HOCHSCHULE RHEINMAIN

PHYSICS LAB 3

Experiment P3-4
Surface Tension of Strain Gauges

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1 Introduction

1.1 Terms and Definitions

Buoyancy force

A body submerged in a fluid experiences a force anti-parallel to the gravitational force and proportional to the mass of the fluid displaced.

Surface Tension

In a simplified manner attracting intermolecular interactions inside a fluid can be seen as isotrope. Thus, they cancel each other out. The closer a particle gets to the interfacing layer of another medium the fewer neighbors there are to exert attracting forces from that direction. The attracting force a particle along the interfacing layer experiences nets to a force normal to the interface layer and directed inside the fluid.

Surfactant

A material or compound able to alter or even nullify the intermolecular forces inside a fluid (typically a liquid).

DU NOÜY-Method to Measure the Surface Tension of a Liquid

A method to measure the surface tension of a liquid by measuring the force withdrawing a submerged ring from that liquid [3].

Other Ways to Measure the Surface Tension of a Liquid

ANDREAS-HAUSER-TUCKER method or Pendant-drop method: the shape of a droplet is solely dependant of the gravitational force and the surface energy. Knowing other parameters of the liquid in question the surface tension can be acquired by geometrically evaluating photographs of hanging droplets [1].
Capillary-tubes method: one opening of a tube of sufficiently small inner diameter is brought in contact with the specimen. The liquid will - to some extent - travel along the tube depending on the adhesive forces between the liquid and the tube wall, the gravitational force and the cohesive forces along the liquids surface i.e. the surface tension.
Parallel-plates method: similar to the capillary-tubes method, the surface tension can be determined by evaluating the rise of the liquid between two parallel plates brought in contact with the liquid.
Sessile drops and bubbles: in this method, the shape of a drop or bubble resting on a flat and horizontal surface is examined.
JAEGERS method: a tube is vertically submerged in a liquid leaving the lower and upper end open. Now, if gas pressure is applied to the top of the liquid column inside the tube until a gas bubble escapes the lower end the surface tension can be derived from the gas pressure at that moment.
Oscillating drops and bubbles: much like a pendulum, a drop or bubble is displaced from its shape of equilibrium. Released and left alone, it will oscillate about the shape of equilibrium. Measuring the frequency the surface tension can be derived [4].

Working Principle of a Strain Gauge

Stress induced change of the geometry of a strain gauge translates to a change of its resistance. The impossibility to measure electrical resistance directly it typically gets transduced to a voltage signal in order to be further processed.

Working Principle of an Instrumentation Amplifier

A instrumentation amplifier is a advanced version of the differential amplifier. A slight variation in the topology of the differential amplifier allows to adjust the gain by change of just one resistor. This is benefitial as it is likely to introduce distortion changing two resistors while maintaining balance [5, 6].

Resolution of an Analog-Digital Converter

An analog-to-digital converter (hereafter referred to as ADC) converts a continuous to a discrete, bitwise signal. The number of fractions the ADC is able to divide a given signal to gives the resolution of that ADC.

1.2 Preparation

1.2.1 Force-Time-Curve

In fig. 4 the force-time-diagram is subdivided into the following 8 parts:

1. The ring is located above the interface. No force is caused.
2. The ring touches the interface, creating a small positive adhesive force between the ring and the surface.
3. The ring is about to penetrate the surface while the surface tension creates a small negative force.
4. The ring is now immersed. Its wires produce a small positive force.
5. When diving upwards, the force gradually increases.
6. The ring breaks through the interface and induces an ever-increasing force.
7. The force reaches its maximum.
8. The force decreases a bit with the raising until the lamella collapses.

If the diagram is compared with that in fig. 14, some differences are noted. For example, there are small periodic vibrations that probably have to do with the fluctuation of the ring and other external influences. It is particularly noticeable that after penetrating the surface, the force does not rise again into the positive, but remains in the negative and only returns into the positive from step 6. Otherwise, from a qualitative point of view, everything is similar to that in fig. 4, except that after the lamella has collapsed, the force oscillates around the zero position.

