The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressures

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Source: *The Review of Financial Studies*, April 2012, Vol. 25, No. 4 (April 2012), pp. 1155-1206

Published by: Oxford University Press. Sponsor: The Society for Financial Studies.

Stable URL: https://www.jstor.org/stable/41407858

The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressures

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I propose a new measure that identifies when the market price of an over-the-counter traded asset is below its fundamental value due to selling pressure. The measure is the difference between prices paid by small traders and those paid by large traders. In a model for over-the-counter trading with search frictions and periods with selling pressures, I show that this measure identifies liquidity crises (i.e., high number of forced sellers). Using a structural estimation, the model is able to identify liquidity crises in the U.S. corporate bond market based on the relative prices paid by small and large traders. New light is shed on two crises, the downgrade of General Motors and Ford in 2005 and the subprime crisis (*JEL* D4, D83, G01, G12).

1. Introduction

We know that asset prices can temporarily decrease below their fundamental value when there is selling pressure—i.e., when many investors seek to sell the asset at the same time. Duffie (2010) reviews recent evidence in his 2010 Presidential Address to the American Finance Association. Identifying when this occurs is difficult. This is because the event that causes selling pressure typically reveals new information about the fundamental value of the asset. Disentangling selling pressure effects from information effects is at best challenging.

The main contribution of this article is to propose a measure that identifies when there is selling pressure in over-the-counter markets. Selling pressure is

I am very grateful to David Lando and Lasse Heje Pedersen for many helpful comments and discussions and to Jens Dick-Nielsen for discussions regarding the TRACE database. I thank Viral Acharya, Darrell Duffie, René Garcia, Soren Hvidkjær, Jesper Rangvid, Norman Schürhoff, Carsten Sorensen, and seminar participants at the Second International Financial Research Forum in Paris, Second Erasmus Liquidity Conference in Rotterdam, 2009 MTS Conference on Financial Markets in London, Doctoral Course at the University of Lausanne, 2009 European Finance Association meeting in Bergen, Nykredit Symposium 2009, D-Caf meeting in Helsingor 2010, Danish Central Bank, Aarhus School of Business, University of Copenhagen, Copenhagen Business School, London Business School, New York Fed, University of Toronto, HEC Paris, Boston College, University of Chicago, Rochester, Boston University, University of Lausanne, Goethe-Universität Frankfurt, Manchester Business School, and University of Cologne for useful suggestions. I thank the Danish Social Science Research Council for financial support. I also thank the editor, Matthew Spiegel, and an anonymous referee for many valuable suggestions. Send correspondence to Peter Feldhütter, London Business School, Regent's Park, London NW1 4SA, United Kingdom; telephone: +44 (0) 20 7000 8277; fax: +44 (0)20 7000 7001. E-mail: pfeldhutter@london.edu.

doi:10.1093/rfs/hhr093 Advance Access publication October 7, 2011

defined as times when the number of sellers relative to the number of buyers is unusually high. In over-the-counter markets, an asset simultaneously trades at different prices because prices are negotiated bilaterally. The price difference between small trades and large trades at a given point in time identifies selling pressure. If large traders trade at unusually low prices relative to small traders, there is selling pressure.

In contrast to the existing literature, the measure does not rely on realized returns. There are two approaches in the current literature. One approach is to look at asset returns around an event that is unlikely to contain new information about asset value. If cumulative returns are negative around the event and rebound fully or partially during a period after, there has been selling pressure. Examples of this approach include Coval and Stafford (2007) and Chen, Noronha, and Singhal (2004). This approach is limited to information-free events. Another approach is to control for new information and see if abnormal returns are negative around the event and subsequently rebound. Mitchell, Pedersen, and Pulvino (2007) and Newman and Rierson (2004) take this approach. If the event reveals new information about fundamental asset value, it can be challenging to adjust abnormal returns correctly. For example, Ellul, Jotikasthira, and Lundblad (2011) and Ambrose, Cai, and Helwege (2009) both study selling pressure in U.S. corporate bonds around downgrades. They use similar datasets and reach conflicting conclusions regarding the importance of selling pressure. Clearly a downgrade contains information about firm quality, and it is difficult to control for the impact of this information. In this article, selling pressure is identified through differences in prices occurring simultaneously. Changes in fundamental value are automatically controlled for since the information effect is the same for both small and large trades. Furthermore, selling pressure can be identified in "real-time." Previous approaches identify selling pressure ex post through the subsequent reversal of returns after an event.

In a theoretical model, I find support for the small trade minus large trade measure as an identifier of selling pressure. My model is a variant of the search model in Duffie, Gârleanu, and Pedersen (2005). Empirically, I study the corporate bond market, so the model is adapted to the structure of this market. There is a corporate bond traded in the model. Investors switch randomly between needing liquidity or not. Investors trade through a dealer whom they find at random with different search intensities. A high search intensity implies that the investor finds a counterparty fast. I interpret such an investor as a sophisticated/large one. An investor with a low search intensity is interpreted as an unsophisticated/small investor. Once an investor meets a dealer, they bargain, and the resulting price reflects their alternatives to immediate trade. One alternative is to cut off negotiations and search for a new counterparty. This alternative is particularly strong for large investors who find counterparties fast. Therefore, large investors negotiate tighter bid-ask spreads. The alternative of searching for a new counterparty is also strong for buyers in a market in which there is selling pressure. This is because there are currently many sellers. The combined advantages of being a large buyer and a buyer in a market experiencing selling pressure lead to significant price discounts. These price discounts are larger than for a small buyer in a market with selling pressure, because the "threat" of looking for another seller is less forceful for a small buyer.

