Sn❄w666

GELSON IEZZI

COMPLEMENTO PARA
O PROFESSOR

FUNDAMENTOS DE MATEMÁTICA 3 ELEMENTAR

TRIGONOMETRIA

ATUAL
EDITORA

EDITORA AFILIADA

SARAIVA S.A. Livreiros Editores, São Paulo, 2000.
Av. Marquês de São Vicente, 1697 — Barra Funda
01139-904 — São Paulo — SP
Fone: (0xx11) 3613-3000
Fax: (0xx11) 3611-3308 — Fax Vendas: (0xx11) 3611-3268
www.editorasaraiva.com.br

Dados Internacionais de Catalogação na Publicação (CIP)
(Câmara Brasileira do Livro, SP, Brasil)

Fundamentos de matemática elementar : complemento para o professor. — São Paulo: Atual, 1993.

Conteúdo: v. 1. Conjuntos e funções / Gelson Iezzi, Carlos Murakami. — v. 2. Logaritmos / Gelson Iezzi, Osvaldo Dolce, Carlos Murakami. — v. 3. Trigonometria / Gelson Iezzi. — v. 4. Seqüências, matrizes, determinantes, sistemas / Gelson Iezzi, Samuel Hazzan. — v. 5. Combinatória, probabilidade / Samuel Hazzan. — v. 6. Complexos, polinômios, equações — v. 7. Geometria analítica / Gelson Iezzi. — v. 8. Limites, derivadas, noções de integral — Gelson Iezzi, Carlos Murakami, Nilson José Machado. — v. 9. Geometria plana — v. 10. Geometria espacial : posição e métrica / Osvaldo Dolce, José Nicolau Pompeo.

1. Matemática (2.° grau) 2. Matemática (2.° grau) — Problemas e exercícios etc. 3. Matemática (Vestibular) — Testes I. Iezzi, Gelson, 1939- II. Murakami, Carlos, 1943- III. Dolce, Osvaldo, 1938- IV. Hazzan, Samuel, 1946- V. Machado, Nilson José, 1947- VI. Pompeo, José Nicolau, 1945-

93-1795 CDD-510.7

Índices para catálogo sistemático:
1. Matemática : Ensino de 2.° grau 510.7

Complemento para o Professor — Fundamentos de Matemática Elementar 3

Editora: Bárbara Ferreira Arena
Editor de campo: Valdir Montanari
Coordenadora editorial: Sandra Lucia Abrano
Chefe de preparação de texto e revisão: Noé Ribeiro
Coordenadora de revisão: Maria Luiza Xavier Souto
Revisores: Alice Kobayashi
Magna Reimberg Teobaldo
Maria Cecília Fernandes Vannucchi
Maria da Penha Faria
Vera Lúcia Pereira Della Rosa
Editor de arte: Zildo Braz
Chefe de arte: Glair Alonso Arruda
Assistentes de arte: Lu Bevilacqua Ghion
Ricardo Yorio
Rosi Meire Martins Ortega
Gerente de produção: Antonio Cabello Q. Filho
Coordenadora de produção: Silvia Regina E. Almeida
Produção gráfica: José Rogerio L. de Simone
Maurício T. de Moraes
Capa: Ettore Bottini
Foto de capa: Hilton Ribeiro
Consultora técnica: Erileide Maria de Sobral de Souza
Fotolito: Binhos
Composição e arte-final: Paika Realizações Gráficas

Visite nosso *site*: www.atualeditora.com.br
Central de atendimento ao professor: (0xx11) 3613-3030

Apresentação

Este livro é o *Complemento para o Professor* do volume 3, Trigonometria, da coleção *Fundamentos de Matemática Elementar*.

Cada volume desta coleção tem um complemento para o professor, com o objetivo de apresentar a solução dos exercícios mais complicados do livro e sugerir sua passagem aos alunos.

É nossa intenção aperfeiçoar continuamente os *Complementos*. Estamos abertos às sugestões e críticas, que nos devem ser encaminhadas através da Editora.

Agradecemos à professora Erileide Maria de Sobral Souza a colaboração na redação das soluções que são apresentadas neste *Complemento*.

Os Autores.

Sumário

Capítulo II – Razões trigonométricas no triângulo retângulo

6. $\text{tg}\ \hat{B} = \dfrac{b}{c} = \dfrac{\sqrt{5}}{2} \Rightarrow b = \dfrac{c\sqrt{5}}{2}$

$b^2 + c^2 = a^2 \Rightarrow \dfrac{5c^2}{4} + c^2 = 36 \Rightarrow c = 4$ e então $b = 2\sqrt{5}$

14. a) $\text{sen}\ 20°15' = \text{sen}\ 20° + \dfrac{15}{60}(\text{sen}\ 21° - \text{sen}\ 20°) =$

$= 0{,}34202 + \dfrac{15}{60}(0{,}35837 - 0{,}34202) = 0{,}34610$

b) $\cos 15°30' = \cos 15° + \dfrac{30}{60}(\cos 16° - \cos 15°) =$

$= 0{,}96593 + \dfrac{30}{60}(0{,}96126 - 0{,}96593) = 0{,}96358$

c) $\text{tg}\ 12°40' = \text{tg}\ 12° + \dfrac{40}{60}(\text{tg}\ 13° - \text{tg}\ 12°) =$

$= 0{,}21256 + \dfrac{40}{60}(0{,}23087 - 0{,}21256) = 0{,}22476$

d) $\text{sen}\ 50°12' = \text{sen}\ 50° + \dfrac{12}{60}(\text{sen}\ 51° - \text{sen}\ 50°) =$

$= 0{,}76604 + \dfrac{12}{60}(0{,}77715 - 0{,}76604) = 0{,}76826$

e) $\cos 70°27' = \cos 70° + \dfrac{27}{60}(\cos 71° - \cos 70°) =$

$0{,}34202 + \dfrac{27}{60}(0{,}32557 - 0{,}34202) = 0{,}33462$

f) $\text{tg}\ 80°35' = \text{tg}\ 80° + \dfrac{35}{60}(\text{tg}\ 81° - \text{tg}\ 80°) =$

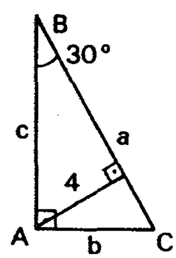
$5{,}67128 + \dfrac{35}{60}(6{,}31375 - 5{,}67128) = 6{,}04605$

15. $b = 4 \cdot \text{tg}\ 35° \Rightarrow b = 2{,}80084$ cm
$a = 4/\cos 35° \Rightarrow a = 4{,}88311$ cm

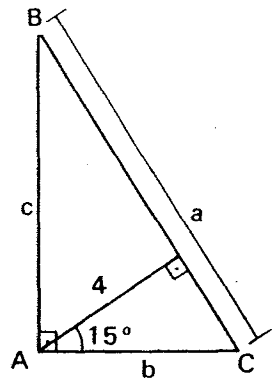
16. $c \cdot \text{sen}\ 30° = 4 \Rightarrow c = 8$

$b = c \cdot \text{tg}\ 30° \Rightarrow b = \dfrac{8\sqrt{3}}{3}$

$a^2 = b^2 + c^2 \Rightarrow a = \dfrac{16\sqrt{3}}{3}$

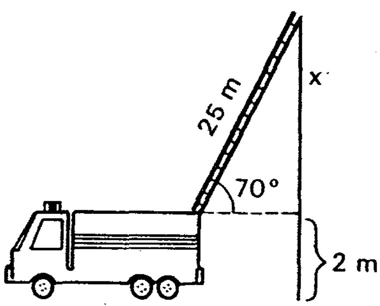


17. $b \cdot \cos 15^\circ = 4 \Rightarrow b = 4\sqrt{2}(\sqrt{3} - 1)$
$c \cdot \cos 75^\circ = 4 \Rightarrow c = 4\sqrt{2}(\sqrt{3} + 1)$
$a^2 = b^2 + c^2 \Rightarrow a = 16$



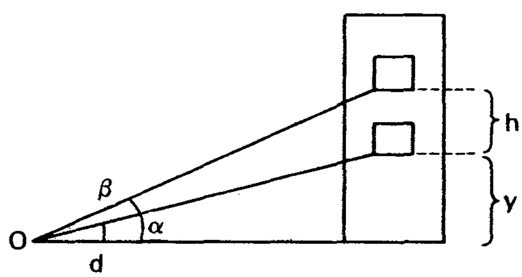
18. $h = y \cdot \text{tg } 30^\circ$ e $h = x \cdot \text{tg } 60^\circ \Rightarrow \dfrac{x}{y} = \dfrac{1}{3}$

19. $x = 25 \cdot \text{sen } 70^\circ \Rightarrow x = 23{,}49$ m
$x + 2 = 25{,}49$ m



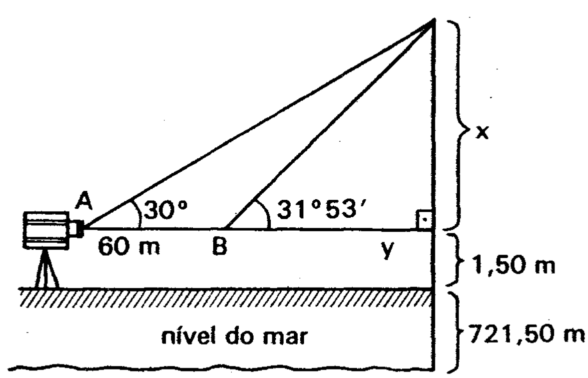
21. $d = \dfrac{h + y}{\text{tg } \beta}$ e $d = \dfrac{y}{\text{tg } \alpha} \Rightarrow$

$h = d(\text{tg } \beta - \text{tg } \alpha)$

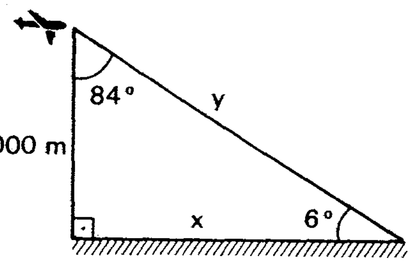


22. $AB = \dfrac{H - h}{\text{tg } \beta}$ e $AB = \dfrac{h}{\text{tg } \alpha} \Rightarrow H = h\left[\dfrac{\text{tg } \beta}{\text{tg } \alpha} + 1\right]$

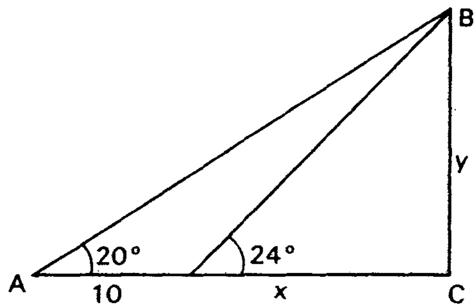
23. $x = y \cdot \text{tg } 31^\circ 53'$ e $x = (60 + y) \cdot \text{tg } 30^\circ \Rightarrow x = 497{,}14$ m
$497{,}14 + 1{,}50 + 721{,}50 = 1220{,}14$ m



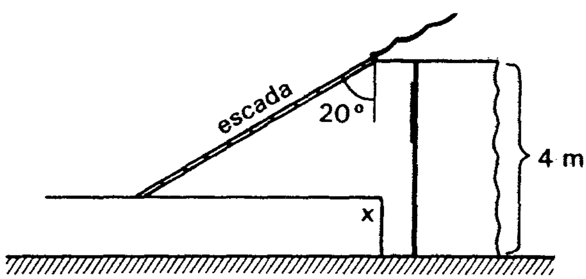
24. $x = 7\,000 \cdot \text{tg } 84° \Rightarrow x = 66{,}60$ km
$y = 7\,000/\cos 84° \Rightarrow y = 66{,}97$ km



25. $y = x \cdot \text{tg } 24°$ e $y = (10 + x) \cdot \text{tg } 20° \Rightarrow x = 44{,}72$ m



26. $4 - x = 3 \cdot \cos 20° \Rightarrow x = 1{,}18$ m



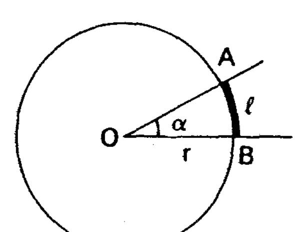
Capítulo III — Arcos e ângulos

35. $\left.\begin{array}{l} a - b = \dfrac{\pi}{12} \\ a + b = \dfrac{7\pi}{4} \end{array}\right\} \Rightarrow a = \dfrac{165\pi}{180} = \dfrac{11\pi}{12}$ rad, $b = \dfrac{150\pi}{180} = \dfrac{5\pi}{6}$ rad

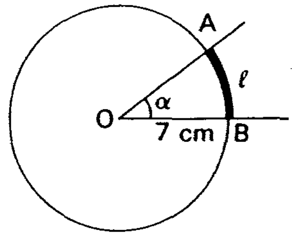
36. $\left.\begin{array}{l} a + b + c = 13° \\ a + b + 2c = 15° \\ a + 2b + c = 20° \end{array}\right\} \Rightarrow a = 4°, b = 7°$ e $c = 2°$

39. $\ell = \dfrac{2\pi r}{3}$

$\alpha = \dfrac{\ell}{r}$ rad $\Rightarrow \alpha = \dfrac{2\pi}{3}$ rad ou 120°

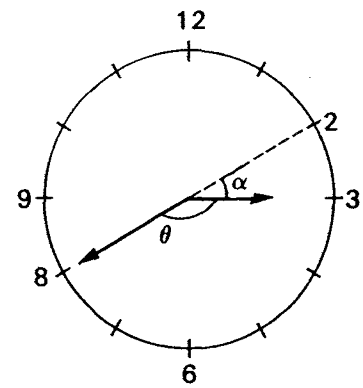


40. $\ell = \alpha \cdot r = 4{,}5 \cdot 7 = 31{,}5$ cm



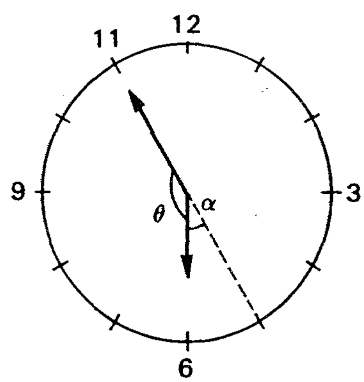
42. a) $\alpha = \dfrac{40}{60} \cdot 30° = 20°$

$\theta = 180° - 20° = 160°$



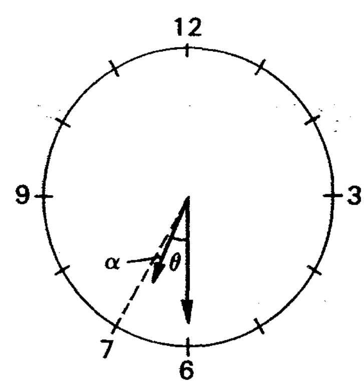
b) $\alpha = \dfrac{55}{60} \cdot 30° = 27{,}5°$

$\theta = 180° - 27{,}5° = 152{,}5°$ ou $152°30'$



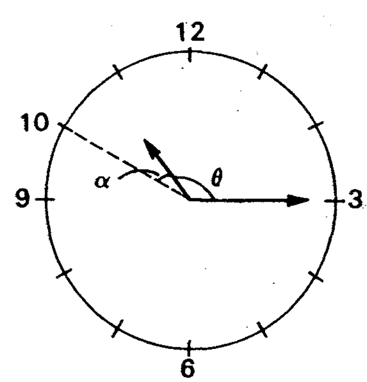
c) $\alpha = \dfrac{30 \cdot 30°}{60} = 15°$

$\theta = 30° - 15° = 15°$



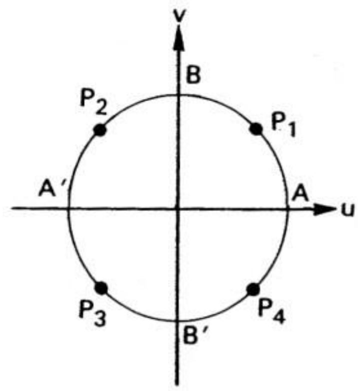
d) $\alpha = \dfrac{15}{60} \cdot 30° = 7{,}5°$

$\theta = 150° - 7{,}5° = 142{,}5°$ ou $142°30'$



44. $\widehat{AP}_1 = \frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$ rad

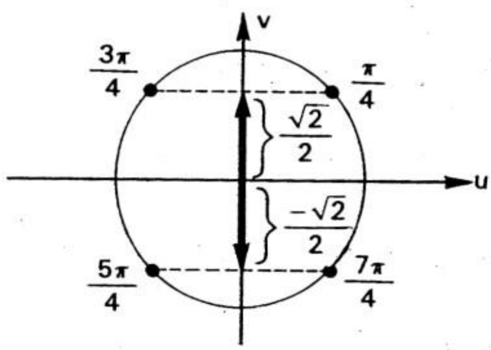
Imagens de x	A	P_1	B	P_2	A′	P_3	B′	P_4
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$



Capítulo IV — Razões trigonométricas na circunferência

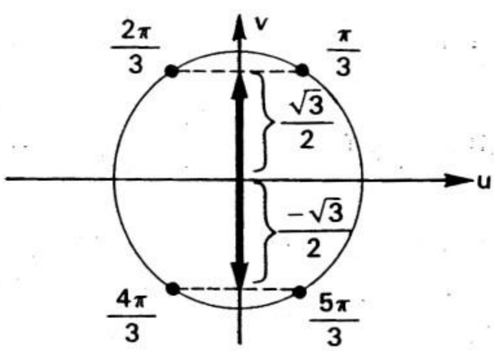
49. $\text{sen}\ \frac{\pi}{4} = \text{sen}\ \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\text{sen}\ \frac{5\pi}{4} = \text{sen}\ \frac{7\pi}{4} = \frac{-\sqrt{2}}{2}$



51. $\text{sen}\ \frac{\pi}{3} = \text{sen}\ \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\text{sen}\ \frac{4\pi}{3} = \text{sen}\ \frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$



52. a) $\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{3} + \sqrt{2}}{2}$

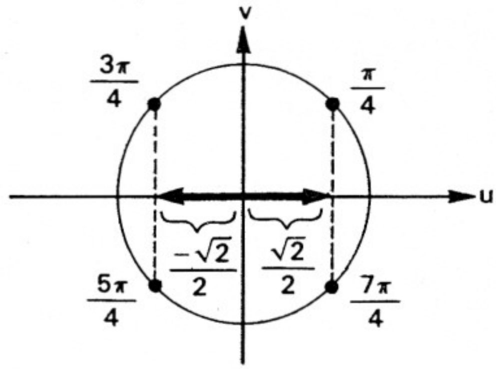
b) $2 \cdot \frac{1}{2} + \frac{1}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) = \frac{4 - \sqrt{2}}{4}$

c) $3(1) - 2\left(\frac{-\sqrt{2}}{2}\right) + \frac{1}{2} \cdot 0 = 3 + \sqrt{2}$

d) $-\frac{2}{3}(-1) + \frac{3}{5} \cdot \left(\frac{-\sqrt{3}}{2}\right) - \frac{6}{7}\left(-\frac{1}{2}\right) = \frac{230 - 63\sqrt{3}}{210}$

57. $\cos\frac{\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

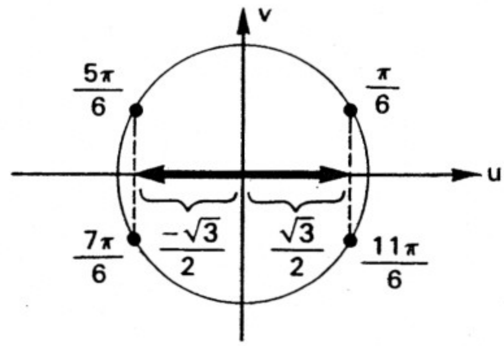
$\cos\frac{3\pi}{4} = \cos\frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$



58. $\cos\frac{\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = \frac{-\sqrt{3}}{2}$

$\cos\frac{7\pi}{6} = \cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$



60. a) $\frac{1}{2} + \frac{\sqrt{2}}{2} - 1 = \frac{-1 + \sqrt{2}}{2}$

b) $2 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{4\sqrt{3} + \sqrt{2}}{4}$

c) $3 \cdot 0 - 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}(-1) = \frac{2\sqrt{2} - 1}{2}$

d) $-\frac{2}{3} \cdot 0 + \frac{3}{5}\left(\frac{1}{2}\right) - \frac{6}{7}\left(-\frac{\sqrt{3}}{2}\right) = \frac{21 + 30\sqrt{3}}{70}$

63. $0° < 45° < 90° \Rightarrow (\text{sen } 45° > 0 \text{ e } \cos 45° > 0) \Rightarrow y_1 > 0$

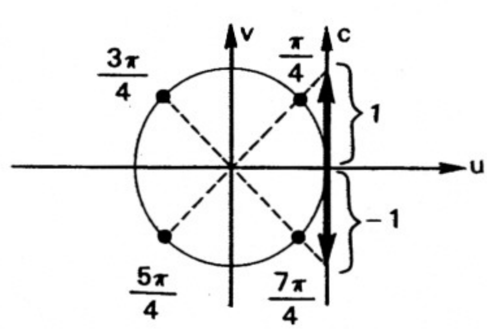
$180° < 225° < 270° \Rightarrow (\text{sen } 225° < 0 \text{ e } \cos 225° < 0) \Rightarrow y_2 < 0$

$\frac{3\pi}{2} < \frac{7\pi}{4} < 2\pi \Rightarrow \left(\text{sen }\frac{7\pi}{4} < 0 \text{ e } \cos\frac{7\pi}{4} > 0 \text{ e } \left|\text{sen }\frac{7\pi}{4}\right| = \left|\cos\frac{7\pi}{4}\right|\right) \Rightarrow y_3 = 0$

$270° < 300° < 315° < 360° \Rightarrow (|\text{sen } 300°| > |\cos 300°|;$
$\text{sen } 300° < 0 \text{ e } \cos 300° > 0) \Rightarrow y_4 < 0$

66. $\text{tg}\frac{\pi}{4} = \text{tg}\frac{5\pi}{4} = 1$

$\text{tg}\frac{3\pi}{4} = \text{tg}\frac{7\pi}{4} = -1$



69. a) $\sqrt{3} + 1 - 0 = 1 + \sqrt{3}$

b) $2 \cdot \dfrac{\sqrt{3}}{3} + \dfrac{1}{2}(-1) = \dfrac{4\sqrt{3} - 3}{6}$

c) $-2(1) + \dfrac{1}{2} \cdot (0) - \dfrac{1}{3}\left(-\dfrac{\sqrt{3}}{3}\right) = \dfrac{-18 + \sqrt{3}}{9}$

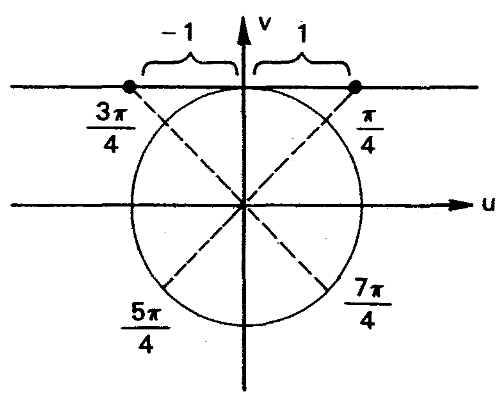
d) $\dfrac{3}{5}(-\sqrt{3}) - \dfrac{6}{7}\left(\dfrac{\sqrt{3}}{3}\right) - \dfrac{2}{3}(0) = \dfrac{-31\sqrt{3}}{35}$

71. a) $\left.\begin{array}{l} 180^\circ < 269^\circ < 270^\circ \Rightarrow \text{tg } 269^\circ > 0 \\ 90^\circ < 178^\circ < 180^\circ \Rightarrow \text{sen } 178^\circ > 0 \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} 0 < \dfrac{5\pi}{11} < \dfrac{\pi}{2} \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \text{tg } \dfrac{12\pi}{7} < 0 \end{array}\right\} \Rightarrow y_2 < 0$

74. $\text{cotg } \dfrac{\pi}{4} = \text{cotg } \dfrac{5\pi}{4} = 1$

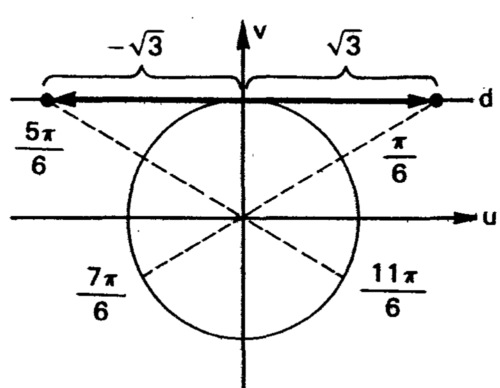
$\text{cotg } \dfrac{3\pi}{4} = \text{cotg } \dfrac{7\pi}{4} = -1$



75. $\text{cotg } \dfrac{\pi}{6} = \text{cotg } \dfrac{7\pi}{6} = \sqrt{3}$

$\text{cotg } \dfrac{5\pi}{6} = -\text{cotg } \dfrac{\pi}{6} = -\sqrt{3}$

$\text{cotg } \dfrac{5\pi}{6} = \text{cotg } \dfrac{11\pi}{6} = -\sqrt{3}$



77. a) $\dfrac{\sqrt{3}}{3} + 1 + \sqrt{3} = \dfrac{3 + 4\sqrt{3}}{3}$

b) $2\left(\dfrac{-\sqrt{3}}{3}\right) - \dfrac{1}{2}(-\sqrt{3}) = \dfrac{-\sqrt{3}}{6}$

c) $\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} - (-\sqrt{3}) + \sqrt{3} = \dfrac{5\sqrt{3} + \sqrt{2}}{2}$

d) $\dfrac{3}{5}\left(-\dfrac{\sqrt{3}}{3}\right) - \dfrac{6}{7} \cdot \sqrt{3} - \dfrac{2}{3}(-1) + \dfrac{4}{5} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{-111\sqrt{3} + 42\sqrt{2} + 70}{105}$

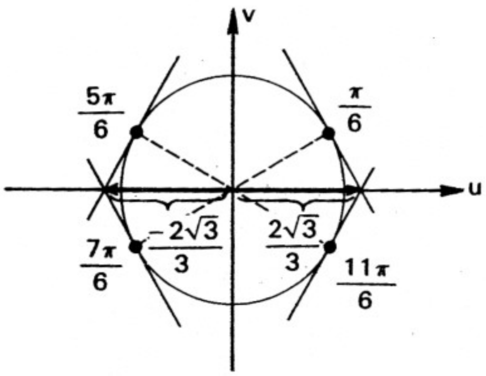
79. a) $\left.\begin{array}{l} 180^\circ < 269^\circ < 270^\circ \Rightarrow \text{cotg } 269^\circ > 0 \\ 90^\circ < 178^\circ < 180^\circ \Rightarrow \text{sen } 178^\circ > 0 \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} \dfrac{\pi}{2} < \dfrac{5\pi}{11} < \pi \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \text{cotg } \dfrac{12\pi}{7} < 0 \end{array}\right\} \Rightarrow y_2 < 0$

81. $\sec \dfrac{5\pi}{6} = -\sec \dfrac{\pi}{6} = \dfrac{-2\sqrt{3}}{3}$

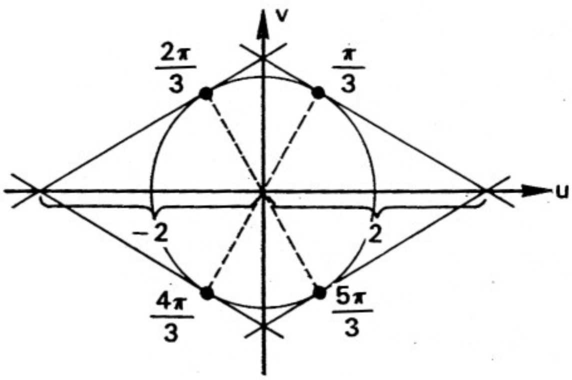
$\sec \dfrac{7\pi}{6} = -\sec \dfrac{\pi}{6} = \dfrac{-2\sqrt{3}}{3}$

$\sec \dfrac{11\pi}{6} = \sec \dfrac{\pi}{6} = \dfrac{2\sqrt{3}}{3}$



82. $\sec \dfrac{5\pi}{3} = \sec \dfrac{\pi}{3} = 2$

$\sec \dfrac{2\pi}{3} = \sec \dfrac{4\pi}{3} = -2$

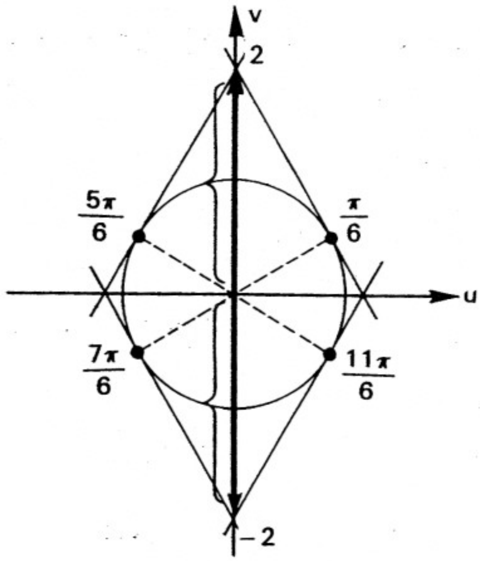


84. a) $\left.\begin{array}{l} 180^\circ < 269^\circ < 270^\circ \Rightarrow \sec 269^\circ < -1 \\ 90^\circ < 178^\circ < 180^\circ \Rightarrow 0 < \text{sen } 178^\circ < 1 \end{array}\right\} \Rightarrow y_1 < 0$

b) $\left.\begin{array}{l} \pi < \dfrac{5\pi}{11} < \dfrac{\pi}{2} \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \sec \dfrac{17\pi}{7} > 0 \end{array}\right\} \Rightarrow y_2 > 0$

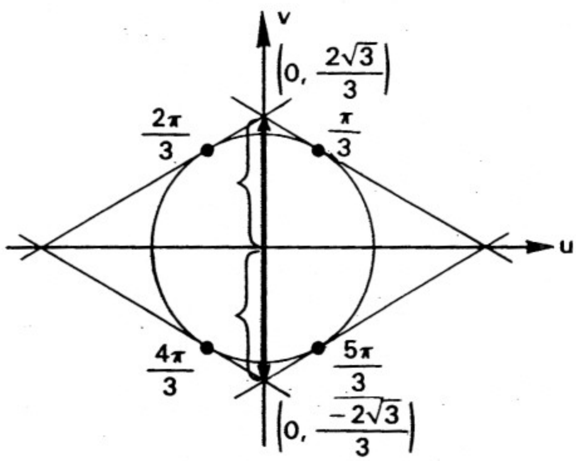
86. $\operatorname{cossec} \dfrac{5\pi}{6} = \operatorname{cossec} \dfrac{\pi}{6} = 2$

$\operatorname{cossec} \dfrac{7\pi}{6} = \operatorname{cossec} \dfrac{11\pi}{6} = -2$



87. $\operatorname{cossec} \dfrac{2\pi}{3} = \operatorname{cossec} \dfrac{\pi}{3} = \dfrac{2\sqrt{3}}{3}$

$\operatorname{cossec} \dfrac{4\pi}{3} = \operatorname{cossec} \dfrac{5\pi}{3} = \dfrac{-2\sqrt{3}}{3}$



89. a) $\left.\begin{array}{l} 90° < 91° < 180° \Rightarrow \cos 91° < 0 \text{ e } \operatorname{cossec} 91° > 0 \\ |\cos 91°| < |\operatorname{cossec} 91°| \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} 90° < 107° < 180° \Rightarrow \operatorname{sen} 107° > 0 \text{ e } \sec 107° < 0 \\ |\operatorname{sen} 107°| < |\sec 107°| \end{array}\right\} \Rightarrow y_2 < 0$

c) $\left.\begin{array}{l} 0 < \dfrac{\pi}{7} < \dfrac{\pi}{2} \Rightarrow \operatorname{cotg} \dfrac{\pi}{7} > 0 \\ \pi < \dfrac{7\pi}{6} < \dfrac{3\pi}{2} \Rightarrow \operatorname{tg} \dfrac{7\pi}{6} > 0 \\ \pi < \dfrac{9\pi}{8} < \dfrac{3\pi}{2} \Rightarrow \sec \dfrac{9\pi}{8} < 0 \end{array}\right\} \Rightarrow y_3 < 0$

90. $\left(2 + \dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2} - 2\right) = 1{,}25\,(\sqrt{2} - 4)$

Capítulo V – Relações fundamentais

92. $\text{cossec } x = \dfrac{-25}{24} \Rightarrow \text{sen } x = \dfrac{-24}{25}$

$\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \dfrac{-7}{25}$

e daí $\text{tg } x = \dfrac{24}{7}$, $\text{cotg } x = \dfrac{7}{24}$ e $\sec x = \dfrac{-25}{7}$

94. $\text{cotg } x = \dfrac{2\sqrt{m}}{m-1} \Rightarrow \text{tg } x = \dfrac{(m-1)\sqrt{m}}{2m}$

$\text{tg}^2 x + 1 = \sec^2 x \Rightarrow \sec x = \pm\dfrac{m+1}{2\sqrt{m}} \Rightarrow \cos x = \pm\dfrac{2\sqrt{m}}{m+1}$

95. $\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \pm\dfrac{(a^2-b^2)}{a^2+b^2} \Rightarrow \sec x = \pm\dfrac{(a^2+b^2)}{a^2-b^2}$

97. $\text{sen } x = \dfrac{1}{3} \Rightarrow \text{cossec } x = 3$

$$\left.\begin{aligned} \text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x &= \dfrac{2\sqrt{2}}{3} \\ \text{sen } x &= \dfrac{1}{3} \end{aligned}\right\} \Rightarrow \text{cotg } x = 2\sqrt{2}$$

$y = \dfrac{2 \text{ cossec } x}{\text{cossec}^2 x - \text{cotg}^2 x} = \dfrac{2 \cdot 3}{9-8} \Rightarrow y = 6$

99. $\cos x = \dfrac{2}{5} \Rightarrow \sec x = \dfrac{5}{2}$; $\sec^2 x = \text{tg}^2 x + 1 \Rightarrow \text{tg}^2 x = \dfrac{21}{4}$

$y = \left(1 + \dfrac{21}{4}\right)^2 + \left(1 - \dfrac{21}{4}\right)^2 \Rightarrow y = \dfrac{457}{8}$

101. $5 \sec x - 3(\sec^2 x - 1) = 1 \Rightarrow 3 \sec^2 x - 5 \sec x - 2 = 0$

e daí $\sec x = \dfrac{-1}{3} \Rightarrow \cos x = -3$ (não convém), $\sec x = 2 \Rightarrow \cos x = \dfrac{1}{2}$

$\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \text{sen } x = \pm\dfrac{\sqrt{3}}{2}$

102. $\text{sen}^2 x + \cos^2 x - 5 \text{ sen } x \cos x = 3 \Rightarrow -5 \text{ sen } x \cos x = 2 \Rightarrow$

$\Rightarrow \dfrac{-5 \text{ sen } x \cos x}{\cos^2 x} = \dfrac{2}{\cos^2 x} \Rightarrow -5 \text{ tg } x = 2(1 + \text{tg}^2 x) \Rightarrow$

$2 \text{ tg}^2 x + 5 \text{ tg } x + 2 = 0 \Rightarrow \text{tg } x = -2$ ou $\text{tg } x = \dfrac{-1}{2}$

104. $\text{cotg}\, x = \dfrac{1}{\text{tg}\, x} \Rightarrow \dfrac{m}{3} = \dfrac{1}{m-2} \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow m = 3$ ou $m = -1$

105. $\text{cossec}\, x = \dfrac{a+1}{\sqrt{a+2}} \Rightarrow \text{sen}\, x = \dfrac{\sqrt{a+2}}{a+1}$

$\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \left(\dfrac{\sqrt{a+2}}{a+1}\right)^2 + \left(\dfrac{1}{a+1}\right)^2 = 1 \Rightarrow a^2 + a - 2 = 0 \Rightarrow a = 1$

108. $(\text{sen}\, x + \cos x)^2 = 1 + 2\, \text{sen}\, x \cos x \Rightarrow a^2 = 1 + 2b \Rightarrow a^2 - 2b = 1$

110. $\text{sen}\, x + \cos x = a \Rightarrow \text{sen}^2 x + 2 \cdot \text{sen}\, x \cdot \cos x + \cos^2 x = \alpha^2 \Rightarrow$

$\Rightarrow \text{sen}\, x \cdot \cos x = \dfrac{a^2 - 1}{2}$

$y = (\text{sen}\, x + \cos x)(\text{sen}^2 x - \text{sen}\, x \cdot \cos x + \cos^2 x) =$

$= a \cdot \left(1 - \dfrac{a^2-1}{2}\right) = \dfrac{a(3-a^2)}{2}$

Capítulo VI — Arcos notáveis

112. $\ell_8 = \sqrt{1(2 - \sqrt{4-2})} = \sqrt{2 - \sqrt{2}}$; $\text{sen}\, \dfrac{\pi}{8} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

$\text{sen}^2 \dfrac{\pi}{8} + \cos^2 \dfrac{\pi}{8} = 1 \Rightarrow \cos \dfrac{\pi}{8} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$; $\text{tg}\, \dfrac{\pi}{8} = \sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}} \Rightarrow$

$\Rightarrow \text{tg}\, \dfrac{\pi}{8} = -1 + \sqrt{2}$

113. $\text{sen}\, \dfrac{\pi}{5} = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$ e $\text{sen}^2 \dfrac{\pi}{5} + \cos^2 \dfrac{\pi}{5} = 1 \Rightarrow \cos \dfrac{\pi}{5} = \dfrac{\sqrt{6+2\sqrt{5}}}{4}$

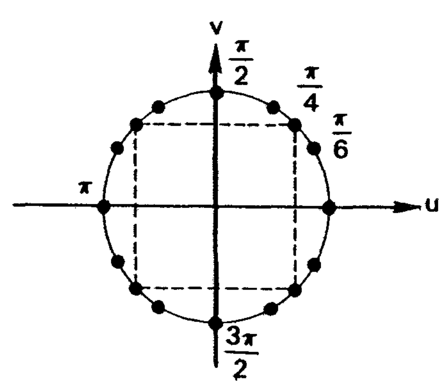
$\text{tg}\, \dfrac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}$; $\text{sen}\, \dfrac{\pi}{10} = \dfrac{\sqrt{5}-1}{4}$ e $\text{sen}^2 \dfrac{\pi}{10} + \cos^2 \dfrac{\pi}{10} = 1 \Rightarrow$

$\cos \dfrac{\pi}{10} = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$; $\text{tg}\, \dfrac{\pi}{10} = \dfrac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}} = \dfrac{\sqrt{25 - 10\sqrt{5}}}{5}$

115. $A = \left\{0, \dfrac{1}{2}, \dfrac{\sqrt{3}}{2}, 1, \dfrac{-1}{2}, \dfrac{-\sqrt{3}}{2}, -1\right\}$

$B = \left\{1, \dfrac{\sqrt{2}}{2}, 0, -\dfrac{\sqrt{2}}{2}, -1\right\}$

$A \cap B = \{-1, 0, 1\}$



Capítulo VII – Redução ao 1º quadrante

118. $\operatorname{sen}\left(x + \frac{\pi}{2}\right) = \operatorname{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{3}{5}$

119. a) $\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

b) $\cos\left(x + \frac{\pi}{2}\right) = -\cos\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = -\cos\left(\frac{\pi}{2} - x\right) = -\operatorname{sen} x = -\frac{1}{2}$

c) $\operatorname{sen}\left(x + \frac{\pi}{2}\right) = \operatorname{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{\sqrt{3}}{2}$

d) $\operatorname{tg}\left(x + \frac{\pi}{2}\right) = \dfrac{\operatorname{sen}\left(x + \frac{\pi}{2}\right)}{\cos\left(x + \frac{\pi}{2}\right)} = -\sqrt{3}$

e) $\operatorname{cotg}\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\operatorname{tg}\left(x + \frac{\pi}{2}\right)} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

f) $\sec\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\cos\left(x + \frac{\pi}{2}\right)} = -2$

g) $\operatorname{cossec}\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\operatorname{sen}\left(x + \frac{\pi}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

120. a) $[\operatorname{sen} x + \operatorname{sen} x][\operatorname{cotg} x + \operatorname{cotg} x] = 2 \operatorname{sen} x \cdot 2 \operatorname{cotg} x =$

$4 \operatorname{sen} x \cdot \dfrac{\cos x}{\operatorname{sen} x} = 4 \cos x.$

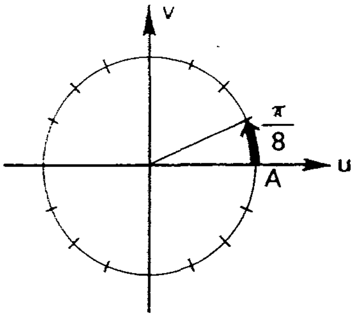
b) $\dfrac{\operatorname{tg} x - \sec x}{[\operatorname{tg} x + \operatorname{cossec} x] \cdot \operatorname{sen} x} = \dfrac{\dfrac{\operatorname{sen} x - 1}{\cos x}}{\left[\dfrac{\operatorname{sen}^2 x + \cos x}{\cos x \cdot \operatorname{sen} x}\right] \cdot \operatorname{sen} x} = \dfrac{\operatorname{sen} x - 1}{\operatorname{sen}^2 x + \cos x}$

121. $\dfrac{\operatorname{sen} x - \operatorname{sen} x + \operatorname{tg} x}{-\operatorname{tg} x - \cos x + \cos x} = \dfrac{\operatorname{tg} x}{-\operatorname{tg} x} = -1$

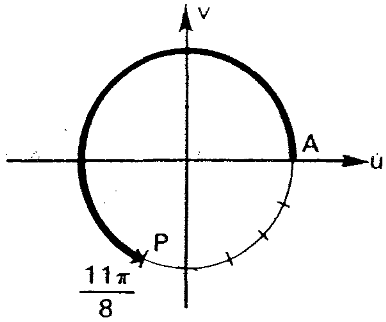
122. $\dfrac{-\operatorname{sen} x - \cos x + \cos x + 3 \operatorname{sen} x}{-\cos x - \cos x - \operatorname{sen} x + \operatorname{sen} x} = \dfrac{2 \operatorname{sen} x}{-2 \cos x} = -\operatorname{tg} x$

Capítulo VIII – Funções circulares

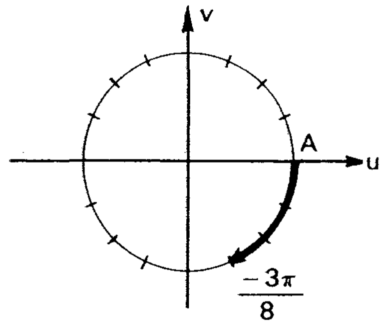
124.



