Archimedes’ Principle

Lab Report

Cieran Wong

Lab Partner: Bennet Jackson

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TA: Wyatt Wallis

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Archimedes’ Principle

Abstract

Archimedes’ principle describes the phenomenon of the force of buoyancy, density of two objects and their interaction, and how the force of buoyancy changes based on how deep an object is submerged. From our experiment, we created a trial to find the density of water and then using that knowledge and application, created a separate experiment to find the density of seven different cubes made of different material to prove Archimedes’ principle. We found our actual density values and compared them to the theoretical density values to gain a better idea of our experiment and its success. Although both our theoretical value for the density of water and the density of the cubes were not close to the theoretical values, we learned how to prove Archimedes’ principle. For our errors, we had percent error of about 15% for water and 153% or a 2.5 multiplier for the cubes.

Introduction

Archimedes’ principle stems from a very well-known story of Archimedes himself in a bathtub. He noticed that when he submerged himself in the tub, the water level rose, and the level changed based on how much of his body was being submerged. Thus, we get to study the idea of buoyancy, an upward force that the fluid exerts on an object which we now know is equal to the weight of the displaced fluid. This force is also dependent on the density of the fluid and the density of the object. If an object with a density less than the density of a fluid, it is probable that the object will float, and the reverse happens if the density of an object is greater than that of the fluid, the object will sink. In our experiment, we will examine the effect that objects of different density and mass, whilst still the same size, has in a beaker of water. We will identify the forces acting on the object and the water and attempt to explain and derive the forces of buoyancy and density of the objects. To achieve this, we will first test out and find the density of water, then using that value, calculate the densities of all the other objects.

Procedure

The first half of the lab involves submerging a metal cylinder at different levels to derive the density of water from a plotted graph. For this part, we will need a metal cylinder, a beaker with measurements on the side to identify the change in volume, a force sensor, and a provided computer to collect the force data. Fill the beaker with water, enough to be able to see the change in volume using the measurements on the side but not too much to the point where if the cylinder were submerged, the water would overflow. Tying a string to the metal cylinder and hooking it up to the force sensor, submerge the cylinder at five different points, that is to say, slightly submerge, half submerge, fully submerge, and two other submerge points (not submerged and three quarters submerged). The purpose of this data collection is to identify the buoyancy force that the fluid is exerting onto the cylinder and using that value, we are able to determine the density of the fluid. From our data collection, we end up with five values which we can plot (volume of displacement vs buoyant force) to get a slope. The slope is our density of our fluid multiplied by gravity, therefore, to find the density, divide by gravity.

The second half of our lab involves finding the density of seven cubes made of different material, as per the brief above. For this section of the lab, we will be tying some string around the cubes and suspending them in water, whilst weighing them to see the change in mass with and without the cube immersed. Instead of examining the forces acting on the cube, we will be looking at the forces acting on the fluid, water in our case. The reason why we would want to do this is because it is slightly easier to picture the forces acting on the water as opposed to the forces acting on the cube.

Theory

The experiment is split into two parts: 1) calculating the density of water, and 2) calculating the density of the provided objects. The first part involves proving the known density of water through employing the use of a metal cylinder to test the force exerted by the cylinder onto the water. From there, we can identify the buoyancy force, the change in volume, and using Equation (1), rearrange to find the density of water.

Where B is the buoyancy force, V is the change in volume, and g is gravity.

The second part involves seven cubes made of different material, all with different masses and theoretically different densities. Their measurements stay the same as they are all a cubic inch which is (arguably the worst unit to use, switch to metric) about 16.3871 cubic centimeters. As their size is a constant, the only variables that are changing are their mass and density, both variables that go hand in hand due to the type of material used. As we are able to measure the mass of said object, the only unknown would their density which we can calculate using Equation (2):

With V being the volume of the object (16.3871 cm^3), m the mass of the object, and p the density.

For the forces acting on the water, we have the normal force, gravity, and the buoyant force. Using Newton’s second law of F=ma, we are able to rearrange and solve for the density of the object so long it is held in suspension and acceleration is equal to zero. Equation (3) shows this very example:

For Equation (4), the buoyant force is equal to the mass of the object multiplied by gravity, which is also equal to the density of the fluid multiplied by gravity and the change in volume of the cylinder when submerged. Therefore, we can set the two equations equal to each other and thus we get Equation (4).

Equation (5) rearranges Equation (4) to find the density of water. We can then take (5) and (2) and perform the following substitution to get (6).

Equation (6) is our final equation for us to calculate the density of the object using the mass of the object, the density of water, the normal force exerted by the object onto the water, and the force of gravity on water, as well as gravity. This equation will help us calculate the actual density of each object using the values we have found; density of water from Part I, the normal force, and the mass of the object.

