POLITECNICO
MILANO 1863

DEPARTMENT OF MECHANICAL ENGINEERING

EXERCISE CLASS 1 (part 1/2)

Review of basic statistical concepts: assumptions check and hypotesis testing (1 sample)

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MIUR 2018-2022

Probability Distributions

A **sample** is a collection of measurements selected from some larger source or *population*

Statistical methods allow us to study a sample and to **draw conclusions about their source** (i.e., about the process that generated them)

A ***probability distribution*** is a **mathematical model** that relates the value of the variable with the **probability of occurrence** of the value in the population.

Such a model could serve as a basis for judgement of observed data

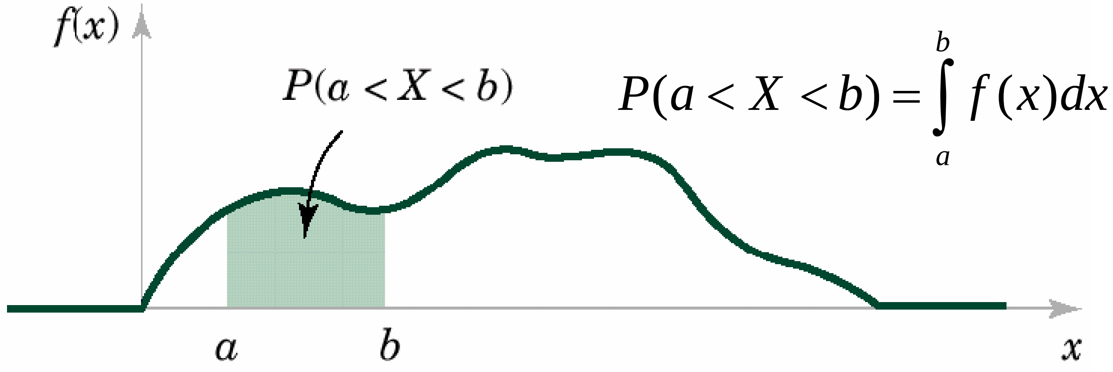
Probability Distributions

- *Continuous distribution*: variable expressed on a continuous scale

Probability density function *f(x)*

If X is a continuous variable, then, for every $x_1 < x_2$:

$$P(x_1 \leq X \leq x_2) = P(x_1 < X \leq x_2) =$$
$$= P(x_1 \leq X < x_2) = P(x_1 < X < x_2)$$

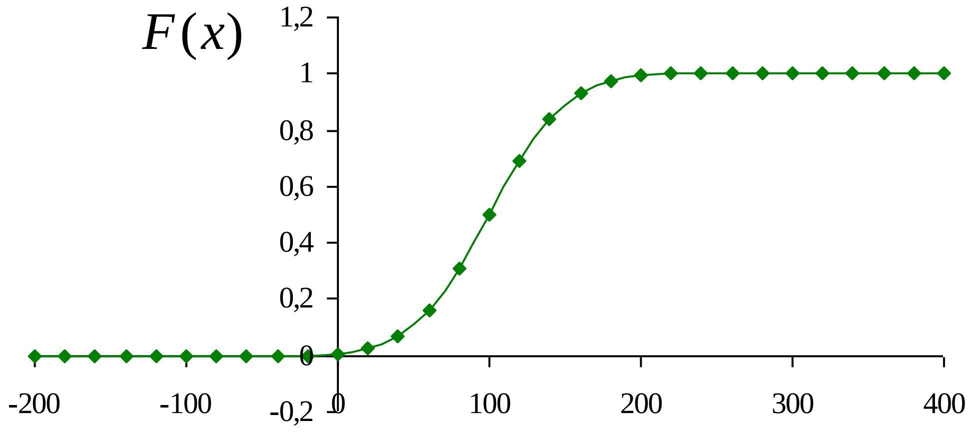


$$P(a < X < b) = \int_a^b f(x)dx$$

Cumulative distribution function F(x):

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du$$

For $-\infty < x < \infty$



Normal (Gaussian) distribution

A random variable X with probability density function:

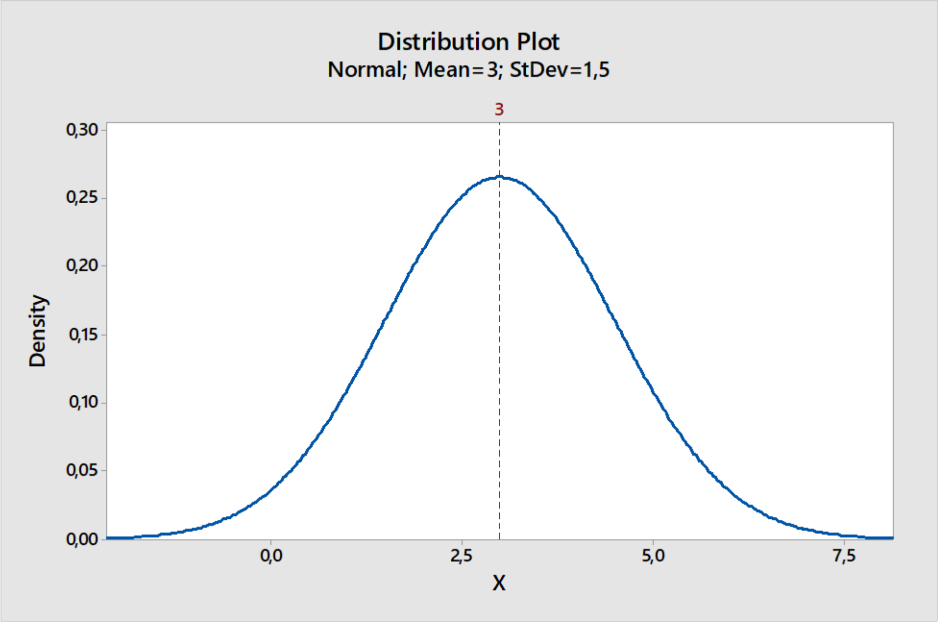
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad \text{for } -\infty < x < \infty$$

has a normal distribution with parameters μ and σ where

$$-\infty < \mu < \infty \text{ and } \sigma > 0$$

Also,

$$E(x) = \mu \quad \text{and } V(x) = \sigma^2$$



Normal (Gaussian) distribution

A normal random variable with $\mu = 0$ and $\sigma^2 = 1$ is called a *standard normal variable*.
A standard normal variable is usually denoted as *Z*.

Suppose *X* is a normal random variable with mean μ and variance σ^2.
Then

$$P(X \le x) = P\left(\frac{X-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) = P\left(Z \le z\right) = \Phi(z)$$

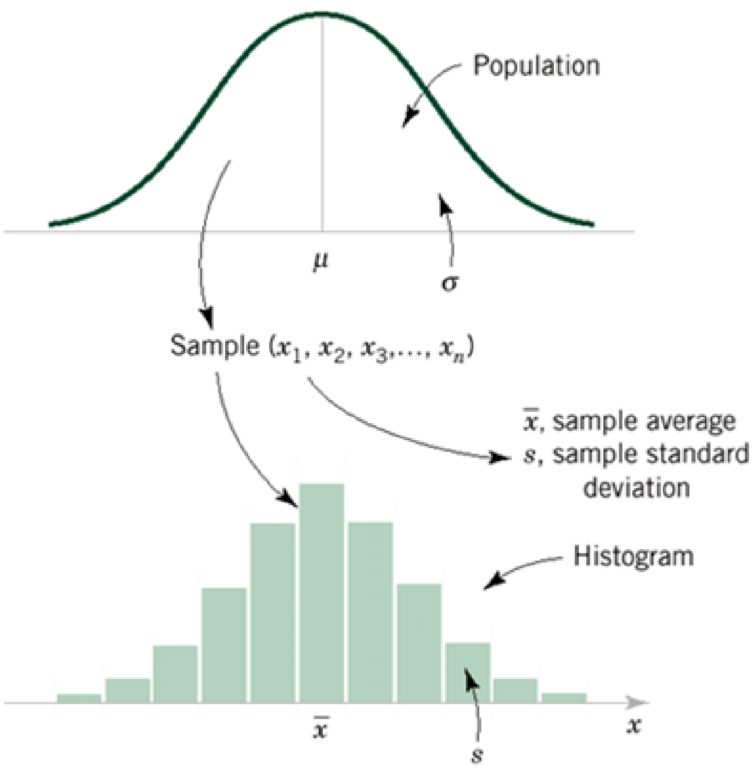
Z is a standard normal random variable and $z = \left(x - \mu\right)/\sigma$
is the *z*-value obtained by standardizing *x*.

Statistical inference

We want to infer properties of the source population by analysing data that are sampled from that distribution

Point estimators

A **point estimate** of some population parameter θ is a single numerical value $\hat{\theta}$ of a statistic $\widehat{\Theta}$



The **point estimator** $\widehat{\Theta}$ is an unbiased estimator of the parameter θ if:

$$E(\widehat{\Theta}) = \theta$$

If the estimator is not unbiased, then the difference $E(\widehat{\Theta}) - \theta$ is called **bias** of the estimator $\widehat{\Theta}$

Statistical inference

Remind:

Unknown Parameter θ	Statistic $\hat{\Theta}$	Point Estimate $\hat{\theta}$
μ	$\overline{X} = \dfrac{\sum X_i}{n}$	$\overline{x}$
σ^2	$S^2 = \dfrac{\sum (X_i - \overline{X})^2}{n - 1}$	s^2
$\mu_1 - \mu_2$	$\overline{X}_1 - \overline{X}_2 = \dfrac{\sum X_{1i}}{n_1} - \dfrac{\sum X_{2i}}{n_2}$	$\overline{x}_1 - \overline{x}_2$
$p_1 - p_2$	$\hat{P}_1 - \hat{P}_2 = \dfrac{X_1}{n_1} - \dfrac{X_2}{n_2}$	$\hat{p}_1 - \hat{p}_2$

