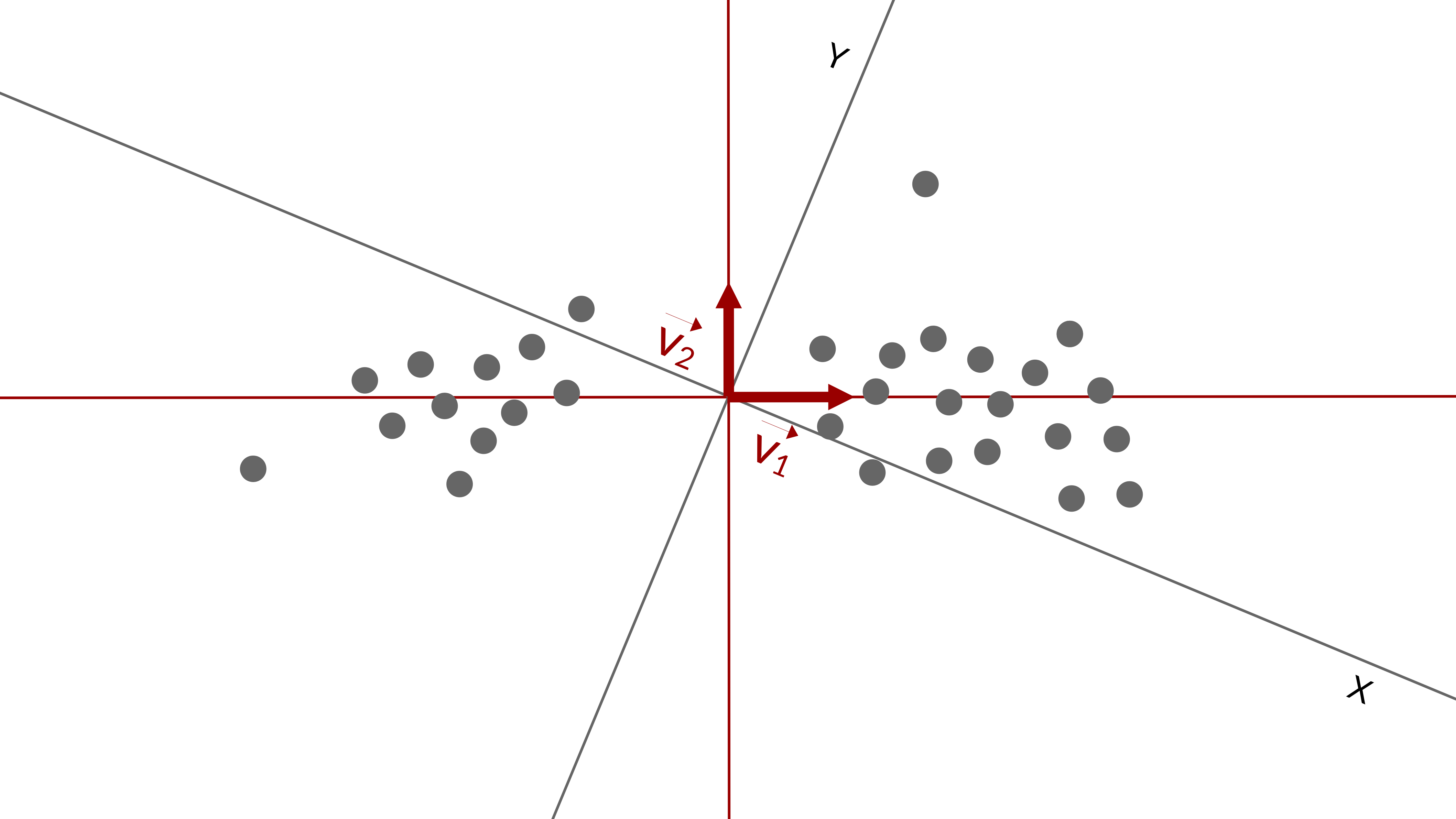
Vectores normales (unitarios)