Sample Calculations and Results

Taking Equation (1) from the Theory section, we are able to find the density of water by obtaining a graph from our results, as shown below:

Our slope, as previously mentioned, is equal to the density of the fluid multiplied by gravity, therefore, to obtain the density value, we must divide by gravity as per the calculations shown below:

Table 1 has our obtained values for Part I: Finding the density of water.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Water change level (m)** | **Volume (mL)** | **Force Buoyant (N)** | **Force Tension (N)** | **Fmg (N)** |
| 0 | 0 | 0 | 1.56 | 1.5975585 |
| 0.0035 | 1.42503E-05 | 0.0824415 | 1.68 |  |
| 0.006 | 0.000024429 | 0.1824415 | 1.78 |  |
| 0.01 | 0.000040715 | 0.3524415 | 1.95 |  |
| 0.015 | 6.10725E-05 | 0.4824415 | 2.08 |  |

Compared to the theoretical value for the density of water, we have an error of about 18%.

Table 2 has both our theoretical calculated and the actual calculated densities for all seven cubes, as well as their description:

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Description** | **Density (kg/m^3)** | **Theory P (Kg/m^3)** |
| PVC | Gray | 514.7830411 | 1314.826113 |
| Nylon | White | 405.7415538 | 1125.686394 |
| Acrylic | Clear | 431.0316043 | 1165.344722 |
| Copper | Red - brown | 3288.35179 | 8880.414887 |
| Brass | Gold | 3180.59865 | 8395.363026 |
| Steel | Black (painted) | 2916.409931 | 7528.981086 |
| Aluminum | Silver | 1037.086805 | 2623.550946 |

Comparing the two columns, our actual is off by a multiplier of 2.5 or 153% error.

Equation (2) as shown below describes how we got our theoretical values to compare with our actual values, using Aluminum as an example:

Table 3 shows the values we used to get the final values in Table 2:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Name** | | **Mass (kg)** | | **Mass after submerge (Kg)** | | **Normal force (N)** | | **Denominator** | **Numerator** |
| PVC | | 0.02155 | | 0.56935 | | 5.57963 | | 0.4102505 | 0.21119 |
| Nylon | | 0.01845 | | 0.57296 | | 5.615008 | | 0.4456285 | 0.18081 |
| Acrylic | | 0.0191 | | 0.5718 | | 5.60364 | | 0.4342605 | 0.18718 |
| Copper | | 0.14555 | | 0.57175 | | 5.60315 | | 0.4337705 | 1.42639 |
| Brass | | 0.1376 | | 0.57075 | | 5.59335 | | 0.4239705 | 1.34848 |
| Steel | | 0.1234 | | 0.5698 | | 5.58404 | | 0.4146605 | 1.20932 |
| Aluminum | | 0.043 | | 0.56895 | | 5.57571 | | 0.4063305 | 0.4214 |
| **Different masses** | | **Mass (grams)** | | **Mass (kg)** | |
| Water bottle w/h20 | | 556.3 | | 0.5563 | |
| Water bottle w/o h20 | | 29.35 | | 0.02935 | |
| Water | | 526.95 | | 0.52695 | |

The following calculations using Equation (6) from the Theory section shows how we got the actual density of Aluminum:

Discussion and Conclusion

From the data we have collected, we can see that our value for the density of water is quite off from the known density of water. We can attribute this error due to possibly incorrectly measuring the volume of the beaker used and misidentifying the values on the measuring line right on the beaker. The method we used to calculate the volume of the beaker that had changed due to submerging the cylinder was to take the area of a circle and multiply it by the change in height. This method may have its flaws as our measurement for the area and the change in height may be incorrect as we are simply using our eyes as a form of visual confirmation to properly confirm that the measurements are correct. This may have caused us to have incorrect values which ultimately resulted in the density of water being incorrect.

Our calculated theoretical densities were also two and a half times larger than the actual densities we found. We could not pinpoint exactly where we messed up, however, we believe it to be somewhere in our calculations even though we had used the theoretical density of water and not the density that we had found in Part I. We ran through the calculations multiple times, using different values and still got the same results. Not too sure if the conversions were incorrect or if we had substituted the wrong values.

We can see from Equation (1), that the relationship between the buoyant force and the volume of the submerged cylinder is equal to the density of the fluid, where the buoyant force is divided by the change in volume multiplied by gravity in the denominator. From the graph derived from the data collected, we are able to identify that as the volume of the submerged cylinder increases, the buoyant force exerted by the water onto the cylinder increases as well, indicating that the two are directly proportional. We can also see this through the graph we had obtained from Part I.

We would say the data does somewhat support Archimedes’ principle, as due to our large percent error, it is difficult to fully confirm that it does support it. In general, however, through our calculations and the data we have derived, it is plausible to conclude that the data does support Archimedes’ principle. Our value for the density of water that we found was about 843.629 kg/m^3, giving us a percent error of about 18%. When comparing our data we had values that were about 2.5 times less than the expected values that we had calculated as a theoretical for comparison with our actual.

We can conclude that the experiment did help us prove Archimedes’ principle, however, in the future when performing this experiment again, we would want to be a lot more careful with calculations and perform repeated trials to ensure that our data not only makes sense, but the actual values are close to the theoretical in the sense that we have an error percent of less than 5%.