In equity markets, it is well known that block trades sell at a discount. This is documented by Kraus and Stoll (1972) and supported by information-based models such as Kyle's (1985) model. The predictions in my search-based model are different from predictions in information-based models. In a market where the numbers of sellers and buyers are balanced, large traders in a search-based model transact at better prices. The reason is that they negotiate tighter bid-ask spreads due to their stronger outside options. In information-based models, large traders sell at lower prices than small traders. This is because they are likely to have private information about asset value. In addition, asset price dynamics under selling pressure are different in the two models. In information-based models, a block trade occurs at a discount and subsequent small trades also occur at (slightly smaller) discounts. In the search model here, the pattern is different: At times when there is selling pressure, small traders trade at high prices and large traders trade at low prices; the time ordering of trades does not matter. This is because, compared with small buyers, large buyers can more quickly "shop around" among numerous sellers. As a consequence, they negotiate larger price discounts.

Another contribution to the literature is that I propose and carry out a maximum likelihood approach to estimate parameters of the model. I use transaction data from the TRACE database for the period from October 2004 to June 2009. The TRACE database contains practically all corporate bond transactions even though trading occurs over the counter. There is a growing literature on search models, but to my knowledge no one has structurally estimated a model before. The estimation approach allows me to empirically identify periods of selling pressure.

A third contribution to the literature is that I shed new light on recent selling-pressure episodes in the U.S. corporate bond market. There are two major incidents of selling pressure according to the empirical results. In May 2005, S&P downgraded General Motors (GM) and Ford to speculative grade, causing strong selling pressure in their bonds. In the preceding months, selling pressure intensified as a downgrade became more likely, consistent with findings in Acharya, Schaefer, and Zhang (2008). My results show that selling pressure was largely isolated to GM and Ford bonds. The time pattern of selling pressure in GM bonds was different from that in Ford bonds. Selling pressure in GM bonds peaked in May because GM was downgraded by both S&P and Fitch and dropped out of the important Lehman investment-grade index. In contrast, selling pressure in Ford bonds decreased in May because Ford was downgraded only by S&P and remained in the Lehman index. The second period with

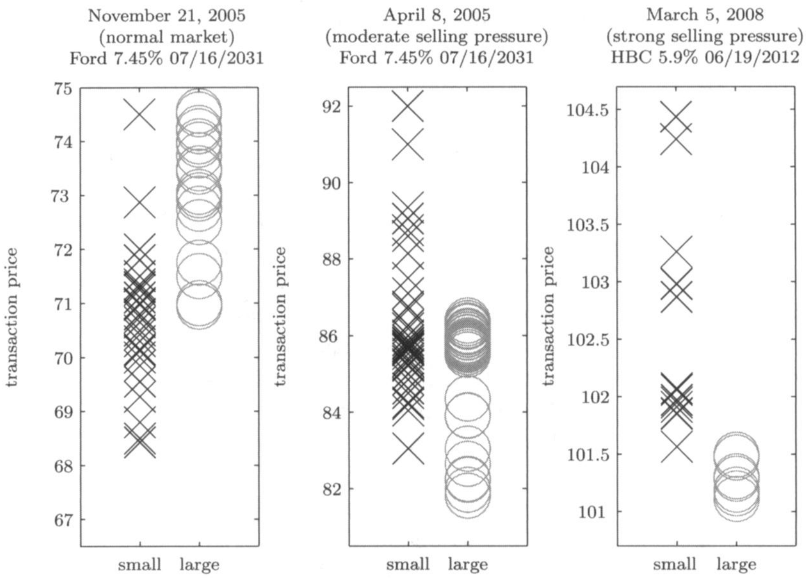

Figure 1
Small and large trades in a normal market, under moderate selling pressure, and under strong selling pressure
This graph shows three examples of all trades smaller than \$100,000 (marked with crosses) and trades of at least \$1,000,000 (marked with circles) for a bond during a day.

selling pressure takes off at the beginning of the subprime crisis in summer 2007. During the crisis, there are three peaks in the selling pressure: when Bear Stearns is taken over, when Lehman Brothers defaults, and at the beginning of 2009 when stock markets lose 30% in two months.

Figure 1 illustrates how the relation in prices between small and large trades identifies selling pressure. The left-hand graph shows that prices for large trades are on average higher than those for small trades in a normal market. The middle graph shows a day where large transaction prices are on average lower than small transaction prices, indicating moderate selling pressure. The right-hand graph shows how prices of large trades are markedly lower than prices of small trades when there is a large imbalance in the number of sellers versus buyers.

