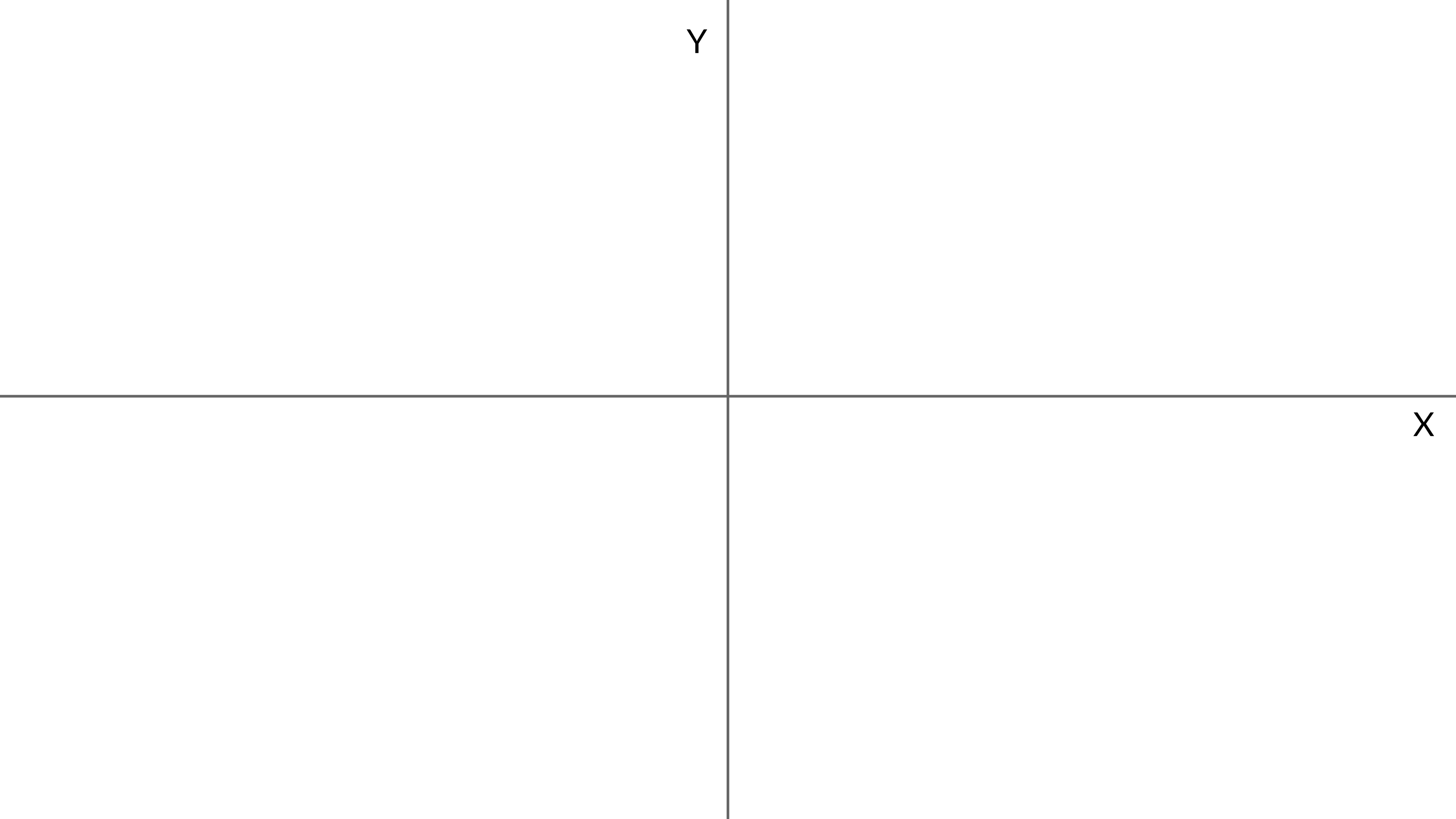
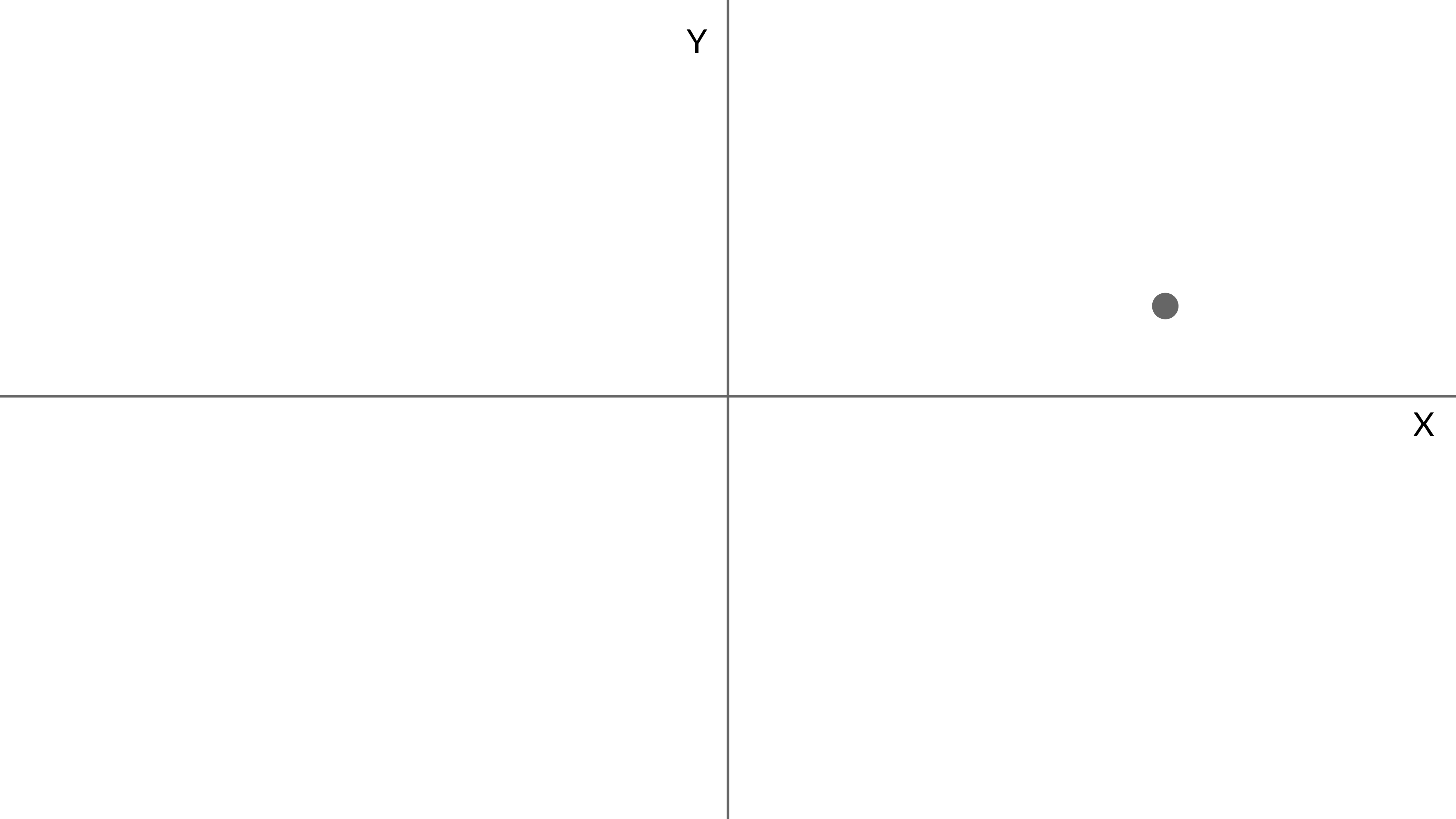
Grupo de Ciencia Computacional HIMFG