$\widehat{AP} = \frac{1}{16}$ do ciclo, no sentido anti-horário

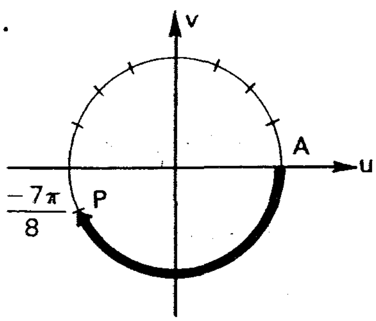


$\widehat{AP} = \frac{11}{16}$ do ciclo, no sentido anti-horário

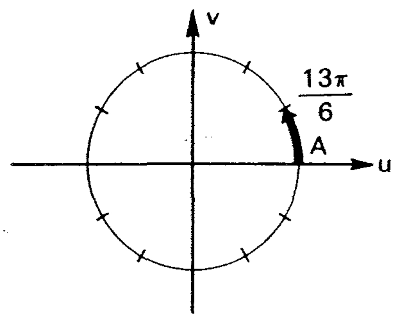


$\widehat{AP} = \frac{3}{16}$ do ciclo, no sentido horário

125.

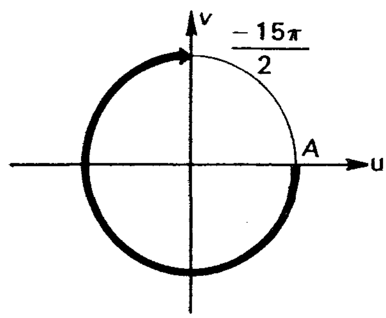


$\widehat{AP} = \frac{7}{16}$ do ciclo, no sentido horário



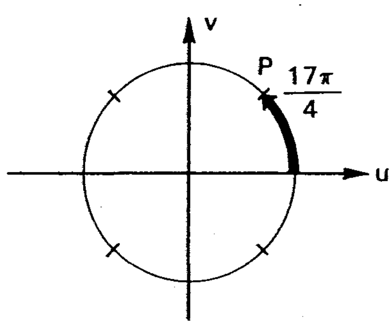
$\frac{13\pi}{6}$ e $\frac{\pi}{6}$ são côngruos

$\widehat{AP} = \frac{1}{12}$ do ciclo, no sentido anti-horário



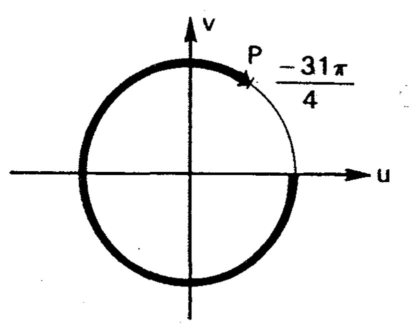
$\frac{-15\pi}{2}$ e $\frac{-3\pi}{2}$ são côngruos

$\widehat{AP} = \frac{3}{4}$ do ciclo, no sentido horário



$\frac{17\pi}{4}$ e $\frac{\pi}{4}$ são côngruos

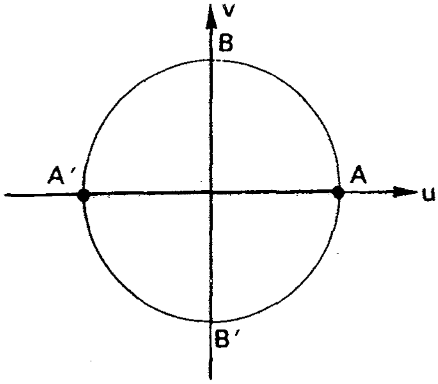
$\widehat{AP}$ é $\frac{1}{8}$ do ciclo, no sentido anti-horário

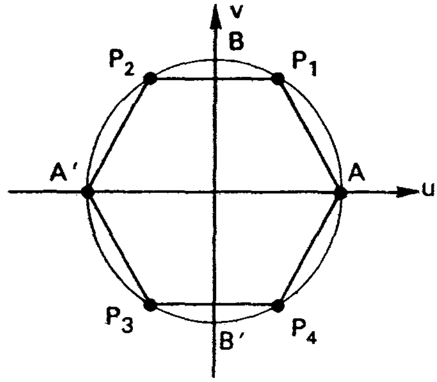


$\frac{-31\pi}{4}$ e $\frac{-7\pi}{4}$ são côngruos

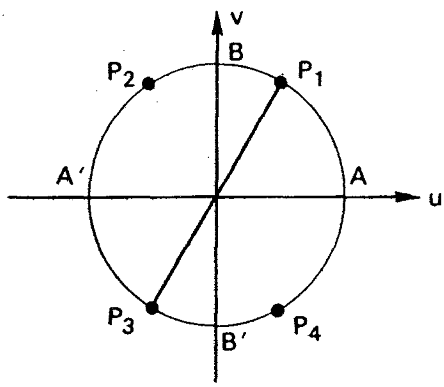
$\widehat{AP}$ é $\frac{7}{8}$ do ciclo, no sentido horário

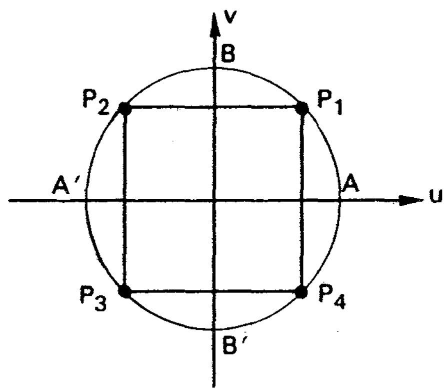
126. $E = \{A', A\}$ $F = \{A, P_1, P_2, A', P_3, P_4\}$





$G = \{P_1, P_3\}$ $H = \{P_1, P_2, P_3, P_4\}$





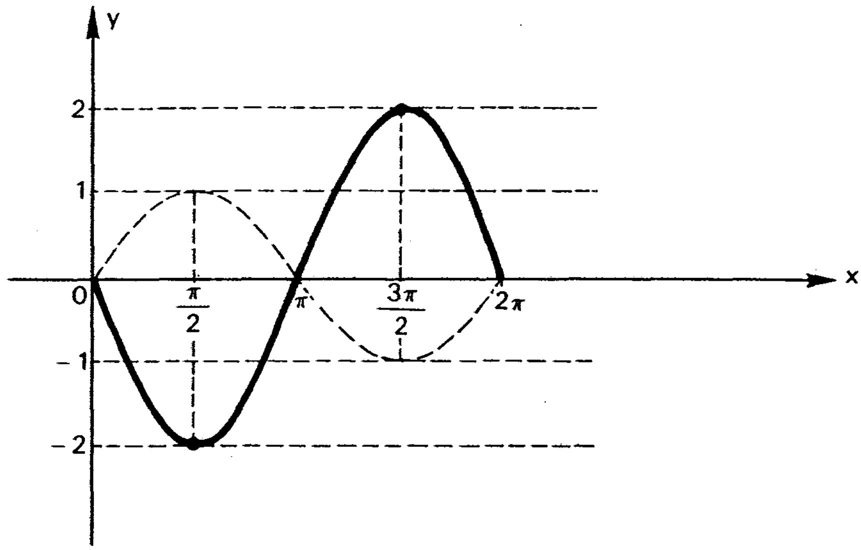
127. a) $830° = 2(360°) + 110° \Rightarrow \text{sen } 830° = \text{sen } 110° = \text{sen } 70°$
$1195° = 3(360°) + 115° \Rightarrow \text{sen } 1195° = \text{sen } 115° = \text{sen } 65°$
$0° < x < 90° \Rightarrow \text{sen } x$ é crescente $\Rightarrow \text{sen } 830° > \text{sen } 1195°$

b) $-535° = -1360° - 175° \Rightarrow \cos(-535°) = \cos(-175°) = -\cos 5°$
$\cos 190° = -\cos 10°$; $0° < x < 90° \Rightarrow \cos x$ é decrescente $\Rightarrow$
$\Rightarrow \cos 5° > \cos 10° \Rightarrow \cos 190° > \cos(-535°)$

130.

x	sen x	y = −2 sen x
0	0	0
$\frac{\pi}{2}$	1	−2
π	0	0
$\frac{3\pi}{2}$	−1	2
2π	0	0

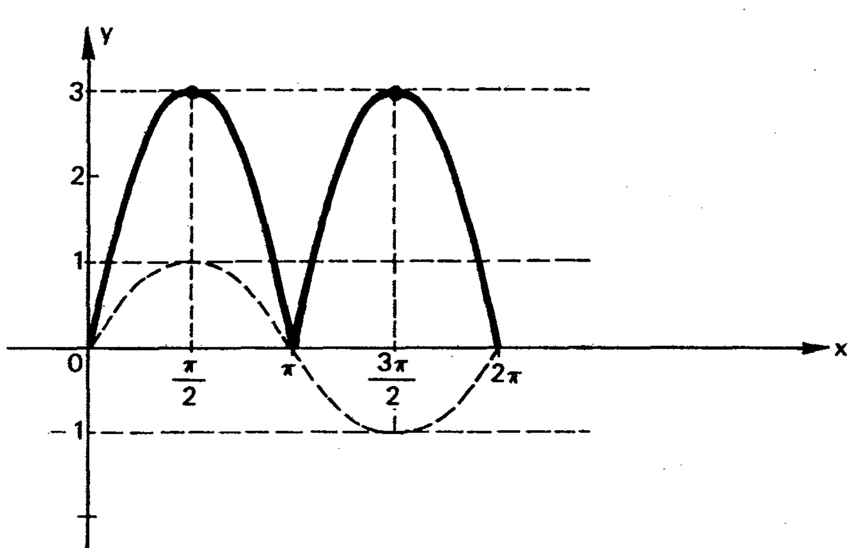
$Im(f) = [-2, 2]$
$p = 2\pi$



132.

x	sen x	y = | 3 sen x |
0	0	0
$\frac{\pi}{2}$	1	3
π	0	0
$\frac{3\pi}{2}$	−1	3
2π	0	0

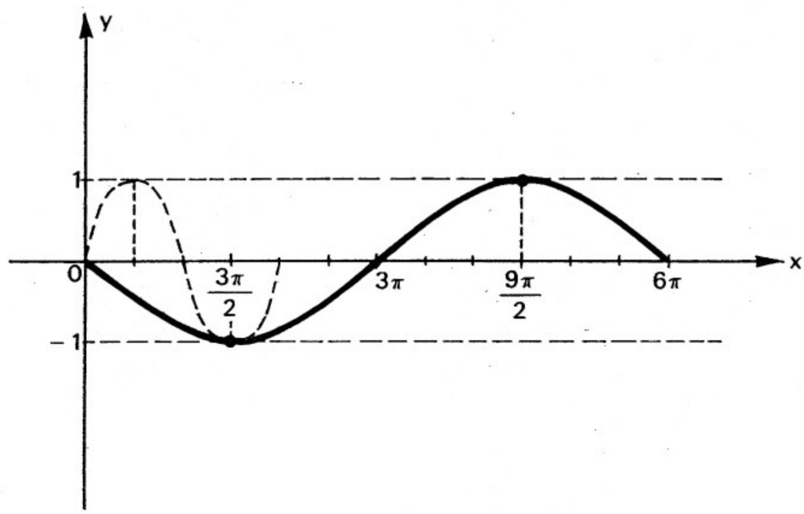
$\text{Im}(f) = [0, 3]$
$p = \pi$



136.

x	$t = \frac{x}{3}$	$y = -\text{sen } t$
0	0	0
$\frac{3\pi}{2}$	$\frac{\pi}{2}$	-1
3π	π	0
$\frac{9\pi}{2}$	$\frac{3\pi}{2}$	1
6π	2π	0

$Im(f) = [-1, 1]$
$p = 6\pi$

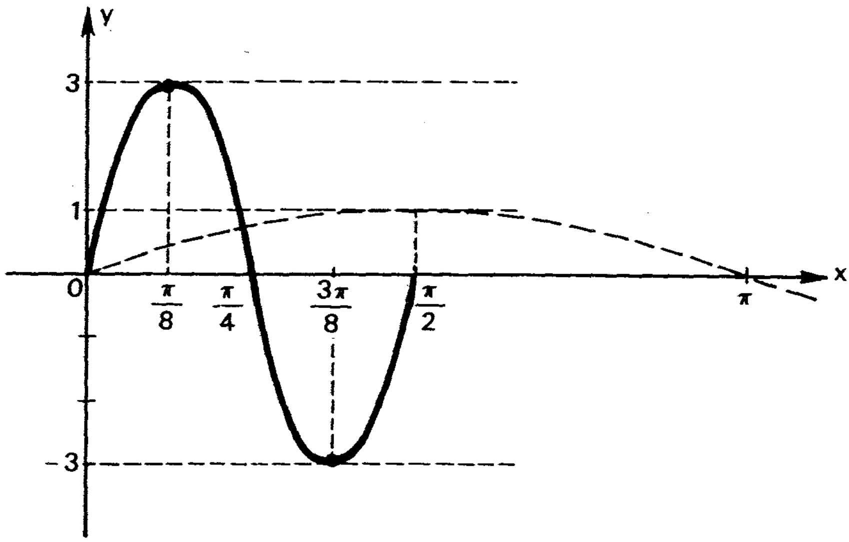


137.

x	$t = 4x$	$y = 3 \text{ sen } t$
0	0	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	3
$\frac{\pi}{4}$	π	0
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	-3
$\frac{\pi}{2}$	2π	0

$Im(f) = [-3, 3]$

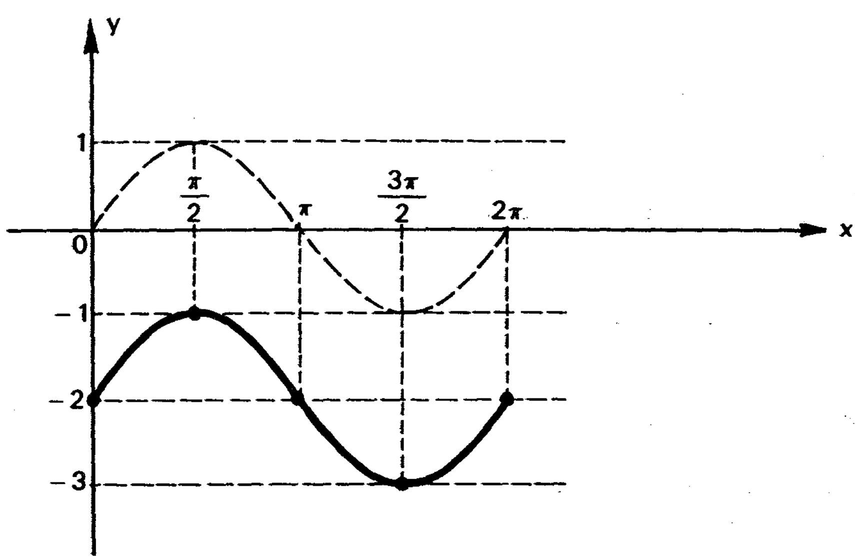
$p = \frac{\pi}{2}$



139.

x	sen x	y = −2 + sen x
0	0	−2
$\frac{\pi}{2}$	1	−1
π	0	−2
$\frac{3\pi}{2}$	−1	−3
2π	0	−2

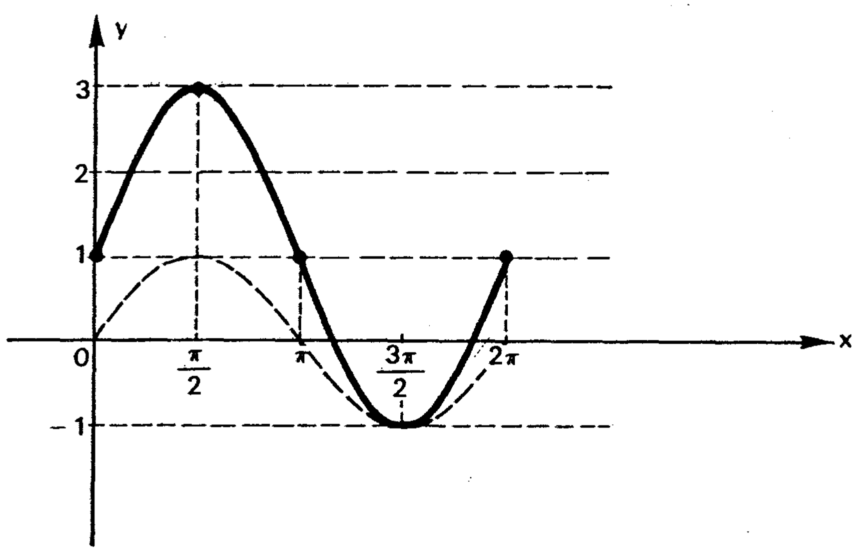
$Im(f) = [-3, -1]$
$p = 2\pi$



140.

x	sen x	y = 1 + 2 sen x
0	0	1
$\frac{\pi}{2}$	1	3
π	0	1
$\frac{3\pi}{2}$	-1	-1
2π	0	1

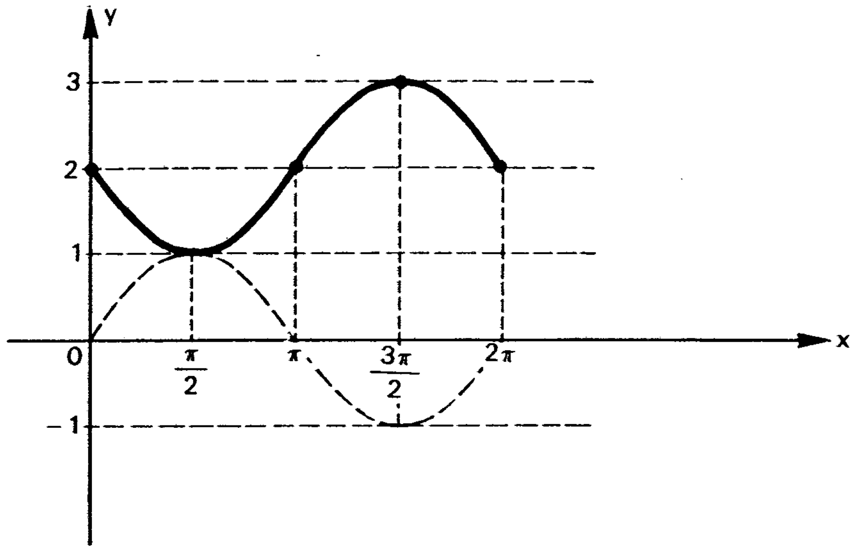
$Im(f) = [-1, 3]$
$p = 2\pi$



141.

x	sen x	y = 2 − sen x
0	0	2
$\frac{\pi}{2}$	1	1
π	0	2
$\frac{3\pi}{2}$	-1	3
2π	0	2

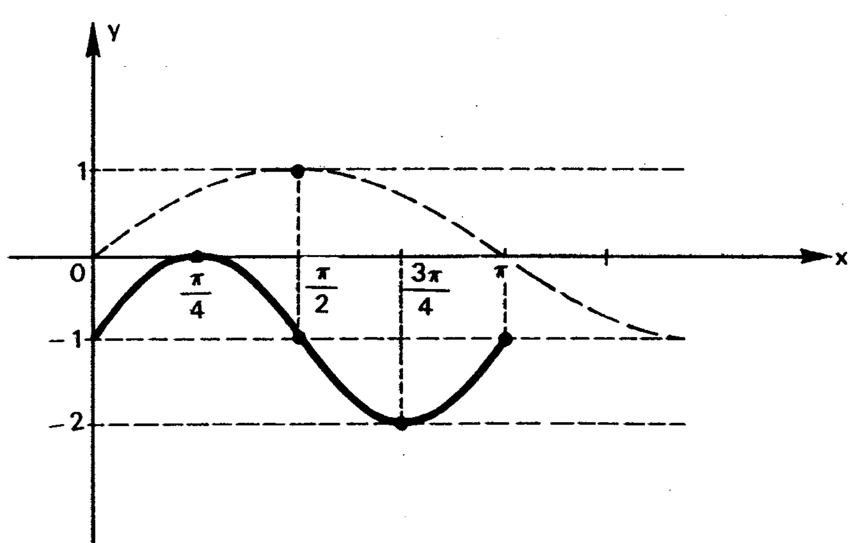
$Im(f) = [1, 3]$
$p = 2\pi$



142.

x	t = 2x	sen t	y = −1 + sen t
0	0	0	−1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1	0
$\frac{\pi}{2}$	π	0	−1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	−1	−2
π	2π	0	−1

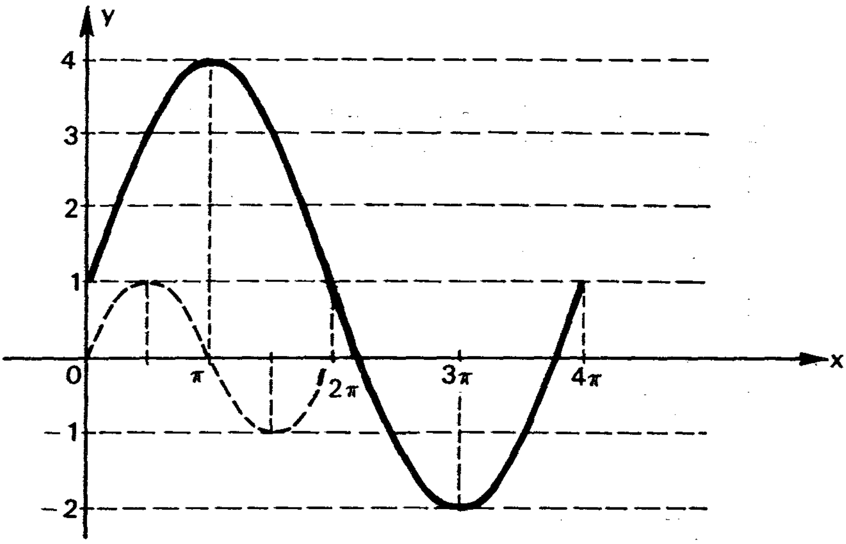
$Im(f) = [-2, 0]$
$p = \pi$



143.

x	$t = \frac{x}{2}$	sen t	y = 1 + 3 sen t
0	0	0	1
π	$\frac{\pi}{2}$	1	4
2π	π	0	1
3π	$\frac{3\pi}{2}$	-1	-2
4π	2π	0	1

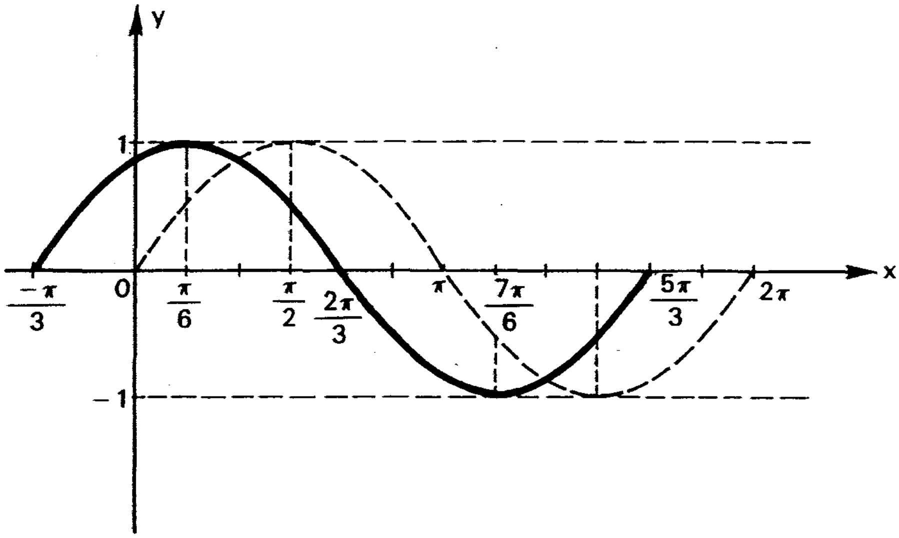
Im(f) = $[-2, 4]$
p = 4π



145.

x	$t = x + \frac{\pi}{3}$	y = sen t
$\frac{-\pi}{3}$	0	0
$\frac{\pi}{6}$	$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	π	0
$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	2π	0

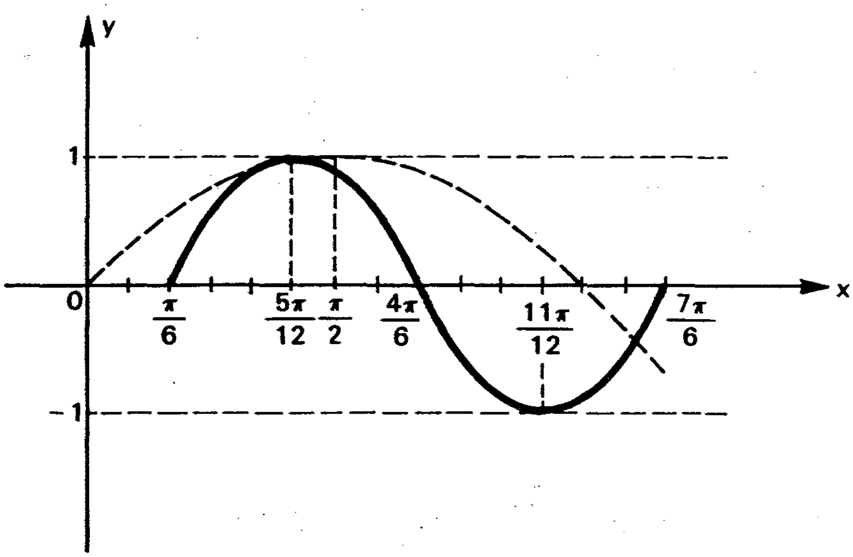
Im(f) = $[-1, 1]$
p = 2π



146.

x	$t = 2x - \frac{\pi}{3}$	y = sen t
$\frac{-\pi}{6}$	0	0
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1
$\frac{4\pi}{6}$	π	0
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	−1
$\frac{7\pi}{6}$	2π	0

$Im(f) = [-1, 1]$
$p = \pi$

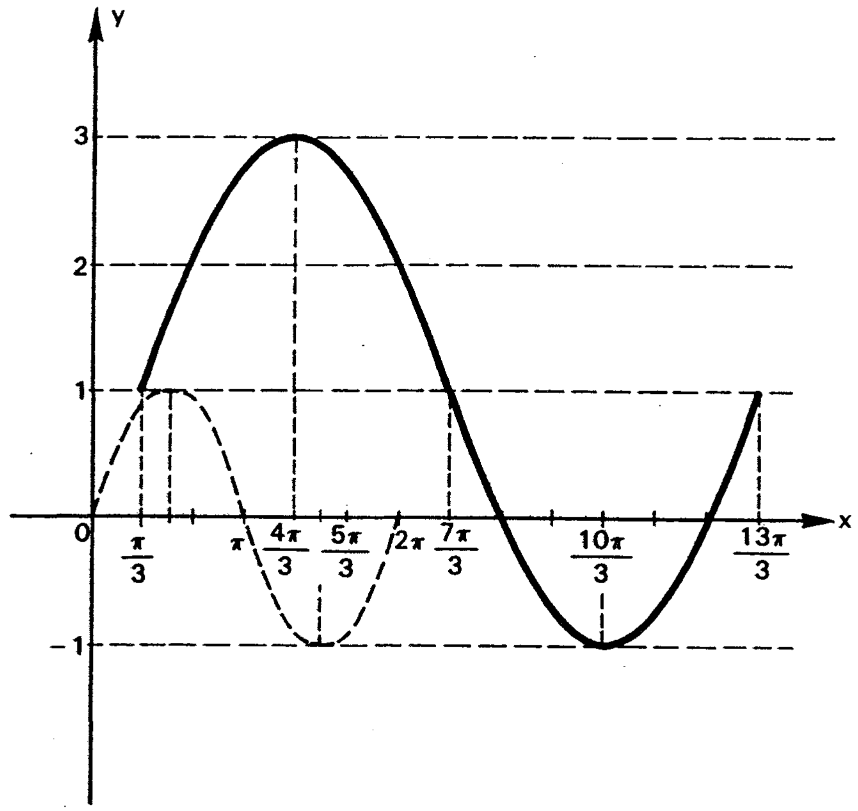


147.

x	$t = \frac{x}{2} - \frac{\pi}{6}$	sen t	y = 1 + 2 sen t
$\frac{\pi}{3}$	0	0	1
$\frac{4\pi}{3}$	$\frac{\pi}{2}$	1	3
$\frac{7\pi}{3}$	π	0	1
$\frac{10\pi}{3}$	$\frac{3\pi}{2}$	-1	-1
$\frac{13\pi}{3}$	2π	0	1

$\text{Im}(f) = [-1, 3]$

$p = 4\pi$

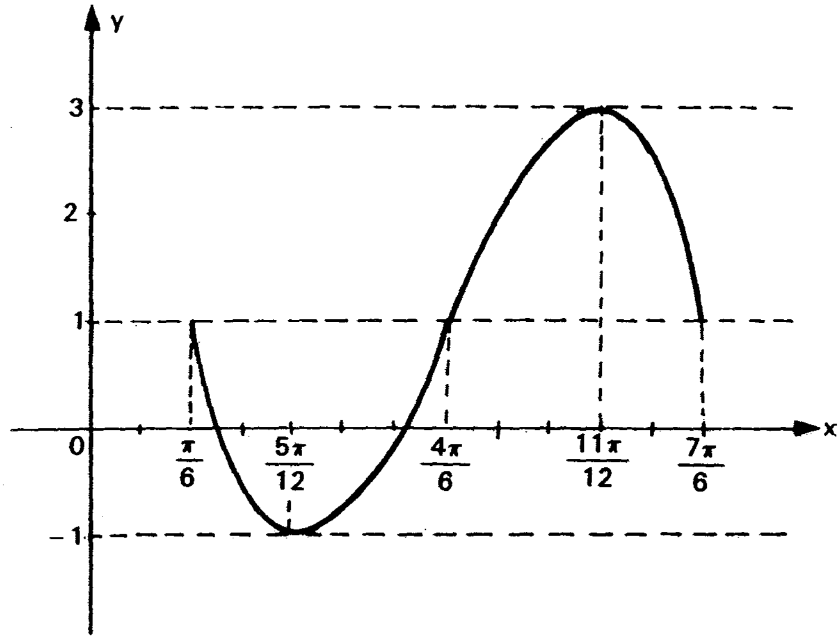


149. $t = 2\pi x + \frac{\pi}{2} \Rightarrow p = \frac{2\pi}{c} = \frac{2\pi}{2\pi} \Rightarrow p = 1$

150.

x	$t = 2x - \frac{\pi}{3}$	sen t	$y = 1 - 2\,\text{sen}\,t$
$\frac{\pi}{6}$	0	0	1
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1	-1
$\frac{4\pi}{6}$	π	0	1
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	-1	3
$\frac{7\pi}{6}$	2π	0	1

$\text{Im}(f) = [-1, 3]$
$p = \pi$



152. a) $-1 \leqslant 2 - 5m \leqslant 1 \Rightarrow -3 \leqslant -5m \leqslant -1 \Rightarrow \frac{1}{5} \leqslant m \leqslant \frac{3}{5}$

b) $\frac{m-1}{m-2} \geqslant -1 \Rightarrow \frac{2m-3}{m-2} \geqslant 0 \Rightarrow m \leqslant \frac{3}{2}$ ou $m > 2$ (A)

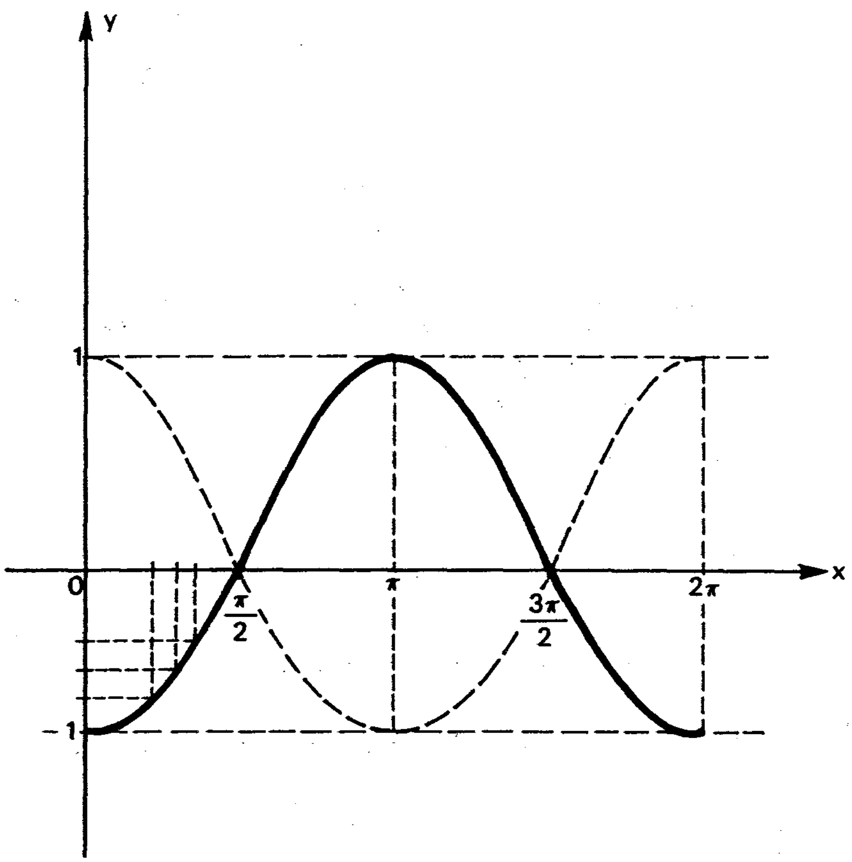
$\frac{m-1}{m-2} \leqslant 1 \Rightarrow \frac{1}{m-2} \leqslant 0 \Rightarrow m < 2$ (B)

Fazendo a interseção de (A) com (B), vem $m \leqslant \frac{3}{2}$.

