POLITECNICO
MILANO 1863

DEPARTMENT OF MECHANICAL
ENGINEERING

EXERCISE CLASS 3

Control Charts for small shifts
&
Attribute Control Charts

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Milano

Control Charts for small shifts

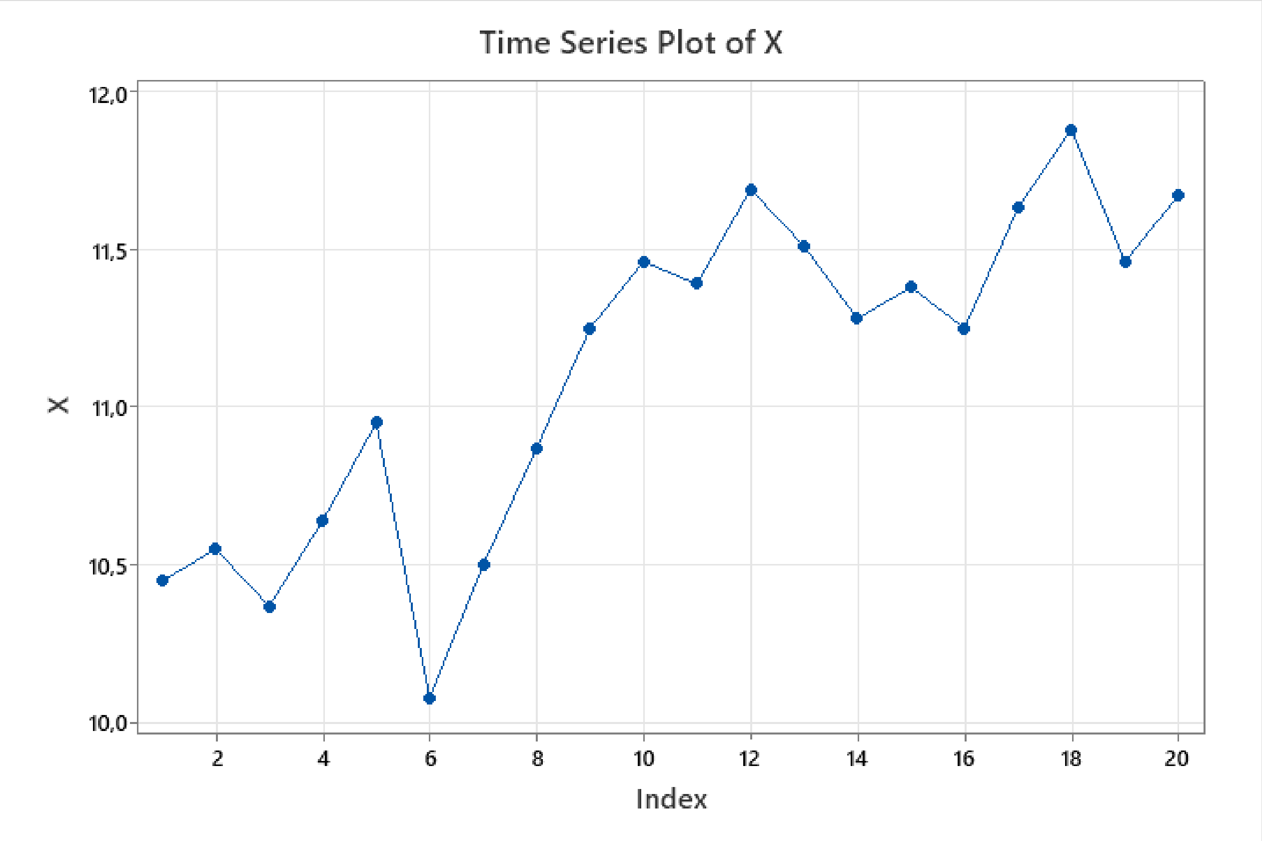
EXAMPLE 1

comparison with traditional control charts

The data stored in `*small_shifts_example1.csv*` represent the mean values of a quantity measured in samples of size **n = 5** taken from a population with **sigma = 1**

1. Design a control chart for individuals (I chart) with the information provided.

2. Design a CUSUM chart (with parameters h = 4 and k = 0.5) and an EWMA (with param. lambda = 0.2) and discuss the results (neglect possible non-random patterns).



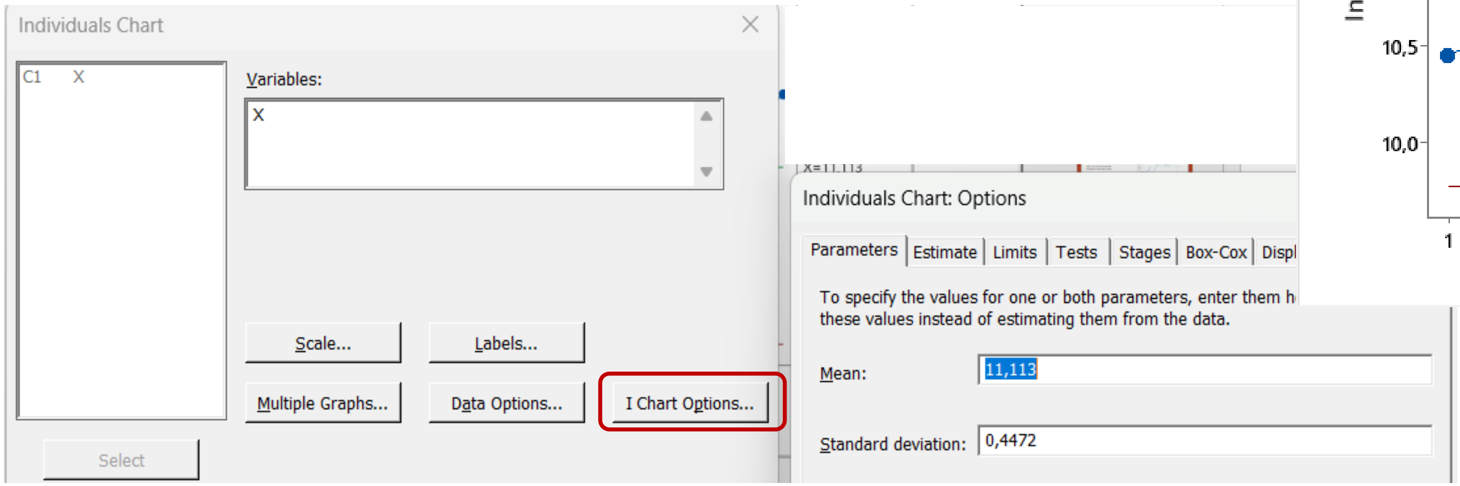
EXAMPLE 1

What happens if we design a CC for the mean (**I-chart**)?
We can compute:

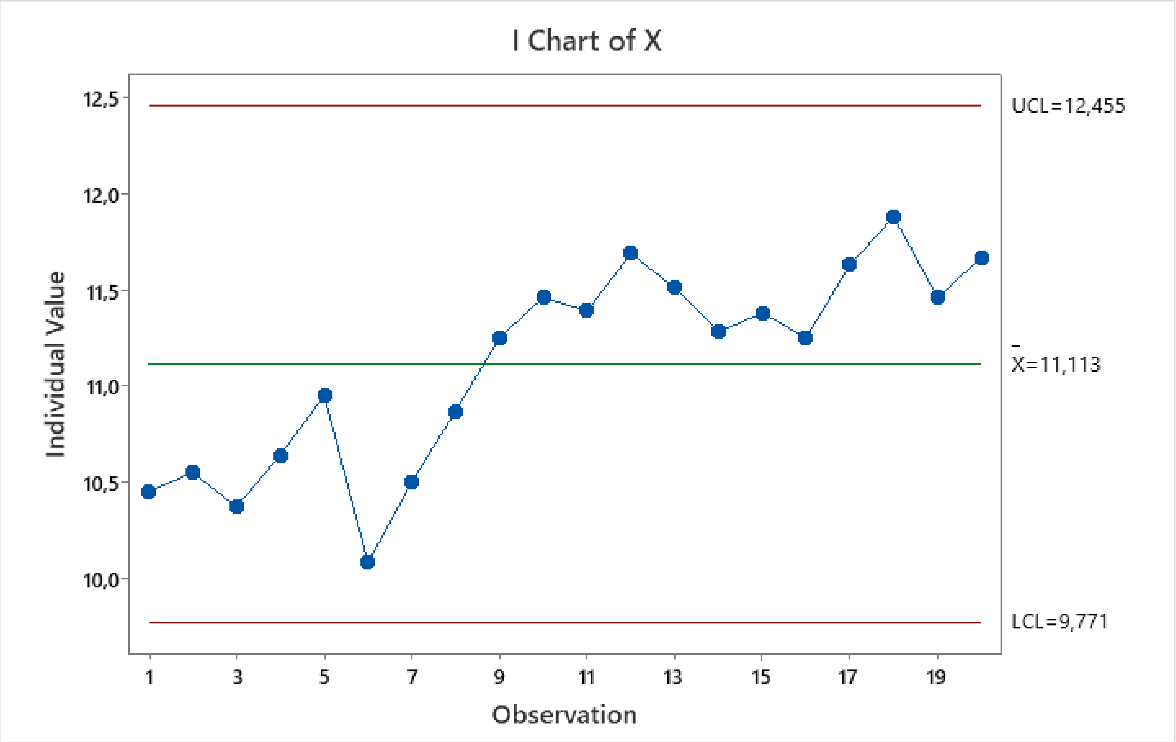
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{5}} = 0{,}4472$$

$$\mu_0 = \bar{\bar{X}} = 11{,}113$$

Then, we can compute the control limits for the I chart.



Stat → Control Charts → Variable Charts for Individuals → Individuals



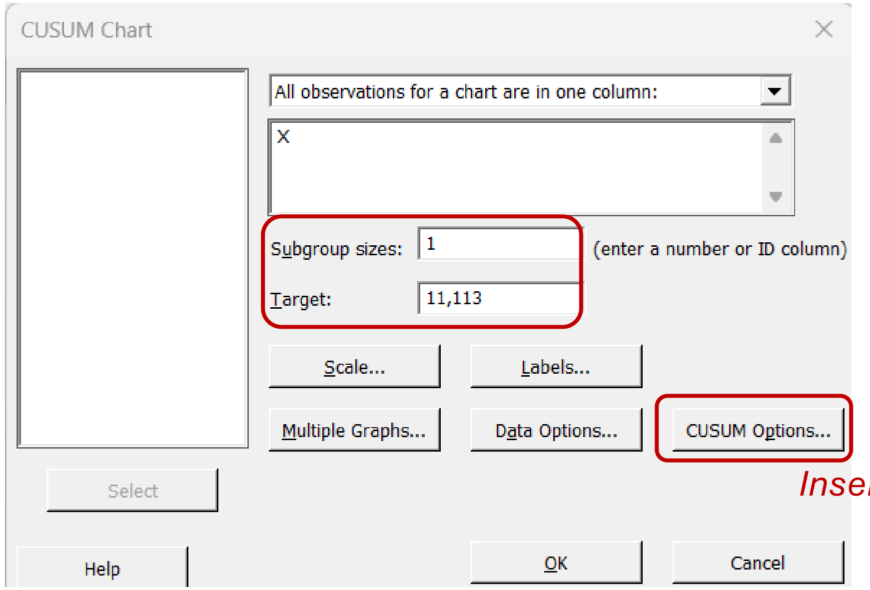
No alarms. But we may detect an OOC state based on the **systematic pattern** in the chart.