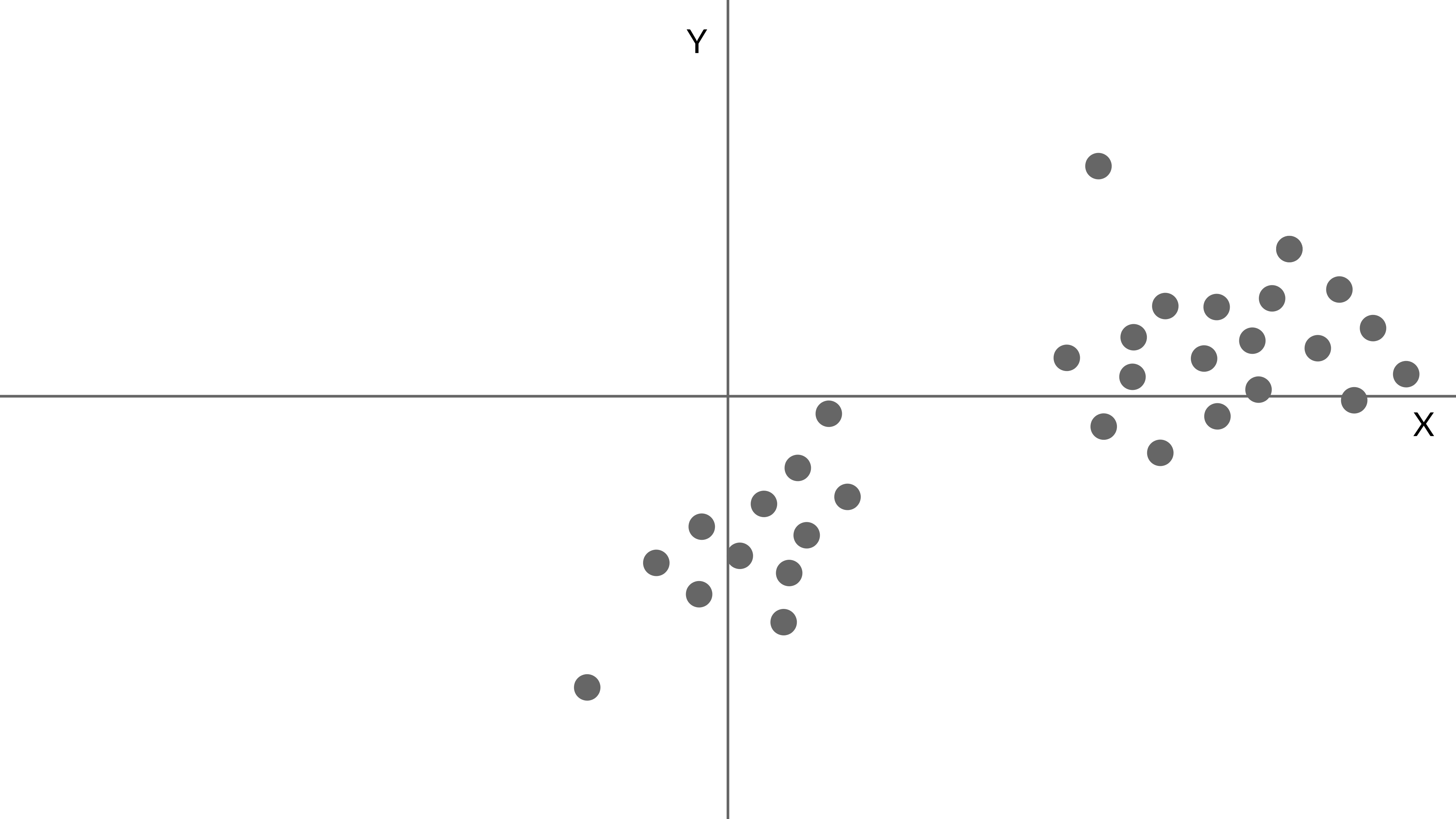
Análisis de Componentes Principales

creative commons

PCA en 2D

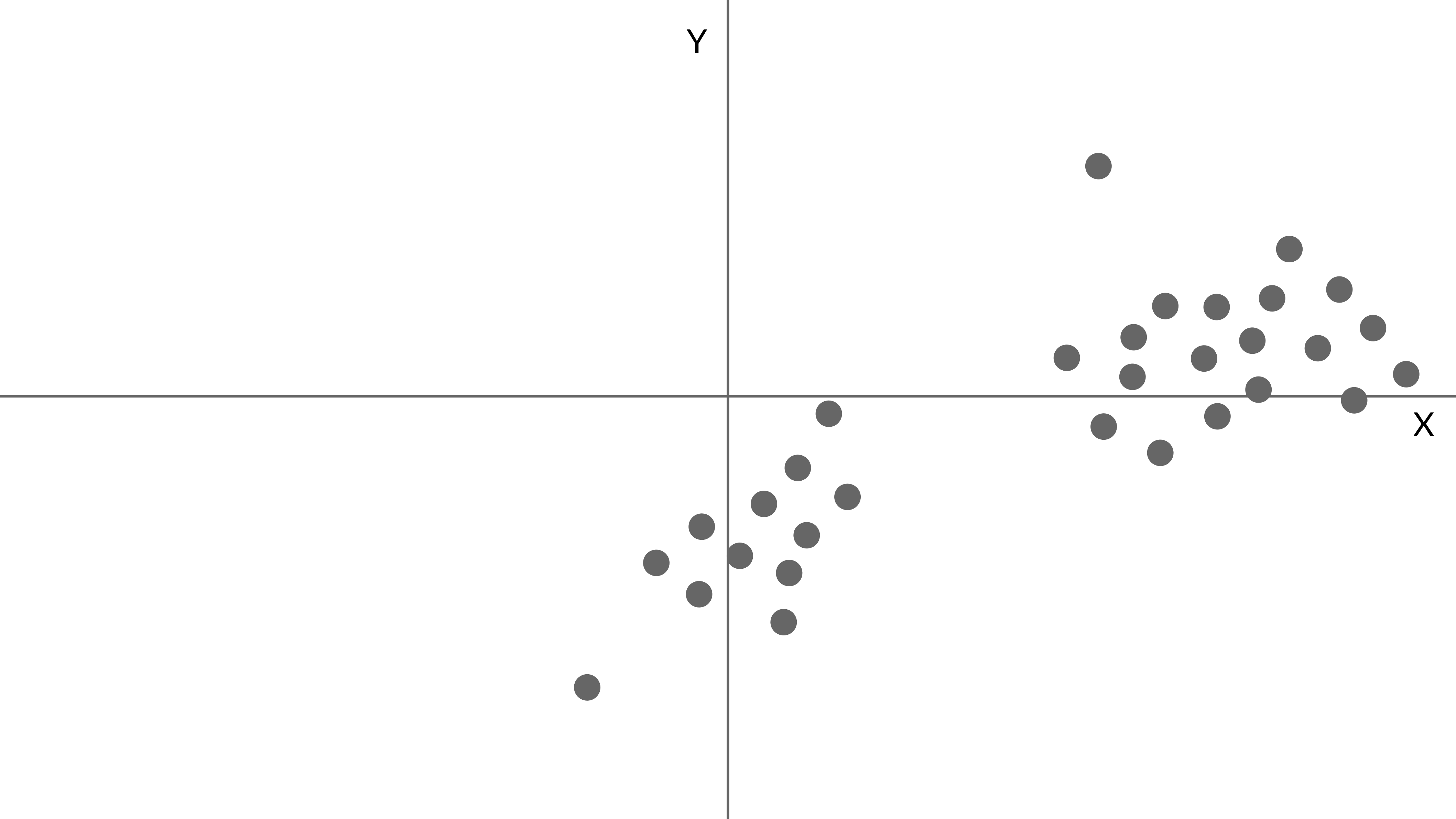

Y
X

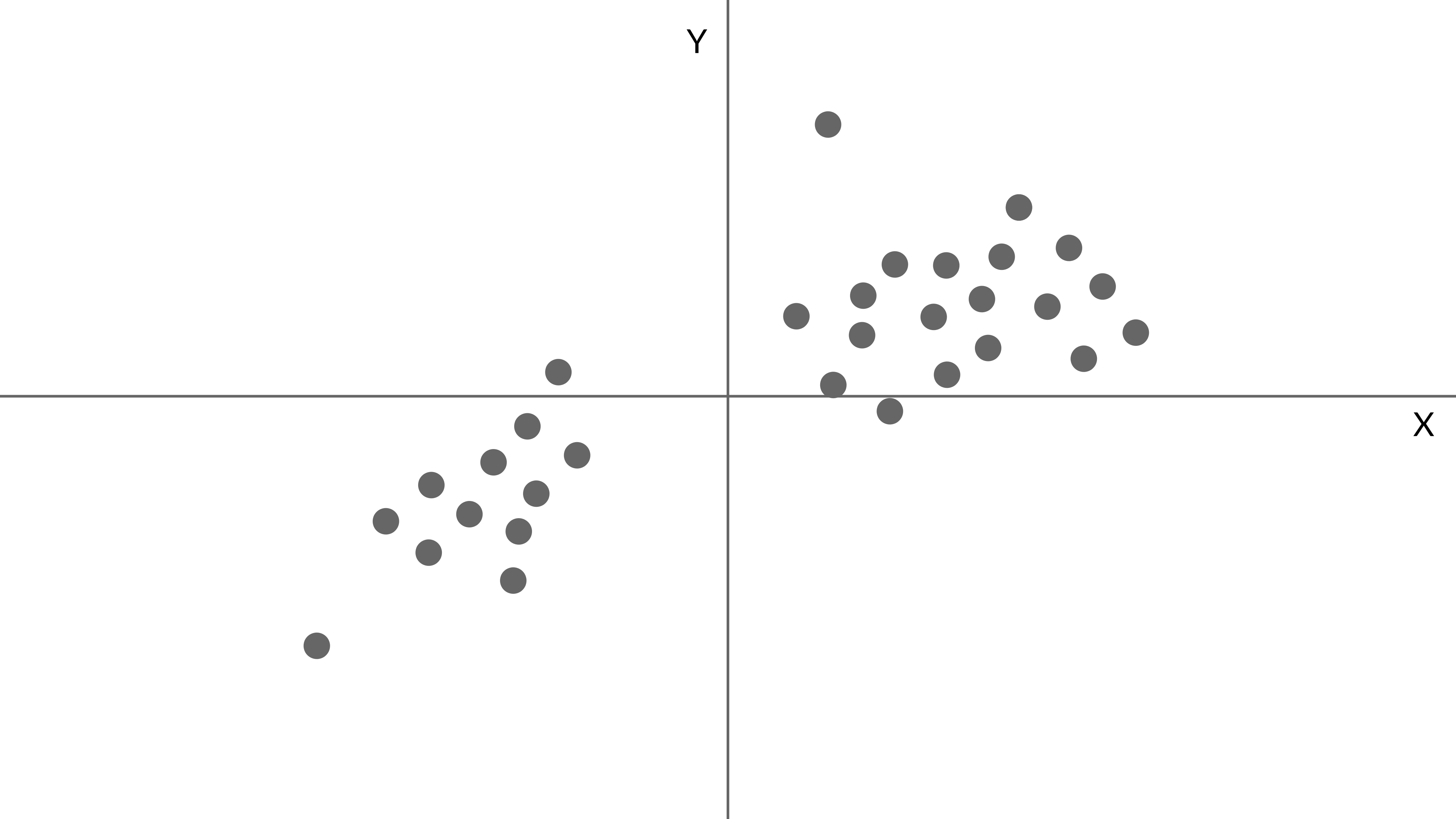

Y
X


Y
X

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle yx \rangle - \langle y \rangle \langle x \rangle \\ \langle xy \rangle - \langle x \rangle \langle y \rangle & \langle y^2 \rangle - \langle y \rangle^2 \end{pmatrix}$$


Y
X


Y
X

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle yx \rangle - \langle y \rangle \langle x \rangle \\ \langle xy \rangle - \langle x \rangle \langle y \rangle & \langle y^2 \rangle - \langle y \rangle^2 \end{pmatrix}$$

Matriz de Covarianza

$$\begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix}$$

Valores propios y vectores propios