Example 1

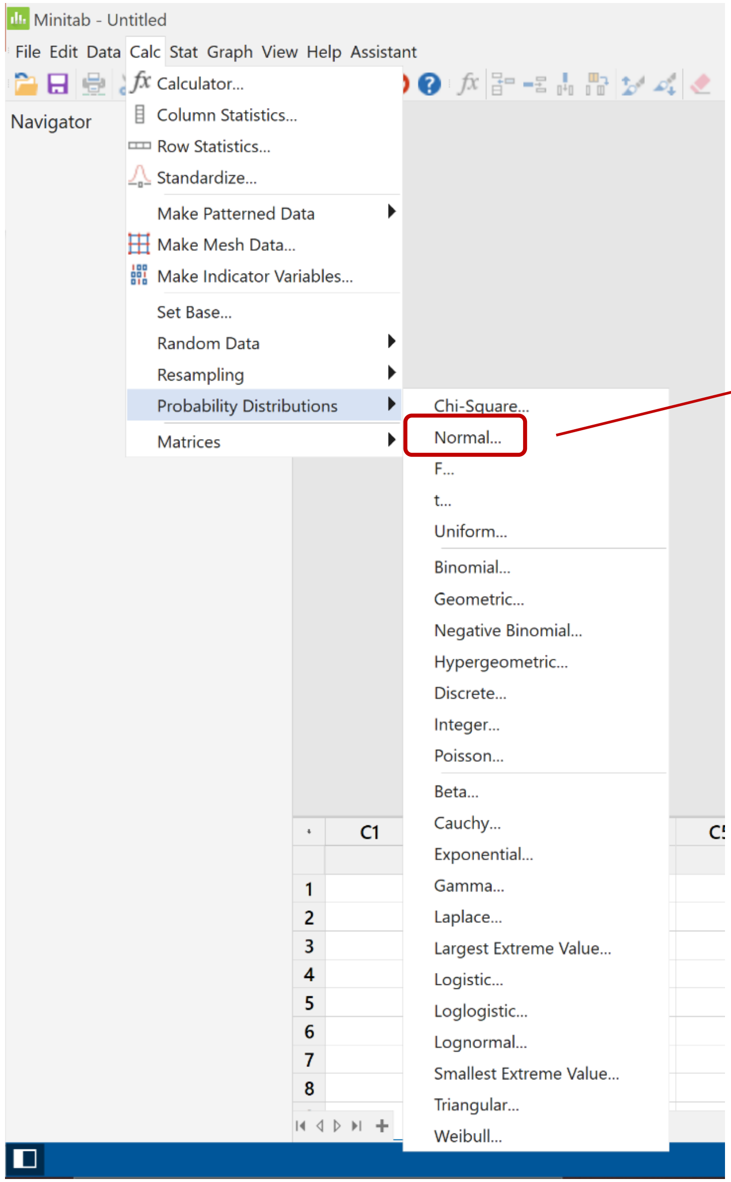
A synthetic fiber used in manufacturing industry has an ultimate tensile strength that is normally distributed with mean 75.5 psi and standard deviation 3.5 psi.
Compute the probability that a random sample of 6 observations has a sample mean larger than 75.75 psi.

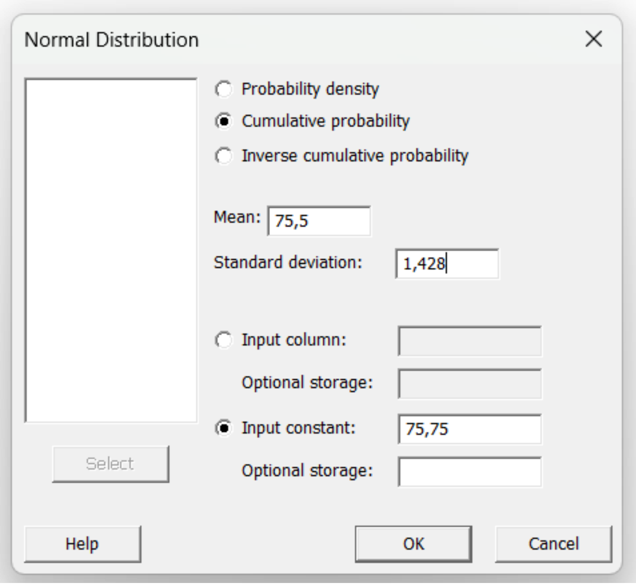
$$\mu = 75.5$$

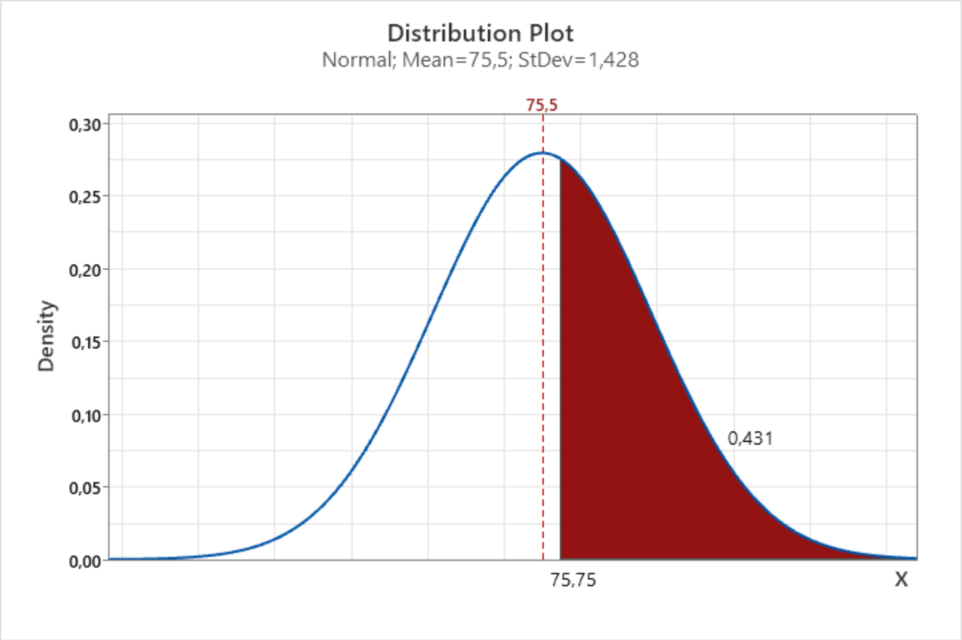
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{6}}$$ = 1,428

$$P(\bar{X} \geq \mu_0) = P(\frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \geq \frac{\mu_0 - \mu}{\sigma_{\bar{X}}}) = P(Z \geq \frac{75.75 - 75.5}{1.429}) = 1 - P(Z \leq 0.175)$$

Example 1







Minitab outcome:

Normal with mean = 75,5 and standard deviation = 1,428

x	P(X ≤ x)
75,75	0,569488

The probability of observing a sample mean larger than 75,75 is:

$P(X \geq x) = 1 - P(X \leq x) =$

$= 1 - 0{,}569 =$ **43,1 %**

Example 2

A random sample of size 16 is drawn from a normal population with mean 75 and standard deviation 8. A second sample of size 9 is drawn from a normal population with mean 70 and standard deviation 12.

a. Compute the probability that the **sample mean difference** between the first and the second sample is greater than 4 (assume that the two populations are independent).

b. Compute the probability that the sample mean difference between the first and the second sample ranges between 3.5 and 5.5 (same assumption).

Example 2

a. Compute the probability that the **sample mean difference** between the first and the second sample is **greater** than 4 (assume that the two populations are independent).

$$n_1 = 16 \qquad n_2 = 9$$
$$\mu_1 = 75 \qquad \mu_2 = 70$$
$$\sigma_1 = 8 \qquad \sigma_2 = 12$$

Remind:

$$V(x_1 - x_2) = \sigma_1^2 + \sigma_2^2 - 2Cov(x_1, x_2)$$

If they are independent, cov=0

$$\bar{X}_1 - \bar{X}_2 \sim N(\mu_{\bar{X}_1} - \mu_{\bar{X}_2}, \sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_2}) = N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}) = N(75 - 70, \frac{8^2}{16} + \frac{12^2}{9})$$

$$\bar{X}_1 - \bar{X}_2 \sim N(5,20)$$

$$P(\bar{X}_1 - \bar{X}_2 > 4)$$
$$P(Z > \tfrac{4-5}{\sqrt{20}}) = P(Z > -0.2236) = 1 - P(Z \le -0.2236)$$
$$\boxed{= 1 - 0.4115 = 0.5885}$$

Minitab outcome:

Normal with mean = 5 and standard deviation = 4,472

x	P(X ≤ x)
4	0,411529

Example 2

b. Compute the probability that the sample mean difference between the first and the second sample **ranges between 3.5 and 5.5** (same assumption).

$$Pr(3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5) = Pr(\frac{3.5-5}{\sqrt{20}} \leq Z \leq \frac{5.5-5}{\sqrt{20}}) = Pr(Z \leq \frac{5.5-5}{\sqrt{20}}) - Pr(Z \leq \frac{3.5-5}{\sqrt{20}})$$

Normal with mean = 5 and standard deviation = 4,472

x	P(X ≤ x)
3,5	0,368654

Normal with mean = 5 and standard deviation = 4,472

x	P(X ≤ x)
5,5	0,544512

Solution:

P (3,5 ≤ X1 – X2 ≤ 5,5) = 0,544 – 0,369 = **17,5 %**

HYPOTESIS TESTING

Hypotesis testing

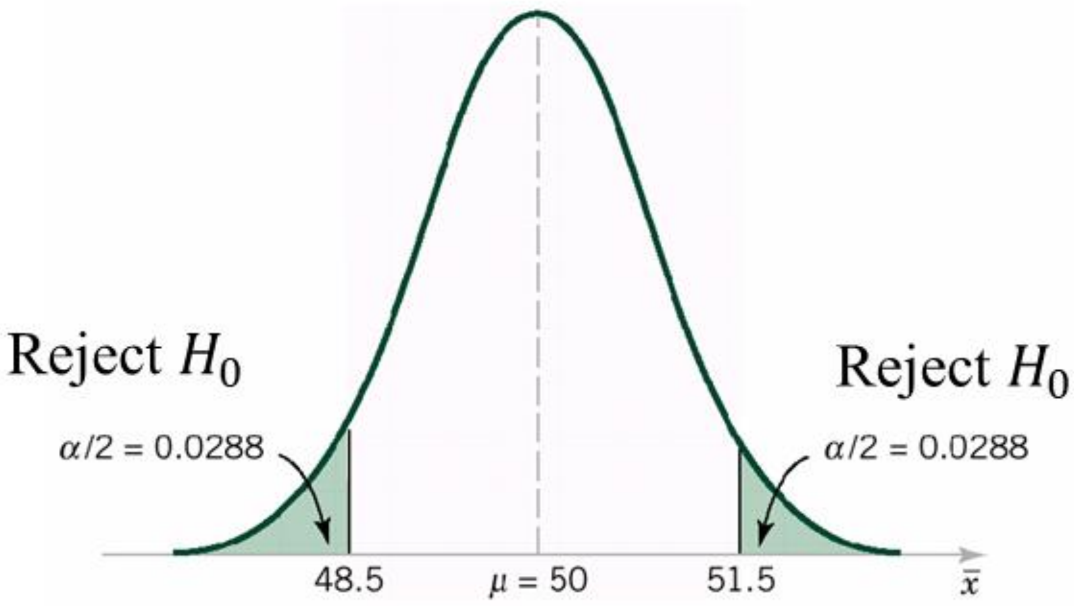
A **statistical hypotesis** is a statement either about the **parameters** of a probability distribution or the parameters of a model.