$$0 = v_1^x \cdot v_2^x + v_1^y \cdot v_2^y$$

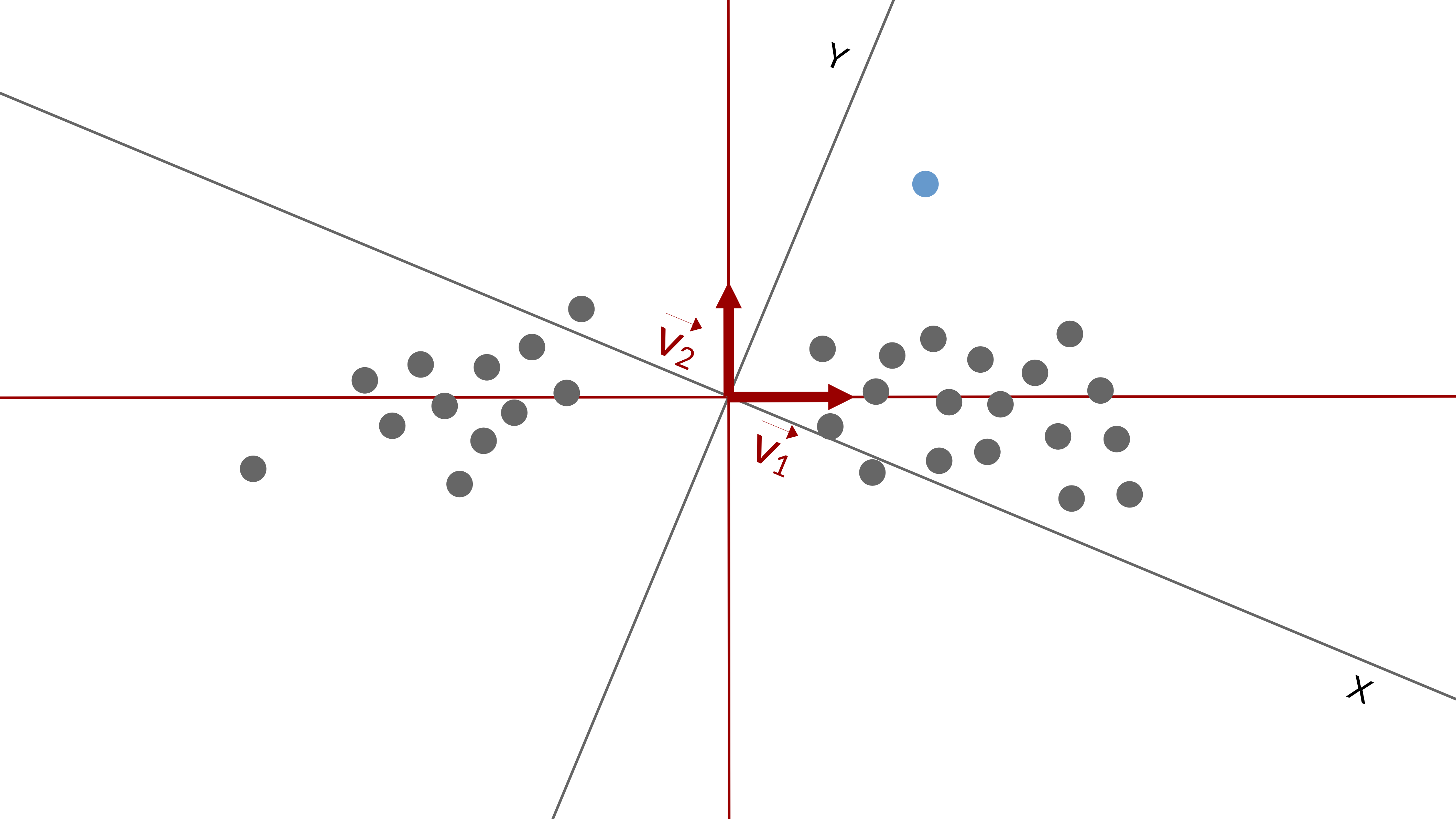
Perpendiculares


Y
X
$\vec{v}_2$
$\vec{v}_1$

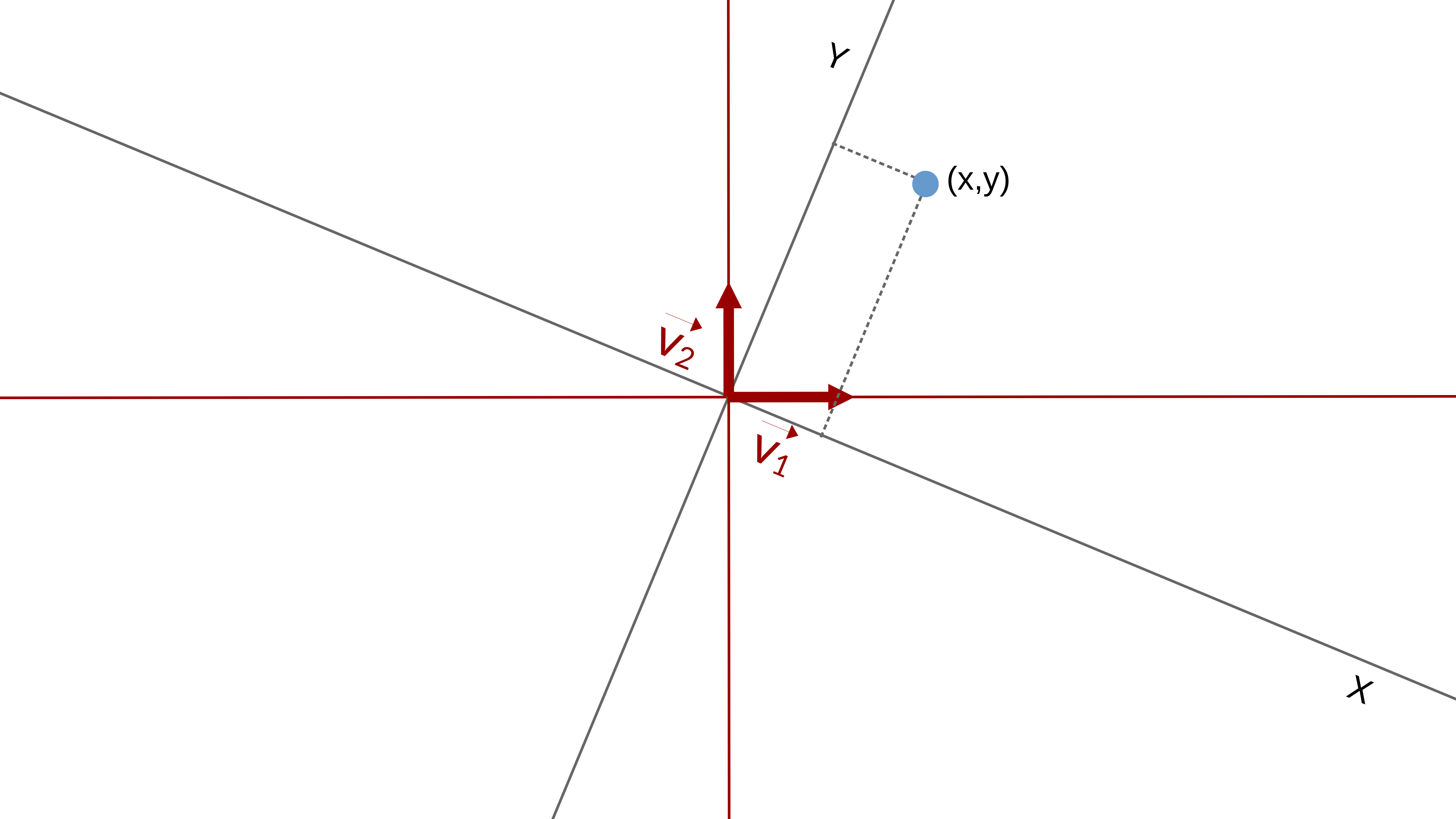

Componentes Principales
Y
X
$\vec{v}_2$
$\vec{v}_1$



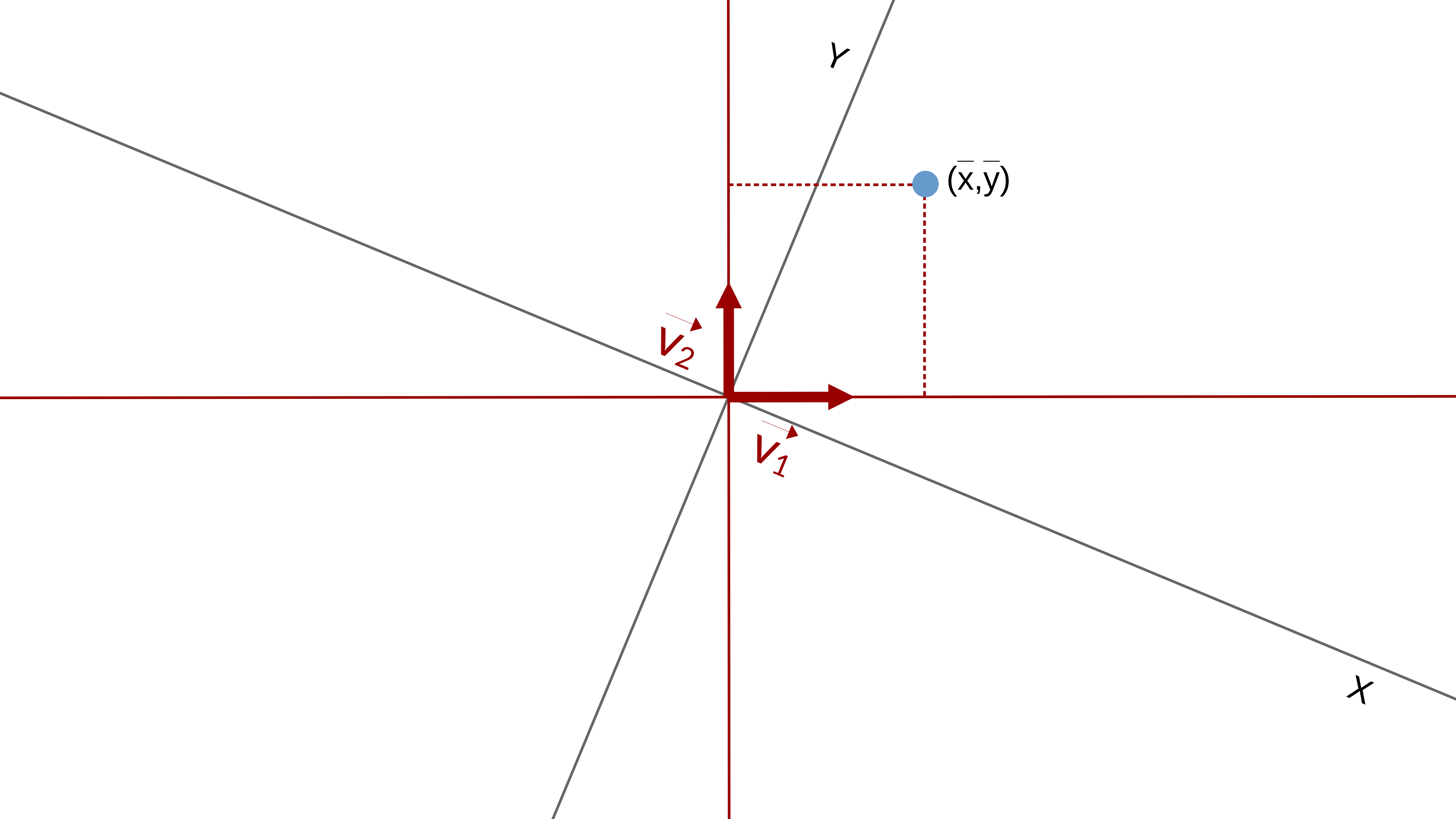
Y
$\vec{V_2}$
$\vec{V_1}$
X



Y
$\vec{V_2}$
$\vec{V_1}$
X



Y
(x,y)
$\vec{v}_2$
$\vec{v}_1$
X

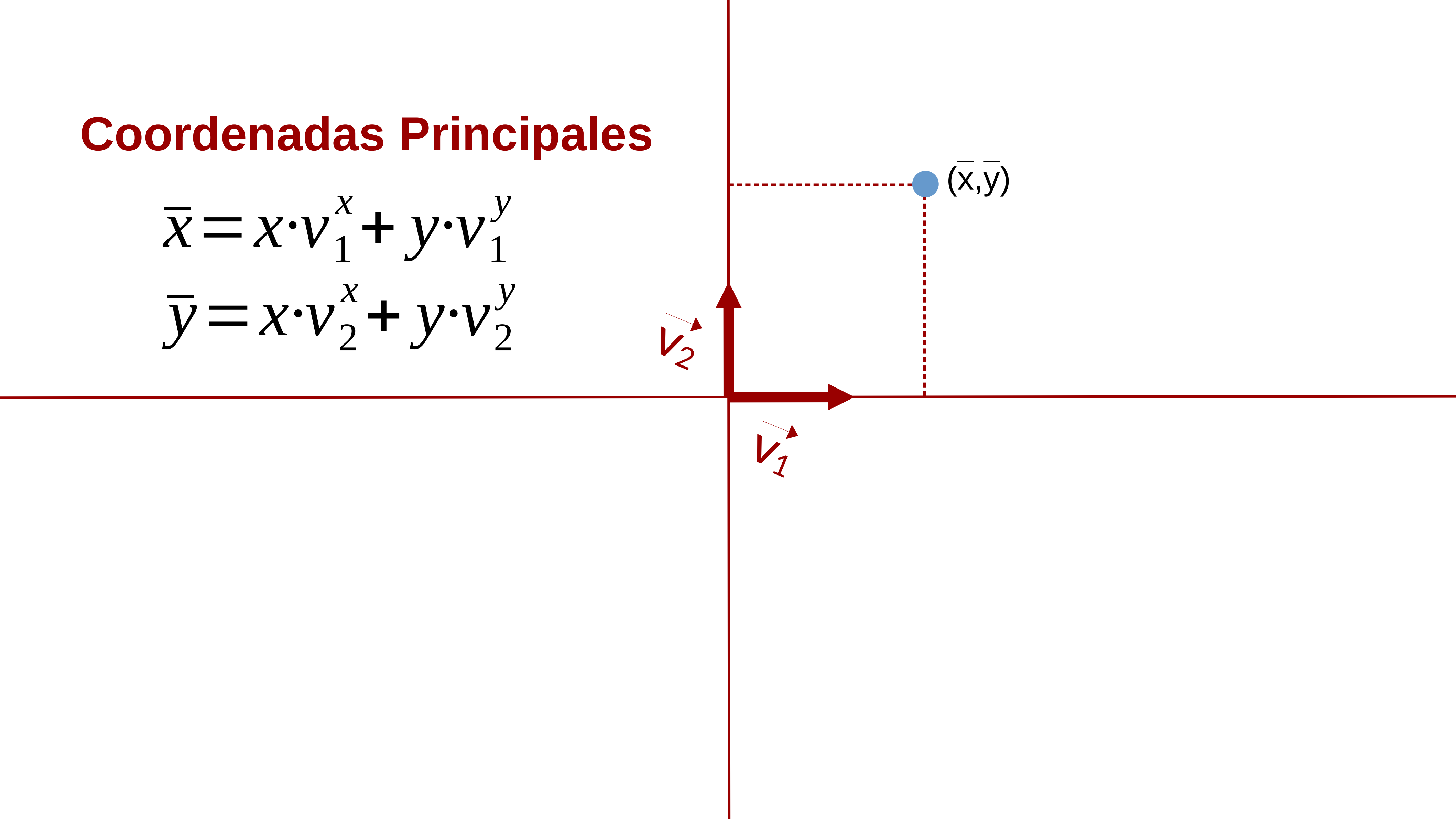


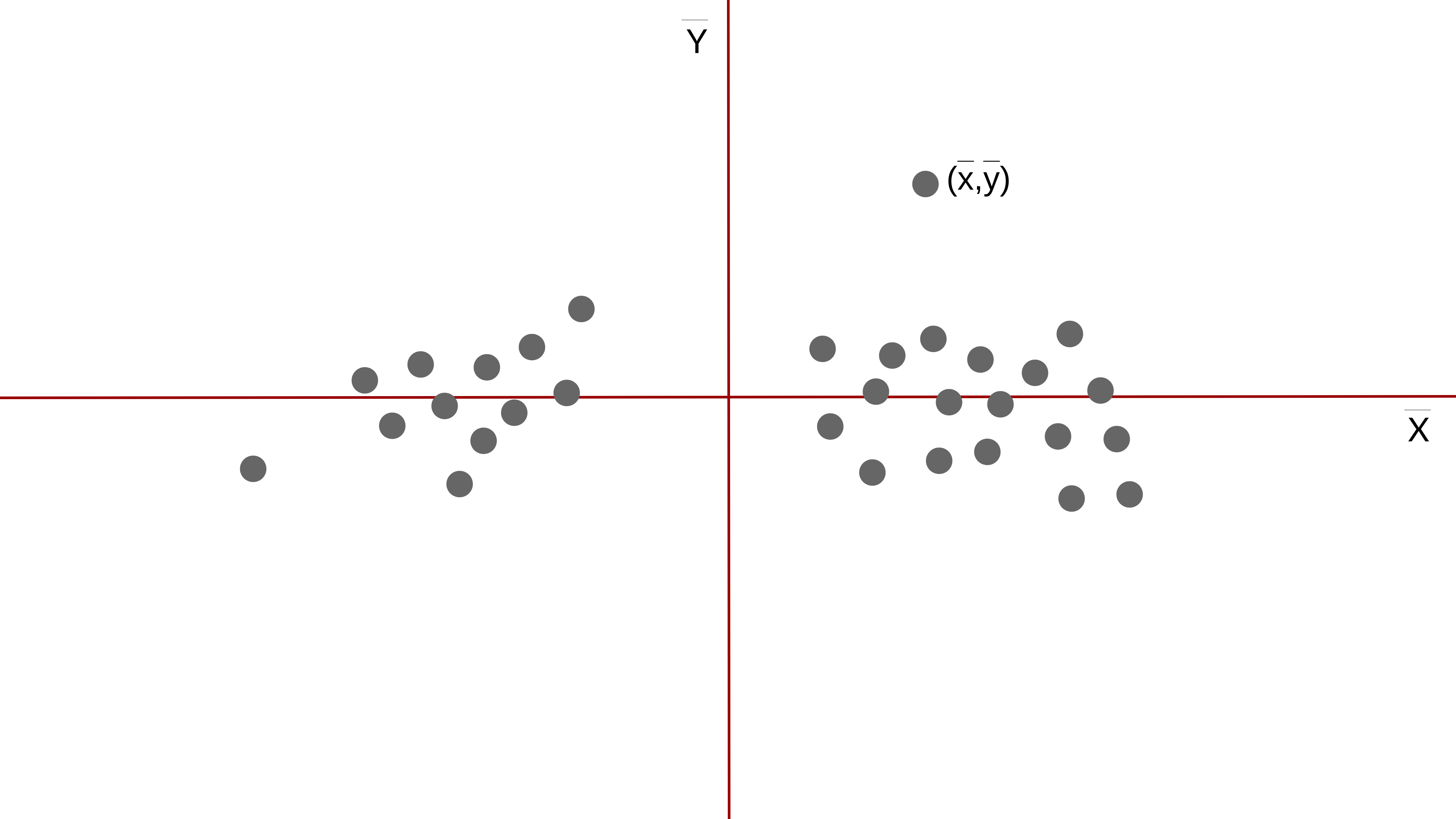
Y
$(\bar{x},\bar{y})$
$\vec{V_2}$
$\vec{V_1}$
X