153.

x	cos x	y = − cos x
0	1	−1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{-1}{2}$
$\frac{\pi}{2}$	0	0
π	−1	1
$\frac{3\pi}{2}$	0	0
2π	1	−1

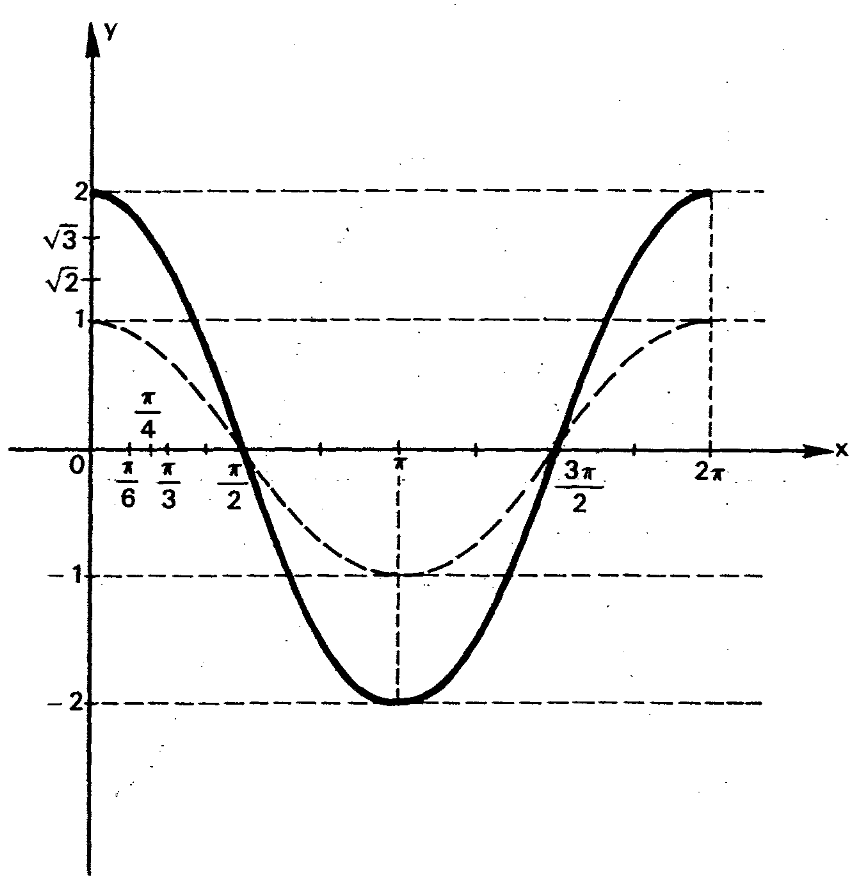
$\mathrm{Im}(f) = [-1, 1]$
$p = 2\pi$



154.

x	cos x	y = 2 cos x
0	1	2
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	1
$\frac{\pi}{2}$	0	0
π	-1	-2
$\frac{3\pi}{2}$	0	0
2π	1	2

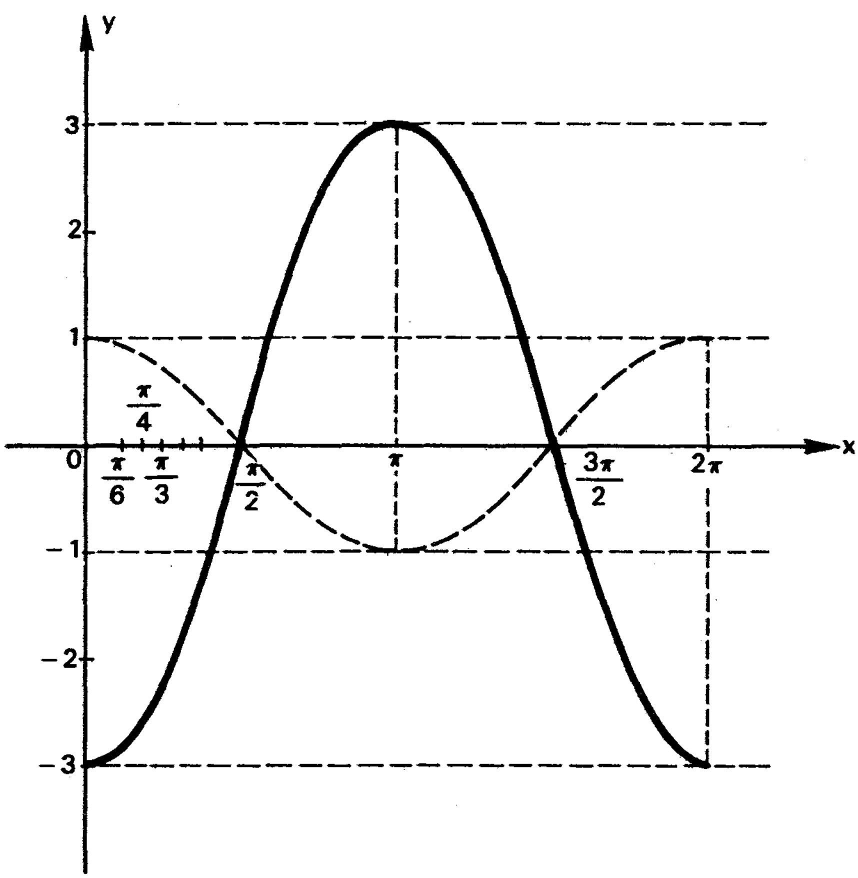
$\text{Im}(f) = [-2, 2]$
$p = 2\pi$



155.

x	$\cos x$	$y = -3\cos x$
0	1	-3
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{-3\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{-3\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{-3}{2}$
$\frac{\pi}{2}$	0	0
π	-1	3
$\frac{3\pi}{2}$	0	0
2π	1	-3

$\mathrm{Im}(f) = [-3, 3]$
$p = 2\pi$

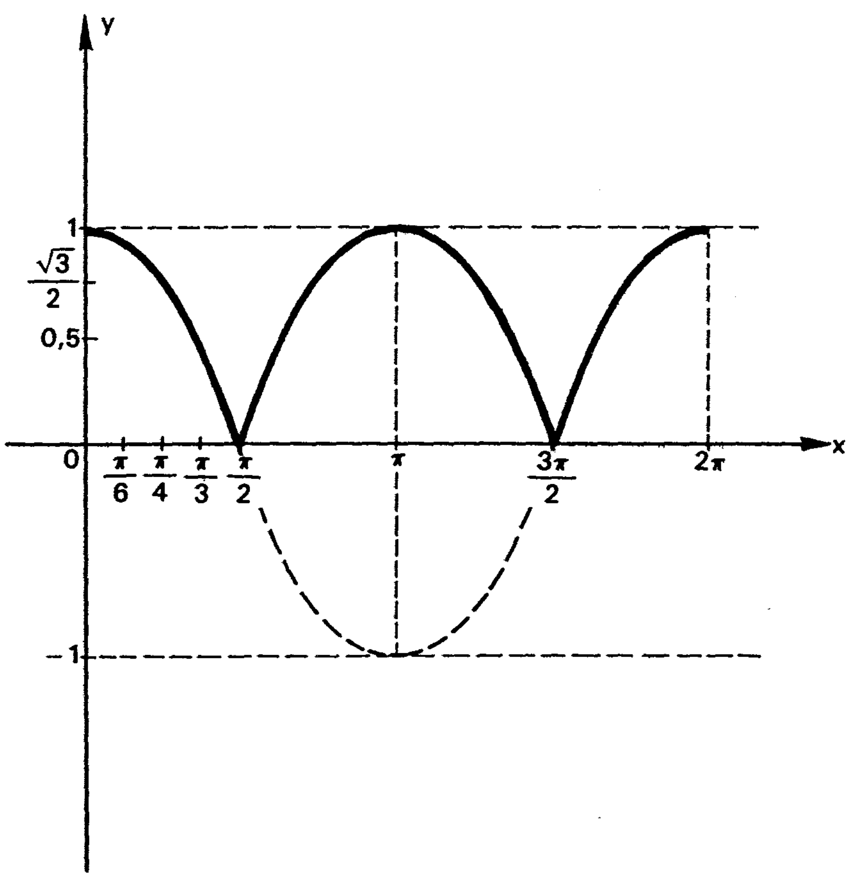


156.

x	$\cos x$	$y = \|\cos x\|$
0	1	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0	0
π	-1	1
$\frac{3\pi}{2}$	0	0
2π	1	1

$\mathrm{Im}(f) = [0, 1]$

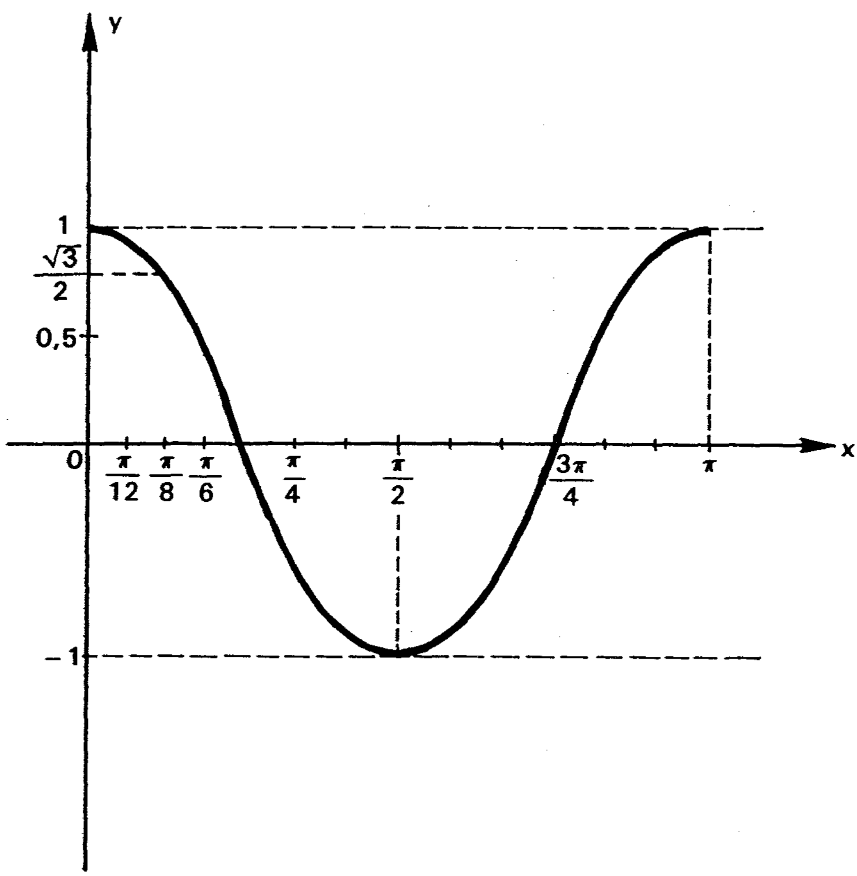
$p = \pi$



157.

x	t = 2x	y = cos t
0	0	1
$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1

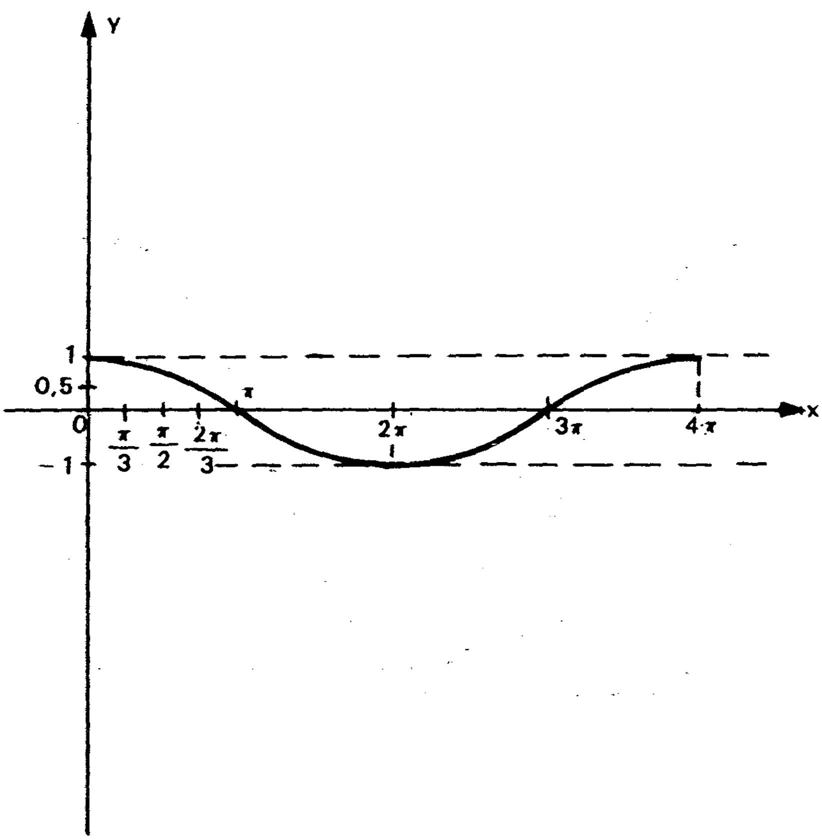
$\mathrm{Im}(f) = [-1, 1]$
$p = \pi$



158.

x	$t = \frac{x}{2}$	$y = \cos t$
0	0	1
$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{2\pi}{3}$	$\frac{\pi}{3}$	$\frac{1}{2}$
π	$\frac{\pi}{2}$	0
2π	π	-1
3π	$\frac{3\pi}{2}$	0
4π	2π	1

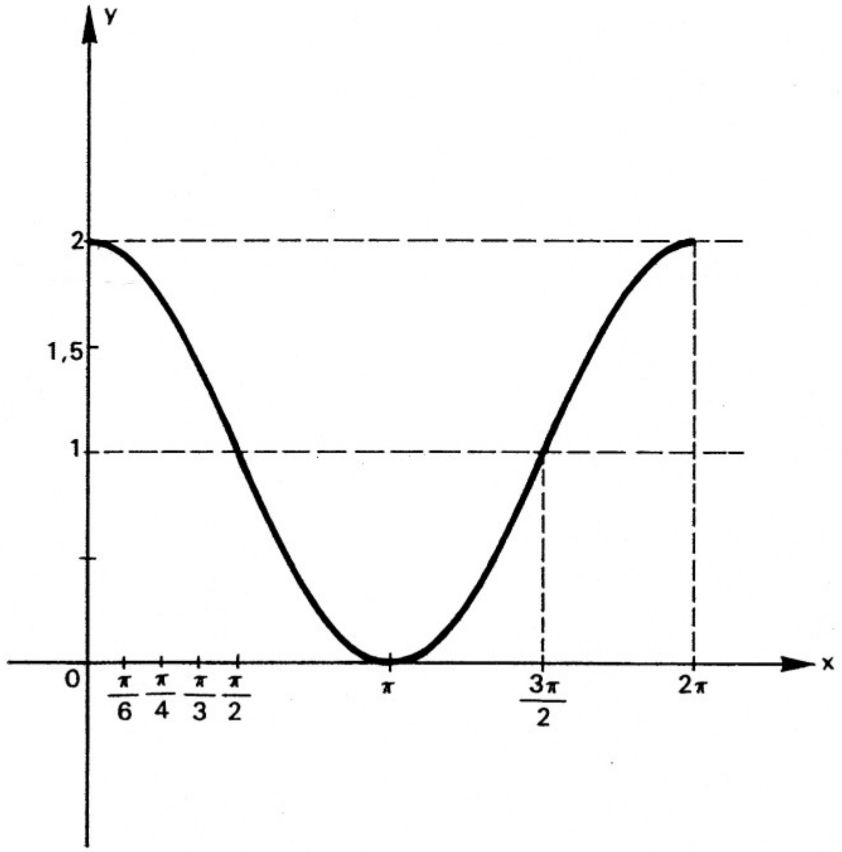
$\mathrm{Im}(f) = [-1, 1]$
$p = 4\pi$



159.

x	cos x	y = 1 + cos x
0	1	2
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$1 + \frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$1 + \frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	1,5
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	1
2π	1	2

$\text{Im}(f) = [0, 2]$
$p = 2\pi$

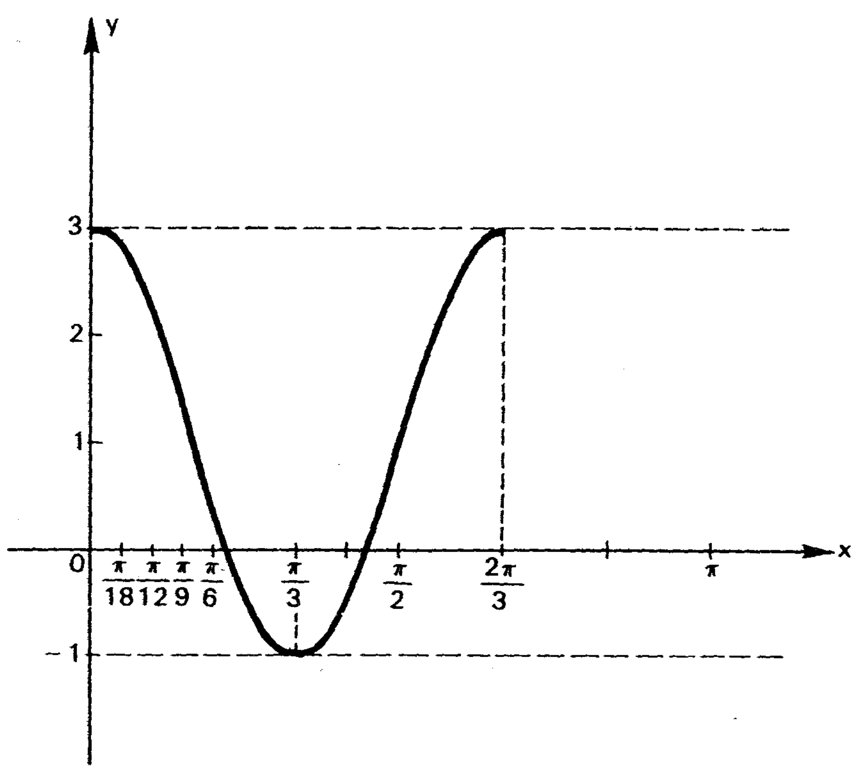


160.

x	$t = 3x$	$\cos t$	$y = 1 + 2\cos t$
0	0	1	3
$\frac{\pi}{18}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$1 + \sqrt{3}$
$\frac{\pi}{12}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$1 + \sqrt{2}$
$\frac{\pi}{9}$	$\frac{\pi}{3}$	$\frac{1}{2}$	2
$\frac{\pi}{6}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{3}$	π	-1	-1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	0	1
$\frac{2\pi}{3}$	2π	1	3

$\text{Im}(f) = [-1, 3]$

$p = \frac{2\pi}{3}$

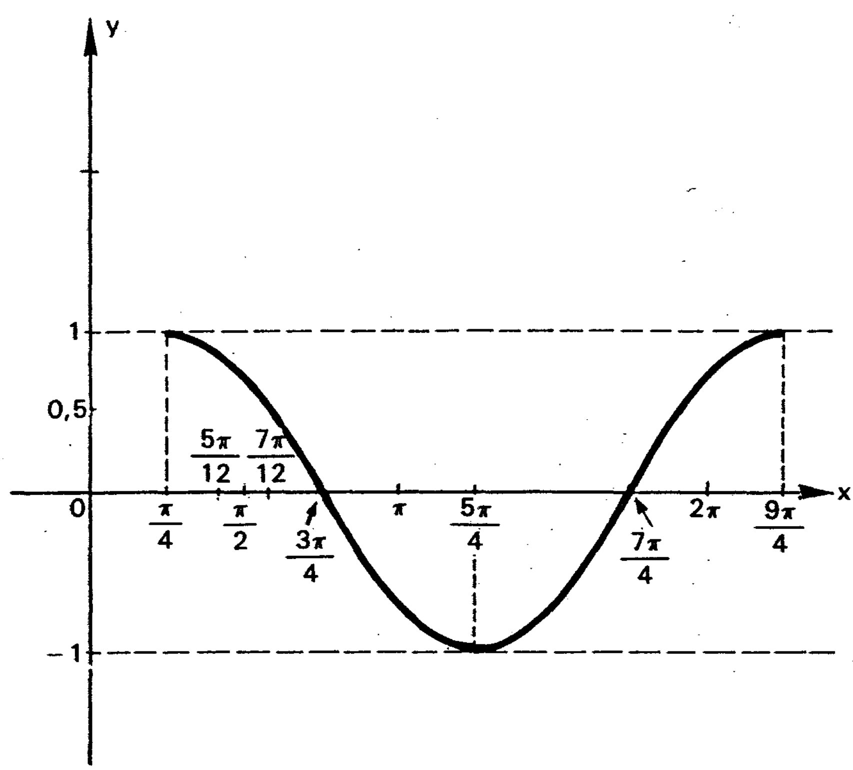


161.

x	$t = x - \frac{\pi}{4}$	$y = \cos t$
$\frac{\pi}{4}$	0	1
$\frac{5\pi}{12}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{7\pi}{12}$	$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{3\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{5\pi}{4}$	π	-1
$\frac{7\pi}{4}$	$\frac{3\pi}{2}$	0
$\frac{9\pi}{4}$	2π	1

$\mathrm{Im}(f) = [-1, 1]$

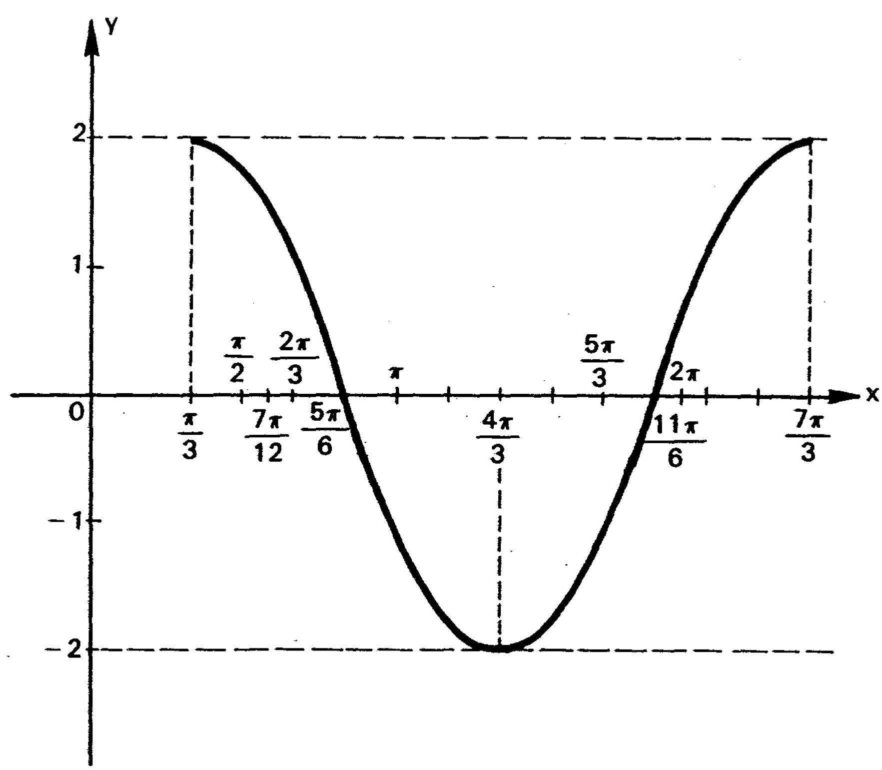
$p = \frac{9\pi}{4} - \frac{\pi}{4} = 2\pi$



162.

x	$t = x - \frac{\pi}{3}$	$y = 2 \cos t$
$\frac{\pi}{3}$	0	2
$\frac{\pi}{2}$	$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{7\pi}{12}$	$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{2\pi}{3}$	$\frac{\pi}{3}$	1
$\frac{5\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{4\pi}{3}$	π	-2
$\frac{11\pi}{6}$	$\frac{3\pi}{2}$	0
$\frac{7\pi}{3}$	2π	2

$\mathrm{Im}(f) = [-2, 2]$
$p = 2\pi$

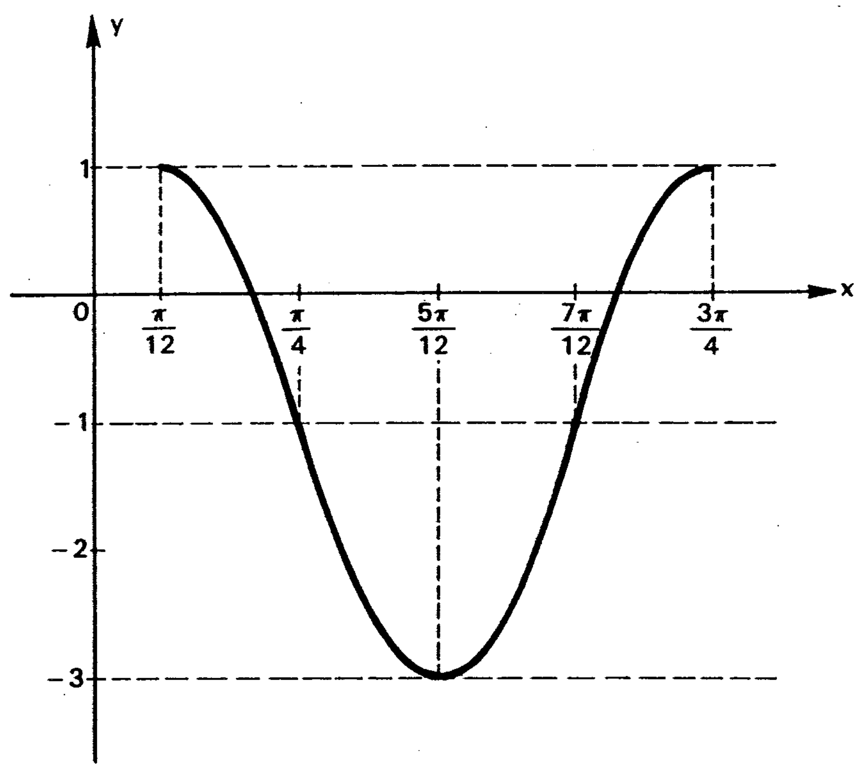


163.

x	$t = 3x - \frac{\pi}{4}$	$\cos t$	$y = -1 + 2\cos t$
$\frac{\pi}{12}$	0	1	1
$\frac{5\pi}{36}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-1 + \sqrt{3}$
$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-1 + \sqrt{2}$
$\frac{7\pi}{36}$	$\frac{\pi}{3}$	$\frac{1}{2}$	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0	-1
$\frac{5\pi}{12}$	π	-1	-3
$\frac{7\pi}{12}$	$\frac{3\pi}{2}$	0	-1
$\frac{3\pi}{4}$	2π	1	1

$\mathrm{Im}(f) = [-3, 1]$

$p = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$



164. $\dfrac{t+2}{2t-1} \geqslant -1 \Rightarrow \dfrac{3t+1}{2t-1} \geqslant 0 \Rightarrow t \leqslant \dfrac{-1}{3}$ ou $t > \dfrac{1}{2}$ (A)

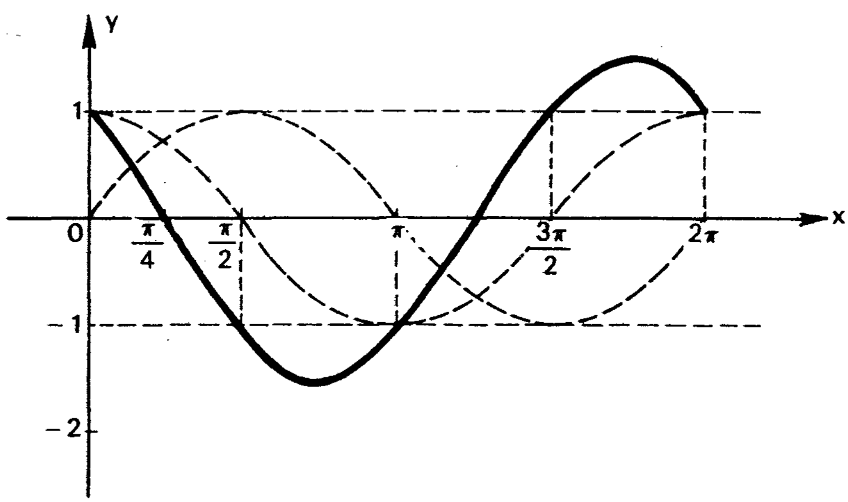
$\dfrac{t+2}{2t-1} \leqslant 1 \Rightarrow \dfrac{-t+3}{2t-1} \leqslant 0 \Rightarrow t < \dfrac{1}{2}$ ou $t \geqslant 3$ (B)

Fazendo a interseção de (A) com (B) vem $t \leqslant -\dfrac{1}{3}$ ou $t \geqslant 3$.

166.

x	cos x	sen x	y = cos x − sen x
0	1	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0
$\frac{\pi}{2}$	0	1	−1
π	−1	0	−1
$\frac{3\pi}{2}$	0	−1	1
2π	1	0	1

$p = 2\pi$



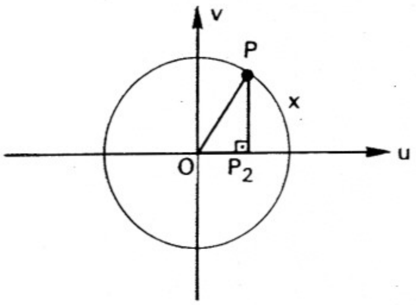
167. Solução 1

$0 < x < \dfrac{\pi}{2} \Rightarrow \text{sen}\, x > 0$ e $\text{sen}\, x < 1 \Rightarrow \text{sen}^2 x < \text{sen}\, x$ (1)

$0 < x < \dfrac{\pi}{2} \Rightarrow \cos x > 0$ e $\cos x < 1 \Rightarrow \cos^2 x < \cos x$ (2)

De (1) + (2) $\Rightarrow \text{sen}\, x + \cos x > 1$.

Solução 2



No triângulo OP_2P, temos:
$\overline{OP_2} + \overline{P_2P} > \overline{OP}$
(um lado é sempre menor que a soma dos outros dois)
e então
$\cos x + \operatorname{sen} x > 1.$

168. $t = 4x;\ t = 0 \Rightarrow x = 0;\ t = 2\pi \Rightarrow x = \frac{\pi}{2};\ p = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

169. A seqüência é *cos α, −cos α, cos α, −cos α, ...*; então: a soma dos *12* termos iniciais é zero.

171. a) $t = 3x;\ 3x \neq \frac{\pi}{2} + k\pi;\ D(f) = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k\frac{\pi}{3},\ k \in \mathbb{Z}\}$

b) $t = 2x - \frac{\pi}{3};\ 2x - \frac{\pi}{3} \neq \frac{\pi}{2} + k\pi;\ D(f) = \{x \in \mathbb{R} \mid x \neq \frac{5\pi}{12} + k\frac{\pi}{2},\ k \in \mathbb{Z}\}$

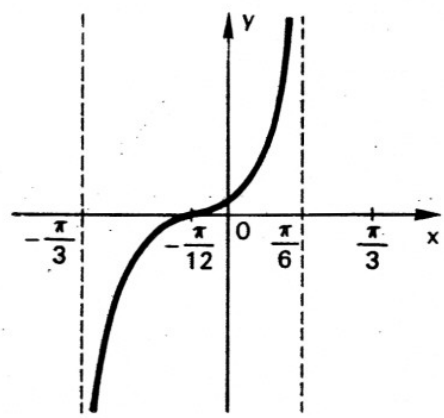
172. $\alpha^2 - 5\alpha + 4 \geqslant 0 \Rightarrow \alpha \leqslant 1$ ou $\alpha \geqslant 4$

174. $t = 2x + \frac{\pi}{6};\ 2x + \frac{\pi}{6} \neq \frac{\pi}{2} + k\pi;$

$D(f) = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k\frac{\pi}{2},\ k \in \mathbb{Z}\}$

$-\frac{\pi}{2} < t < \frac{\pi}{2};\ \Rightarrow\ -\frac{\pi}{3} < x < \frac{\pi}{6};$

$p = \frac{\pi}{6} - \left(-\frac{\pi}{3}\right) = \frac{\pi}{2}$



175. $t = x - \frac{\pi}{3},\ x - \frac{\pi}{3} \neq k\pi,\ D(f) = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{3} + k\pi,\ k \in \mathbb{Z}\}$

$0 < x - \frac{\pi}{3} < \pi \Rightarrow \frac{\pi}{3} < x < \frac{4\pi}{3};\ p = \frac{4\pi}{3} - \frac{\pi}{3} = \pi$

$t = 2x,\ 2x \neq \frac{\pi}{2} + k\pi,\ D(g) = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\frac{\pi}{2},\ k \in \mathbb{Z}\}$

$-\frac{\pi}{2} < 2x < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{4} < x < \frac{3\pi}{4};\ p = \frac{3\pi}{4} - \left(-\frac{\pi}{4}\right) = \pi$

$$t = x + \frac{\pi}{4},\ x + \frac{\pi}{4} \neq k\pi,\ D(h) = \{x \in \mathbb{R} \mid x \neq -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}\}$$

$$0 < x + \frac{\pi}{4} < 2\pi \Rightarrow -\frac{\pi}{4} < x < \frac{7\pi}{4};\ p = \frac{7\pi}{4} - \left(-\frac{\pi}{4}\right) = 2\pi$$

176. a) $2 - m \geqslant 0 \Rightarrow m \leqslant 2$

b) $3m - 2 \leqslant -1 \Rightarrow m \leqslant \frac{1}{3}$ ou $3m - 2 \geqslant 1 \Rightarrow m \geqslant 1$

c) $\frac{2m-1}{1-3m} \leqslant -1 \Rightarrow \frac{-m}{1-3m} \leqslant 0 \Rightarrow 0 \leqslant m < \frac{1}{3}$

ou

$\frac{2m-1}{1-3m} \geqslant 1 \Rightarrow \frac{5m-2}{1-3m} \geqslant 0 \Rightarrow \frac{1}{3} < m \leqslant \frac{2}{5}$

177. $$\frac{1}{\left(1 + \frac{1}{\cos x}\right)} \cdot \frac{(1 + \cos x)^{1/2}}{(1 - \cos x)^{1/2}} = \frac{\cos x}{\sqrt{1 - \cos^2 x}} = \frac{\cos x}{\sqrt{\operatorname{sen}^2 x}} = \frac{\cos x}{|\operatorname{sen} x|}$$

178. $$\frac{\frac{1}{\operatorname{sen} x} - \operatorname{sen} x}{\frac{1}{\cos x} - \cos x} = \frac{\cos^2 x}{\operatorname{sen} x} \cdot \frac{\cos x}{\operatorname{sen}^2 x} = \operatorname{cotg}^3 x$$

179. $$\frac{1 - \operatorname{sen}^2 \theta}{1 - \operatorname{sen} \theta} = \frac{(1 + \operatorname{sen} \theta)(1 - \operatorname{sen} \theta)}{1 - \operatorname{sen} \theta} = 1 + \operatorname{sen} \theta$$

180. $$\frac{(\overset{1}{\cancel{\cos^2 x + \operatorname{sen}^2 x}})(\cos^2 x - \operatorname{sen}^2 x)}{(1 + \operatorname{tg}^2 x)(1 - \operatorname{tg}^2 x)} = \frac{\cos^2 x - \cos^2 x \cdot \operatorname{tg}^2 x}{\sec^2 x \cdot (1 - \operatorname{tg}^2 x)} = \frac{\cos^2 x}{\sec^2 x} = \cos^4 x$$

181. $$\frac{\operatorname{sen}^2 x + (1 + \cos x)^2}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2 + 2\cos x}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2}{\operatorname{sen} x} = 2 \operatorname{cossec} x$$

182. $\operatorname{sen} x - \operatorname{cossec} x = t \Rightarrow (\operatorname{sen} x - \operatorname{cossec} x)^2 = t^2 \Rightarrow \operatorname{sen}^2 x + \operatorname{cossec}^2 x = t^2 + 2$

183. $$\frac{\sec^2 x}{\operatorname{cossec}^2 x} = \frac{\operatorname{sen}^2 x}{1 - \operatorname{sen}^2 x} = \frac{\left(\frac{n-1}{n}\right)^2}{1 - \left(\frac{n-1}{n}\right)^2} = \frac{(n-1)^2}{2n-1}$$

184. a) $\operatorname{tg}(-x) = \frac{\operatorname{sen}(-x)}{\cos(-x)} = -\frac{\operatorname{sen} x}{\cos x} = -\operatorname{tg}(x)$; a função é ímpar

b) $\operatorname{cotg}(-x) = \frac{1}{\operatorname{tg}(-x)} = -\frac{1}{\operatorname{tg} x} = -\operatorname{cotg}(x)$; a função é ímpar

c) $\sec(-x) = \dfrac{1}{\cos(-x)} = \dfrac{1}{\cos x} = \sec x$; a função é par

d) $\operatorname{cossec}(-x) = \dfrac{1}{\operatorname{sen}(-x)} = -\dfrac{1}{\operatorname{sen} x} = -\operatorname{cossec} x$; a função é ímpar

185. a) $0 \in D(f)$, f é ímpar $\Rightarrow f(-0) = -f(0) \Rightarrow f(0) + f(0) = 0 \Rightarrow f(0) = 0$

b) $\left.\begin{array}{l} f \text{ é ímpar} \Rightarrow f(-x) = -f(x) \\ f \text{ é par} \Rightarrow f(-x) = f(x) \end{array}\right\} \Rightarrow -f(x) = f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) \equiv 0, \forall x$

186. $f(x) = f(-x) = 3 \Rightarrow f(x)$ é par; $g(x)$ é par, $\forall n$, pois

$$g(x) = \underbrace{f(x) \cdot f(x) \cdot \ldots \cdot f(x)}_{n \text{ fatores}} = \underbrace{f(-x) \cdot f(-x) \cdot f(-x) \cdot \ldots \cdot f(-x)}_{n \text{ fatores}} = g(-x), \forall x$$

Capítulo IX — Transformações

188. a) $\operatorname{cotg}(120° + 45°) = \dfrac{-\dfrac{\sqrt{3}}{3} \cdot 1 - 1}{-\dfrac{\sqrt{3}}{3} + 1} = -(2 + \sqrt{3})$;

b) $\cos(225° + 30°) = -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{2} + \sqrt{6}}{4}$;

$\sec 255° = \dfrac{1}{\cos 255°} = -\sqrt{2} + \sqrt{6}$

c) $\operatorname{sen}(45° - 30°) = \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$;

$\operatorname{cossec} 15° = \dfrac{1}{\operatorname{sen} 15°} = \sqrt{6} + \sqrt{2}$

189. $\operatorname{tg}(A - B) = \dfrac{2 - 1}{1 + 2 \cdot 1} = \dfrac{1}{3}$

190. $\left.\begin{array}{l} \operatorname{sen}(60° + 45°) = \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4} \text{ (A)} \\ \cos(30° + 45°) = \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4} \text{ (B)} \end{array}\right\} A - B = \dfrac{\sqrt{2}}{2}$

193. $\cos x = +\sqrt{1 - \operatorname{sen}^2 x} = \dfrac{8}{17}$; $\cos y = -\sqrt{1 - \operatorname{sen}^2 x} = -\dfrac{4}{5}$;

$\operatorname{tg} x = \dfrac{15}{8}$; $\operatorname{tg} y = \dfrac{3}{4}$

$$\text{sen}(x+y) = \frac{15}{17}\cdot\left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\left(\frac{8}{17}\right) = -\frac{84}{85};$$

$$\cos(x+y) = \frac{13}{85};\ \text{tg}(x+y) = -\frac{84}{13}$$

195. a) $f(x) = \cos 2x \cdot \cos 2x - \text{sen}\, 2x \cdot \text{sen}\, 2x = \cos 4x$

$D(f) = \mathbb{R};\ Im(f) = [-1, 1];\ p = \dfrac{\pi}{2}$

b) $g(x) = 2\cdot\text{sen}\,\dfrac{\pi}{3}\cos x - 2\cos\dfrac{\pi}{3}\,\text{sen}\, x = 2\cdot\text{sen}\left(\dfrac{\pi}{3} - x\right)$

$D(g) = \mathbb{R};\ Im(g) = [-2, 2];\ p = 2\pi$

c) $$h(x) = \frac{\dfrac{\text{sen}\, x}{\cos x} + \dfrac{\cos x}{\cos x}}{\dfrac{\cos x}{\cos x} - \dfrac{\text{sen}\, x}{\cos x}} = \frac{\text{tg}\, x + 1}{1 - \text{tg}\, x} = \text{tg}\left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi \Rightarrow D(h) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\pi\right\}$$

$$\left.\begin{array}{l} x + \dfrac{\pi}{4} = -\dfrac{\pi}{2} \Rightarrow x = -\dfrac{3\pi}{4} \\ x + \dfrac{\pi}{4} = \dfrac{\pi}{2} \Rightarrow x = \dfrac{\pi}{4} \end{array}\right\} \Rightarrow p = \frac{\pi}{4} - \left(-\frac{3\pi}{4}\right) = \pi$$

196. $f(x) = \text{sen}\, x(\cos 2x\cos 3x - \text{sen}\, 2x\,\text{sen}\, 3x) + \cos x(\text{sen}\, 2x\cos 3x + \text{sen}\, 3x\cos 2x) \Rightarrow$

$\Rightarrow f(x) = \text{sen}\, x\cos 5x + \cos x\,\text{sen}\, 5x = \text{sen}\, 6x;\ p = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$

197. $\text{tg}(75^\circ - 60^\circ) = \dfrac{(2+\sqrt{3}) - \sqrt{3}}{1 + (2+\sqrt{3})\sqrt{3}} = 2 - \sqrt{3}$

199. Solução 1

$$\text{tg}\, x + \frac{1}{\text{tg}\, x} = 3 \Rightarrow \text{tg}^2 x - 3\,\text{tg}\, x + 1 = 0 \Rightarrow \text{tg}\, x = \frac{3 \pm \sqrt{5}}{2}$$

$$\text{sen}^2 x = \frac{\text{tg}^2 x}{1 + \text{tg}^2 x} \Rightarrow \text{sen}^2 x = \frac{\left(\dfrac{3 \pm \sqrt{5}}{4}\right)^2}{1 + \left(\dfrac{3 + \sqrt{5}}{4}\right)^2} \Rightarrow \text{sen}\, x = \sqrt{\frac{3 \pm \sqrt{5}}{6}}$$

$$\cos^2 x = \frac{1}{1 + \text{tg}^2 x} \Rightarrow \cos x = \pm\sqrt{\frac{4}{4 + (3 \pm \sqrt{5})^2}} \Rightarrow \cos x = \sqrt{\frac{4}{6(3 \pm \sqrt{5})}}$$

$$\text{sen}\, 2x = 2\,\text{sen}\, x\cos x \Rightarrow \text{sen}\, 2x = \frac{2}{3}$$

Solução 2

$$\text{tg}\, x + \text{cotg}\, x = 3 \Rightarrow \frac{\text{sen}^2 x + \cos^2 x}{\cos x \cdot \text{sen}\, x} = 3 \Rightarrow \text{sen}\, x \cdot \cos x = \frac{1}{3} \Rightarrow$$

$$\Rightarrow 2 \cdot \text{sen}\, x \cdot \cos x = \frac{2}{3} \Rightarrow \text{sen}\, 2x = \frac{2}{3}$$

201. a) $\text{sen}\left(\frac{\pi}{2} + 2\alpha\right) = \cos 2\alpha = 1 - 2\,\text{sen}^2\,\alpha \Rightarrow \text{sen}\left(\frac{\pi}{2} + 2\alpha\right) = \frac{1}{9}$

b) $\left.\begin{array}{l} \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\,\text{sen}\,\alpha \\ \cos\alpha = \sqrt{1 - \text{sen}^2\,\alpha} \Rightarrow \cos\alpha = \frac{\sqrt{5}}{3} \end{array}\right\} \Rightarrow \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{10} - 2\sqrt{2}}{6}$

203. $\text{sen}\, x = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \Rightarrow \text{sen}\, x = -\frac{4}{5}$; $\text{sen}\, 3x = 3 \cdot \left(\frac{-4}{5}\right) - 4 \cdot \left(\frac{-4}{5}\right)^3 = \frac{-44}{125}$

204. $\cos x = -\sqrt{1 - \left(\frac{12}{13}\right)^2} \Rightarrow \cos x = \frac{-5}{13}$; $\cos 3x = 4 \cdot \left(\frac{-5}{13}\right)^3 - 3 \cdot \left(\frac{-5}{13}\right) = \frac{2\,035}{2\,197}$

205. $\text{tg}\, x = \sqrt{\sec^2 x - 1} = \frac{\sqrt{7}}{3} \Rightarrow \text{tg}\, 3x = \dfrac{3 \cdot \frac{\sqrt{7}}{3} - \left(\frac{\sqrt{7}}{3}\right)^3}{1 - 3\left(\frac{\sqrt{7}}{3}\right)^2} = \frac{-5\sqrt{7}}{9}$

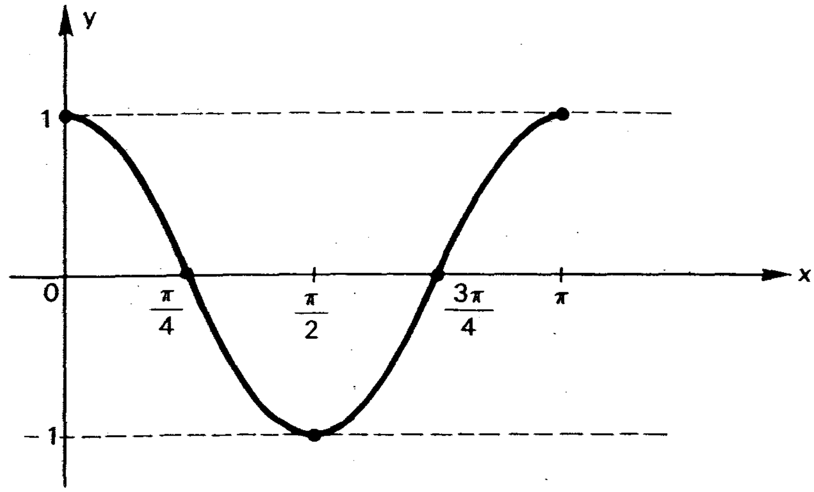
206. $\left(\text{sen}^2\frac{\pi}{12} - \cos^2\frac{\pi}{12}\right) + \text{tg}\,\frac{\pi}{3} + \text{tg}\,\frac{14\pi}{3} = -\cos 2 \cdot \frac{\pi}{12} + \text{tg}\,\frac{\pi}{3} + \text{tg}\,\frac{2\pi}{3} =$