Taking a random sample from a population, a **test statistic** is computed. Then, it is possible **to reject or fail to reject the null hypothesis H0**.

Part of the testing procedure is specifying the set of values for the test statistic that leads to the rejection of H0 (**critical region**)

Example: hypotesis testing on a population mean

- H_0: $\mu = 50$ is the *null hypotesis*
- H_1: $\mu \neq 50$ is the *alternative hypotesis*



The critical region for H_0: $\mu = 50$ versus H_1: $\mu \neq 50$ and $n = 10$.

Hypotesis testing

- One sample tests:
  - Test for mean (known variance): one-sample z-test
  - Test for mean (unknown variance): one-sample t-test
  - Test for variance: chi-squared test (variance)

- Two samples tests
  - Test for mean difference (known var): two-sample z-test
  - Test for mean difference (unknown var): two-sample t-test
  - Test for mean of paired data (unknown var): paired t-test
  - Test for equality of variances: F-test (variances)

Hypotesis testing - errors

Two kinds of errors may be committed when testing hypothesis

Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	no error	type II error
Reject H_0	type I error	no error

$$\alpha = P(type\ I\ error) = P(reject\ H_0 | H_0 \text{ is true})$$

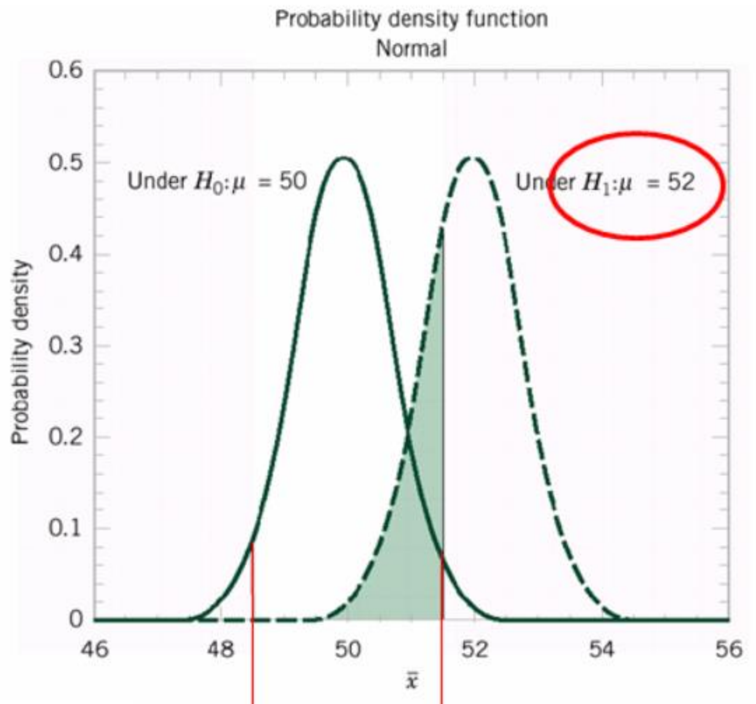
'Probability of rejecting a good product'

Also known as: ***significance level***

$$\beta = P(type\ II\ error) = P(fail\ to\ reject\ H_0 | H_0 \text{ is false})$$

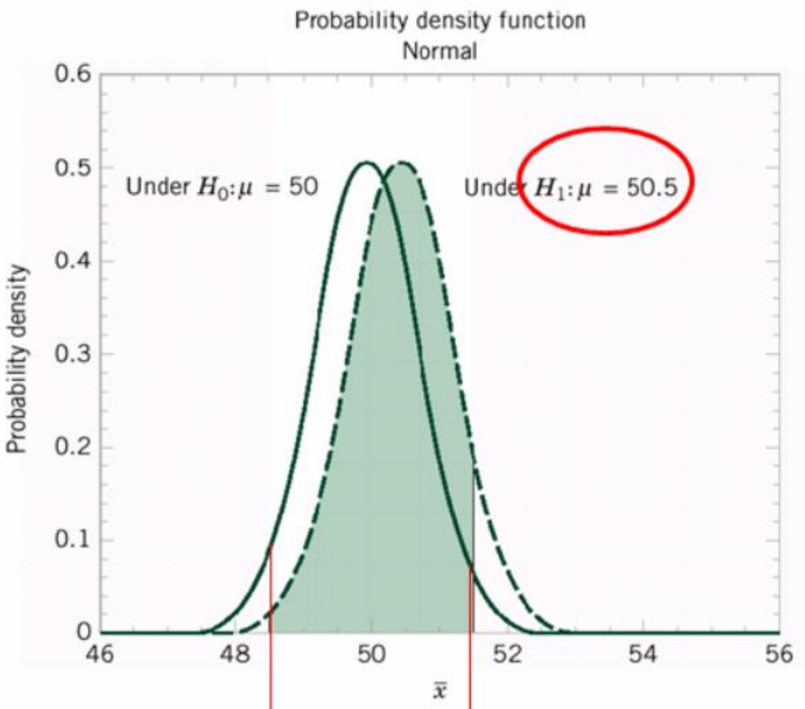
'Probability of accepting a nonconforming product'

Hypotesis testing - errors



The probability of type II error when $\mu = 52$ and $n = 10$.

Acceptance region



The probability of type II error when $\mu = 50.5$ and $n = 10$.

Acceptance region

Hypotesis testing – general procedure

1. From the problem context, identify the parameter of interest.
2. State the null hypothesis, H_0.
3. Specify an appropriate alternative hypothesis, H_1.
4. Choose a significance level α.
5. State an appropriate test statistic.
6. State the rejection region for the statistic.
7. Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
8. Decide whether or not H_0 should be rejected and report that in the problem context.

Thus:
Specify a value of α (Type I error) and design a procedure such that β (Type II error) has a suitably small value

Hypotesis testing – Confidence Intervals

There is a direct link between hypothesis testing and ***confidence intervals***

Let $[L, U]$ be a $100(1-\alpha)\%$ confidence interval for the parameter θ, then the hypothesis test:

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta \neq \theta_0$$

with significance level α will yield the rejection of the null hypothesis H_0 **if and only if** θ_0 is NOT included into the $100(1-\alpha)\%$ confidence interval $[L, U]$.

REMIND:

$L \leq \theta \leq U$ such that $P(L \leq \theta \leq U) = 1 - \alpha$
is called $100(1-\alpha)\%$ confidence interval for the (unknown) parameter θ

Interpretation: if, in repeated random samplings, a large number of such intervals is constructed, $100(1-\alpha)\%$ of them will contain the true value of θ.

Hypotesis testing – p-value

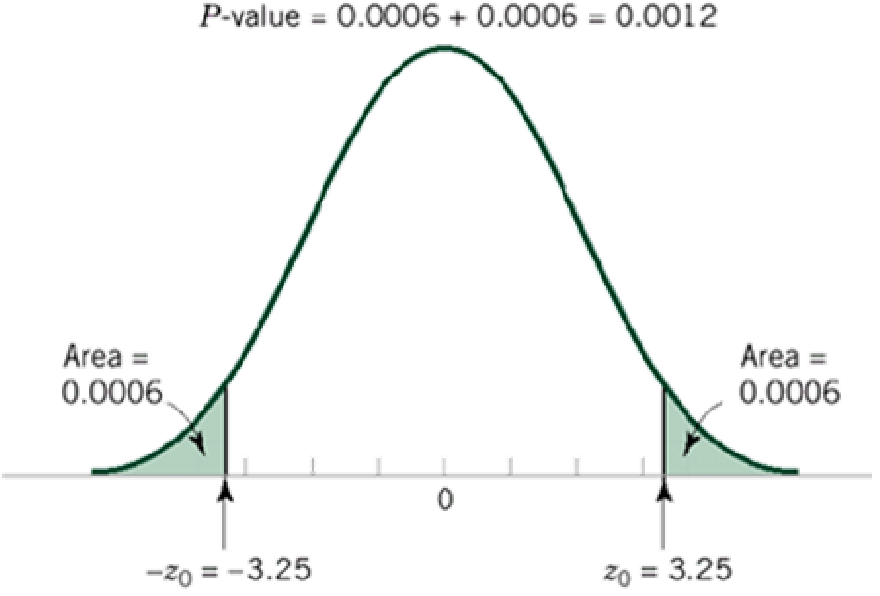
The ***P-value*** is the smallest level of significance that would lead to rejection of the null hypothesis H_0

It is the probability that the test statistic will take on a value that is at least as extreme as the observed value of the statistic when the null hypothesis is true

$$P = \begin{cases} 2[1 - \Phi(|z_0|)] & \text{for a two-tailed test: } H_0\text{: } \mu = \mu_0, \quad H_1\text{: } \mu \neq \mu_0 \\ 1 - \Phi(z_0) & \text{for an upper-tailed test: } H_0\text{: } \mu = \mu_0, \quad H_1\text{: } \mu > \mu_0 \\ \Phi(z_0) & \text{for a lower-tailed test: } H_0\text{: } \mu = \mu_0, \quad H_1\text{: } \mu < \mu_0 \end{cases}$$

E.g.: p-value=0.0012 means that only (100*0.0012)%=0.12% of population is more extreme than Z_0

Very small p-value: reject H_0



MODELLING PROCESS DATA:
THE IMPORTANCE OF THE ASSUMPTIONS

Assumptions - IID data

Statistical inference and hypothesis testing require data to be independent and identically distributed ("**iid**").
This means that the assumptions of independence and normality must be verified.

IID = INDEPENDENCE + NORMALITY

Without these conditions, results may be biased or unreliable.

As consequence, **methods to check these assumptions must be always applied before any analysis**.

Assumptions	Hypothesis test (to check the assumption)
"independence" (random pattern)	- Runs test - Bartlett's test - LBQ's test
Normal distribution	Normality test

Exercise 1

A study was performed by ComputerTek Co to determine the time series of order processing durations. Data in the file `days.csv` refer to the period 1995, July – 1997, October. Each datum represents the time (in days) to ship the order.