$$\lambda \vec{v} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \vec{v}$$

Valores propios y vectores propios

$$\begin{pmatrix} \lambda v_x \\ \lambda v_y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 v_1^x \\ \lambda_1 v_1^y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_1^x \\ v_1^y \end{pmatrix}$$

$$\begin{pmatrix} \lambda_2 v_1^x \\ \lambda_2 v_2^y \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle yx \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \begin{pmatrix} v_2^x \\ v_2^y \end{pmatrix}$$

$$\lambda_1 > \lambda_2$$

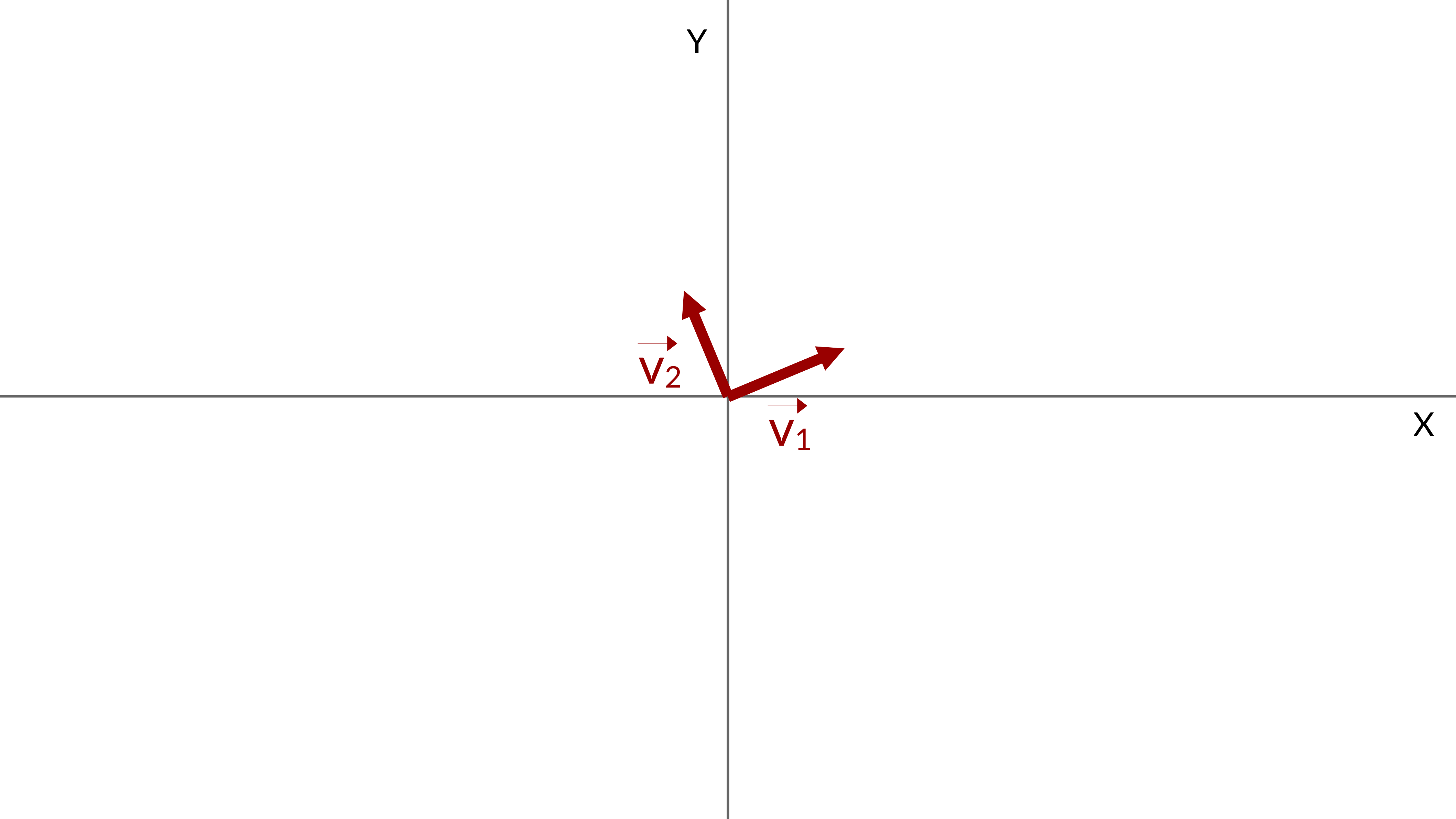
$$1 = v_1^x + v_1^y$$

$$1 = v_2^x + v_2^y$$

Vectores normales (unitarios)

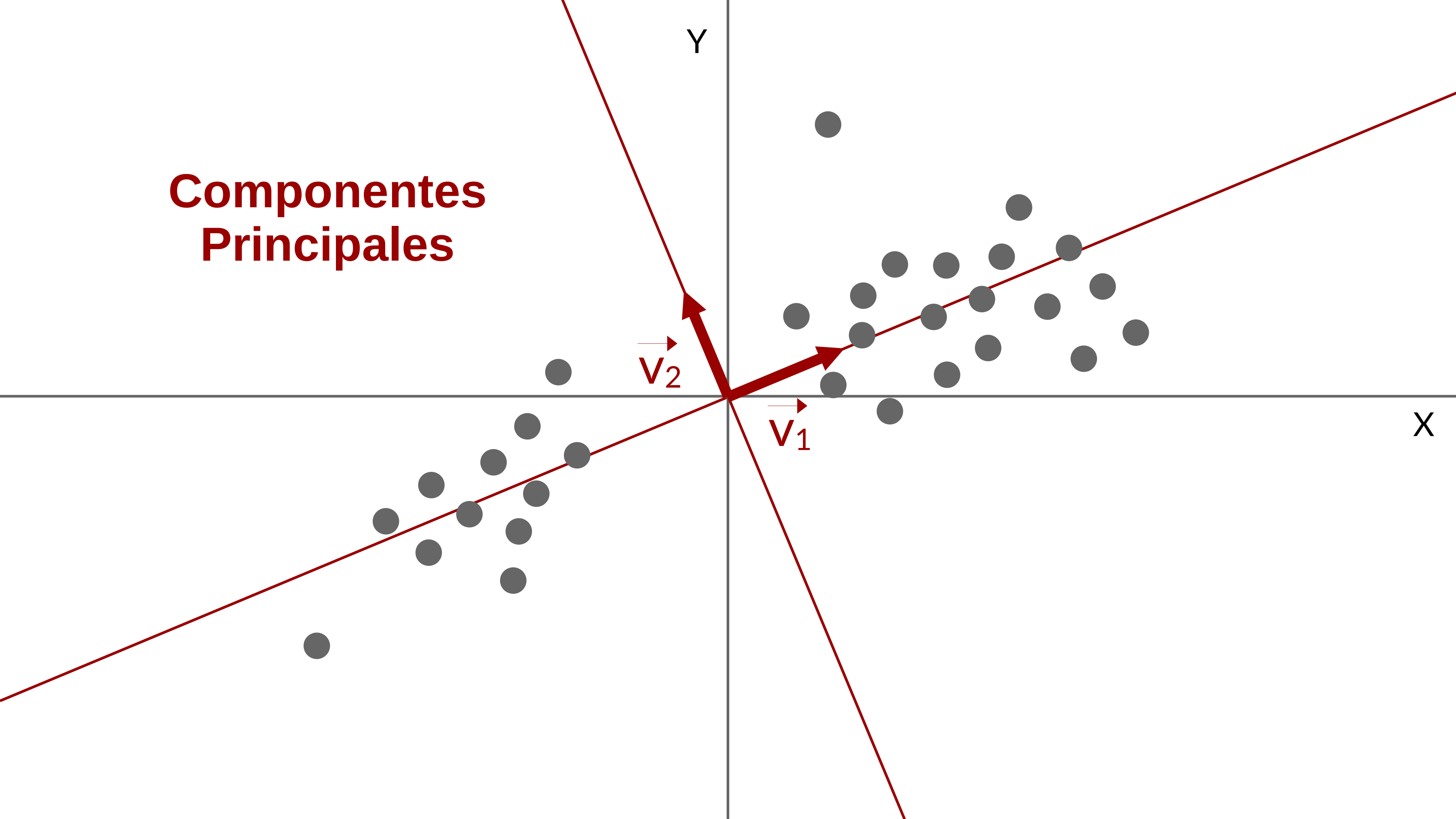
$$0 = v_1^x \cdot v_2^x + v_1^y \cdot v_2^y$$