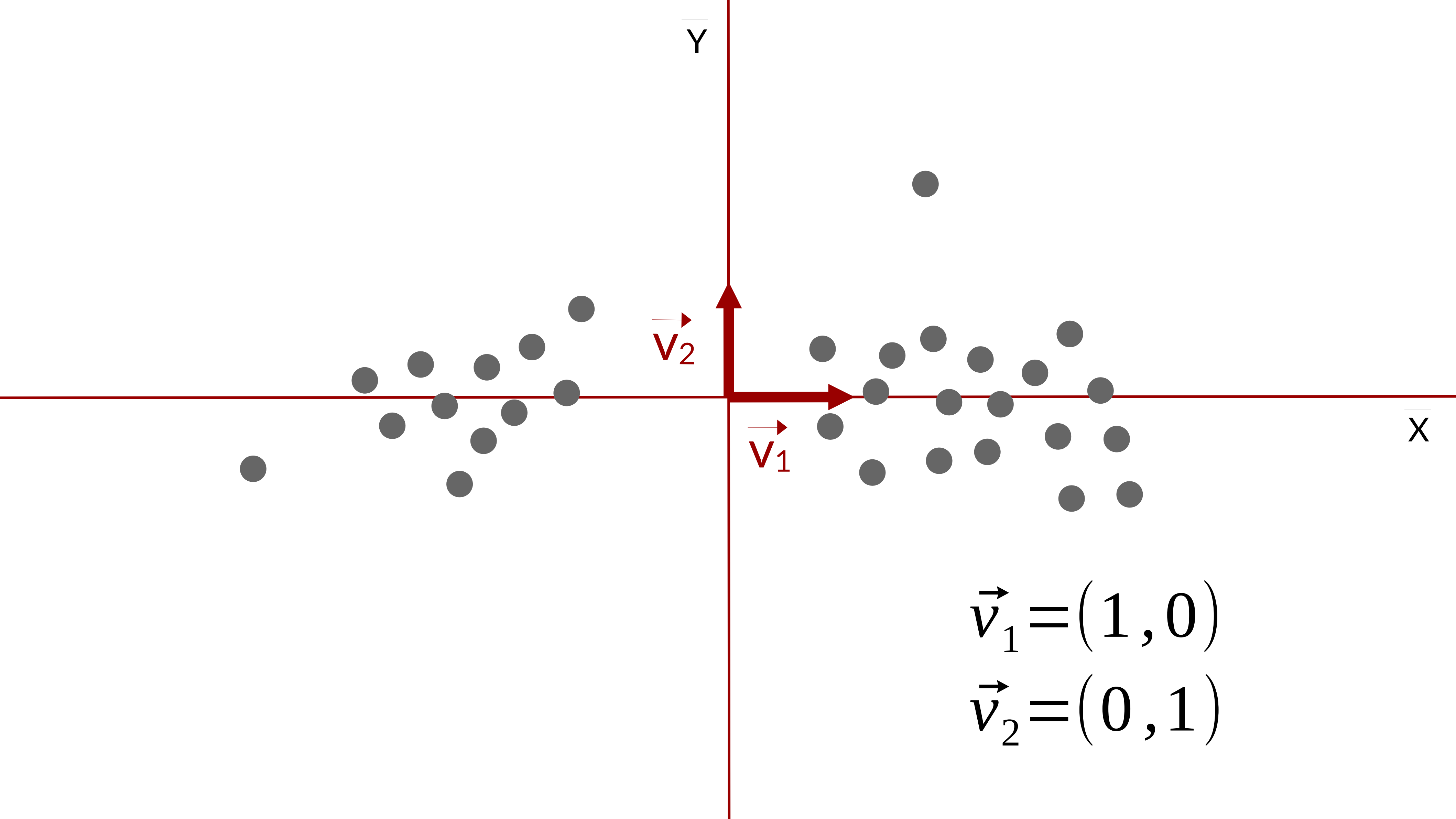
Coordenadas Principales

$$\bar{x} = x \cdot v_1^x + y \cdot v_1^y$$

$$\bar{y} = x \cdot v_2^x + y \cdot v_2^y$$




$\overline{Y}$
$(\bar{x},\bar{y})$
$\overline{X}$



$$\vec{v}_1 = (1, 0)$$
$$\vec{v}_2 = (0, 1)$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \langle \overline{x}^2 \rangle & \langle \overline{y}\, \overline{x} \rangle \\ \langle \overline{x}\, \overline{y} \rangle & \langle \overline{y}^2 \rangle \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda v_{\bar{x}} \\ \lambda v_{\bar{y}} \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} v_{\bar{x}} \\ v_{\bar{y}} \end{pmatrix}$$

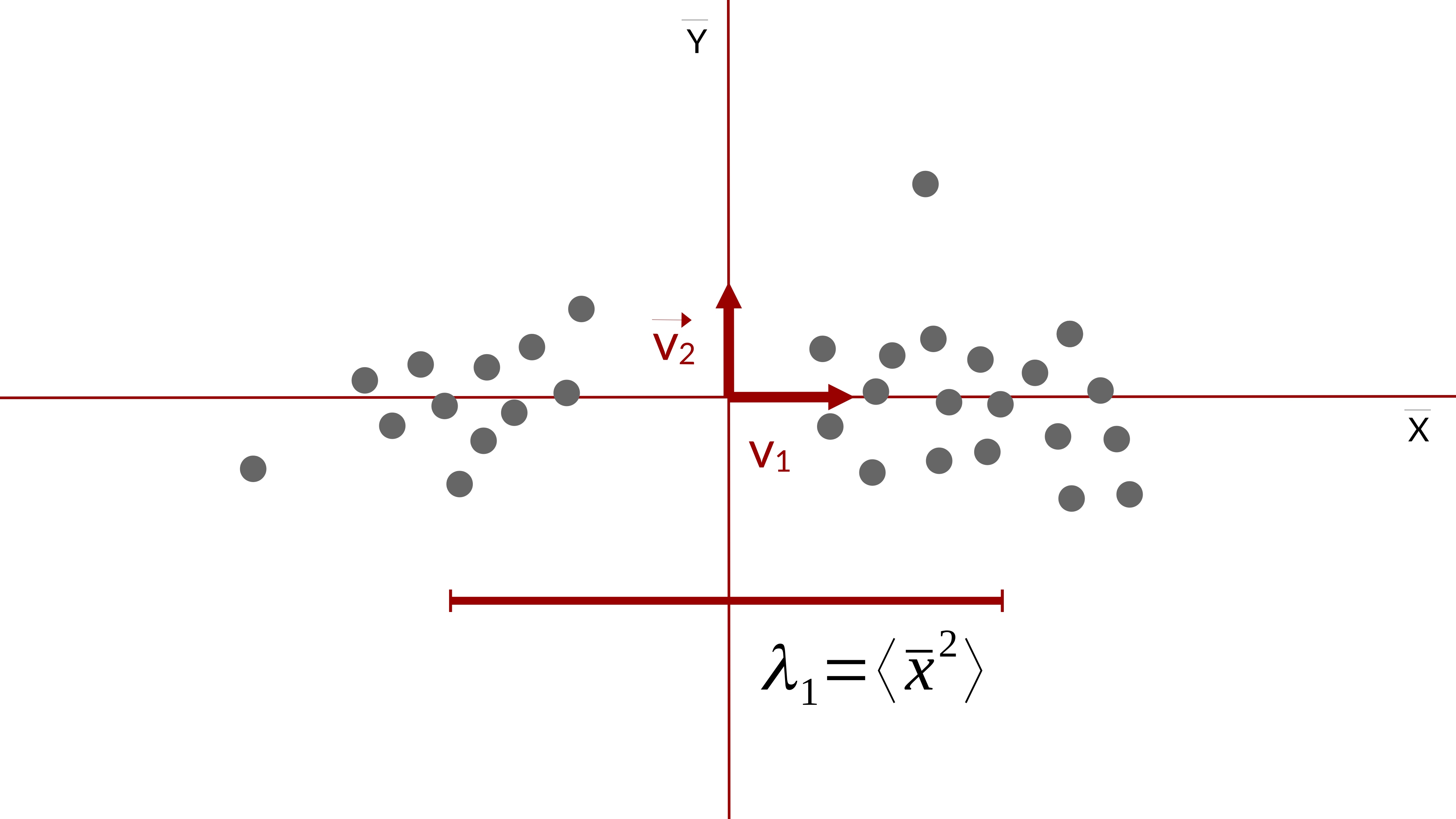
Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda_1 \cdot 1 \\ \lambda_1 \cdot 0 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{\lambda_1 = \langle \bar{x}^2 \rangle}$$