INDEPENDENCE:
1. Determine the value of n and m in observed runs
2. Assuming that the runs distribution is random, which is the expected number of runs? And the 95% confidence interval for the number of runs?
3. Test the null hypothesis of observation **randomness** (significance level 5%)

NORMALITY:
4. Test the null hypothesis of observation **normality** (significance level 5%)

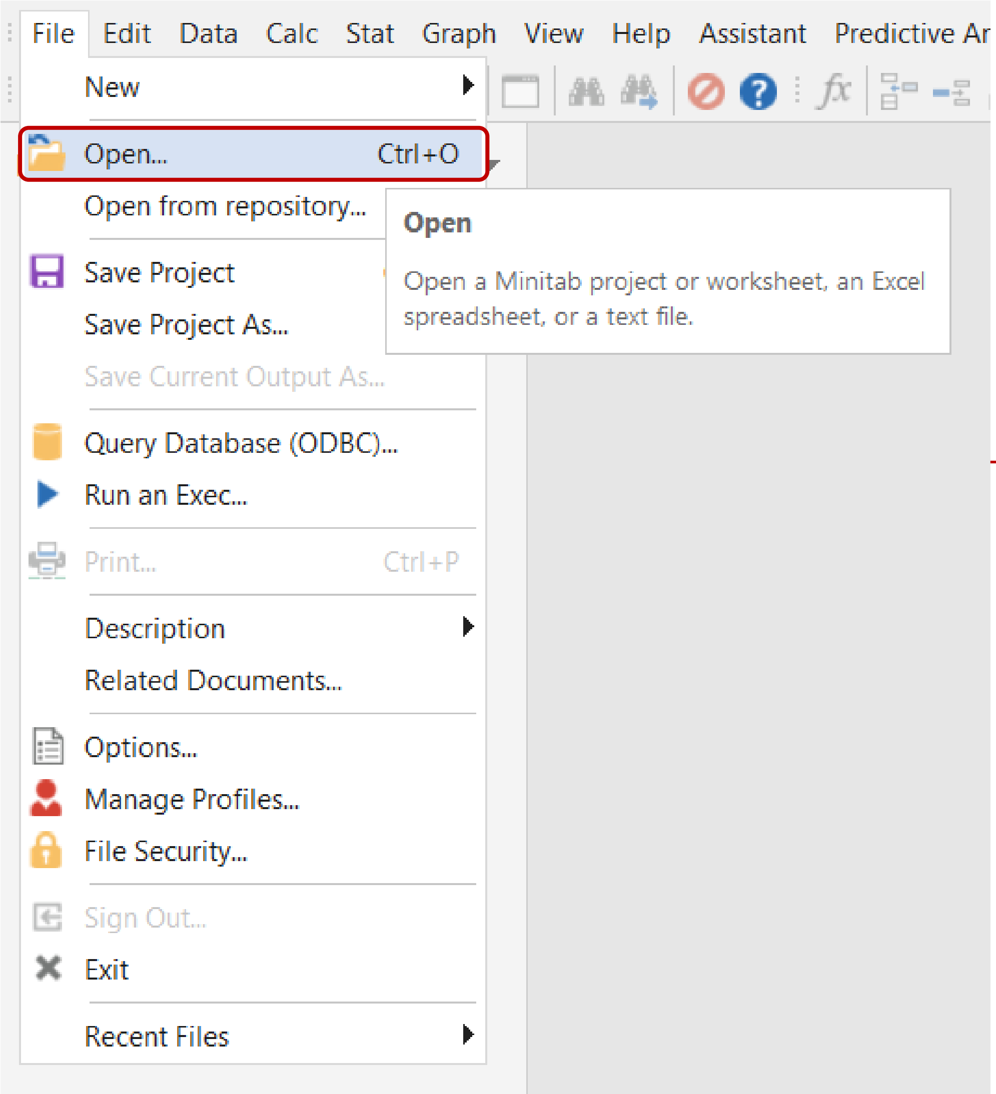
AFTER ASSUMPTIONS CHECK:
5. Assume to know that the variable representing the days to ship the order is normally distributed standard deviation (known) 7 [days].
   5.1 Is there statistical evidence to state that the mean life of neon lights is **larger** than 31 days (confidence level: 95%)?
   5.2 Is there statistical evidence to state that the mean life of neon lights is **different** from 31 days (confidence level: 95%)?
   5.3 Compute the power curve for the test of Point 5.1.

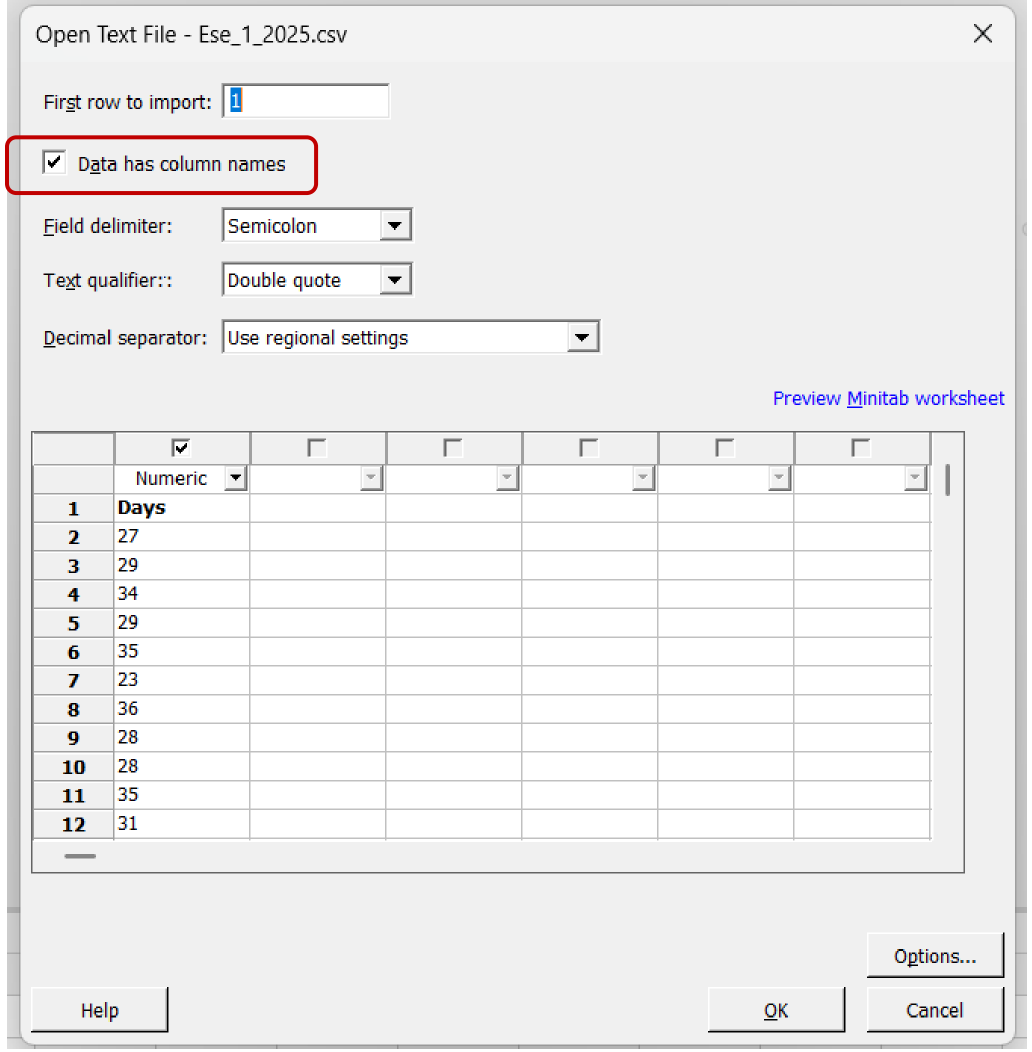
6. Assume to know that the variable representing the days to ship the order is normally distributed with mean 30 [days], and the standard deviation is **unknown**. Is there statistical evidence to state that the mean life of neon lights is **larger** than 31 days (confidence level: 95%)?

7. Compute the upper limit of the one-sided interval for the variance (99%) and the two-sided confidence interval for the standard deviation (98%)

Data exploration

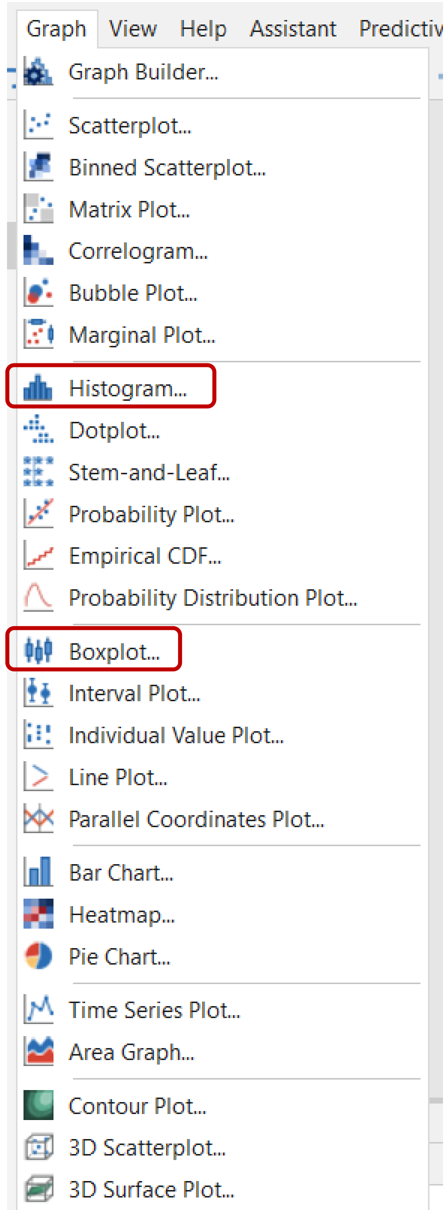
Import the file `Ese_1.csv`:

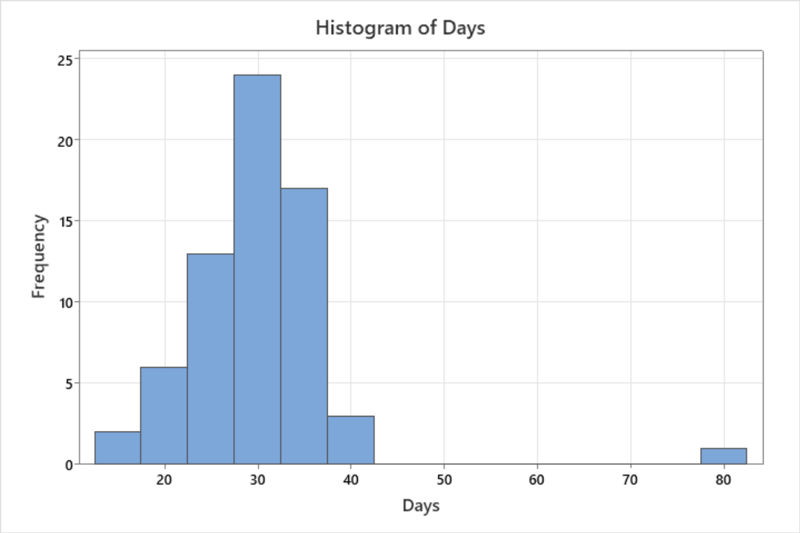


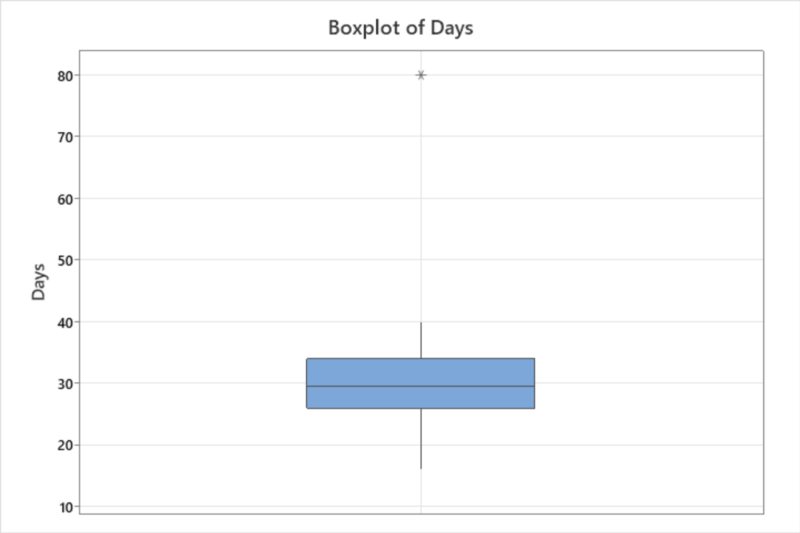


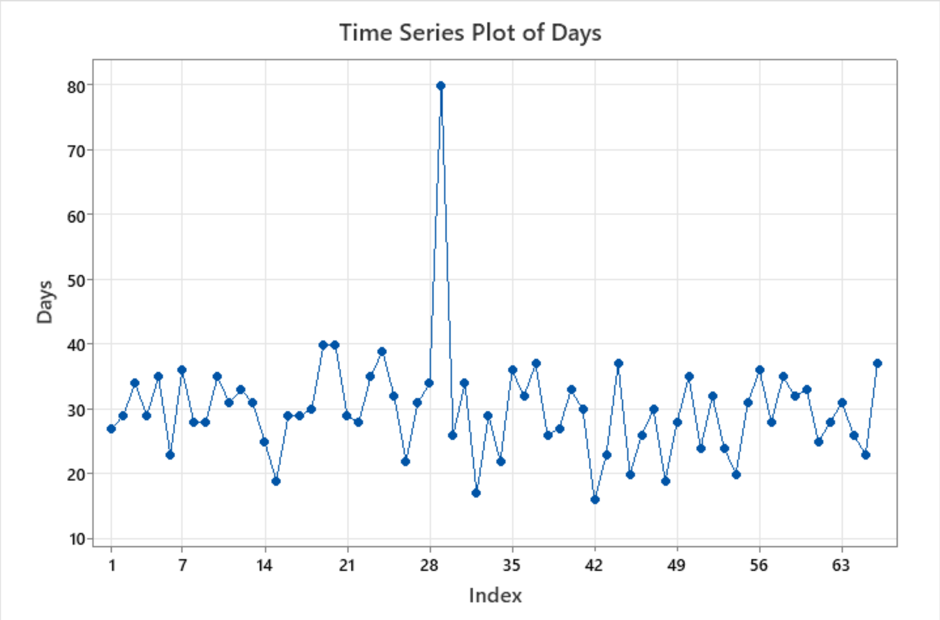
Data exploration

Graph → Histogram /Boxplot → Simple → click on column name ('Days'), then 'Select'





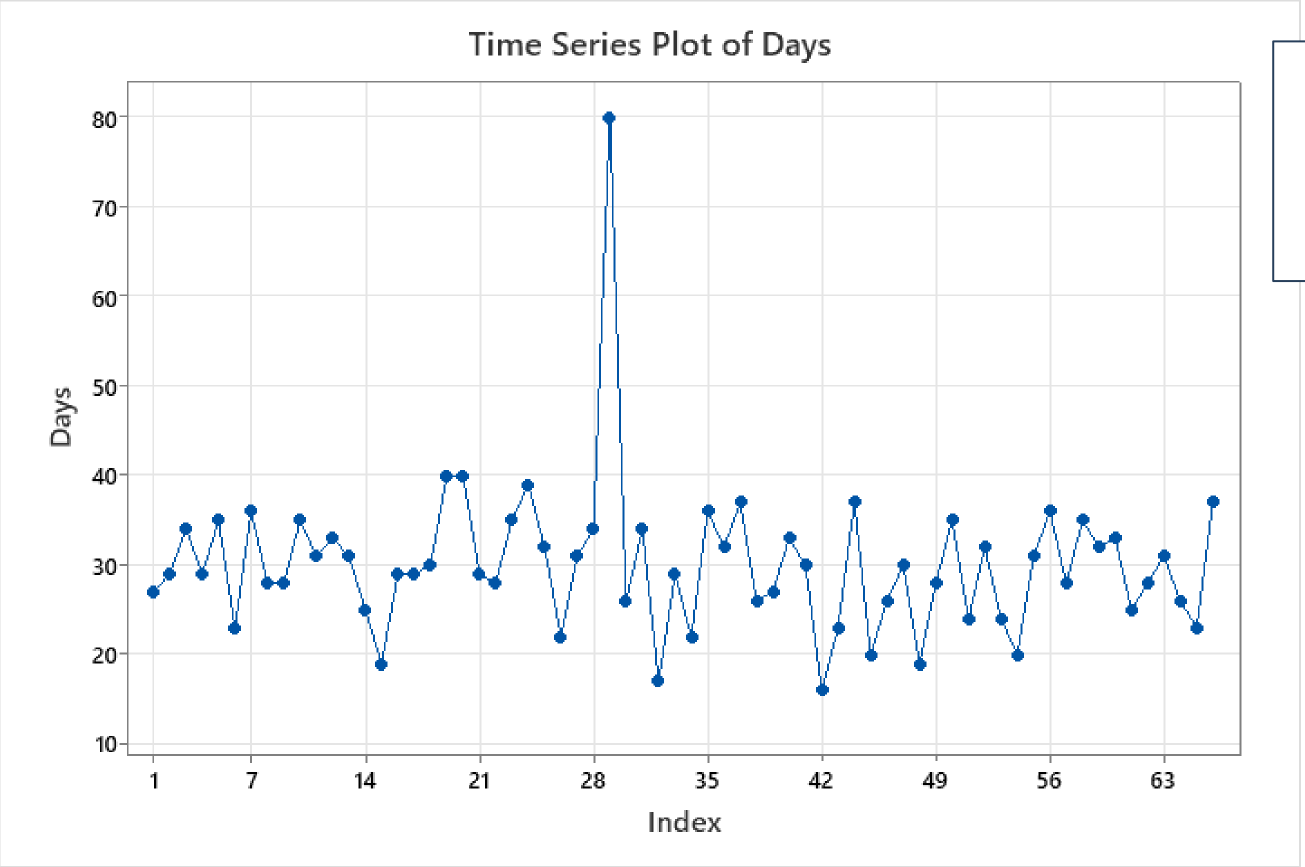




Stat → Time Series → Time Series plot

Point 1

Determine the value of n and m in observed runs



Stat → Basic Statistics → Display Descriptive Statistics

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Days	66	0	30,1364	1,03232	8,38659	16	26	29,5	34	80

n : n° of observations

m : n° of +

Number of observations: **n = 66**

Number of + (above mean: 30,136): **m = 30**

Number of runs (Y): 34

Point 2

Assuming that the runs distribution is random, which is the expected number of runs?
And the 95% confidence interval for the number of runs?

Expected number of runs: $E(Y) = \frac{2m(n\text{-}m)}{n} + 1$ **= 33,73**

Confidence interval (CI) computation

$$E(Y) \mp z_{\alpha/2}\sqrt{V(Y)}$$

Where the variable: $Y \sim N(E(Y), V(Y))$

is a Normal approximation of a Poisson distribution

$$\hookrightarrow \sqrt{V(Y)} = \sqrt{\frac{2m(n-m)[2m(n-m)-n]}{n^2(n-1)}} = 4$$

CI = (25.894, 41.561)

Inverse Cumulative Distribution Function:

Normal with mean = 0 and standard deviation = 1

P(X ≤ x)	x
0,025	$\boxed{-1,95996}$ $= Z_{\alpha/2}$

Calc → Probability Distributions → Normal

95% CI → α/2 = 0,025

Point 3 Test the null hypothesis of observation **randomness** (significance level 5%)

We can verify if the process is random by using :

- time series plot (qualitative)
- runs test (quantitative)
- ACF/PACF (qualitative)
- Bartlett's test (quantitative)
- LBQ test (quantitative)

Stat → Nonparamentrics → Runs test

RUNS TEST

Test

Null hypothesis	H_0: The order of the data is random
Alternative hypothesis	H_1: The order of the data is not random

