

# Exercises - 2

## CCS with Keys

### Syntax & Semantics

Processes in CSSK are exactly the same as CCS processes, with three exceptions:

- They are denoted by  $X, Y$  instead of  $P, Q$  (this is just a convention, and we will keep on using  $P$  and  $Q$  to denote CCS processes),
- They can contain “keyed prefixes”  $\alpha[k].X$ , which represent “past actions”,
- They generally do not include operators to represent infinite behaviours such as recursion, replication or iteration.

$$X, Y ::= 0 \parallel \alpha.X \parallel X + Y \parallel X|Y \parallel X[\alpha/\beta] \parallel X \backslash \alpha \parallel \alpha[k].X$$

The operator  $\alpha[k].X$  marks that the channel  $\alpha$  was already used, and tags it with the key  $k$ .

**Standard and reachable processes:** The set of keys occurring in  $X$ ,  $\text{key}(X)$ , is defined inductively:

$$\begin{aligned} \text{key}(0) &= \emptyset & \text{key}(X + Y) &= \text{key}(X) \cup \text{key}(Y) \\ \text{key}(\alpha.X) &= \text{key}(X) & \text{key}(X|Y) &= \text{key}(X) \cup \text{key}(Y) \\ \text{key}(\alpha[k].X) &= \{k\} \cup \text{key}(X) \end{aligned}$$

We say that  $X$  is *standard* and write  $\text{std}(X)$  iff  $\text{key}(X) = \emptyset$ —that is, if  $X$  is a CCS process.

**Exercise:** For each of the processes below, decide if they are standard, and if they are not, write their set of keys.

- $a[k].b|\bar{b}$
- $a|b.c$
- $(b[k].\bar{c} + a)|(c + \bar{b})$
- $(a[k].c[k'])(\bar{a}[k].d)$

CSSK has *two* LTSes: one for the forward transitions, given in Figure 1, and one for the reverse transitions.

**Exercise:** Write the term you would obtain in CCSK for each of the transition listed in the last exercise of the previous exercise sheet.

The LTS for the reverse transitions (denoted  $\rightsquigarrow$ , or, in ASCII notation,  $\sim\sim\sim>$ ) is the exact symmetric of the one given in Figure 1. That is, for every rule

$$\begin{array}{c} X \xrightarrow{a} Y \\ \hline X' \xrightarrow{a} Y' \end{array}$$

, there is a rule

$$\begin{array}{c} Y \sim\sim\sim> Y \\ \hline Y' \sim\sim\sim> X' \end{array}$$

**Exercise:** Write the reverse LTS for CCSK.

**Exercise:** Write all of the forward and backward transitions that the following processes can perform:

### Action, Prefix and Restriction

$$\begin{array}{c}
 \text{std}(X) \xrightarrow{\alpha.X \xrightarrow{\alpha[k]} \alpha[k].X} \text{act.} \qquad k \neq k' \xrightarrow{X \xrightarrow{\beta[k]} X' \quad \alpha[k'].X \xrightarrow{\beta[k]} \alpha[k'].X'} \text{pre.} \\
 a \notin \{\alpha, \bar{\alpha}\} \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X \setminus a \xrightarrow{\alpha[k]} X' \setminus a} \text{res.}
 \end{array}$$

### Parallel Group

$$\begin{array}{c}
 k \notin \text{key}(Y) \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X \mid Y \xrightarrow{\alpha[k]} X' \mid Y} |_{\text{L}} \qquad k \notin \text{key}(X) \xrightarrow{Y \xrightarrow{\alpha[k]} Y' \quad X \mid Y \xrightarrow{\alpha[k]} X \mid Y'} |_{\text{R}} \\
 \xrightarrow{X \xrightarrow{\lambda[k]} X' \quad Y \xrightarrow{\bar{\lambda}[k]} Y' \quad X \mid Y \xrightarrow{\tau[k]} X' \mid Y'} \text{syn.}
 \end{array}$$

### Sum Group

$$\text{std}(Y) \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X + Y \xrightarrow{\alpha[k]} X' + Y} +_{\text{L}} \qquad \text{std}(X) \xrightarrow{Y \xrightarrow{\alpha[k]} Y' \quad X + Y \xrightarrow{\alpha[k]} X + Y'} +_{\text{R}}$$

Figure 1: The semantics of CSSK

- $a[k].b|\bar{b}$
- $(b[k].\bar{c} + a)|(c + \bar{b})$
- $(a[k].c[k'])(\bar{a}[k].d)$

**Exercise:** Looking back at the vending machine example from the previous exercise sheet, explain intuitively why  $1p.1p.big.collect.V + 1p.little.collect.V$  could be an ok way of answering exercise 4 in a reversible system, and why this process would not fulfill the requirement of exercise 4 in a non-reversible set-up.