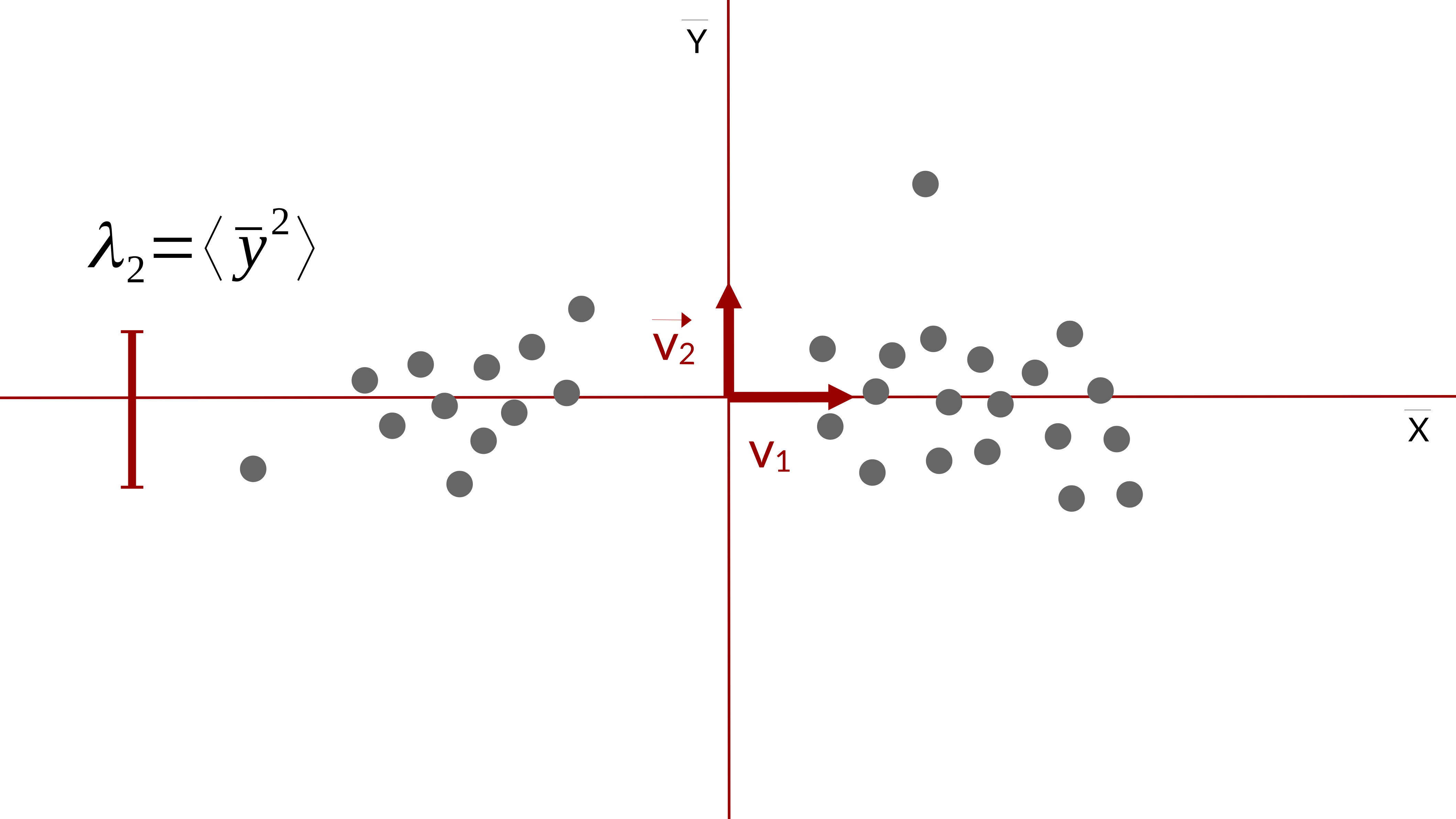


$\bar{Y}$
$\vec{V}_2$
V_1
$\bar{X}$
$\lambda_1 = \langle \bar{x}^2 \rangle$

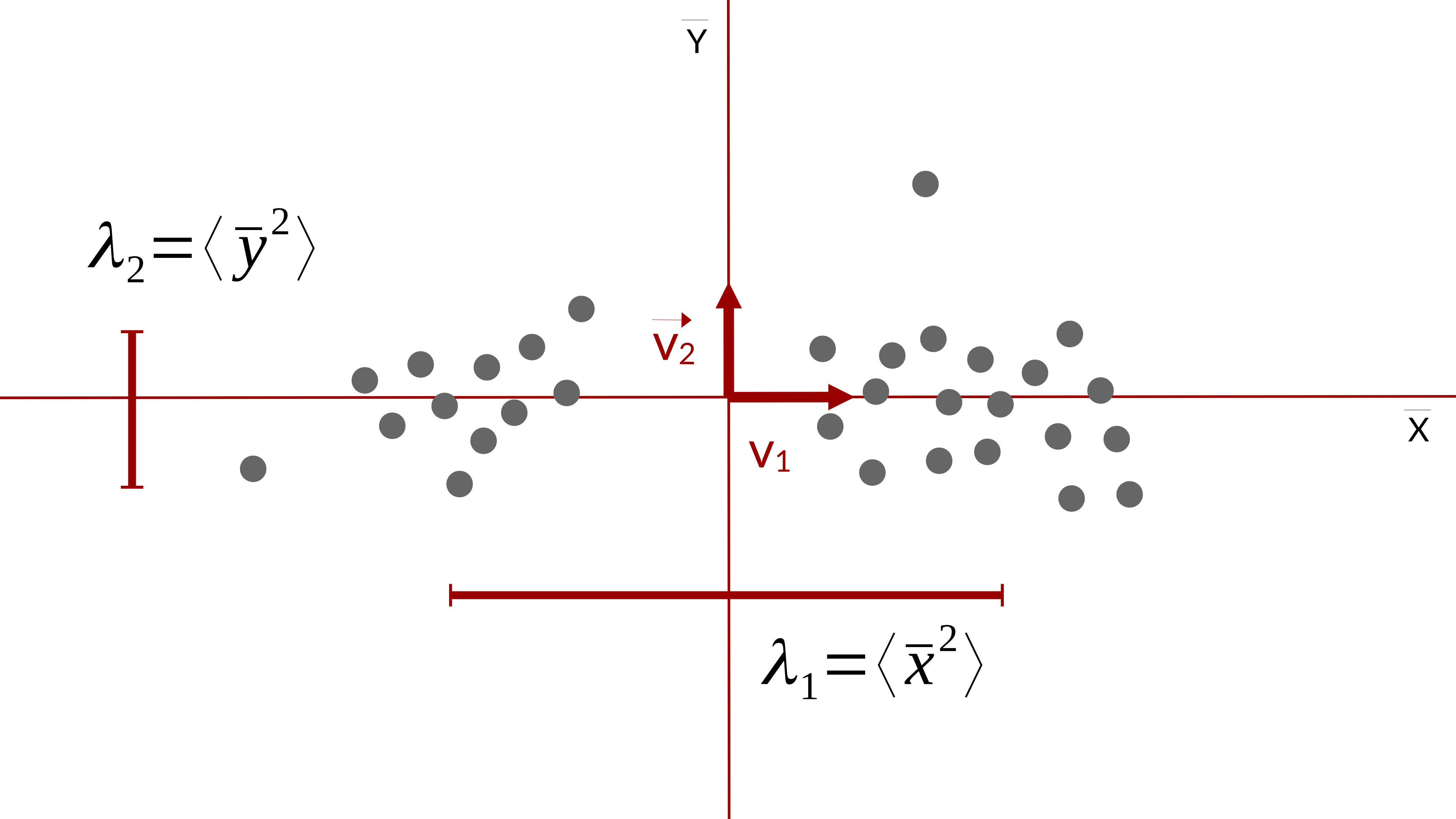
Nueva Matriz de Covarianza

$$\begin{pmatrix} 0 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

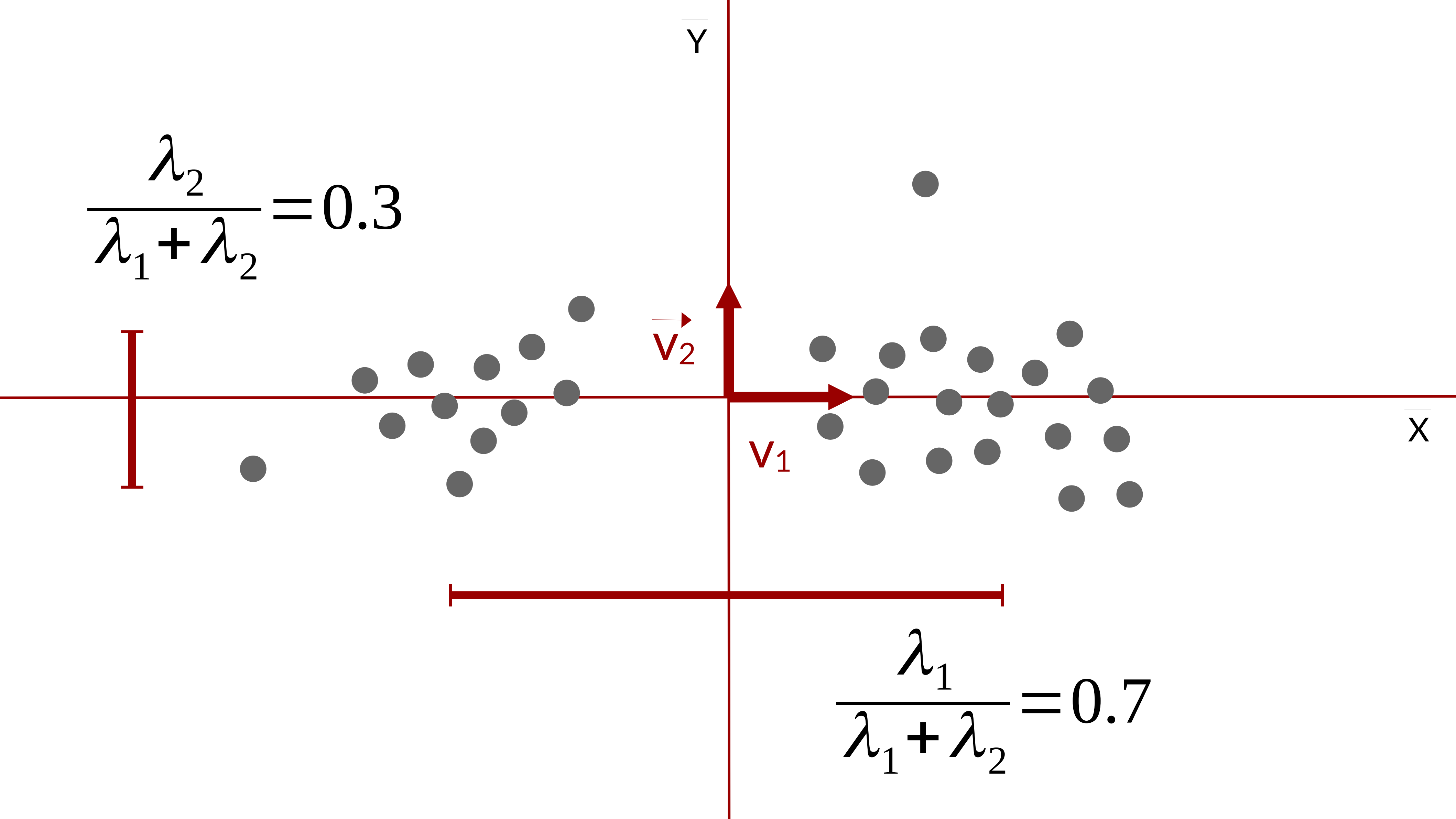
$$\boxed{\lambda_2 = \langle \bar{y}^2 \rangle}$$



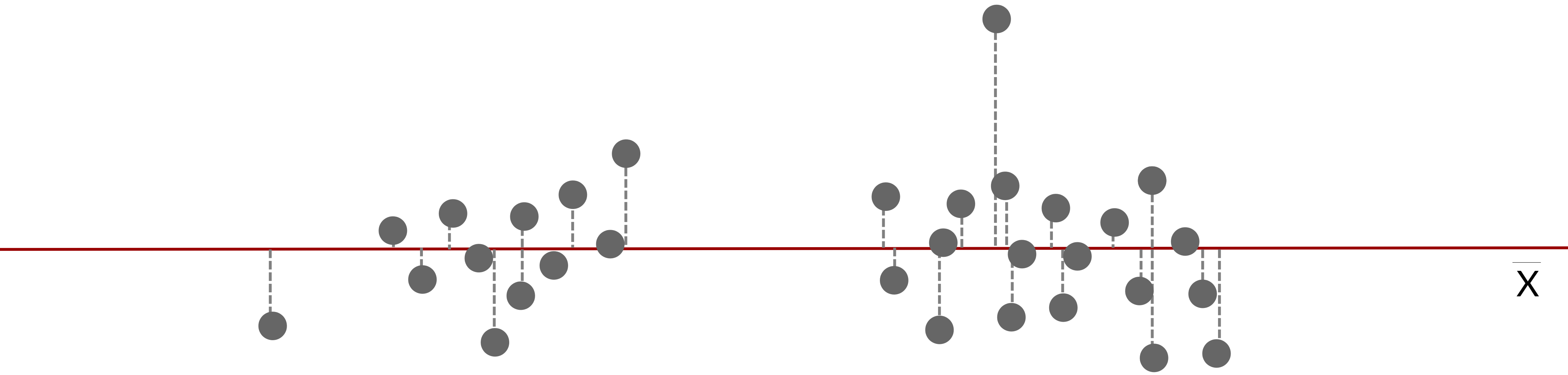
$\lambda_2 = \langle \bar{y}^2 \rangle$
$\bar{Y}$
$\bar{X}$
$\vec{V_2}$
V_1