Number of Runs		
Observed	Expected	P-Value
34	33,73	0,946

Test statistic: $$Z_0 = \frac{Y - E(Y)}{\sqrt{V(Y)}} \approx 0$$

$$z_{\alpha/2} = 1.95996$$

$$|Z_0| > z_{\alpha/2}$$

P-value > Significance level

Descriptive Statistics

		Number of Observations	
N	K	≤ K	> K
66	30,1364	36	30

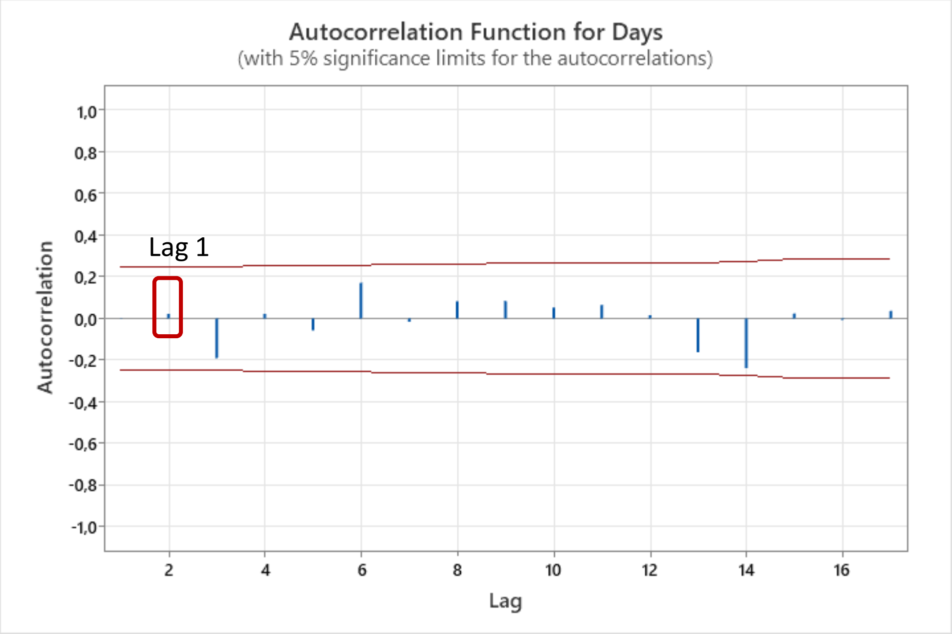
K = sample mean

Therefore, there is statistical evidence to state that process is random (95%).

Point 3 Test the null hypothesis of observation **randomness** (significance level 5%)

We can verify if the process is random by using :

- time series plot (qualitative)
- runs test (quantitative)
- ACF/PACF (qualitative)
- Bartlett's test (quantitative)
- LBQ test (quantitative)



Stat → Time Series → Autocorrelation

BARTLETT's TEST
for a specific lag

$$H_0 : \rho_k = 0 \qquad H_1 : \rho_k \neq 0$$

ρ_k: *true autocorr at lag k*
r_k: *sample autocorr at lag k*

To test the absence of autocorrelation at <u>1 predefined lag</u>.

In this case, **lag 1**

Test statistic: r_k

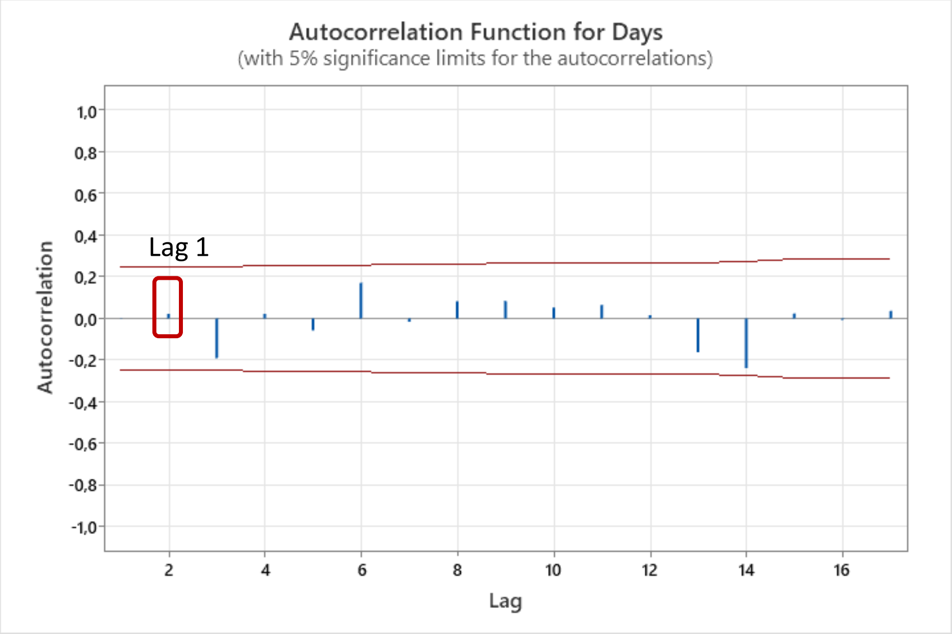
Lag	ACF
1	-0,001466
2	0,022790
3	-0,189604

Critical (rejection) region: $|r_k| > \dfrac{z_{\alpha/2}}{\sqrt{n}}$ = 0,24

Point 3 Test the null hypothesis of observation **randomness** (significance level 5%)

We can verify if the process is random by using :

- time series plot (qualitative)
- runs test (quantitative)
- ACF/PACF (qualitative)
- Bartlett's test (quantitative)
- LBQ test (quantitative)



Stat → Time Series → Autocorrelation

Ljung-Box (LBQ) TEST

for a global test

$$H_0 : \rho_k = 0, k = 1, \dots, L \qquad H_1 : \exists k \in [1, L] \,/\, \rho_k \neq 0$$

Test statistic: $LBQ = n(n+2)\sum_{k=1}^{L} \frac{r_k^2}{n-k}$

L = 6

(number of lags to test)

$$LBQ \sim \chi_L^2 \Rightarrow \text{rejection region} : LBQ > \chi_{\alpha,L}^2$$

Lag	ACF	T	LBQ
1	-0,001466	-0,01	0,00
2	0,022790	0,19	0,04
3	-0,189604	-1,54	2,60
4	0,021469	0,17	2,63
5	-0,057189	-0,45	2,87
6	0,170806	1,34	5,05

LBQ

Inverse Cumulative Distribution Function:

Chi-Square with 6 DF

P(X ≤ x)	x
0,05	1,63538

$\chi_{\alpha,L}^2$

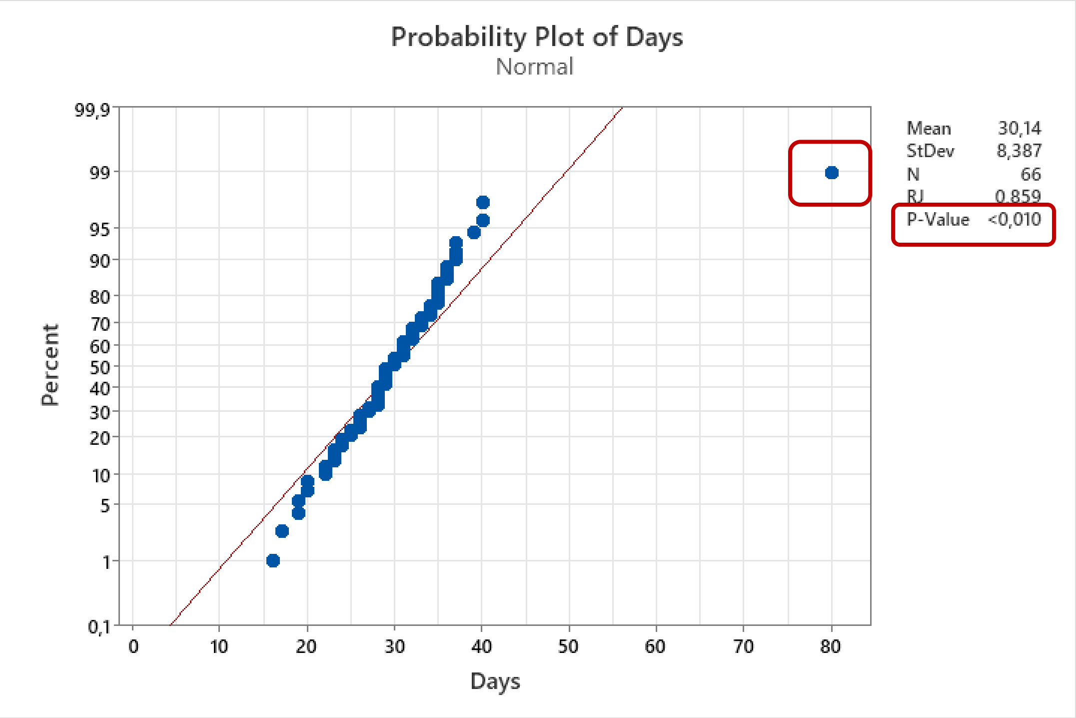
Calc → Probability Distributions → Chi Squared

Point 4 Test the null hypothesis of observation **normality** (significance level 5%)

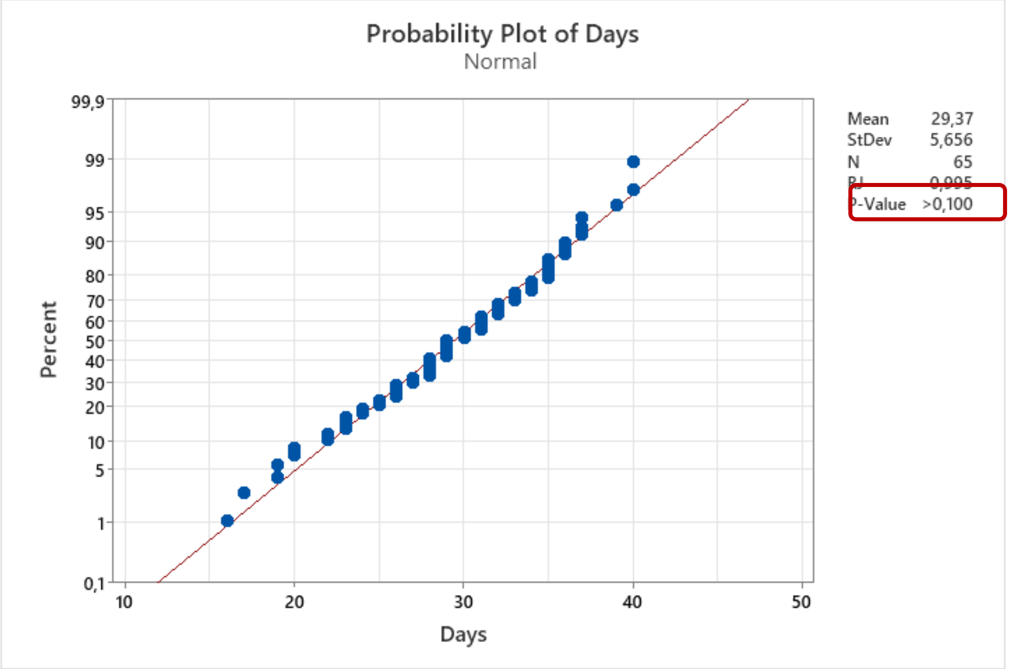
Normality can be tested with:

- **Shapiro-wilk test**
- Anderson-Darling test

H0: process is Normal
H1: process in not Normal



How much is this result influenced by the **outlier**?
We can try to remove the outlier and check for normality again:



Stat → Basic Statistics → Normality test

Point 4 Test the null hypothesis of observation **normality** (significance level 5%)

Normality can be tested with:

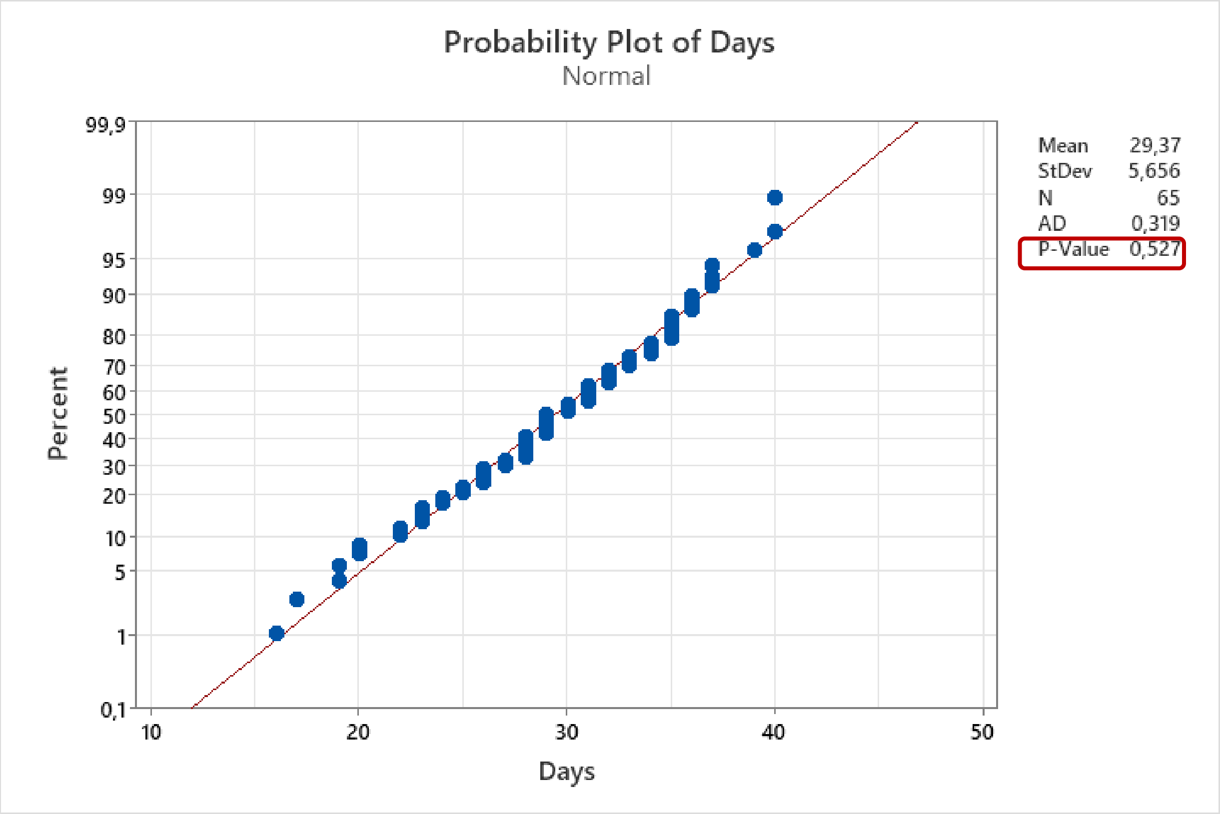
- Shapiro-wilk test
- **Anderson-Darling test**

H0: process is Normal
H1: process in not Normal

P-value > significance level

↓

No statistical evidence to reject the null hypothesis of Normality



Stat → Basic Statistics → Normality test

Point 5

Assume to know that the variable representing the days to ship the order is normally distributed with **standard deviation (known)** 7 [days].

ONE-SAMPLE Z-TEST

- One sample tests:
  - Test for mean (known variance): one-sample z-test
  - Test for mean (unknown variance): one-sample t-test
  - Test for variance: chi-squared test (variance)
- Two samples tests
  - Test for mean difference (known var): two-sample z-test
  - Test for mean difference (unknown var): two-sample t-test
  - Test for mean of paired data (unknown var): paired t-test
  - Test for equality of variances: F-test (variances)

Assumptions

- $X_1, X_2, \ldots, X_n$ is a random sample of size n from a population.
- Population is **normal**.
- The **variance** of the population is **known**.

  Under those assumptions, the quantity Z follows a standard normal distribution $N(0,1)$.

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Null hypothesis: $H_0 : \mu = \mu_0$

Where:

- μ is the population mean
- μ_0 is the hypothesized population mean
- n is the sample size

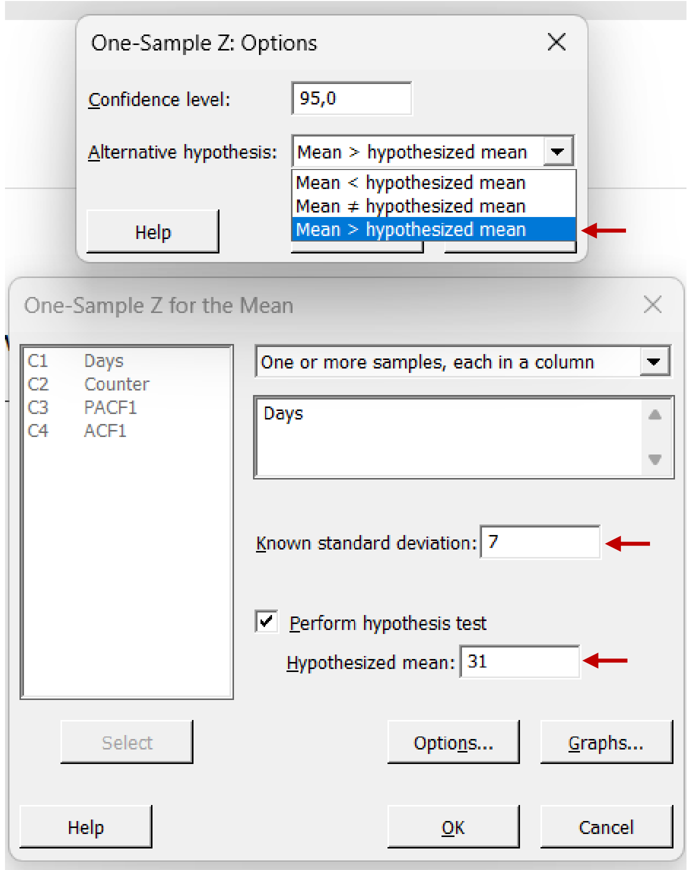
Test statistic: $Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

Alternative hypotheses	Rejection criterion
$H_1 : \mu \neq \mu_0$	$\lvert Z_0 \rvert > z_{\alpha/2}$
$H_1 : \mu > \mu_0$	$Z_0 > z_\alpha$
$H_1 : \mu < \mu_0$	$Z_0 < -z_\alpha$

Point 5.1

Assume to know that the variable representing the days to ship the order is normally distributed with **standard deviation (known)** 7 [days].
Is there statistical evidence to state that the mean life of neon lights is **larger** than 31 days (confidence level: 95%)? Compute the associated one-sided confidence interval.

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu > \mu_0$$



Stat → Basic Statistics → 1-Sample Z

One-sided confidence interval

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
65	29,369	5,656	0,868	27,941

μ: population mean of Days
Known standard deviation = 7

Test

Null hypothesis H_0: μ = 31
Alternative hypothesis H_1: μ > 31

Z-Value	P-Value
-1,88	0,970

p-value > 0,05

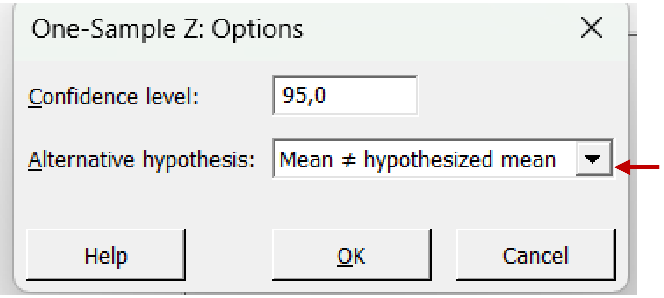
↓

No statistical evidence to reject the null hypothesis

Point 5.2

Assume to know that the variable representing the days to ship the order is normally distributed with **standard deviation (known)** 7 [days].
Is there statistical evidence to state that the mean life of neon lights is **different** from 31 days (confidence level: 95%)? Compute the associated two-sided confidence interval.

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu \neq \mu_0$$



Stat → Basic Statistics → 1-Sample Z

Two-sided confidence interval

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
65	29,369	5,656	0,868	(27,668; 31,071)

μ: population mean of Days
Known standard deviation = 7

Test

Null hypothesis	H_0: μ = 31
Alternative hypothesis	H_1: μ ≠ 31

Z-Value	P-Value
-1,88	0,060

p-value > 0,05

↓

No statistical evidence to reject the null hypothesis

Point 5.3

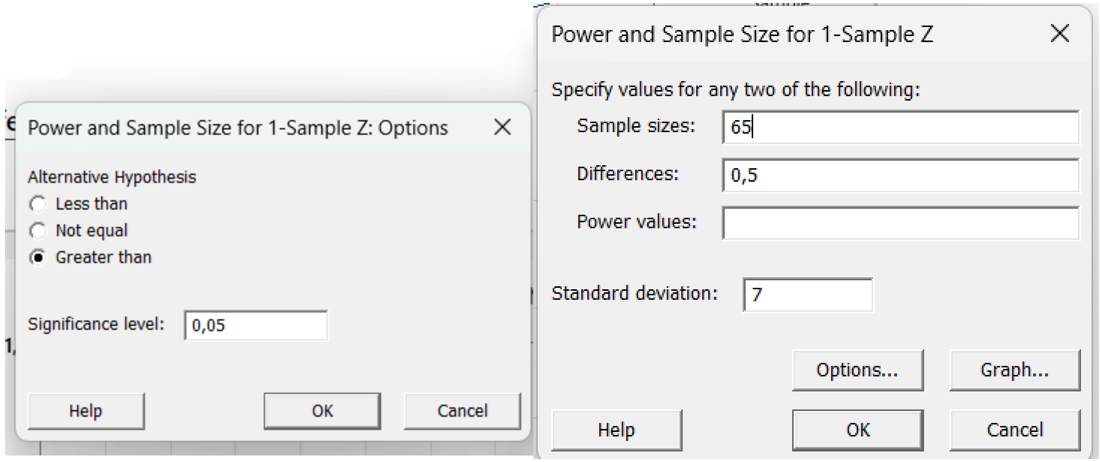
Assume to know that the variable representing the days to ship the order is normally distributed with **standard deviation (known)** 7 [days].
Compute the **power curve** for the test of Point 5.1.