Figure 2 shows another example of the price pattern when there is selling pressure. In the figure, all transaction prices in a Citigroup bond on March 11–12, 2009, are graphed with time stamps. At a given point in time, the bond trades at multiple prices, reflecting that bond trading is over-the-counter with bilateral negotiation. Large traders transact at around \$70, while small traders transact at an average close to \$75. This indicates strong selling pressure. Note that a large trade at a low price is not followed by small trades at low prices, so the price differences between small and large trades are not due to price impact

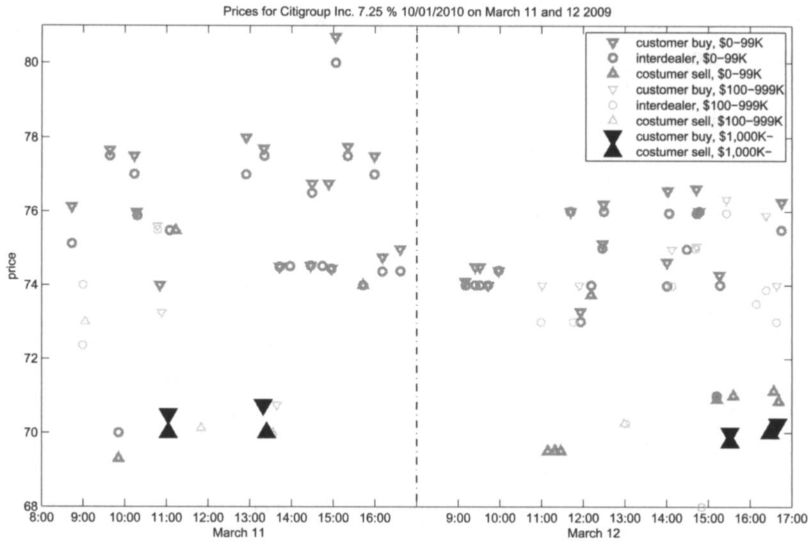

Figure 2
Transactions under strong selling pressure
This graph shows all customer buys from a dealer, customer sells to a dealer, and interdealer trades on March 11–12, 2009, for a Citigroup bond with coupon 7.25% and maturity October 1, 2010. Average transaction price during the two days is 74.8 for trades with a notional below $100K, 73.8 for trades with a notional in the range $100–$999K, and 70.2 for trades of $1,000K or more.

of large trades. In addition, all transactions are publicly disseminated with at most a fifteen-minute lag, so the market is transparent.[1] The graph also shows that there is information in medium-sized trades, since they trade at average prices between those of small and large trades.

2. The Model

The U.S. corporate bond market is a principal source of financing for firms. It is comparable to the U.S. Treasury market measured in amount outstanding, but trading volume is more than thirty times lower.[2] An investor sequentially contacts one or several dealers over the telephone to trade a corporate bond. Dealers typically do not make a market, and a price quote is firm for only a short period of time. This limits the ability to obtain multiple quotations before

[1] At http://cxa.marketwatch.com/finra/BondCenter/Default.aspx, the latest trades in any U.S. corporate bond are one click away.

[2] Principal outstanding volume by the end of 2010 was $7,536 billion in the U.S. corporate bond market and $8,853 billion in the U.S. Treasury market, while average daily trading volume in 2010 was $16.3 billion in the U.S. corporate bond market and $528.2 billion in the U.S. Treasury market. Source: Securities and Financial Markets Association (www.sifma.org).

committing to a trade.[3] Hence, prices are outcomes of a bargaining game, determined in part by the ease with which investors find counterparties and the relative number of investors currently looking to buy or sell. The following model captures these important features of the market.

The economy is populated by two kinds of agents, investors and dealers, who are risk-neutral and infinitely lived. They consume a nonstorable consumption good used as numéraire, and their time preferences are given by the discount rate $r > 0$. Time is continuous, starts at $t = 0$, and goes on forever.

Investors have access to a risk-free bank account paying interest rate r. The bank account can be viewed as a liquid security that can be traded instantly. To rule out Ponzi schemes, the value W_t of an investor's bank account is bounded from below. In addition, investors have access to an over-the-counter corporate bond market for a credit-risky bond. There is a continuum of credit-risky firms that issue these bonds. If a firm defaults, it is replaced by an identical new firm. The bond pays coupons at the constant rate of C units of consumption per year. The bond has expected maturity T and face value F, meaning that it matures randomly according to a Poisson process with intensity $\lambda_T = 1/T$ and pays F at maturity. The bond defaults with intensity λ_D and pays a fraction $(1 - f)F$ of face value in default. The total amount outstanding of the bond at time 0 is A where $0 < A < 1$. When bonds mature or default, firms issue new bonds to replace them, so the total issue intensity is $(\lambda_D + \lambda_T)A$. This implies that the amount outstanding of bonds at any point in time is A. When bonds are issued, they are sold through the dealers. I do not model the interaction between dealers and firms, so the issued bonds appear as extra bonds dealers sell. A bond trade occurs when an investor finds a dealer in a search process that will be described in a moment.

Investors hold at most one unit of the bond and cannot short-sell. Because agents are risk-neutral, investors hold either zero or one unit of the bond in equilibrium. An investor is of type "high" or "low." The "high" type has no holding cost when owning the asset, while the "low" type has a holding cost of $\delta > 0$ per time unit. The holding cost can be interpreted as a funding liquidity shock that hits the investor. Each investor receives a preference shock with Poisson arrival rate λ. Conditional on the shock, the probability that the investor will become type "high" is $1 - \pi$, while it is π to become a "low" type. The switching processes are for all investors pairwise independent.