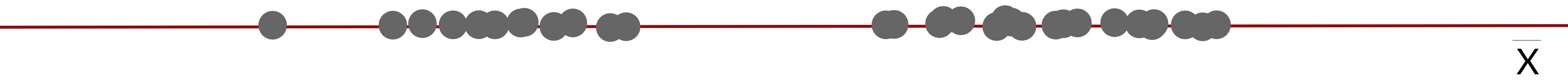


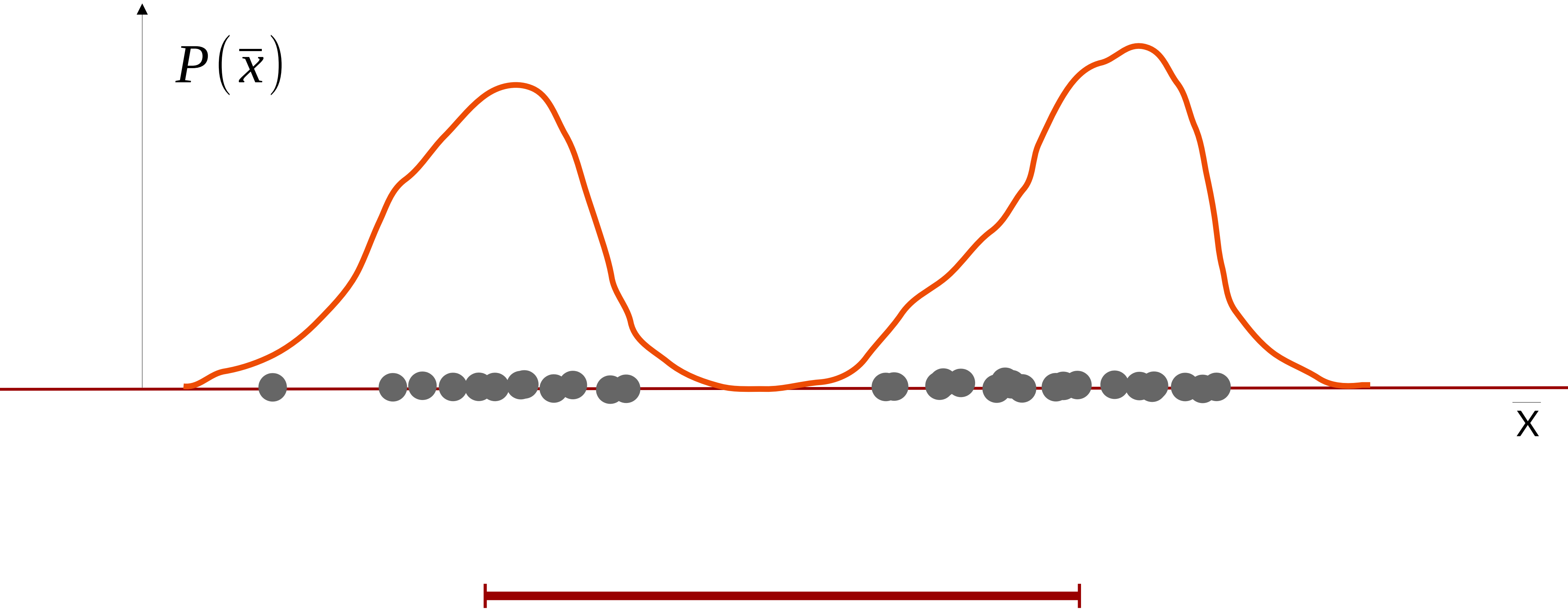
$\bar{Y}$
$\lambda_2 = \langle \bar{y}^2 \rangle$
$\vec{V}_2$
V_1
$\bar{X}$
$\lambda_1 = \langle \bar{x}^2 \rangle$



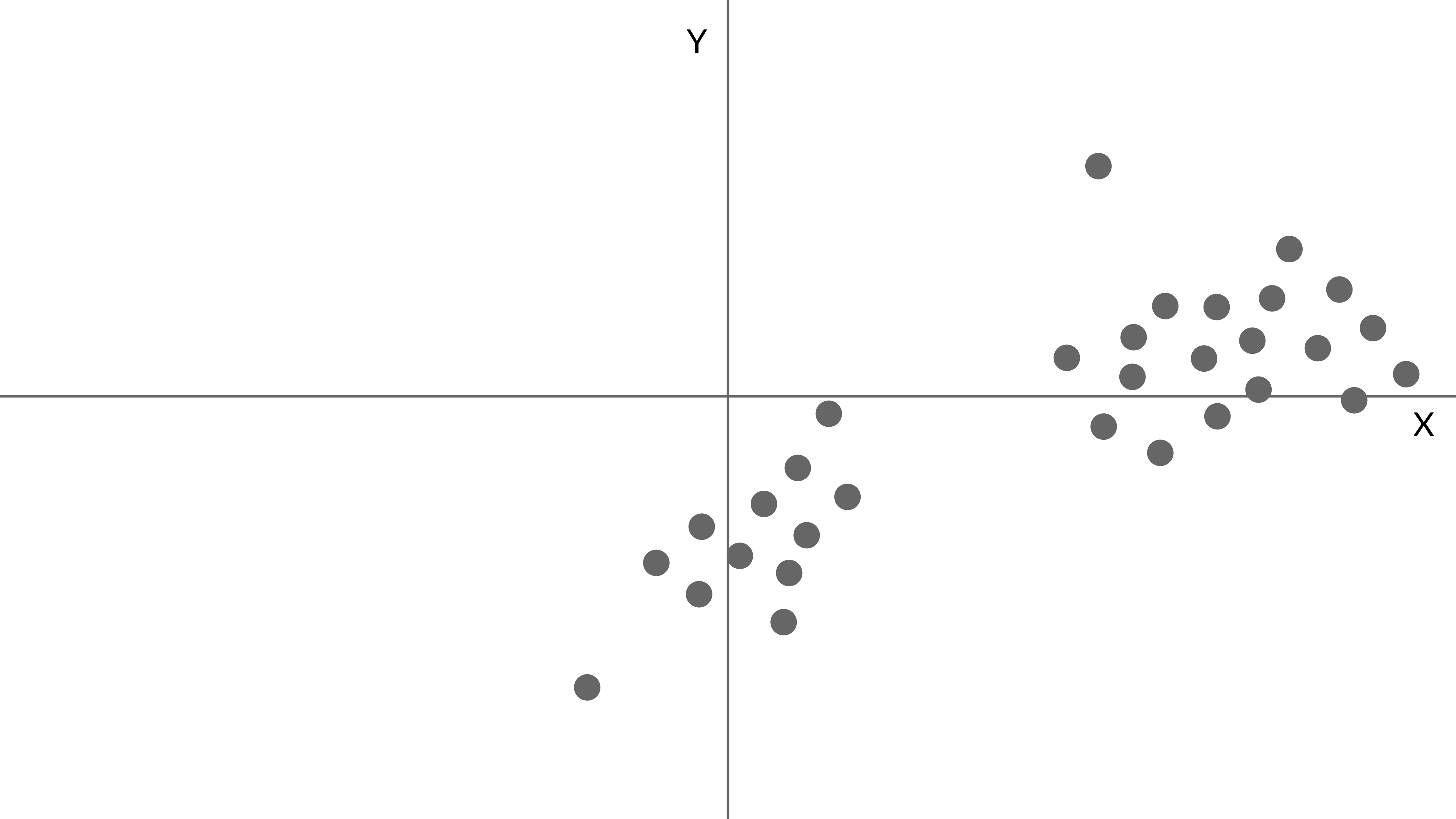
$\overline{Y}$
$\frac{\lambda_2}{\lambda_1+\lambda_2}=0.3$
$\vec{v}_2$
v_1
$\overline{X}$
$\frac{\lambda_1}{\lambda_1+\lambda_2}=0.7$