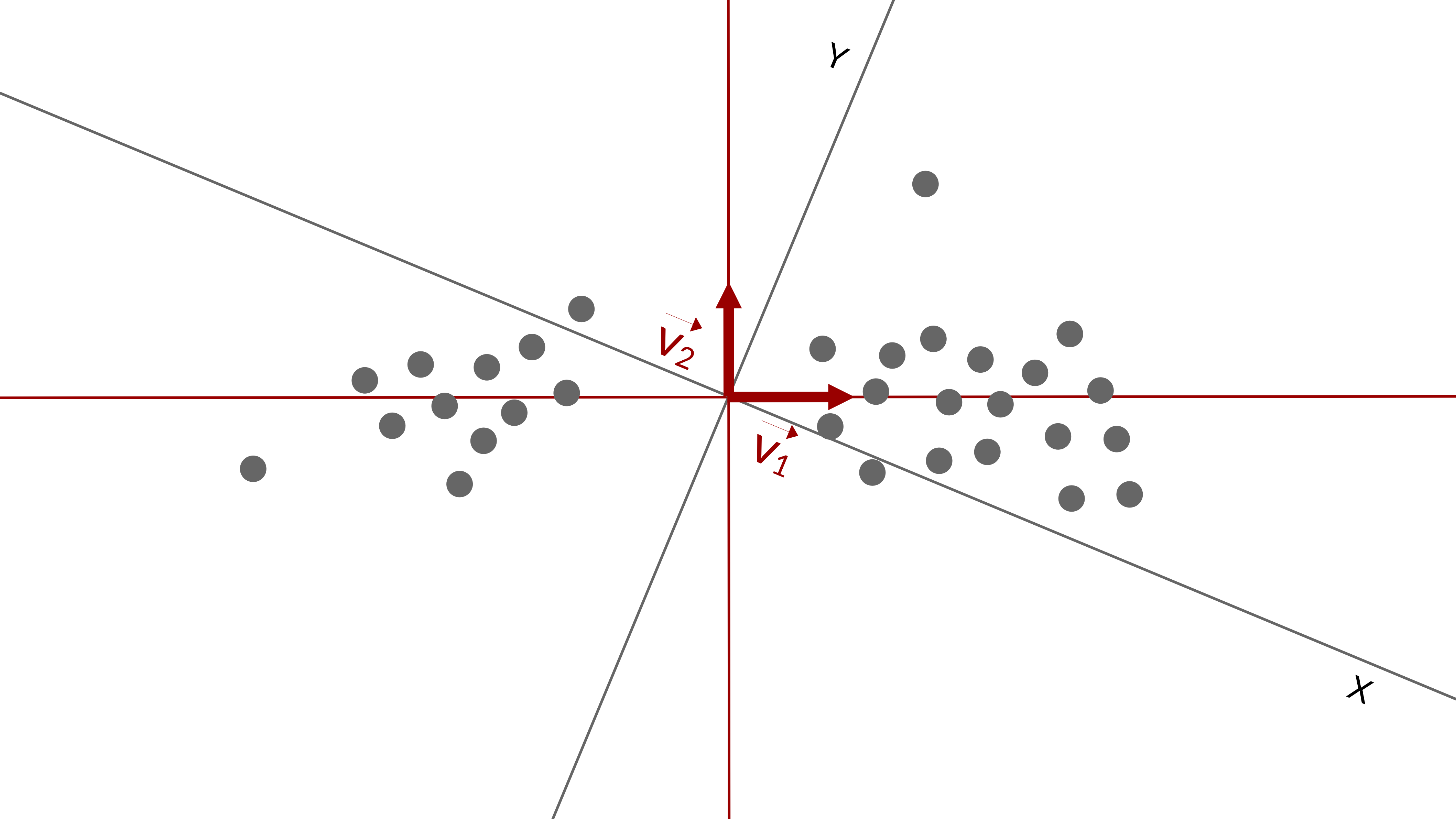
Perpendiculares



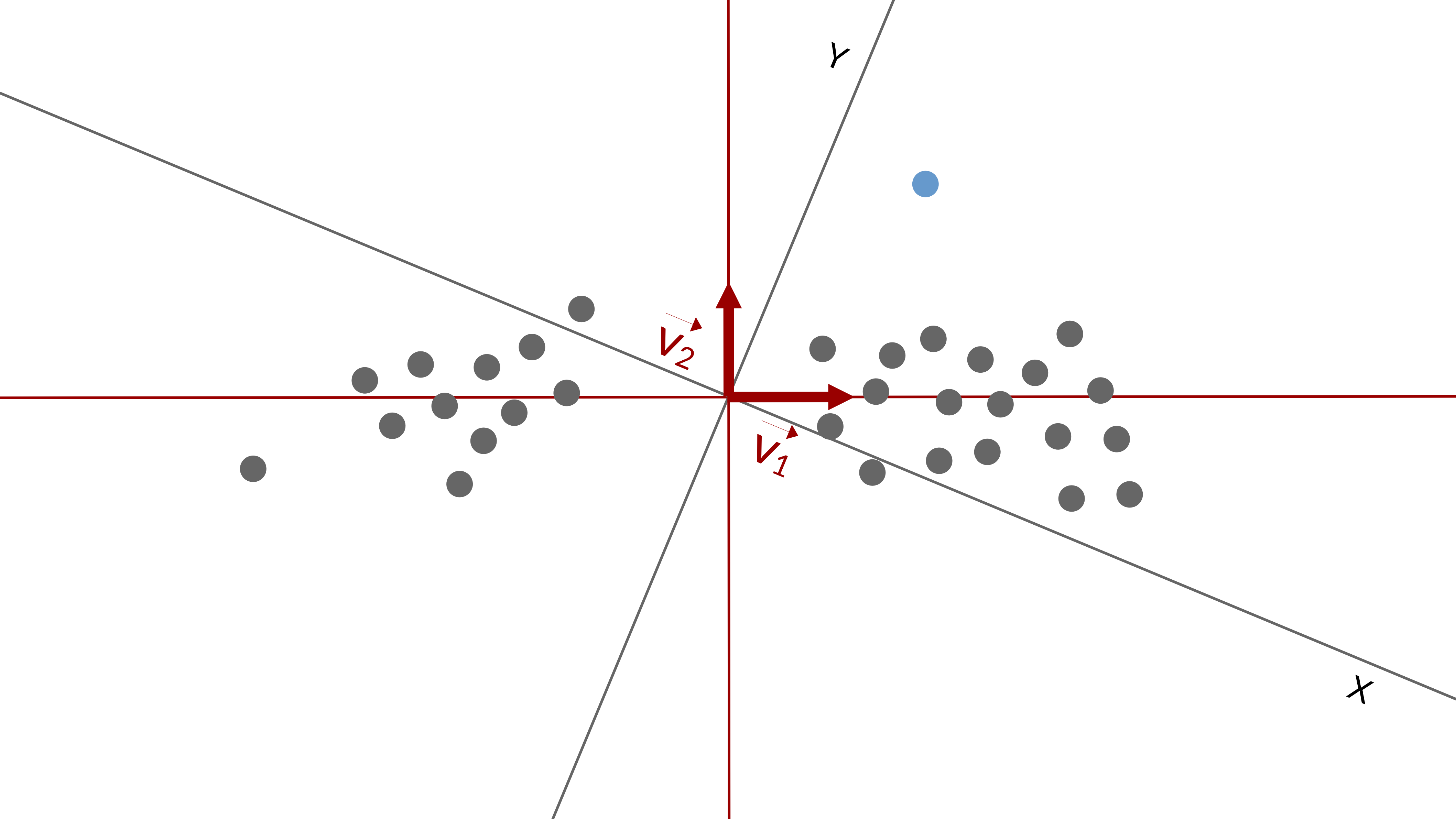
Y
X
$\vec{V}_1$
$\vec{V}_2$

Componentes Principales

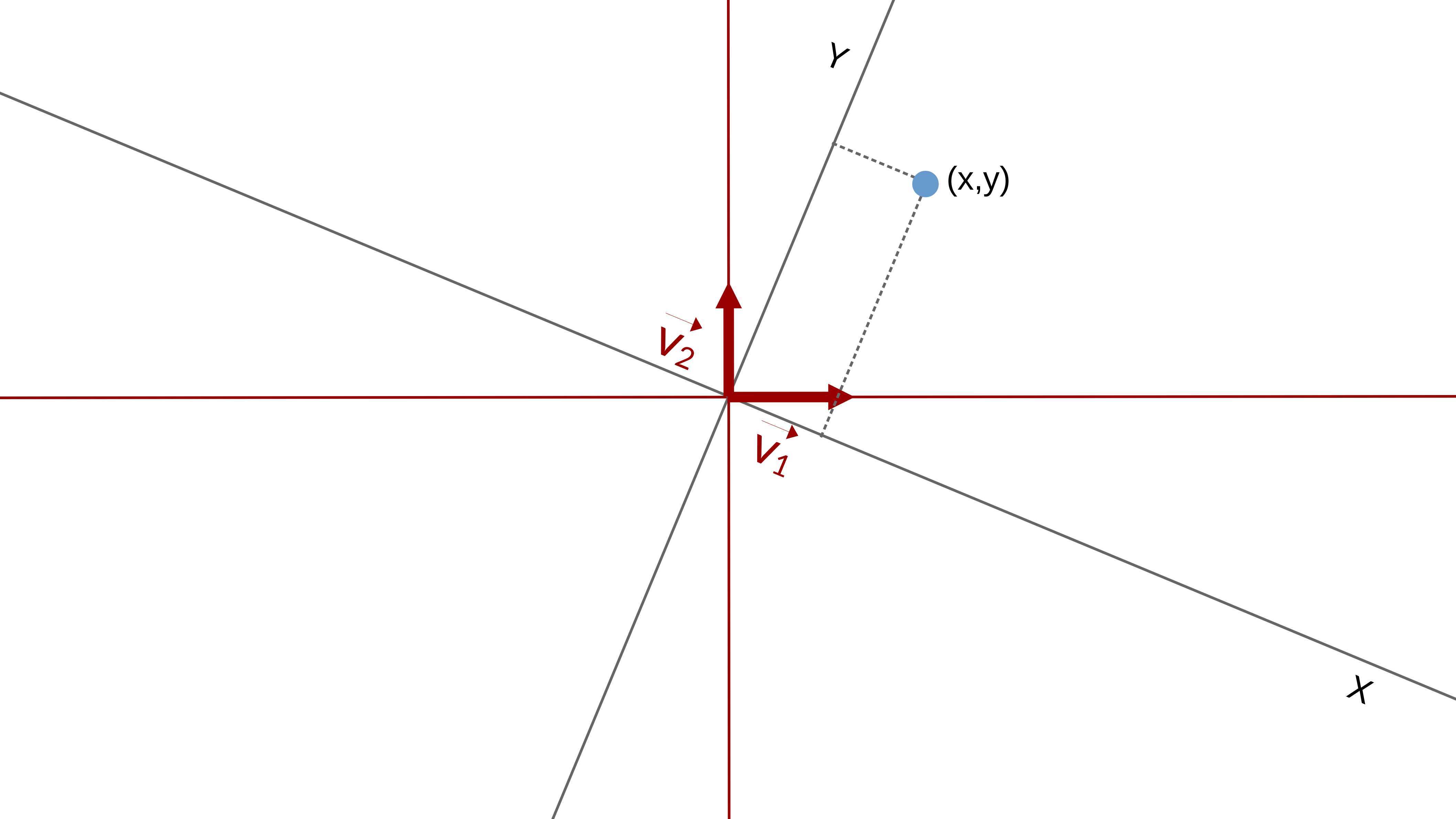