The power of a statistical test is the <u>probability of rejecting the null hypotesis when the alternative hypotesis is true</u>.

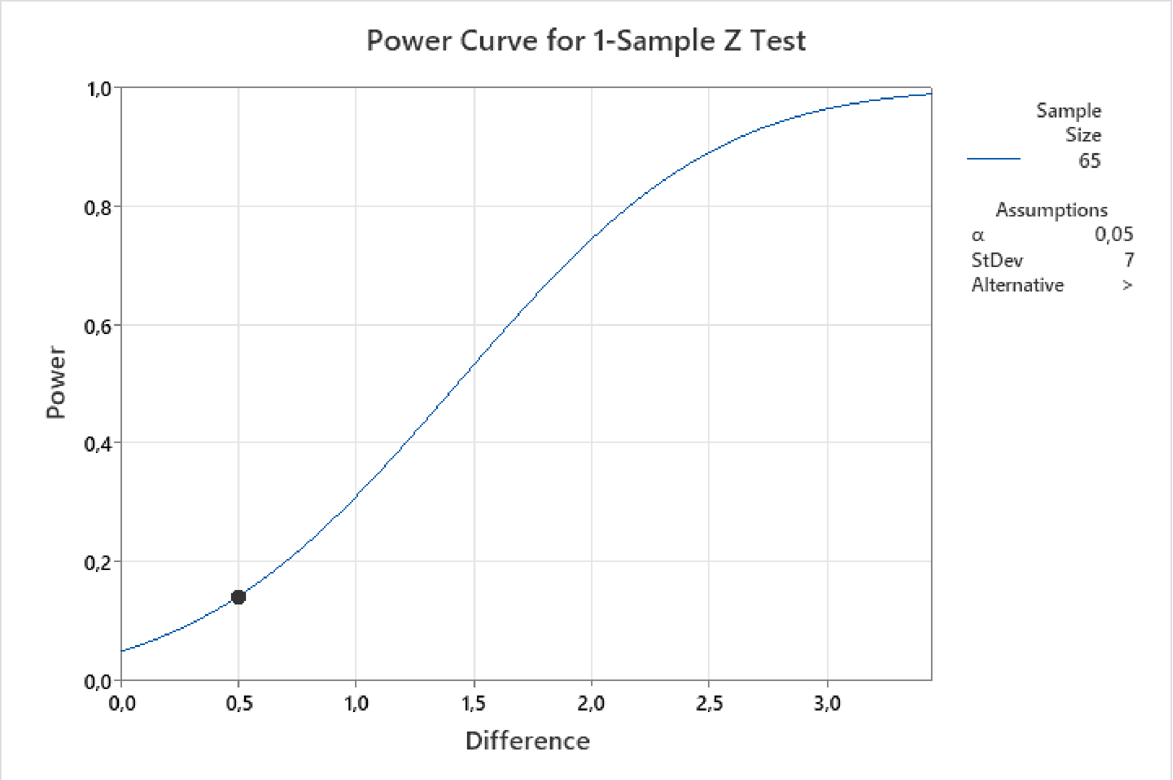
$$power = 1 - \beta = P(reject H_0 | H_0 false) = 1 - \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) + \Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

where *Z_alpha/2* is the critical value, *delta* is the difference between the hypothesized mean and the true mean, *sigma* is the standard deviation of the population, and *n* is the sample size.

- Reducing *beta*, the power increases.
- The estimation of beta depends on H1.



Stat → Power and Sample size → 1-Sample Z



Point 6

Assume to know that the variable representing the days to ship the order is normally distributed, and the standard deviation is **unknown**

ONE-SAMPLE t-TEST

- One sample tests:
  - Test for mean (known variance): one-sample z-test
  - Test for mean (unknown variance): one-sample t-test
  - Test for variance: chi-squared test (variance)

Assumptions

- $X_1, X_2, \ldots, X_n$ is a random sample of size n from a population.
- Population is **normal**.
- The **variance** of the population is **unknown**

Under those assumptions, the quantity T follows a Student-t distribution with $n-1$ degrees of freedom.

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Where:

- S is the sample standard deviation

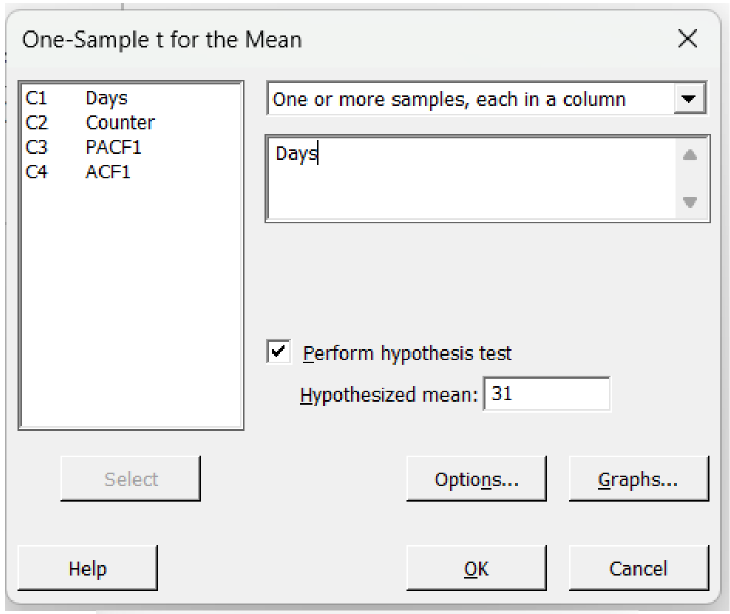
Null hypothesis: $H_0 : \mu = \mu_0$

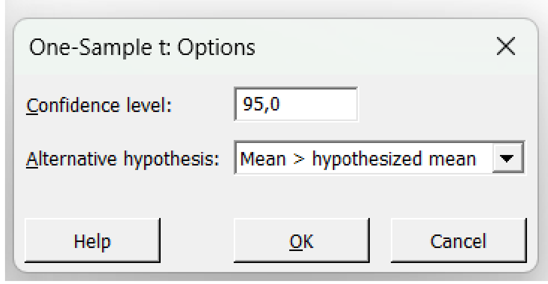
Test statistic: $t_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

Alternative hypotheses	Rejection criterion
$H_1 : \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$
$H_1 : \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$
$H_1 : \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$

Point 6

Assume to know that the variable representing the days to ship the order is normally distributed, and the standard deviation is **unknown.**
Is there statistical evidence to state that the mean life of neon lights is **larger** than 31 days (confidence level: 95%)?





Stat → Basic Statistics → 1-Sample t

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
65	29,369	5,656	0,701	28,198

μ: population mean of Days

Test

Null hypothesis $H_0: \mu = 31$
Alternative hypothesis $H_1: \mu > 31$

T-Value	P-Value
-2,32	0,988

Point 7

Compute the upper limit of the one-sided **interval for the variance** (99%) and the two-sided confidence **interval for the standard deviation** (98%)

CHI-SQUARED TEST

One sample tests:

- Test for mean (known variance): one-sample z-test
- Test for mean (unknown variance): one-sample t-test
- Test for variance: chi-squared test (variance)

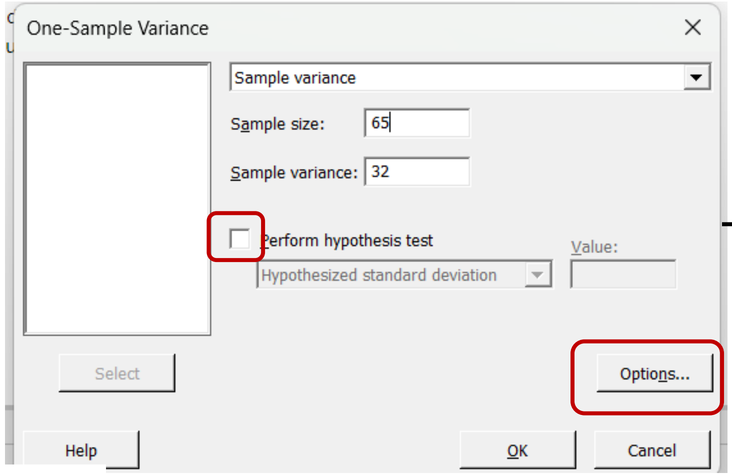
The Chi-squared test statistic is:

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

where S^2 is the sample variance and σ^2 is the population variance.

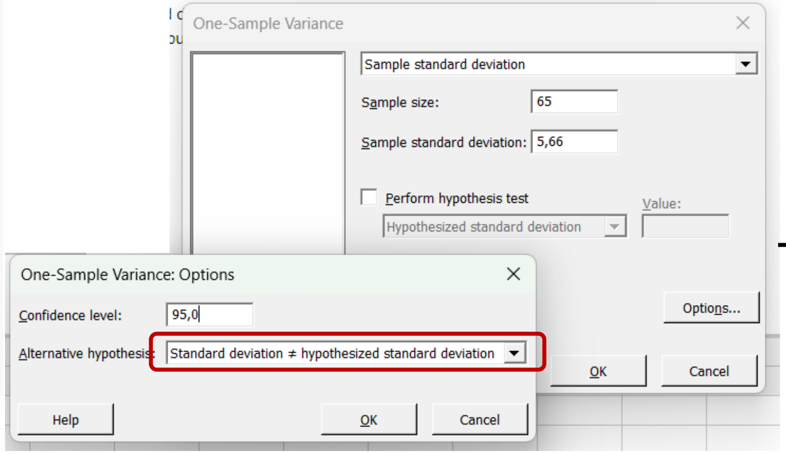
The one-sided CI on the variance is computed from:

$$\sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}$$



Descriptive Statistics

N	StDev	Variance	95% Upper Bound for σ² using Chi-Square
65	5,66	32,0	43,95



Descriptive Statistics

N	StDev	Variance	95% CI for σ using Chi-Square
65	5,66	32,0	(4,83; 6,84)

Stat → Basic Statistics → 1 Variance