1.2.2 Surface Tension Formula

The change of area ΔA corresponds to the sum of inner and outer radius - r_i and r_a respectively - of the ring multiplied by the change in height Δx:

$$\Delta A = 2\pi \cdot (r_i + r_a) \cdot \Delta x \tag{1.1}$$

Since the diameter $D = 2R$ of the ring is much bigger than the thickness $t = r_a - r_i$, eq. (1.1) can be simplified to

$$\begin{gathered} 2R \gg r_a - r_i \quad \Rightarrow \quad r_i \approx r_a \approx R \\ \Leftrightarrow \\ \Delta A = 4\pi \cdot R \cdot \Delta x \end{gathered} \tag{1.2}$$

Considering, that the amount of energy ΔE is a result of the assumed constant force F_0 multiplied by the change in height, it ensues:

$$\Delta E = F_0 \cdot \Delta x \tag{1.3}$$

Since the surface tension σ is defined as the energy required to increase the surface divided by the change in surface area

$$\sigma = \frac{\Delta E}{\Delta A} \tag{1.4}$$

and eq. (1.2) as well as eq. (1.3) are inserted into eq. (1.4), it follows:

$$\begin{aligned} \sigma &= \frac{F_0 \cdot \Delta x}{4\pi \cdot R \cdot \Delta x} \\ \sigma &= \frac{F_0}{4\pi \cdot R} \end{aligned} \tag{1.5}$$

1.2.3 Gravitational Force of the Du Noüy-Ring in Water and Air

Air:

$$F_{Ring,Air} = -m_{Ring} \cdot g \tag{1.6}$$

Water:

$$\begin{aligned} F_{Ring,H_2O} &= (m_{H_2O} - m_{Ring}) \cdot g \\ &= (\rho_{H_2O} - \rho_{Al})\, V_{Ring} \cdot g \end{aligned} \tag{1.7}$$

1.2.4 Deriving an Equation for the bridge voltage

$$U_{Br} = U_2 - U_4 = U_3 - U_1 \quad \text{with} \quad U_{Br} = 0 \text{for} \frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{1.8}$$

$$U_{Br} = U_2 - U_4 = U_0 \cdot \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \tag{1.9}$$

Defining

$$R_2 = R_3 = R + \Delta R \tag{1.10}$$

$$R_1 = R_4 = R - \Delta R \tag{1.11}$$

leads to

$$\begin{aligned} U_{Br} &= U_0 \cdot \left(\frac{R + \Delta R}{R - \Delta R + R + \Delta R} - \frac{R - \Delta R}{R + \Delta R + R - \Delta R} \right) \\ &= U_0 \cdot \left(\frac{R + \Delta r}{2R} + \frac{-R + \Delta R}{2R} \right) \\ &= U_0 \cdot \frac{\Delta R}{R} \end{aligned} \tag{1.12}$$

thus, the bridge voltage is linearly proportional to the factor $\frac{\Delta R}{R}$.
Now, with the knowledge that the resistance of a body relates with its geometry by $R = \varrho \cdot \frac{l}{b \cdot d}$ and the change of its dimensions under stress follow

$$\frac{\Delta b}{b} = -\nu \frac{\Delta l}{l}, \quad \frac{\Delta d}{d} = -\nu \frac{\Delta l}{l} \tag{1.13}$$

the bridge voltage can be expressed as

$$U_{Br} = U_0 \cdot \left(\frac{\Delta s}{s} + \nu \cdot \frac{3\Delta l}{l} \right) \tag{1.14}$$

