Moment of Inertia

Lab Report

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Course: PHYS 141

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Due: 8 AM on November 10, 2022

Moment of Inertia

Abstract

The goals for this lab were to examine and determine the relationship between angular acceleration and inertia, using the mass and acceleration to find inertia, and to see if there was any effect on changing the center of mass and distribution of mass on the moment of inertia; we also changed the distance from the center to observe if there was any impact on the moment of inertia. We found the inertia of the bar to change when we changed the distance from the center, and the inertia of the disk and ring were significantly different even though their mass was the same. We may conclude that the distribution and distance are factors of change when it comes to inertia if we keep the mass of the object and the angular acceleration the same.

Introduction

In this lab we will be exploring the relationship between angular acceleration and moment of inertia based on the understanding that the larger an object’s inertia, the smaller its acceleration when a force is applied to said object. When applied to rotating objects, we employ the use of torque, where moment of inertia is now rotational inertia and acceleration has become angular acceleration. If we were to keep inertia constant, the torque would be dependent on the angular acceleration of the object. Our goals for the lab are to determine this relationship and determine the moment of inertia of various objects by measuring the torque and angular acceleration. To examine the relationship between torque and angular acceleration, we will be changing the mass of the hanging mass. Additionally, we will also be analyzing how moment of inertia changes for one of our objects when rotated about different points from the center.

Procedure

Per the introduction, we will be looking at the moment of inertia of three different objects, a bar, a disk, and a ring. For the ring, we will need to place it on the disk, therefore, to calculate the moment of inertia of the ring, we will need to take the total moment of inertia and subtract the inertia of the disk. More will be discussed in the Theory section of the report. The setup for this lab is particularly simple, however, attention to detail regarding measurements must be kept in mind and paid extra attention to. Our equipment for the experiment includes the following: solid disk, ring, bar, rotating stage, super pulley, smart gate, mass set, string, and a sparklink. The disk, ring, and bar are our three objects that we will be using to find their moment of inertia. The rotating stage is where we shall place the objects on and serves as the foundation for this experiment. Connected to the rotating stage is a smart gate to measure the revolutions of the rotating stage, and a super pulley that will have string over it, the string being connected to the rotating stage and a hanging mass. Our sparklink will be connected to the provided computer and will allow us to use the smart gate to measure the angular acceleration of the object. As we would like to find the relationship between angular acceleration and torque, using those values to find moment of inertia, we will need to run at least three trials for four different masses, which will bring us to a total of twelve trials per object. Note that we will keep the four masses constant throughout, say for example we used 20 grams, 50 grams, 100 grams, and 200 grams. These masses will be kept the same throughout the experiment, regardless of what object is on the rotating stage.

The lab can be split up into three main parts, one for each object. The part with the bar will be split into two kinds of runs, the first being having the bar rotating about its center, whilst changing the mass of the hanging mass to analyze the angular acceleration changing and see its impact on torque. The second part of the bar involves adjusting the bar rotating from different points along its length whilst still changing the mass of the hanging mass. For this part, measure the distance that the bar will be rotating at from the center. For example, rotating about the center we have a distance of 0, rotating at 5 cm on the number line would indicate that we are rotating 5 cm away from the center of the bar. Change the distance from the center twice, using the 0 distance as one of the trials. We will finish with a total of 36 trials: three different distances, four different masses, three trials per mass.

The second part of the lab will be using the disk instead of the bar. The trials are run the exact same way as the first part of the bar section. Twelve trials, three trials for four different masses.

The third part of the lab has the ring placed upon the disk. Understandably, we must now account for both the mass of the disk and the ring. Again, twelve total trials.

To record the data for the whole lab, place a mass on the string, have the string wound up as much as possible around the stage, when ready to record the data, release the hanging mass and run the software till right about when the string is fully unwound. Stop the data collection else the graph will start decreasing as the rotating stage will begin turning in the opposite direction as the string start winding up again. To reset and prep for the next trial, have the string wound up in the same position as before, failure to do so will result in inaccurate data as there is a slight angle between the pulley and the rotating stage when the string is unwound from a different direction. For example, letting it unwind from the right has no angle, while unwinding from the left creates a slight angle which may increase the number of forces acting upon the rotating stage and the object on the stage. Swapping back and forth between the two may potentially change the data, therefore it is imperative that the same wind up be used each time.

