#### Extreme Value in Financial Statistics

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#### Introduction

What we have Data about past events e.g. the record of the prices of a stock over time.

What we want to know Values taken at <u>future extreme events</u> e.g. maximum price between T and  $T + \Delta T$ .

## Settings

```
Observations (X_n)_{n\geq 0} i.i.d. rvs \sim F_X.

Maxima (M_n)_{n\geq 0}=(\max_{0\leq i\leq n}(X_i))_{n\geq 0}

Standardized maxima (M_n^*)_{n\geq 0}=(\frac{M_n-b_n}{a_n})_{n\geq 0},\ a_n>0,\ b_n\in\mathbb{R}
```

### Convergence in distribution of $(M_n^*)_{n\geq 0}$ ?

- Possible limits ?
  - ⇒ extremal limit pb
- Under what conditions ?
  - → domain of attraction pb

## Fisher-Tippett-Gnedenko Theorem

#### Theorem (Fisher-Tippett-Gnedenko)

If the sequence of standardized maxima converges to a non-degenerate distribution, then this distribution is either a Gumbel, a Frechet or a Weibull distribution.

#### Extreme value distributions

- Fréchet  $\Phi_{\alpha}(x) = \exp(-x^{\alpha})$
- Weibull  $\Psi_{\alpha}(x) = \exp(-|x|^{\alpha})$
- **Gumbel**  $\Delta(x) = \exp(-\exp(-x))$

#### Domain of attraction

- Domain of attraction of an EV distribution  $\implies$  set of  $F_X$  such that the standardized maxima converge to this EV distribution.
  - Notation :  $\mathcal{D}(\cdot)$  where  $\cdot = \Phi_{\alpha}$ ,  $\Psi_{\alpha}$  or  $\Delta$ .
- Hazard function

$$r(x) = \frac{f_X(x)}{1 - F_X(x)}$$

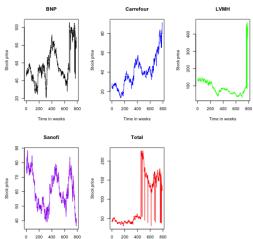
#### Von Mises' Theorem

#### Theorem (Von Mises' Theorem)

- If  $x^+ = +\infty$  and  $xr(x) \xrightarrow[x \to +\infty]{} \alpha > 0$ , then  $F_X \in \mathcal{D}(\Phi_\alpha)$ .
- If  $x^+ < +\infty$  and  $(x^+ x)r(x) \xrightarrow[x \to x^+]{} \alpha > 0$ , then  $F_X \in \mathcal{D}(\Psi_\alpha)$ .
- If  $\exists$  neighbourhood of  $x^+$  where  $r(x) \ge 0$ , differentiable and  $\frac{\mathrm{d}r}{\mathrm{d}x}(x) \xrightarrow{} 0$ , then  $F_X \in \mathcal{D}(\Delta)$ .

## Looking into financial data

Our starting point : 5 stocks listed on the Paris Stock Exchange. Stock price data over 15 years.



## The Black-Scholes model for stock prices

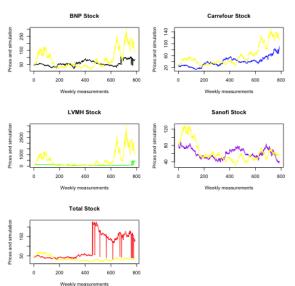
The price increment at time t should be proportional to the price at time t. **SDE** :

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dB_t \\ S(0) = s_0 \end{cases}$$

**Solution**: Geometric Brownian Motion

$$S_t = s_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma B_t)$$

## Simulation using a GBM



## Simulation using a GBM - observations

- The simulation may overestimate wrt to the actual prices.
- ullet On the whole  $\Longrightarrow$  rather satisfactory...
- ... except for the LVMH and even worse the Total stocks.
- sudden & large variations 

  model performs very poorly.

## Statistics of extremes & financial data (I)

Making the junction



#### References include



Jan Beirlant, Yuri Goegebeur, Johan Segers & Jozef Teugels Statistics of Extremes - Theory and Applications

Ruey S. Tsay

Analysis of Financial Time Series

# Thanks for your attention !