$= -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

208. a) $f(x) = (\cos^2 x + \text{sen}^2 x)(\cos^2 x - \text{sen}^2 x) = \cos 2x$

$\text{Im}(f) = [-1, 1]$, $D(f) = \mathbb{R}$ $p = \pi$

x	$t = 2x$	$\cos t$
0	0	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1



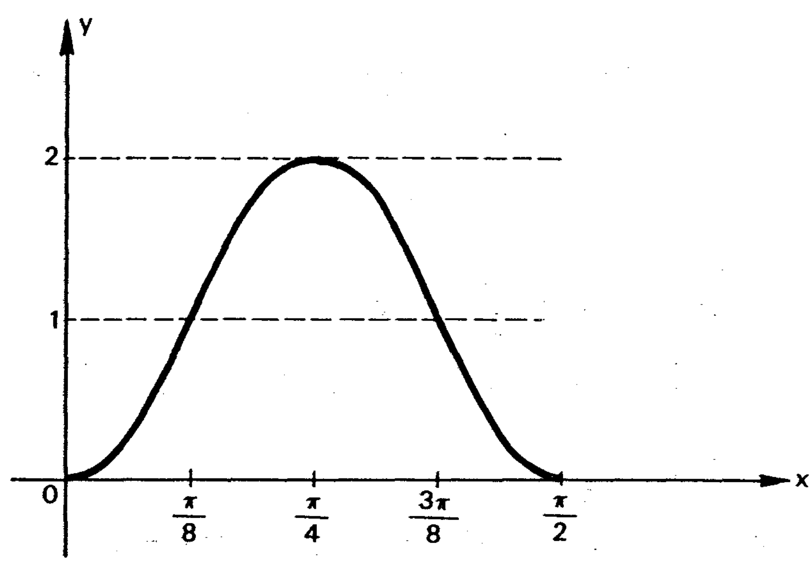
b) $g(x) = 2(2 \text{ sen } x \cos x)^2 = 2 \text{ sen}^2 2x = 1 - \cos 4x$

x	t = 4x	cos t	y = 1 − cos 4x
0	0	1	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{4}$	π	−1	2
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	1
$\frac{\pi}{2}$	2π	1	0

$\text{Im}(g) = [0, 2]$

$D(g) = \mathbb{R}$

$p = \frac{\pi}{2}$



c) $h(x) = (\cos^2 x + \operatorname{sen}^2 x)^2 - 2 \cdot \cos^2 x \cdot \operatorname{sen}^2 x = 1 - 2 \cdot \left(\frac{\operatorname{sen} 2x}{2}\right)^2 =$

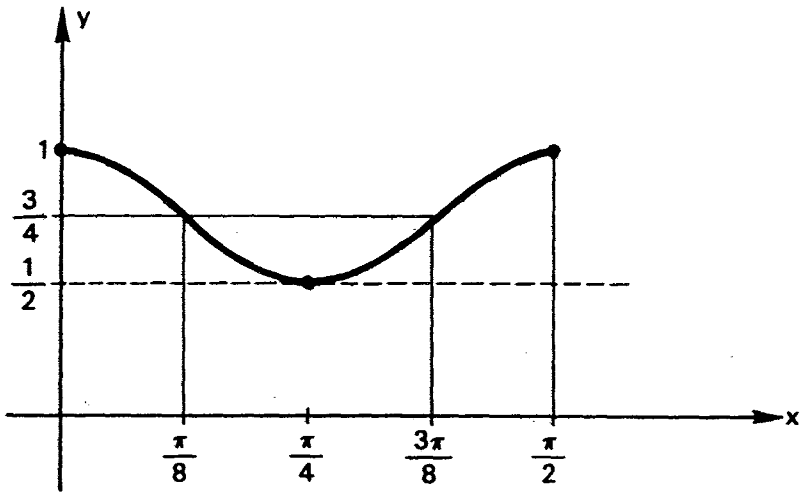
$= 1 - \frac{1}{2} \cdot \operatorname{sen}^2 2x = 1 - \frac{1}{2} \cdot \left(\frac{1 - \cos 4x}{2}\right) = \frac{3}{4} + \frac{1}{4} \cdot \cos 4x$

x	t = 4x	cos t	y = h(x)
0	0	1	1
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{4}$	π	-1	$\frac{1}{2}$
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{2}$	2π	1	1

$\operatorname{Im}(h) = \left[\frac{1}{2}, 1\right]$

$D(h) = \mathbb{R}$

$p = \frac{\pi}{2}$



209. a) $f(x) = \frac{1}{2} \operatorname{sen} 2x \quad p = \frac{2\pi}{2} \Rightarrow p = \pi$

b) $g(x) = \frac{1 - \operatorname{tg}^2 2x}{\sec^2 2x} = (1 - \operatorname{tg}^2 2x) \cdot \cos^2 2x = \cos^2 2x - \operatorname{sen}^2 2x = \cos 4x$

$p = \frac{2\pi}{4} \Rightarrow p = \frac{\pi}{2}$

c) $h(x) = (\cos^2 x + \operatorname{sen}^2 x)(\cos^4 x - \cos^2 x \cdot \operatorname{sen}^2 x + \operatorname{sen}^4 x) =$

$= (\cos^4 x + \operatorname{sen}^4 x) - \cos^2 x \cdot \operatorname{sen}^2 x = (\cos^2 x + \operatorname{sen}^2 x)^2 = 3 \cdot \cos^2 x \cdot \operatorname{sen}^2 x$

$$= 1 - \frac{3}{4}(\text{sen } 2x)^2 = 1 - \frac{3}{4}\left(\frac{1 - \cos 4x}{2}\right) = \frac{5}{8} + \frac{3}{8} \cdot \cos 4x$$

$$p = \frac{2\pi}{4} = \frac{\pi}{2}$$

210. $\text{sen } 2a = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$; $\cos 2a = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$;

$$\text{sen } 2a + \cos 2a = \frac{31}{25}$$

211. $\sec a = \frac{1}{\cos a} \Rightarrow \cos a = \frac{-2}{3}$; $\text{sen } a = \sqrt{1 - \left(\frac{-2}{3}\right)^2} = \frac{\sqrt{5}}{3}$;

$\text{sen } b = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}}$; $\text{sen } 2a = 2 \cdot \frac{\sqrt{5}}{3} \cdot \left(\frac{-2}{3}\right) = \frac{-4\sqrt{5}}{9}$

$$\cos 2b = \left(\frac{1}{3}\right)^2 - \left(\sqrt{\frac{8}{9}}\right)^2 = \frac{1}{9} - \frac{8}{9} = \frac{-7}{9} \Rightarrow \cos 2b = \frac{-7}{9}$$

212. $\text{tg } x = a \text{ cotg } x + b \cdot \frac{1}{\text{tg } 2x} \Rightarrow \text{tg } x = \frac{a}{\text{tg } x} + \frac{b(1 - \text{tg}^2 x)}{2 \text{ tg } x} \Rightarrow$

$\Rightarrow 2 \text{ tg}^2 x - 2a = -b \text{ tg}^2 x + b \Rightarrow (2 = -b \text{ e } -2a = b) \Rightarrow b = -2 \text{ e } a = 1$

215. $\cos\theta = -\sqrt{1 - \text{sen}^2 \theta} \Rightarrow \cos\theta = -\frac{4}{5}$; $\text{sen } \frac{\theta}{2} = +\sqrt{\frac{1 - \cos\theta}{2}} \Rightarrow$

$\Rightarrow \text{sen } \frac{\theta}{2} = \frac{3}{\sqrt{10}}$ $\qquad A = 25 \cdot \frac{3}{5} + \sqrt{10} \cdot \frac{3}{\sqrt{10}} \Rightarrow A = 18$

216. $\cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \cos a_n}{2}} \Rightarrow \cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \frac{n}{n+1}}{2}} \Rightarrow$

$$\cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{2n + 1}{2n + 2}}$$

$$\cos\left(\frac{a_n}{2}\right) = \frac{\sqrt{4n^2 + 6n + 2}}{2n + 2}$$

218. $\text{sen } \frac{x}{2} = +\sqrt{\frac{1 - \cos x}{2}} = \frac{\sqrt{3}}{3}$; $\text{tg } \frac{x}{2} = +\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sqrt{2}}{2}$

219. $\cos\frac{x}{2} = +\sqrt{\frac{1 + \cos x}{2}} = \frac{7}{5\sqrt{2}}$; $\text{sen } \frac{x}{4} = +\sqrt{\frac{1 - \cos\frac{x}{2}}{2}} = +\sqrt{\frac{10 - 7\sqrt{2}}{20}}$

$\cos\frac{x}{4} = +\sqrt{\frac{1 + \cos\frac{x}{2}}{2}} = \sqrt{\frac{10 + 7\sqrt{2}}{20}}$; $\text{tg } \frac{x}{4} = \sqrt{\frac{10 - 7\sqrt{2}}{10 + 7\sqrt{2}}} = 5\sqrt{2} - 7$

220. $\sec x = 4 \Rightarrow \cos x = \dfrac{1}{4}$

$$\frac{3\pi}{2} < x < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi$$

$$\operatorname{tg}\left(\frac{\pi}{2} + \frac{x}{2}\right) = -\operatorname{cotg}\frac{x}{2} = -\left(-\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{\sqrt{15}}{3}$$

222. $f(x) = \dfrac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}} = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}} = |\operatorname{tg} x|$

$\operatorname{Im}(f) = \mathbb{R}_+$; $p = \pi$; $D(f) = \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\}$

223. $f(x) = \sqrt{1+\cos 4x} = \sqrt{2}\cdot\sqrt{\dfrac{1+\cos 4x}{2}} = \sqrt{2}\cdot|\cos 2x|$

$p = \dfrac{\pi}{2}$

224. $\operatorname{tg} a = \dfrac{2\cdot\operatorname{tg}\frac{a}{2}}{1-\operatorname{tg}^2\frac{a}{2}} = \dfrac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \dfrac{4}{3}$

225. $\operatorname{cotg}\dfrac{a}{2} = \dfrac{1}{\operatorname{tg}\frac{a}{2}} \Rightarrow \operatorname{tg}\dfrac{a}{2} = \dfrac{\sqrt{3}}{3}$; $\operatorname{sen} a = \dfrac{2\cdot\operatorname{tg}\frac{a}{2}}{1+\operatorname{tg}^2\frac{a}{2}} = \dfrac{\sqrt{3}}{2}$

229. a) $y = 2\operatorname{sen}\left(\dfrac{a+b+c-a+b-c}{2}\right)\cdot\cos\left(\dfrac{a+b+c+a-b+c}{2}\right) =$

$= 2\operatorname{sen} b\cos(a+c)$

b) $y = 2\cdot\cos\left(\dfrac{a+2b+a}{2}\right)\cdot\cos\left(\dfrac{a+2b-a}{2}\right) = 2\cos(a+b)\cos b$

c) $y = (\operatorname{sen}(a+3r) + \operatorname{sen} a) + (\operatorname{sen}(a+2r) + \operatorname{sen}(a+r)) =$

$= 2\cdot\operatorname{sen}\dfrac{2a+3r}{2}\cdot\cos\dfrac{3r}{2} + 2\cdot\operatorname{sen}\dfrac{2a+3r}{2}\cdot\cos\dfrac{r}{2} =$

$= 2\cdot\operatorname{sen}\dfrac{2a+3r}{2}\cdot\left(\cos\dfrac{3r}{2} + \cos\dfrac{r}{2}\right) = 4\cdot\operatorname{sen}\dfrac{2a+3r}{2}\cdot\cos r\cdot\cos\dfrac{r}{2}$

d) $y = (\cos(a+3b) + \cos a) + (\cos(a+2b) + \cos(a+b)) =$

$= 2\cdot\cos\dfrac{2a+3b}{2}\cdot\cos\dfrac{3b}{2} + 2\cdot\cos\dfrac{2a+3b}{2}\cdot\cos\dfrac{b}{2} =$

$= 2\cdot\cos\dfrac{2a+3b}{2}\cdot\left(\cos\dfrac{3b}{2} + \cos\dfrac{b}{2}\right) = 4\cdot\cos\dfrac{2a+3b}{2}\cdot\cos b\cdot\cos\dfrac{b}{2}$

e) $y = (\cos p + \cos q)(\cos p - \cos q)$

$$y = 2 \cdot \cos\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)\left[-2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \operatorname{sen}\left(\frac{p-q}{2}\right)\right] =$$

$$= -\operatorname{sen}(p+q) \cdot \operatorname{sen}(p-q)$$

f) $y = (\operatorname{sen} p + \operatorname{sen} q)(\operatorname{sen} p - \operatorname{sen} q)$

$$y = 2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right) \cdot 2 \operatorname{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right) =$$

$$= \operatorname{sen}(p+q) \cdot \operatorname{sen}(p-q)$$

g) $y = \dfrac{1 + \cos 2p}{2} - \dfrac{1 - \cos 2q}{2} = \dfrac{1}{2}(\cos 2p + \cos 2q) =$

$$= \cos(p+q) \cdot \cos(p-q)$$

h) $y = \dfrac{2 \operatorname{sen}(a+b) \cos(a-b)}{-2 \operatorname{sen}(a+b) \operatorname{sen}(a-b)} = -\operatorname{cotg}(a-b)$

i) $y = \dfrac{\operatorname{sen}\frac{\pi}{2} + \operatorname{sen} a}{\operatorname{sen}\frac{\pi}{2} - \operatorname{sen} a} = \dfrac{2 \operatorname{sen}\left(\frac{\pi}{4} + \frac{a}{2}\right) \cdot \cos\left(\frac{\pi}{4} - \frac{a}{2}\right)}{2 \operatorname{sen}\left(\frac{\pi}{4} - \frac{a}{2}\right) \cdot \cos\left(\frac{\pi}{4} + \frac{a}{2}\right)} = \operatorname{tg}^2\left(\frac{\pi}{4} + \frac{a}{2}\right)$

231. a) $\dfrac{p+q}{2} = \dfrac{7\pi}{8}$; $\dfrac{p-q}{2} = \dfrac{\pi}{8}$; $p = \pi$ e $q = \dfrac{3\pi}{4}$;

$$y = \frac{1}{2} \cdot \left(2 \cos\frac{7\pi}{8} \cdot \cos\frac{\pi}{8}\right) = \frac{1}{2}\left(\cos\pi + \cos\frac{3\pi}{4}\right) = \frac{-2 - \sqrt{2}}{4}$$

b) $\dfrac{p+q}{2} = \dfrac{13\pi}{12}$; $\dfrac{p-q}{2} = \dfrac{7\pi}{12}$; $p = \dfrac{5\pi}{3}$ e $q = \dfrac{\pi}{2}$;

$$y = \frac{-1}{2}\left(-2 \operatorname{sen}\frac{13\pi}{12} \cdot \operatorname{sen}\frac{7\pi}{12}\right) = \frac{-1}{2}\left(\cos\frac{5\pi}{3} - \cos\frac{\pi}{2}\right) = -\frac{1}{4}$$

c) $\dfrac{p+q}{2} = \dfrac{5\pi}{24}$; $\dfrac{p-q}{2} = \dfrac{\pi}{24}$; $p = \dfrac{\pi}{4}$ e $q = \dfrac{\pi}{6}$;

$$y = \frac{1}{2}\left(2 \operatorname{sen}\frac{5\pi}{24} \cdot \cos\frac{\pi}{24}\right) = \frac{1}{2}\left(\operatorname{sen}\frac{\pi}{4} + \operatorname{sen}\frac{\pi}{6}\right) = \frac{1 + \sqrt{2}}{4}$$

233. $(\operatorname{tg} 81° + \operatorname{tg} 9°) - (\operatorname{tg} 63° + \operatorname{tg} 27°) = \dfrac{\operatorname{sen} 90°}{\cos 81° \cdot \cos 9°} - \dfrac{\operatorname{sen} 90°}{\cos 63° \cdot \cos 27°} =$

$$= \frac{1}{\frac{1}{2}(\cos 90° + \cos 72°)} - \frac{1}{\frac{1}{2}(\cos 90° + \cos 36°)} = \frac{2}{\operatorname{sen} 18°} - \frac{2}{\operatorname{sen} 54°} =$$

$$= \frac{2(\operatorname{sen} 54° - \operatorname{sen} 18°)}{\operatorname{sen} 18° \cdot \operatorname{sen} 54°} = \frac{2 \cdot 2 \cdot \operatorname{sen} 18° \cdot \cos 36°}{\operatorname{sen} 18° \cdot \operatorname{sen} 54°} = 4$$

235. $f(x) = \operatorname{sen} 2x + \operatorname{sen}\left(\dfrac{\pi}{2} + 2x\right) = 2 \operatorname{sen}\left(2x + \dfrac{\pi}{4}\right) \cdot \cos\left(\dfrac{-\pi}{4}\right)$

$f(x) = \sqrt{2} \cdot \operatorname{sen}\left(2x + \dfrac{\pi}{4}\right)$; $D(f) = \mathbb{R}$; $Im(f) = [-\sqrt{2}, \sqrt{2}]$, $p = \pi$

236. $$f(x) = \frac{\operatorname{tg}\frac{\pi}{4} + \operatorname{tg} x}{\operatorname{tg}\frac{\pi}{4} - \operatorname{tg} x} = \frac{\dfrac{\operatorname{sen}\left(\frac{\pi}{4} + x\right)}{\cos\frac{\pi}{4}\cdot\cos x}}{\dfrac{\operatorname{sen}\left(\frac{\pi}{4} - x\right)}{\cos\frac{\pi}{4}\cdot\cos x}} = \frac{\operatorname{sen}\left(\frac{\pi}{4} + x\right)}{\operatorname{sen}\left(\frac{\pi}{4} - x\right)} = \operatorname{tg}\left(\frac{\pi}{4} + x\right);\ p = \pi$$

237. $$|\operatorname{sen} x - \operatorname{sen} y| = \left|2 \operatorname{sen}\frac{x-y}{2}\cdot\cos\frac{x+y}{2}\right| =$$
$$= |2|\cdot\left|\operatorname{sen}\frac{x-y}{2}\right|\cdot\left|\cos\frac{x+y}{2}\right| \leqslant |2|\cdot\left|\frac{x-y}{2}\right|\cdot 1 =$$
$$= \left|2\cdot\left(\frac{x-y}{2}\right)\right| = |x-y| \Rightarrow |\operatorname{sen} x - \operatorname{sen} y| \leqslant |x-y|$$

238. a) $\operatorname{tg} b\,(\operatorname{tg} a + \operatorname{tg} c) + \operatorname{tg} c\cdot\operatorname{tg} a = \operatorname{cotg}(a+c)\cdot(\operatorname{tg} a + \operatorname{tg} c) + \operatorname{tg} c\cdot\operatorname{tg} a =$
$$= \frac{\cos(a+c)}{\operatorname{sen}(a+c)}\cdot\frac{\operatorname{sen}(a+c)}{\cos a\cos c} + \frac{\operatorname{sen} c\operatorname{sen} a}{\cos c\cos a} =$$
$$= \frac{\cos a\cos c - \operatorname{sen} a\operatorname{sen} c + \operatorname{sen} c\operatorname{sen} a}{\cos a\cos a} = 1$$

b) $\cos^2 a + co^2 b + \cos^2 c - 2\cdot\operatorname{sen} a\cdot\operatorname{sen} b\cdot\operatorname{sen} c =$
$$= \frac{1+\cos 2a}{2} + \frac{1+\cos 2b}{2} + (1 - \operatorname{sen}^2 c) + (-2\cdot\operatorname{sen} a\cdot\operatorname{sen} b)\cdot\operatorname{sen} c =$$
$$= 1 + \cos(a+b)\cos(a-b) + 1 - \operatorname{sen}^2 c + [\cos(a+b) - \cos(a-b)]\operatorname{sen} c =$$
$$= 2 + \operatorname{sen} c\cdot\cos(a-b) - \operatorname{sen}^2 c + [\operatorname{sen} c - \cos(a-b)]\operatorname{sen} c = 2$$

241. $\operatorname{sen} 4A + \operatorname{sen} 4B = -\operatorname{sen} 4C \Rightarrow$
$\Rightarrow 2\operatorname{sen}(2A+2B)\cdot\cos(2A-2B) = -2\operatorname{sen} 2C\cdot\cos 2C \Rightarrow$
$\Rightarrow \operatorname{sen}(360^\circ - 2C)\cdot\cos(2A-2B) = -\operatorname{sen} 2C\cdot\cos 2C \Rightarrow$
$\Rightarrow \cos(2A-2B) = \cos 2C \Rightarrow 2A - 2B = 2C \Rightarrow$
$\Rightarrow \left(A = B + C = \frac{\pi}{2} \text{ ou } 2A - 2B = -2C\right) \Rightarrow B = A + C = \frac{\pi}{2}$

242. $A + B + C = \pi \Rightarrow C = \pi - (A+B) \Rightarrow 3C = 3\pi - (3A+3B) \Rightarrow$
$\Rightarrow \operatorname{sen} 3C = -\operatorname{sen}(3A+3B)$, então:
$(\operatorname{sen} 3A + \operatorname{sen} 3B) + \operatorname{sen} 3C = 0 \Rightarrow$
$$\Rightarrow 2\cdot\operatorname{sen}\frac{3(A+B)}{2}\cdot\cos\frac{3(A-B)}{2} - 2\cdot\operatorname{sen}\frac{3(A+B)}{2}\cdot\cos\frac{3(A+B)}{2} = 0 \Rightarrow$$

$$\Rightarrow\ 2\cdot \operatorname{sen}\frac{3(A+B)}{2}\cdot\left(\cos\frac{3(A-B)}{2}-\cos\frac{3(A+B)}{2}\right)=0\ \Rightarrow$$

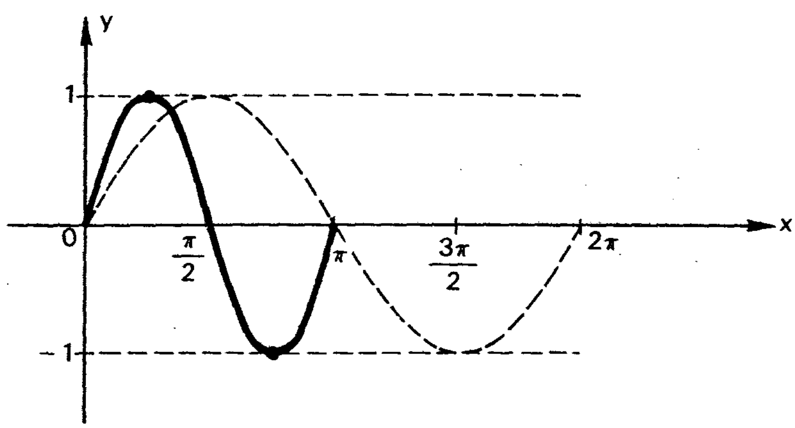
$$\Rightarrow\ 4\cdot \operatorname{sen}\frac{3(A+B)}{2}\cdot \operatorname{sen}\frac{3A}{2}\cdot \operatorname{sen}\frac{3B}{2}=0\ \Rightarrow$$

$$\Rightarrow\ \operatorname{sen}\frac{3(A+B)}{2}=0 \text{ ou } \operatorname{sen}\frac{3A}{2}=0 \text{ ou } \operatorname{sen}\frac{3B}{2}=0\ \Rightarrow$$

$$\Rightarrow\ C=\frac{\pi}{3} \text{ ou } A=\frac{\pi}{3} \text{ ou } B=\frac{\pi}{3} \text{ (respectivamente)}$$

244. $4\operatorname{sen}(x+60^\circ)\cdot\cos(x+30^\circ)=$
$=4(\operatorname{sen}x\cos 60^\circ+\operatorname{sen}60^\circ\ \cos x)\cdot(\cos x\cos 30^\circ-\operatorname{sen}x\operatorname{sen}30^\circ)=$
$=(\operatorname{sen}x+\sqrt{3}\cos x)\cdot(\sqrt{3}\cos x-\operatorname{sen}x)=(\sqrt{3}\cos x)^2-(\operatorname{sen}x)^2=$
$=3\cos^2 x-\operatorname{sen}^2 x$

245. a) $f(x)=\operatorname{sen}2x$, $Im(f)=[-1,1]$, $D(f)=\mathbb{R}$, $p=\pi$



b) $f(x)=\operatorname{sen}2x+\operatorname{sen}\left(2x+\frac{\pi}{2}\right)=2\operatorname{sen}\left(2x+\frac{\pi}{4}\right)\cdot\cos\left(\frac{-\pi}{4}\right)=$

$=\sqrt{2}\operatorname{sen}\left(2x+\frac{\pi}{4}\right)$

246. $\operatorname{sen}u+\cos u=\operatorname{sen}u+\operatorname{sen}\left(\frac{\pi}{2}-u\right)=\sqrt{2}\cdot\cos\left(u-\frac{\pi}{4}\right)=\sqrt{2}\cos v$ (1)

$$\sqrt{2}\cdot\operatorname{sen}u\cdot\cos u=\frac{2\cdot\operatorname{sen}u\cdot\cos u}{\sqrt{2}}=\frac{\operatorname{sen}2u}{\sqrt{2}}=\frac{\operatorname{sen}\left(\frac{\pi}{2}-2v\right)}{\sqrt{2}}=\frac{\cos 2v}{\sqrt{2}}$$

$$S=\frac{\sqrt{2}\cdot\cos v}{\frac{\cos 2v}{\sqrt{2}}}=\frac{2\cdot\cos v}{\cos 2v}=\frac{2x}{2x^2-1}$$

247. $n<20\cdot\cos^2 15=20\cdot\frac{1+\cos 30^\circ}{2}=20\cdot\left(\frac{1}{2}+\frac{\sqrt{3}}{4}\right)\cong 18{,}66$

então $n=18$

248. $f(x) = \cos 2x + \operatorname{sen} 2x = \operatorname{sen}\left(\frac{\pi}{2} + 2x\right) + \operatorname{sen} 2x = \sqrt{2}\operatorname{sen}\left(\frac{\pi}{4} + 2x\right)$

$\operatorname{Im}(f) = [-\sqrt{2}, \sqrt{2}]$

Capítulo X – Identidades

253. a) $f(x) = (\cos^2 x + \operatorname{sen}^2 x)^2 = 1 = g(x)$

b) $f(x) = \dfrac{\operatorname{sen} x}{\operatorname{cossec} x} + \dfrac{\cos x}{\sec x} = \dfrac{\operatorname{sen} x}{\dfrac{1}{\operatorname{sen} x}} + \dfrac{\cos x}{\dfrac{1}{\cos x}} = \operatorname{sen}^2 x + \cos^2 x = 1 = g(x)$

254. $f(x) = \operatorname{tg} x + \operatorname{cotg} x = \dfrac{\operatorname{sen} x}{\cos x} + \dfrac{\cos x}{\operatorname{sen} x} = \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x} =$

$= \dfrac{1}{\cos x} \cdot \dfrac{1}{\operatorname{sen} x} = \sec x \cdot \operatorname{cossec} x = g(x)$

255. $f(x) = \left(\dfrac{\operatorname{sen} x}{\cos x} + \dfrac{\cos x}{\operatorname{sen} x}\right)\left(\dfrac{1}{\cos x} - \cos x\right)\left(\dfrac{1}{\operatorname{sen} x} - \operatorname{sen} x\right) =$

$= \left(\dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x}\right)\left(\dfrac{1 - \cos^2 x}{\cos x}\right)\left(\dfrac{1 - \operatorname{sen}^2 x}{\operatorname{sen} x}\right) =$

$= \dfrac{\operatorname{sen}^2 x \cdot \cos^2 x}{\operatorname{sen}^2 x \cdot \cos^2 x} = 1 = g(x)$

256. $f(x) = \dfrac{1}{\cos^2 x} + \dfrac{1}{\operatorname{sen}^2 x} = \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\cos^2 x \cdot \operatorname{sen}^2 x} = \dfrac{1}{\cos^2 x} \cdot \dfrac{1}{\operatorname{sen}^2 x} =$

$= \sec^2 x \cdot \operatorname{cossec}^2 x + g(x)$

257. $f(x) = \dfrac{\operatorname{cotg}^2 x}{\operatorname{cossec}^2 x} = \dfrac{\cos^2 x}{\operatorname{sen}^2 x} : \dfrac{1}{\operatorname{sen}^2 x} = \cos^2 x = g(x)$

258. $f(x) = \dfrac{(\operatorname{sen} x - \cos x)(\operatorname{sen}^2 x + \cos^2 x + \operatorname{sen} x \cos x)}{\operatorname{sen} x - \cos x} = 1 + \operatorname{sen} x \cos x = g(x)$

259. $f(x) = 1 + \operatorname{cotg}^2 x + \operatorname{tg}^2 x = \sec^2 x + \operatorname{cotg}^2 x = g(x)$

260. $f(x) = 2\left(\operatorname{sen} x + \dfrac{\operatorname{sen} x}{\cos x}\right)\left(\cos x + \dfrac{\cos x}{\operatorname{sen} x}\right) = 2\left(\dfrac{\operatorname{sen} x (\cos x + 1)}{\cos x}\right)\left(\dfrac{\cos x (\operatorname{sen} x + 1)}{\operatorname{sen} x}\right) =$

$= 2(\cos x + 1)(\operatorname{sen} x + 1) = h(x)$

$g(x) = (1 + \operatorname{sen} x + \cos x)^2 =$

$= 1 + \operatorname{sen}^2 x + \cos^2 x + 2 \operatorname{sen} x \cos x + 2 \operatorname{sen} x + 2 \cos x =$

$= 2(1 + \operatorname{sen} x + \cos x + \operatorname{sen} x \cos x) = 2(1 + \operatorname{sen} x)(1 + \cos x) = h(x)$

261. $f(x) = 1 + 2\,\text{cotg}\,x + \text{cotg}^2\,x + 1 - 2\,\text{cotg}\,x + \text{cotg}^2\,x = 2 + 2\,\text{cotg}^2\,x =$
$= 2\,(1 + \text{cotg}^2\,x) = 2\,\text{cossec}^2\,x = g(x)$

262. $f(x) = \dfrac{(1 - \cos^2 x)^2}{(1 - \text{sen}^2\,x)^2} = \left(\dfrac{\text{sen}^2\,x}{\cos^2 x}\right)^2 = \text{tg}^4\,x = g(x)$

263. $f(x) - g(x) = \text{cotg}^2\,x - 2\,\text{cotg}\,x \cdot \cos x + \cos^2 x + 1 - 2 \cdot \text{sen}\,x + \text{sen}^2\,x -$
$- 1 + 2 \cdot \text{cossec}\,x - \text{cossec}^2\,x =$
$= -2 \cdot \dfrac{\cos x}{\text{sen}\,x} \cdot \cos x - 2 \cdot \text{sen}\,x + 2 \cdot \dfrac{1}{\text{sen}\,x} =$
$= \dfrac{-2 \cdot \cos^2 x - 2 \cdot \text{sen}^2\,x + 2}{\text{sen}\,x} = 0$

264. $f(x) - g(x) =$
$= \dfrac{\cos x + \cos y}{\text{sen}\,x - \text{sen}\,y} - \dfrac{\text{sen}\,x + \text{sen}\,y}{\cos y - \cos x} = \dfrac{\cos^2 y - \cos^2 x - \text{sen}^2\,x + \text{sen}^2\,y}{(\text{sen}\,x - \text{sen}\,y)(\cos y - \cos x)} = 0$

265. $f(x) = \dfrac{\cos x + \dfrac{\cos x}{\text{sen}\,x}}{\dfrac{\text{sen}\,x}{\cos x} + \dfrac{1}{\cos x}} = \dfrac{(\text{sen}\,x \cdot \cos x + \cos x) \cdot \cos x}{\text{sen}\,x\,(\text{sen}\,x + 1)} =$
$= \dfrac{\cos^2 x\,(\text{sen}\,x + 1)}{\text{sen}\,x(\text{sen}\,x + 1)} = \cos x \cdot \text{cotg}\,x = g(x)$

266. $f(x) = \dfrac{\text{sen}^2\,x - \cos^2 y + \cos^2 x \cos^2 y}{\cos^2 x \cdot \cos^2 y} = \dfrac{\text{sen}^2\,x - \cos^2 y\,(1 - \cos^2 x)}{\cos^2 x \cdot \cos^2 y} =$
$= \dfrac{\text{sen}^2\,x\,(1 - \cos^2 y)}{\cos^2 x \cdot \cos^2 y} = \text{tg}^2\,x \cdot \text{tg}^2\,y = g(x)$

267. $g(x) = \text{cossec}^2\,x - 2 \cdot \text{cossec}\,x \cdot \text{cotg}\,x + \text{cotg}^2\,x = \dfrac{1}{\text{sen}^2\,x} - \dfrac{2\cos x}{\text{sen}^2\,x} + \dfrac{\cos^2 x}{\text{sen}^2\,x} =$
$= \dfrac{(1 - \cos x)^2}{1 - \cos^2 x} = \dfrac{(1 - \cos x)^2}{(1 + \cos x)(1 - \cos x)} = f(x)$

268. $f(x) = \dfrac{\dfrac{\cos x}{\text{sen}\,x} + \dfrac{\cos y}{\text{sen}\,y}}{\dfrac{\text{sen}\,x}{\cos x} + \dfrac{\text{sen}\,y}{\cos y}} =$
$= \dfrac{(\cos x \cdot \text{sen}\,y + \cos y \cdot \text{sen}\,x)}{\text{sen}\,x \cdot \text{sen}\,y} \cdot \dfrac{(\cos x \cdot \cos y)}{(\text{sen}\,x \cdot \cos y + \text{sen}\,y \cdot \cos x)} =$
$= \text{cotg}\,x \cdot \text{cotg}\,y = g(x)$

269. $f(x) = \sec^2 x \cdot \sec^2 y + 2 \cdot \sec x \cdot \sec y \cdot tg\, x \cdot tg\, y + tg^2 x \cdot tg^2 y =$

$= (1 + tg^2 x)(1 + tg^2 y) + 2 \cdot \sec x \cdot \sec y \cdot tg\, x \cdot tg\, y + tg^2 x \cdot tg^2 y =$

$= 1 + \underline{\underline{tg^2 x}} + \underline{tg^2 y} + \underline{tg^2 x \cdot tg^2 y} + 2 \cdot \sec x \cdot \sec y \cdot tg\, x \cdot tg\, y + \underline{\underline{tg^2 x \cdot tg^2 y}} =$

$= 1 + tg^2 y\,(1 + tg^2 x) + 2 \cdot \sec x \cdot \sec y \cdot tg\, x \cdot tg\, y + tg^2 x\,(1 + tg^2 y) =$

$= 1 + tg^2 y \cdot \sec^2 x + 2 \cdot \sec x \cdot \sec y \cdot tg\, x \cdot tg\, y + tg^2 x \cdot \sec^2 y = g(x)$

270. $f(x) - g(x) = \dfrac{(\sec x - tg\, x)(\sec x + tg\, x) - 1}{\sec x + tg\, x} = \dfrac{\sec^2 x - tg^2 x - 1}{\sec x + tg\, x} = 0$

271. $f(x) = (\mathrm{cossec}^2 x - \mathrm{cotg}^2 x)(\mathrm{cossec}^4 x + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x + \mathrm{cotg}^4 x) =$

$= (1 + \mathrm{cotg}^2 x - \mathrm{cotg}^2 x)[(1 + \mathrm{cotg}^2 x)^2 + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x + \mathrm{cotg}^4 x] =$

$= 1 + 2\,\mathrm{cotg}^2 x + 2\,\mathrm{cotg}^4 x + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x =$

$= 1 + 2\,\mathrm{cotg}^2 x\,(1 + \mathrm{cotg}^2 x) + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x =$

$= 1 + 3\,\mathrm{cotg}^2 x \cdot \mathrm{cossec}^2 x = g(x)$

273. $f(x) = tg^2\,(45° + x) = \left[\dfrac{tg\, 45° + tg\, x}{1 - tg\, 45°\, tg\, x}\right]^2 = \left[\dfrac{\cos x + \mathrm{sen}\, x}{\cos x - \mathrm{sen}\, x}\right]^2 =$

$= \dfrac{1 + 2\,\mathrm{sen}\, x \cos x}{1 - 2\,\mathrm{sen}\, x \cos x} = g(x)$

275. $\dfrac{\pi}{4} < a < \dfrac{\pi}{2} \Rightarrow 0{,}7 < \mathrm{sen}\, a < 1;\ 0 < \cos a < 0{,}71$

$\dfrac{\pi}{4} < b < \dfrac{\pi}{2} \Rightarrow 0{,}7 < \mathrm{sen}\, b < 1;\ 0 < \cos b < 0{,}71$

$\mathrm{sen}\,(a + b) - \mathrm{sen}\, a - \dfrac{4}{5}\,\mathrm{sen}\, b = \mathrm{sen}\, a\,(\cos b - 1) + \mathrm{sen}\, b\,(\cos a - 0{,}8) < 0 \Rightarrow$

$\Rightarrow \mathrm{sen}\,(a + b) < \mathrm{sen}\, a + \dfrac{4}{5}\,\mathrm{sen}\, b$

276. $f(x) = \left[\mathrm{sen}\, A + \mathrm{sen}\left(\dfrac{\pi}{2} - A\right)\right]^4 = \left[\sqrt{2} \cdot \cos\left(A - \dfrac{\pi}{4}\right)\right]^4 =$

$= 4 \cos^4\left(A - \dfrac{\pi}{4}\right) = g(x)$

278. a) $\mathrm{sen}\, B + \mathrm{sen}\, C - \mathrm{sen}\, A = 2\,\mathrm{sen}\,\dfrac{B + C}{2} \cdot \cos\dfrac{B - C}{2} - 2\,\mathrm{sen}\,\dfrac{A}{2} \cdot \cos\dfrac{A}{2} =$

$= 2 \cos\dfrac{A}{2} \cdot \cos\dfrac{B - C}{2} - 2\,\mathrm{sen}\,\dfrac{A}{2} \cdot \cos\dfrac{A}{2} =$

$= 2 \cos\dfrac{A}{2}\left[\cos\dfrac{B - C}{2} - \cos\dfrac{B + C}{2}\right] =$

$= 2 \cos\dfrac{A}{2}\left[-2\,\mathrm{sen}\,\dfrac{B}{2}\,\mathrm{sen}\left(\dfrac{-C}{2}\right)\right] = 4 \cos\dfrac{A}{2}\,\mathrm{sen}\,\dfrac{B}{2}\,\mathrm{sen}\,\dfrac{C}{2}$

b) $\cos B + \cos C - \cos A =$

$$= 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} - \left(1 - 2\,\text{sen}^2\frac{A}{2}\right) =$$

$$= -1 + 2\,\text{sen}\frac{A}{2}\left(\cos\frac{B-C}{2} + \text{sen}\frac{A}{2}\right) =$$

$$= -1 + 2\,\text{sen}\frac{A}{2}\left(\cos\frac{B-C}{2} + \cos\frac{B+C}{2}\right) =$$

$$= -1 + 2\,\text{sen}\frac{A}{2}\left[2\cos\frac{B}{2}\cdot\cos\left(\frac{-C}{2}\right)\right] =$$

$$= -1 + 4\cos\frac{B}{2}\cdot\cos\frac{C}{2}\cdot\text{sen}\frac{A}{2}$$

c) $\cos 2A + \cos 2B + \cos 2C = 2\cos^2 A - 1 + 2\cos(B + C)\cdot\cos(B - C) =$

$= 2\cos^2 A - 1 + 2(-\cos A)\cos(B - C) =$

$= -1 + 2\cos A\,[\cos A - \cos(B - C)]$

$= -1 - 2\cos A\,[\cos(B + C) + \cos(B - C)] =$

$= -1 - 2\cos A\,(2\cos B\cos C) = -1 - 4\cos A\cos B\cos C$

d) $\text{sen}^2 A + \text{sen}^2 B + \text{sen}^2 C = \dfrac{1 - \cos 2A}{2} + \dfrac{1 - \cos 2B}{2} + \dfrac{1 - \cos 2C}{2} =$

$$= \frac{3 + 1 + 4\cos A\cos B\cos C}{2} = 2(1 + \cos A\cos B\cos C)$$

e) $A + B + C = \pi \Rightarrow A + B = \pi - C;$

$\text{cotg}(A + B) = \text{cotg}(\pi - C) = -\text{cotg}\,C \Rightarrow \dfrac{\text{cotg}\,A\cdot\text{cotg}\,B - 1}{\text{cotg}\,A + \text{cotg}\,B} =$