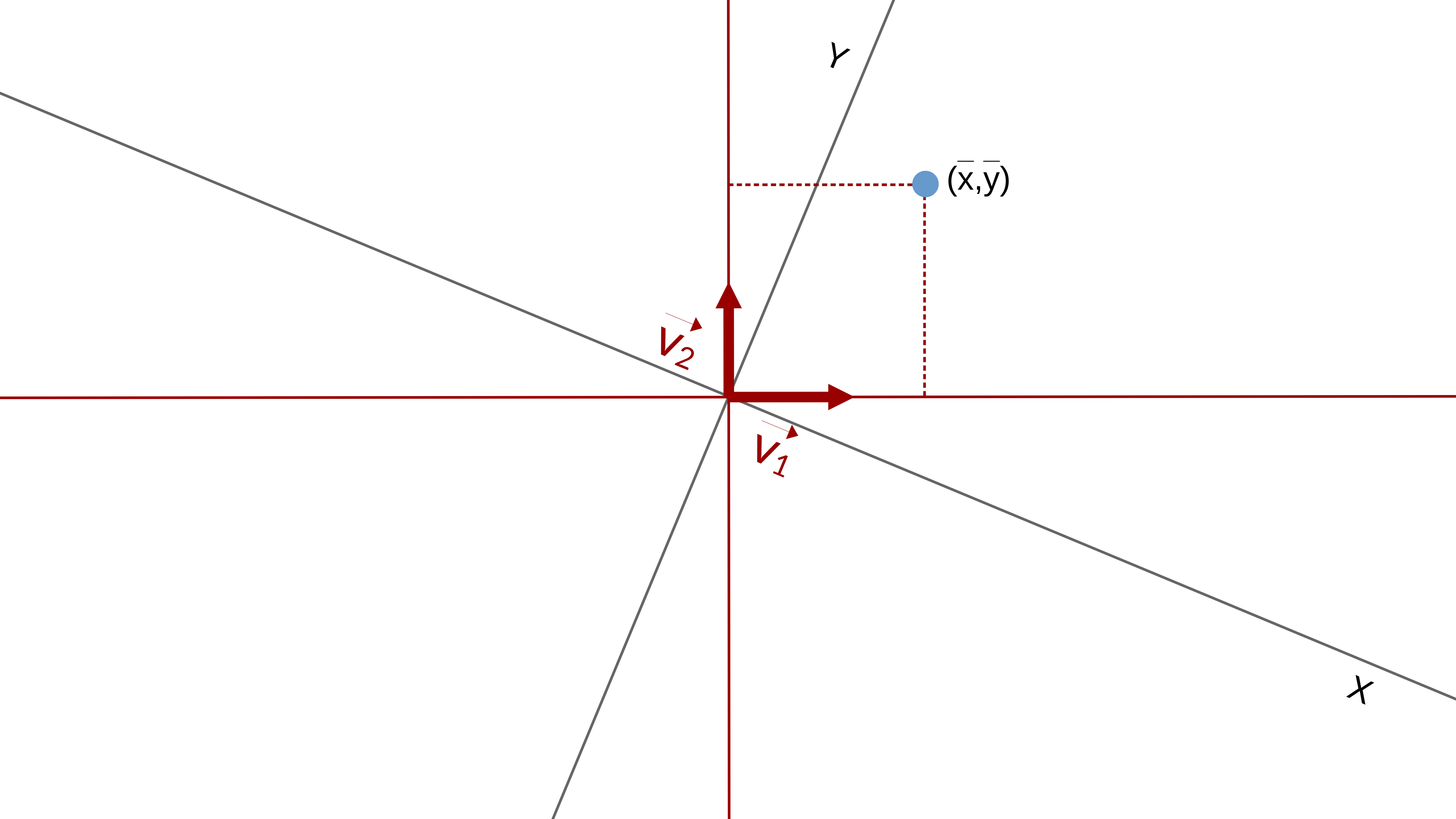


Y
$\vec{V_2}$
$\vec{V_1}$
X



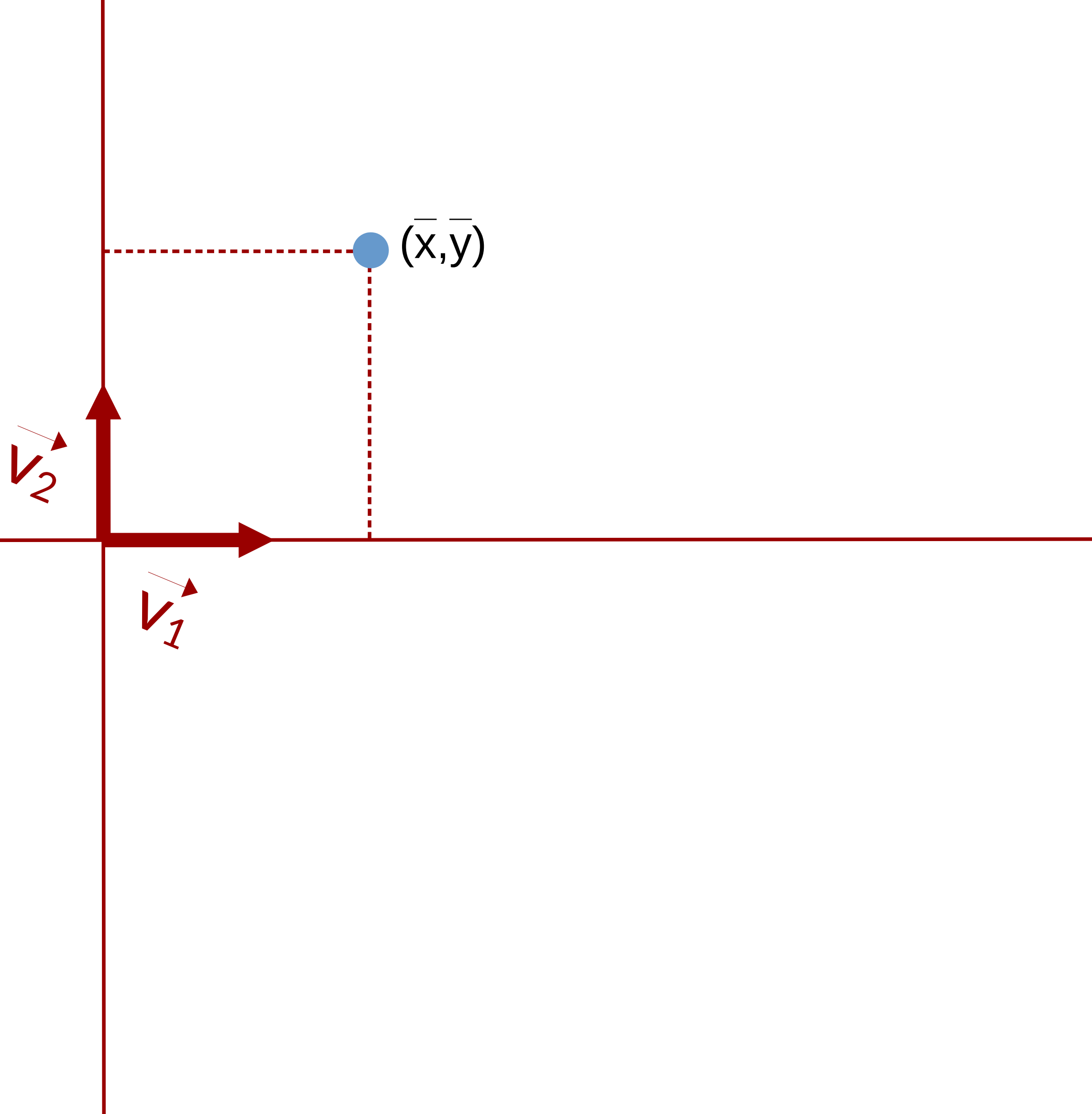
Y
$\vec{V_2}$
$\vec{V_1}$
X

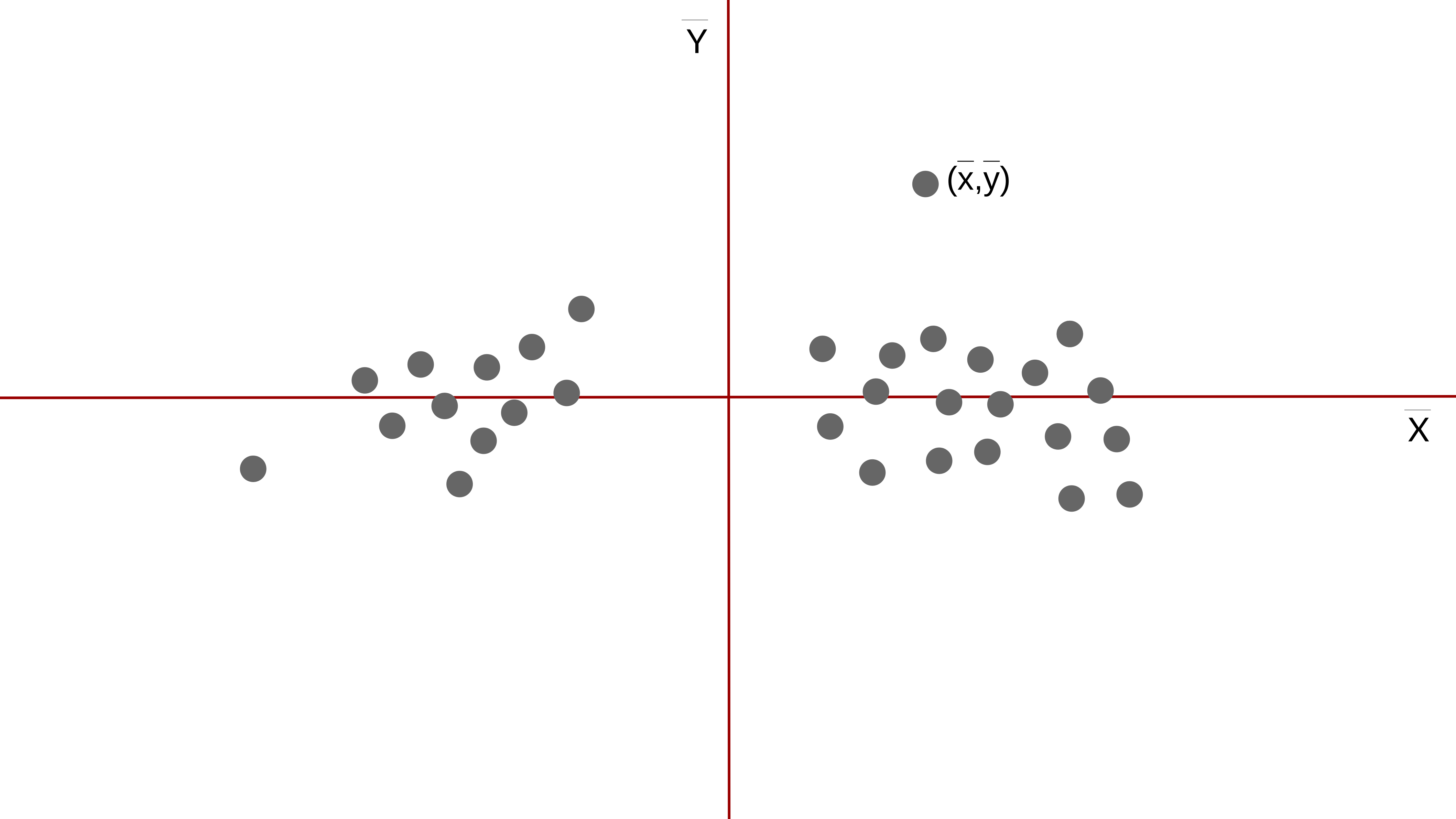

Y
(x,y)
V2
V1
X


Y