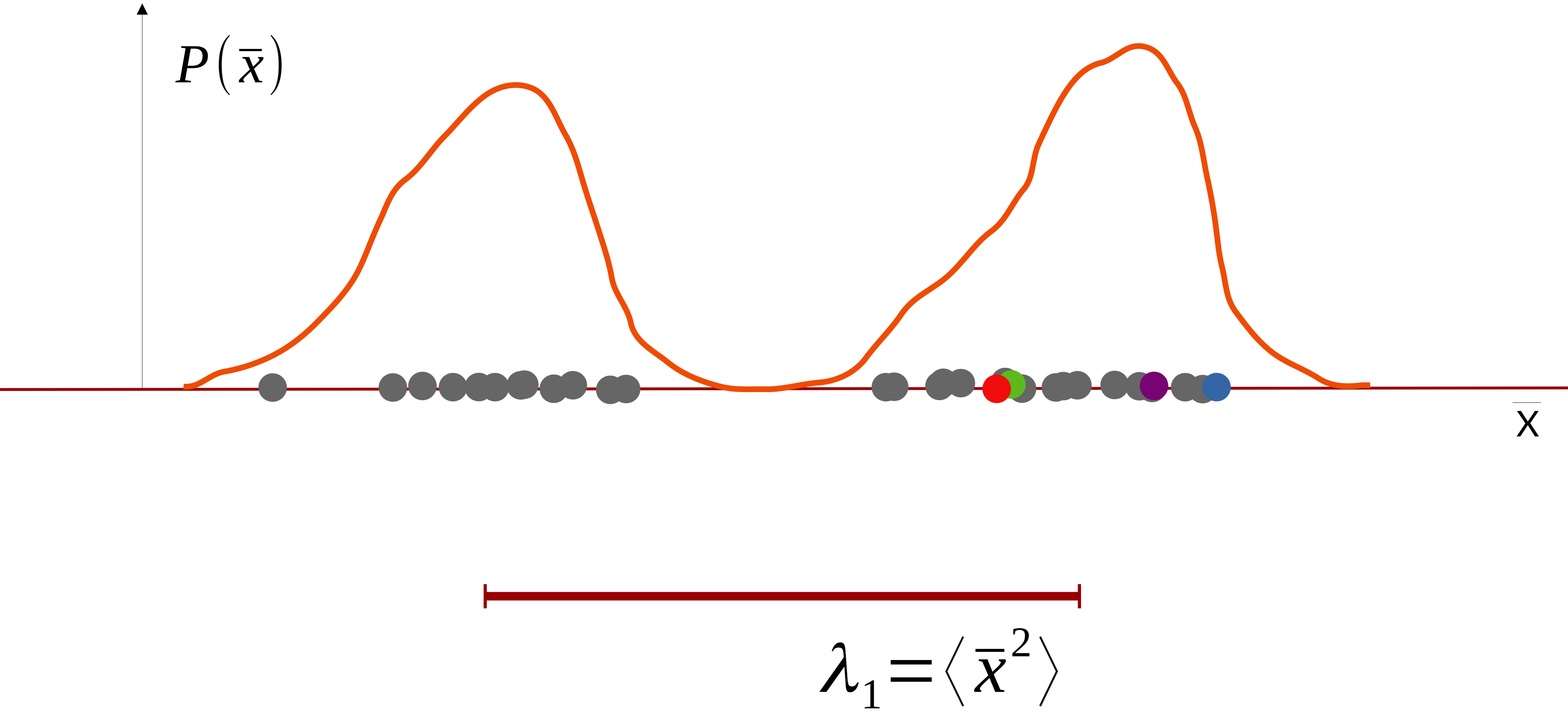

$\overline{X}$

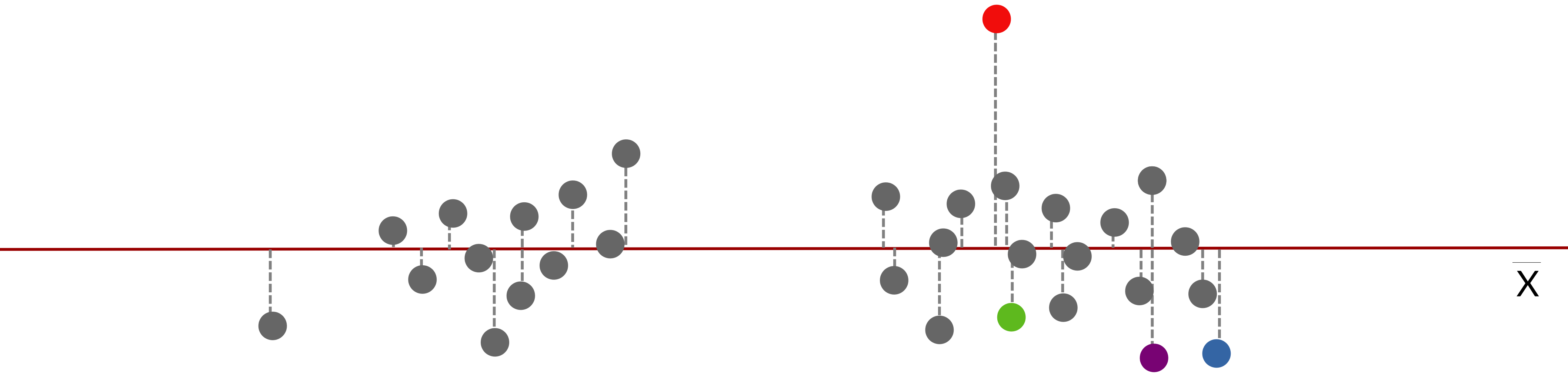

$\overline{X}$

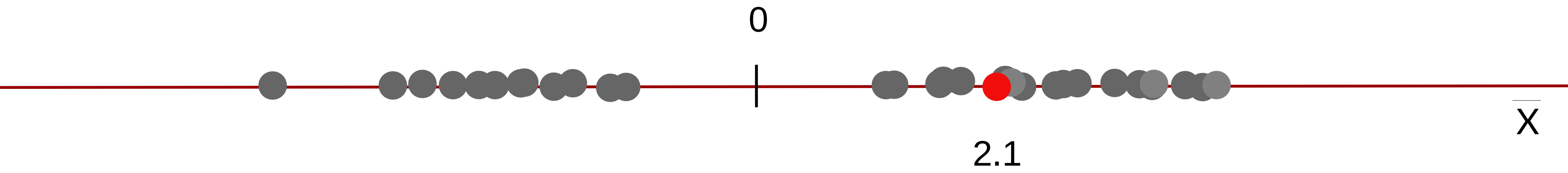


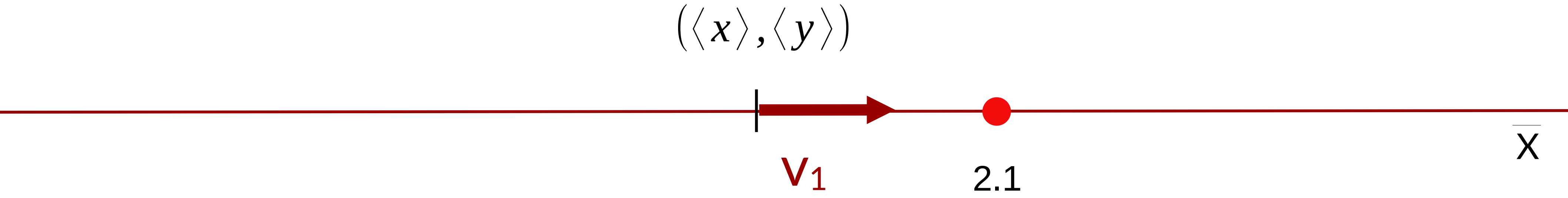
$$\lambda_1 = \langle \bar{x}^2 \rangle$$

