Exercises - 3

Forward-Only CCS

A binary relation $\mathcal{R}$ on CCS processes is a *simulation* if for all P, Q such that $P\mathcal{R}Q$[1] and for every α, P' such that $P \to^{\alpha} P'$, there exists Q' such that $Q \to^{\alpha} Q'$. It is a *bisimulation* if furthermore for every α, Q' such that $Q \to^{\alpha} Q'$, there exists P' such that $P \to^{\alpha} P'$.

Please not that the α on the label of the transitions *needs to be the same*: intuitively, this means "if P can do α, then Q also can do it, and they are still in this relation". The *bi*simulation part of the definition stress that we should not worry about "who is leading the game", i.e. if it is Q that should match what P is doing, or the opposite.

1. Prove that the empty relation is a bisimulation.
2. Prove that the identity relation id ($P \mathrm{id} P$ for all P) is a bisimulation.

One generally write that "P and Q *are* bisimilar" if there exists a bisimulation $\mathcal{R}$ such that $P\mathcal{R}Q$.

1. Prove that $a|a$ and $a.a$ are bisimilar.
2. Prove that $b|a$ and $a.b$ are *not* bisimilar.
3. Prove that $a + a$ and a are bisimilar.
4. Prove that $a + b$ and $a|b$ are *not* bisimilar.
5. Prove that $a.b + b.a$ and $a|b$ are bisimilar.
6. Prove that for any P and Q, $P|Q$ and $Q|P$ are bisimilar.
7. Prove that for any P, $P + 0$ and P are bisimilar.

The following is inspired from this sheet. For each of the following process decide whether they are bisimilar and if not, justify why not:

1. $b.a + b$ and $b.(a + b)$
2. $a.(b.c + b.d)$ and $a.b.c + a.b.d$
3. $(a|b) + c.a$ and $a|(b + c)$

Last but not least, check whenever $2p.\mathit{big}.\mathit{collect} + 1p.1p.\mathit{big}.\mathit{collect} + 1p.\mathit{little}.\mathit{collect}$ and $2p.\mathit{big}.\mathit{collect} + 1p.(1p.\mathit{big}.\mathit{collect} + \mathit{little}.\mathit{collect})$ are bisimilar.

CCSK

Writing $\rightsquigarrow$ the backward transition, a "forward-reverse simulation" [1] is a relation $\mathcal{R}$ on CCSK terms such that whenever $X\mathcal{R}Y$,

- X and Y have the same set of keys,
- for all X', α, m if $X \to^{\alpha[m]} X'$, then there exists Y' such that $Y \to^{\alpha[m]} Y'$ and $X'\mathcal{R}Y'$,
- for all X', α, m if $X \rightsquigarrow^{\alpha[m]} X'$, then there exists Y' such that $Y \rightsquigarrow^{\alpha[m]} Y'$ and $X'\mathcal{R}Y'$.

From there, the forward-reverse __bi__simulation is defined as usual.

We similarly write that X and Y *are* bisimilar if there exists a forward-reverse bisimulation between them.

1. Prove that $a[k] + b$ and $a[k].b$ are *not* bisimilar.
2. Prove that $a[k].(b + c)$ and $a[k].b + c$ are *not* bisimilar.
3. Prove that $a[k]|b[k']$ and $a[k].b[k']$ are *not* bisimilar.
4. Decide whenever $(a.a)|b$ and $(a|a)|b$ are bisimilar.
5. Decide whenever $a.(b + b)$ and $(a.b) + (a.b)$ are bisimilar.
6. Check whenever $2p.\mathit{big}.\mathit{collect}+1p.1p.\mathit{big}.\mathit{collect}+1p.\mathit{little}.\mathit{collect}$ and $2p.\mathit{big}.\mathit{collect}+1p.(1p.\mathit{big}.\mathit{collect}+\mathit{little}.\mathit{collect})$ (this time, seen as reversible processes) are bisimilar.

[1]Which is a notation for "P and Q are in the relationship $\mathcal{R}$", or "$(P, Q) \in \mathcal{R}$".

Discussion

Observe that forcing both terms to have *exactly* the same keys is not really meaningful: it seems strange to decide that $a[k]|\bar{a}[k]$ and $a[k']|\bar{a}[k']$, for $k \neq k'$, are not bisimilar. Two different solutions to that issues have been explored:

- One can either add a rule to rename "bound" keys, e.g. keys that occur in complementary names [1], so that $a[k]|\bar{a}[k]$ and $a[k']|\bar{a}[k']$ would be considered to be "the same term" (up to renaming, of course),
- One can change the definition of bisimulation for reversible calculus, so that only a bijection (and not the identity) between the keys is needed [2].

The second solution let e.g. terms like $a[k]$ and $a[k']$ be bisimilar while the first one won't let them. However, for that second solution, some care is needed so that the bijection is not "changed overnight": you can try to see with $(a|(b+c))+(a|b)+((a+c)|b)$ and $(a|(b+c))+((a+c)|b)$ that they would be bisimilar if the keys can change while the relation is being constructed, but they are not if the bijection needs to be "preserved".

References

[1] I. Lanese, I. Phillips, Forward-reverse observational equivalences in CCSK, in: S. Yamashita, T. Yokoyama (Eds.), Reversible Computation - 13th International Conference, RC 2021, Virtual Event, July 7-8, 2021, Proceedings, Springer, 2021: pp. 126–143. https://doi.org/10.1007/978-3-030-79837-6_8.

[2] C. Aubert, I. Cristescu, How reversibility can solve traditional questions: The example of hereditary history-preserving bisimulation, in: I. Konnov, L. Kovács (Eds.), 31st International Conference on Concurrency Theory, CONCUR 2020, September 1–4, 2020, Vienna, Austria, Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2020: pp. 13:1–13:24. https://doi.org/10.4230/LIPIcs.CONCUR.2020.13.