$(\bar{x},\bar{y})$
$\vec{V_2}$
$\vec{V_1}$
X

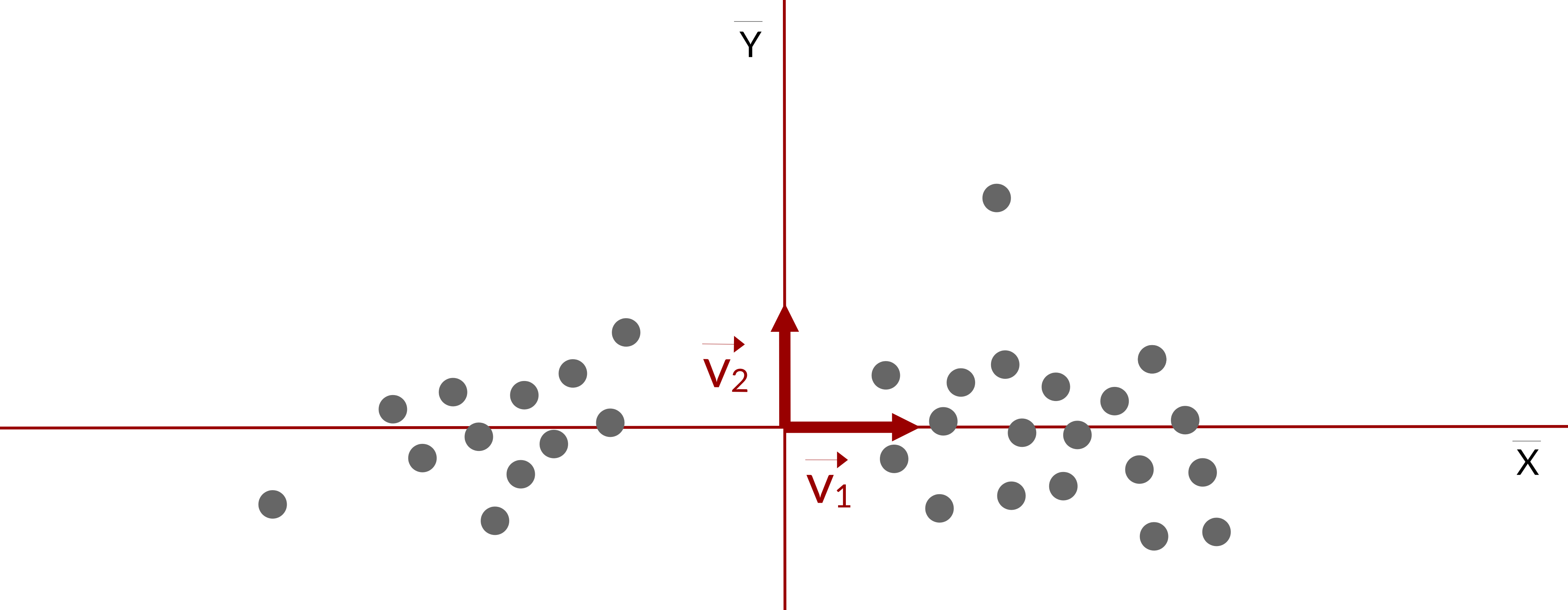
Coordenadas Principales

$$\bar{x} = x \cdot v_1^x + y \cdot v_1^y$$
$$\bar{y} = x \cdot v_2^x + y \cdot v_2^y$$