EXAMPLE 1

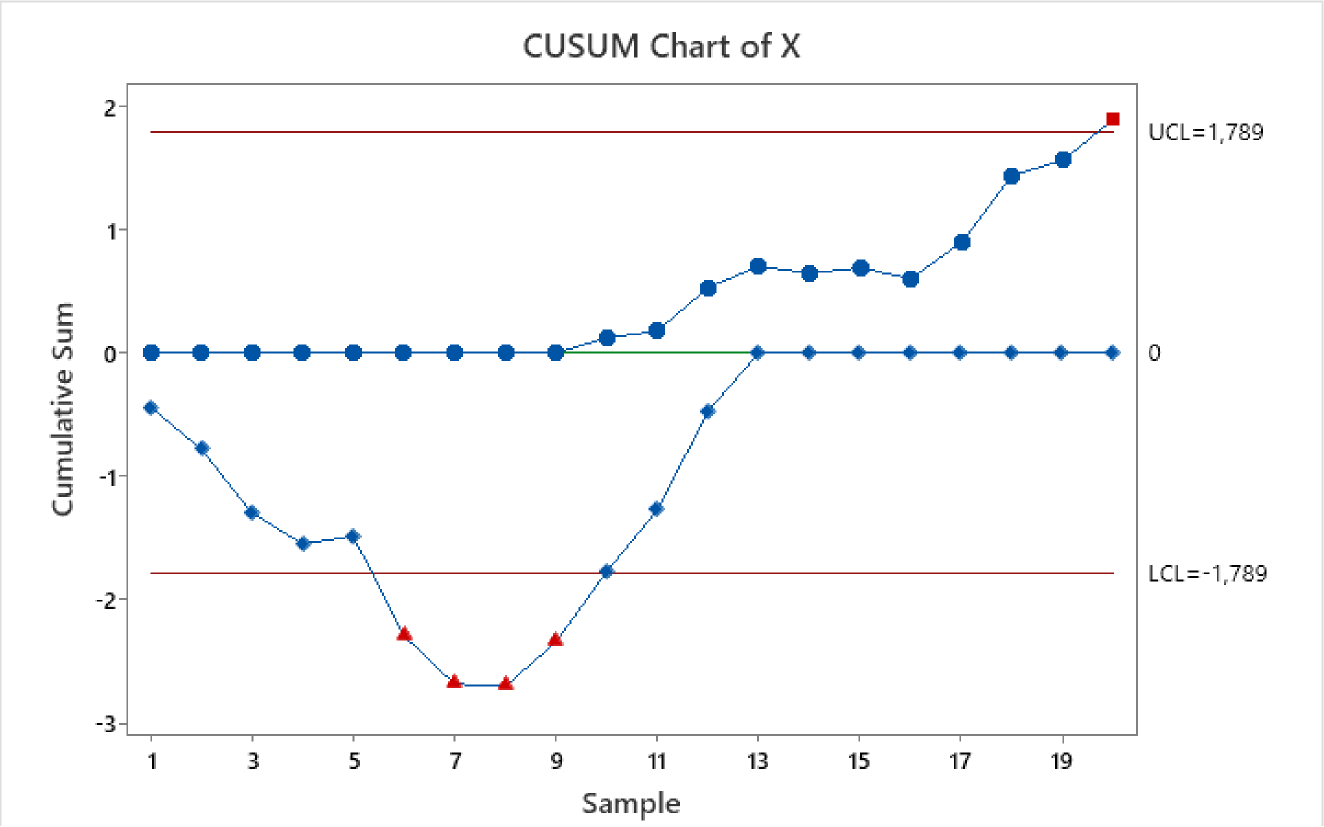
Design a CUSUM chart (with parameters h = 4 and k = 0.5) and an EWMA (with param. lambda = 0.2) and discuss the results (neglect possible non-random patterns).

CUSUM chart:

- $C_i^+ = \max(0, \bar{x}_i - (\mu_0 + K) + C_{i-1}^+)$
- $C_i^- = \max(0, (\mu_0 - K) - \bar{x}_i + C_{i-1}^-)$
- $H = h \cdot \sigma_{\bar{x}} = 4 \cdot 0.4472 = 1.7889$
- $K = k \cdot \sigma_{\bar{x}} = 0.5 \cdot 0.4472 = 0.2236$



Insert $\sigma_{\bar{x}}$



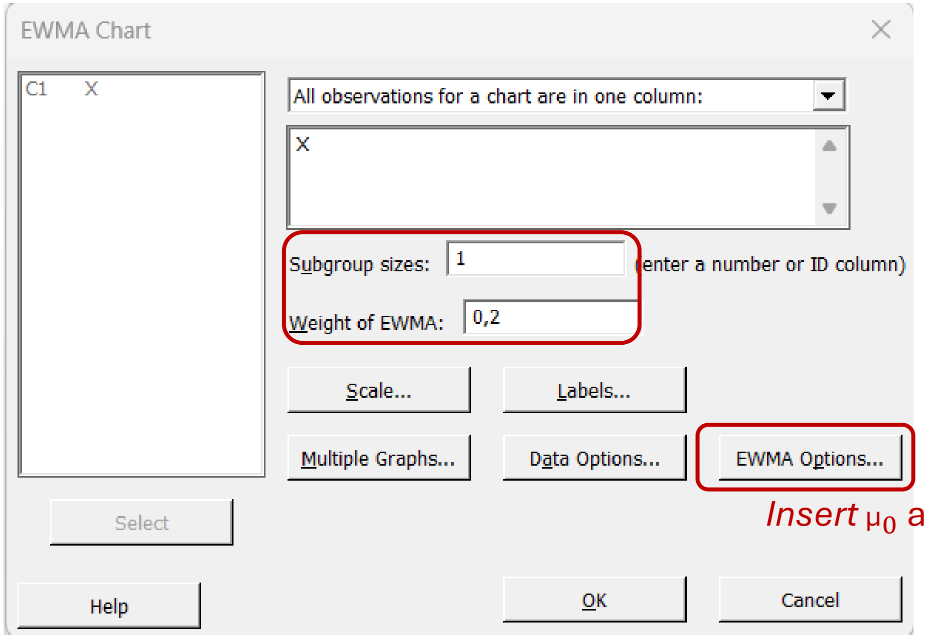
Stat → Control Charts → Time-Weighted Charts → CUSUM

EXAMPLE 1

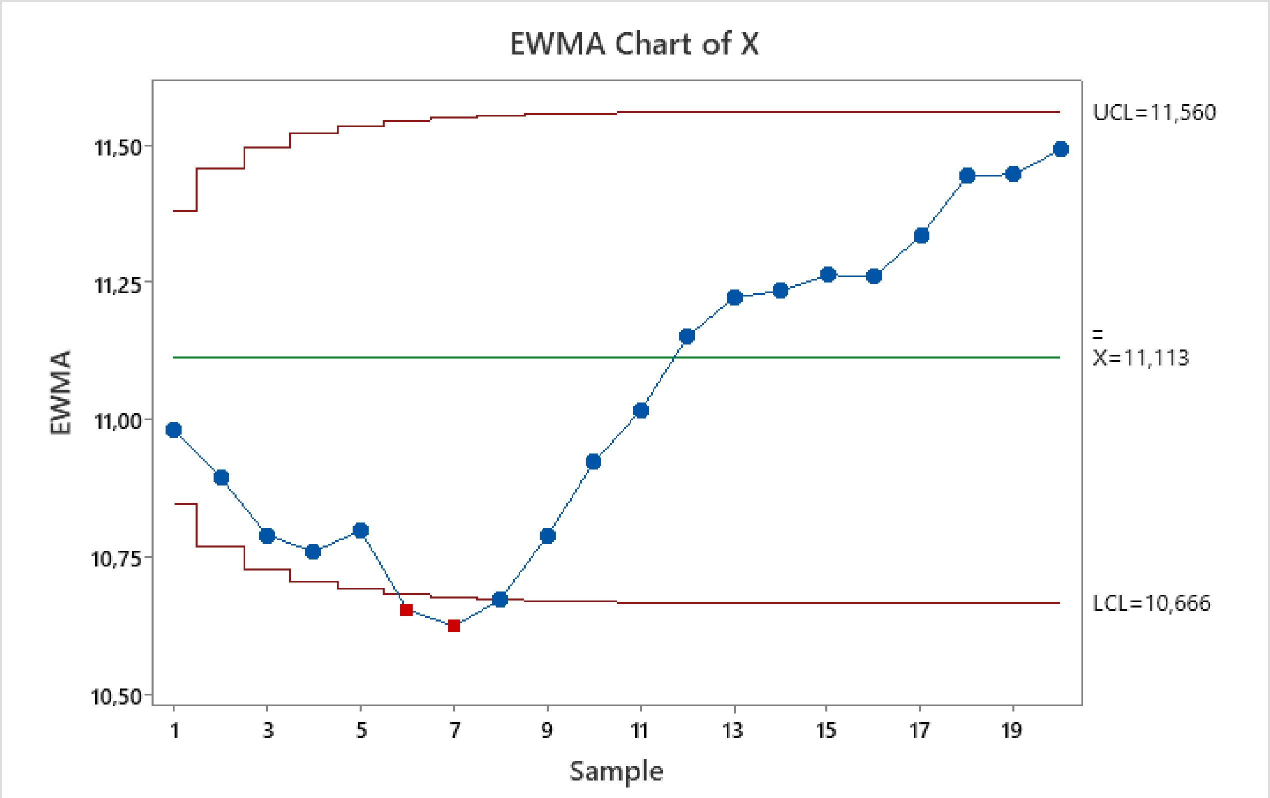
Design a CUSUM chart (with parameters h = 4 and k = 0.5) and an EWMA (with param. lambda = 0.2) and discuss the results (neglect possible non-random patterns).

EWMA chart:

- $z_0 = \bar{\bar{x}} = 11.113$
- $z_i = \lambda \cdot \bar{x}_i + (1 - \lambda) \cdot z_{i-1}$
- $a_t = \frac{\lambda}{2-\lambda} \cdot [1 - (1 - \lambda)^{2t}]$



Insert μ_0 and $\sigma_{\bar{x}}$



Stat → Control Charts → Time-Weighted Charts → EWMA

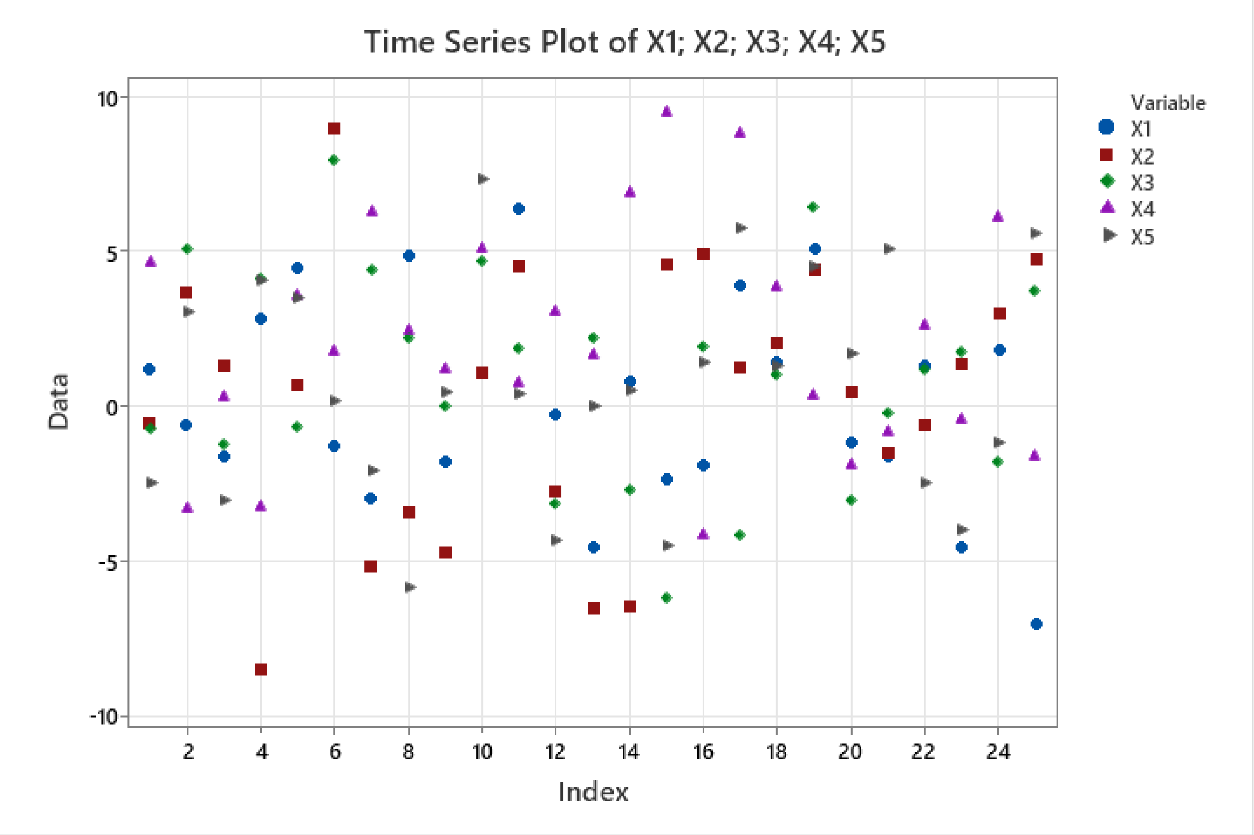
EXERCISE 2

The deviation from the nominal center-to-center distance of a piston rod is known to follow a distribution characterized by:

- ***μ = 0.4417 μm***
- ***σ = 3.4914 μm***

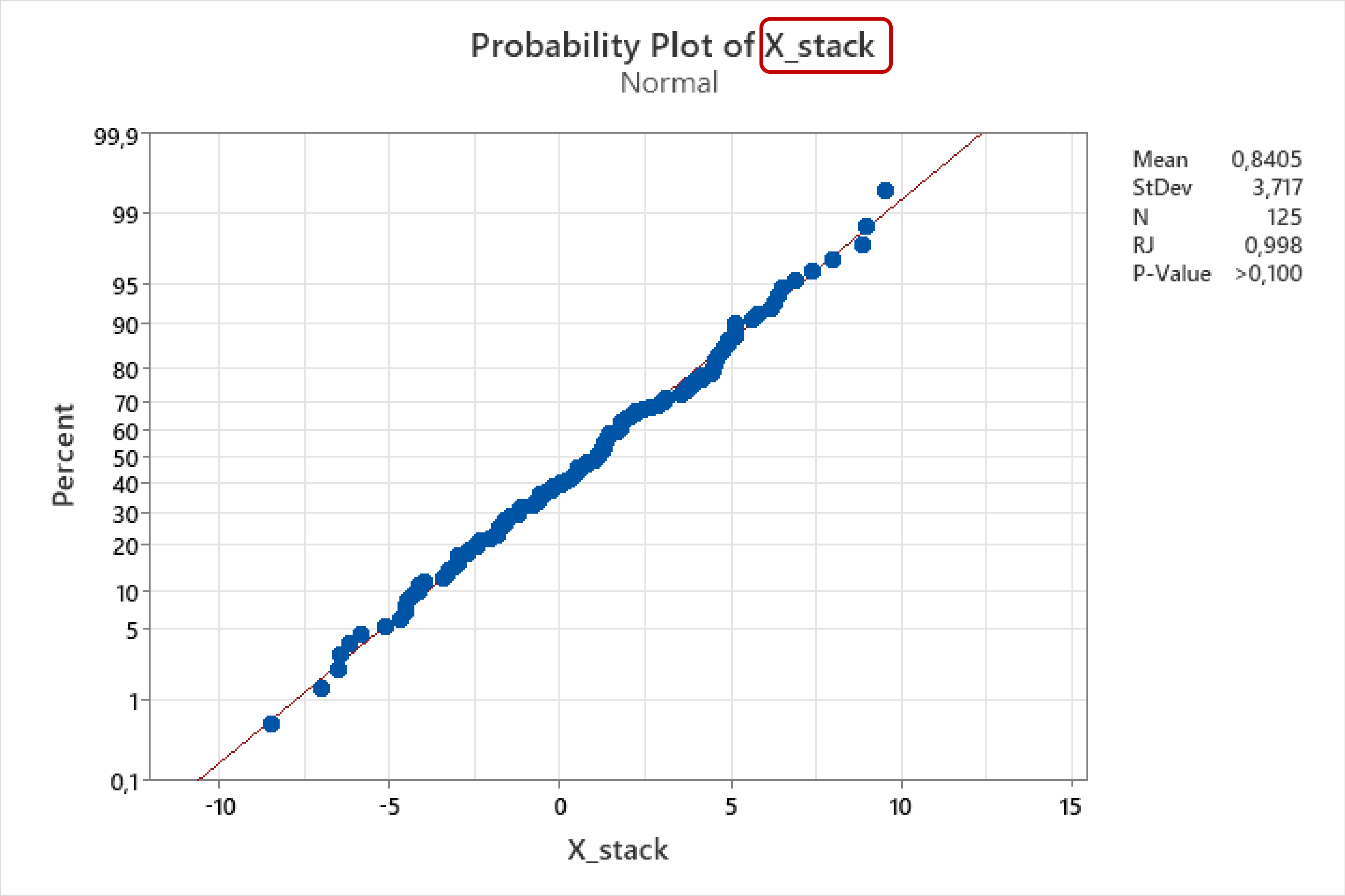
A sample of size *n = 5* is acquired on a daily basis. The measurements of 25 consecutive days are reported in the file '*small_shifts_phase1.csv*'

1. Design a Xbar-S control chart for the process.
2. Design a CUSUM control chart (*h=4, k=0.5*).
3. Design an EWMA control chart (*lambda=0.2*).
4. Import *5 additional samples* that were collected in phase 2 (file '***small_shifts_phase2***.*csv*'). Determine if the process is still in control.



Point 1 Design a **Xbar-S** control chart for the process.

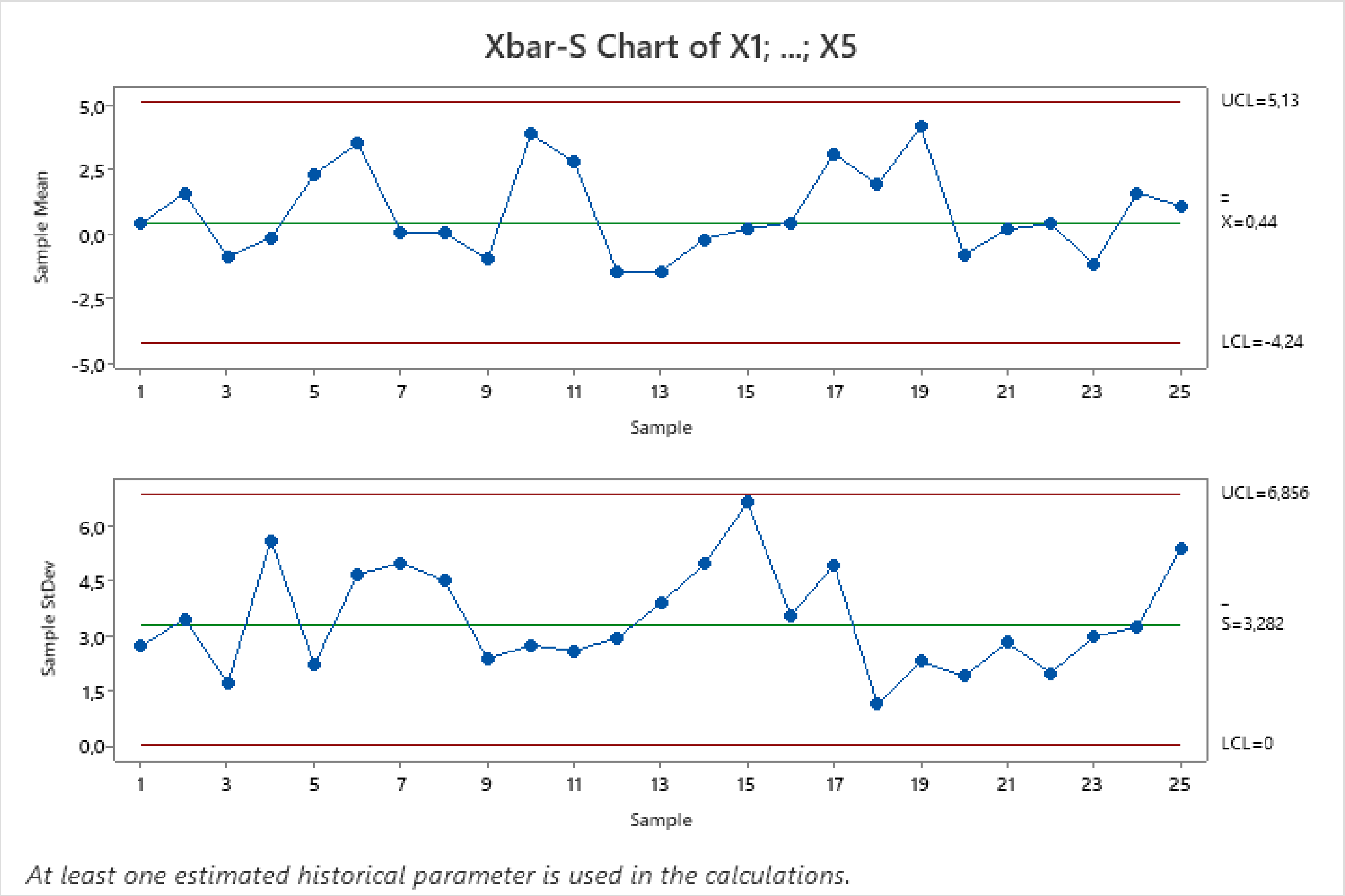
First, check if data is **normally distributed**:



No information is given about the acquisition order of the data. Randomness is only qualitatively assessed from the scatter plot.

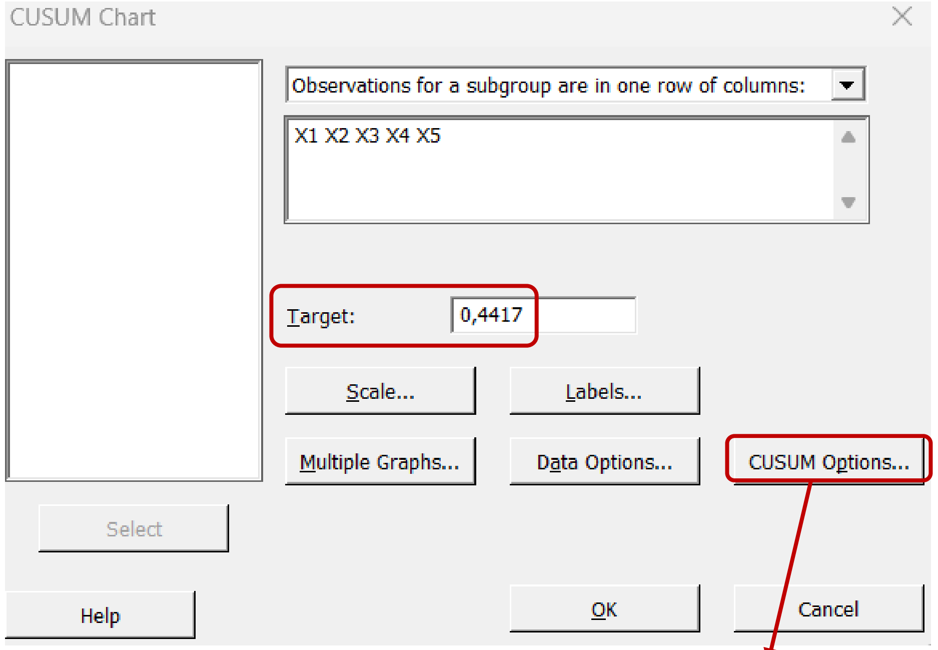
Let's design an Xbar-S control chart for the process.

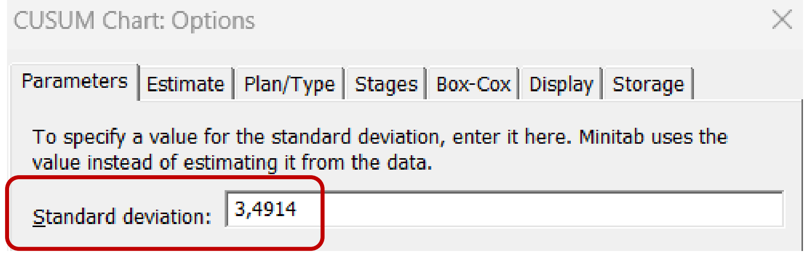
Point 1 Design a **Xbar-S** control chart for the process.

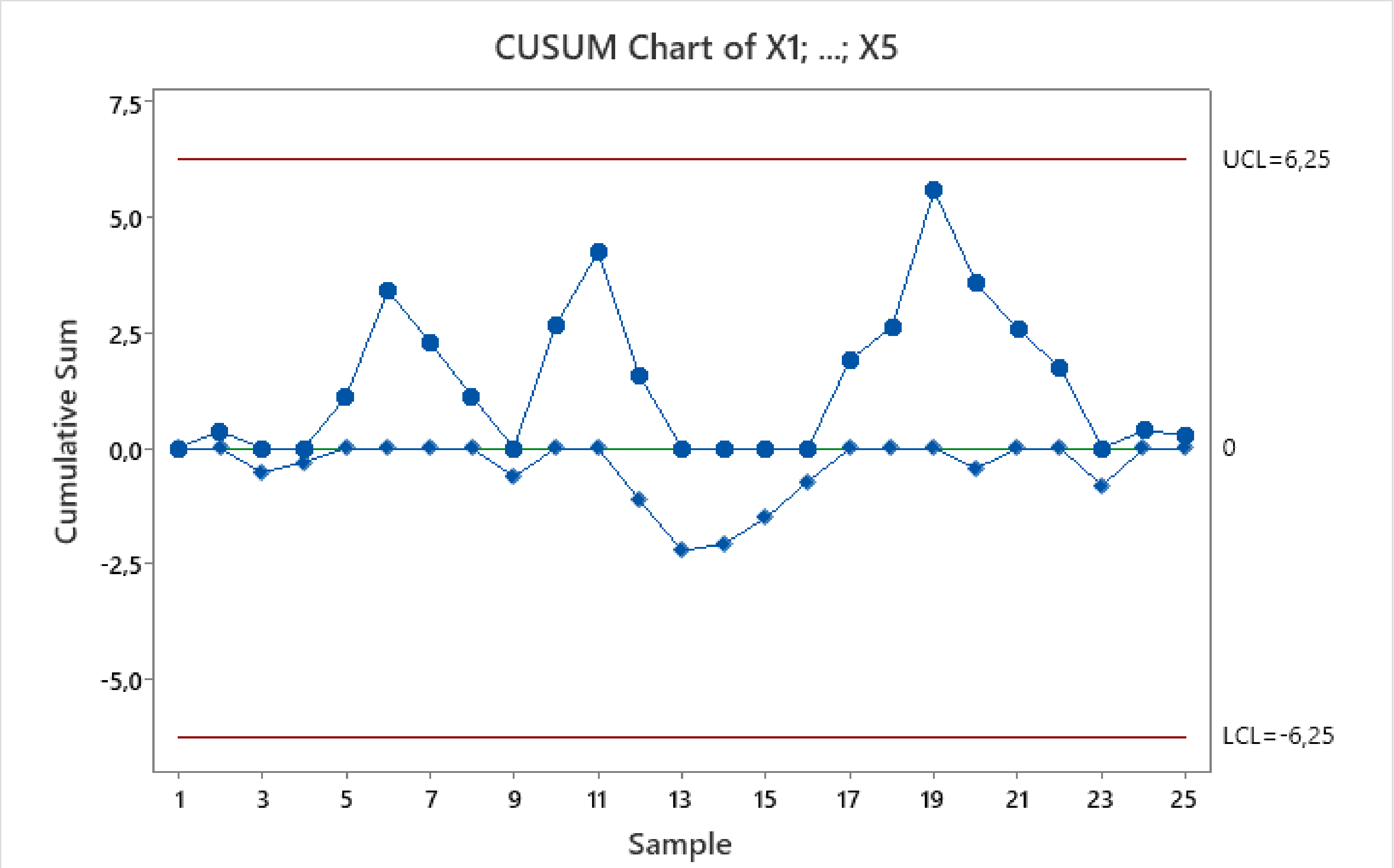


Remember: mean and standard deviation known

Point 2 Design a **CUSUM** control chart (*h=4, k=0.5*).

