AP3 - Lógica -
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19/10/20

1.

$\boxed{1.1.\ (C \vee B) \to (B \vee C)}$

(1) H6 $\to B \to (B \vee C)$, $p = B$, $q = C$

(2) H7 $\to C \to (B \vee C)$, $p = B$, $q = C$

H8 $\to (p \to r) \to ((q \to r) \to ((p \vee q) \to r))$, $p = C$, $r = (B \vee C)$ e $q = B$

$\overset{H8}{\to} (C \to (B \vee C)) \to (B \to (B \vee C)) \to ((C \vee B) \to (B \vee C))$

MP com H7 (2)

$\to (B \to (B \vee C)) \to ((C \vee B) \to (B \vee C))$

MP com H6 (1)

$\Rightarrow \underline{(C \vee B) \to (B \vee C)}$

$\boxed{1.2.\ (A \wedge (B \wedge C)) \to B}$

(1) H4 $\to (B \wedge C) \to B$, $p = B$, $q = C$

(2) H5 $\to (A \wedge (B \wedge C)) \to (B \wedge C)$, $p = A$, $q = (B \wedge C)$

trans. $(p \to q) \to (q \to r) \to (p \to r)$

trans. (2) e (1),

$\Rightarrow \underbrace{((A \wedge (B \wedge C)) \to (B \wedge C))}_{MP\ H5(2)} \to \underbrace{((B \wedge C) \to B)}_{MP\ H4(1)} \to ((A \wedge (B \wedge C)) \to B)$

Usando transitividade com (H5) e (H4): $\underline{(A \wedge (B \wedge C)) \to B}$

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2.

2.1 - $(C \to (B \to (A \vee (B \wedge C))))$

J3: $\underbrace{((C \wedge B) \to (A \vee (B \wedge C)))}_{\text{o que preciso provar}} \to \underbrace{(C \to (B \to (A \vee (B \wedge C))))}_{\text{o que quero}}$, $r = A \vee (B \wedge C)$, $q = B$, $p = C$

H7 $\to$ $(B \wedge C) \to (A \vee (B \wedge C))$, com $p = A$ e $q = (B \wedge C)$

H3: $\underline{C \to (B \to (B \wedge C))}$ *

J2: $C \to (B \to (B \wedge C)) \to ((C \wedge B) \to (B \wedge C))$

$\underline{\overset{H3}{*} + MP}$ e J2 fica: $((C \wedge B) \to (B \wedge C))$ □

Usando transitividade em □ + H7

$$\left.\begin{array}{l} \square: (C \wedge B) \to (B \wedge C) \\ (B \wedge C) \to (A \vee (B \wedge C)) \end{array}\right\} \xRightarrow{\text{transitividade}} (C \wedge B) \to (A \vee (B \wedge C)) \; ☆$$

Voltando a J3 lá em cima, podemos usar ☆ e Modus Ponens

Nossa parte: $(C \wedge B) \to (A \vee (B \wedge C)) \to (C \to (B \to (A \vee (B \wedge C))))$

☆ + MP $\Rightarrow$ $\boxed{(C \to (B \to (A \vee (B \wedge C))))}$

J1: $((C \wedge B) \to B) \to ((C \wedge B) \to C) \to ((C \wedge B) \to (B \wedge C))$

H4 $\to$ $(C \wedge B) \to B$

H5 $\to$ $(C \wedge B) \to C$

- Utilizando J1 + (H4 + MP) + (H5 + MP) obtenho a comutatividade: $\underline{(C \wedge B) \to (B \wedge C)}$

Instanciação
$p = C \wedge B$
$q = B$
$r = C$

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2.2. $(A \to (B \to (C \to D))) \to (A \to ((B \wedge C) \to D)))$

J2: $(p \to (q \to r)) \to ((p \wedge q) \to r)$, $p = A, B = q, r = (C \to D)$

J2
$\Rightarrow (A \to (B \to (C \to D))) \to ((A \wedge B) \to (C \to D))$

J2 $\Rightarrow (B \to (C \to D)) \to ((B \wedge C) \to D)$, $p = B, q = C, r = D$

L1 adicionando A.

$\Rightarrow (A \to (B \to (C \to D))) \to (A \to ((B \wedge C) \to D)))$

3.

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4.

1. $(A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))$

H8 $\Rightarrow$ $p = B$, $q = C$, $r = ((A \wedge B) \vee (A \wedge C))$

$$\underbrace{(B \to ((A \wedge B) \vee (A \wedge C)))}_{(1)} \to \underbrace{(C \to ((A \wedge B) \vee (A \wedge C)))}_{(2)} \to \underbrace{((B \vee C) \to ((A \wedge B) \vee (A \wedge C)))}_{(3)}$$

H5, com $p = A$, $q = (B \vee C)$

H5 $\to$ $(A \wedge (B \vee C)) \to (B \vee C)$ (*) transitividade

Só preciso construir (1) e (2) e usar MP, em seguida usar transitividade com (*) e 3.

$B \to ((A \wedge B) \vee (A \wedge C))$ (1) // construir (1)

H6 com $p = (A \wedge B)$ e $q = (A \wedge C)$:
$(A \wedge B) \to ((A \wedge B) \vee (A \wedge C))$ $\square$

J1: $(B \to A) \to ((B \to B) \to (B \to (A \wedge B)))$

A1: $A \to (B \to A)$ e R2 p/ tirar o A e H12 só com $B \to A$, agora posso usar MP

J1 fica com: $((B \to B) \to (B \to (A \wedge B)))$,

$\Rightarrow B \to B$

MP $\Rightarrow B \to (A \wedge B)$ $\square\square$

trans $\to$ $\square\square + \square$ = $\overset{p}{B} \to \overset{q}{(A \wedge B)}$ e $\overset{q}{(A \wedge B)} \to \overset{r}{((A \wedge B) \vee (A \wedge C))}$

obtenho $B \to ((A \wedge B) \vee (A \wedge C))$ consegui 1 agora posso

fazer MP já em uma (1)

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$C \to ((A \wedge B) \vee (A \wedge C))$. // construir o (2)

$(A \wedge C) \to ((A \wedge B) \vee (A \vee C))$ ☒

I1:

$(C \to A) \to ((C \to C) \to (C \to (A \wedge C)))$

H1: $A \to (C \to A)$ e R2 p/ tirar o A do fundo e ficar só com $C \to A$

MP $\Rightarrow (C \to C) \to (C \to (A \wedge C))$

$\Rightarrow C \to C$

$\overset{MP}{\Rightarrow}$ $\underline{(C \to (A \wedge C))}$ ☒☒

trans. ☒☒ + ☒

$\Rightarrow$ $(C \to (A \wedge C))$ e $(A \wedge C) \to ((A \wedge B) \vee (A \vee C))$

$\overset{trans}{\Rightarrow}$ $\underline{C \to ((A \wedge B) \vee (A \vee C))}$ com isso posso MP lá em cima

e obtenho $(B \vee C) \to ((A \wedge B) \vee (A \wedge C))$

e agora posso usar transitividade com (*)

$\overset{p}{(A \wedge (B \vee C))} \to \overset{q}{(B \vee C)}$

$\overset{q}{(B \vee C)} \to \overset{r}{((A \wedge B) \vee (A \wedge C))}$

$\overset{trans}{\rightarrow}$ $p \to r$

$$\boxed{(A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))}$$

■