Exercises - 2

CCS with Keys

Syntax & Semantics

Processes in CSSk are exactly the same as CCS processes, with three exceptions:

- They are denoted by X, Y instead of P, Q (this is just a convention, and we will keep on using P and Q to denote CCS processes),
- They can contain "keyed prefixes" $\alpha[k].X$, which represent "past actions",
- They generally do not include operators to represent infinite behaviours such as recursion, replication or iteration.

$$X, Y ::= 0 \parallel \alpha.X \parallel X + Y \parallel X|Y \parallel X[\alpha/\beta] \parallel X\backslash\alpha \parallel \alpha[k].X$$

The operator $\alpha[k].X$ marks that the channel α was already used, and tags it with the key k.

Standard and reachable processes: The set of keys occurring in X, $\mathrm{key}(X)$, is defined inductively:

$$\begin{aligned} \mathrm{key}(0) &= \emptyset & \mathrm{key}(X+Y) &= \mathrm{key}(X) \cup \mathrm{key}(Y) \\ \mathrm{key}(\alpha.X) &= \mathrm{key}(X) & \mathrm{key}(X|Y) &= \mathrm{key}(X) \cup \mathrm{key}(Y) \\ \mathrm{key}(\alpha[k].X) &= \{k\} \cup \mathrm{key}(X) \end{aligned}$$

We say that X is *standard* and write $\mathrm{std}(X)$ iff $\mathrm{key}(X) = \emptyset$—that is, if X is a CCS process.

Exercise: For each of the processes below, decide if they are standard, and if they are not, write their set of keys.

- $a[k].b|\overline{b}$
- $a|b.c$
- $(b[k].\overline{c} + a)|(c + \overline{b})$
- $(a[k].c[k'])|(\overline{a}[k].d)$

CSSK has *two* LTSes: one for the forward transitions, given in Figure 1, and one for the reverse transitions.

Exercise: Write the term you would obtain in CCSK for each of the transition listed in the last exercise of the previous exercise sheet.

The LTS for the reverse transitions (denoted $\rightsquigarrow$, or, in ASCII notation, `~~~>`) is the exact symmetric of the one given in Figure 1. That is, for every rule

```
X ---a---> Y
------------
X' --a---> Y'
```

, there is a rule

```
Y ~~a~~> Y
-------------
Y' ~~a~~> X'
```

Exercise: Write the reverse LTS for CCSK.

Exercise: Write all of the forward and backward transitions that the following processes can perform:

Action, Prefix and Restriction

$$\mathrm{std}(X)\ \frac{}{\alpha.X \xrightarrow{\alpha[k]} \alpha[k].X}\ \text{act.} \qquad k \neq k'\ \frac{X \xrightarrow{\beta[k]} X'}{\alpha[k'].X \xrightarrow{\beta[k]} \alpha[k'].X'}\ \text{pre.}$$

$$a \notin \{\alpha, \overline{\alpha}\}\ \frac{X \xrightarrow{\alpha[k]} X'}{X \backslash a \xrightarrow{\alpha[k]} X' \backslash a}\ \text{res.}$$

Parallel Group

$$k \notin \mathrm{key}(Y)\ \frac{X \xrightarrow{\alpha[k]} X'}{X \mid Y \xrightarrow{\alpha[k]} X' \mid Y}\ |_{\mathrm{L}} \qquad k \notin \mathrm{key}(X)\ \frac{Y \xrightarrow{\alpha[k]} Y'}{X \mid Y \xrightarrow{\alpha[k]} X \mid Y'}\ |_{\mathrm{R}}$$

$$\frac{X \xrightarrow{\lambda[k]} X' \qquad Y \xrightarrow{\overline{\lambda}[k]} Y'}{X \mid Y \xrightarrow{\tau[k]} X' \mid Y'}\ \text{syn.}$$

Sum Group

$$\mathrm{std}(Y)\ \frac{X \xrightarrow{\alpha[k]} X'}{X + Y \xrightarrow{\alpha[k]} X' + Y}\ +_{\mathrm{L}} \qquad \mathrm{std}(X)\ \frac{Y \xrightarrow{\alpha[k]} Y'}{X + Y \xrightarrow{\alpha[k]} X + Y'}\ +_{\mathrm{R}}$$

Figure 1: The semantics of CSSK

- $a[k].b|\overline{b}$
- $(b[k].\overline{c} + a)|(c + \overline{b})$
- $(a[k].c[k'])|(\overline{a}[k].d)$
- $a[k].c|b[k'].\overline{c}$
- $a[k].b[k'].0|\overline{b}[k'].c[k''].d$

Exercise: Looking back at the vending machine example from the previous exercise sheet, explain intuitively why $1p.1p.big.collect.V + 1p.little.collect.V$ could be an ok way of answering exercise 4 in a reversible system, and why this process would not fulfill the requirement of exercise 4 in a non-reversible set-up.