I assume that there is a unit mass of independent nonatomic dealers who maximize profits. An investor with level of sophistication i, $i \in \{1, 2, ..., N\}$ meets a dealer with intensity ρ^i, which can be interpreted as the sum of the intensity of dealers' search for investors and investors' search for dealers. This captures that a sophisticated investor quickly finds a trading partner, while an unsophisticated investor spends considerable time finding someone to trade with. The search intensity is observable to everyone. When I refer to a

[3] See Bessembinder and Maxwell (2008) for further details about the U.S. corporate bond market.

large/sophisticated investor, this means an investor with a high search intensity ρ^j. Likewise, a small/unsophisticated investor refers to an investor with a small search intensity ρ^j. Without loss of generality, assume that $\rho^i < \rho^j$ when $i < j$. This assumption implies that investors with intensity ρ^1 are the most unsophisticated and those with intensity ρ^N are the most sophisticated. There is a mass of $\frac{1}{N}$ investors with search intensity ρ_i, so the total mass of investors is 1. When an investor and a dealer meet, they bargain over the terms of trade. Dealers have a fraction, $z \in [0, 1]$, of the bargaining power when facing an investor. I assume that dealers immediately unload their positions in an interdealer market, so they have no inventory.

In the Appendix, I show that if bond supply A is low, unsophisticated investors never own any bonds in steady state. If bond supply is high, unsophisticated investors—no matter if they are liquidity-shocked or not—always buy in steady state. To ensure that we in steady state see both buy and sell prices for investors with search intensity ρ^i for any i, I assume that the bond supply is given as $A = \frac{1-\pi}{N}\left(\sum_{j=2}^{N} \frac{\rho^j}{\rho^j+\lambda_T+\lambda_D} + (1-\omega)\frac{\rho^1}{\rho^1+\lambda_T+\lambda_D}\right)$ for small ω. The assumption is not important for how prices react to a liquidity shock, which is the mechanism through which selling pressure is identified. However, it does provide simple pricing formulas.[4] The following theorem states equilibrium bid and ask prices in the economy, and a proof is given in the Appendix.

Theorem 2.1. Prices in steady state. In steady state, the bid B^j and ask A^j prices for investors with search intensity ρ^j are given as

$$\begin{aligned} A^j_{ss} &= \Psi - \delta\frac{\lambda\pi}{(\Delta + (1-z)\rho^1 + \lambda)\Delta} \\ &\quad - \delta\frac{z\lambda\pi(1-z)(\rho^j - \rho^1)}{(\Delta + (1-z)\rho^j)(\Delta + (1-z)\rho^j + \lambda)(\Delta + (1-z)\rho^1 + \lambda)} \\ B^j_{ss} &= A^j_{ss} - \frac{\delta z}{\Delta + (1-z)\rho^j + \lambda}, \end{aligned}$$

where

$$\begin{aligned} \Psi &= \frac{C + \lambda_D(1-f)F + \lambda_T F}{r + \lambda_D + \lambda_T} \\ \Delta &= r + \lambda_T + \lambda_D. \end{aligned}$$

The last part of the expression for the ask price, $\delta\frac{z\lambda\pi(1-z)(\rho^j-\rho^1)}{(\Delta+(1-z)\rho^j)(\Delta+(1-z)\rho^j+\lambda)(\Delta+(1-z)\rho^1+\lambda)}$, shows how ask prices vary with

[4] While the assumption is not important for how prices react to a liquidity shock, it does influence the relation between prices paid by small and larger traders in steady state. See Appendix A.5 for a discussion.

search intensity ρ^j. Relative to the most unsophisticated investor with search intensity ρ^1, more sophisticated investors have lower ask prices. How much lower depends, among other things, on two important parameters δ and π. A higher δ implies higher differences because a liquidity shock is "more expensive." A higher π has the same effect because it makes liquidity shocks more frequent. As an obvious consequence of the theorem, we have the following corollary:

Corollary 2.1. Bid-ask spreads. The bid-ask spread for investors with search intensity ρ_j is given as

$$A_{ss}^j - B_{ss}^j = \frac{z\delta}{\lambda + \rho^j(1-z) + r + \lambda_T + \lambda_D}. \tag{1}$$

We see that sophisticated investors trade at tighter bid-ask spreads than unsophisticated investors because bid-ask spreads decrease in ρ. The prices buyers and sellers negotiate with dealers reflect their alternatives to immediate trade. An alternative is to cut off negotiations and find a new dealer. Since sophisticated investors find a new dealer more easily, their alternative to trade is stronger and they negotiate better bid and ask prices (see also Duffie, Gârleanu, and Pedersen 2005).

We also see that bid-ask spreads increase in the maturity of the bond $1/\lambda_T$. A seller can let a bond mature instead of selling, giving him a strong alternative to trade in case of a short-maturity bond. A buyer is aware that a short-maturity bond will mature soon and that he will receive coupons only for a short period. Thus, neither buyer nor seller is willing to give large price concessions to the dealer, leading to tight bid-ask spreads.