1.2.5 Schematic Diagram of a Full Bridge

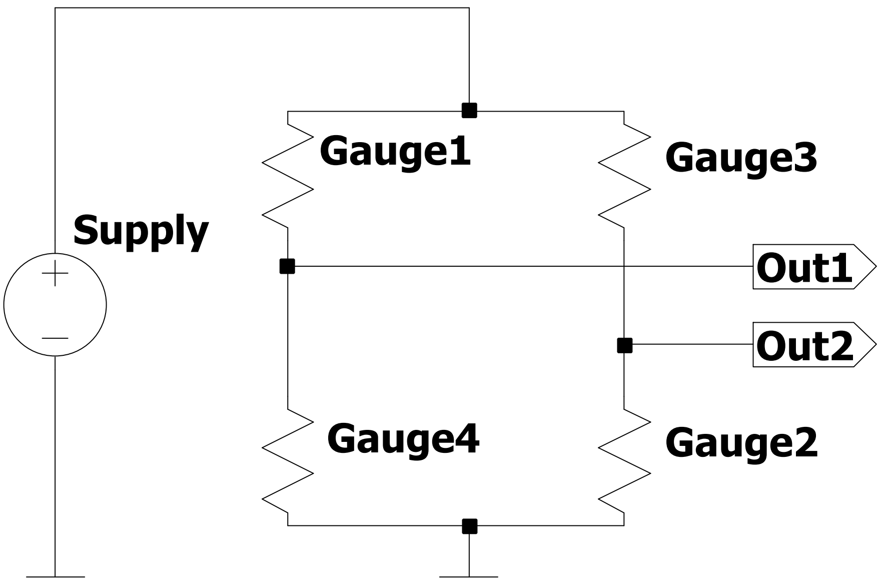


Figure 1.1: Strain gauge in full bridge configuration.

1.2.6 Characteristics of HX711

Resolution:	$n = 24\,\mathrm{bit}$	(1.15)
Full-scale range:	$U_{FSR} = \pm 20\,\mathrm{mV}$	(1.16)
Minimum voltage change:	$U_{LSB} = U_{FSR} \cdot 2^{-24} \approx 1.2\,\mathrm{nV}$	(1.17)

1.2.7 Impact of Statistical Effects on the effective Resolution

The resolution n of an ADC as per specification implies an ideal, i.e. free-of-noise dc signal. The effective resolution — or ENOB (**E**ffective **N**umber **O**f **B**its) — can be calculated by eq. (1.18)

$$ENOB = n - \log_2 \sigma \tag{1.18}$$

where σ is the RMS (**R**oot **M**ean **S**quare) or standard deviation of the signals noise.