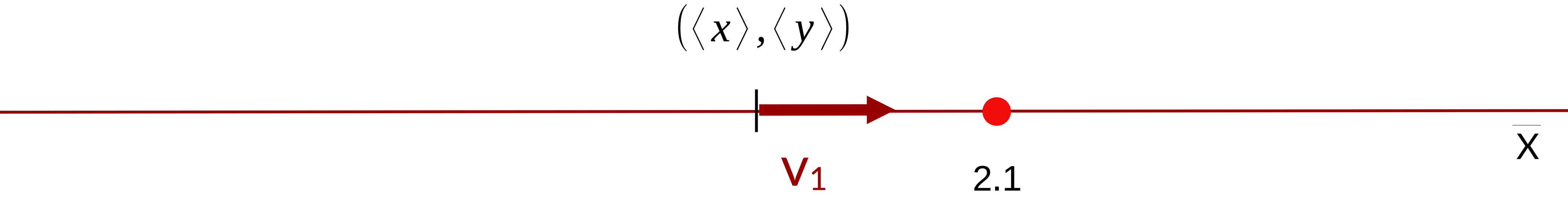

Y
X


$P(\bar{x})$
$\bar{x}$
$\lambda_1 = \langle \bar{x}^2 \rangle$

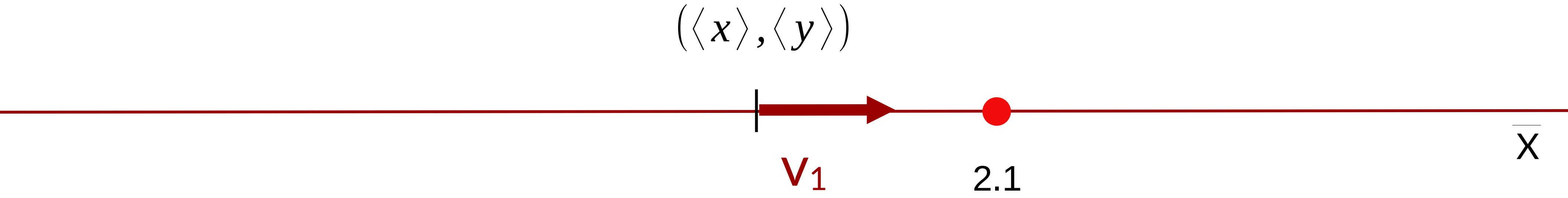

$\overline{X}$


0
2.1
$\overline{X}$

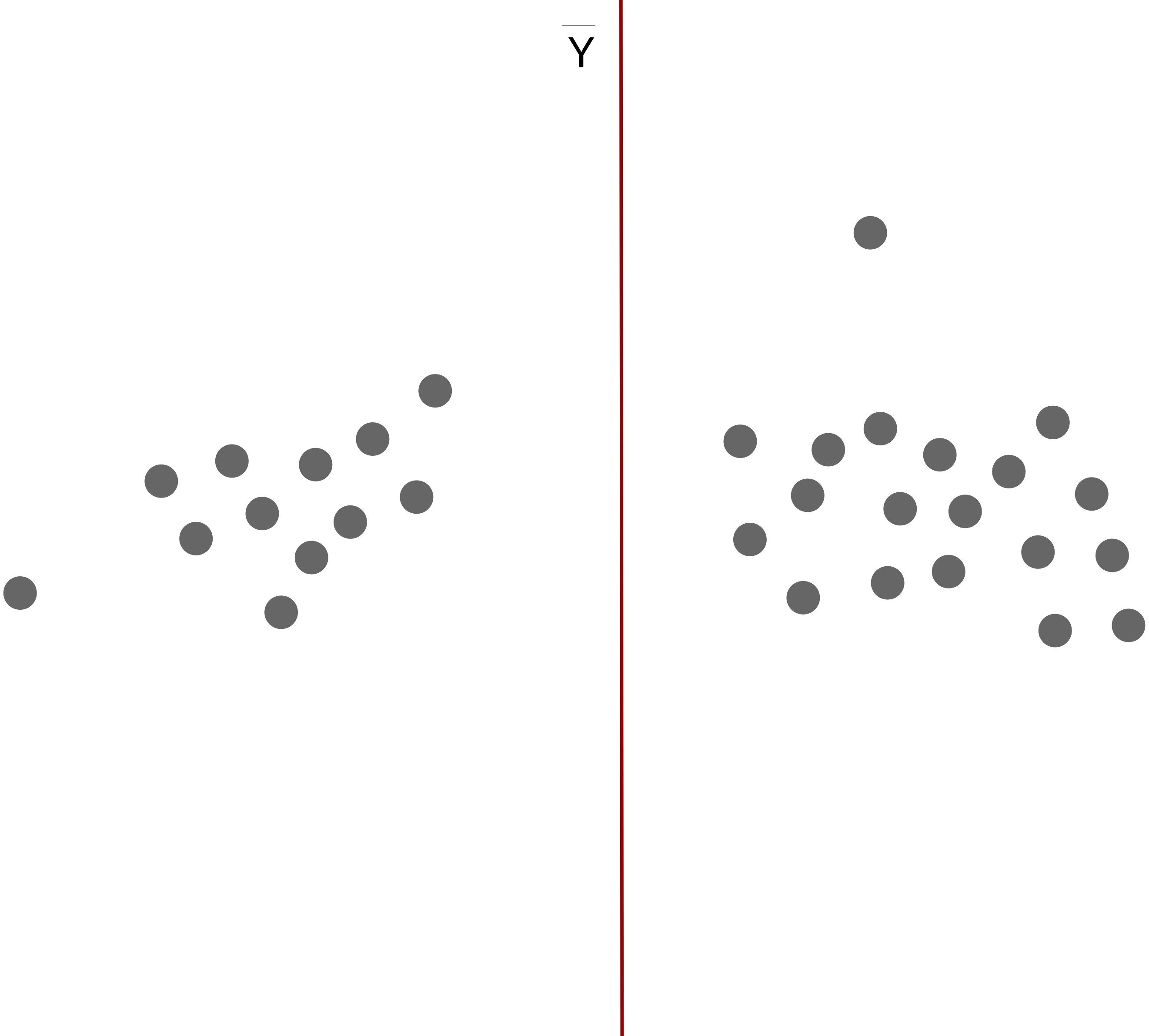

$(\langle x \rangle, \langle y \rangle)$
V_1
2.1
$\overline{X}$

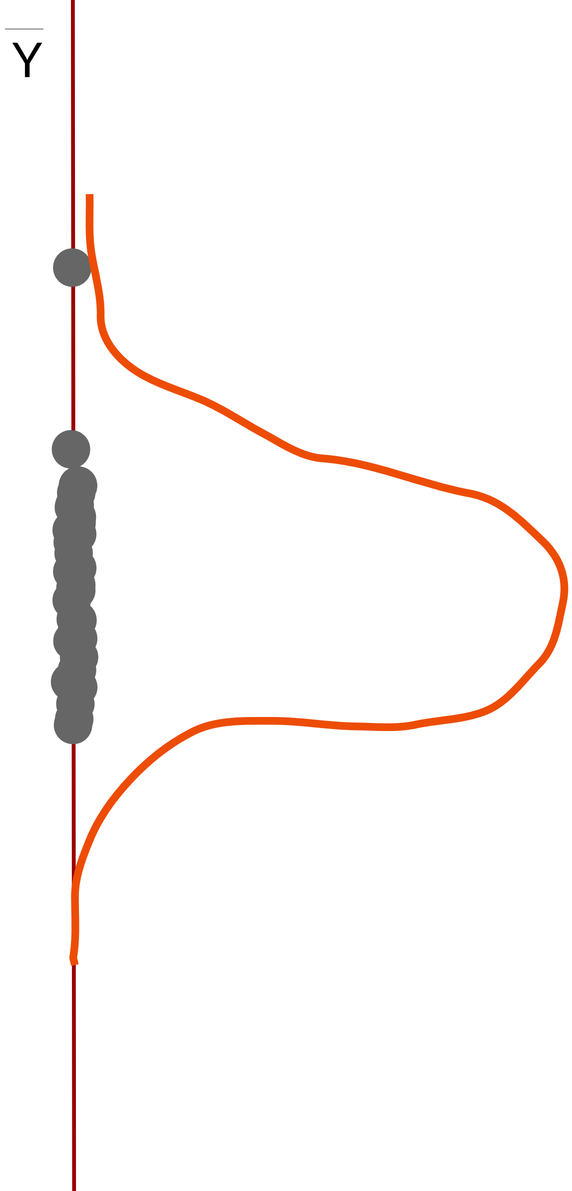


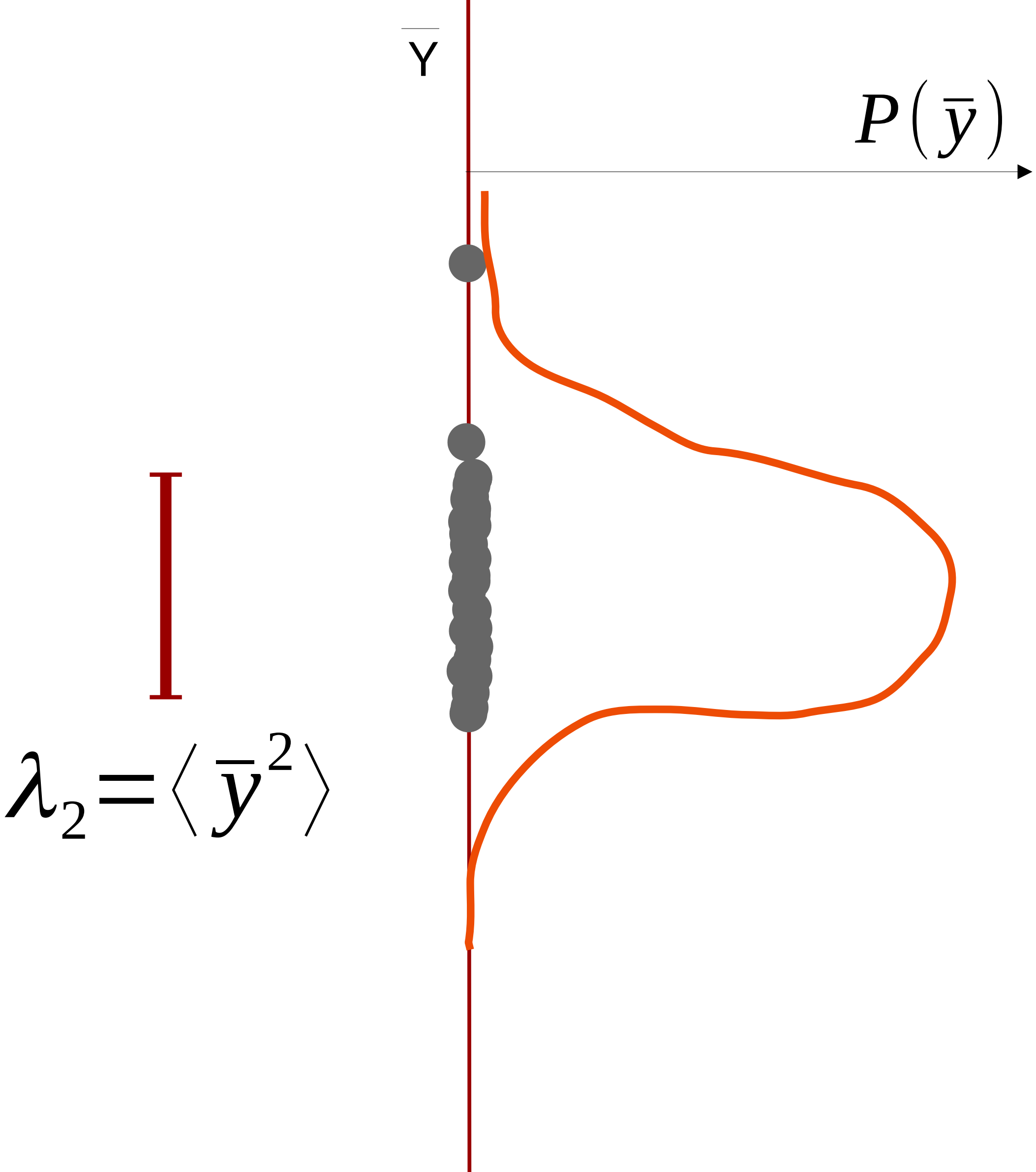
$$(x, y) = (\langle x\rangle, \langle y\rangle) + 2.1\,(v_1^x, v_1^y)$$



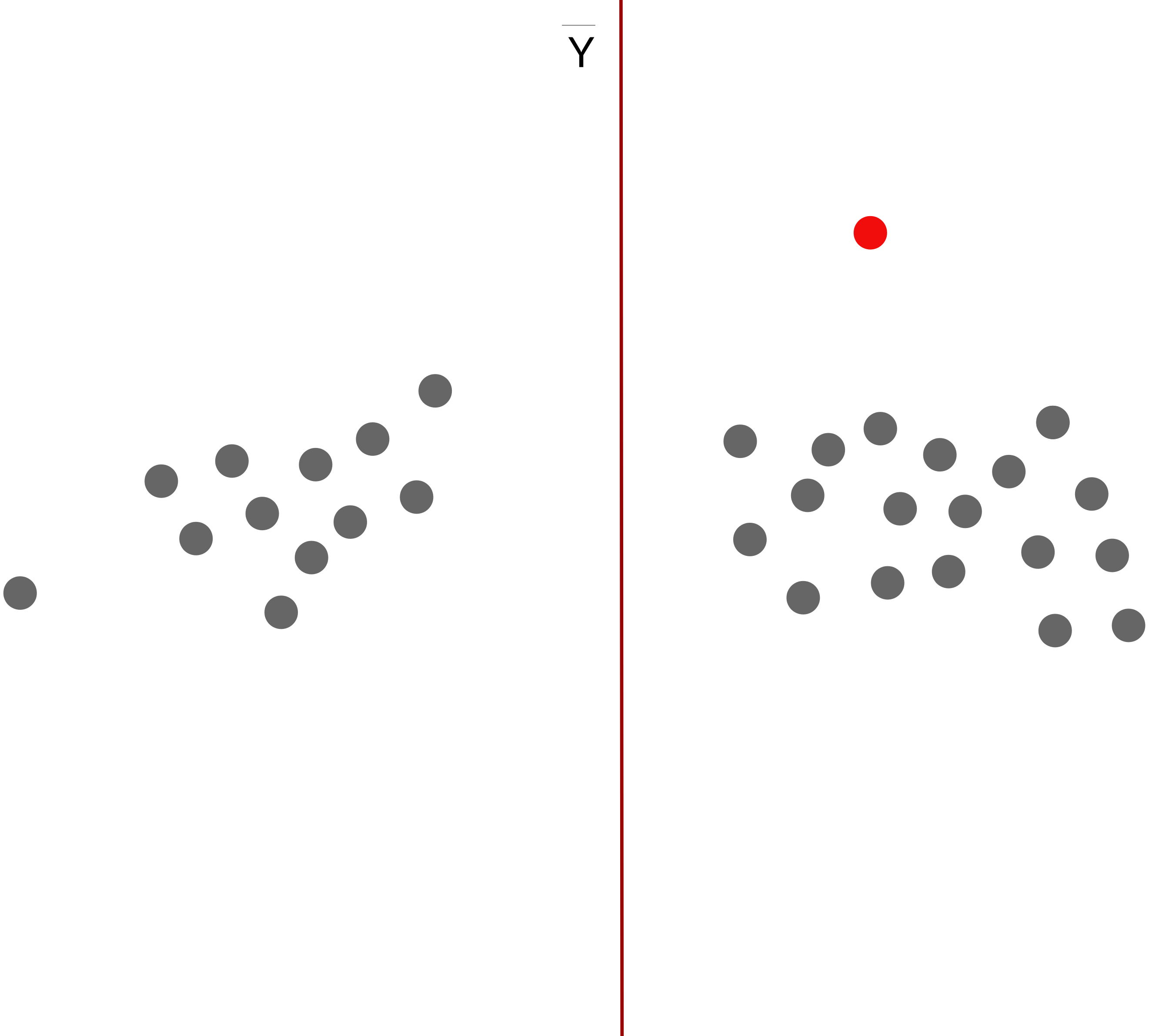
$$\cancel{(x, y) = (\langle x \rangle, \langle y \rangle) + 2.1(v_1^x, v_1^y)}$$

