Extreme Value in Financial Statistics

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Table of contents

Introduction

What we have Data about past events e.g. the record of the prices of a stock over time.

What we want to know Values taken at future extreme events e.g. maximum price between T and $T + \Delta T$.

Settings

Observations $(X_n)_{n\geq 0}$ i.i.d. rvs $\sim F_X$.
Maxima $(M_n)_{n\geq 0} = (\max_{0\leq i\leq n}(X_i))_{n\geq 0}$
Standardized maxima $(M_n^*)_{n\geq 0} = (\frac{M_n - b_n}{a_n})_{n\geq 0}$, $a_n > 0$, $b_n \in \mathbb{R}$

Convergence in distribution of $(M_n^*)_{n\geq 0}$?

- Possible limits ?
  $\Longrightarrow$ extremal limit pb
- Under what conditions ?
  $\Longrightarrow$ domain of attraction pb

Fisher-Tippett-Gnedenko Theorem

Theorem (Fisher-Tippett-Gnedenko)

If the sequence of standardized maxima converges to a non-degenerate distribution, then this distribution is either a Gumbel, a Frechet or a Weibull distribution .

Extreme value distributions

- **Fréchet** $\Phi_\alpha(x) = \exp(-x^\alpha)$
- **Weibull** $\Psi_\alpha(x) = \exp(-|x|^\alpha)$
- **Gumbel** $\Delta(x) = \exp(-\exp(-x))$

Domain of attraction

- Domain of attraction of an EV distribution $\Longrightarrow$ set of F_X such that the standardized maxima converge to this EV distribution.
  Notation : $\mathcal{D}(\cdot)$ where $\cdot = \Phi_\alpha$, Ψ_α or Δ.

- Hazard function

$$r(x) = \frac{f_X(x)}{1 - F_X(x)}$$

Von Mises' Theorem

Theorem (Von Mises' Theorem)

- *If* $x^+ = +\infty$ *and* $xr(x) \xrightarrow[x\to+\infty]{} \alpha > 0$, *then* $F_X \in \mathcal{D}(\Phi_\alpha)$.
- *If* $x^+ < +\infty$ *and* $(x^+ - x)r(x) \xrightarrow[x\to x^+]{} \alpha > 0$, *then* $F_X \in \mathcal{D}(\Psi_\alpha)$.
- *If* $\exists$ *neighbourhood of* x^+ *where* $r(x) \geq 0$, *differentiable and* $\frac{\mathrm{d}r}{\mathrm{d}x}(x) \xrightarrow[x\to x^+]{} 0$, *then* $F_X \in \mathcal{D}(\Delta)$.

Looking into financial data (I)

The frame about real-world financial data

Statistics of extremes & financial data (I)

Making the junction

Paragraphs of Text

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Bullet Points

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- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Jones].

References include


Owen Jones, Robert Maillardet & Andrew Robinson
Introduction to Scientific Programming and Simulation Using R


Jan Beirlant, Yuri Goegebeur, Johan Segers & Jozef Teugels
Statistics of Extremes - Theory and Applications


Ruey S. Tsay
Analysis of Financial Time Series

Thanks for your attention !