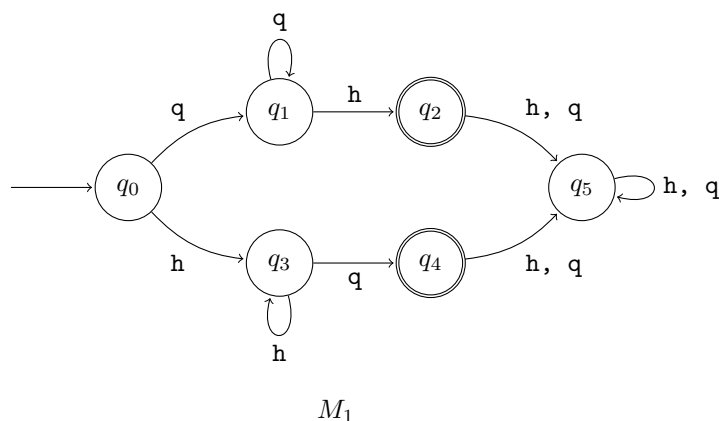**Problem 1** (200 pts).

Consider the following DFA M_1:

$$M_1$$

(a) (40 pts) Give the formal description of M_1.

(b) (40 pts) Provide the state sequence and outcome of computation on string $s_1 = $ qqqqhhh.

(c) (40 pts) Provide the state sequence and outcome of computation on string $s_2 = $ hhhhqqq.

(d) (40 pts) Give an example, if one exists, of each of the following:

    (a) The shortest accepting string

    (b) The longest accepting string

    (c) The shortest rejecting string

    (d) The longest rejecting string

(e) (40 pts) Describe $L(M_1)$.

Problem 2 (120 pts).

(a) (40 pts) Recall the argument, covered in lecture, which shows that regular languages are closed under union. Modify this argument to show that regular languages are closed under intersection.

(b) (40 pts) Recall that we showed that regular languages are closed under complement. Argue that regular languages are closed under set difference.

(c) (40 pts) Argue that regular languages are closed under symmetric difference.

Problem 3 (200 pts).

Given $\Sigma = \{$0, 1, 2$\}$, construct DFAs which accept the following languages. Explain your strategy and argue your construction works.

  (a) (60 pts) $L_1 = \{w \in \Sigma^* \mid w \text{ does not contain } 01 \text{ as a substring}\}$

  (b) (60 pts) $L_2 = \{w \in \Sigma^* \mid \text{neither of the first two symbols in } w \text{ are } 2\}$

  (c) (80 pts) $L_3 = L_1 \Delta L_2$

Problem 4 (120 pts).

Define DFA M_2 by

$$\Sigma = \{0,1, 2\}$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$
$$\delta(q_i, a) = \begin{cases} q_{(i+2) \bmod 6} & \text{if } a = 0 \\ q_{(i+2) \bmod 6} & \text{if } a = 1 \\ q_{(i+4) \bmod 6} & \text{if } a = 2 \end{cases}$$
$$q_0 = q_0$$
$$F = \{q_3\}$$

Describe $L(M_2)$.

Problem 5 (160 pts).

For strings s, t, u of equal length $k \geq 0$ on alphabet $\Sigma = \{$0$,$1$\}$, define the operator $\star$ as follows:

$$s \star t \star u = s_1 t_1 u_1 s_2 t_2 u_2 \cdots s_k t_k u_k$$

That is, the star operator interleaves the symbols in s, t, and u, in the order they appear in their respective strings, one at a time.

Now let $\text{Bin}(w) \in \mathbb{N} \cup \{0\}$ be the numeric value of string $w \in \Sigma^*$, let $\text{Str}(n) \in \Sigma^*$ be the string representation of $n \in \mathbb{N} \cup \{0\}$ over Σ, and define

$$L_4 = \{w \in \Sigma^* : w = s \star t \star u, \text{ and } \text{Bin}(s) \oplus \text{Bin}(t) = \text{Bin}(u)\}$$

  (a) (100 pts) Provide a DFA which recognizes L_4.

  (b) (60 pts) Prove or disprove the following: if $v, w \in L_4$, then $\text{Str}(\text{Bin}(v) \oplus \text{Bin}(w)) \in L_4$.