1.2.8 Step Response of an Exponential Filter

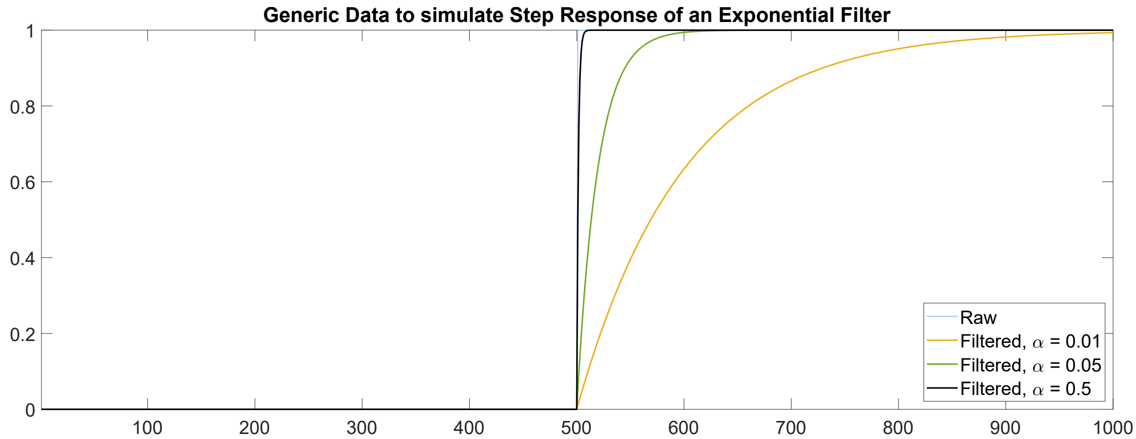
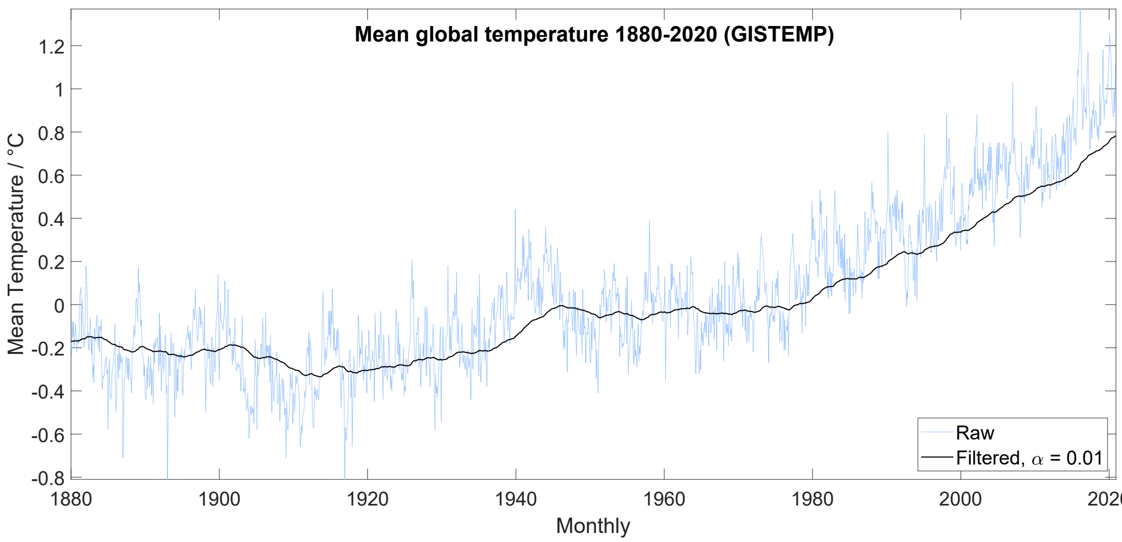


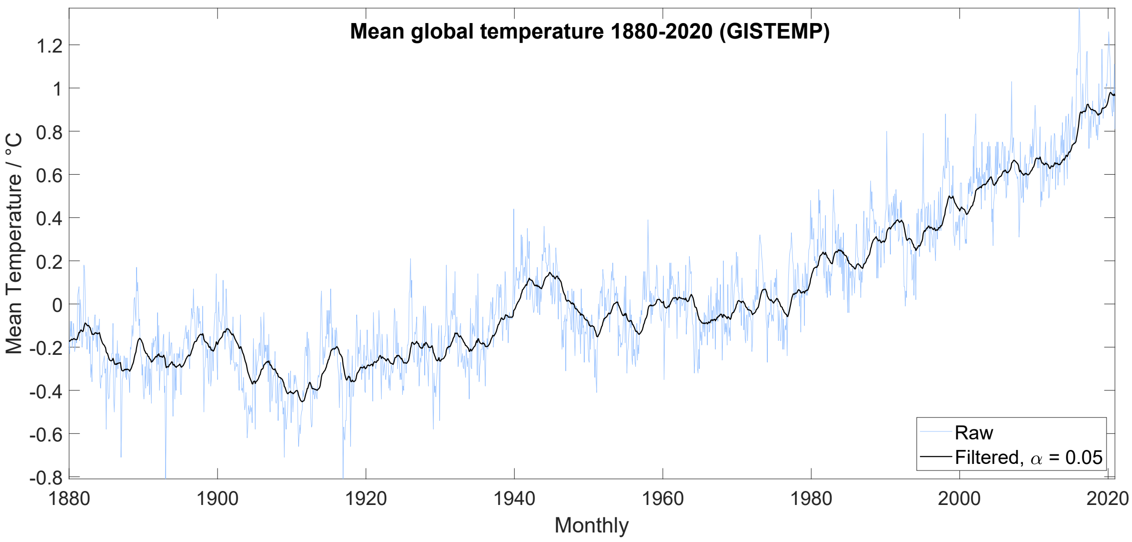
Figure 1.2: An exponential filter algorithm applied to a square step signal (light blue) and its responses for $\alpha = 0.5$ (black), $\alpha = 0.05$ (green) and $\alpha = 0.01$ (orange).