Next, a liquidity shock to investors is defined. I assume that the model is in steady state and a sudden liquidity shock occurs. If a shock of size $0 \leq s \leq 1$ occurs, a "high" investor (no liquidity need) becomes a "low" investor (liquidity need) with probability s:

Definition 2.1. Liquidity shock. When a liquidity shock of size $0 < s \leq 1$ occurs, any high investor becomes a low investor with probability s.

Goldstein, Hotchkiss, and Sirri (2007) find that dealers do not split trades and perform a matching/brokerage function in illiquid bonds. According to market participants, risk limits often prohibit dealers from taking bonds on the book and splitting trades when there is a liquidity shock.[5] To capture

[5] According to conversations with market participants, a corporate bond trade is often carried out as follows, especially during a crisis. If an investor wants to sell a significant amount of a corporate bond, he contacts a salesperson from a given bank. The salesperson asks the marketmaker in the bank if he wants to buy it. Often, and in particular during a crisis, the marketmaker cannot take the bond on the book due to the risk. The salesperson therefore searches directly for a buyer, and once there is a match, the transaction is carried out. Typically,

this, I assume that markets are segmented for a while after a shock. This means that after a liquidity shock, an investor with search intensity ρ^i trades only (through the dealer) with investors with the same search intensity. More specifically, assume that if a liquidity shock larger than ω occurs, markets become segmented until the shock size has diminished to ω. While markets are segmented, I assume that the proportion of new bond issues that investors with search intensity ρ^i buy is the same proportion as in steady state. Prices following a liquidity shock are given in the following theorem. A proof is in the Appendix.

Theorem 2.2. Prices after a liquidity shock. Assume that the economy is in steady state and a liquidity shock of size $0 < s \leq 1$ occurs. Then,

$$B^j(s) = B^j_{ss}, \quad s \leq \omega \frac{\rho^1}{\sum_{j=1}^N \rho^j}$$

$$B^j(s) = B^j_{ss} - V(s) + zS^j(s), \quad \omega \geq s > \omega \frac{\rho^1}{\sum_{j=1}^N \rho^j}$$

$$B^j(s) = e^{-\Delta t_2(s)}\Big(B^j(\omega) + (1-z)[R + S^j(\omega) - \frac{\delta}{\lambda + \rho^j(1-z) + \Delta}]\Big)$$
$$+(1 - e^{-\Delta t_2(s)})P^j_{shock}, \quad s > \omega,$$

while

$$A^j(s) = B^j(s) + \frac{z\delta}{\lambda + \rho^j(1-z) + \Delta},$$

where

$$V(s) = \delta \frac{\Delta + \rho^1(1-z)\big(1 - e^{-\Delta t_1(s)}\big)}{(\rho^1(1-z) + \lambda + \Delta)\Delta}$$

$$S^j(s) = \delta \frac{\rho^1(1-z) + \Delta + (\rho^j - \rho^1)(1-z)e^{-(\rho^j(1-z)+\Delta)t_1(s)}}{(\rho^1(1-z) + \lambda + \Delta)(\rho^j(1-z) + \Delta)}$$

$$R = \delta \frac{(1-z)(\rho^j - \rho^1)\pi\lambda}{(\Delta + \lambda + (1-z)\rho^1)(\Delta + (1-z)\rho^j)(\Delta + \lambda + (1-z)\rho^j}$$

$$P^j_{shock} = \frac{C + \lambda_D(1-f)F + \lambda_T F}{\Delta} - \delta \frac{\rho^j(1-z) + \pi\lambda + \Delta}{\Delta(\rho^j(1-z) + \lambda + \Delta)}$$

$$t_1(s) = \log\Big(\frac{s}{\omega}\frac{\sum_{j=1}^N \rho^j}{\rho^1}\Big)/\lambda$$

the bid-ask fee is collected by the salesperson, not the marketmaker. Consistent with this, Bessembinder and Maxwell (2008) note, "In interviews, numerous corporate bond market participants ... told that, post-TRACE, bond dealers no longer hold large inventories of bonds for some of the most active issues; for less-active bonds, they now serve only as brokers."

$$t_2(s) = \log(\frac{s}{\omega})/\lambda$$
$$\Delta = r + \lambda_T + \lambda_D.$$

Prices decrease because low-type sellers arriving at dealers outnumber high-type buyers arriving at dealers for a while. During this period, some low-type investors are buying such that bond demand equals bond supply. To make markets clear, they buy at their reservation price, the price at which they are indifferent between buying or not. The term B^j_{shock} is the reservation price of a low-type investor with search intensity ρ^j in a situation in which there are more sellers than buyers permanently. The weight on B^j_{shock} depends on the time low-type sellers outnumber high-type buyers. The larger the shock s is, the longer the period is, and the lower the prices following the shock are. The period is determined through the fraction $\frac{s}{\omega}$. For this reason, I refer to $\frac{s}{\omega}$ as selling pressure in the empirical analysis rather than the shock size s itself.

