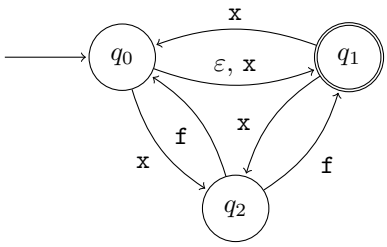
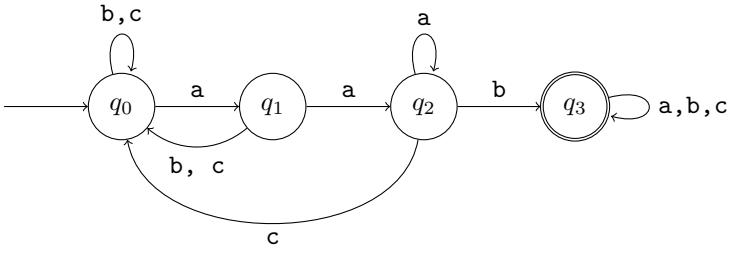
Problem 1 (180 pts)**.**

Convert the following NFA to a DFA using the subset construction technique found in the lectures and textbook:



Problem 2 (180 pts)**.**

Convert the following DFA into a regular expression using the GNFA construction technique found in the lectures and textbook. Then describe the language in words or set-builder notation.



Problem 3 (180 pts)**.**

(a) (90 pts) Using the techniques described in class and the textbook (Theorems 1.45, 1.47, 1.49), construct an NFA recognizing $(\mathtt{1}(\mathtt{01} \cup \mathtt{10})^* \cup \mathtt{0}(\mathtt{11} \cup \mathtt{00})^+)^*$.

(b) (90 pts) Convert the following language into a regular expression, and then give a 4-state NFA which accepts it:

$$L_1 = \{w \in \{0,1\}^* : w \text{ starts with a 1 and has even length, or starts with a 0 and has odd length}\}$$

Problem 4 (180 pts)**.**

Consider a set of two interlocked gears mounted to a wall, one with three teeth (call this G_3), and one with five teeth (call this G_5). For the sake of simplicity, assume that the gears rotate in only one direction. Further suppose that each gear has a specially marked tooth which is initially pointed straight up. Now let $\Sigma_2 = \{\texttt{t, f}\}$ be an alphabet, where for any $w \in \Sigma_2^*$, every occurrence of $\texttt{t} \in w$ indicates that G_3 makes a full revolution, and every occurrence of $\texttt{f} \in w$ indicates that G_5 makes a full revolution.

- (90 pts) Show that the following language L_2 is regular:

$$L_2 = \{w \in \Sigma_2^* : \text{ both gears will be pointing straight up after each revolution represented by } w\}$$

- (90 pts) Now suppose that a third gear with 7 teeth, G_7, is added to the system, and let $\Sigma_3 = \Sigma_2 \cup \{\texttt{s}\}$, where occurrences of the added symbol $\texttt{s}$ represent a full revolution of G_7. Show that the following language L_3 is regular:

$$L_3 = \{w \in \Sigma_3^* : \text{ any two gears will be pointing straight up after each revolution represented by } w\}$$

Hint: any time gear G_k makes a full rotation, each of the gears in the system advance by k teeth in some fixed constant direction.

Problem 5 (80 pts)**.**

Consider the the construction of NFA N which recognizes N_1^* for a given NFA N_1 on Page 62 of our textbook and Slide 27 of the Lecture 3 slides. Suppose we modify this construction in the following manner:

- We do not add a new start state and instead make the start state of N_1 the start state of N.
- Additionally, we make this (the original) start state of N_1, an accept state.

Explain why this is problematic. In particular, try to think of an NFA N_1 which under this modified construction admits strings that should not be in N_1^*.