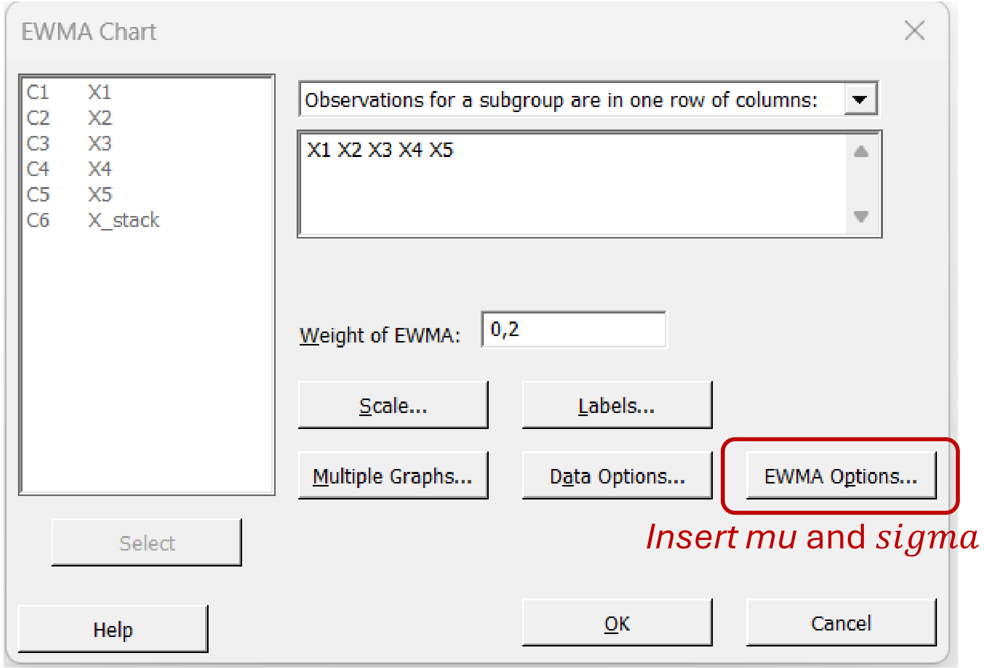


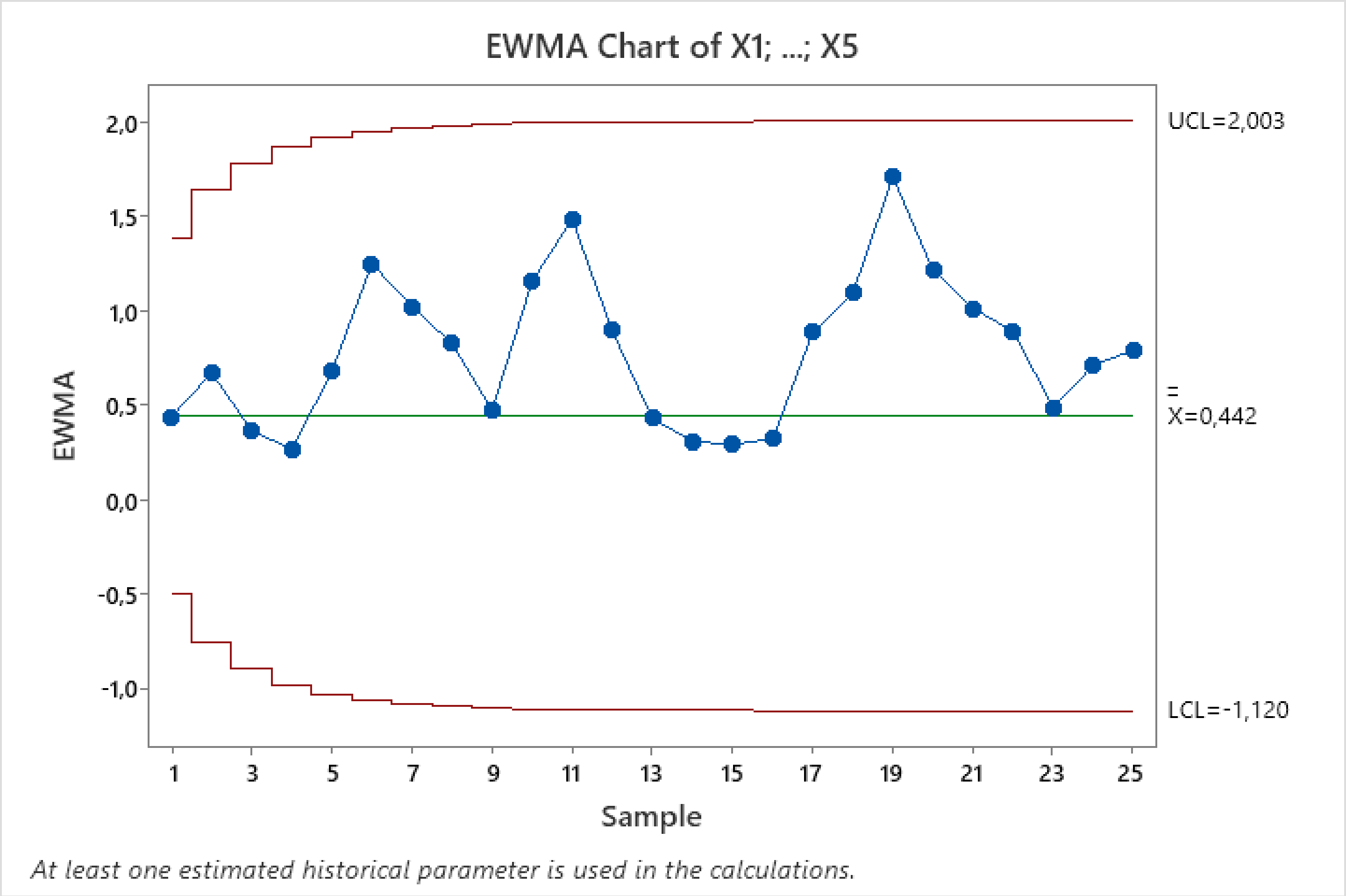




Stat → Control Charts → Time-Weighted Charts → CUSUM

Point 3 Design an **EWMA** control chart (*lambda=0.2*).

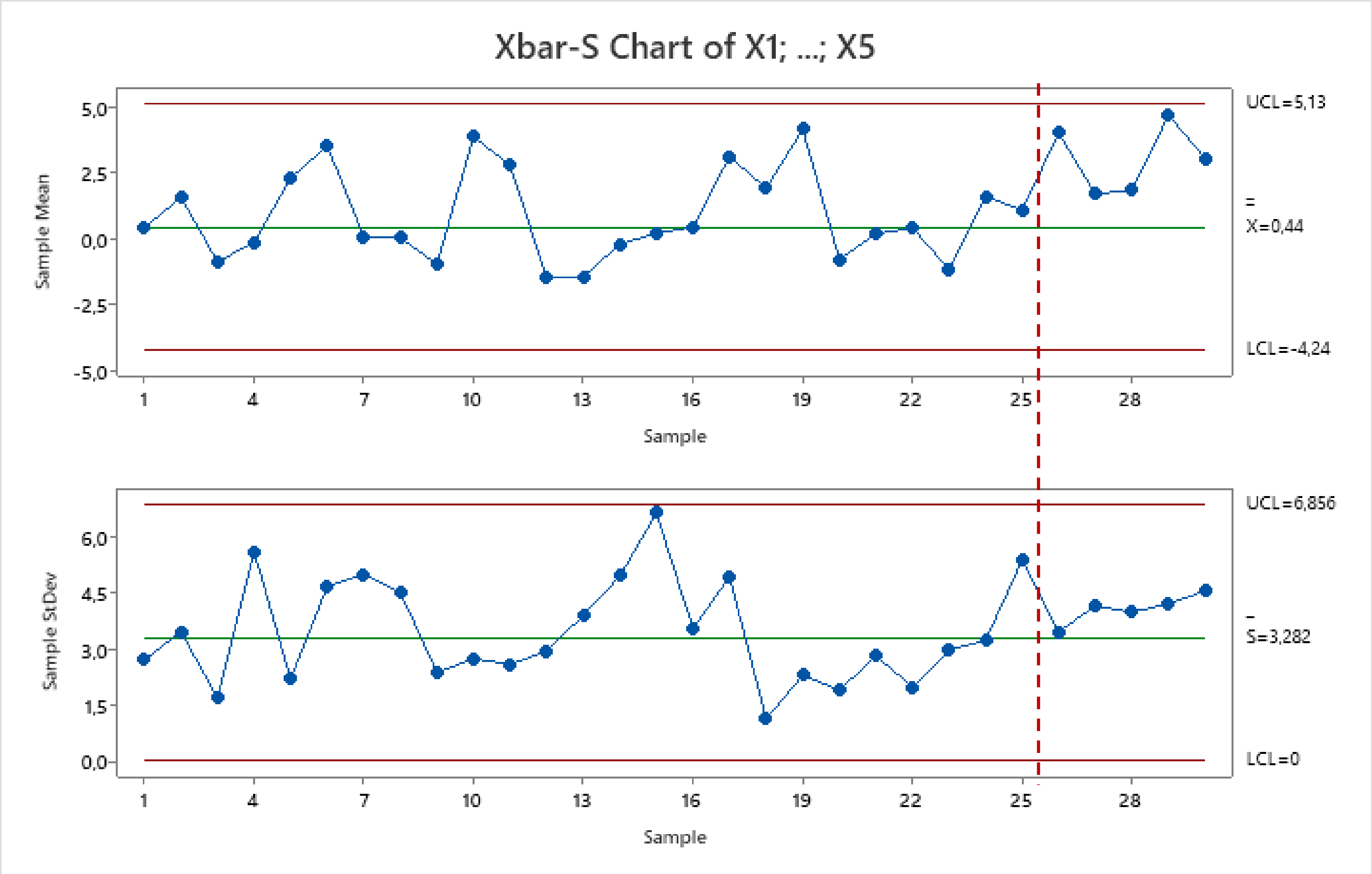




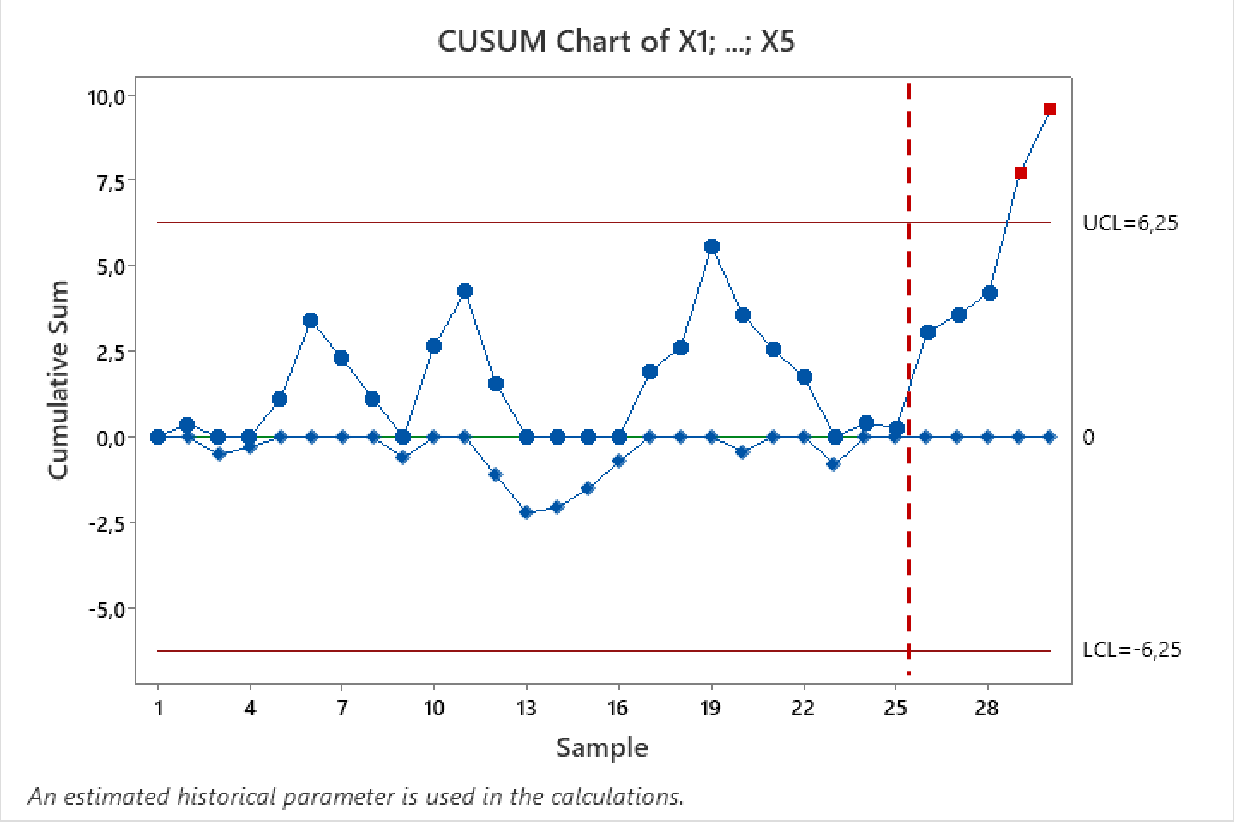
Stat → Control Charts → Time-Weighted Charts → EWMA