$= -\text{cotg}\,C \Rightarrow \text{cotg}\,A\cdot\text{cotg}\,B - 1 = -\text{cotg}\,A\cdot\text{cotg}\,C - \text{cotg}\,B\cdot\text{cotg}\,C \Rightarrow$

$$\Rightarrow \frac{1}{\text{tg}\,A\cdot\text{tg}\,B} + \frac{1}{\text{tg}\,B\cdot\text{tg}\,C} + \frac{1}{\text{tg}\,C\cdot\text{tg}\,A} = 1$$

279. a) $f(a) = \text{sen}\,4a = 2\,\text{sen}\,2a\cdot\cos 2a = 4\,\text{sen}\,a\cdot\cos a\,(\cos^2 a - \text{sen}^2 a) =$

$= 4\,\text{sen}\,a\cdot\cos^3 a - 4\,\text{sen}^3 a\cdot\cos a = g(a)$

b) $f(a) = \cos 4a = 2\cos^2 2a - 1 = 2(2\cos^2 a - 1)^2 - 1 =$

$= 8\cos^4 a - 8\cos^2 a + 1 = g(a)$

c) $f(a) = \dfrac{2\cdot\text{tg}\,2a}{1 - \text{tg}^2 2a} = (2\cdot\text{tg}\,2a)\cdot\dfrac{1}{1 - \text{tg}^2 2a} = \dfrac{4\cdot\text{tg}\,a}{1 - \text{tg}^2 a}\cdot\dfrac{1}{1 - \left(\dfrac{2\cdot\text{tg}\,a}{1 - \text{tg}^2}\right)^2} =$

$$\frac{4\cdot\text{tg}\,a}{1 - \text{tg}^2 a}\cdot\frac{(1 - \text{tg}^2 a)^2}{(1 - \text{tg}^2 a) - (2\cdot\text{tg}\,a)^2} = \frac{4\cdot\text{tg}\,a - 4\cdot\text{tg}^3 a}{1 - 6\cdot\text{tg}^2 a + \text{tg}^4 a} = g(a)$$

280. Prova pelo princípio da indução finita.

Para $n = 1$, $\dfrac{\text{sen}\,2a}{2\,\text{sen}\,a} = \dfrac{2\,\text{sen}\,a\cos a}{2\,\text{sen}\,a} = \cos a.$

Admitindo para $n = k$:

Hip. Ind. $\cos a \cdot \cos 2a \cdot \ldots \cdot \cos (2^{k-1} \cdot a) = \dfrac{\text{sen } 2^k \cdot a}{2^k \cdot \text{sen } a}$.

provemos que vale para $n = k + 1$:

$$1 \cdot \cos a \cdot \cos 2a \cdot \ldots \cdot \cos (2^{k-1} \cdot a) \cdot \cos 2^k \cdot a \underset{\text{H.I.}}{=} \frac{\text{sen } 2^k \cdot a}{2^k \cdot \text{sen } a} \cdot \cos 2^k \cdot a =$$

$$= \frac{1 \cdot \text{sen } 2 \cdot (2^k \cdot a)}{2 \cdot 2^k \cdot \text{sen } a} = \frac{\text{sen } 2^{k+1} \cdot a}{2^{k+1} \cdot \text{sen } a}.$$

281. a) $\text{cotg } \dfrac{\alpha}{2} - \text{cotg } \alpha = \dfrac{1}{\text{tg } \dfrac{\alpha}{2}} - \dfrac{1}{\text{tg } \alpha} =$

$$= \frac{1}{\text{tg } \dfrac{\alpha}{2}} - \frac{1 - \text{tg}^2 \dfrac{\alpha}{2}}{2 \cdot \text{tg } \dfrac{\alpha}{2}} = \frac{-1 + \text{tg}^2 \dfrac{\alpha}{2}}{2 \cdot \text{tg } \dfrac{\alpha}{2}} =$$

$$= \frac{\sec^2 \dfrac{\alpha}{2}}{2 \cdot \text{tg } \dfrac{\alpha}{2}} = \frac{1}{2 \cdot \text{sen } \dfrac{\alpha}{2} \cdot \cos \dfrac{\alpha}{2}} = \frac{1}{\text{sen } \alpha}$$

b) $\dfrac{1}{\text{sen } a} + \dfrac{1}{\text{sen } 2a} + \dfrac{1}{\text{sen } 4a} + \ldots + \dfrac{1}{\text{sen } 2^n \cdot a} =$

$$= \left(\text{cotg } \frac{a}{2} - \text{cotg } a\right) + (\text{cotg } a - \text{cotg } 2a) + \ldots + (\text{cotg } 2^{n-2}a - \text{cotg } 2^{n-1}a) +$$

$$+ (\text{cotg } 2^{n-1}a - \text{cotg } 2^n a) = \text{cotg } \frac{a}{2} - \text{cotg } 2^n a$$

282. $\cos^2 x + \text{sen}^2 x + 2 \text{ sen } x \cos x + k \text{ sen } x \cos x - 1 = 0 \Rightarrow$
$\Rightarrow \text{ sen } x \cos x \,(2 + k) = 0 \Rightarrow 2 + k = 0 \Rightarrow k = -2$

285. a) $\dfrac{(-\text{sen } x)(-\text{sen } x)}{-\dfrac{\text{sen } x}{\cos x}(-\cos x)} = \dfrac{\text{sen}^2 x}{\text{sen } x} = \text{sen } x$

b) $\dfrac{\text{sen } x \,(-\text{cotg } x)}{-\text{tg } x \,(-\text{sen } x)} = \dfrac{-\text{cotg } x}{\text{tg } x} = -\text{cotg}^2 x$

c) $\dfrac{-\sec x \,(-\text{cotg } x)}{\text{cossec } x \,(-\text{cotg } x)} = \dfrac{-\text{sen } x}{\cos x} = -\text{tg } x$

d) $-\cos x + \cos x + \text{cotg } x = \text{cotg } x$

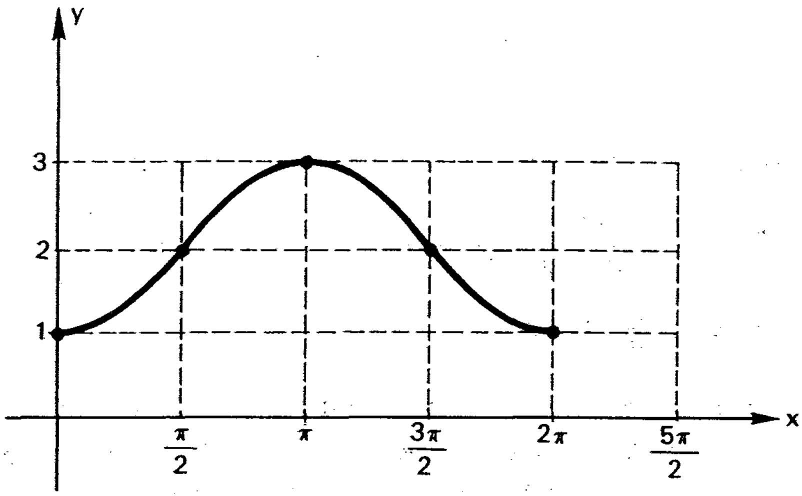
286. $1 - \text{sen } x \cdot \text{sen } x = 1 - \text{sen}^2 x = \cos^2 x$

287. $-1 + \dfrac{(-\text{sen } x) \cdot (-\text{tg } x)}{-\cos x} = -1 - \dfrac{\text{sen}^2 x}{\cos^2 x} = -1 - \text{tg}^2 x = -\sec^2 x$

288. $\dfrac{-a^2 - (a - b)^2(-1) + 2ab}{b^2} = \dfrac{b^2}{b^2} = 1$

289. $y = -\cos x + 2$

$Im(f) = [1, 3]$; $p = 2\pi$, $D(f) = \mathbb{R}$



Capítulo XI — Equações

293. $\text{sen}^2\, x = t \Rightarrow 4t^2 - 11t + 6 = 0 \Rightarrow t = 2$ ou $t = \dfrac{3}{4}$; $\text{sen}^2\, x = 2$ não serve;

$\text{sen}^2\, x = \dfrac{3}{4} \Rightarrow \text{sen}\, x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow x = \pm\dfrac{\pi}{3} + k\pi$

295. a) $5x = 3x + 2k\pi \Rightarrow x = k\pi$

ou

$5x = \pi - 3x + 2k\pi \Rightarrow x = \dfrac{\pi}{8} + \dfrac{k\pi}{4}$

b) $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$

ou

$3x = \pi - 2x + 2k\pi \Rightarrow x = \dfrac{\pi}{5} + 2k\dfrac{\pi}{5}$

296. $\text{sen}\, x < 0 \Rightarrow 2 \cdot \text{sen}\, x \cdot (-\text{sen}\, x) + 3 \cdot \text{sen}\, x = 2 \Rightarrow$

$\Rightarrow \nexists x \in \mathbb{R} \mid 2\, \text{sen}\, x \mid \text{sen}\, x \mid + 3\, \text{sen}\, x - 2 = 0$

$\text{sen}\, x > 0 \Rightarrow \text{sen}\, x = t \Rightarrow 2t^2 + 3t - 2 = 0 \Rightarrow t = \dfrac{1}{2}$ ou $t = -2$;

$\text{sen}\, x = -2$ não serve; $\text{sen}\, x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$

297. $\operatorname{sen}(x + y) = \operatorname{sen} 0 \Rightarrow \left.\begin{matrix} x + y = k\pi \\ x - y = \pi \end{matrix}\right\} \oplus\ x = \frac{\pi}{2} + \frac{k\pi}{2}$ e $y = \frac{-\pi}{2} + \frac{k\pi}{2}$

302. a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \frac{k\pi}{2}$

b) $5x = x + \frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{\pi}{12} + \frac{k\pi}{12}$ ou $5x = -x - \frac{\pi}{3} + 2k\pi \Rightarrow$

$\Rightarrow x = \frac{-\pi}{18} + \frac{k\pi}{3}$

303. a) $(\operatorname{sen} x + \cos x)(\sec x + \operatorname{cossec} x) = 4 \Rightarrow (\operatorname{sen} x + \cos x)\left(\frac{\operatorname{sen} x + \cos x}{\cos x \operatorname{sen} x}\right) =$

$\frac{\operatorname{sen}^2 x + 2 \operatorname{sen} x \cos x + \cos^2 x}{\cos x \operatorname{sen} x} = 4 \Rightarrow \operatorname{sen} 2x = 1 \Rightarrow 2x = \frac{\pi}{2} + 2k\pi$

$\Rightarrow x = \frac{\pi}{4} + k\pi$

b) $\operatorname{sen} x = \cos y \Rightarrow \cos^2 x = \operatorname{sen}^2 y \Rightarrow \cos x = \pm \operatorname{sen} y$

Notemos que $\cos x = -\operatorname{sen} y \Rightarrow \sec x = -\operatorname{cossec} y \Rightarrow$

$\Rightarrow (\operatorname{sen} x + \cos x)(\sec x + \operatorname{cossec} y) = 0 \neq 4$

então só interessa a hipótese *cos x = sen y*.

Temos:

$(\operatorname{sen} x + \cos y)(\sec x + \operatorname{cossec} y) = 4 \Rightarrow$

$\Rightarrow (\operatorname{sen} x + \operatorname{sen} x)(\sec x + \sec x) = 4 \Rightarrow \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

e daí $y = \frac{\pi}{4} + k\pi$.

304. $\operatorname{sen}^2 x = y \Rightarrow y^3 + y^2 + y = 3 \Rightarrow (y - 1)(y^2 + 2y + 3) = 0 \Rightarrow y =$

$\operatorname{sen} x = \pm 1 \Rightarrow x = \frac{\pi}{2} + k\pi \Rightarrow x = (2k + 1)\frac{\pi}{2}$

305. $2 \operatorname{sen} \frac{\pi}{4} \cdot \cos x = \sqrt{2} \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi$

306. $x + y = \pi \Rightarrow x = \pi - y \Rightarrow \operatorname{sen} x = \operatorname{sen} y$

$\operatorname{sen} x + \operatorname{sen} y = \log_{10} t^2 \Rightarrow 2 \operatorname{sen} x = 2 \log_{10} t \Rightarrow \operatorname{sen} x = \log_{10} t$

$-1 < \log_{10} t \leqslant 1 \Rightarrow 0{,}1 < t \leqslant 10$

310. a) $1 + \operatorname{tg}^2 x - 2 \operatorname{tg} x = 0$, $\operatorname{tg} x = t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1$

$\operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

b) $\operatorname{cossec}^2 x = 1 - \operatorname{cotg} x \Rightarrow 1 + \operatorname{cotg}^2 x = 1 - \operatorname{cotg} x \Rightarrow$

$\Rightarrow \operatorname{cotg}^2 x = \operatorname{cotg} x = 0 \Rightarrow \operatorname{cotg} x = 0$ ou $\operatorname{cotg} x = -1 \Rightarrow$

$\Rightarrow x = \frac{\pi}{2} + k\pi$ ou $x = \frac{3\pi}{4} + k\pi$

c) $\text{sen } 2x \cdot \cos\left(x + \frac{\pi}{4}\right) - \cos 2x \cdot \text{sen}\left(x + \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow \text{sen}\left[2x - \left(x + \frac{\pi}{4}\right)\right] = 0 \Rightarrow \text{sen}\left(x - \frac{\pi}{4}\right) = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$

d) $1 + \text{sen } 2x - \text{tg } x - \text{tg } x \text{ sen } 2x = 1 + \text{tg } x \Rightarrow \text{sen } 2x = \frac{2 \text{ tg } x}{1 - \text{tg } x} \Rightarrow$

$\frac{2 \text{ tg } x}{1 + \text{tg}^2 x} = \frac{2 \text{ tg } x}{1 - \text{tg } x} \Rightarrow \text{tg}^2 x + \text{tg } x = 0 \Rightarrow$

$\Rightarrow \text{tg } x (\text{tg } x + 1) = 0 \Rightarrow \text{tg } x = 0 \Rightarrow x = k\pi$ ou $\text{tg } x = -1 \Rightarrow$

$\Rightarrow x = \frac{3\pi}{4} + k\pi$

311. $\frac{1}{\text{tg } x} = \frac{2 \text{ tg } x}{1 + \text{tg}^2 x} \Rightarrow \text{tg}^2 x = 1 \Rightarrow \text{tg } x = \pm 1 \Rightarrow x = \pm\frac{\pi}{4} + k\pi$

312. $\text{tg}^2 \frac{\pi}{2} p = 1 \Rightarrow \text{tg} \frac{\pi}{2} p = \pm 1 \Rightarrow \frac{\pi}{2} p = \pm\frac{\pi}{4} + k\pi \Rightarrow p = \pm\frac{1}{2} + 2k \;\; k \in \mathbb{Z}$

313. $\text{tg } x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

ou

$\text{sen } x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \pm\frac{\pi}{3} + 2k\pi$ ou $x = \pm\frac{2\pi}{3} + 2k\pi$

então as raízes positivas são $\frac{\pi}{4}, \frac{5\pi}{4}, \ldots, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$ e daí

$a = \frac{\pi}{4}$ (a menor delas). Temos:

$\text{sen}^4 a - \cos^2 a = \text{sen}^4 \frac{\pi}{4} - \cos^2 \frac{\pi}{4} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

314. $\Delta = 4 \text{ tg}^2 a + 4 = 4 \sec^2 a,\; x = \frac{2 \text{ tg } a \pm 2 \sec a}{2} \Rightarrow x = \frac{\text{sen } a \pm 1}{\cos a}$

316. a) $\text{sen } x + \text{sen}\left(\frac{\pi}{2} - x\right) = -1 \Rightarrow \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = -1 \Rightarrow$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\Rightarrow \left(\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4} \Rightarrow x = \pi + 2k\pi \text{ ou } \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{5\pi}{4} \Rightarrow\right.$

$\left.\Rightarrow x = \frac{3\pi}{2} + 2k\pi\right)$

b) $\operatorname{sen} x - \dfrac{1}{\sqrt{3}} \cos x = -1 \Rightarrow \operatorname{sen} x - \dfrac{\operatorname{sen} \dfrac{\pi}{6}}{\cos \dfrac{\pi}{6}} \cdot \cos x = 1 \Rightarrow$

$\Rightarrow \operatorname{sen}\left(x - \dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \left(\operatorname{sen}\left(x - \dfrac{\pi}{6}\right) = \operatorname{sen} \dfrac{4\pi}{3} \Rightarrow x = \dfrac{3\pi}{2} + 2k\pi \text{ ou } \operatorname{sen}\left(x - \dfrac{\pi}{6}\right) = \operatorname{sen} \dfrac{5\pi}{3}\right.$

$\left.\Rightarrow x = \dfrac{11\pi}{6} + 2k\pi\right)$

318. a) $\operatorname{sen} 4x = u$ e $\cos x = v$, $\begin{cases} u + v = 1 \; (1) \\ u^2 + v^2 = 1 \; (2) \end{cases}$, (1) em (2) $\Rightarrow u^2 + (1 - u)^2 = 1$

$\Rightarrow 2u^2 - 2u = 0$. Então: $u = 0$ e $v = 1 \Rightarrow \operatorname{sen} 4x = 0$ e $\cos 4x = 1 \Rightarrow x = \dfrac{k\pi}{2}$

ou $u = 1$ e $v = 0 \Rightarrow \operatorname{sen} 4x = 1$ e $\cos 4x = 0 \Rightarrow x = \dfrac{\pi}{8} + \dfrac{k\pi}{2}$

b) $\operatorname{sen} x = 0$ e $\cos x = \pm 1 \Rightarrow x = k\pi$

$\operatorname{sen} x = \pm 1$ e $\cos x = 0 \Rightarrow x = \dfrac{\pi}{2} + k\pi$

319. $\operatorname{sen} 2x = u$ e $\cos 2x = v$, $\begin{cases} u + v = 1 \; (1) \\ u^2 + v^2 = 1 \; (2) \end{cases}$, (1) em (2) $\Rightarrow u^2 + (1 - u)^2 = 1$

$\Rightarrow 2u^2 - 2u = 0$. Então: $u = 0$ e $v = 1 \Rightarrow \operatorname{sen} 2x = 0$ e $\cos 2x = 1 \Rightarrow x = k\pi$

ou $u = 1$ e $v = 0 \Rightarrow \operatorname{sen} 2x = 1$ e $\cos 2x = 0 \Rightarrow x = \dfrac{\pi}{4} + k\pi$

321. a) $\operatorname{sen} x = \dfrac{2t}{1 + t^2}$ e $\cos x = \dfrac{1 - t^2}{1 + t^2} \Rightarrow m \dfrac{(1 - t^2)}{1 + t^2} - (m + 1) \dfrac{2t}{1 + t^2} = m \Rightarrow$

$\Rightarrow t(-2mt - 2m - 2) = 0 \Rightarrow t = 0$ ou $-2mt - 2m - 2 = 0 \Rightarrow$

$\Rightarrow t = \dfrac{-(m + 1)}{m}$, $m \neq 0$

$m = 0 \Rightarrow 0 \cdot \cos x - \operatorname{sen} x = 0$, $\exists x$, $\forall m \in \mathbb{R}$.

b) $\operatorname{sen} x = \dfrac{2t}{1 + t^2}$ e $\cos x = \dfrac{1 - t^2}{1 + t^2} \Rightarrow \dfrac{2t}{1 + t^2} + \dfrac{1 - t^2}{1 + t^2} = m \Rightarrow$

$\Rightarrow (-m - 1)t^2 + 2t + 1 - m = 0 \Rightarrow \Delta = -4m^2 + 8 \geqslant 0 \Rightarrow$

$\Rightarrow -\sqrt{2} \leqslant m \leqslant \sqrt{2}$

323. a) $2 \operatorname{sen}\left(\dfrac{mx + nx}{2}\right) \cdot \cos\left(\dfrac{mx - nx}{2}\right) = 0$. Então: $\operatorname{sen} \dfrac{(m + n)x}{2} = 0 \Rightarrow$

$\Rightarrow x = \dfrac{2k\pi}{m + n}$

ou $\cos \dfrac{(m - n)x}{2} = 0 \Rightarrow x = \dfrac{\pi}{m - n} + \dfrac{2k\pi}{m - n}$

b) $2 \cos \dfrac{ax + bx}{2} \cdot \cos \dfrac{ax - bx}{2} = 0$. Então: $\cos \dfrac{(a + b)x}{2} = 0 \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{a + b} + \dfrac{2k\pi}{a + b}$

ou $\cos \dfrac{(a - b)x}{2} = 0 \Rightarrow x = \dfrac{\pi}{a - b} + \dfrac{2k\pi}{a - b}$

c) $\text{sen } 2x - \text{sen}\left(\dfrac{\pi}{4} - x\right) = 0 \Rightarrow 2 \text{ sen}\left(\dfrac{3x}{2} - \dfrac{\pi}{8}\right) \cos\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) = 0$

Então:

$\text{sen}\left(\dfrac{3x}{2} - \dfrac{\pi}{8}\right) = 0 \Rightarrow x = \dfrac{\pi}{12} + \dfrac{2k\pi}{3}$ ou $\cos\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) = 0 \Rightarrow x = \dfrac{3\pi}{4} + 2k\pi$

325. a) $2 \text{ sen } 3x \cdot \cos 2x - 2 \text{ sen } 3x = 0 \Rightarrow 2 \text{ sen } 3x (\cos 2x - 1) = 0$. Então:

$\text{sen } 3x = 0 \Rightarrow x = \dfrac{k\pi}{3}$ ou $\cos 2x = 1 \Rightarrow x = k\pi$.

b) $2 \cos (2x + a) \cos (x + a) + \cos (2x + a) = 0 \Rightarrow$

$\Rightarrow \cos (2x + a)[2 \cos (x + a) + 1] = 0$. Então: $\cos (2x + a) = 0 \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{4} - \dfrac{a}{2} + \dfrac{k\pi}{2}$

ou $2 \cos (x + a) + 1 = 0 \Rightarrow \cos (x + a) = -\dfrac{1}{2} \Rightarrow x = \dfrac{2\pi}{3} - a + 2k\pi$

ou $x = \dfrac{4\pi}{3} - a + 2k\pi$

c) $2 \text{ sen } 4x \cdot \cos 3x - 2 \text{ sen } 4x \cdot \text{sen}(-x) = 0 \Rightarrow 2 \text{ sen } 4x (\cos 3x + \text{sen } x) = 0 \Rightarrow$

$\Rightarrow 2 \text{ sen } 4x \left(\cos 3x + \cos\left(\dfrac{\pi}{2} - x\right)\right) = 0 \Rightarrow$

$4 \text{ sen } 4x \cdot \cos\left(x + \dfrac{\pi}{4}\right) \cdot \cos\left(2x - \dfrac{\pi}{4}\right) = 0$

Então:

$\text{sen } 4x = 0 \Rightarrow x = \dfrac{k\pi}{4}$ ou $\cos\left(x + \dfrac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{4} + k\pi$ ou $\cos\left(2x - \dfrac{\pi}{4}\right) = 0 \Rightarrow x = -\dfrac{\pi}{8} + \dfrac{k\pi}{2}$

326. $\dfrac{1 + \cos (2x + 2a)}{2} + \dfrac{1 + \cos (2x - 2a)}{2} = 1 \Rightarrow \cos (2x + 2a) + \cos (2x - 2a) = 0$

$\Rightarrow 2\cos(2x)\cdot\cos(2a) = 0 \Rightarrow \cos(2x) = 0 \Rightarrow x = \dfrac{\pi}{4} + k\pi$ ou $x = \dfrac{3\pi}{4} + k\pi$

(supondo $\cos 2a \neq 0$).

327. $(\text{sen } 3x - \text{sen } x) + (\cos 2x - \cos 0) = 0 \Rightarrow$

$\Rightarrow 2 \cdot \text{sen } x \cdot \cos 2x - 2 \cdot \text{sen}^2 x = 0 \Rightarrow 2 \cdot \text{sen } x \cdot (\cos 2x - \text{sen } x) = 0$

Então:

$\text{sen } x = 0 \Rightarrow x = k\pi$

ou

$\cos 2x = \text{sen } x \Rightarrow \cos 2x = \cos\left(\dfrac{\pi}{2} - x\right) \Rightarrow 2x = \pm\left(\dfrac{\pi}{2} - x\right) + 2k\pi \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{6} + \dfrac{2k\pi}{3}$ ou $x = -\dfrac{\pi}{2} + 2k\pi$

328. $2 \cdot \text{sen } x \cdot \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} \Rightarrow \text{sen } x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$

329. a) Substituindo $x = 2\pi$ e $y = \dfrac{\pi}{2}$ na equação (1):

$\text{sen}\left(2\pi + \dfrac{\pi}{2}\right) + \text{sen}\left(2\pi - \dfrac{\pi}{2}\right) = 1 - 1 \neq 2.$

b) $\left.\begin{array}{l} 2 \cdot \text{sen } x \cdot \cos y = 2 \\ \text{sen } x + \cos y = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \text{sen } x \cdot \cos y = 1 \; (A) \\ \text{sen } x + \cos y = 2 \; (B) \end{array}\right\}$, (A) em (B) $\Rightarrow$ $\Rightarrow \text{sen}^2 x - 2\,\text{sen } x + 1 = 0$

Então:

$\text{sen } x = 1$ (C) $\Rightarrow x = \dfrac{\pi}{2} + 2k\pi$, (C) em (A) $\Rightarrow \cos y = 1 \Rightarrow y = 2k\pi$

330. $(\cos x + 1)(\text{sen } x + 1) = 0$. Então: $\cos x + 1 = 0 \Rightarrow x = \pi + 2k\pi$ ou

$\text{sen } x + 1 = 0 \Rightarrow x = \dfrac{3\pi}{2} + 2k\pi$

333. a) $(\cos^2 x + \text{sen}^2 x)^2 - 2 \cdot \text{sen}^2 x \cdot \cos^2 x = \dfrac{5}{8} \Rightarrow$

$\Rightarrow 1 - 2 \cdot \text{sen}^2 x \cdot \cos^2 x = \dfrac{5}{8} \Rightarrow 1 - \dfrac{1}{2}\text{sen}^2 2x = \dfrac{5}{8} \Rightarrow \text{sen}^2 2x = \dfrac{3}{4}.$

Então: $\text{sen } 2x = \dfrac{+\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{6} + k\pi$ ou $x = \dfrac{\pi}{3} + k\pi$;

$\text{sen } 2x = \dfrac{-\sqrt{3}}{2} \Rightarrow x = \dfrac{2\pi}{3} + k\pi$ ou $x = \dfrac{5\pi}{6} + k\pi$

b) $(\cos^2 x + \text{sen}^2 x)(\cos^4 x - \cos^2 x \cdot \text{sen}^2 x + \text{sen}^4 x) = \dfrac{5}{8} \Rightarrow$

$$\Rightarrow (\text{sen}^4 x + \cos^4 x) - \text{sen}^2 x \cdot \cos^2 x = \frac{5}{8} \Rightarrow$$

$$\Rightarrow \left(1 - \frac{1}{2} \cdot \text{sen}^2 2x\right) - \frac{1}{4} \cdot \text{sen}^2 2x = \frac{5}{8} \Rightarrow 1 - \frac{3}{4} \cdot \text{sen}^2 2x = \frac{5}{8}.$$

Então: $\text{sen } 2x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{8} + k\pi$ ou $x = \frac{3\pi}{8} + k\pi$ ou

$\text{sen } 2x = \frac{-\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{8} + k\pi$ ou $x = \frac{7\pi}{8} + k\pi$

c) $1 - \frac{1}{2} \cdot \text{sen}^2 2x = \frac{1}{2}$. Então: $\text{sen } 2x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

ou $\text{sen } 2x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$

d) $1 - \frac{3}{4} \cdot \text{sen}^2 x = \frac{7}{16}$. Então: $\text{sen } x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} = k\pi$ ou

$\text{sen } x = \frac{-\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3} + k\pi$

e) $(\text{sen } x + \cos x)(\text{sen}^2 x + \cos^2 x - \text{sen } x \cdot \cos x) = 1 \Rightarrow$

$\Rightarrow (\text{sen } x + \cos x)(1 - \text{sen } x \cdot \cos x) = 1$. Fazendo $\text{sen } x + \cos x = y$,

temos $(\text{sen } x + \cos x)^2 = y^2$ e daí: $\text{sen } x \cdot \cos x = \frac{y^2 - 1}{2}$.

A equação fica $y \cdot \left(1 - \frac{y^2 - 1}{2}\right) = 1 \Rightarrow (y - 1)^2(y + 2) = 0$

$\Rightarrow y = 1$ ou $y = -2$ não serve, pois $-\sqrt{2} \leqslant y \leqslant \sqrt{2}$; então $\text{sen } x + \cos x = 1 \Rightarrow$

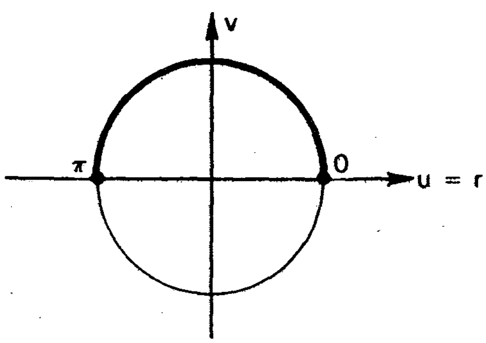
$$\Rightarrow \sqrt{2} \cdot \cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow x = \frac{\pi}{2} + 2k\pi \text{ ou } x = 2k\pi$$

Capítulo XII — Inequações

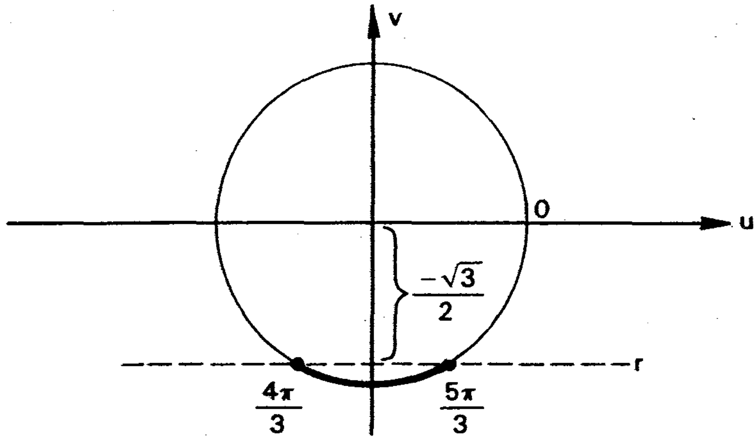
335. $\text{sen } x = 0 \Rightarrow x = 2k\pi$ ou $x = \pi + 2k\pi$

$S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \pi + 2k\pi\}$



336. $\operatorname{sen} x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{4\pi}{3} + 2k\pi$ ou $x = \dfrac{5\pi}{3} + 2k\pi$

$S = \{x \in \mathbb{R} \mid \dfrac{4\pi}{3} + 2k\pi \leqslant x \leqslant \dfrac{5\pi}{3} + 2k\pi\}$

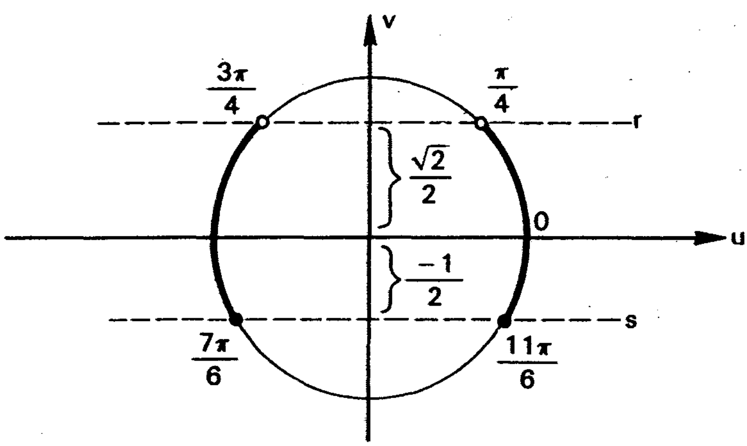


337. $\operatorname{sen} x = \dfrac{\sqrt{2}}{2} \Rightarrow x = \dfrac{\pi}{4} + 2k\pi$ ou $x = \dfrac{3\pi}{4} + 2k\pi$

$\operatorname{sen} x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6} + 2k\pi$ ou $x = \dfrac{11\pi}{6} + 2k\pi$

$S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x < \dfrac{\pi}{4} + 2k\pi$ ou $\dfrac{3\pi}{4} + 2k\pi < x \leqslant \dfrac{7\pi}{6} + 2k\pi$

ou $\dfrac{11\pi}{6} + 2k\pi \leqslant x < 2\pi + 2k\pi\}$



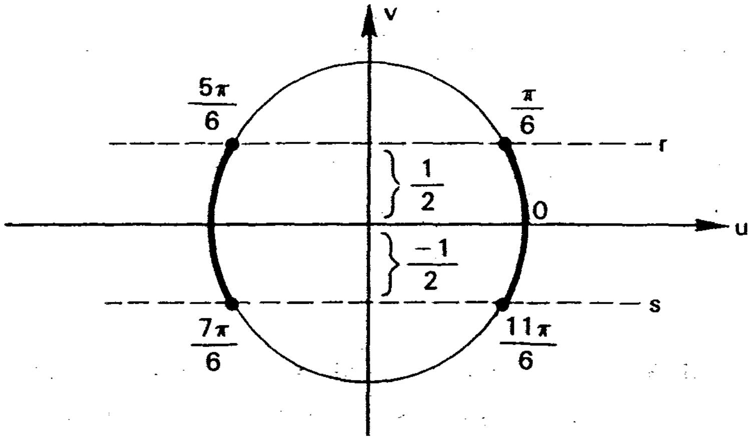
339. $\operatorname{sen} x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$

$\operatorname{sen} x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6} + 2k\pi$ ou $x = \dfrac{11\pi}{6} + 2k\pi$

$|\operatorname{sen} x| \leqslant \dfrac{1}{2} \Leftrightarrow -\dfrac{1}{2} \leqslant \operatorname{sen} x \leqslant \dfrac{1}{2}$

$S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi$ ou $\frac{5\pi}{6} + 2k\pi \leqslant x \leqslant \frac{7\pi}{6} + 2k\pi$

ou $\frac{11\pi}{6} + 2k\pi \leqslant x < 2\pi + 2k\pi\}$

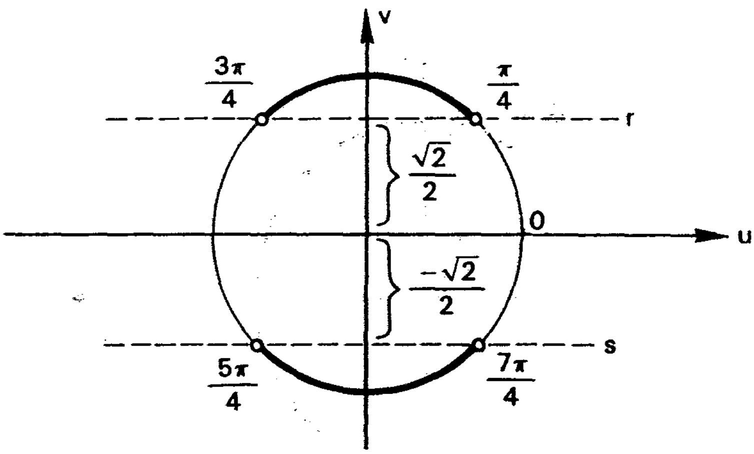


340. $|\text{sen } x| > \frac{\sqrt{2}}{2} \Leftrightarrow \text{sen } x > \frac{\sqrt{2}}{2}$ ou $\text{sen } x < -\frac{\sqrt{2}}{2}$

$\text{sen } x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} + 2k\pi$ ou $x = \frac{3\pi}{4} + 2k\pi$

$\text{sen } x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{4} + 2k\pi$ ou $x = \frac{7\pi}{4} + 2k\pi$

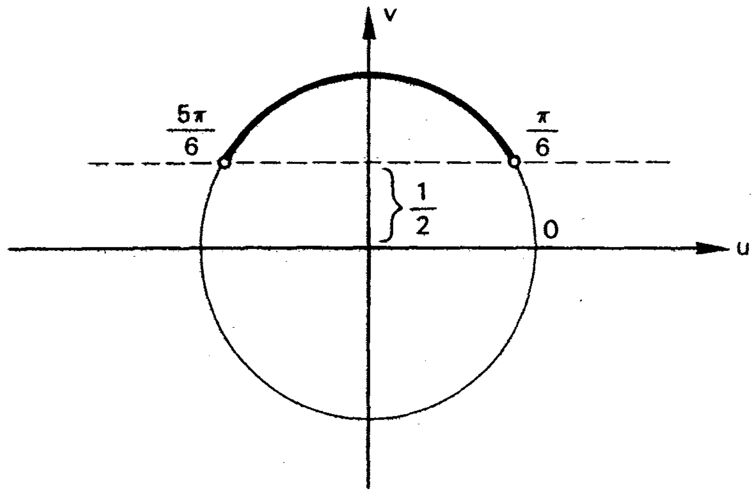
$S = \{x \in \mathbb{R} \mid \frac{\pi}{4} + 2k\pi < x < \frac{3\pi}{4} + 2k\pi$ ou $\frac{5\pi}{4} + 2k\pi < x < \frac{7\pi}{4} + 2k\pi\}$



342. a) $2 \text{ sen } x - 1 > 0 \Rightarrow \text{sen } x > \frac{1}{2}$

$\text{sen } x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

$S = \{x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi\}$



b) $2 \cdot \log_2 (2 \cdot \text{sen } x - 1) = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

$\Rightarrow \log_2 (2 \cdot \text{sen } x - 1)^2 = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

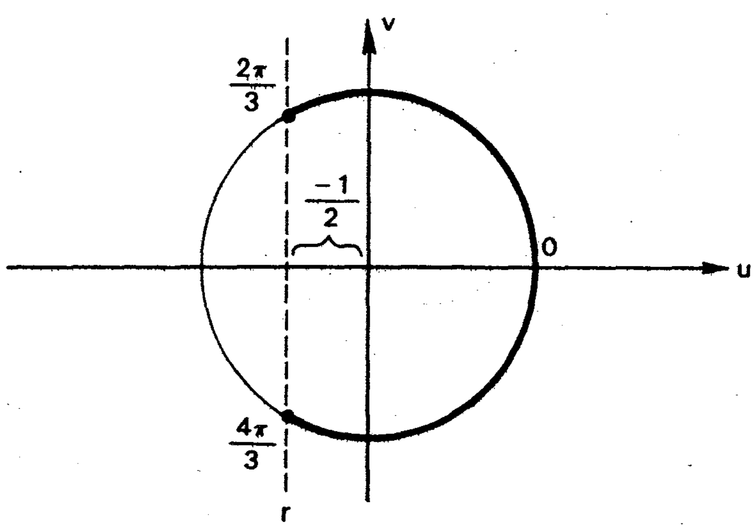
$\Rightarrow (2 \cdot \text{sen } x - 1)^2 = (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

$\Rightarrow \text{sen}^2 x = 1 \Rightarrow \text{sen } x = \pm 1$

Só convém *sen x = 1* devido à parte a), então $x = \frac{\pi}{2} + 2k\pi$.

