

Exercises - 2

CCS with Keys

Syntax & Semantics

Processes in CSSK are exactly the same as CCS processes, with three exceptions:

- They are denoted by X, Y instead of P, Q (this is just a convention, and we will keep on using P and Q to denote CCS processes),
- They can contain “keyed prefixes” $\alpha[k].X$, which represent “past actions”,
- They generally do not include operators to represent infinite behaviours such as recursion, replication or iteration.

$$X, Y ::= 0 \parallel \alpha.X \parallel X + Y \parallel X|Y \parallel X[\alpha/\beta] \parallel X \setminus \alpha \parallel \alpha[k].X$$

The operator $\alpha[k].X$ marks that the channel α was already used, and tags it with the key k .

Standard and reachable processes: The set of keys occurring in X , $\text{key}(X)$, is defined inductively:

$$\begin{aligned} \text{key}(0) &= \emptyset & \text{key}(X + Y) &= \text{key}(X) \cup \text{key}(Y) \\ \text{key}(\alpha.X) &= \text{key}(X) & \text{key}(X|Y) &= \text{key}(X) \cup \text{key}(Y) \\ \text{key}(\alpha[k].X) &= \{k\} \cup \text{key}(X) \end{aligned}$$

We say that X is *standard* and write $\text{std}(X)$ iff $\text{key}(X) = \emptyset$ —that is, if X is a CCS process.

Exercise: For each of the processes below, decide if they are standard, and if they are not, write their set of keys.

- $a[k].b|\bar{b}$
- $a|b.c$
- $(b[k].\bar{c} + a)|(c + \bar{b})$
- $(a[k].c[k'])|(\bar{a}[k].d)$

CSSK has *two* LTSes: one for the forward transitions, given in Figure 1, and one for the reverse transitions.

Exercise: Write the term you would obtain in CCSK for each of the transition listed in the last exercise of the previous exercise sheet.

The LTS for the reverse transitions (denoted \rightsquigarrow , or, in ASCII notation, $\sim\sim\sim>$) is the exact symmetric of the one given in Figure 1. That is, for every rule

$$\begin{array}{c} X \xrightarrow{a} Y \\ \hline X' \xrightarrow{a} Y' \end{array}$$

, there is a rule

$$\begin{array}{c} Y \sim\sim\sim> Y \\ \hline Y' \sim\sim\sim> X' \end{array}$$

Exercise: Write the reverse LTS for CCSK.

Exercise: Write all of the forward and backward transitions that the following processes can perform:

Action, Prefix and Restriction

$$\begin{array}{c}
 \text{std}(X) \xrightarrow{\alpha.X \xrightarrow{\alpha[k]} \alpha[k].X} \text{act.} \qquad k \neq k' \xrightarrow{X \xrightarrow{\beta[k]} X' \quad \alpha[k'].X \xrightarrow{\beta[k]} \alpha[k'].X'} \text{pre.} \\
 a \notin \{\alpha, \bar{\alpha}\} \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X \setminus a \xrightarrow{\alpha[k]} X' \setminus a} \text{res.}
 \end{array}$$

Parallel Group

$$\begin{array}{c}
 k \notin \text{key}(Y) \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X \mid Y \xrightarrow{\alpha[k]} X' \mid Y} |_{\text{L}} \qquad k \notin \text{key}(X) \xrightarrow{Y \xrightarrow{\alpha[k]} Y' \quad X \mid Y \xrightarrow{\alpha[k]} X \mid Y'} |_{\text{R}} \\
 \xrightarrow{X \xrightarrow{\lambda[k]} X' \quad Y \xrightarrow{\bar{\lambda}[k]} Y' \quad X \mid Y \xrightarrow{\tau[k]} X' \mid Y'} \text{syn.}
 \end{array}$$

Sum Group

$$\text{std}(Y) \xrightarrow{X \xrightarrow{\alpha[k]} X' \quad X + Y \xrightarrow{\alpha[k]} X' + Y} +_{\text{L}} \qquad \text{std}(X) \xrightarrow{Y \xrightarrow{\alpha[k]} Y' \quad X + Y \xrightarrow{\alpha[k]} X + Y'} +_{\text{R}}$$

Figure 1: The semantics of CSSK

- $a[k].b|\bar{b}$
- $(b[k].\bar{c} + a)|(c + \bar{b})$
- $(a[k].c[k'])|(\bar{a}[k].d)$

Exercise: Looking back at the vending machine example from the previous exercise sheet, explain intuitively why $1p.1p.big.collect.V + 1p.little.collect.V$ could be an ok way of answering exercise 4 in a reversible system, and why this process would not fulfill the requirement of exercise 4 in a non-reversible set-up.