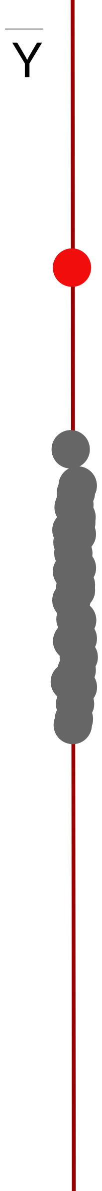

$\bar{Y}$

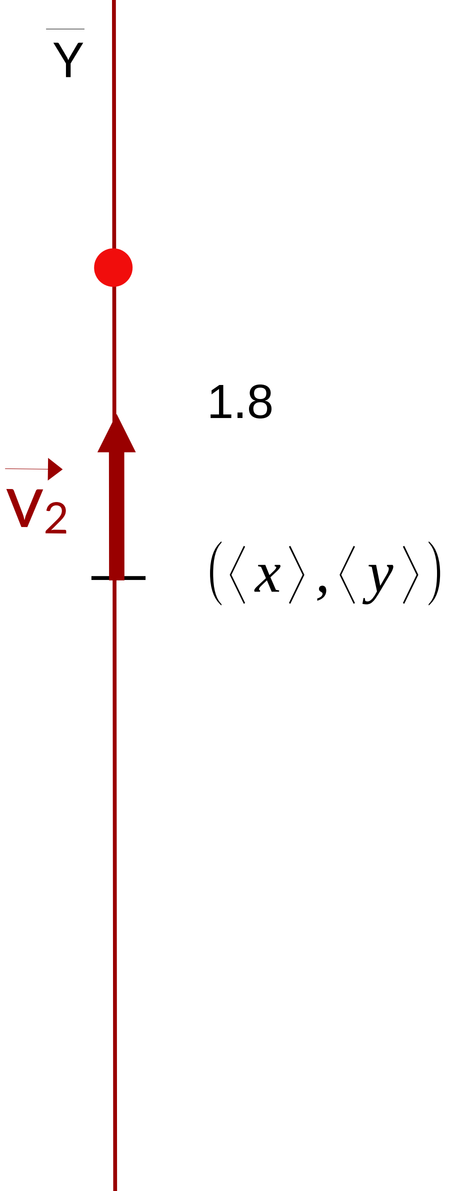

$\overline{Y}$

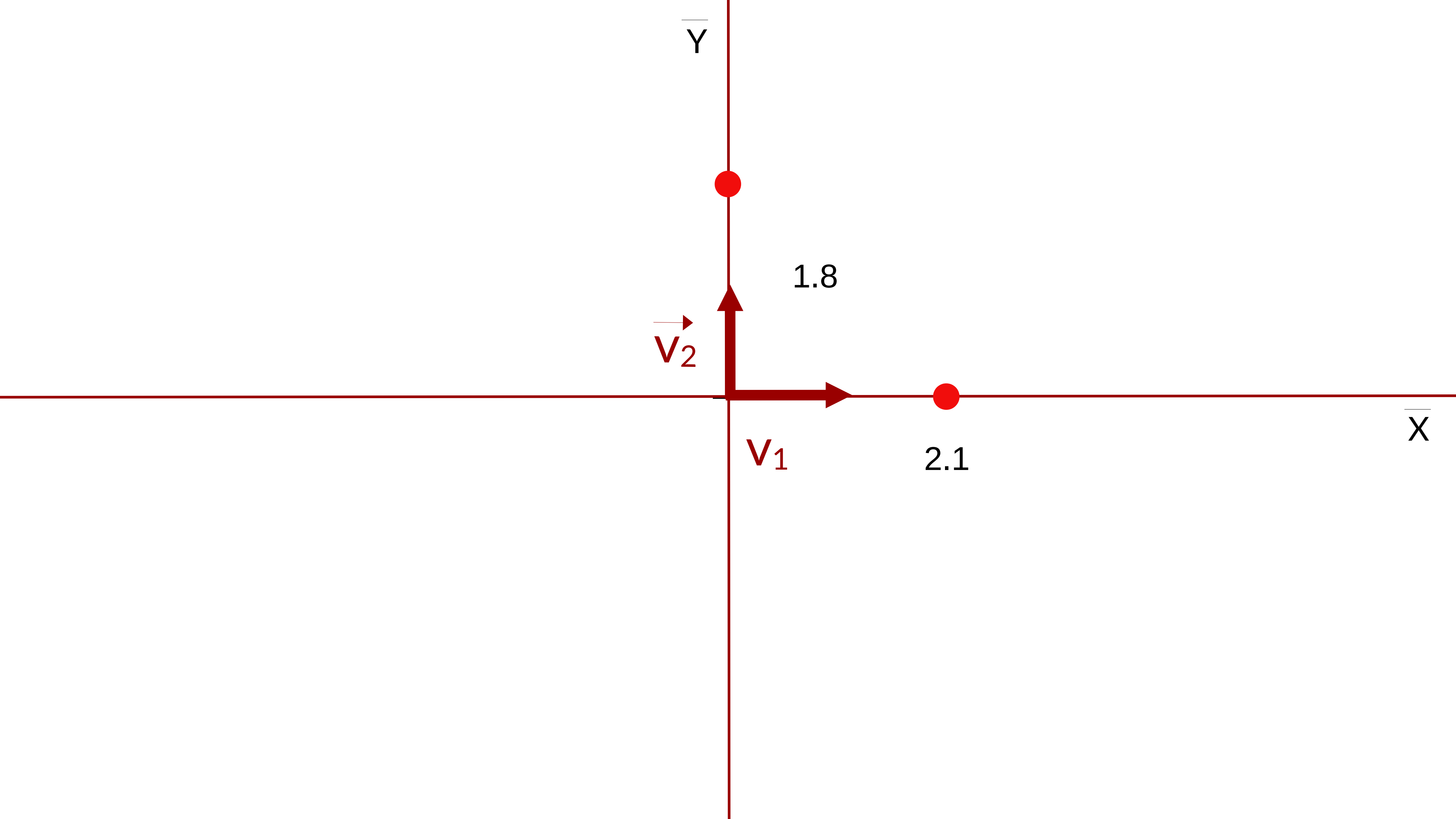


$\overline{Y}$
$P(\bar{y})$
$\lambda_2 = \langle \bar{y}^2 \rangle$

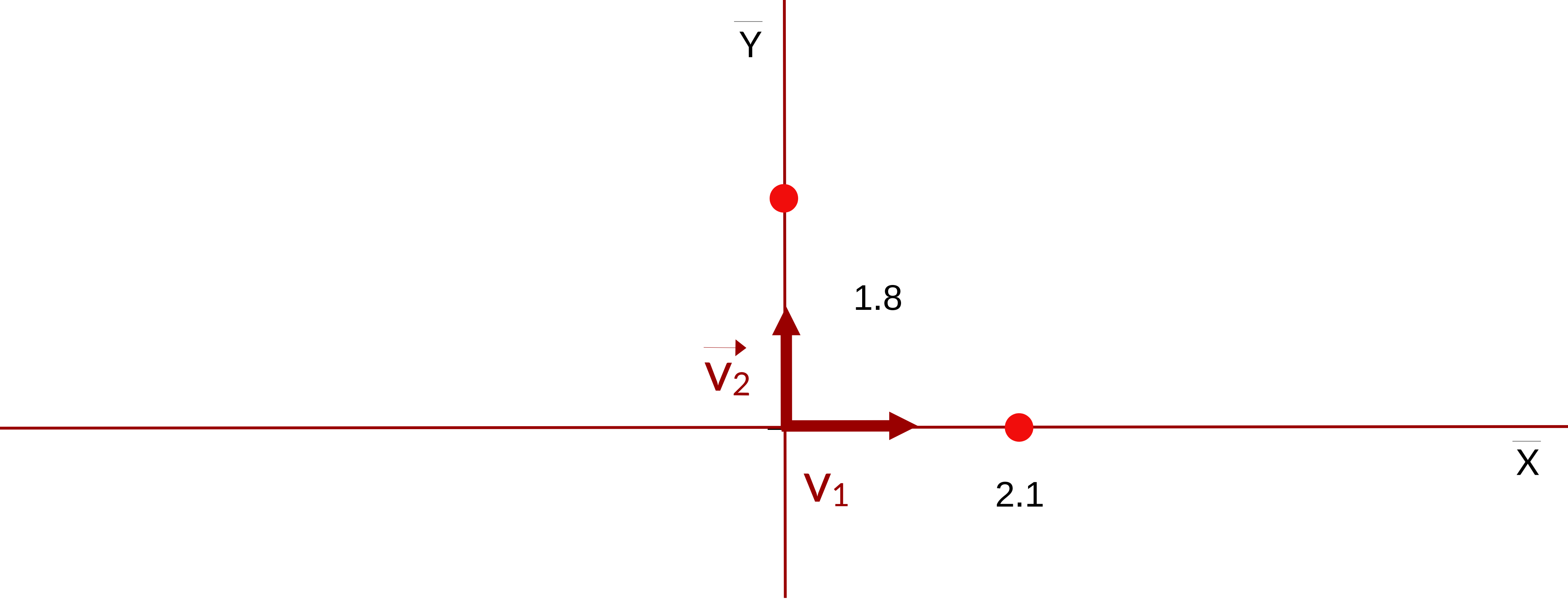

$\bar{Y}$


$\overline{Y}$

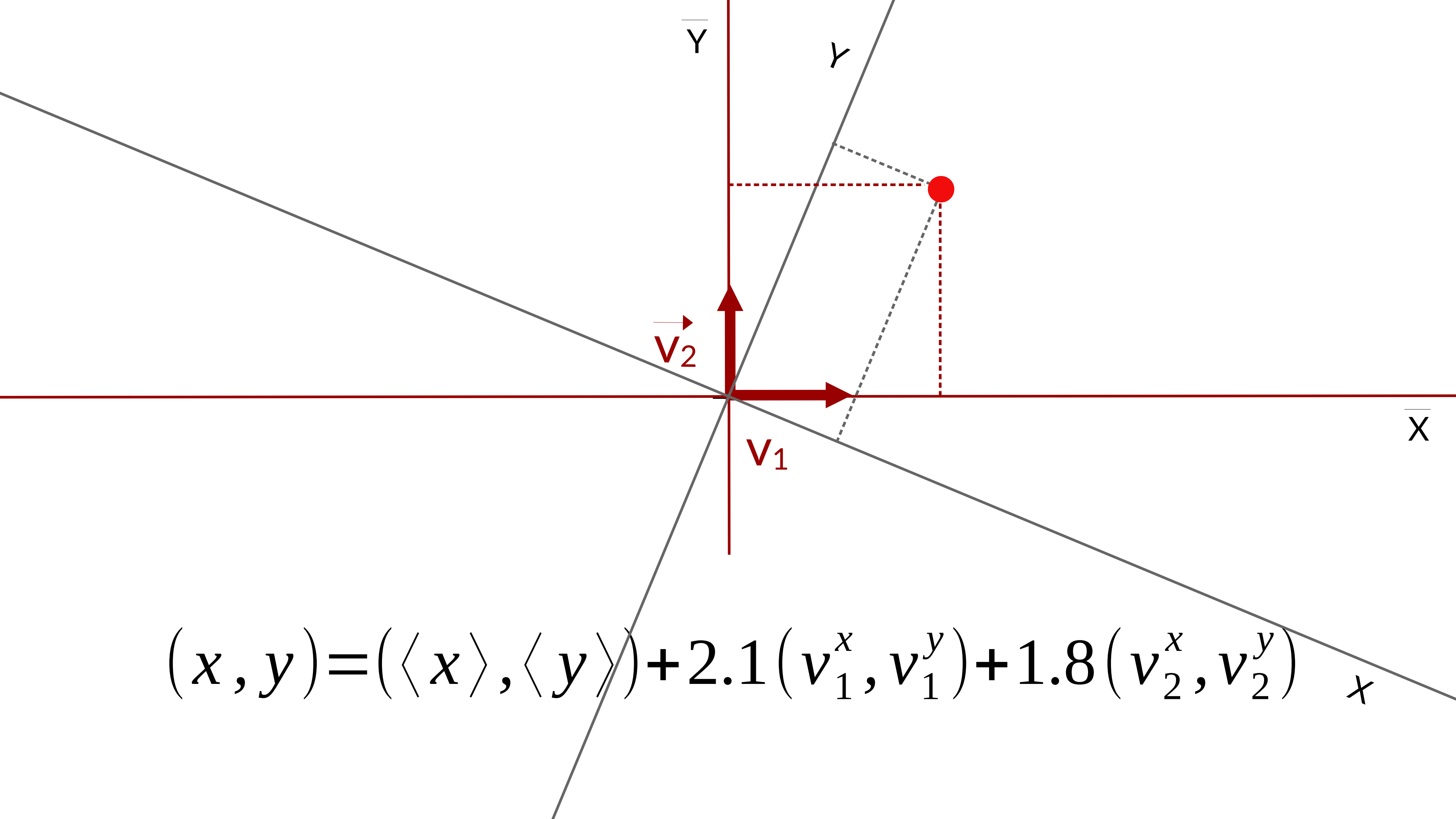

$\overline{Y}$
1.8
$\vec{V}_2$
$(\langle x \rangle, \langle y \rangle)$



$\overline{Y}$
1.8
$\vec{V_2}$
V_1
2.1
$\overline{X}$



$$\left(x,y\right)=\left(\langle x\rangle,\langle y\rangle\right)+2.1\left(v_1^x,v_1^y\right)+1.8\left(v_2^x,v_2^y\right)$$



$$(x, y) = (\langle x \rangle, \langle y \rangle) + 2.1(v_1^x, v_1^y) + 1.8(v_2^x, v_2^y)$$

PCA en nD

Matriz de Covarianza

$$
\begin{pmatrix}
\langle a^2 \rangle & \langle ba \rangle & \langle ca \rangle & \langle da \rangle & \ldots \\
\langle ab \rangle & \langle b^2 \rangle & \langle cb \rangle & \langle db \rangle & \ldots \\
\langle ac \rangle & \langle bc \rangle & \langle c^2 \rangle & \langle cb \rangle & \ldots \\
\langle ad \rangle & \langle bd \rangle & \langle cd \rangle & \langle d^2 \rangle & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots
\end{pmatrix}
$$