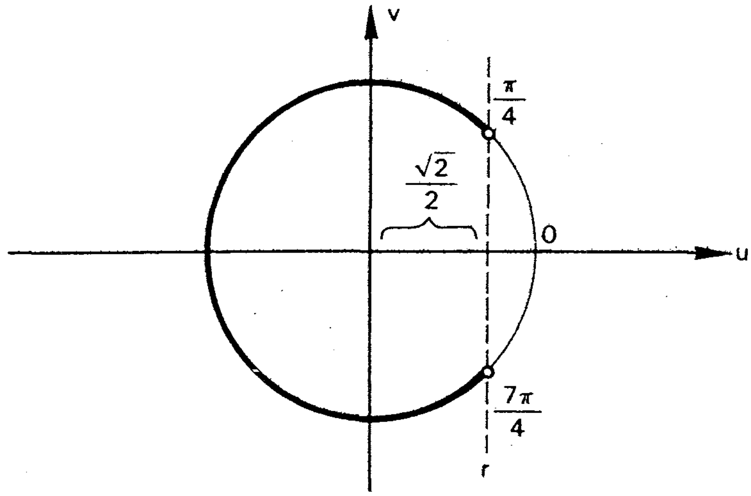
344. $\cos x = \frac{-1}{2} \Rightarrow x = \frac{2\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$

$S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \frac{2\pi}{3} + 2k\pi$ ou $\frac{4\pi}{3} + 2k\pi \leqslant x < 2\pi + 2k\pi\}$



345. $\cos x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} + 2k\pi$ ou $x = \frac{7\pi}{4} + 2k\pi$

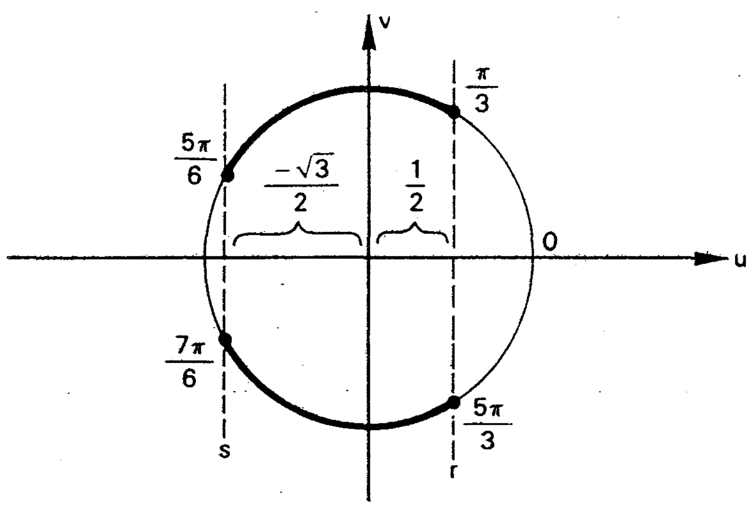
$S = \{x \in \mathbb{R} \mid \frac{\pi}{4} + 2k\pi < x < \frac{7\pi}{4} + 2k\pi\}$



346. $\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2k\pi$ ou $x = \frac{5\pi}{3} + 2k\pi$

$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6} + 2k\pi$ ou $x = \frac{7\pi}{6} + 2k\pi$

$S = \{x \in \mathbb{R} \mid \frac{\pi}{3} + 2k\pi \leqslant x \leqslant \frac{5\pi}{6} + 2k\pi$ ou $\frac{7\pi}{6} + 2k\pi \leqslant x \leqslant \frac{5\pi}{3} + 2k\pi\}$

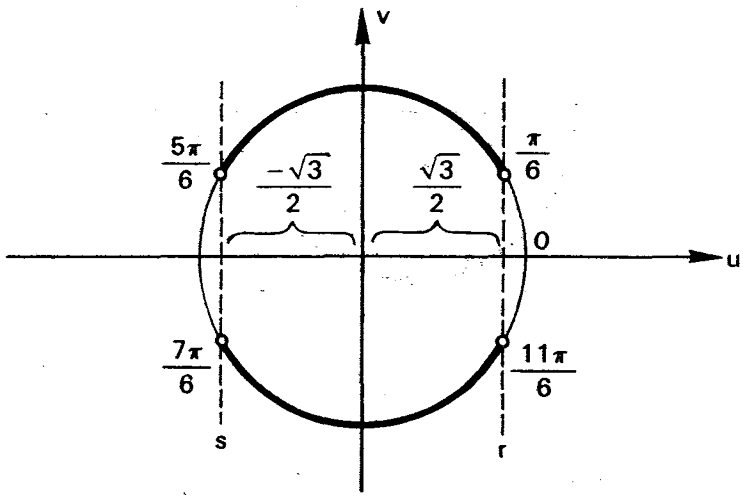


347. $|\cos x| < \frac{\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2} < \cos x < \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{7\pi}{6} + 2k\pi$ ou $x = \frac{11\pi}{6} + 2k\pi$

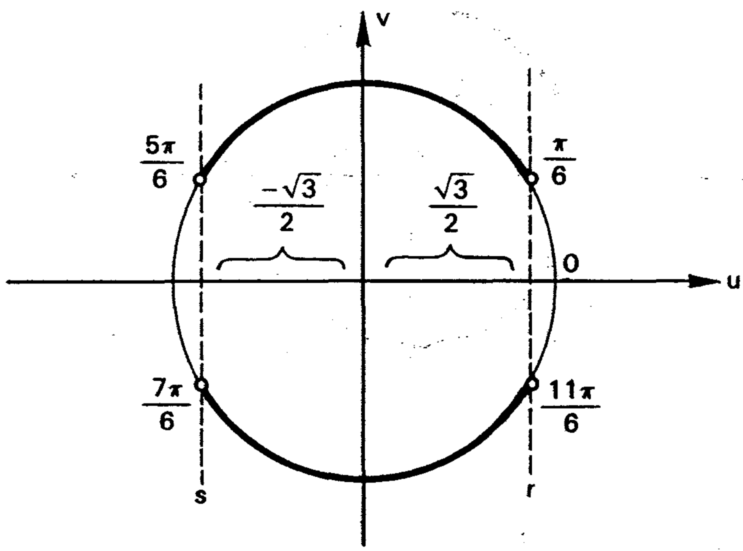
$$S = \{x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \text{ ou } \frac{7\pi}{6} + 2k\pi < x < \frac{11\pi}{6} + 2k\pi\}$$



348. $|\cos x| > \frac{5}{3}$, impossível, pois $-1 \leqslant \cos x \leqslant 1$; $S = \varnothing$

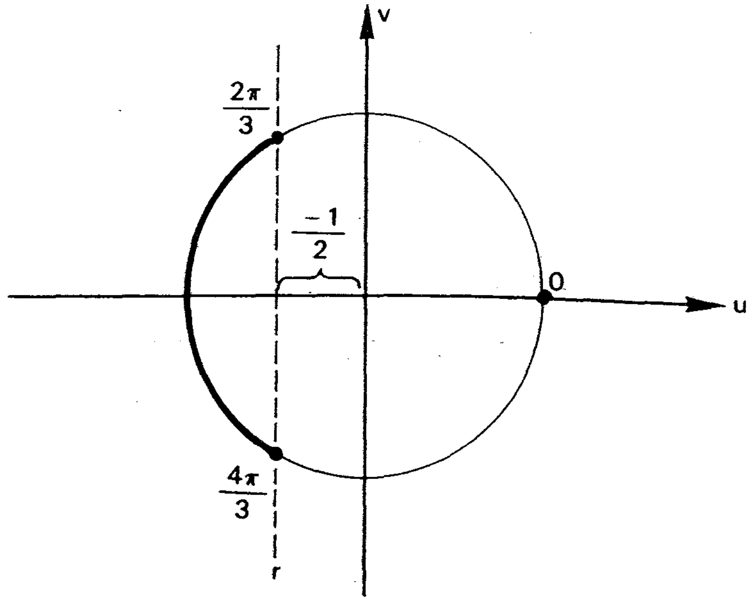
350. $\cos^2 x < \frac{3}{4} \Leftrightarrow -\frac{\sqrt{3}}{2} < \cos x < \frac{\sqrt{3}}{2}$

$$S = \{x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \text{ ou } \frac{7\pi}{6} + 2k\pi < x < \frac{11\pi}{6} + 2k\pi\}$$



351. $2\cos^2 x - 1 - \cos x \geqslant 0 \Leftrightarrow \cos x \leqslant -\frac{1}{2}$ ou $\cos x = 1$

$$S = \{x \in \mathbb{R} \mid \frac{2\pi}{3} + 2k\pi \leqslant x \leqslant \frac{4\pi}{3} + 2k\pi \text{ ou } x = 2k\pi\}$$



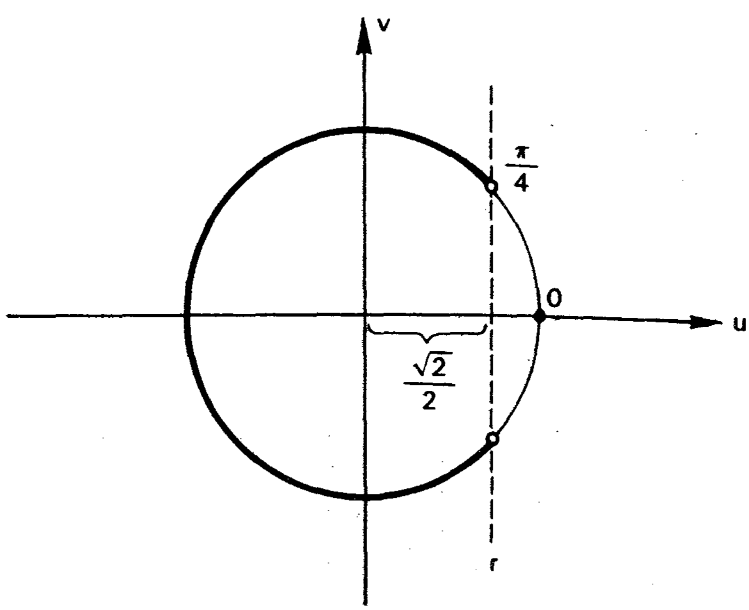
353. $\text{sen}\, x + \text{sen}\left(\frac{\pi}{2} - x\right) < 1 \Rightarrow \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) < 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) < \frac{\sqrt{2}}{2}$

Fazendo $x - \frac{\pi}{4} = y$, temos:

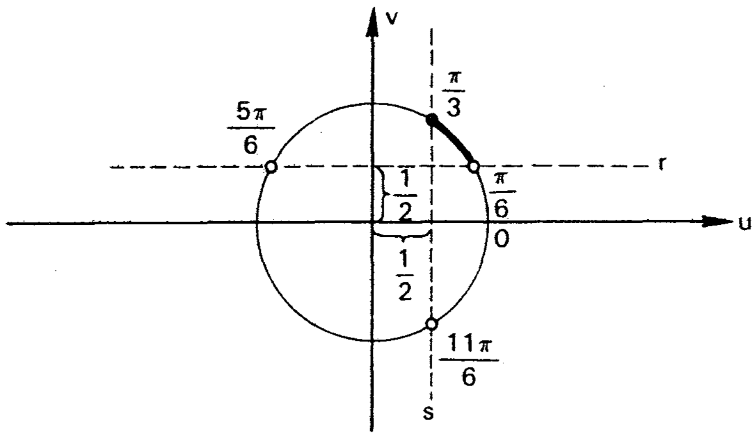
$\frac{\pi}{4} + 2k\pi < y < \frac{7\pi}{4} + 2k\pi$

então $\frac{\pi}{4} + 2k\pi < x - \frac{\pi}{4} < \frac{7\pi}{4} + 2k\pi$

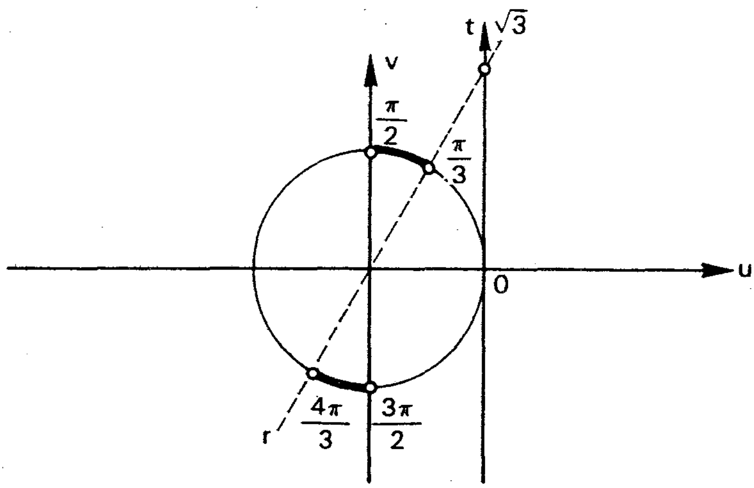
$S = \{x \in \mathbb{R} \mid \frac{\pi}{2} + 2k\pi < x < 2\pi + 2k\pi\}$



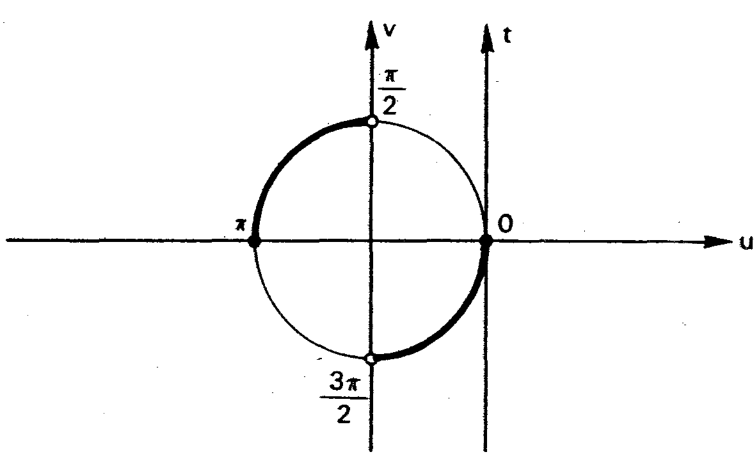
355. $S = \{x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x \leqslant \frac{\pi}{3} + 2k\pi\}$



357. $S = \{x \in \mathbb{R} \mid \frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi\}$

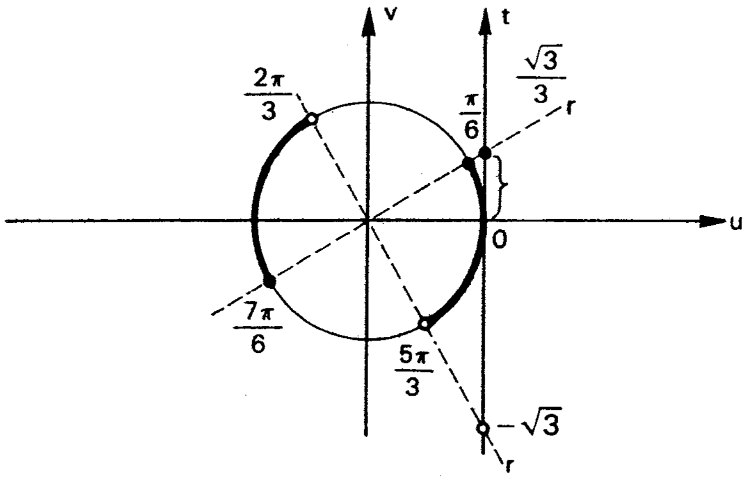


358. $S = \{x \in \mathbb{R} \mid \frac{\pi}{2} + k\pi < x \leqslant \pi + k\pi\}$



359. $S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi$ ou

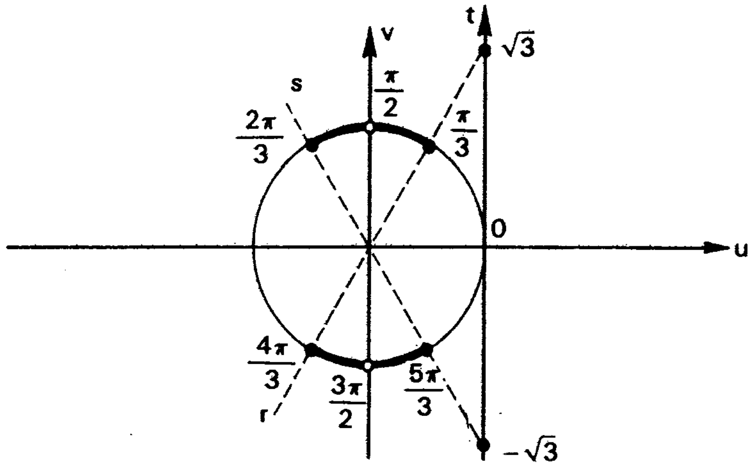
$\frac{2\pi}{3} + 2k\pi < x \leqslant \frac{7\pi}{6} + 2k\pi$ ou $\frac{5\pi}{3} + 2k\pi < x \leqslant 2\pi$ ou $2k\pi\}$



360. $\operatorname{tg} x \leqslant -\sqrt{3}$ ou $\operatorname{tg} x \geqslant \sqrt{3}$

$S = \{x \in \mathbb{R} \mid \frac{\pi}{3} + k\pi \leqslant x < \frac{\pi}{2} + k\pi$ ou

$\frac{\pi}{2} + k\pi < x \leqslant \frac{2\pi}{3} + k\pi\}$



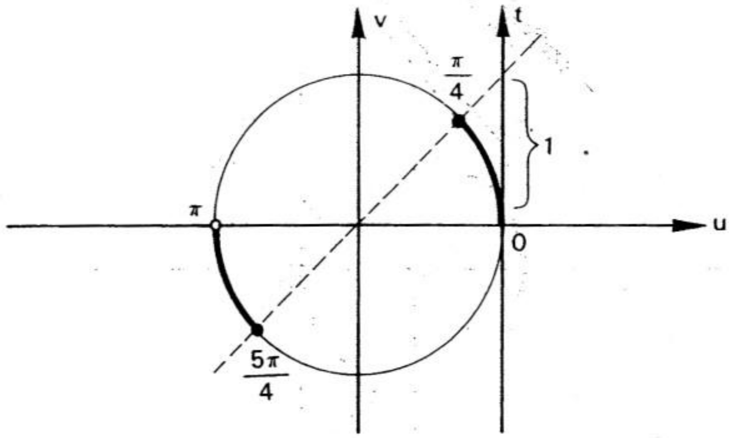
362. C.E. $\operatorname{tg} x > 0$ (A)

$\log y = \log a^{\log \operatorname{tg} x} \geqslant 0 \Rightarrow \log a^{\log \operatorname{tg} x} \geqslant \log 1 \Rightarrow \log a^{\log \operatorname{tg} x} \geqslant \log a^0 \Rightarrow$

$\Rightarrow a^{\log \operatorname{tg} x} \geqslant a^0 \Rightarrow \log \operatorname{tg} x \leqslant 0 \Rightarrow \log \operatorname{tg} x \leqslant \log 1 \Rightarrow \operatorname{tg} x \leqslant 1$ (B)

De Ⓐ e Ⓑ $\Rightarrow \quad 0 < \text{tg } x \leqslant 1$

$$S = \left\{ x \in \mathbb{R} \mid 0 < x \leqslant \frac{\pi}{4} \text{ ou } \pi < x \leqslant \frac{5\pi}{4} \right\}$$



Capítulo XIII – Funções circulares inversas

363. a) $\alpha = \text{arc sen } 0 \Leftrightarrow \text{sen } \alpha = 0 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = 0$

b) $\alpha = \text{arc sen } \frac{\sqrt{3}}{2} \Leftrightarrow \text{sen } \alpha = \frac{\sqrt{3}}{2} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{3}$

c) $\alpha = \text{arc sen } \left(-\frac{1}{2}\right) \Leftrightarrow \text{sen } \alpha = -\frac{1}{2} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{6}$

d) $\alpha = \text{arc sen } 1 \Leftrightarrow \text{sen } \alpha = 1 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2}$

e) $\alpha = \text{arc sen } (-1) \Leftrightarrow \text{sen } \alpha = -1 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{2}$

367. a) $\alpha = \text{arc sen } \left(-\frac{2}{3}\right) \Rightarrow \text{sen } \alpha = -\frac{2}{3}, -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

$\beta = \text{arc sen } \frac{1}{4} \Rightarrow \text{sen } \beta = \frac{1}{4}, -\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$

$\text{sen}^2 \alpha = \frac{\text{tg}^2 \alpha}{1 + \text{tg}^2 \alpha} \Rightarrow \text{tg } \alpha = \frac{-2\sqrt{5}}{5}; \text{sen}^2 \beta = \frac{\text{tg}^2 \beta}{1 + \text{tg}^2 \beta} \Rightarrow \text{tg } \beta = \frac{\sqrt{15}}{15}$

$\text{tg } (\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \cdot \text{tg } \beta} = \frac{\sqrt{5}(-6 + \sqrt{3})}{15 + 2\sqrt{3}}$

b) $\alpha = \text{arc sen } \left(-\frac{3}{5}\right) \Rightarrow \text{sen } \alpha = -\frac{3}{5} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

$\cos \alpha = \sqrt{1 - \text{sen}^2 \alpha} = \frac{4}{5}; \text{sen } 2\alpha = 2 \cdot \text{sen } \alpha \cdot \cos \alpha = -\frac{24}{25}$

c) $\beta = \text{arc sen } \frac{12}{13} \Rightarrow \text{sen } \beta = \frac{12}{13}$ e $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$;

$\cos \beta = \sqrt{1 - \text{sen}^2 \beta} = \frac{5}{13}$; $\cos 3\beta = 4 \cos^3 \beta - 3 \cos \beta = -\frac{2\,035}{2\,197}$

368. $\text{arc sen } \frac{1}{2} = \alpha \Rightarrow \text{sen } \alpha = \frac{1}{2}$ e $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{6}$

$\text{arc sen x} = 2 \cdot \frac{\pi}{6} \Rightarrow \text{arc sen x} = \frac{\pi}{3} \Rightarrow \text{sen } \frac{\pi}{3} = x \Rightarrow x = \frac{\sqrt{3}}{2}$

370. a) $\beta = \text{arc cos } 1 \Rightarrow \cos \beta = 1$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \beta = 0$

b) $\beta = \text{arc cos } \frac{1}{2} \Rightarrow \cos \beta = \frac{1}{2}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \frac{\pi}{3}$

c) $\beta = \text{arc cos } \frac{\sqrt{2}}{2} \Rightarrow \cos \beta = \frac{\sqrt{2}}{2}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \frac{\pi}{4}$

d) $\beta = \text{arc cos } 0 \Rightarrow \cos \beta = 0$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \frac{\pi}{2}$

e) $\beta = \text{arc cos } (-1) \Rightarrow \cos \beta = -1$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \pi$

372. $\beta = \text{arc cos } \left(-\frac{3}{5}\right) \Rightarrow \cos \beta = -\frac{3}{5}$ e $0 \leqslant \beta \leqslant \pi$

$\text{sen } \beta = \sqrt{1 - \cos^2 \beta} \Rightarrow \text{sen } \beta = \frac{4}{5}$

373. $\beta = \text{arc cos } \frac{2}{7} \Rightarrow \cos \beta = \frac{2}{7}$ e $0 \leqslant \beta \leqslant \pi$; $\text{sen } \beta = \sqrt{1 - \cos^2 \beta} = \frac{3\sqrt{5}}{7}$

$\text{cotg } \beta = \frac{\cos \beta}{\text{sen } \beta} = \frac{2\sqrt{5}}{15}$

374. $\left.\begin{array}{l} \text{arc sen x} = A \Rightarrow \text{sen A} = x \\ \text{arc cos x} = B \Rightarrow \cos B = x \end{array}\right\} \Rightarrow \text{sen A} = \cos B \Rightarrow B = \frac{\pi}{2} - A = \text{arc cos x}$

376. a) $\text{arc cos } \frac{3}{5} = \beta \Rightarrow \cos \beta = \frac{3}{5}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow$

$\Rightarrow \text{sen } \beta = \sqrt{1 - \cos^2 \beta} = \frac{4}{5}$

$\text{arc cos } \frac{5}{13} = \alpha \Rightarrow \cos \alpha = \frac{5}{13}$ e $0 \leqslant \alpha \leqslant \pi \Rightarrow$

$\Rightarrow \text{sen } \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{12}{13}$

$\text{sen } (\beta - \alpha) = \text{sen } \beta \cdot \cos \alpha - \text{sen } \alpha \cdot \cos \beta = -\frac{16}{65}$

b) $\text{arc sen } \frac{7}{25} = \beta \Rightarrow \text{sen } \beta = \frac{7}{25}$ e $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2} \Rightarrow \cos \beta = \frac{24}{25}$

$\text{arc cos } \frac{12}{13} = \alpha \Rightarrow \cos \alpha = \frac{12}{13}$ e $0 \leqslant \alpha \leqslant \pi \Rightarrow \text{sen } \alpha = \frac{5}{13}$

$\cos(\beta - \alpha) = \cos \beta \cdot \cos \alpha + \text{sen } \beta \cdot \text{sen } \alpha = \frac{323}{325}$

c) $\text{arc cos } \left(-\frac{3}{5}\right) = \beta \Rightarrow \cos \beta = -\frac{3}{5}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \text{sen } \beta = \frac{4}{5}$ e

$\text{tg } \beta = \frac{\text{sen } \beta}{\cos \beta} = -\frac{4}{3} \Rightarrow \text{tg } 2\beta = \frac{2 \text{ tg } \beta}{1 - \text{tg}^2 \beta} = \frac{24}{7}$

d) $\text{arc cos } \frac{7}{25} = \beta \Rightarrow \cos \beta = \frac{7}{25}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow$

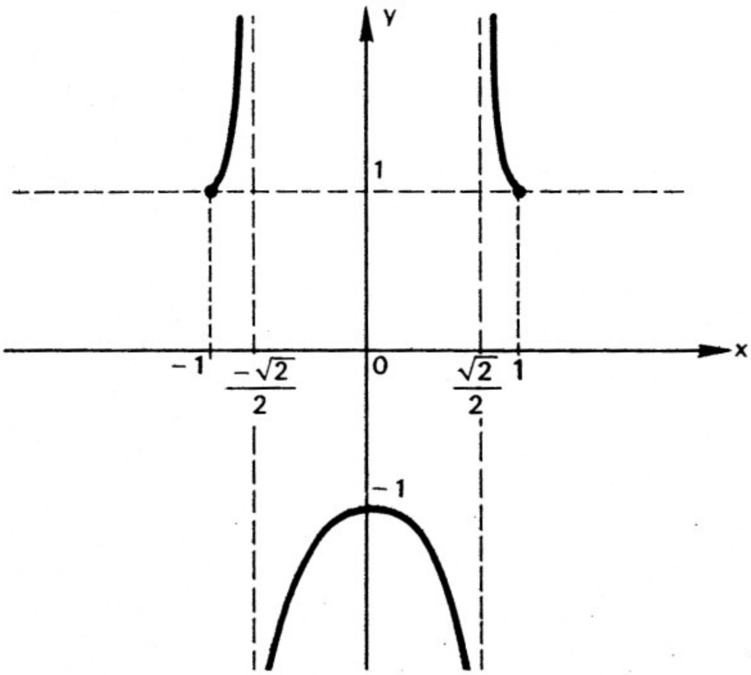
$\Rightarrow \cos \frac{\beta}{2} = \sqrt{\frac{1 + \cos \beta}{2}} = \frac{4}{5}$

377. a) $\cos(2 \text{ arc cos } x) = 0$

$\text{arc cos } x = \beta \Rightarrow \cos \beta = x$ e $0 \leqslant \beta \leqslant \pi$

$\cos 2\beta = 0 \Rightarrow 2 \cos^2 \beta - 1 = 0 \Rightarrow 2x^2 - 1 = 0 \Rightarrow x = \pm\frac{\sqrt{2}}{2}$

b) $g(x) = \frac{1}{f(x)} = \frac{1}{\cos 2\beta} = \frac{1}{2x^2 - 1}$, $-1 \leqslant x \leqslant 1$ e $x \neq +\frac{\sqrt{2}}{2}$ e $x \neq -\frac{\sqrt{2}}{2}$



378. $\beta = \text{arc cos } \sqrt{2} \Rightarrow \cos \beta = \sqrt{2}$, $\nexists\, \beta$ pois $-1 \leqslant \cos \beta \leqslant 1, \forall \beta \in \mathbb{R}$

380. $\text{arc tg } 0 = \alpha \Rightarrow \text{tg } \alpha = 0$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow \alpha = 0$

$\text{arc tg } \sqrt{3} = \beta \Rightarrow \text{tg } \beta = \sqrt{3}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{3}$

$\text{arc tg } (-1) = \alpha \Rightarrow \text{tg } \alpha = -1$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{4}$

$\text{arc tg } \left(-\frac{\sqrt{3}}{3}\right) = \beta \Rightarrow \text{tg } \beta = -\frac{\sqrt{3}}{3}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow \beta = -\frac{\pi}{6}$

382. $\text{arc tg } \left(-\frac{4}{3}\right) = \beta \Rightarrow \text{tg } \beta = -\frac{4}{3}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\cos \beta = \sqrt{\frac{1}{1 + \text{tg}^2 \beta}} = \frac{3}{5}$$

384. a) $\text{arc tg } 2 = \alpha \Rightarrow \text{tg } \alpha = 2$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\text{arc tg } 3 = \beta \Rightarrow \text{tg } \beta = 3$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\cos \alpha = \sqrt{\frac{1}{1 + \text{tg}^2 \alpha}} = \frac{\sqrt{5}}{5}; \ \cos \beta = \sqrt{\frac{1}{1 + \text{tg}^2 \beta}} = \frac{\sqrt{10}}{10}$$

$$\text{sen } \alpha = \sqrt{\frac{\text{tg}^2 \alpha}{1 + \text{tg}^2 \alpha}} = \frac{2\sqrt{5}}{5}; \ \text{sen } \beta = \sqrt{\frac{\text{tg}^2 \beta}{1 + \text{tg}^2 \beta}} = \frac{3\sqrt{10}}{10}$$

$$\text{sen } (\alpha + \beta) = \text{sen } \alpha \cdot \cos \beta + \text{sen } \beta \cdot \cos \alpha = \frac{\sqrt{2}}{2}$$

b) $\text{arc tg } 2 = \beta \Rightarrow \text{tg } \beta = 2$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow \text{sen } \beta = \frac{2\sqrt{5}}{5}, \cos \beta = \frac{\sqrt{5}}{5}$

$\text{arc tg } \frac{1}{2} = \gamma \Rightarrow \text{tg } \gamma = \frac{1}{2}$ e $-\frac{\pi}{2} < \gamma < \frac{\pi}{2} \Rightarrow$

$\Rightarrow \text{sen } \gamma = \frac{\sqrt{5}}{5}, \cos \gamma = \frac{2\sqrt{5}}{5}$

$$\cos (\beta - \gamma) = \cos \beta \cdot \cos \gamma + \text{sen } \beta \cdot \text{sen } \gamma = \frac{4}{5}$$

c) $\text{arc tg } \frac{1}{5} = \beta \Rightarrow \text{tg } \beta = \frac{1}{5}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \text{tg}^2 \beta = \frac{2 \text{ tg } \beta}{1 - \text{tg}^2 \beta} = \frac{5}{12}$$

d) $\text{arc tg } \frac{24}{7} = \beta \Rightarrow \text{tg } \beta = \frac{24}{7}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \cos\beta = \sqrt{\frac{1}{1 + \mathrm{tg}^2\,\beta}} = \frac{7}{25}$$

$$\cos 3\beta = 4 \cdot \cos^3\beta - 3 \cdot \cos\beta = -\frac{11\,753}{15\,625}$$

386. a) $\left.\begin{array}{l} \operatorname{arc\,tg}\frac{1}{2} = \beta \Rightarrow \mathrm{tg}\,\beta = \frac{1}{2} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2} \\ \operatorname{arc\,tg}\frac{1}{3} = \alpha \Rightarrow \mathrm{tg}\,\alpha = \frac{1}{3} \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 < \beta + \alpha < \pi$ (A); $\gamma = \frac{\pi}{4}$ (B)

$\mathrm{tg}\,(\beta + \alpha) = \dfrac{\mathrm{tg}\,\beta + \mathrm{tg}\,\alpha}{1 - \mathrm{tg}\,\beta \cdot \mathrm{tg}\,\alpha} = 1 = \mathrm{tg}\,\gamma$ (C).

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

b) $\left.\begin{array}{l} \operatorname{arc\,sen}\frac{1}{\sqrt{5}} = \beta \Rightarrow \mathrm{sen}\,\beta = \frac{1}{\sqrt{5}} \text{ e } -\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2} \\ \operatorname{ar\,cos}\frac{3}{\sqrt{10}} = \alpha \Rightarrow \cos\alpha = \frac{3}{\sqrt{10}} \text{ e } 0 \leqslant \alpha \leqslant \pi \Rightarrow 0 < \alpha < \frac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 \leqslant \beta + \alpha \leqslant \pi$ (A); $\gamma = \frac{\pi}{4}$ (B)

$\mathrm{sen}^2\,\beta = \dfrac{\mathrm{tg}^2\,\beta}{1 + \mathrm{tg}^2\,\beta} \Rightarrow \mathrm{tg}\,\beta = \frac{1}{2}$; $\cos^2\alpha = \dfrac{1}{1 + \mathrm{tg}^2\,\alpha} \Rightarrow \mathrm{tg}\,\alpha = \frac{1}{3}$

$\mathrm{tg}\,(\beta + \alpha) = \dfrac{\mathrm{tg}\,\beta + \mathrm{tg}\,\alpha}{1 - \mathrm{tg}\,\beta \cdot \mathrm{tg}\,\alpha} = 1 + \mathrm{tg}\,\gamma$ (C).

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

c) $\left.\begin{array}{l} \operatorname{arc\,cos}\frac{3}{5} = \alpha \Rightarrow \cos\alpha = \frac{3}{5} \text{ e } 0 \leqslant \alpha \leqslant \pi \Rightarrow 0 < \alpha < \frac{\pi}{2} \\ \operatorname{arc\,cos}\frac{12}{13} = \beta \Rightarrow \cos\beta = \frac{12}{13} \text{ e } 0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \frac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 \leqslant \alpha + \beta \leqslant \pi$ (A)

$\operatorname{arc\,cos}\frac{16}{65} = \gamma \Rightarrow \cos\gamma = \frac{16}{65}$ e $0 \leqslant \gamma \leqslant \pi$ (B)

$\mathrm{sen}\,\alpha = \sqrt{1 - \cos^2\alpha} = \frac{4}{5}$, $\mathrm{sen}\,\beta = \sqrt{1 - \cos^2\beta} = \frac{5}{13}$

$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \mathrm{sen}\,\alpha \cdot \mathrm{sen}\,\beta = \frac{16}{65} = \cos\gamma$ (C).

De (A), (B) e (C) $\Rightarrow \alpha + \beta = \gamma$.

d) arc sen $\frac{24}{25} = \alpha \Rightarrow$ sen $\alpha = \frac{24}{25}$ e $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{2}$

arc sen $\frac{3}{5} = \beta \Rightarrow$ sen $\beta = \frac{3}{5}$ e $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2}$ $\Rightarrow$

$\Rightarrow 0 \leqslant \alpha + \beta \leqslant \pi$ (A)

arc tg $\frac{3}{4} = \gamma \Rightarrow$ tg $\gamma = \frac{3}{4}$ e $-\frac{\pi}{2} < \gamma < \frac{\pi}{2}$ (B)

$\text{sen}^2\, \alpha = \frac{\text{tg}^2\, \alpha}{1 + \text{tg}^2\, \alpha} \Rightarrow \text{tg}\, \alpha = \frac{24}{7}$; $\text{sen}^2\, \beta = \frac{\text{tg}^2\, \beta}{1 + \text{tg}^2\, \beta} \Rightarrow \text{tg}\, \beta = \frac{3}{4}$

$$\text{tg}\,(\alpha - \beta) = \frac{\frac{24}{7} - \frac{3}{4}}{1 + \frac{24}{7} \cdot \frac{3}{4}} = \frac{3}{4} = \text{tg}\, \gamma \quad \text{(C)}.$$

De (A), (B) e (C) $\Rightarrow \alpha - \beta = \gamma$.

387. a) arc tg $\frac{2}{3} = \alpha \Rightarrow$ tg $\alpha = \frac{2}{3}$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{4} \Rightarrow 0 < 2\alpha < \frac{\pi}{2}$

arc cos $\frac{12}{13} = \beta \Rightarrow$ cos $\beta = \frac{12}{13}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \frac{\pi}{2}$

$\Rightarrow 0 < 2\alpha + \beta \leqslant \pi$ (A)

$$\text{tg}\, 2\alpha = \frac{2 \cdot \frac{2}{3}}{1 - \left(\frac{2}{3}\right)^2} = \frac{12}{5}; \ \cos 2\alpha = \sqrt{\frac{1}{1 + \left(\frac{12}{5}\right)^2}} = \frac{5}{13};$$

$$\text{sen}\, 2\alpha = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$$\text{sen}\, \beta = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}; \ \gamma = \frac{\pi}{2} \quad \text{(B)}$$

$$\cos\,(2\alpha + \beta) = \frac{5}{13} \cdot \frac{12}{13} - \frac{12}{13} \cdot \frac{5}{13} = 0 = \cos \gamma \quad \text{(C)}.$$

De (A), (B) e (C) $\Rightarrow 2\alpha + \beta = \gamma$.

b) arc sen $\frac{1}{4} = \alpha \Rightarrow$ sen $\alpha = \frac{1}{4}$ e $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{6} \Rightarrow$

$\Rightarrow 0 < 3\alpha < \frac{\pi}{2}$

arc cos $\frac{11}{16} = \beta \Rightarrow$ cos $\beta = \frac{11}{16}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \frac{\pi}{2}$

$\Rightarrow 0 < 3\alpha + \beta < \pi$ (A); $\gamma = \frac{\pi}{2}$ (B)

$$\cos\alpha = \sqrt{1 - \frac{1}{16}} = \frac{\sqrt{15}}{4},\ \text{sen}\ \beta = \sqrt{1 - \frac{121}{256}} = \frac{3\sqrt{15}}{16};$$

$$\cos 3\alpha = 4 \cdot \left(\frac{\sqrt{15}}{4}\right)^3 - \frac{3\sqrt{15}}{4} = \frac{3\sqrt{15}}{16}$$

$$\text{sen}\ 3\alpha = 3 \cdot \frac{1}{4} - 4\left(\frac{1}{4}\right)^3 = \frac{11}{16},\ \cos(3\alpha + \beta) = \frac{3\sqrt{15}}{16} \cdot \frac{11}{16} - \frac{11}{16} \cdot \frac{3\sqrt{15}}{16} =$$

$$= 0 = \cos\gamma \ \text{(C)}$$

De (A), (B) e (C) $\Rightarrow\ 3\alpha + \beta = \gamma.$

388. $\text{arc tg}\left(\frac{1 + e^x}{2}\right) = \alpha \ \Rightarrow\ \text{tg}\ \alpha = \frac{1 + e^x}{2}$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\text{arc tg}\left(\frac{1 - e^x}{2}\right) = \beta \ \Rightarrow\ \text{tg}\ \beta = \frac{1 - e^x}{2}$ e $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\alpha + \beta = \frac{\pi}{4} \ \Rightarrow\ \text{tg}(\alpha + \beta) = \frac{\left(\frac{1 + e^x}{2}\right) + \left(\frac{1 - e^x}{2}\right)}{1 - \left(\frac{1 + e^x}{2}\right)\left(\frac{1 - e^x}{2}\right)} =$$

$$= \frac{2}{1 + e^x} = 1 \ \Rightarrow\ 1 + e^x = 2 \ \Rightarrow\ x = 0,\ S = \{0\}$$

389. $\alpha = \text{arc tg}\,(7x - 1) \ \Rightarrow\ \text{tg}\ \alpha = 7x - 1$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ (A)

$\beta = \text{arc sec}\,(2x + 1) \ \Rightarrow\ \sec\beta = 2x + 1$ e $0 \leqslant \beta \leqslant \pi$ (B)

$\alpha = \beta \ \Rightarrow\ \text{tg}\ \alpha = \text{tg}\ \beta \ \Rightarrow\ \text{tg}^2\,\alpha = \text{tg}^2\,\beta \ \Rightarrow\ \text{tg}^2\,\alpha = \sec^2\beta - 1 \ \Rightarrow$

$\Rightarrow\ (7x - 1)^2 = (2x + 1)^2 - 1 \ \Rightarrow\ 45x^2 - 18x + 1 = 0 \ \Rightarrow$

$\Rightarrow\ x = \frac{1}{3}$ ou $x = \frac{1}{15}$ (não satisfaz $\text{tg}\ \alpha = -\text{tg}\ \beta$) $\therefore\ x = \frac{1}{3}$

390. $\alpha = \text{arc sen}\left(\frac{4}{5}\right) \ \Rightarrow\ \text{sen}\ \alpha = \frac{4}{5}$ e $\frac{\pi}{2} < \alpha < \pi$

$\beta = \text{arc tg}\left(-\frac{4}{3}\right) \ \Rightarrow\ \text{tg}\ \beta = -\frac{4}{5}$ e $\frac{3\pi}{2} < \beta \leqslant 2\pi$

$$\cos\alpha = \sqrt{1 - \frac{16}{25}} = -\frac{3}{5};\ \text{sen}\ \beta = \sqrt{\frac{\frac{16}{9}}{1 + \frac{16}{9}}} = -\frac{4}{5};$$

$$\cos\beta = \sqrt{1 - \left(-\frac{4}{5}\right)} = \frac{3}{5}$$

$$25 \cdot \cos(\alpha + \beta) = 25 \cdot \left[-\frac{3}{5} \cdot \frac{3}{5} - \frac{4}{5}\left(-\frac{4}{5}\right)\right] = 7$$

Apêndice A — Resolução de equações e inequações em intervalos determinados

393. $\text{sen } 3x = \text{sen } \frac{\pi}{6} \Rightarrow \begin{cases} 3x = \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{\pi}{18} + 2k\frac{\pi}{3} \\ \text{ou} \\ 3x = \pi - \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{5\pi}{18} + 2k\frac{\pi}{3} \end{cases}$

$k = 0 \Rightarrow x = \frac{\pi}{18}$ ou $x = \frac{5\pi}{18}$; $k = 1 \Rightarrow x = \frac{13\pi}{18}$ ou $x = \frac{17\pi}{18}$

$S = \left\{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}\right\}$

395. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = \pi - 2x + 2k\pi \Rightarrow x = \frac{\pi}{5} + \frac{2k\pi}{5}$

$S = \left\{0, \frac{\pi}{5}, \frac{3\pi}{5}, \pi\right\}$

397. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = -2x + 2k\pi \Rightarrow x = \frac{2k\pi}{5}$

$S = \left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}\right\}$

398. $\text{sen } x \cdot (4 \text{ sen}^2 x - 1) = 0$. Então: $\text{sen } x = 0 \Rightarrow x = k\pi$ ou $\text{sen } x = \frac{1}{2} \Rightarrow$

$\Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$ ou $\text{sen } x = -\frac{1}{2} \Rightarrow$

$\Rightarrow x = \frac{7\pi}{6} + 2k\pi$ ou $x = -\frac{\pi}{6} + 2k\pi \Rightarrow$

$\Rightarrow S = \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi\right\}$

400. $\text{tg } x + \frac{1}{\text{tg } x} = 2 \Rightarrow \text{tg}^2 x - 2 \text{ tg } x + 1 = 0 \Rightarrow$

$\Rightarrow \text{tg } x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

401. $\sqrt{\text{sen}^2 x} = \sqrt{\cos^2 x} \Rightarrow \text{sen}^2 x = \cos^2 x \Rightarrow \cos^2 x - \text{sen}^2 x = 0 \Rightarrow$

$\Rightarrow \cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + k\pi \Rightarrow x = \frac{\pi}{4} + \frac{k\pi}{2} \Rightarrow S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

402. $\operatorname{sen} x + \operatorname{sen} y = 2 \operatorname{sen}\left(\dfrac{x+y}{2}\right) \cdot \cos\left(\dfrac{x-y}{2}\right) = 1 \Rightarrow \cos\left(\dfrac{\frac{\pi}{3} - 2y}{2}\right) = 1 \Rightarrow$

$\Rightarrow \cos\left(\dfrac{\pi}{6} - y\right) = \cos 0 \Rightarrow \dfrac{\pi}{6} - y = 2k\pi \Rightarrow y = \dfrac{\pi}{6} - 2k\pi$

$x + y = \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$

403. a) $\cos 2x = \cos \dfrac{\pi}{6} \Rightarrow \begin{cases} 2x = \dfrac{\pi}{6} + 2k\pi \Rightarrow & x = \dfrac{\pi}{12} + k\pi \\ \text{ou} & \text{ou} \\ 2x = -\dfrac{\pi}{6} + 2k\pi \Rightarrow & x = -\dfrac{\pi}{12} = k\pi \end{cases}$

$S = \left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}$

b) $2x = x + 2k\pi \Rightarrow x = 2k\pi$ ou $2x = -x + 2k\pi \Rightarrow x = \dfrac{2k\pi}{3}$

$S = \left\{0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 2\pi\right\}$

c) $\cos\left(x + \dfrac{\pi}{6}\right) = \cos \dfrac{\pi}{2} \Rightarrow x = \dfrac{\pi}{3} + k\pi \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{4\pi}{3}\right\}$

404. a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \dfrac{k\pi}{2}$

$S = \left\{0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, \cancel{2\pi}\right\}$

b) $5x = x + \dfrac{\pi}{3} + 2k\pi \Rightarrow x = \dfrac{\pi}{12} + \dfrac{k\pi}{2}$ ou $5x = -x - \dfrac{\pi}{3} + 2k\pi \Rightarrow$

$\Rightarrow x = -\dfrac{\pi}{18} + \dfrac{k\pi}{3}$

$k = \{0, 1, 2, 3, 4, 5, 6\} \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{12}, \dfrac{7\pi}{12}, \dfrac{13\pi}{12}, \dfrac{19\pi}{12}, \dfrac{5\pi}{18}, \dfrac{11\pi}{18}, \dfrac{17\pi}{18}, \dfrac{23\pi}{18}, \dfrac{29\pi}{18}, \dfrac{35\pi}{18}\right\}$

405. $\cos^2 x - \dfrac{2}{\cos^2 x} = 1 \Rightarrow \cos^4 x - \cos^2 x - 2 = 0 \Rightarrow \cos^2 x = 2$ ou $\cos^2 x = -1, S = \varnothing$

406. O 1º membro é a soma dos *10* termos de P.G., com $a_1 = 1$ e $q = \cos x$; então sua soma é $\dfrac{1 \cdot \cos^{10} x - 1}{\cos x - 1} = 0$, e daí $\cos x = -1$; a equação tem uma única solução.