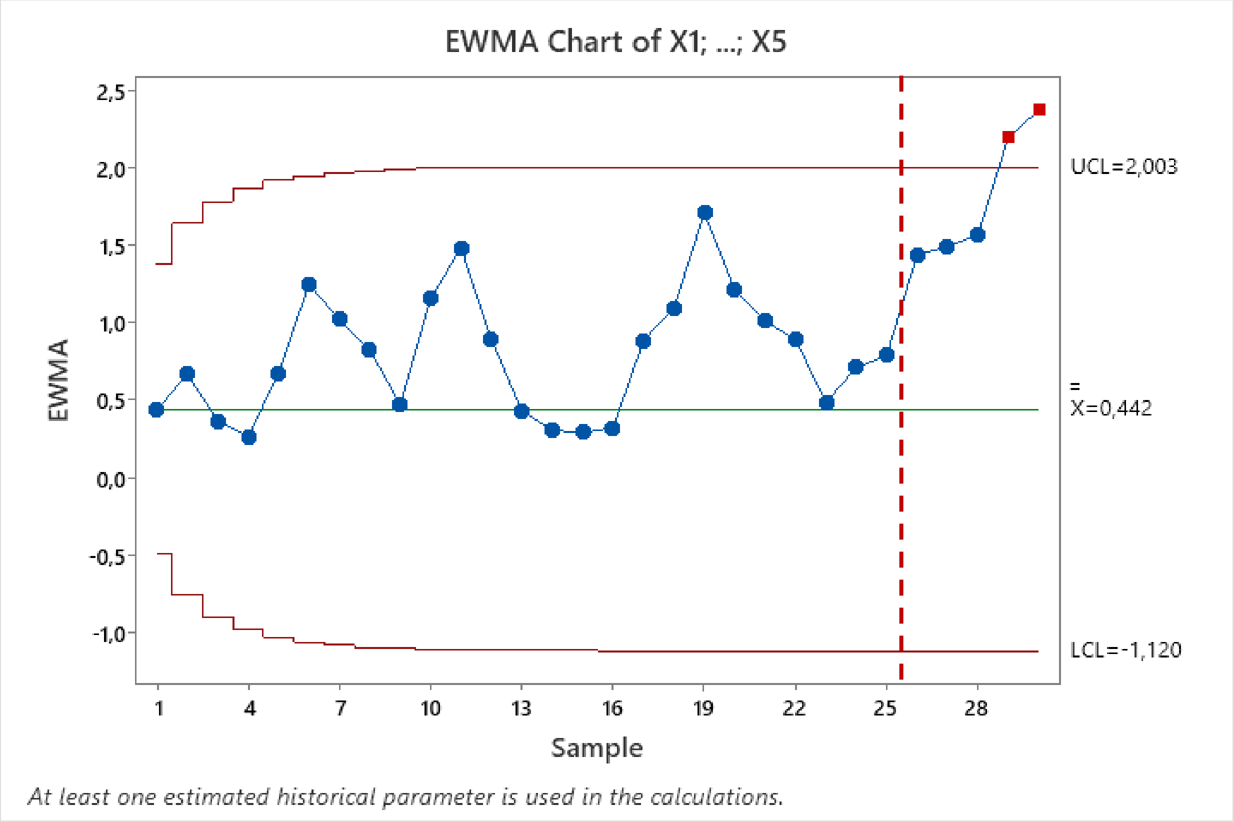
Point 4 Import *5 additional samples* that were collected in phase 2 (file '*small_shifts_phase2.csv'*). Determine if the process is still in control. In the presence of OOCs, estimate the new process mean.

Xbar – S:



Point 4 Import *5 additional samples* that were collected in phase 2 (file ‘*small_shifts_phase2.csv’*). Determine if the process is still in control. In the presence of OOCs, estimate the new process mean.

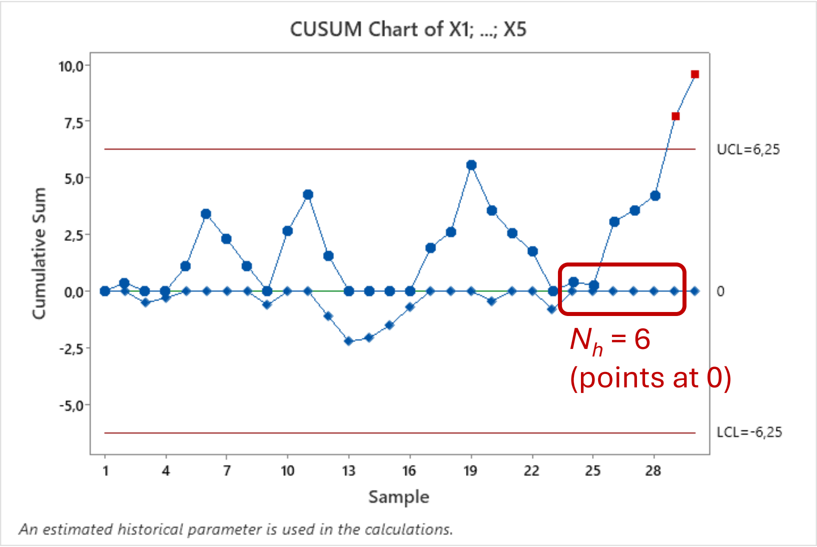




Point 4 Import *5 additional samples* that were collected in phase 2 (file '*small_shifts_phase2.csv'*). Determine if the process is still in control. **In the presence of OOCs, estimate the new process mean.**

The new estimated process mean is calculated by the following formula: $\hat{\mu} = \mu_0 + K + \frac{C_i^+}{N_h}$

Where N_h is the number of consecutive non-zero values of the upper cumulator when the first OOC is detected:



- $\mu_0 = 0.4417$
- $K = k * \sigma_{\bar{x}} = k * \frac{\sigma}{\sqrt{n}} = 0.5 * \frac{3.4914}{\sqrt{5}} = 0.78$
- $N_h = 6$
- $C_i^+ = \max(0, \bar{x}_i - (\mu_0 + K) + C_{i-1}^+)$ = 7.707

$\longrightarrow$ $\hat{\mu} = 2.507$

Attributes Control Charts

Monitoring Discrete & Categorical data

The basic control charts for attributes used in practice are:

- $\boldsymbol{p}$**-chart**: monitor the fraction nonconforming
- $\boldsymbol{np}$**-chart**: monitor the number of nonconforming

Non-conforming units

- $\boldsymbol{c}$**-chart**: monitor the number of defects per unit
- $\boldsymbol{u}$**-chart**: monitor the average number of defects per unit

Defects per units

→ $\boldsymbol{p}$ & $\boldsymbol{np}$ charts make use of the Binomial distribution, while
→ $\boldsymbol{c}$ & $\boldsymbol{u}$ charts employ the Poisson distribution.

The attribute control chart construction relies on the same (Shewhart) principles that we had for the continuous variables.

p and np control charts (known parameters)

For the p-**chart** we will have (typically, $L = 3$) :

$$
\begin{aligned}
LCL &= p - L\sqrt{\frac{p(1-p)}{n}} \\
CL &= p \\
UCL &= p + L\sqrt{\frac{p(1-p)}{n}}
\end{aligned}
$$

For the np-**chart** we will have (typically, $L = 3$)

$$
\begin{aligned}
LCL &= np - L\sqrt{np(1-p)} \\
CL &= np \\
UCL &= np + L\sqrt{np(1-p)}
\end{aligned}
$$

p and np control charts (unknown parameters)

- For unknown parameter settings, we estimate p using $\overline{p}$ from a phase I sample of size m, where:
- For each sample $i = 1,2,\ldots,m$ of size n each, we observe D_i non-conforming, and:

$$\hat{p}_i = \frac{D_i}{n}$$

- The average of these individual sample fractions non-conforming is:

$$\overline{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = \frac{\sum_{i=1}^{n} D_i}{mn}$$

- When p is too small/large (rare/almost sure) events, then n needs to be large.

EXERCISE 1

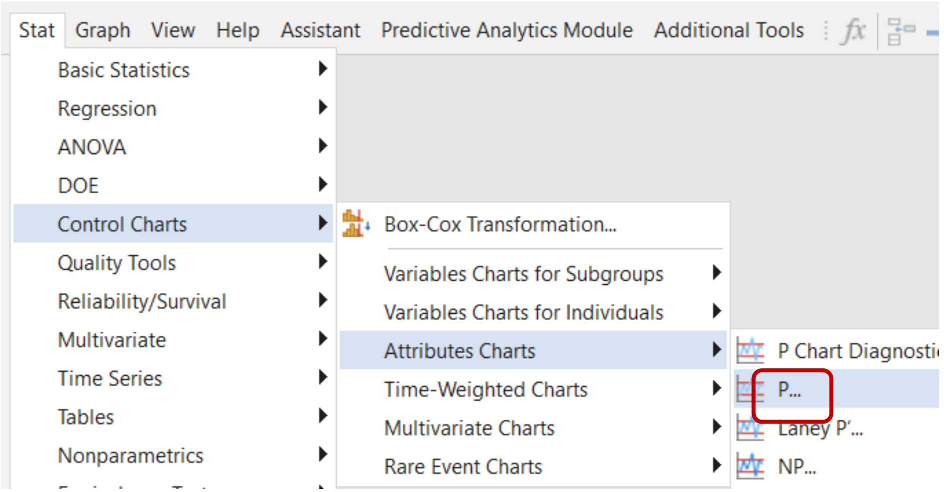
The supervisor for a call center wants to evaluate the process for answering customer phone calls. The supervisor records in the file *UnansweredCalls.csv* the total number of incoming calls and the number of unanswered calls for 21 days.

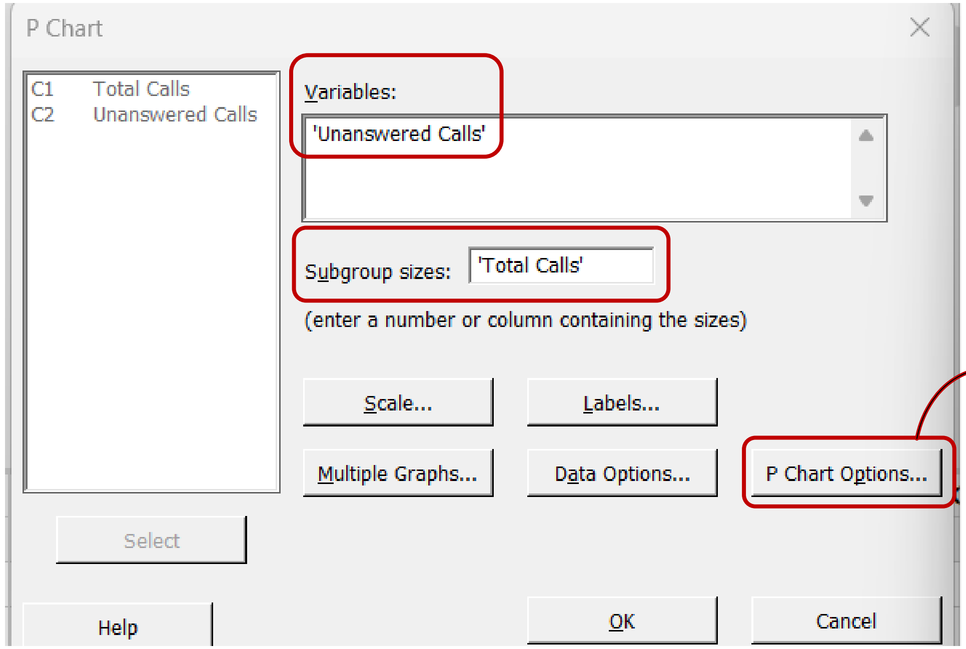
Create a p-chart to monitor the proportion of unanswered calls.

