Extreme Value in Financial Statistics

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Introduction

What we have Data about past events e.g. the record of the prices of a stock over time.

What we want to know Values taken at future extreme events e.g. maximum price between T and $T + \Delta T$.

Settings

Observations $(X_n)_{n\geq 0}$ i.i.d. rvs $\sim F_X$.
Maxima $(M_n)_{n\geq 0} = (\max_{0\leq i\leq n}(X_i))_{n\geq 0}$
Standardized maxima $(M_n^*)_{n\geq 0} = (\frac{M_n - b_n}{a_n})_{n\geq 0}$, $a_n > 0$, $b_n \in \mathbb{R}$

Convergence in distribution of $(M_n^*)_{n\geq 0}$?

- Possible limits ?

  $\Longrightarrow$ extremal limit pb

- Under what conditions ?

  $\Longrightarrow$ domain of attraction pb

Fisher-Tippett-Gnedenko Theorem

Theorem (Fisher-Tippett-Gnedenko)

If the sequence of standardized maxima converges to a non-degenerate distribution, then this distribution is either a Gumbel, a Frechet or a Weibull distribution .

Extreme value distributions

- **Fréchet** $\Phi_\alpha(x) = \exp(-x^\alpha)$
- **Weibull** $\Psi_\alpha(x) = \exp(-|x|^\alpha)$
- **Gumbel** $\Delta(x) = \exp(-\exp(-x))$

Domain of attraction

- Domain of attraction of an EV distribution $\Longrightarrow$ set of F_X such that the standardized maxima converge to this EV distribution.
  Notation : $\mathcal{D}(\cdot)$ where $\cdot = \Phi_\alpha$, Ψ_α or Δ.

- Hazard function

$$r(x) = \frac{f_X(x)}{1-F_X(x)}$$

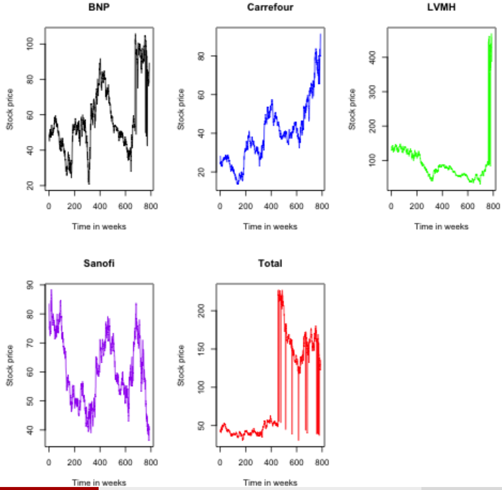
Von Mises' Theorem

Theorem (Von Mises' Theorem)

- *If* $x^+ = +\infty$ *and* $xr(x) \xrightarrow[x \to +\infty]{} \alpha > 0$, *then* $F_X \in \mathcal{D}(\Phi_\alpha)$.
- *If* $x^+ < +\infty$ *and* $(x^+ - x)r(x) \xrightarrow[x \to x^+]{} \alpha > 0$, *then* $F_X \in \mathcal{D}(\Psi_\alpha)$.
- *If* $\exists$ *neighbourhood of* x^+ *where* $r(x) \geq 0$, *differentiable and* $\frac{\mathrm{d}r}{\mathrm{d}x}(x) \xrightarrow[x \to x^+]{} 0$, *then* $F_X \in \mathcal{D}(\Delta)$.

Looking into financial data

Our starting point : 5 stocks listed on the Paris Stock Exchange. Stock price data over 15 years.



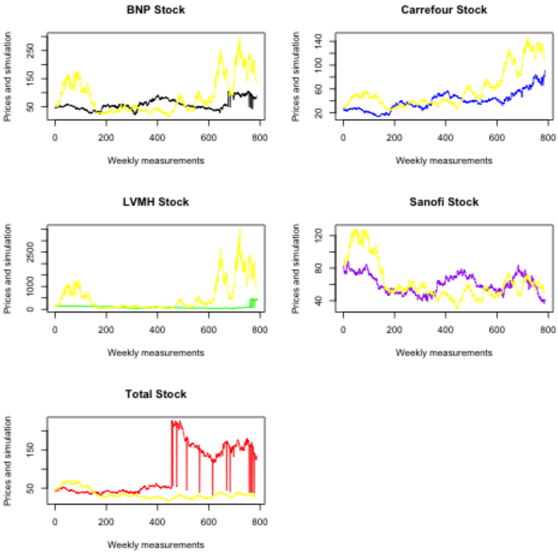
The Black-Scholes model for stock prices

The price increment at time t should be proportional to the price at time t.
SDE :

$$\begin{cases} \mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t \\ S(0) = s_0 \end{cases}$$

Solution : Geometric Brownian Motion
$S_t = s_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma B_t)$

Simulation using a GBM



Simulation using a GBM - observations

- The simulation may overestimate *wrt* to the actual prices.
- On the whole $\implies$ rather satisfactory...
- ... except for the LVMH and even worse the Total stocks.
- sudden & large variations $\implies$ model performs very poorly.

Statistics of extremes & financial data (I)

Making the junction

References include

- Owen Jones, Robert Maillardet & Andrew Robinson
  Introduction to Scientific Programming and Simulation Using R
- Jan Beirlant, Yuri Goegebeur, Johan Segers & Jozef Teugels
  Statistics of Extremes - Theory and Applications
- Ruey S. Tsay
  Analysis of Financial Time Series

Thanks for your attention !