Without the assumption of segmented markets, both high- and low-type unsophisticated investors buy bonds following a liquidity shock, as Appendix A shows. Furthermore, high-type sophisticated investors buy bonds, while low-type sophisticated investors sell bonds. That is, there would be no sell trades by investors with low search intensities. As the next theorem shows, bid and ask prices facing sophisticated investors decrease more than bid and ask prices facing unsophisticated investors.

Theorem 2.3. Relation between prices after a liquidity shock. Assume that the economy is in steady state and a liquidity shock of size $\omega < s \leq 1$ occurs. Assume that $\rho^i(1-z) + r + \lambda_T + \lambda_D > 1 + e^{-(r+\lambda_D+\lambda_T+\rho^1(1-z))t_1(\omega)-1}$. For $\rho_i < \rho_j$, prices immediately after the shock satisfy that $M_i(s) - M_j(s)$ is increasing in s, where M can be either the bid or ask price.

The theorem shows that the difference between the price paid by unsophisticated investors and the price paid by sophisticated investors is a monotonically increasing function of the shock size. The price can be either the bid or ask. The reason is that prices are outcomes of bargaining, and they reflect investors' alternatives to immediate trade. Buying investors have the alternative to search for a new counterparty, and this alternative is strong for sophisticated investors since they find new counterparties quickly. Therefore, they can negotiate higher price discounts. One might think that sophisticated sellers have an equally strong outside option. However, because sellers temporarily outnumber buyers, they sell at their reservation value and their outside options are irrelevant while the shock lasts. The condition $\rho^i(1-z) + r + \lambda_T + \lambda_D > 1 + e^{-(r+\lambda_D+\lambda_T+\rho^1(1-z))t_1(\omega)-1}$ is a sufficient condition for the theorem to hold, not a necessary condition.

3. Data

Corporate bond transaction data only recently became available on a large scale through TRACE. TRACE covers all trades in the secondary over-the-counter market for corporate bonds and accounts for more than 99% of the total secondary trading volume in corporate bonds. The public dissemination starts in July 1, 2002, with dissemination of a small subset of trades, and from October 1, 2004, almost all trades are disseminated.

I use a sample of noncallable, nonconvertible, straight coupon bullet bonds with maturity less than thirty years. I collect information for each bond from Bloomberg.[6] Their trading history is collected from TRACE covering the period from October 1, 2004, to June 30, 2009, and after filtering out erroneous trades, 10,050,090 trades are left. Error trades are filtered out using the approach in Dick-Nielsen (2009). Summary statistics are given in Panel B in Table 1. An average bond has a maturity of around five years and trades around 140 times each quarter, and each trade has a size of around \$225,000. Trade sizes are downward-biased because trade sizes in TRACE are capped at \$5,000,000 for investment-grade bonds and \$1,000,000 for speculative-grade bonds. Also, while the average bond trades around 140 times each quarter, the median bond trades only 18 times each quarter, so a small number of bonds trade often while the majority of bonds trade infrequently.

To estimate the search model outlined in the previous section, an estimate of round-trip costs in the dealer market is needed. The round-trip cost is the difference between the price at which a dealer sells a bond to a customer and the price at which a dealer buys a bond from a customer. Two main approaches to estimate round-trip costs exist in the literature. The first is on a given day to subtract average buy prices from average sell prices (Hong and Warga 2000; Chakravarty and Sarkar 2003). The second is a regression-based methodology in which each transaction price is regressed on a benchmark price and a buy/sell indicator (Schultz 2001; Bessembinder, Maxwell, and Venkataraman 2006; Goldstein, Hotchkiss, and Sirri 2007; Edwards, Harris, and Piwowar 2007). However, both approaches require a buy/sell indicator for each trade, which is not publicly available for the period up to October 2008. I estimate bid-ask spreads by a different procedure, which I describe below.

The methodology for estimating round-trip costs in this article is based on what I denote *imputed roundtrip trades* (IRT). IRTs are based on a situation that occurs with some regularity: A bond that does not trade for hours or days suddenly has two or three trades with the same volume within, say, five minutes. Such trades are likely part of a pre-matched arrangement in which a dealer has matched a buyer and seller. Once there is a match, a trade between the seller and the dealer and a trade between the buyer and the dealer are carried

[6] More precisely, I require for each bond a "no" in the fields "callable" and "convertible" and a "yes" in the fields "fixed" and "bullet." Approximately 5% of all bonds are convertible, 50% callable, 80% fixed, and 50% bullet.

Table 1
Summary statistics

Panel A: Straight coupon bullet bonds in the sample

	2004	2005				2006				2007				2008				2009	
	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2
# bonds	3,627	4,032	4,003	3,967	4,053	4,012	3,969	3,803	3,807	3,623	3,733	3,483	3,622	3,821	2,916	2,871	2,854	3,041	2,977
# trades (quarterly)	21	23	24	22	23	24	23	23	24	25	26	30	31	37	39	43	55	62	74
Trade size (in 1000)	181	173	190	197	185	202	179	164	185	203	173	153	151	153	162	151	151	141	160
Maturity (in years)	5.6	5.4	5.2	5.3	5.2	5.2	5.2	5.1	5.2	5.2	5.1	5.1	5.0	4.8	5.5	5.2	5.5	5.3	5.4
Price	104	103	102	102	99	99	98	99	100	101	100	99	99	100	99	96	86	89	93