$\overline{Y}$
$(\overline{x},\overline{y})$
$\overline{X}$



$$\vec{v}_1 = (1, 0)$$
$$\vec{v}_2 = (0, 1)$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \langle \bar{x}^2 \rangle & \langle \bar{y}\, \bar{x} \rangle \\ \langle \bar{x}\, \bar{y} \rangle & \langle \bar{y}^2 \rangle \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda v_{\bar{x}} \\ \lambda v_{\bar{y}} \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} v_{\bar{x}} \\ v_{\bar{y}} \end{pmatrix}$$

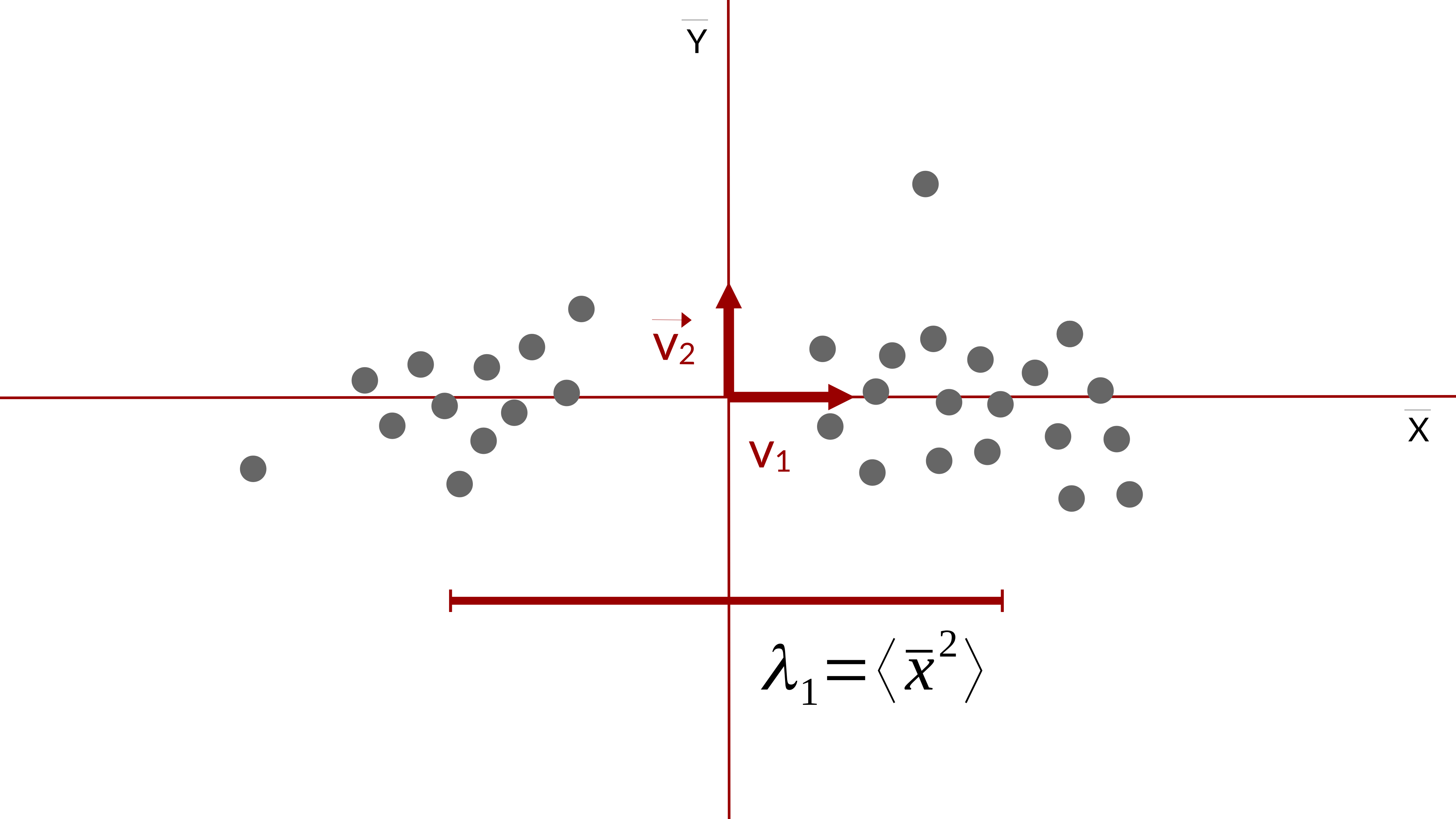
Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda_1 \cdot 1 \\ \lambda_1 \cdot 0 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Nueva Matriz de Covarianza

$$\begin{pmatrix} \lambda_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

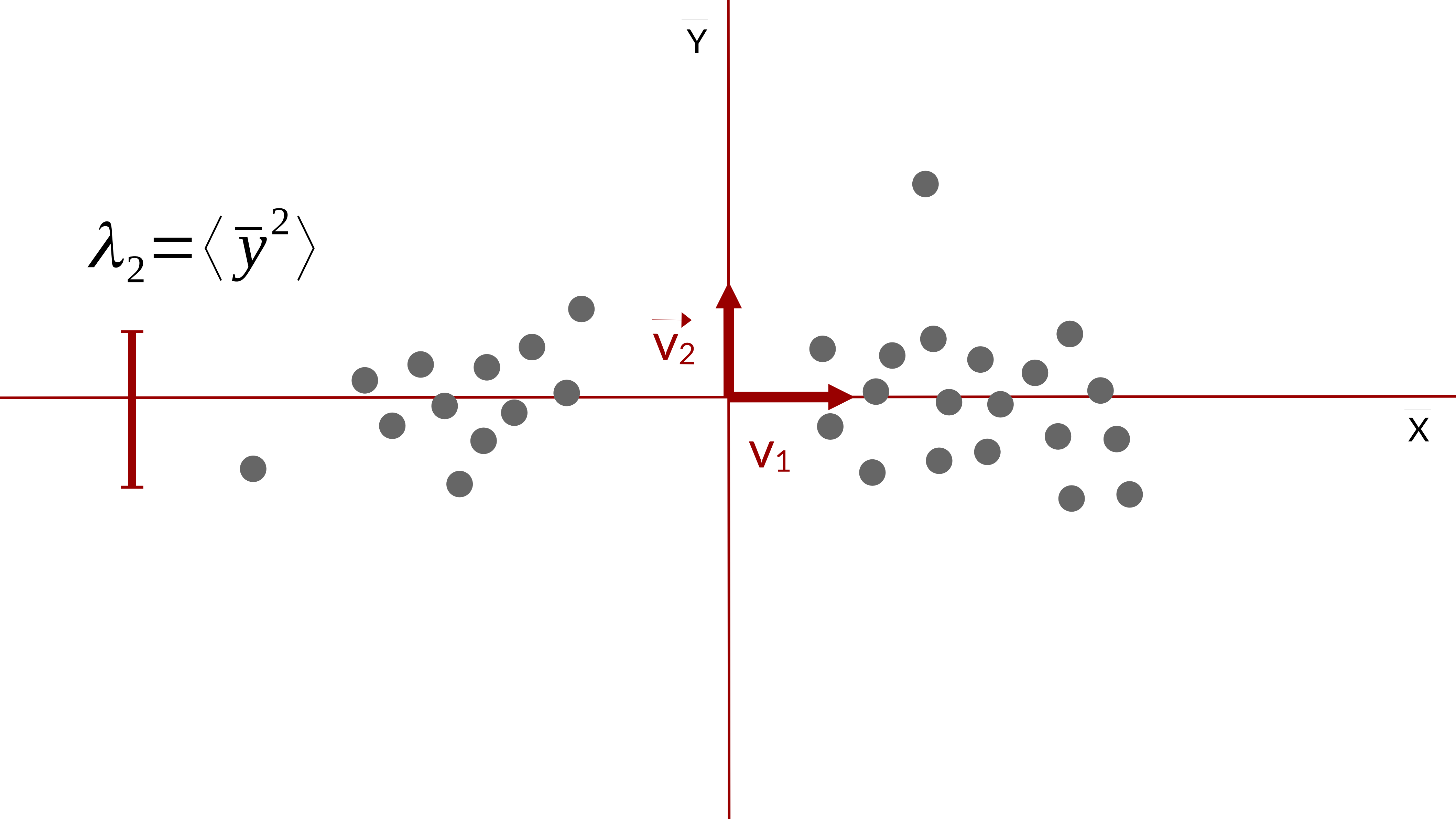
$$\boxed{\lambda_1 = \langle \bar{x}^2 \rangle}$$