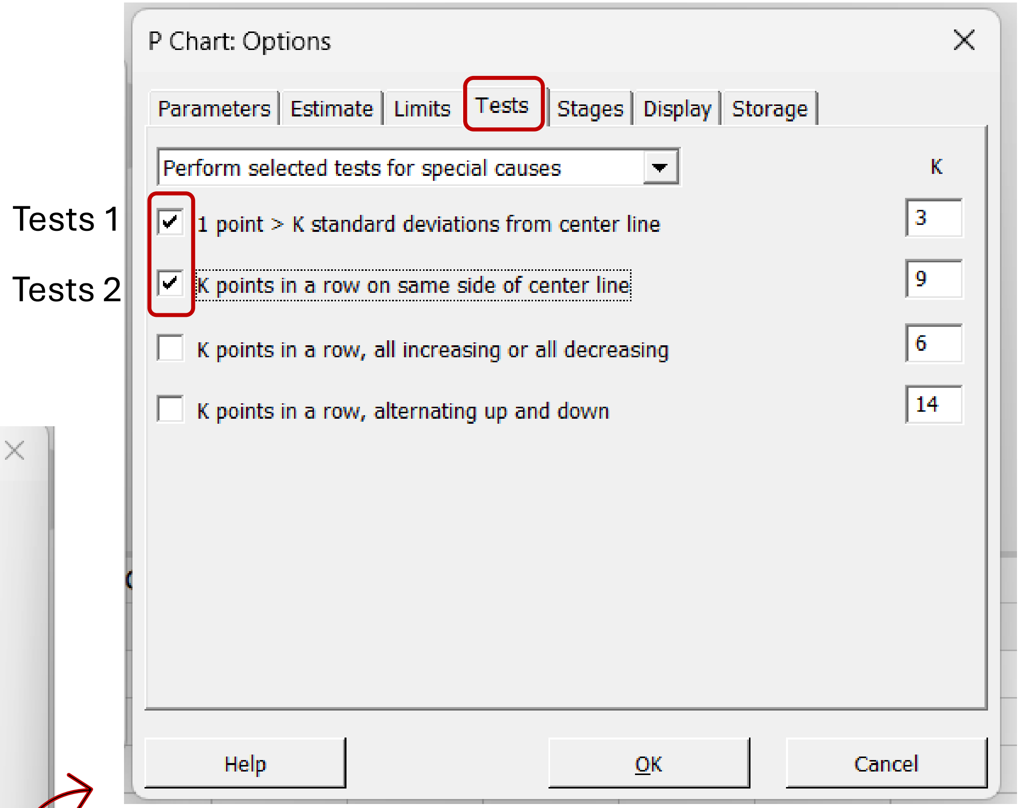
EXERCISE 1

Create a p-chart to monitor the proportion of unanswered calls.

Stat → Control Charts → Attributes Chart → P



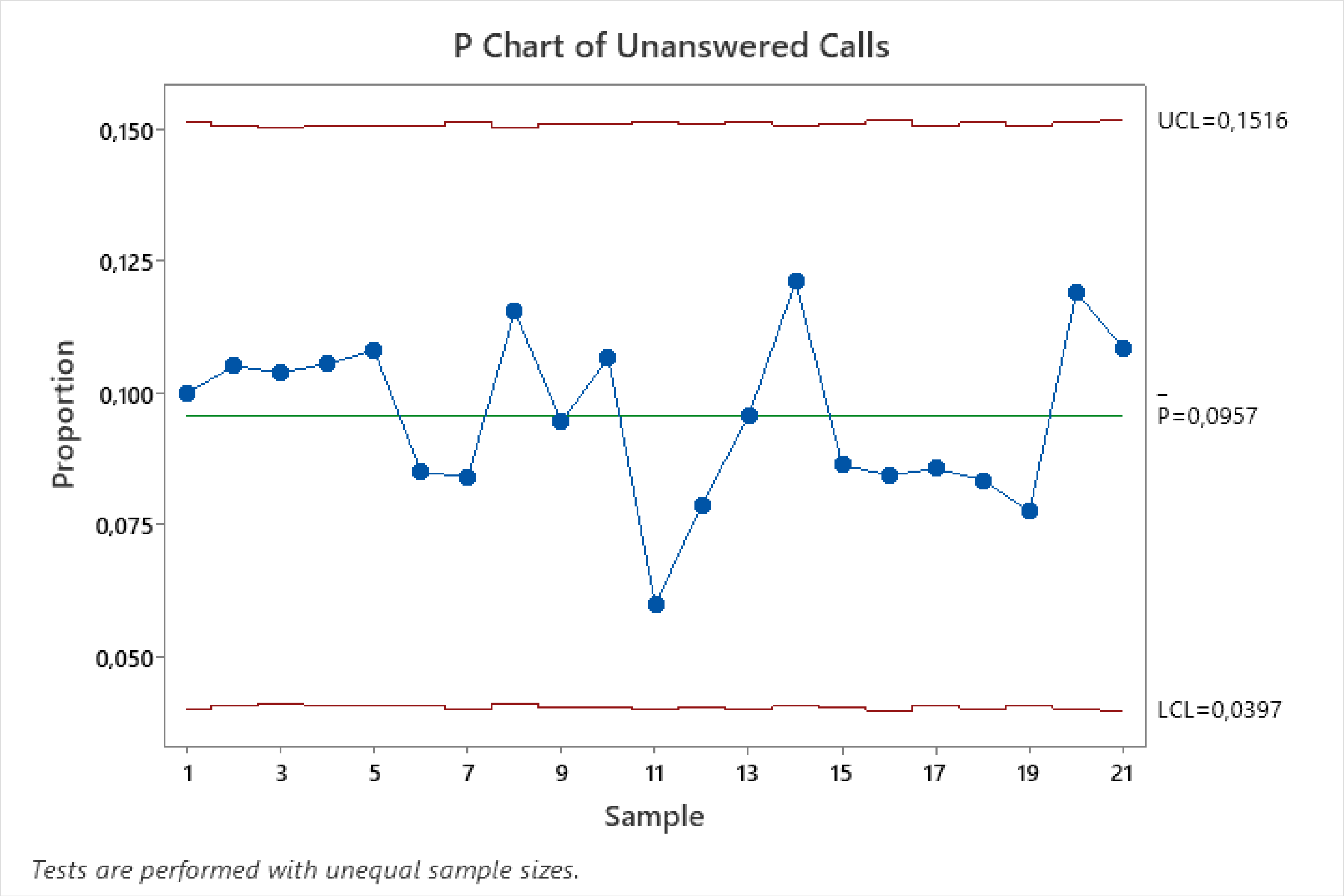




If not sure which tests apply in a specific situation, use Tests 1 and 2 when you first establish the control limits

EXERCISE 1

Create a p-chart to monitor the proportion of unanswered calls.



The chart shows that, on average, 9.57% of calls are unanswered.

None of the subgroup proportions are outside of the control limits. Furthermore, the points inside the limits display a random pattern.

This P chart does not provide any evidence for lack of control. Thus, the process is in control.

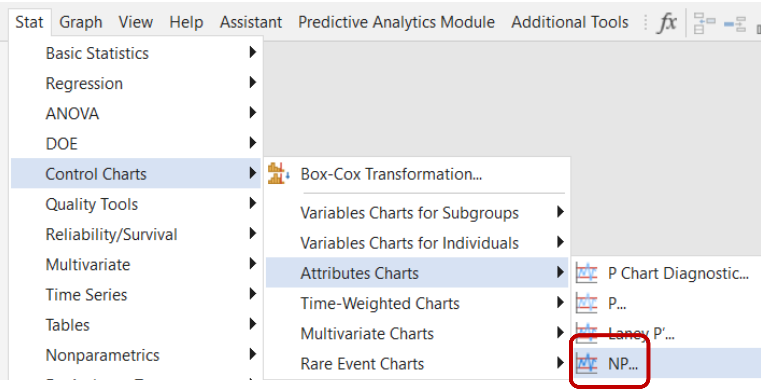
EXERCISE 2

A quality engineer assesses whether the process used to manufacture light bulbs is in control. The engineer tests 500 light bulbs each hour for three 8-hour shifts and records the number of bulbs that did not light (defectives) in the file *LightBulbs.csv*.

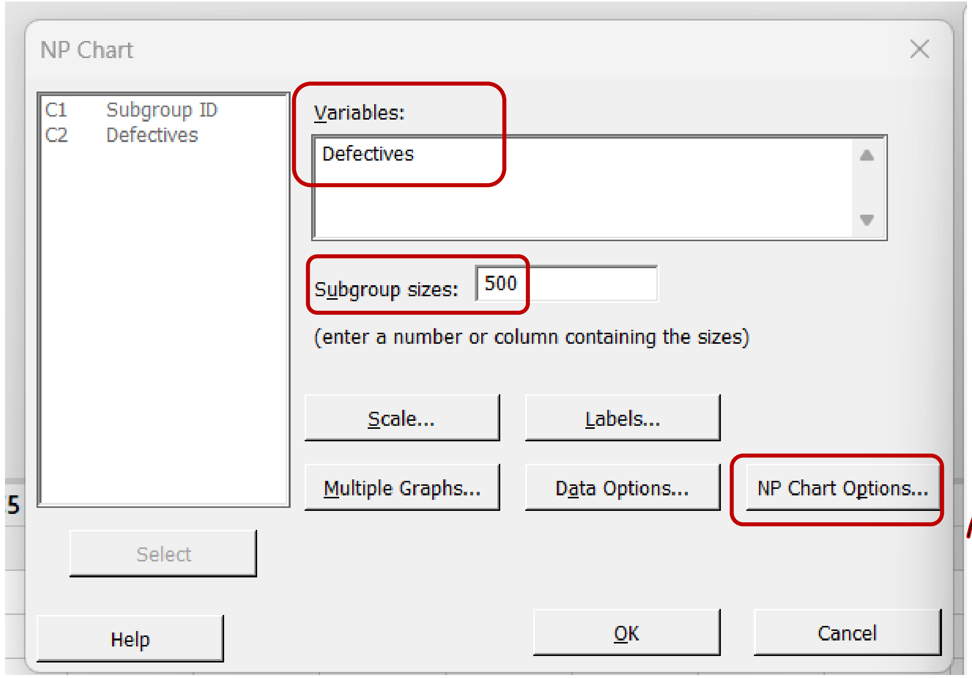
Create an NP chart to monitor the number of defective light bulbs

