funções hiperbólicas

Definição: As funções hiperbólicas são
denotadas e definidas por:

i) $\operatorname{Senh} x = \dfrac{e^x - e^{-x}}{2}$

ii) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

iii) $\operatorname{tgh} x = \dfrac{\operatorname{senh} x}{\cosh x}$

iv) $\operatorname{sech} x = \dfrac{1}{\cosh x}$

v) $\operatorname{cotgh} x = \dfrac{\cosh x}{\operatorname{senh} x}$

vi) $\operatorname{cossech} x = \dfrac{1}{\operatorname{senh} x}$

Gráficos:

i)

ii)
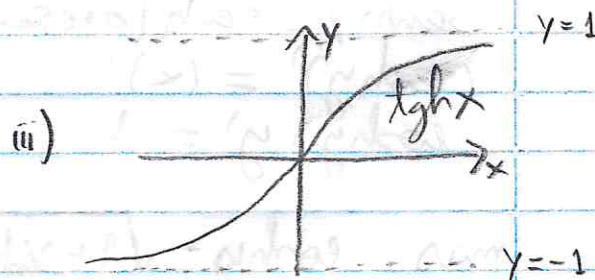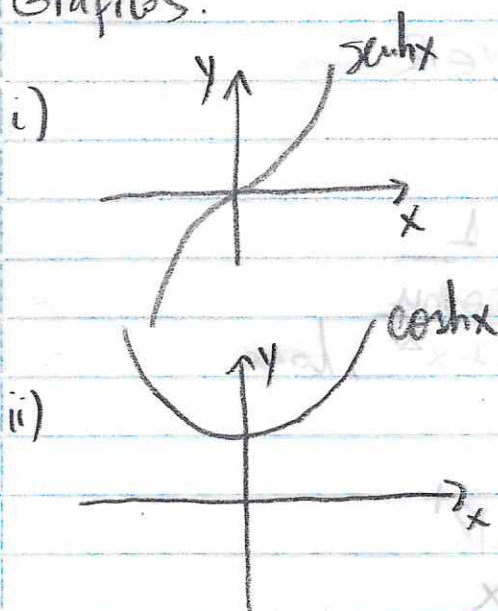


Identidades Hiperbólicas:
- a) $\operatorname{senh}(-x) = -\operatorname{senh} x$    e    $\cosh(-x) = \cosh x$
- b) $\cosh^2 x - \operatorname{senh}^2 x = 1$
- c) $1 - \operatorname{tgh}^2 x = \operatorname{sech}^2 x$    e    $\operatorname{cotgh}^2 x - 1 = \operatorname{cossech}^2 x$
- d) $\operatorname{senh}(x+y) = \operatorname{senh} x \cdot \cosh y + \operatorname{senh} y \cdot \cosh x$
- e) $\cosh(x+y) = \cosh x \cdot \cosh y + \operatorname{senh} x \cdot \operatorname{senh} y$

Derivada das funções hiperbólicas:

1) $D_x(\operatorname{senh}x) = \cosh x$
2) $D_x(\cosh x) = \operatorname{senh}x$
3) $D_x(tgh\,x) = sech^2 x$
4) $D_x(cotgh\,x) = -cosech^2 x$
5) $D_x(sech\,x) = -sech\,x \cdot tgh\,x$
6) $D_x(cosech\,x) = -cosech\,x \cdot cotgh\,x$

Derivada das funções hiperbólicas Inversas:

1) $y = \operatorname{arcsenh}x$      com   $y \in \mathbb{R}$

$\operatorname{senh}y = \operatorname{senh}(\operatorname{arcsenh}x) = x$

$(\operatorname{senh}y)' = (x)'$

$\cosh y \cdot y' = 1 \qquad y' = \dfrac{1}{\cosh y}$

mas $\cosh y = \sqrt{1 + \operatorname{senh}^2 y} = \sqrt{1 + x^2}$, logo

$$y' = \dfrac{1}{\sqrt{1 + x^2}}$$

2) $y = \operatorname{arccosh}x$    com    $y \in \mathbb{R}^+$

$\cosh y = \cosh(\operatorname{arccosh}x) = x$

$(\cosh y)' = (x)'$

$\operatorname{senh}y \cdot y' = 1 \qquad y' = \dfrac{1}{\operatorname{senh}y}$

mas $\operatorname{senh}y = \sqrt{\cosh^2 y - 1} = \sqrt{x^2 - 1}$, logo

$$y' = \dfrac{1}{\sqrt{x^2 - 1}}$$

3) $y = \operatorname{arctgh}x$     com    $y \in \mathbb{R}$

4) $y = \text{arc cotgh} x$ $\qquad y \in \mathbb{R}$

5) $y = \text{arcsech} x$ $\qquad y \in \mathbb{R}$

6) $y = \text{arccosech} x$ $\qquad y \in \mathbb{R}$

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