$\bar{Y}$
$\vec{v}_2$
v_1
$\bar{X}$
$\lambda_1 = \langle \bar{x}^2 \rangle$

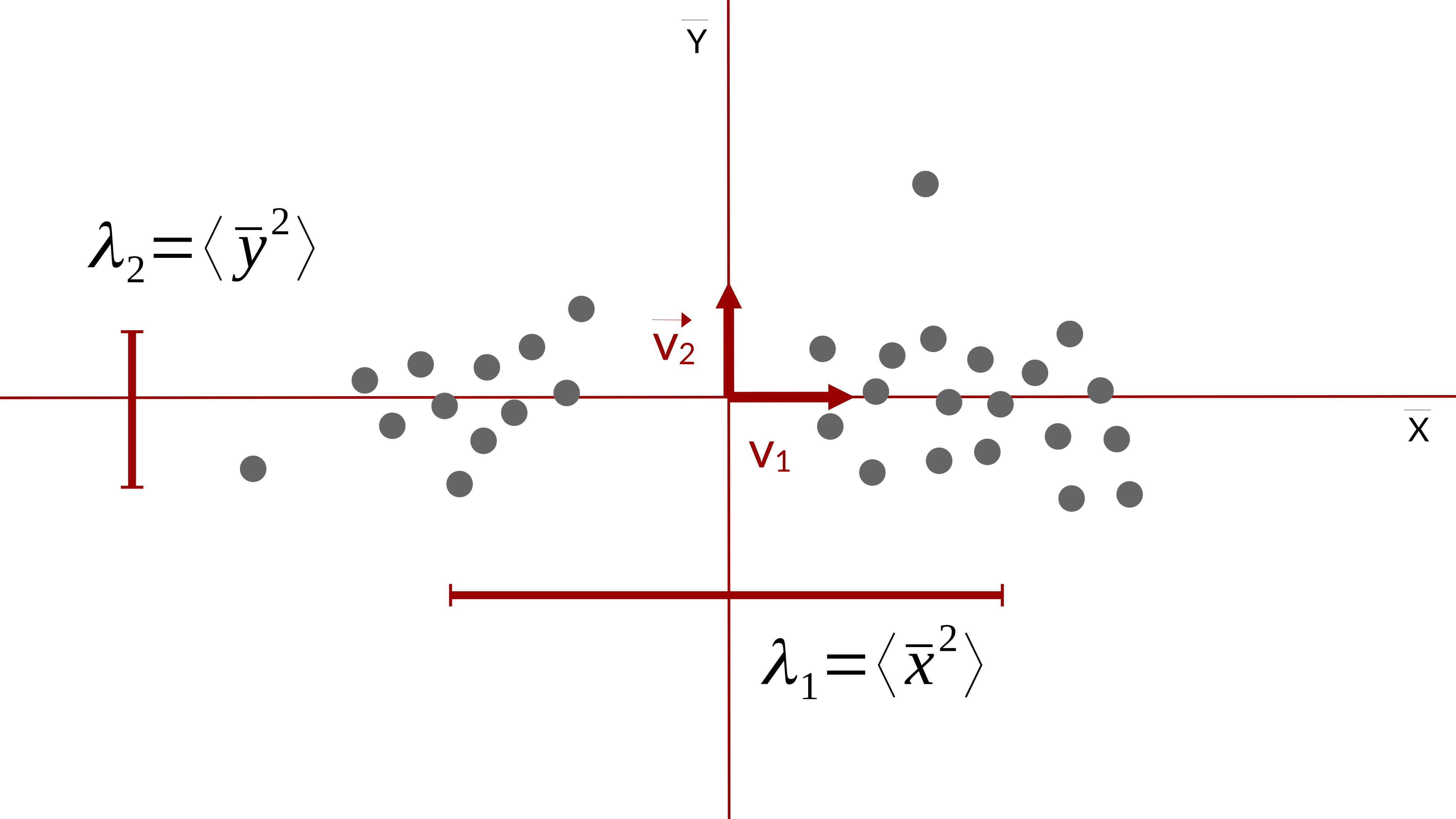
Nueva Matriz de Covarianza

$$\begin{pmatrix} 0 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \langle \bar{x}^2 \rangle & 0 \\ 0 & \langle \bar{y}^2 \rangle \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

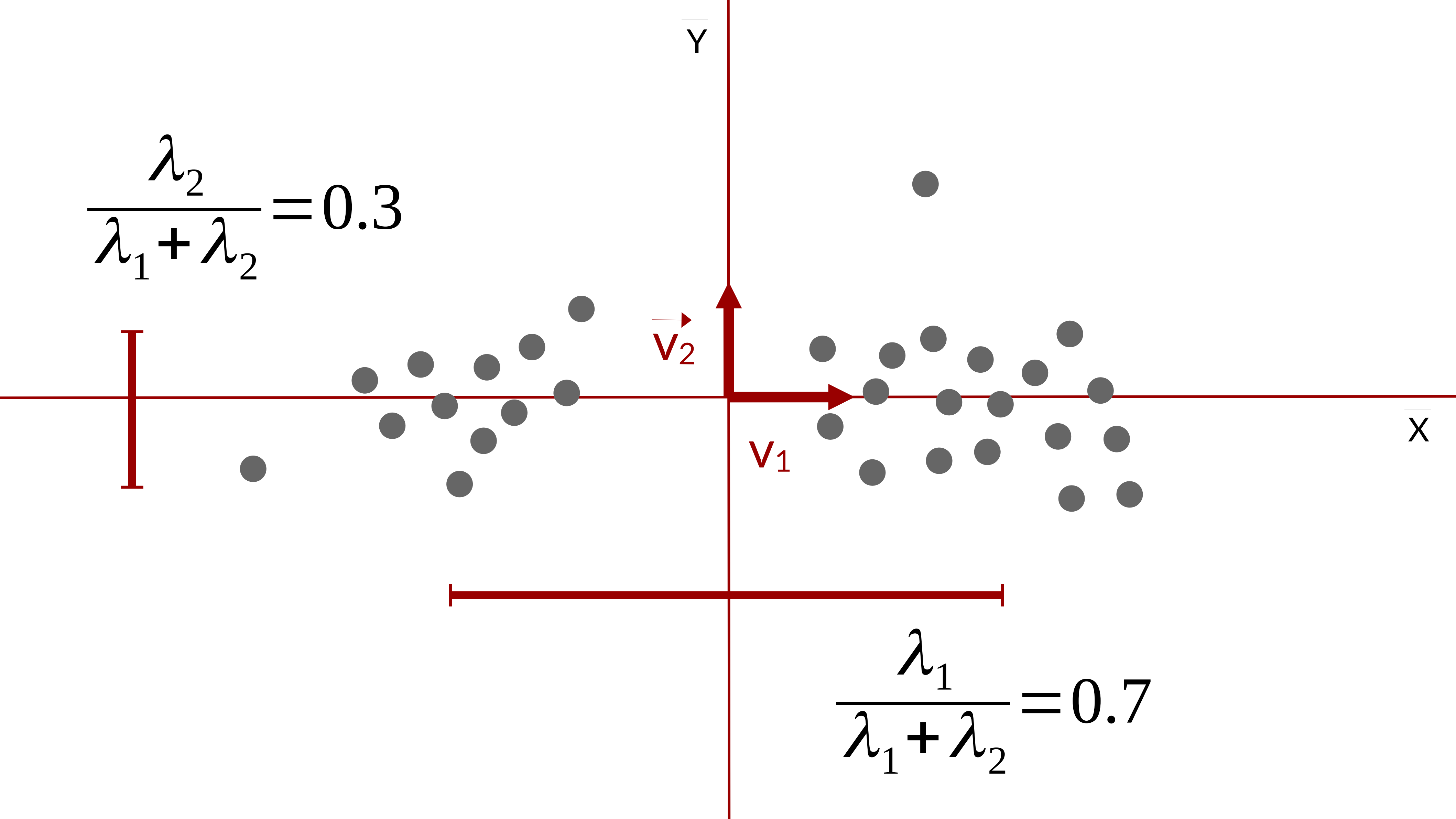
$$\boxed{\lambda_2 = \langle \bar{y}^2 \rangle}$$



$\bar{Y}$
$\lambda_2 = \langle \bar{y}^2 \rangle$
$\vec{v}_2$
v_1
$\bar{X}$



$\bar{Y}$
$\lambda_2 = \langle \bar{y}^2 \rangle$
$\vec{v}_2$
v_1
$\bar{X}$
$\lambda_1 = \langle \bar{x}^2 \rangle$


$\frac{\lambda_2}{\lambda_1+\lambda_2}=0.3$
Y
V_2
V_1
X
$\frac{\lambda_1}{\lambda_1+\lambda_2}=0.7$