EXERCISE 2

Create an NP chart to monitor the number of defective light bulbs

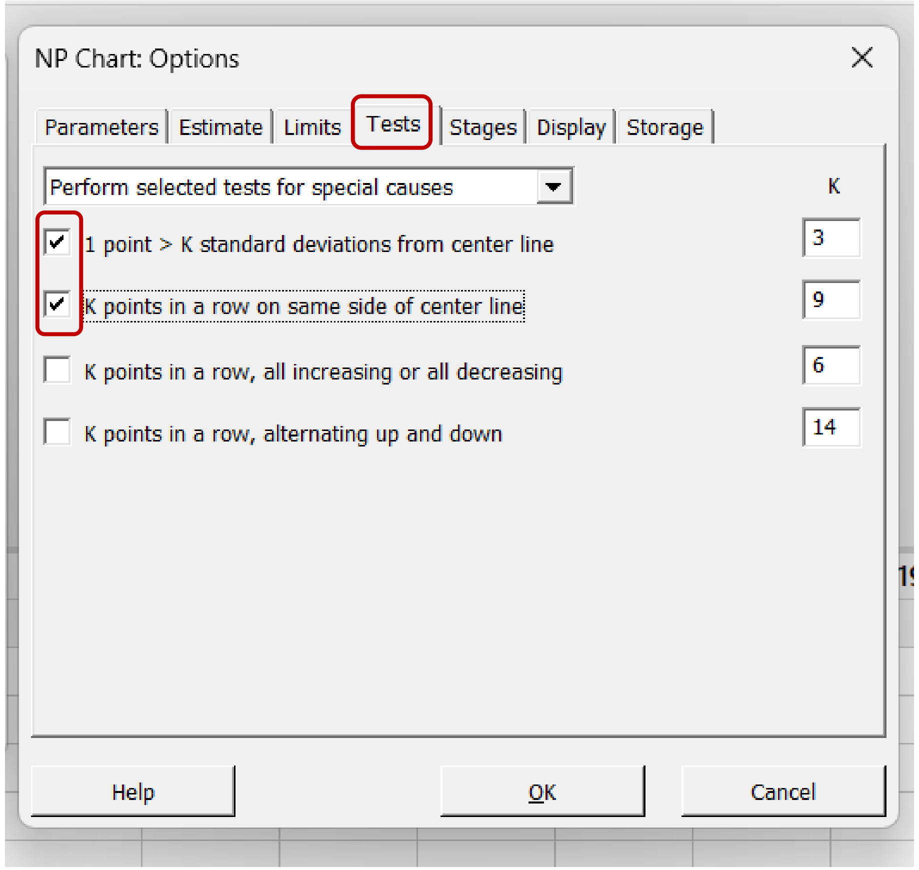


Stat → Control Charts → Attributes Chart → NP



Tests 1

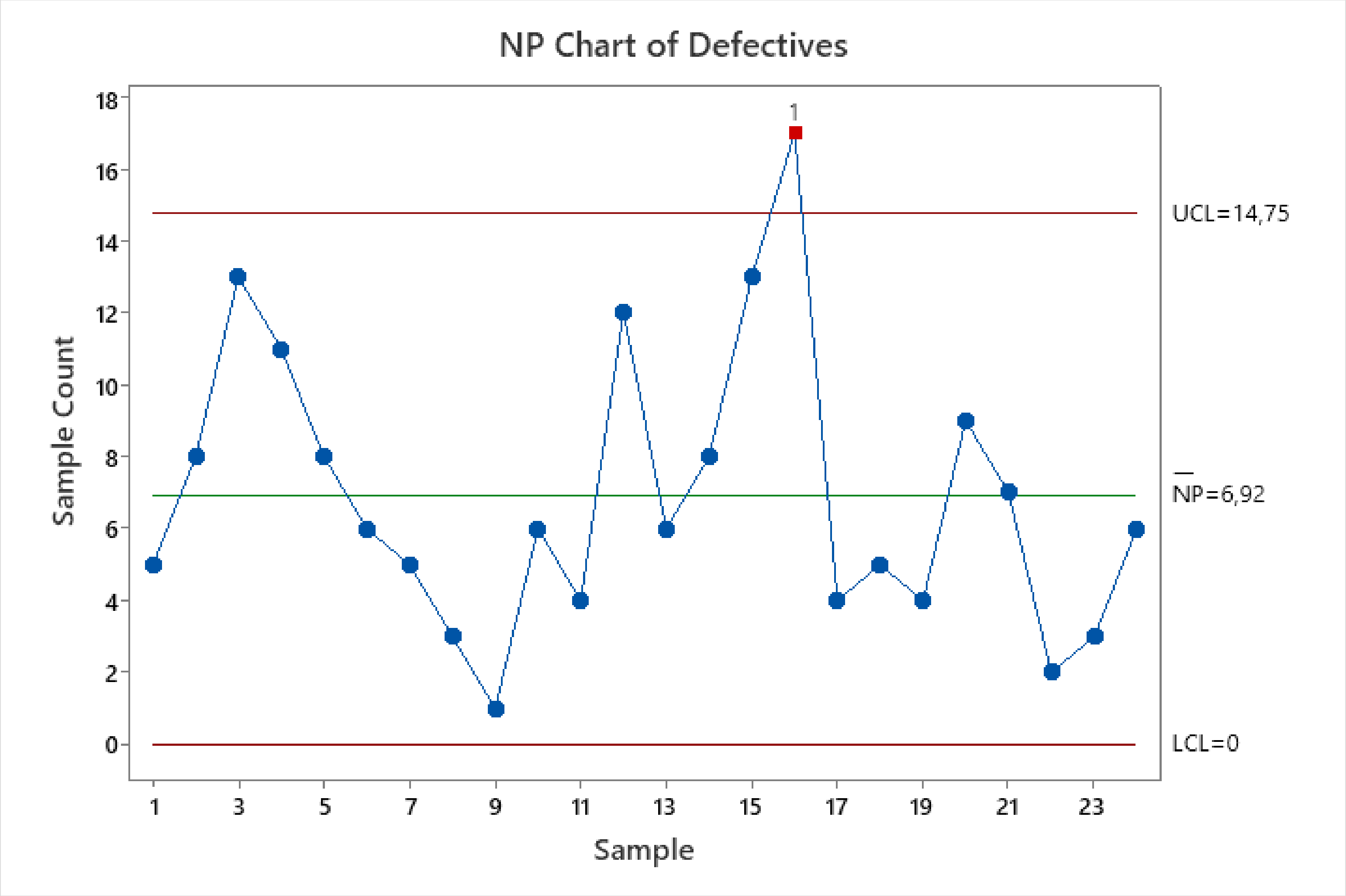
Tests 2



If you are not sure which tests apply in a specific situation, use Tests 1 and 2 when you first establish the control limits

EXERCISE 2

Create an NP chart to monitor the number of defective light bulbs



One point is out of control.
It can be concluded that the process is not stable and should be improved.

c control charts

- When the c parameter is known, we will have (typically, $L = 3$)

$$\begin{aligned} LCL &= c - L\sqrt{c} \\ CL &= c \\ UCL &= c + L\sqrt{c} \end{aligned}$$

- When c is unknown we will estimate it via $\bar{c}$, from a phase I sample of size m (i.e. average number of non-conformities out of samples m. Then we will have (typically, $L = 3$)

$$\begin{aligned} LCL &= \bar{c} - L\sqrt{\bar{c}} \\ CL &= \bar{c} \\ UCL &= \bar{c} + L\sqrt{\bar{c}} \end{aligned}$$

- When the c is small, probability-based control limits are preferable.

u control charts

- In certain cases, we do not monitor a single inspection unit but several. Then we can employ a control chart based on the **average** number of nonconformities per inspection unit.
- If we find x total nonconformities in a sample of n inspection units, then the average number of nonconformities per inspection unit is:

$$u = \frac{x}{n}$$

- If $\overline{u}$ is a phase I based estimate we will have (typically, $L = 3$):

$$LCL = \overline{u} - L\sqrt{\overline{u}/_n}$$
$$CL \;\; = \; \overline{u}$$
$$UCL = \overline{u} + L\sqrt{\overline{u}/_n}$$

EXERCISE 3

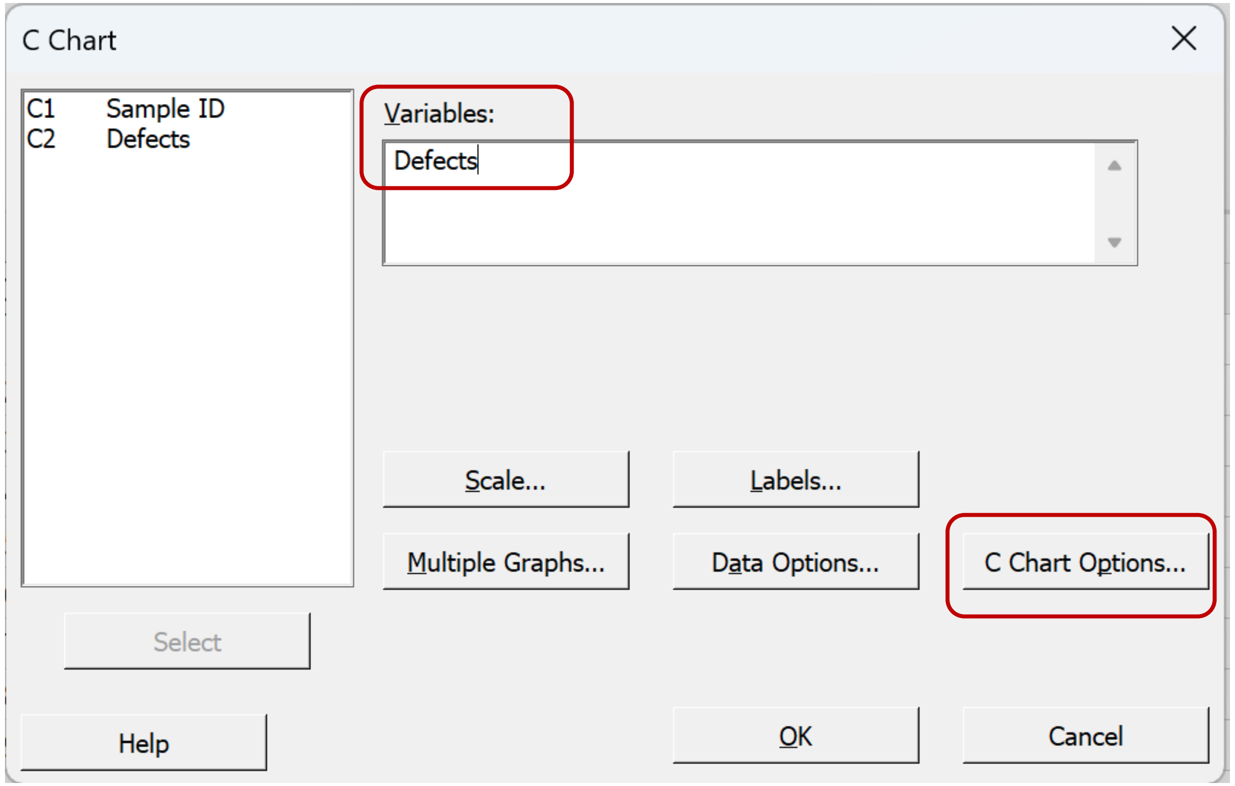
A quality engineer for a wallpaper manufacturer wants to assess the stability of the printing process. Every hour, the engineer takes a sample of 100 feet of wallpaper and counts the number of printing defects, which include print smears, pattern distortions, and missing ink. The collected data are reported in *WallpaperDefects.csv*

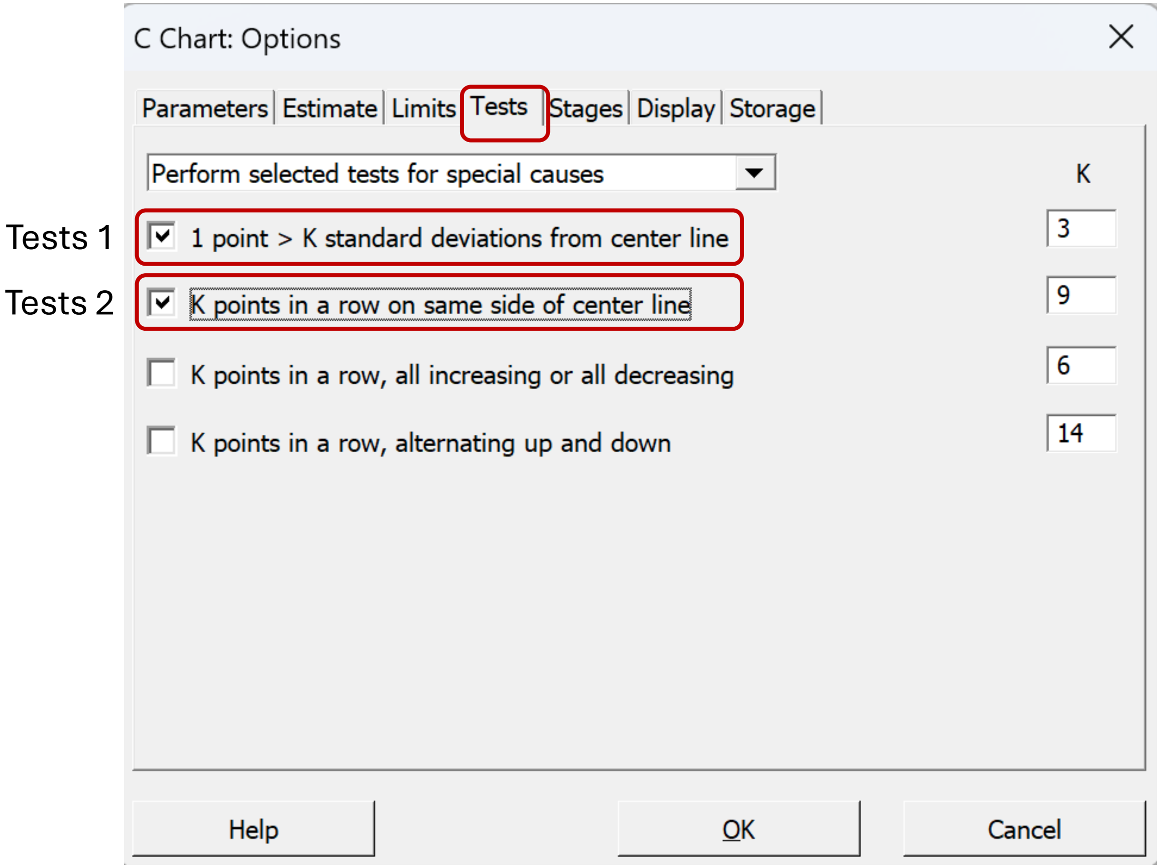
Design a C chart to monitor the number of defects.

EXERCISE 3

Design a C chart to monitor the number of defects.