407. a) $\operatorname{tg} 2x = \operatorname{tg} \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{6} + \dfrac{k\pi}{2} \Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}\right\}$

b) $2x = x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$

c) $\text{tg } 3x = \text{tg } \frac{\pi}{4} \Rightarrow 3x = \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{12} + \frac{k\pi}{3} \Rightarrow$

$\Rightarrow S = \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}\right\}$

d) $\text{tg } 3x = \text{tg } 2x \Rightarrow 3x = 2x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$

e) $2x = x + \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{4} + k\pi$, mas $x + \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi$, então $S = \varnothing$

f) $\text{tg } 4x = \text{tg } \frac{\pi}{4} \Rightarrow 4x = \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{16} + \frac{k\pi}{4} \Rightarrow$

$S = \left\{\frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}\right\}$

g) $2x = x + \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

h) $\text{tg } 2x = \text{tg } \frac{\pi}{3} \Rightarrow 2x = \frac{\pi}{3} + k\pi \Rightarrow x = \frac{\pi}{6} + \frac{k\pi}{2}$

ou

$\text{tg } 2x = \text{tg } \frac{5\pi}{3} \Rightarrow 2x = \frac{5\pi}{3} + k\pi \Rightarrow x = \frac{5\pi}{6} + \frac{k\pi}{2}$

$S = \left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}\right\}$

408. a) $1 + \text{tg}^2 x - 2 \text{ tg } x = 0 \Rightarrow \text{tg } x = 1 \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

b) $1 = \text{sen}^2 x - \text{sen } x \cdot \cos x \Rightarrow \text{sen}^2 x + \cos^2 x = \text{sen}^2 x - \text{sen } x \cdot \cos x \Rightarrow$

$\Rightarrow \cos x (\text{sen } x + \cos x) = 0$

Então:

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$ ou $\text{sen } x + \cos x = 0 \Rightarrow \text{sen } x = -\cos x \Rightarrow$

$\Rightarrow \text{tg } x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

c) $\text{sen } 2x \cdot \cos\left(x + \frac{\pi}{4}\right) - \cos 2x \cdot \text{sen}\left(x + \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow \text{sen}\left[2x - \left(x + \frac{\pi}{4}\right)\right] = 0 \Rightarrow \text{sen}\left(x - \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow x - \frac{\pi}{4} = k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

d) $1 + \text{sen } 2x - \text{tg } x - \text{tg } x \text{ sen } 2x = 1 + \text{tg } x \Rightarrow \text{sen } 2x (1 - \text{tg } x) = 2 \text{ tg } x \Rightarrow$

$\frac{2 \text{ tg } x}{1 + \text{tg}^2 x} = \frac{2 \text{ tg } x}{1 - \text{tg } x} \Rightarrow \text{tg } x (\text{tg } x + 1) = 0$. Então: $\text{tg } x = 0 \Rightarrow x = k\pi$ ou

$\text{tg } x = -1 \Rightarrow x = \frac{3\pi}{4}$ e daí $S = \left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi\right\}$

e) $\sec x\,(3 \sec x - 2) = 0 \Rightarrow \sec x = 0$ ou $\sec x = \frac{2}{3}$ e daí $S = \varnothing$

f) $2 \text{ sen}^2 x = \frac{\text{sen } x}{\cos x} \cdot \cos x \Rightarrow \text{sen } x\,(2 \text{ sen } x - 1) = 0$. Então: $\text{sen } x = 0$

$\Rightarrow x = k\pi$ ou $\text{sen } x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

$S = \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi\right\}$

409. $\frac{\pi}{2} p = \frac{\pi}{4} + \frac{k\pi}{2} \Rightarrow p = \frac{1}{2} + k$

$\text{tg } \frac{\pi}{2} p = \frac{1}{\text{tg } \frac{\pi}{2} p} \Rightarrow \text{tg } \frac{\pi}{2} p = \pm 1$

410. $\text{sen } x = \text{tg } x \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi \Rightarrow \nexists\, x \mid 0 < x < \pi \Rightarrow$ nenhum ponto

411. $1 + \text{tg}^2 x - \text{tg } x = 1 \Rightarrow \text{tg } x\,(\text{tg } x - 1) = 0 \Rightarrow \text{tg } x = 0$ ou $\text{tg } x = 1$

$\Rightarrow x = k\pi$ ou $x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$

412. $6x = 2x + k\pi \Rightarrow x = \frac{k\pi}{4} \Rightarrow x \in \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\right\}$

$x = \frac{\pi}{4} \Rightarrow \nexists \text{ tg } 2x;\ x = \frac{3\pi}{4} \Rightarrow \nexists \text{ tg } 2x \Rightarrow S = \left\{0, \frac{\pi}{2}, \pi\right\}$

413. a) $(\text{sen}^2 x + \cos^2 x)^2 - 2 \text{ sen}^2 x \cos^2 x = \frac{5}{8} \Rightarrow \text{sen } 2x = \pm\frac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \left(x = \frac{\pi}{6} + k\pi \text{ ou } x = \frac{\pi}{3} + k\pi \text{ ou } x = \frac{2\pi}{3} + k\pi \text{ ou } x = \frac{5\pi}{6} + k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$

b) $(\text{sen}^2 x + \cos^2 x)(\text{sen}^4 x - \text{sen}^2 x \cos^2 x + \cos^4 x) = \frac{5}{8} \Rightarrow$

$\Rightarrow \text{sen } 2x = \pm\frac{\sqrt{2}}{2} \Rightarrow \left(x = \frac{\pi}{8} + k\pi \text{ ou } x = \frac{3\pi}{8} + k\pi \text{ ou } x = \frac{5\pi}{8} + k\pi\right.$

$\left.\text{ou } \frac{7\pi}{8} + k\pi\right) \Rightarrow S = \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

c) $\operatorname{sen}^2 2x = 1 \Rightarrow \operatorname{sen} 2x = \pm 1 \Rightarrow \left(x = \frac{\pi}{4} + k\pi \text{ ou } x = \frac{3\pi}{4} + k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

d) $\operatorname{sen}^2 x = \frac{3}{4} \Rightarrow \operatorname{sen} x = \pm\frac{\sqrt{3}}{2} \Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi \text{ ou}\right.$

$\left. x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

e) $(\operatorname{sen} x + \cos x)(\operatorname{sen}^2 x - \operatorname{sen} x \cdot \cos x + \cos^2 x) = 1 \Rightarrow$

$\Rightarrow (\operatorname{sen} x + \cos x)(1 - \operatorname{sen} x \cdot \cos x) = 1$; fazendo $\operatorname{sen} x + \cos x = y$

e $\operatorname{sen} x \cdot \cos x = \frac{y^2 - 1}{2}$, vem $y\left(1 - \frac{y^2 - 1}{2}\right) = 1 \Rightarrow$

$\Rightarrow y = 1$ ou $y = -2$ (não serve, pois $-\sqrt{2} \leqslant y \leqslant \sqrt{2}$)

$\operatorname{sen} x + \cos x = 1 \Rightarrow \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\Rightarrow \left(x = \frac{\pi}{2} + 2k\pi \text{ ou } x = 2k\pi\right) \Rightarrow S = \left\{0, \frac{\pi}{2}, 2\pi\right\}$

414. $2x = x + 2k\pi \Rightarrow x = 2k\pi$ ou $2x = \pi - x + 2k\pi \Rightarrow x = \frac{\pi}{3} + \frac{2k\pi}{3} \Rightarrow$

$\Rightarrow S = \left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\} \Rightarrow$ quatro soluções

415. $\frac{3\operatorname{sen}^2 x}{\cos^2 x} + 5 = \frac{7}{\cos x} \Rightarrow 3(1 - \cos^2 x) + 5\cos^2 x - 7\cos x = 0 \Rightarrow$

$\Rightarrow 2\cos^2 x - 7\cos x + 3 = 0 \Rightarrow \left(\cos x = 3 \text{ impossível ou } \cos x = \frac{1}{2}\right) \Rightarrow$

$\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{-\frac{\pi}{3}, +\frac{\pi}{3}\right\}$

416. $\operatorname{sen} \pi x = -\cos \pi x \Rightarrow \operatorname{sen} \pi x = \operatorname{sen}\left(\frac{3\pi}{2} - \pi x\right) \Rightarrow x = \frac{3}{4} + k \Rightarrow$

$\Rightarrow S = \left\{\frac{3}{4}, \frac{7}{4}\right\}$

417. $1 + \operatorname{tg}^2 x = \operatorname{tg} x + 1 \Rightarrow \operatorname{tg} x(\operatorname{tg} x - 1) = 0 \Rightarrow (\operatorname{tg} x = 0 \text{ ou } \operatorname{tg} x = 1) \Rightarrow$

$\Rightarrow \left(x = k\pi \text{ ou } x = \frac{\pi}{4} + k\pi\right) \Rightarrow S = \left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$

418. a) $2(1 - \cos^2 x) - 3\cos x - 3 = 0 \Rightarrow 2\cos^2 x + 3\cos x + 1 = 0 \Rightarrow$

$\Rightarrow \left(\cos x = -\frac{1}{2} \text{ ou } \cos x = -1\right) \Rightarrow$

$\Rightarrow \left(x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi \text{ ou } x = \pi + 2k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

b) $\cos^2 x - 2\cos x + 1 = 0 \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi \Rightarrow S = \{0, 2\pi\}$

c) $2\cos^2 x + 5\cos x - 3 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$

$\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

d) $4\cos^2 x - 8\cos x + 3 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$

$\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

419. $\cos x = \pm\frac{1}{2} \Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi \text{ ou } x = \frac{2\pi}{3} + 2k\pi\right.$

$\left.\text{ou } x = \frac{4\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\} \Rightarrow$

$\Rightarrow \text{soma} = \frac{\pi + 2\pi + 4\pi + 5\pi}{3} = 4\pi$

420. $2\cos^2 x + 3(1 - \cos^2 x) - 5 - 3\cos x = 0 \Rightarrow \cos^2 x + 3\cos x + 2 = 0 \Rightarrow$
$\Rightarrow \cos x = -1 \Rightarrow x = \pi + 2k\pi \Rightarrow S = \{\pi\} \Rightarrow$ uma solução

421. a) $\operatorname{sen}(x + y) + \operatorname{sen}(x - y) = \operatorname{sen}\frac{5\pi}{2} + \operatorname{sen}\left(\frac{3\pi}{2}\right) = 1 - 1 = 0 \neq 2$

b) $\left.\begin{array}{l} 2\operatorname{sen} x \cos y = 2 \\ \operatorname{sen} x + \cos y = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \operatorname{sen} x \cos y = 1 \quad (A) \\ \operatorname{sen} x + \cos y = 2 \quad (B) \end{array}\right\}$ (A) em (B) $\Rightarrow$

$\Rightarrow \operatorname{sen}^2 x - 2\operatorname{sen} x + 1 = 0 \Rightarrow \operatorname{sen} x = 1$ (C) $\Rightarrow x = \frac{\pi}{2} + 2k\pi$

(C) em (A) $\Rightarrow \cos y = 1 \Rightarrow y = 2k\pi$

$k = \{0, 1\} \Rightarrow S = \left\{\left(\frac{\pi}{2}, 0\right)\right\}$ ou $\left\{\left(\frac{\pi}{2}, 2\pi\right)\right\}$

422. $\left\{\begin{array}{l} \operatorname{sen} a + \cos b = 1 \quad (A) \\ \operatorname{sen} a + \operatorname{sen} b = 1 \quad (B) \end{array}\right.$ (A) em (B) $\Rightarrow \operatorname{sen} b - \cos b = 0 \Rightarrow$

$\Rightarrow \operatorname{sen}\left(b - \frac{\pi}{4}\right) = 0 \Rightarrow b = \frac{\pi}{4} + k\pi \Rightarrow b = \frac{\pi}{4}$ (C),

(C) em (A) $\Rightarrow \operatorname{sen} a = \frac{2 - \sqrt{2}}{2} \Rightarrow a = \operatorname{arc\,sen}\frac{2 - \sqrt{2}}{2}$

$S = \left\{\left(\operatorname{arc\,sen}\frac{2 - \sqrt{2}}{2}, \frac{\pi}{4}\right)\right\}$

423. *1º caso: sen $x \geqslant 0$*

$|\text{sen } x| = \text{sen } x \Rightarrow 2\,\text{sen}^2 x + \text{sen } x - 1 = 0 \Rightarrow \text{sen } x = \dfrac{1}{2} \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$

2º caso: sen < 0

$|\text{sen } x| = \text{sen } x \Rightarrow 2\,\text{sen}^2 x - \text{sen } x - 1 = 0 \Rightarrow \text{sen } x = -\dfrac{1}{2} \Rightarrow$

$\Rightarrow x = \dfrac{7\pi}{6} + 2k\pi$ ou $x = \dfrac{11\pi}{6} + 2k\pi$

Então: $S = \left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$.

424. $\cos x = \pm\dfrac{1}{2} \Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = \dfrac{5\pi}{3} + 2k\pi \text{ ou } x = \dfrac{2\pi}{3} + 2k\pi\right.$

$\left.\text{ou } x = \dfrac{4\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right\} \Rightarrow$ a soma é π

425. $\log 2\,\text{sen}^2 x = 0 \Rightarrow 2\,\text{sen}^2 x = 1 \Rightarrow \text{sen } x = \pm\dfrac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{4} + 2k\pi \text{ ou } x = \dfrac{3\pi}{4} + 2k\pi \text{ ou } x = \dfrac{5\pi}{4} + 2k\pi \text{ ou } x = \dfrac{7\pi}{4} + 2k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}$

426. $\text{sen } x = y \Rightarrow 2y^2 - 5y + 2 = 0 \Rightarrow y = 2$ não serve ou $y = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \text{sen } x = \dfrac{1}{2} \Rightarrow \left(x = \dfrac{\pi}{6} + 2k\pi \text{ ou } x = \dfrac{5\pi}{6} + 2k\pi\right) \Rightarrow$

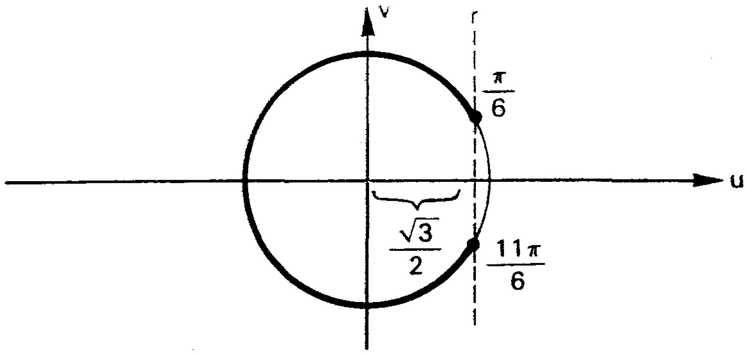
$\Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\} \Rightarrow$ a soma é π

428. $2x = t \Rightarrow \cos t \leqslant \dfrac{\sqrt{3}}{2}$

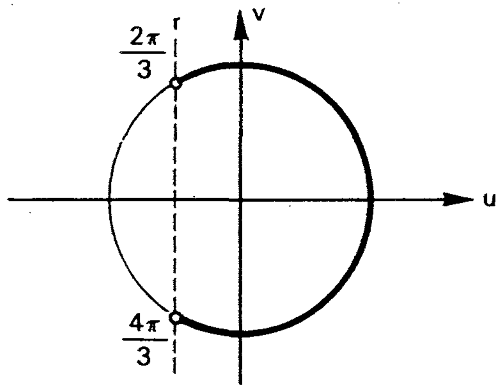
$\dfrac{\pi}{6} + 2k\pi \leqslant t \leqslant \dfrac{11\pi}{6} + 2k\pi$

$\dfrac{\pi}{12} + k\pi \leqslant x \leqslant \dfrac{11\pi}{12} + k\pi$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{12} \leqslant x \leqslant \dfrac{11\pi}{12} \text{ ou } \dfrac{13\pi}{12} \leqslant x \leqslant \dfrac{23\pi}{12}\right\}$



429. $4x = t \Rightarrow \cos t > -\dfrac{1}{2}$



$2k\pi \leqslant t < \dfrac{2\pi}{3} + 2k\pi$ ou

$\dfrac{4\pi}{3} + 2k\pi < t < 2\pi + 2k\pi$

$\dfrac{k\pi}{2} \leqslant x < \dfrac{\pi}{6} + \dfrac{k\pi}{2}$ ou

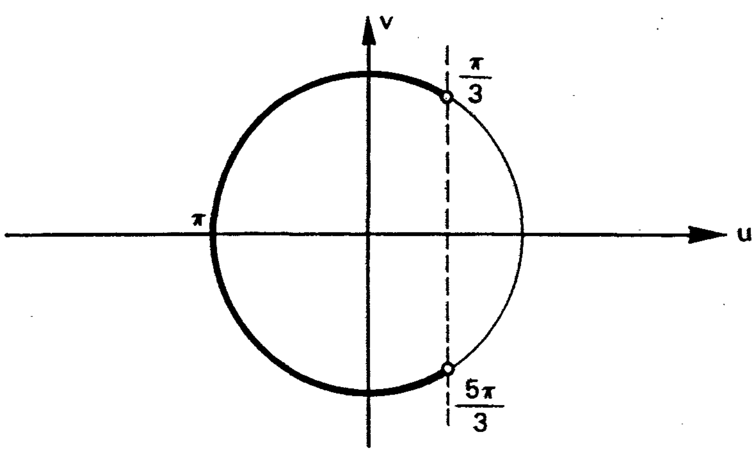
$\dfrac{\pi}{3} + \dfrac{k\pi}{2} < x < \dfrac{\pi}{2} + \dfrac{k\pi}{2}$

$k = \{0, 1, 2\} \Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \dfrac{\pi}{6} \text{ ou } \dfrac{\pi}{3} < x < \dfrac{2\pi}{3} \text{ ou } \dfrac{5\pi}{6} < x < \dfrac{7\pi}{6} \text{ ou } \dfrac{4\pi}{3} < x < \dfrac{5\pi}{3} \text{ ou } \dfrac{11\pi}{6} < x \leqslant 2\pi\right\}$

431. $\cos x = y \Rightarrow \dfrac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \dfrac{1}{2}$ ou $y > 1$

$-1 < \cos x < \dfrac{1}{2}$ ou $\cos x > 1$ impossível

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{3} < x < \pi\right\}$

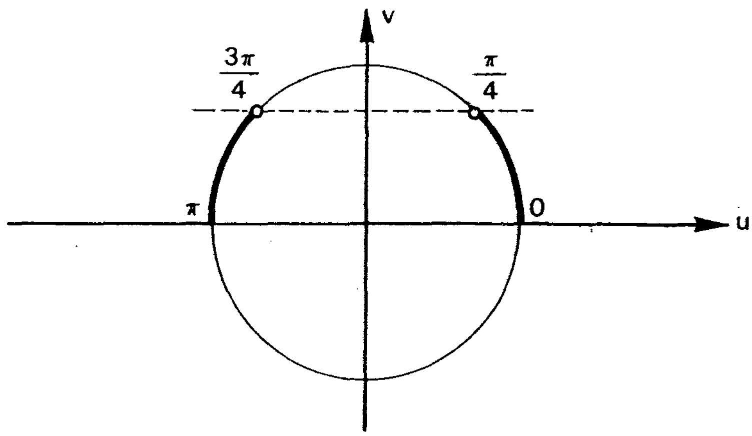


432. $\dfrac{1 - 2\,\text{sen}^2 x + \text{sen}\, x + 1}{1 - 2\,\text{sen}^2 x} - 2 \geqslant 0 \;\Rightarrow\; \dfrac{2\,\text{sen}^2 x + \text{sen}\, x}{1 - 2\,\text{sen}^2 x} \geqslant 0,\ \text{sen}\, x = y \;\Rightarrow$

$\Rightarrow\; \dfrac{2y^2 + y}{1 - 2y^2} \geqslant 0 \;\Rightarrow\; -\dfrac{\sqrt{2}}{2} < y \leqslant -\dfrac{1}{2} \text{ ou } 0 \leqslant y < \dfrac{\sqrt{2}}{2} \;\Rightarrow$

$\Rightarrow\; -\dfrac{\sqrt{2}}{2} < \text{sen}\, x \leqslant -\dfrac{1}{2} \text{ ou } 0 \leqslant \text{sen}\, x < \dfrac{\sqrt{2}}{2} \;\Rightarrow$

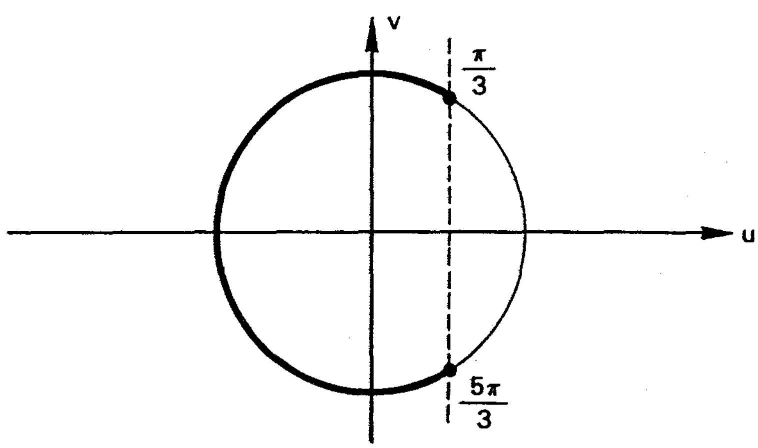
$\Rightarrow\; S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \dfrac{\pi}{4} \text{ ou } \dfrac{3\pi}{4} < x \leqslant \pi\right\}$



433. $2^{\cos 2x} \leqslant 2^{1/2} \;\Rightarrow\; \cos 2x \leqslant \dfrac{1}{2},\ 2x = t \;\Rightarrow\; \cos t \leqslant \dfrac{1}{2} \;\Rightarrow$

$\Rightarrow\; \dfrac{\pi}{3} + 2k\pi \leqslant t \leqslant \dfrac{5\pi}{3} + 2k\pi \;\Rightarrow\; \dfrac{\pi}{6} + k\pi \leqslant x \leqslant \dfrac{5\pi}{6} + k\pi$

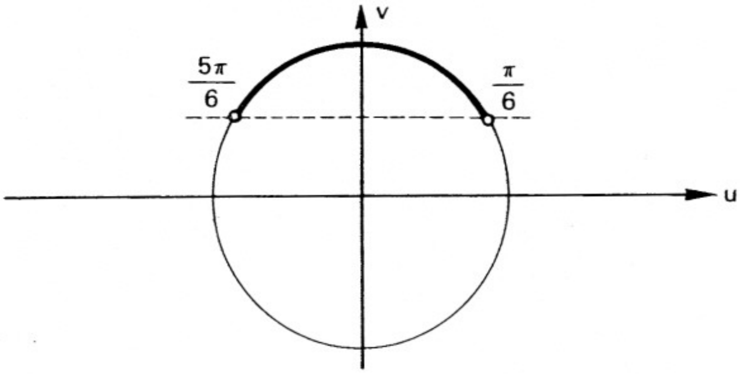
$\Rightarrow\; S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}\right\}$



435. $2x = t \;\Rightarrow\; \text{sen}\, t > \dfrac{1}{2} \;\Rightarrow\; \dfrac{\pi}{6} + 2k\pi < t < \dfrac{5\pi}{6} + 2k\pi \;\Rightarrow$

$\Rightarrow\; \dfrac{\pi}{12} + k\pi < x < \dfrac{5\pi}{12} + k\pi$

$$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{12} < x < \frac{5\pi}{12} \text{ ou } \frac{13\pi}{12} < x < \frac{17\pi}{12}\right\}$$



436. $3x = t \Rightarrow \text{sen } t \leqslant \frac{\sqrt{3}}{2} \Rightarrow$

$$\Rightarrow \left(2k\pi \leqslant t \leqslant \frac{\pi}{3} + 2k\pi \text{ ou } \frac{2\pi}{3} + 2k\pi \leqslant t < 2\pi + 2k\pi\right) \Rightarrow$$

$$\Rightarrow \left(\frac{2k\pi}{3} \leqslant x \leqslant \frac{\pi}{9} + \frac{2k\pi}{3} \text{ ou } \frac{2\pi}{9} + \frac{2k\pi}{3} \leqslant x < \frac{2\pi}{3} + \frac{2k\pi}{3}\right)$$

$$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \frac{\pi}{9} \text{ ou } \frac{2\pi}{9} \leqslant x \leqslant \frac{7\pi}{9} \text{ ou } \frac{8\pi}{9} \leqslant x \leqslant \frac{13\pi}{9} \text{ ou } \frac{14\pi}{9} \leqslant x < 2\pi\right\}$$

437. $\frac{1}{4} \leqslant \frac{1}{2} \text{ sen } 2x < \frac{1}{2} \Rightarrow \frac{1}{2} \leqslant \text{sen } t < 1,\ t = 2x,$

$$\frac{\pi}{6} + 2k\pi \leqslant t \leqslant \frac{5\pi}{6} + 2k\pi,\ t \neq \frac{\pi}{2} + 2k\pi \Rightarrow \frac{\pi}{12} + k\pi \leqslant x \leqslant \frac{5\pi}{12} + k\pi,$$

$$x \neq \frac{\pi}{4} + k\pi \Rightarrow$$

$$\Rightarrow S = \left\{x \in \mathbb{R} \mid \frac{\pi}{12} \leqslant x \leqslant \frac{5\pi}{12},\ x \neq \frac{\pi}{4} \text{ ou } \frac{13\pi}{12} \leqslant x \leqslant \frac{17\pi}{12},\ x \neq \frac{5\pi}{4}\right\}$$

438. $\frac{4 \text{ sen}^2 x - 1}{\cos x} \geqslant 0 \Rightarrow \frac{4(1 - \cos^2 x) - 1}{\cos x} \geqslant 0 \Rightarrow \frac{3 - 4\cos^2 x}{\cos x} \geqslant 0 \Rightarrow$

$$\Rightarrow \left(\cos x \leqslant -\frac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leqslant \frac{\sqrt{3}}{2}\right) \Rightarrow$$

$$\Rightarrow \left(\frac{5\pi}{6} \leqslant x \leqslant \frac{7\pi}{6} \text{ ou } \frac{\pi}{6} \leqslant x < \frac{\pi}{2} \text{ ou } \frac{3\pi}{2} < x \leqslant \frac{11\pi}{6}\right)$$

439. $-1 \leqslant \text{sen } 2x \leqslant +1 \Rightarrow -1 - 2 \leqslant \text{sen } 2x - 2 \leqslant 1 - 2 \Rightarrow \text{sen } 2x - 2 < 0$ (A)

$$\frac{\text{sen } 2x - 2}{\cos 2x + 3\cos x - 1} \geqslant 0 \overset{(A)}{\Rightarrow} \cos 2x + 3\cos x - 1 < 0 \Rightarrow$$

$$\Rightarrow 2\cos^2 x + 3\cos x - 2 < 0 \Rightarrow -2 < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{5\pi}{3}$$

440. a) $\Delta \geqslant 0 \Rightarrow [-(4\cos\alpha)]^2 - 4(2\cos^2\alpha)(4\cos^2\alpha - 1) \geqslant 0 \Rightarrow$
$\Rightarrow -32\cos^4\alpha + 24\cos^2\alpha \geqslant 0$ fazendo $\cos\alpha = t \Rightarrow -32t^4 + 24t^2 \geqslant 0 \Rightarrow$

$$\Rightarrow -8t^2(4t^2 - 3) \geqslant 0 \Rightarrow -\frac{\sqrt{3}}{2} \leqslant t \leqslant \frac{\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2} \leqslant \cos\alpha \leqslant \frac{\sqrt{3}}{2}$$

$$\Rightarrow \alpha \in \mathbb{R} \mid \frac{\pi}{6} \leqslant \alpha \leqslant \frac{5\pi}{6}$$

b) $-\dfrac{b}{a} < 0 \Rightarrow \dfrac{4\cos\alpha}{2\cos^2\alpha} < 0 \Rightarrow \dfrac{2}{\cos\alpha} < 0 \Rightarrow \cos\alpha < 0 \Rightarrow \dfrac{\pi}{2} < \alpha \leqslant \pi$ (A)

$$\frac{c}{a} > 0 \Rightarrow \frac{4\cos^2\alpha - 1}{2\cos^2\alpha} > 0 \Rightarrow 4\cos^2\alpha - 1 > 0$$

Fazendo $\cos\alpha = y \Rightarrow 4y^2 - 1 > 0 \Rightarrow \left(y < -\dfrac{1}{2} \text{ ou } y > \dfrac{1}{2}\right) \Rightarrow$

$$\Rightarrow \left(\cos\alpha < -\frac{1}{2} \text{ ou } \cos\alpha > \frac{1}{2}\right) \Rightarrow \left(\frac{2\pi}{3} < \alpha \leqslant \pi \text{ ou } 0 \leqslant \alpha < \frac{\pi}{3}\right) \text{ (B)}$$

Soluções reais $\Rightarrow \dfrac{\pi}{6} \leqslant \alpha \leqslant \dfrac{5\pi}{6}$ (C),

$$A \cap B \cap C = \left\{\alpha \in \mathbb{R} \mid \frac{2\pi}{3} < \alpha \leqslant \frac{5\pi}{6}\right\}.$$

441. $(\cos x > 0 \text{ e } 2 \cdot \cos x - 1 > 0 \text{ e } 1 + \cos x > 0) \Rightarrow \cos x > \dfrac{1}{2}$ (A)

$\log_{\cos x}(2\cos x - 1)(1 + \cos x) > \log_{\cos x}\cos x \Rightarrow 2\cos^2 x + \cos x - \cos x - 1 < 0 \Rightarrow$

$$\Rightarrow 2 \cdot \cos^2 x - 1 < 0 \Rightarrow -\frac{\sqrt{2}}{2} < \cos x < \frac{\sqrt{2}}{2} \text{ (B).}$$

De (A) e (B) vem $\dfrac{1}{2} < \cos x < \dfrac{\sqrt{2}}{2}$, então:

$$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{4} < x < \frac{\pi}{3} \text{ ou } \frac{5\pi}{3} < x < \frac{7\pi}{4}\right\}.$$

442. Uma condição necessária é *sen x > 0*. Então: $(\sqrt{1 - \cos x})^2 < \operatorname{sen}^2 x \Rightarrow$
$\Rightarrow 1 - \cos x < 1 - \cos^2 x \Rightarrow \cos^2 x - \cos x < 0 \Rightarrow 0 < \cos x < 1 \Rightarrow$
$\Rightarrow 0 < x < \dfrac{\pi}{2}$.

444. Fazendo $2x = y \Rightarrow \operatorname{tg} y \geqslant -\sqrt{3} \Rightarrow \left(2k\pi \leqslant y < \dfrac{\pi}{2} + 2k\pi \text{ ou}\right.$

$$\left.\frac{2\pi}{3} + 2k\pi \leqslant y < \frac{3\pi}{2} + 2k\pi \text{ ou } \frac{5\pi}{3} + 2k\pi \leqslant y < 2\pi + 2k\pi\right) \Rightarrow$$

$$\Rightarrow \left(k\pi \leqslant x < \frac{\pi}{4} + k\pi \text{ ou } \frac{2\pi}{6} + k\pi \leqslant x < \frac{3\pi}{4} + k\pi \text{ ou } \frac{5\pi}{6} + k\pi \leqslant x < \pi + k\pi\right) \Rightarrow$$

$$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \frac{\pi}{4} \text{ ou } \frac{\pi}{3} \leqslant x < \frac{3\pi}{4} \text{ ou } \frac{5\pi}{6} \leqslant x < \frac{5\pi}{4} \text{ ou}\right.$$

$\frac{4\pi}{3} \leqslant x < \frac{7\pi}{4}$ ou $\left.\frac{11\pi}{6} \leqslant x < 2\pi\right\}$.

445. $tg^2\, 2x - tg\, 2x \leqslant 0 \Rightarrow 0 \leqslant tg\, 2x \leqslant 1 \Rightarrow$

$\Rightarrow k\pi \leqslant 2x \leqslant \frac{\pi}{4} + k\pi \Rightarrow \frac{k\pi}{2} \leqslant x \leqslant \frac{\pi}{8} + \frac{k\pi}{2} \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \frac{\pi}{8}\right.$ ou $\frac{\pi}{2} \leqslant x \leqslant \frac{5\pi}{8}$ ou $\pi \leqslant x \leqslant \frac{9\pi}{8}$ ou

$\left.\frac{3\pi}{2} \leqslant x \leqslant \frac{13\pi}{8}\right\}$

446. Fazendo $tg\, 2x = t \Rightarrow t^2 - 3 < 0 \Rightarrow -\sqrt{3} < t < \sqrt{3} \Rightarrow -\sqrt{3} < tg\, 2x < \sqrt{3}$.

Fazendo $2x = y \Rightarrow -\sqrt{3} < tg\, y < \sqrt{3} \Rightarrow \frac{2\pi}{3} + 2k\pi < y < \frac{4\pi}{3} + 2k\pi$ ou

$2k\pi \leqslant y < \frac{\pi}{3} + 2k\pi$ ou $\frac{5\pi}{3} + 2k\pi < y < 2\pi + 2k\pi \Rightarrow$

$\Rightarrow \frac{\pi}{3} + k\pi < x < \frac{2\pi}{3} + k\pi$ ou $k\pi \leqslant x < \frac{\pi}{6} + k\pi$ ou $\frac{5\pi}{6} + k\pi < x < \pi + k\pi \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \frac{\pi}{6}\right.$ ou $\frac{\pi}{3} < x < \frac{2\pi}{3}$ ou $\frac{5\pi}{6} < x < \frac{7\pi}{6}$ ou

$\frac{4\pi}{3} < x < \frac{5\pi}{3}$ ou $\left.\frac{11\pi}{6} < x \leqslant 2\pi\right\}$.

447. $\text{sen}\, x - \cos x > 0 \Rightarrow \text{sen}\, x - \text{sen}\left(\frac{\pi}{2} - x\right) > 0 \Rightarrow \text{sen}\left(x - \frac{\pi}{4}\right) > 0 \Rightarrow$

$\Rightarrow 0 < x - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < x < \frac{5\pi}{4}$

448. $\cos x + \dfrac{\text{sen}\, \frac{\pi}{3}}{\cos \frac{\pi}{3}} \cdot \text{sen}\, x > \sqrt{2} \Rightarrow$

$\Rightarrow \cos x \cdot \cos\frac{\pi}{3} + \text{sen}\, \frac{\pi}{3} \cdot \text{sen}\, x > \sqrt{2} \cos\frac{\pi}{3} \Rightarrow$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) > \frac{\sqrt{2}}{2}$ fazendo $x - \frac{\pi}{3} = t \Rightarrow \cot t > \frac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow \left(0 \leqslant t < \frac{\pi}{4}\right.$ ou $\left.-\frac{\pi}{4} < t \leqslant 0\right)$. E daí: $0 \leqslant x - \frac{\pi}{3} < \frac{\pi}{4} \Rightarrow$

$\Rightarrow \frac{\pi}{3} < x < \frac{7\pi}{12}$ ou $-\frac{\pi}{4} < x - \frac{\pi}{3} \leqslant 0 \Rightarrow \frac{\pi}{12} < x \leqslant \frac{\pi}{3}$; portanto,

$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{12} < x < \frac{7\pi}{12}\right\}$

449. $\pi \leqslant x \leqslant 2\pi \Rightarrow \text{sen } x \leqslant 0 \Rightarrow |\cos x| \geqslant \text{sen } x$.

Se *sen x* $\geqslant 0$, a inequação equivale a $\cos^2 x \geqslant \text{sen}^2 x$ e daí $2 \cdot \text{sen}^2 x - 1 \leqslant 0$, portanto $-\dfrac{\sqrt{2}}{2} \leqslant \text{sen } x \leqslant \dfrac{\sqrt{2}}{2}$.

Tendo em vista a hipótese, temos $0 \leqslant \text{sen } x \leqslant \dfrac{\sqrt{2}}{2}$, de onde vem

$0 \leqslant x \leqslant \dfrac{\pi}{4}$ ou $\dfrac{3\pi}{4} \leqslant x \leqslant \pi$.

$$S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \frac{\pi}{4} \text{ ou } \frac{3\pi}{4} \leqslant x \leqslant 2\pi\right\}$$

450. $\dfrac{2 \operatorname{tg} x \left(1 + \operatorname{tg}^2 x - \dfrac{1}{3}\right)}{1 + \operatorname{tg}^2 x} \leqslant 0 \Rightarrow \operatorname{tg} x \leqslant 0 \Rightarrow$

$\Rightarrow \dfrac{\pi}{2} < x \leqslant \pi$ ou $\dfrac{3\pi}{2} < x \leqslant 2\pi$

451. $\text{sen}^2 x - \dfrac{1}{4} \geqslant 0 \Rightarrow \text{sen } x \geqslant \dfrac{1}{2}$ ou $\text{sen } x \leqslant -\dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}$ ou $\dfrac{7\pi}{6} \leqslant x \leqslant \dfrac{11\pi}{6}$

452. $3^{2 \text{ sen } x - 1} \geqslant 3^0 \Rightarrow 2 \text{ sen } x - 1 \geqslant 0 \Rightarrow \text{sen } x \geqslant \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}$

453. a) $2 \text{ sen } x - 1 > 0 \Rightarrow \text{sen } x > \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

b) $\log_2 (2 \cdot \text{sen } x - 1) = \dfrac{1}{2} \cdot \log_2 (3 \text{ sen}^2 x - 4 \cdot \text{sen } x + 2)$

$\log_2 (2 \cdot \text{sen } x - 1)^2 = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2)$
então
$(2 \cdot \text{sen } x - 1)^2 = 3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2 \Rightarrow$
$\Rightarrow \text{sen}^2 x = 1 \Rightarrow \text{sen } x = \pm 1$

Levando em conta a parte a), resulta $\text{sen } x = 1 \Rightarrow x = \dfrac{\pi}{2}$.