Panel B: All straight coupon bullet bonds in TRACE

	2004	2005				2006				2007				2008				2009	
	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2
# bonds	4,922	5,270	5,378	5,318	5,308	5,173	4,977	4,769	4,726	4,515	4,583	4,358	4,407	4,494	3,429	3,402	3,411	3,528	3,527
# trades (quarterly)	114	129	133	101	114	112	103	104	106	105	109	117	125	173	178	188	276	342	414
Trade size (in 1000)	248	251	237	254	238	268	251	235	250	264	251	228	198	200	221	184	178	177	197
Maturity (in years)	5.9	5.7	5.5	5.5	5.4	5.4	5.4	5.3	5.4	5.4	5.3	5.2	5.2	5.1	5.7	5.5	5.7	5.6	5.7
Price	103	103	101	102	99	99	98	99	100	100	100	99	99	100	99	96	86	89	92

Panel C: IRT over time

	2004	2005				2006				2007				2008				2009	
	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2
All sizes	67	62	64	60	58	55	55	56	54	52	55	55	53	58	56	58	62	66	68
≤ 100K	77	71	72	69	65	63	63	63	62	60	62	60	57	62	61	62	65	71	73
> 100K, < 1,000K	28	30	31	27	27	24	23	24	23	23	28	32	33	42	37	41	47	47	49
≥ 1,000K	12	10	13	11	10	9	8	8	8	8	9	13	15	22	18	22	32	28	25

Panel D: IRT and trade size

Trade size	all	5K	10K	20K	50K	100K	200K	500K	1000K	> 1KK
IRT	59	82	68	68	65	57	48	35	23	16
Obs.(in 1000)	974	61	109	214	271	120	72	49	23	54

(continued)

Table 1
Continued

Panel E: IRT and maturity

Maturity	all	0–0.5y	0.5–1y	1–2y	2–3y	3–4y	4–5y	5–7y	7–10y	10–30y
IRT	59	26	34	44	48	57	57	68	72	103
Obs.(in 1000)	974	37	77	145	126	126	114	118	120	110

Panel F: number of IRT trades out of total number of trades

# trades	all	1	2	3	4	5–6	7–8	9–10	11–15	16–20	21–30	31–40	41–50	51–100	>100
% IRT	22	0	46	43	33	32	29	27	24	21	18	15	13	11	6
Bond days(in 1000)	1437	341	302	175	115	141	84	57	84	44	42	20	11	16	6

Panel G: volume of IRT trades out of total volume

# trades	all	1	2	3	4	5–6	7–8	9–10	11–15	16–20	21–30	31–40	41–50	51–100	>100
% IRT	13	0	19	14	14	14	14	14	13	13	13	13	12	12	12
Bond days(in 1000)	1437	341	302	175	115	141	84	57	84	44	42	20	11	16	6

Panel A shows summary statistics for the bonds in the sample. Panel B shows summary statistics for all straight coupon bullet bonds in TRACE. Panel C shows average round-trip costs (measured by IRT) in cents over time. Panel D shows average round-trip costs in cents as a function of trade size. Panel E shows average IRT in cents as a function of bond maturity. In Panels F and G, a bond day is defined as a day for a bond where there is at least one trade, and the bond days are sorted according to the number of trades occurring on that day. Out of the total number of bond trades on a bond day, Panel F shows the fraction of trades that are part of an IRT. For example, there are 302,000 observations of a bond trading two times in a day, and out of the 604,000 transactions 46% are part of an IRT. Panel G shows the fraction of volume that is part of an IRT. For example, there are 302,000 observations of a bond trading two times in a day, and out of the total volume of the 604,000 transactions, 19% is part of an IRT. The sample bonds are straight coupon bullet bonds, and the sample period is October 1, 2004, to June 30, 2009.

out. If a second dealer is involved in the pre-matching, there is also a trade between the two dealers. Therefore, for a given bond on a given day, if there are exactly two or three trades for a given volume and they occur within fifteen minutes, I define these trades to be part of an IRT. In an IRT, the highest price is assumed to be an investor buying from a dealer, the lowest price assumed to be an investor selling to a dealer, and the investor round-trip cost to be the highest minus the lowest price. I delete IRTs with a zero round-trip cost from the sample.[7] Beginning in November 2008, buy/sell indicators are available, which allows me to check the accuracy of IRTs for this subsample (shown in Appendix C). Appendix C shows that although IRTs tend to underestimate round-trip costs, the empirical results are robust to this bias.

Of the 10,050,090 trades in the full sample, 2,159,447 are part of IRTs, resulting in a total of 973,600 IRTs. Panel A in Table 1 shows summary statistics for the subsample of data consisting of IRTs. We see that average trade sizes are slightly lower compared with the full sample, average maturity is roughly the same, and the number of quarterly trades decreases from an average of around 140 to around 30. Approximately 20% of the bonds drop out of the sample. Panels F and G address whether IRTs occur mostly in the liquid or illiquid segment of the corporate bond market. Panel F shows that most of the IRT trades are in bonds that have few trades each day. However, Panel G shows that the total fraction of trade volume that is captured by IRTs is almost the same across bonds that trade frequently and infrequently. Thus, IRTs capture transaction costs for both liquid and illiquid bonds.