1.2.9 Simulation of the Filter Algorithm

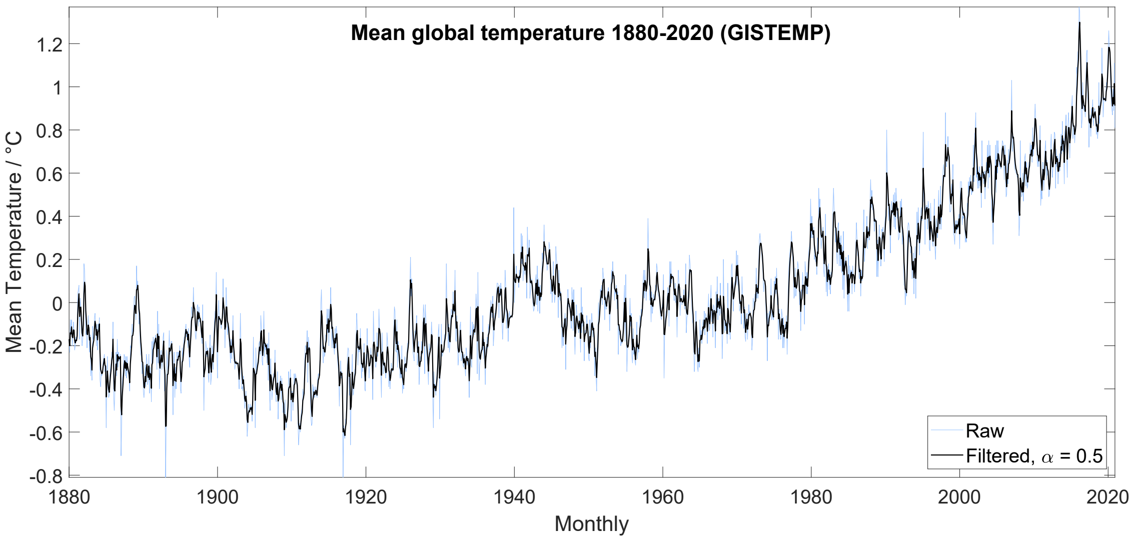
Figure 1.3 shows the effect of an exponential filter algorithm applied to a data set to filter out statistical noise. The algorithm takes a smoothing factor $\alpha \in 0 \leq \alpha \leq 1$ as an input and outputs a smoothed representation of the raw data.



(a) Expo-smoothing at $\alpha = 0.01$.



(b) Expo-smoothing at $\alpha = 0.05$.



(c) Expo-smoothing at $\alpha = 0.5$.

Figure 1.3: Examples of an exponential filter algorithm applied to a noisy set of data. To emphasize its effect on the output, three different values for the smoothing factor α are shown. The raw data is shown in light blue whereas black represents the filtered data. Raw data is provided by the Goddard Institute for Space Studies [2]

As seen in fig. 1.3a a too aggressive filtering of the data could bias the trending of the output with respect to the raw data. Too less of smoothing on the other hand might still output too much noise (cf. fig. 1.3c). Depending on the data in question, the smoothing factor needs to be adjusted as needed.

1.2.10 Expected Forces

2 Set-Up of Experiment

3 Execution

4 Evaluation

5 Conclusion

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Glossar

A First letter of the alphabet

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