454. $\cos^2 x - \dfrac{3}{4} > 0 \Rightarrow -\dfrac{\sqrt{3}}{2} < \cos x < \dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$ ou $\dfrac{7\pi}{6} < x < \dfrac{11\pi}{6}$

455. Fazendo $\cos x = y \Rightarrow \dfrac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \dfrac{1}{2}$ ou $y > 1$

e daí $-1 < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \pi$.

456. $\frac{4 \operatorname{sen}^2 x - 1}{\cos x} \geqslant 0 \Rightarrow \frac{4 - 4\cos^2 x - 1}{\cos x} \geqslant 0$.

Fazendo $\cos x = y \Rightarrow \frac{3 - 4y^2}{y} \geqslant 0 \Rightarrow \left(y \leqslant -\frac{\sqrt{3}}{2} \text{ ou } 0 < y \leqslant \frac{\sqrt{3}}{2}\right) \Rightarrow$

$\Rightarrow \left(\cos x \leqslant -\frac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leqslant \frac{\sqrt{3}}{2}\right) \Rightarrow$

$\Rightarrow \frac{\pi}{6} \leqslant x < \frac{\pi}{2} \text{ ou } \frac{5\pi}{6} \leqslant x \leqslant \frac{7\pi}{6} \text{ ou } \frac{3\pi}{2} < x \leqslant \frac{11\pi}{6}$.

457. $x^2 + x + \left(\operatorname{tg} \alpha - \frac{3}{4}\right) > 0, \forall x \Rightarrow \Delta < 0 \Rightarrow 1 - 4 \operatorname{tg} \alpha + 3 < 0 \Rightarrow$

$\Rightarrow \operatorname{tg} \alpha > 1 \Rightarrow \frac{\pi}{4} < \alpha < \frac{\pi}{2}$

458. $\operatorname{sen}^2 x - 2 \operatorname{sen} x < 0 \Rightarrow 0 < \operatorname{sen} x < 2 \Rightarrow 0 < x < \pi$

459. $\operatorname{sen}^2 x + \cos^2 x + 2 \operatorname{sen} x \cos x > 1 \Rightarrow \operatorname{sen} 2x > 0$. Fazendo $2x = t$, temos:

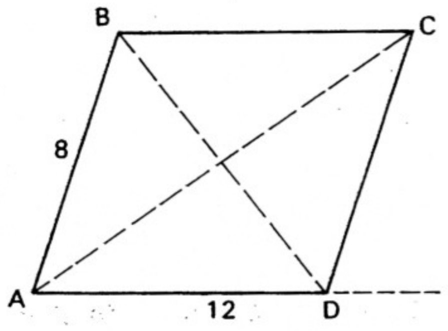
$\Rightarrow \operatorname{sen} t > 0 \Rightarrow 2k\pi < t < \pi + 2k\pi \Rightarrow k\pi < x < \frac{\pi}{2} + k\pi \Rightarrow$

$\Rightarrow 0 < x < \frac{\pi}{2} \text{ ou } \pi < x < \frac{3\pi}{2}$.

Apêndice B — Trigonometria em triângulos quaisquer

461. $c^2 = 4^2 + (3\sqrt{2})^2 - 2 \cdot 4 \cdot 3\sqrt{2} \cdot \cos 45° \Rightarrow c = \sqrt{10}$

462.



$BD = a$ e $AC = b$

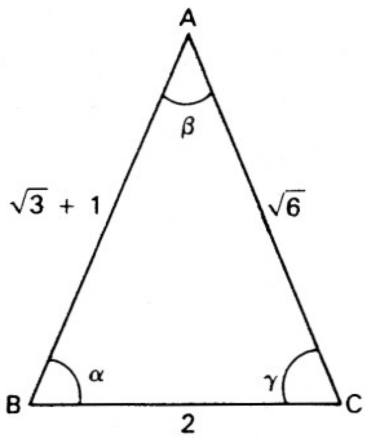
$a^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \cos 60° \Rightarrow$

$\Rightarrow a = 4\sqrt{7}$ m

$b^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \cos 120° \Rightarrow$

$\Rightarrow b = 4\sqrt{19}$ m

463.



$$2^2 = (\sqrt{6})^2 + (\sqrt{3} + 1)^2 - 2(\sqrt{6})(\sqrt{3} + 1) \cos \beta \Rightarrow$$

$$\Rightarrow \cos \beta = \frac{\sqrt{2}}{2} \Rightarrow \beta = 45°$$

$$(\sqrt{6})^2 = (\sqrt{3} + 1)^2 + 2^2 - 2 \cdot 2(\sqrt{3} + 1) \cos \alpha \Rightarrow$$

$$\Rightarrow \cos \alpha = \frac{1}{2} \Rightarrow \alpha = 60°$$

$$\alpha + \beta + \gamma = 180° \Rightarrow \gamma = 75°$$

464. $a, b, c \in \mathbb{Q} \Rightarrow a^2, b^2, c^2 \in \mathbb{Q} \Rightarrow (a^2 + c^2 - b^2) \in \mathbb{Q}$
$a, c \in \mathbb{Q} \Rightarrow 2ac \in \mathbb{Q}$

$$\frac{a^2 + c^2 - b^2}{2ac} \in \mathbb{Q} \text{ e } \cos \beta = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \cos \beta \in \mathbb{Q}$$

465. $(x^2 + x + 1)^2 = (x^2 - 1)^2 + (2x + 1)^2 - 2(x^2 - 1)(2x + 1) \cdot \cos \beta \Rightarrow$

$$\Rightarrow \cos \beta = \frac{2x^3 + x^2 - 2x - 1}{-4x^3 - 2x^2 + 4x + 2} = \frac{1 \cdot (2x^3 + x^2 - 2x - 1)}{-2 \cdot (2x^3 + x^2 - 2x - 1)} = -\frac{1}{2} \Rightarrow$$

$$\Rightarrow \beta = 120°$$

466. $a^2 = c^2 + 1 - 2c \cos 120° \Rightarrow a^2 - c^2 - c = 1 \Rightarrow (2c)^2 - c^2 - c = 1 \Rightarrow$

$$\Rightarrow c = \frac{1 + \sqrt{13}}{6}$$

468. a) $17^2 = 15^2 + 8^2 \Rightarrow$ O triângulo é retângulo.
b) $10^2 > 5^2 + 6^2 \Rightarrow$ O triângulo é obtusângulo.
c) $8^2 < 6^2 + 7^2 \Rightarrow$ O triângulo é acutângulo.

469. Chamando as medidas dos lados de a, aq, aq^2, só falta impor duas condições:
(1) o maior lado é menor que a soma dos outros dois (condição para existência do triângulo): $aq^2 < a + aq$;
(2) o quadrado do maior lado é maior que a soma dos quadrados dos outros dois (condição para o triângulo ser obtusângulo): $(aq^2)^2 > a^2 + (aq)^2$.

De (1) resulta $\dfrac{1 - \sqrt{5}}{2} < q < \dfrac{1 + \sqrt{5}}{2}$.

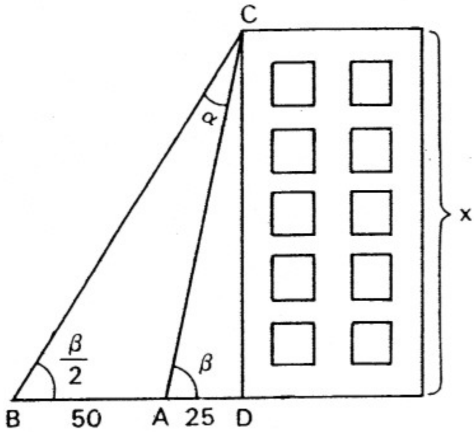
De (2) resulta $q < -\sqrt{\dfrac{1 + \sqrt{5}}{2}}$ ou $q > \sqrt{\dfrac{1 + \sqrt{5}}{2}}$.

Como $q > 0$, temos $\sqrt{\dfrac{1 + \sqrt{5}}{2}} < q < \dfrac{1 - \sqrt{5}.}{2}$

472. $$\left.\begin{array}{l} \text{sen } \hat{B} = \dfrac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60^\circ \text{ ou } \hat{B} = 120^\circ \\ \text{sen } \hat{C} = \dfrac{\sqrt{2}}{2} \Rightarrow \hat{C} = 45^\circ \text{ ou } C = 135^\circ \\ A + B + C = 180^\circ \Rightarrow B + C = 180^\circ - 15^\circ \Rightarrow B + C = 165^\circ \end{array}\right\} \Rightarrow$$

$\Rightarrow \hat{B} = 120^\circ$ e $\hat{C} = 45^\circ$ e $\hat{A} = 15^\circ$

473.



$\beta = \dfrac{\beta}{2} + \alpha$ (ângulo externo ao ΔABC) $\Rightarrow$

$\Rightarrow \alpha = \dfrac{\beta}{2}$

$\therefore$ o triângulo ABC é isósceles $\Rightarrow$

$\Rightarrow BA = AC = 50$ m

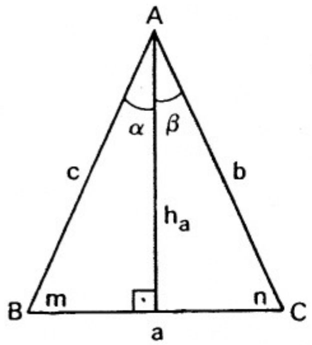
No ΔACD temos $DC^2 = AC^2 - AD^2 = $
$= 50^2 - 25^2$

$DC = 25\sqrt{3}$ m.

474. $$\left.\begin{array}{l} \text{tg } \alpha = \dfrac{m}{h_a} \Rightarrow h_a = \dfrac{m}{\text{tg } \alpha} \\ \text{tg } \beta = \dfrac{n}{h_a} \Rightarrow h_a = \dfrac{n}{\text{tg } \beta} \end{array}\right\} \Rightarrow \dfrac{m}{\text{tg } \alpha} = \dfrac{n}{\text{tg } \beta} \Rightarrow$$

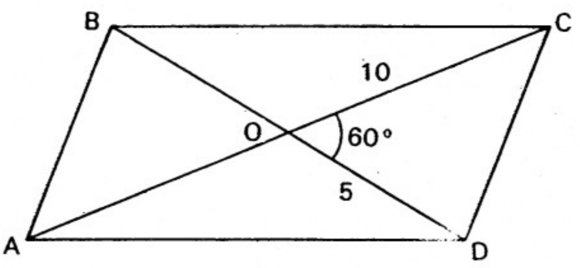
$$\Rightarrow \frac{m + n}{\text{tg } \alpha + \text{tg } \beta} = \frac{m}{\text{tg } \alpha} \Rightarrow \frac{a}{\text{tg } \alpha + \text{tg } \beta} = \frac{m}{\text{tg } \alpha} \Rightarrow$$

$$a = \frac{m}{\text{tg } \alpha} \cdot (\text{tg } \alpha + \text{tg } \beta) \Rightarrow a = h_a(\text{tg } \alpha + \text{tg } \beta)$$



475. $S = \dfrac{4 \cdot 7 \text{ sen } 60^\circ}{2} \Rightarrow S = 7\sqrt{3}$ m^2

476.



Sabe-se que as diagonais de um paralelogramo dividem-se mutuamente ao meio, então:

$$\overline{AO} = \overline{OC} = 10 \text{ e } \overline{BO} = \overline{OD} = 5$$

Além disso, as diagonais dividem o paralelogramo em quatro triângulos de áreas iguais, então:

$$S_{ABCD} = 4 \cdot S_{DOC} = 4 \cdot \frac{\overline{DO} \cdot \overline{OC} \cdot \text{sen } D\hat{O}C}{2} = 4 \cdot \frac{5 \cdot 10 \cdot \sqrt{3}}{4} = 50\sqrt{3}$$

477. $S = \dfrac{8 \cdot 10}{2} \text{sen } \alpha \Rightarrow \text{sen } \alpha = \dfrac{1}{2} \Rightarrow \alpha = 30°,$

$a^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cdot \cos 30° \Rightarrow a = 2\sqrt{41 - 20\sqrt{3}},$

$\dfrac{a}{\text{sen } \alpha} = 2R \Rightarrow \dfrac{2\sqrt{41 - 20\sqrt{3}}}{\frac{1}{2}} = 2R \Rightarrow R = 2\sqrt{41 - 20\sqrt{3}}$

478. $7^2 = c^2 + 8^2 - 2 \cdot 8 \cdot c \cdot \cos 60° \Rightarrow c^2 - 8c + 15 = 0 \Rightarrow c = 5 \text{ ou } c = 3$

$c = 5 \text{ m} \Rightarrow S = \dfrac{5 \cdot 8}{2} \text{sen } 60° \Rightarrow S = 10\sqrt{3} \text{ m}^2 \text{ ou } c = 3 \text{ m} \Rightarrow S = 6\sqrt{3} \text{ m}^2$

479. $A + B + C = 180° \Rightarrow A + B = 180° - C \text{ ou } B + C = 180° - A,$

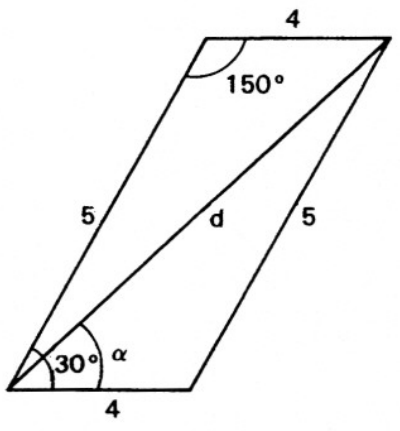
$\cos(180° - C) = -\cos C \Rightarrow \cos(A + B) = \cos(180° - C) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \cos C = -\dfrac{1}{2} \Rightarrow C = 120°$

$\text{sen}(B + C) = \text{sen}(180° - A) = \text{sen } A \Rightarrow \text{sen } A = \dfrac{1}{2} \Rightarrow A = 30°$

$B = 180° - (A + C) = 30°$

480.



$d^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 150° =$

$= 41 + 20\sqrt{3}$

$\dfrac{5}{\text{sen } \alpha} = \dfrac{\sqrt{41 + 20\sqrt{3}}}{\text{sen } 150°} \Rightarrow$

$\Rightarrow \text{sen } \alpha = \dfrac{5}{2\sqrt{41 + 20\sqrt{3}}}$

$\alpha = \text{arc sen } \dfrac{5}{2\sqrt{41 + 20\sqrt{3}}}$

482. $\frac{a}{5} = \frac{b}{7} = \frac{c}{9} = k \Rightarrow a = 5k, b = 7k, c = 9k$

$\cos \hat{B} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{25k^2 + 81k^2 - 49k^2}{2(5k)(9k)} = \frac{57}{90}$

$\hat{B} = \text{arc cos } \frac{57}{90}$

483. $\text{sen } 15° = \text{sen } (45° - 30°) = \text{sen } 45° \cos 30° - \text{sen } 30° \cos 45° = \frac{\sqrt{6} - \sqrt{2}}{4}$

$\frac{1}{\text{sen } 15°} = \frac{\sqrt{3} + 1}{\text{sen } \hat{B}} \Rightarrow \text{sen } \hat{B} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45° \text{ ou } \hat{B} = 135°$

$\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{B} + \hat{C} = 180° - 15° = 165° \Rightarrow$

$\Rightarrow (\hat{C} = 120° \text{ e } \hat{B} = 45°) \text{ ou } (\hat{C} = 30° \text{ e } \hat{B} = 135°)$

484. $c^2 = (2b)^2 + b^2 - 2 \cdot 2b \cdot b \cdot \cos 60° \Rightarrow c^2 = 3b^2 \Rightarrow c = b\sqrt{3}$

$\frac{c}{\text{sen } 60°} = \frac{b}{\text{sen } \hat{B}} \Rightarrow \text{sen } \hat{B} = \frac{1}{2} \Rightarrow \hat{B} = 30°$
(pois $\hat{B} < 120°$)

$\hat{A} = 180° - (\hat{B} + \hat{C}) = 90°$

485. $\hat{B} = 180° - (\hat{A} + \hat{C}) = 180° - 3\hat{A} \Rightarrow \text{sen } \hat{B} = \text{sen } (180° - 3\hat{A}) = \text{sen } 3\hat{A}$

$\frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} \Rightarrow \frac{b}{\hat{C}} = \frac{\text{sen } 3\hat{A}}{\text{sen } 2\hat{A}} = \frac{3 \text{ sen } \hat{A} - 4 \text{ sen}^3 \hat{A}}{2 \text{ sen } \hat{A} \cdot \cos \hat{A}} = \frac{3 - 4 \text{ sen}^2 \hat{A}}{2 \cos \hat{A}} \Rightarrow$

$\Rightarrow \left(\frac{2}{\sqrt{3}}\right)^2 = \left(\frac{3 - 4 \text{ sen}^2 \hat{A}}{2 \cos \hat{A}}\right)^2 \Rightarrow 48 \text{ sen}^4 \hat{A} - 56 \text{ sen}^2 \hat{A} + 11 = 0 \Rightarrow$

$\Rightarrow \text{sen } \hat{A} = \frac{1}{2} \Rightarrow \hat{A} = 30°$, então $\hat{C} = 60°$ e $\hat{B} = 90°$

486. $\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} \Rightarrow \frac{6}{\text{sen } 3\hat{B}} = \frac{3}{\text{sen } \hat{B}} \Rightarrow$

$\Rightarrow 2 = \frac{\text{sen } 3\hat{B}}{\text{sen } \hat{B}} = \frac{3 \text{ sen } \hat{B} - 4 \text{ sen}^3 \hat{B}}{\text{sen } \hat{B}} = 3 - 4 \text{ sen}^2 \hat{B} \Rightarrow \text{sen } \hat{B} = \frac{1}{2} \Rightarrow$

$\Rightarrow \hat{B} = 30°, \hat{A} = 90° \text{ e } \hat{C} = 60°, c^2 = b^2 + a^2 - 2ab \cos \hat{C} \Rightarrow$

$c^2 = 3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 60° \Rightarrow c = 3\sqrt{3} \text{ m}$

487. $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{B} + \hat{C} = 180° - \hat{A} \Rightarrow \text{tg } (\hat{B} + \hat{C}) = -\text{tg } \hat{A} \Rightarrow$

$\Rightarrow \frac{\text{tg } \hat{B} + \text{tg } \hat{C}}{1 - \text{tg } \hat{B} \cdot \text{tg } \hat{C}} = -\text{tg } \hat{A} \Rightarrow \frac{2 \cdot \text{tg } \hat{A}}{1 - \text{tg } \hat{B} \cdot \text{tg } \hat{C}} = -\text{tg } \hat{A} \Rightarrow$

$\Rightarrow 2 = -1 + \text{tg } \hat{B} \cdot \text{tg } \hat{C} \Rightarrow \text{tg } \hat{B} \cdot \text{tg } \hat{C} = 3$

488. $S = \dfrac{3 \cdot 4 \cdot \text{sen}\ \alpha}{2} = 6 \cdot \text{sen}\ \alpha$

$S - 3 = \dfrac{3 \cdot 4 \cdot \text{sen}\ (\alpha - 60^\circ)}{2} = 6 \cdot \text{sen}\ (\alpha - 60^\circ)$

então

$6 \cdot \text{sen}\ \alpha - 3 = 6 \cdot \text{sen}\ (\alpha - 60^\circ) \Rightarrow \text{sen}\ \alpha - \text{sen}\ (\alpha - 60^\circ) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow 2 \cdot \text{sen}\ 30^\circ \cdot \cos\ (\alpha - 30^\circ) = \dfrac{1}{2} \Rightarrow \cos\ (\alpha - 30^\circ) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \alpha - 30^\circ = 60^\circ \Rightarrow \alpha = 90^\circ \Rightarrow S = 6 \cdot \text{sen}\ \alpha = 6\ m^2$

490. $a^2 = b^2 + c^2 \Rightarrow a^2 - c^2 = 3^2 \Rightarrow (a + c)(a - c) = 9 \Rightarrow$

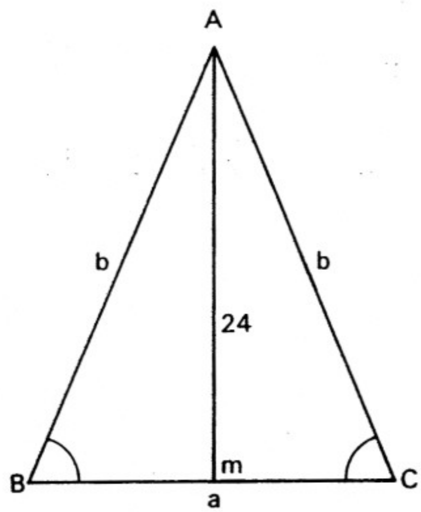
$\Rightarrow (a + c)\sqrt{3} = 9 \Rightarrow a + c = 3\sqrt{3}$ (1)

É dado que $a - c = \sqrt{3}$ (2). De (1) e (2) resulta $a = 2\sqrt{3}$ e $c = \sqrt{3}$, então

$sen\ \hat{B} = \dfrac{b}{a} = \dfrac{\sqrt{3}}{2}$ e $\hat{B} = 60^\circ$.

491. Do sistema $a + b = 18$, $a + c = 25$ e $b^2 + c^2 = a^2$, resulta
$(18 - a)^2 + (25 - a)^2 = a^2$ e daí $a = 13$, portanto
$b = 18 - a = 5$ e $c = 25 - a = 12$.
Finalmente $sen\ \hat{B} = \dfrac{b}{a} = \dfrac{5}{13}$ e $sen\ \hat{C} = \dfrac{c}{a} = \dfrac{12}{13}$.

492.



$2b + a = 64$ (1)

$b^2 = \left(\dfrac{a}{2}\right)^2 + 24^2$ (2)

De (1) $a = 64 - 2b$, que substituído em (2) dá $b^2 = (32 - b)^2 + 576$ e daí $b = 25$.

(1) $a = 64 - 2b = 14$

$\cos\ \hat{B} = \cos\ \hat{C} = \dfrac{a/2}{b} = \dfrac{7}{25}$

$\text{sen}\ \dfrac{\hat{A}}{2} = \dfrac{a/2}{b} = \dfrac{7}{25}$

493. $b = 1$, $c = \text{tg}\ \varphi \Rightarrow a^2 = b^2 = c^2 = 1 + \text{tg}^2\ \varphi = \sec^2 \varphi \Rightarrow a = \sec \varphi$

$\text{tg}\ \hat{B} = \dfrac{b}{c} = \dfrac{1}{\text{tg}\ \varphi} = \text{cotg}\ \varphi$

$\text{tg}\ \hat{C} = \dfrac{c}{b} = \text{tg}\ \varphi$

494. lados: a, b = a + 1, c = a + 2

$$\frac{a}{\text{sen } \hat{A}} = \frac{c}{\text{sen } \hat{C}} \Rightarrow \frac{a}{\text{sen } \hat{A}} = \frac{a+2}{\text{sen } 2\hat{A}} \Rightarrow \cos \hat{A} = \frac{a+2}{2a}$$

$$a^2 = (a+1)^2 + (a+2)^2 - 2(a+1)(a+2) \cdot \frac{a+2}{2a} \Rightarrow a^2 - 3a - 4 = 0 \Rightarrow$$

$$\Rightarrow a = 4 \text{ e daí: } b = 5,\ c = 6,\ \cos \hat{A} = \frac{3}{4},\ \hat{C} = 2 \cdot \text{ arc sen } \frac{\sqrt{7}}{4}$$

495. $\frac{(a+b+c) \cdot r}{2} = \frac{a \cdot h}{2} \Rightarrow h = \frac{2r^2 + 2ra}{a} \Rightarrow h = \frac{12}{5}.$

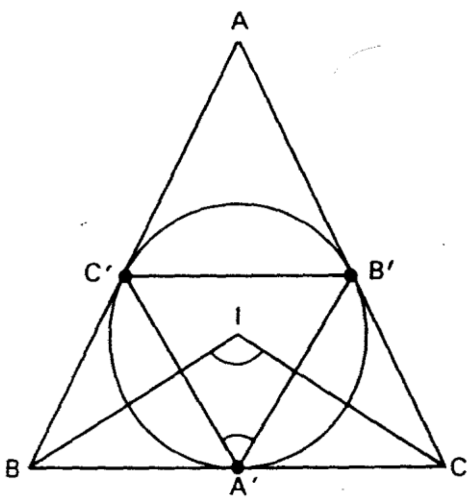
$(a + b + c) \cdot r = a \cdot h \Rightarrow b + c = 7$ (A), $b \cdot c = a \cdot h \Rightarrow b \cdot c = 12$ (B).

De (A) e (B) vem $x^2 - 7x + 12 = 0 \Rightarrow x = 4$ ou $x = 3$.

$$\cos A = \frac{9 + 16 - 25}{24} = 0 \Rightarrow A = 90°,\ \text{sen } \hat{B} = \frac{3}{5} \Rightarrow \hat{B} = \text{arc sen } \frac{3}{5}$$

$$\cos C = \frac{3}{5} \Rightarrow \hat{C} = \text{arc cos } \frac{3}{5}$$

496.



Seja I o centro da circunferência inscrita em ABC. Ligando I com B e com C, temos:

$$\hat{A}' \equiv 180° - B\widehat{I}C \equiv \frac{\hat{B} + \hat{C}}{2}.$$

Analogamente:

$$\hat{B}' \equiv 180° - A\widehat{I}C \equiv \frac{\hat{A} + \hat{C}}{2}.$$

$$\hat{C}' \equiv 180° - A\widehat{I}B \equiv \frac{\hat{A} + \hat{B}}{2}.$$

O triângulo $CA'B'$ é isósceles ($CA' \equiv CB'$), então

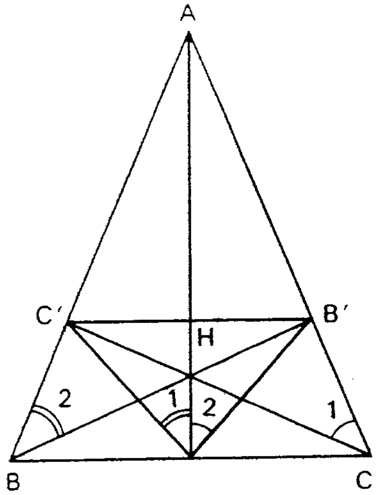
$$c' = A'B' = 2 \cdot CA' \cdot \text{sen } \frac{\hat{C}}{2} = 2(p - c)\text{sen } \frac{\hat{C}}{2}$$

em que $p = \frac{a + b + c}{2}$.

Analogamente, temos:

$$b' = 2(p - b)\text{sen } \frac{\hat{B}}{2} \text{ e } a' = 2(p - a)\text{sen } \frac{\hat{A}}{2}.$$

497.



Seja H o ponto em que as alturas AA', BB' e CC' se interceptam. Os quadriláteros $HA'CB'$ e $HA'BC'$ são inscritíveis porque têm dois ângulos opostos retos e, portanto, suplementares, então:
$\hat{A}_1' \equiv \hat{C} \equiv 90° - \hat{A}$ e $\hat{A}_2' \equiv \hat{B}_2 \equiv 90° - \hat{A}$.
Chamando de $\hat{A}'$, $\hat{B}'$ e $\hat{C}'$ os ângulos do triângulo $A'B'C'$, obtemos:

$\hat{A}' \equiv \hat{A}_1' + \hat{A}_2' \equiv 180° - 2\hat{A}$, $\hat{B}' \equiv 180° - 2\hat{B}$ e $\hat{C}' \equiv 180° - 2\hat{C}$.
Aplicando a lei dos senos ao triângulo $A'B'C$, temos:

$$\frac{A'B'}{\text{sen } C} = \frac{B'C}{\text{sen } B'\hat{A}'C} = \frac{a \cos C}{\text{sen } A} \Rightarrow A'B' = \frac{a \text{ sen } C \cos C}{\text{sen } A}.$$

Analogamente: $B'C' = \dfrac{c \text{ sen } \hat{A} \cos \hat{A}}{\text{sen } B}$ e $A'C' = \dfrac{b \text{ sen } \hat{B} \cos \hat{B}}{\text{sen } C}$.

498. $\cos \hat{A} = \sqrt{1 - \text{sen}^2 \hat{A}} = \dfrac{41}{50}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \hat{A} \Rightarrow 9 = b^2 + (10 - b)^2 - 2b(10 - b) \cdot \frac{41}{50} \Rightarrow$$

$\Rightarrow b^2 - 10b + 25 = 0 \Rightarrow b = 5 \Rightarrow c = 5$
Então $\hat{B} = \hat{C}$ e $\hat{A} + \hat{B} + \hat{C} = \pi$, portanto:

$$\hat{B} = \hat{C} = \frac{\pi}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{1}{2} \cdot \text{arc sen } \frac{3\sqrt{91}}{50}.$$

499. $h_a = c \cdot \text{sen } \hat{B} \Rightarrow n = c \cdot \text{sen } \hat{B} \Rightarrow c = \dfrac{n}{\text{sen } \hat{B}}$

$h_a = b \cdot \text{sen } \hat{C} \Rightarrow n = b \cdot \text{sen } \hat{C} \Rightarrow b = \dfrac{n}{\text{sen } \hat{C}}$

$b + c = m \Rightarrow \dfrac{n}{\text{sen } \hat{B}} + \dfrac{n}{\text{sen } \hat{C}} = 11 \quad (1)$

De (1) vem:
$n(\text{sen } \hat{B} + \text{sen } \hat{C}) = m \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}$

$$2n \cdot \text{sen } \frac{\hat{B} + \hat{C}}{2} \cdot \cos \frac{\hat{B} - \hat{C}}{2} = \frac{m}{2}[\cos(\hat{B} - \hat{C}) - \cos(\hat{B} + \hat{C})] \quad (2)$$

Notemos que:
$\cos(\hat{B} + \hat{C}) = -\cos \hat{A}$ (dado)

$\text{sen } \dfrac{\hat{B} + \hat{C}}{2} = \cos \dfrac{\hat{A}}{2}$ (calculável a partir de $\cos \hat{A}$)

$\cos(\hat{B} - \hat{C}) = 2 \cdot \cos^2 \dfrac{\hat{B} - \hat{C}}{2} - 1$

Então a equação (2) fica:

$$m \cdot \cos^2 \frac{\hat{B} - \hat{C}}{2} - 2n \cdot \cos \frac{\hat{A}}{2} \cdot \cos \frac{\hat{B} - \hat{C}}{2} - m \cdot \text{sen}^2 \frac{\hat{A}}{2} = 0$$

A partir dessa equação obtém-se o ângulo $\frac{\hat{B} - \hat{C}}{2}$.

Como $\frac{\hat{B} + \hat{C}}{2} = \frac{\pi}{2} - \frac{\hat{A}}{2}$, os ângulos $\hat{B}$ e $\hat{C}$ estão determinados e daí

$$b = \frac{n}{\text{sen } \hat{C}}, \ c = \frac{n}{\text{sen } \hat{B}}, \ a = \frac{m \cdot \text{sen } (\hat{B} + \hat{C})}{\text{sen } \hat{B} + \text{sen } \hat{C}}.$$

500. $$\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} \Rightarrow \frac{a + c}{\text{sen } \hat{A} + \text{sen } \hat{C}} = \frac{b}{\text{sen } \hat{B}} \Rightarrow$$

$$\Rightarrow \frac{b}{\text{sen } (\hat{A} + \hat{C})} = \frac{k}{\text{sen } \hat{A} + \text{sen } \hat{C}} \Rightarrow$$

$$\frac{b}{2 \cdot \text{sen } \frac{\hat{A} + \hat{C}}{2} \cdot \cos \frac{\hat{A} + \hat{C}}{2}} = \frac{k}{2 \cdot \text{sen } \frac{\hat{A} + \hat{C}}{2} \cdot \cos \frac{\hat{A} - \hat{C}}{2}} \Rightarrow$$

$$\Rightarrow \frac{b}{\cos \frac{\hat{A} + \hat{C}}{2}} = \frac{k}{\cos \frac{\hat{A} - \hat{C}}{2}} \Rightarrow$$

$$\Rightarrow \frac{b + k}{\cos \frac{\hat{A} + \hat{C}}{2} + \cos \frac{\hat{A} - \hat{C}}{2}} = \frac{k - b}{\cos \frac{\hat{A} - \hat{C}}{2} - \cos \frac{\hat{A} + \hat{C}}{2}} \Rightarrow$$

$$\Rightarrow \frac{b + k}{2 \cdot \cos \frac{\hat{A}}{2} \cdot \cos \frac{\hat{C}}{2}} = \frac{k - b}{2 \cdot \text{sen } \frac{\hat{A}}{2} \cdot \text{sen } \frac{\hat{C}}{2}} \Rightarrow$$

$$\text{cotg } \frac{\hat{C}}{2} = \frac{b + k}{k - b} \cdot \text{tg } \frac{\hat{A}}{2}$$

Conhecendo $\hat{C}$, temos:

$$B = \pi - \hat{A} - \hat{C}, \ a = \frac{b \text{ sen } \hat{A}}{\text{sen } \hat{B}} \text{ e } c = \frac{b \text{ sen } \hat{C}}{\text{sen } \hat{B}}.$$

501. $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{A} = 180° - (\hat{B} + \hat{C})$

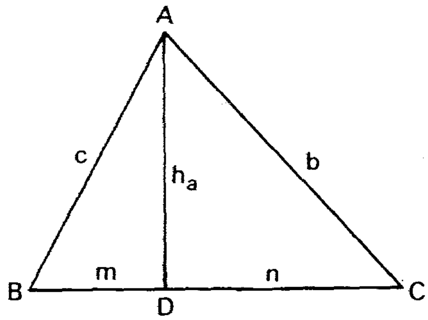
$$\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} = 2R \Rightarrow a = 2R \text{ sen } \hat{A}, \ b = 2R \text{ sen } \hat{B}, \ c = 2R \text{ sen } \hat{C}$$

em que *R* é calculado assim:

$$S = \frac{1}{2} bc \text{ sen } \hat{A} = \frac{1}{2} \cdot 4R^2 \cdot \text{sen } \hat{A} \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}, \text{ então:}$$

$$2R = \sqrt{\frac{2S}{\text{sen } \hat{A} \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}}}$$

502.



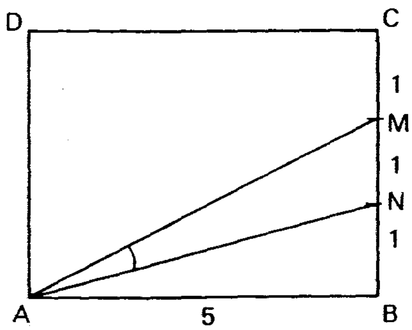
$$a = m + n = \frac{h_a}{\text{tg } \hat{B}} + \frac{h_a}{\text{tg } \hat{C}}$$

$$b = \frac{h_a}{\text{sen } \hat{C}}$$

$$c = \frac{h_a}{\text{sen } \hat{B}}$$

$$\hat{A} = 180° - (\hat{B} + \hat{C})$$

503.

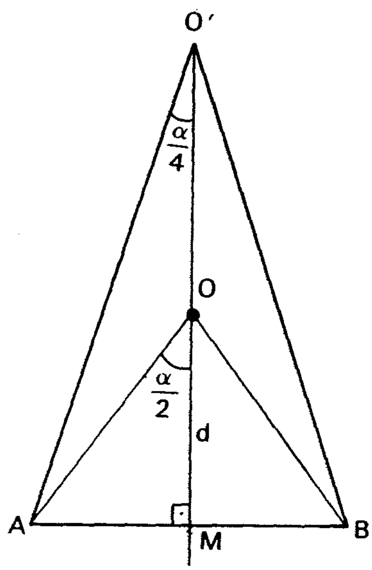


$$\text{tg } M\hat{A}B = \frac{2}{5} \text{ e tg } N\hat{A}B = \frac{1}{5}$$

$$\text{tg } M\hat{A}N = \text{tg } (M\hat{A}B - N\hat{A}B) =$$

$$\frac{\frac{2}{5} - \frac{1}{5}}{1 + \frac{2}{5} \cdot \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{27}{25}} = \frac{5}{27}$$

504.



$$\frac{AB}{2} = d \cdot \text{tg } \frac{\alpha}{2} \text{ e}$$

$$\frac{AB}{2} = (d + OO') \cdot \text{tg } \frac{\alpha}{4}$$

então

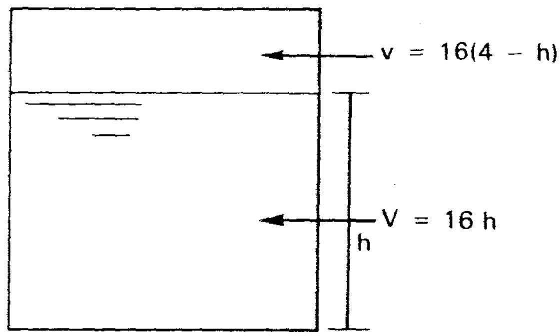
$$d \cdot \text{tg } \frac{\alpha}{2} = (d + OO') \cdot \text{tg } \frac{\alpha}{4}$$

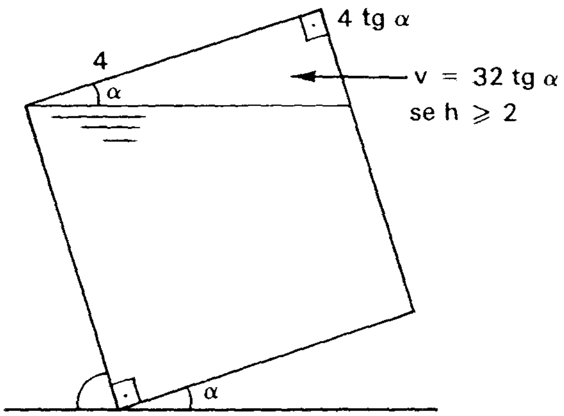
$$OO' = \frac{d\left(\text{tg } \frac{\alpha}{2} - \text{tg } \frac{\alpha}{4}\right)}{\text{tg } \frac{\alpha}{4}} = \frac{d}{\cos \frac{\alpha}{2}}$$

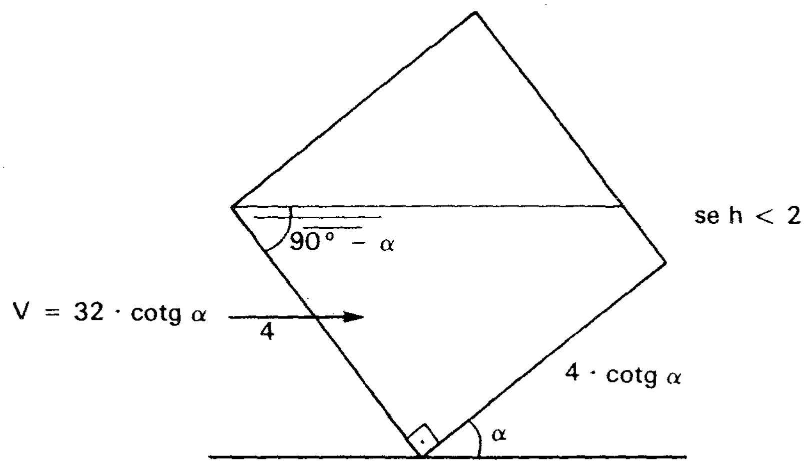
505. $\cos \theta = \dfrac{100 + 49 - 169}{140} = -\dfrac{1}{7} \Rightarrow \sec^2 \theta = 49 \Rightarrow 1 + \text{tg}^2 \theta = 49$

$\Rightarrow \text{tg } \theta = -4\sqrt{3} = \alpha \; |\sqrt{3}\,\alpha| = |\sqrt{3}\,(-4\sqrt{3})| = 12.$

506. *situação inicial* *situação final*







Se $h \geqslant 2$, então $16\,(4 - h) = 32\ tg\ \alpha$ e daí $tg\ \alpha = \dfrac{4 - h}{2}$.

Se $h < 2$, então $16h = 32\ cotg\ \alpha$ e daí $tg\ \alpha = \dfrac{2}{h}$.