Theory

In our Theory section, we will be discussing the three Moment of Inertia equations for the three objects and relating the torque equation to Newton’s second law of F = ma.

Torque equation:

Where r is the radius of the rotating stage where the string is connected to, and T is the tension in the string due to the hanging mass.

From our free body diagram of the hanging mass from the pulley, we get the following equation using Newton’s laws:

Using the relationship in equation (1), we then get the following equation relating angular acceleration and inertia:

From equation (2), solving for α, we get equation (3):

If we inverse equation (3), we get equation (4):

When graphing the inverse of the mass of the hanging mass versus the inverse of the angular acceleration, our slope will be the inertia of the object divided by gravity times the radius of the stage where the string is connected to (I/gr). The radius divided by gravity will by the y-intercept (r/g).

Moment of Inertia equations for the three objects:

Bar:

Where a is the length of the bar in meters and b is the width of the bar in meters.

Disk:

Where R is the radius of the Disk.

Ring:

Where r1 is the inner radius and r2 is the outer radius.

To calculate the Moment of Inertia of the Ring we use the following:

For the parallel component of the bar when adjusting the distance of the center of the beam:

Where m is the mass of the object and d is the adjusted distance from the center.

Sample Calculations and Results

Through both our data collection and theoretical calculations, we obtained two values of inertia per object, a theoretical and an actual.

For the bar, our theoretical was 0.01366 kg m^2, and our actual was 0.01458 kg m^2. Our calculations are as follows, note the mass of the bar was 649.15 grams, the length of the bar was 50 cm, and the width was 5 cm:

Theoretical:

For the actual we took the plotted data of the inverse of the mass versus the inverse of the angular acceleration and found the slope of the graph. That slope is the inertia divided by gravity and the radius of the stage. To get inertia, we than times the slope by gravity and radius to cancel and find inertia.

Actual:

Similarly, when changing the distance from the center of the bar, we found the inertia to be as follows:

For 5 cm:

For 15 cm:

For the disk, we found the radius of the disk to be 11 cm, the mass was 1.42729 kg. For the theoretical we used the given inertia equation of the disk and for the actual, we used the same method as the bar, finding the slope of the graph and canceling out the gravity and radius.

Theoretical Inertia of the disk:

Actual:

The ring was slightly more complicated, we had to take the total inertia of both the disk and the ring and subtract the known inertia of the disk from previous calculations to obtain the actual inertia of the ring. The theoretical inertia was calculated using the given equation. The mass of the ring was 1.4204 kg, the inner radius was 0.054 m, and the outer radius was 0.0635 m.

Theoretical:

Actual:

The total inertia was found again using the same method as before, taking the slope from the graph and canceling out the gravity and radius.

Discussion and Conclusion

From our results, it may be clear that the moment of inertia for the ring is greater than the moment of inertia of the disk, even though their masses are more or less the same. This can be attributed to the center of mass. With the disk, as it is solid all around, the distribution of mass is more even, making it more distributed over the rotating stage, whereas the ring being hollow, is less distributed when placed on the disk.

There are plenty of errors in our calculations. For our bar, the theoretical and actual moment of inertia were fairly close, however, for the disk and the ring, our moment of inertia produced errors of 148.96% and 45.23% respectively. This was calculated by taking the theoretical divided by the actual. We may attribute these errors to our setup of the lab. When performing the tests, we failed to reset the stage to its original position, instead letting the string wind and unwind in both directions. If you may recall, in the procedure section, it was indicated that the stage and string should only be unwound in a single direction to prevent an angle and any external forces from acting on it. However, we failed to do so, as we did not recognize that there was an angle till after our data collection. This may have severely impacted our data and given us the errors we have calculated. Strangely, this error did not impact the bar data to a large extent. We were still some ways off, however that may be attributed to our data collection, when did we time the stopping of the data to ensure that the graph and the slope of the graph did not decrease. That is more of a timing and reflex issue while the disk and the ring may conclude as setup and experimental issues.

From what we have gathered based on our data, we can conclude two things. The first being that when you change the distance from the center, the inertia of the bar increases drastically. This could be due to the center or mass being thrown off and no longer rotating from the center, therefore there needs to be compensation in order to keep the torque the same. The second thing was the more evenly distributed the mass, the larger the inertia as we saw from the theoretical calculations of the disk and the ring. Even though they had the same mass, the inertia of the disk was almost twice that of the ring. Weight distribution plays a huge role here.

Additional Graphs and Tables