EPFL, under the supervision of Pr. Stephan Morgenthaler

Master Thesis Project : Extreme Value Theory
– Introductory Task –

Killian Martin–Horgassan

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1 Summary - tasks assigned

Let us consider a sequence of i.i.d. random variables $X_1, X_2, \ldots \sim \mathcal{N}(0,1)$[1]. Let us define the maximum at stage n as $M_n = \max(X_1, \ldots, X_n)$.
Let us consider the following example :

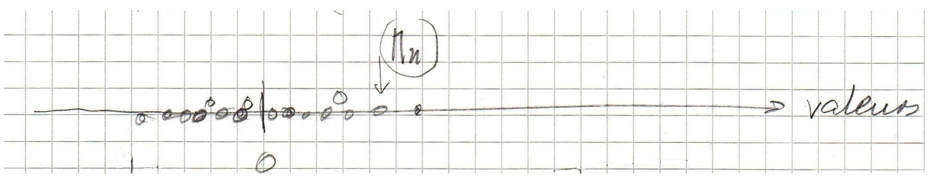
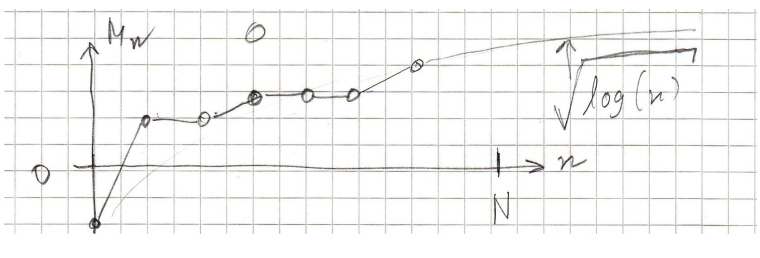


Figure 1.1: Distribution of the $x_1, \ldots, x_n$ around 0.

Let us draw the plotting of M_n against n :



(M_n) is a non-decreasing sequence with growth is of order $\sqrt{\log(n)}$.

[1] could be extended to $\mathcal{N}(\mu, \sigma^2)$

1.1 Sub-task 1

Write a R program that will generate the sequence of the maxima M_1,...,M_n and make the plot of M_n against n. The signature is the following : NormMax(N,m,s,DIST), where :

- N is the number of stages
- m is the mean
- s is the standard deviation
- DIST is the common distribution of the X_i[2]

1.1.1 Sub-task 2

Let us make the following observation, if F_n denotes the cumulated distribution function of M_n then : $F_n(t) = \Pr(\{M_n \leq t\}) = \Pr(\{\max(X_1, ..., X_n) \leq t\}) = \Pr(\{X_1 \leq t\} \cap ... \cap \{X_n \leq t\})$.
By independence of the X_i, it boils down to $F_n(t) = \Phi(t)^n$.

- What is the limit, if any, when n goes to infinity ?
- Is it possible to find two sequences (a_n) and (b_n) such that $\Pr(a_n + b_n M_n \leq t)$ has a limit distribution ?

2 Results

3 Work done

Please report to file 'JournalDeBord.pdf'[3], section *Week I*.

[2] gaussian, exponential or Cauchy for instance.
[3] available on `https://github.com/CillianMH/pdmExtremeValueTheory`