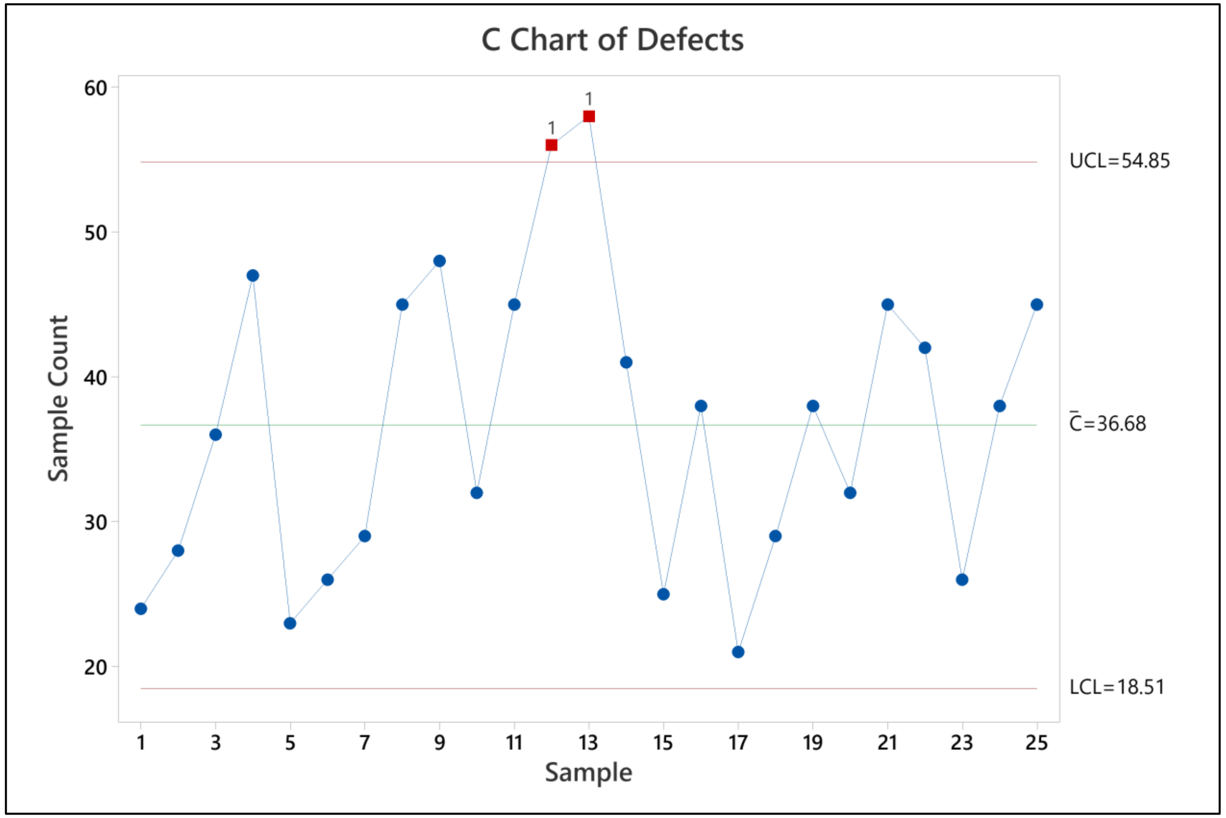
Stat → Control Charts → Attributes Charts → C





EXERCISE 3

Design a C chart to monitor the number of defects.



The average number of defects per sample is 36.68. **Samples 12 and 13 failed Test 1 because they are outside the control limits.** Thus, the process is out of control.

EXERCISE 4

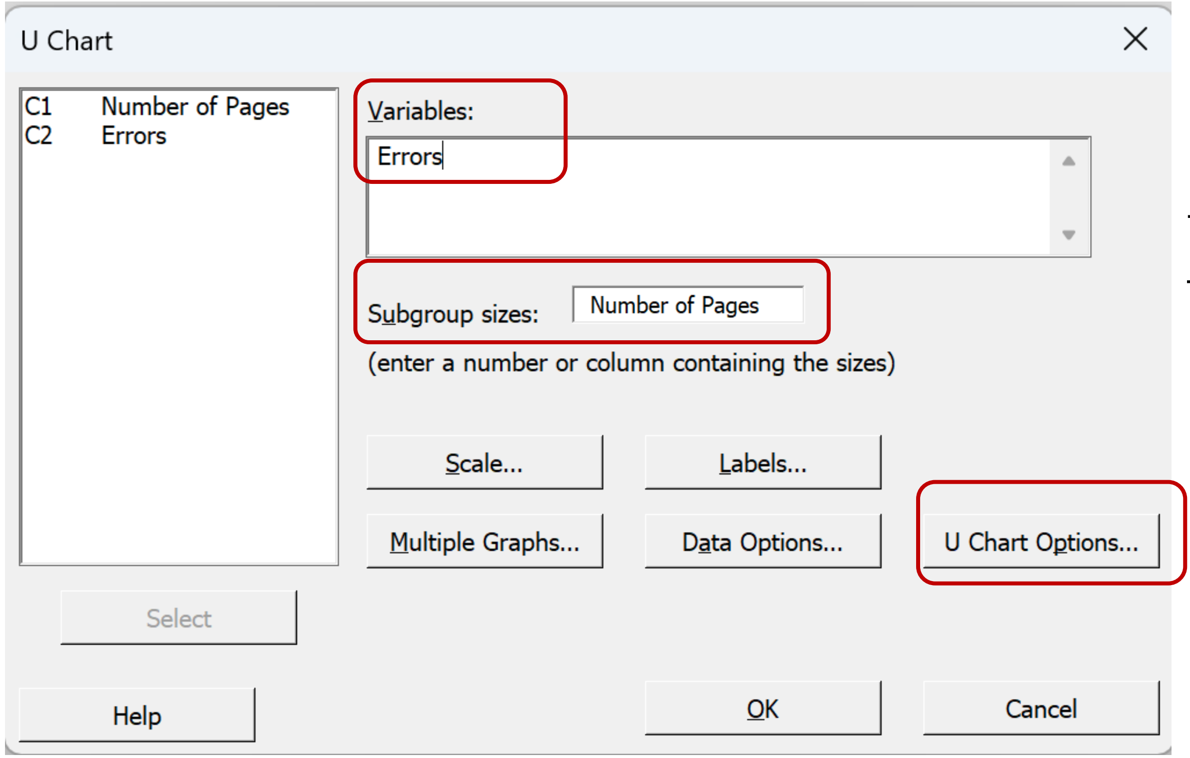
A manager for a transcription company wants to assess the quality of the transcription service. The manager randomly selects 25 sets of pages from consecutive orders and counts the number of typographical errors (defects). Each set has a different number of pages. Data are stored in *TranscriptionErrors.csv*

Design a u chart to monitor the number of errors.

EXERCISE 4

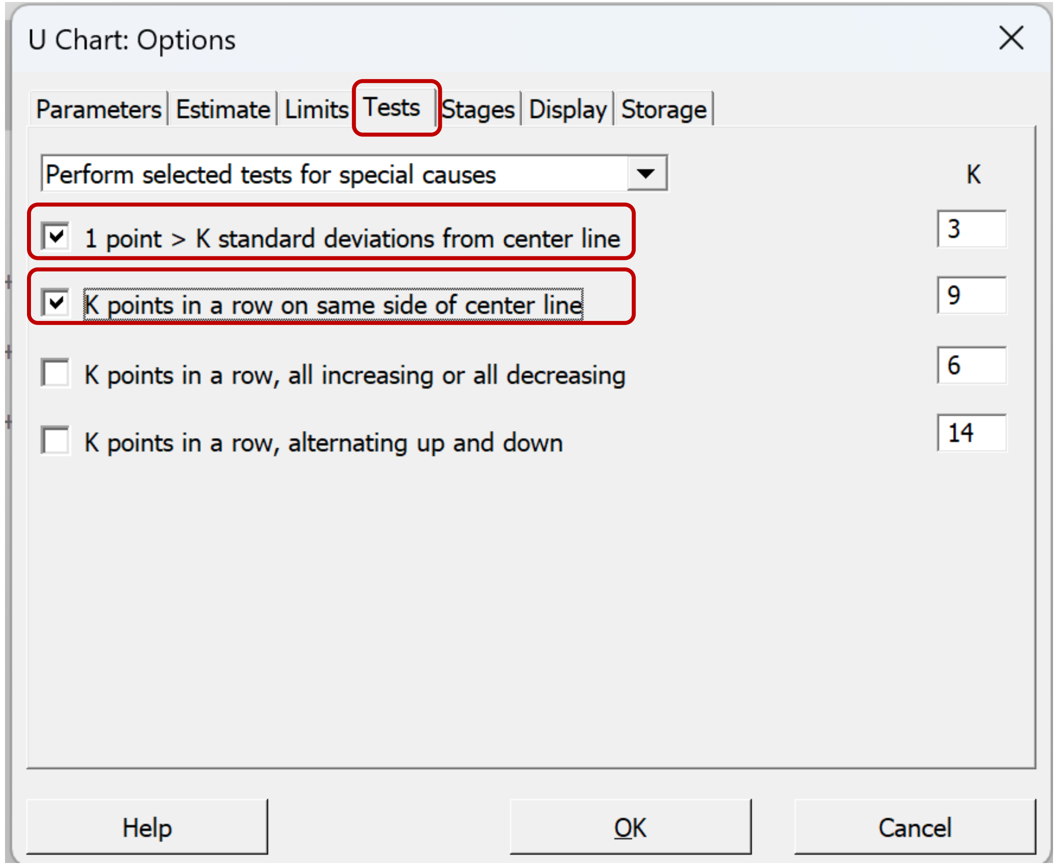
Design a u chart to monitor the number of errors.

Stat → Control Charts → Attributes Charts → U



Tests 1

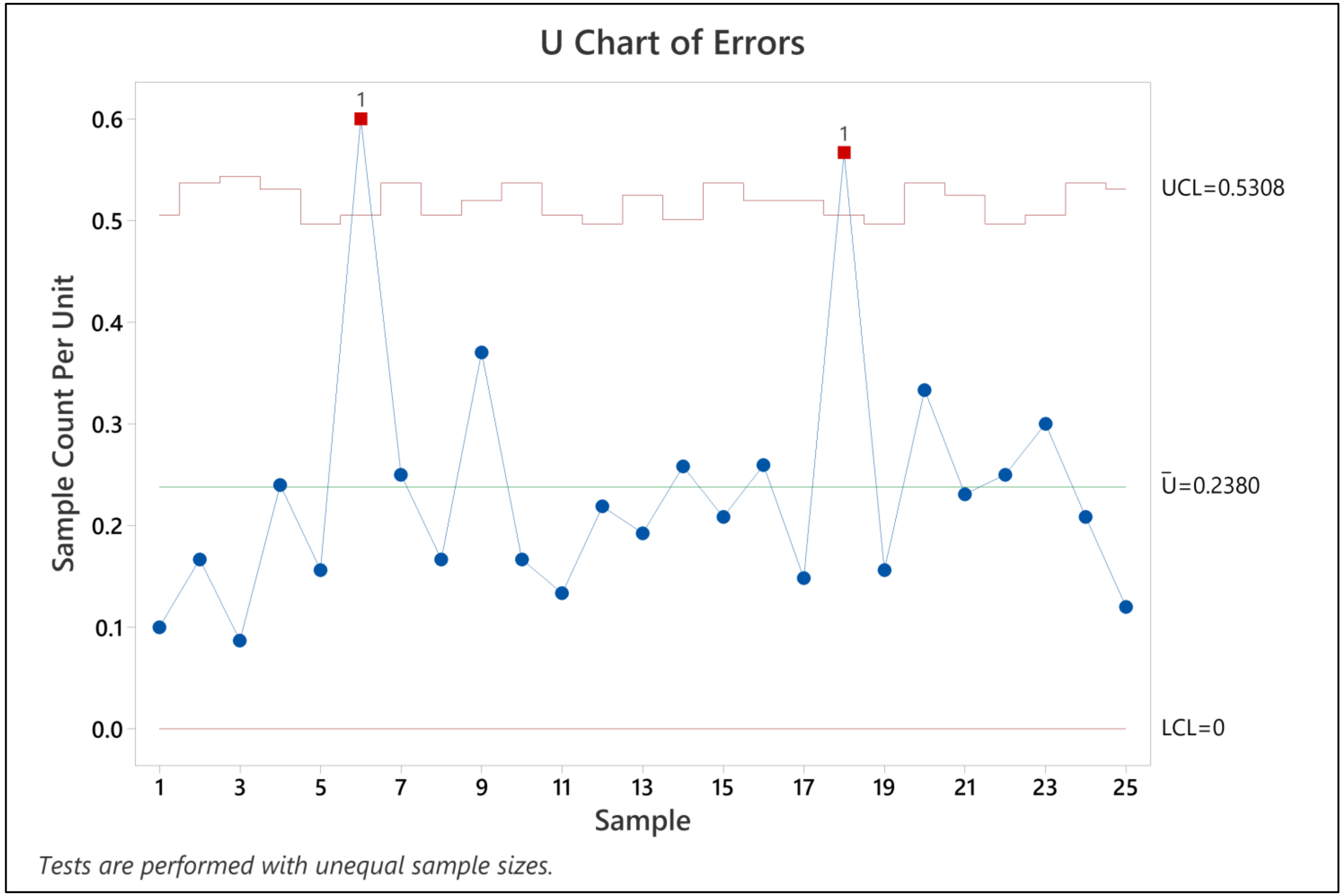
Tests 2



EXERCISE 4

Design a u chart to monitor the number of errors.

Stat → Control Charts → Attributes Charts → U



Because the sample sizes are unequal, the control limits vary. The average number of defects per set of pages is 0.238. **Subgroups 6 and 18 failed Test 1 because they are outside of the control limits.** Thus, the process is out of control.