Summary statistics of trading costs using IRTs are given in Table 1, Panels C–E. Panel D shows that the average transaction cost is 59 cents and is decreasing as a function of trade size. This is consistent with findings in Schultz (2001), Chakravarty and Sarkar (2003), and Edwards, Harris, and Piwowar (2007). Transaction costs are increasing in maturity, as Panel E shows. Costs are around four times as large for long maturities compared with short maturities. Finally, Panel C shows that average transaction costs decrease from 2004 to 2006 and increase during the subprime crisis 2007–2009. There are significant differences in the time-series pattern for large and small trades. Transaction costs for small trades are relatively stable during the sample period. Costs for large trades increase with a factor of 4 from the first quarter in 2007 to the fourth quarter in 2008.

The main point in this article is that price differences between small and large trades identify periods of selling pressure, and this is so for both buy and sell prices. The onset of the subprime crisis caused liquidity in the U.S. corporate bond market to dry up (Bao, Pan, and Wang 2011; Dick-Nielsen, Feldhütter, and Lando forthcoming), and Table 2 shows summary statistics for

[7] IRTs are closely related to Green, Hollifield, and Schürhoff's (2007) "immediate matches." An "immediate match" is a pair of trades where a buy from a customer is followed by a sale to a customer in the same bond for the same par amount on the same day with no intervening trades in that bond. However, since there is no information about the sides in the transactions in the TRACE database, "immediate trades" cannot be calculated.

Table 2
Differences in prices for small and large trades

Panel A: Small buy – large buy (early)

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−1	7	13	11	8
250K–500K	3	15	26	26	17
500–1,000K	2	22	36	35	24
> 1,000K	8	33	35	52	32
Average	3	19	28	31	

Panel B: Small sell – large sell (early)

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−15	−16	−18	−31	−20
250K–500K	−17	−22	−23	−39	−25
500–1,000K	−23	−22	−20	−42	−27
> 1,000K	−21	−12	−21	−21	−19
Average	−19	−18	−20	−33	

Panel C: Buy-diff(early) – buy-diff(late)

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−7	−13	−7	−16	−11
250K–500K	−19	−23	−28	−32	−25
500–1,000K	−33	−30	−33	−46	−36
> 1,000K	−29	−31	−46	−58	−41
Average	−22	−24	−29	−40	

Panel D: Sell-diff(early) – sell-diff(late)

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−17	−23	−22	−34	−24
250K–500K	−30	−36	−41	−51	−40
500–1,000K	−44	−40	−47	−64	−49
> 1,000K	−40	−37	−47	−65	−47
Average	−33	−34	−39	−54	

The sample period is split into an early period, 2004Q4–2007Q2, and a late period, 2007Q3–2009Q2. Panel A shows for the early period the average price difference in cents for an investor buy with a small volume minus an investor buy with a large volume in the same bond on the same day. Panel B shows the same price difference for investor sells. For example, the average price difference for a 7–30-year bond between investor sells with a volume of more than $1,000,000 and investor sells with a volume less than $100,000 is 21 cents. Panel C shows the difference in buy differences between the early period and late period. Panel D shows the differences in sell difference between the early and the late period. For example, the average price difference for a 7–30-year bond between investor sells with a volume of more than $1,000,000 and investor sells with a volume less than $100,000 is 21 cents in the early period and −44 cents in the late period, leading to a difference of 65 cents, as the table shows.

price differences across trade sizes before the onset of the crisis, 2004Q4–2007Q2, and after, 2007Q3–2009Q2. In the table, a difference in trade prices is recorded if two trades occur within the same day in the same bond. Panel A shows that during the period 2004Q4–2007Q2, large buyers paid less than small buyers and the difference was increasing in the maturity of the bond and trade size. For example, the selling price for a long-maturity bond was on average 52 cents higher for a trade of at least \$1,000,000 compared with a trade of \$100,000 or less. Similarly, Panel B shows that large investors sold at higher prices than small investors, and we see that the difference increased in bond maturity and trade size, although the pattern is less pronounced than for investor buy transactions.

The results in Panels A and B of Table 2 are not surprising given that transaction costs increase in bond maturity and decrease in trade size. More strikingly, Panels C and D show how price differences across trade sizes changed after the onset of the subprime crisis. While the average selling price of a long-maturity bond was 21 cents higher in a large trade compared with a small trade before the subprime crisis, Panel D shows that this decreased by 65 cents after the onset of the crisis, such that the average selling price in large trades was now 44 cents lower than in a small trade. We see from Panels C and D that the impact of the subprime crisis on price differences is increasing in bond maturity and trade size. In addition, the impact is similar in buys and sells.[8]

Overall, Panels C and D show that price differences change systematically during the subprime crisis and the size of the change depends on both bond maturity and trade size.

4. Estimation Methodology

Liquidity risk and credit risk are hard to empirically disentangle, since prices decrease in response to an increase in either of them. Assuming that large traders are more sophisticated than small traders, the model in this article predicts that prices of large traders react stronger to selling pressure than those of small traders. Therefore, a liquidity shock can be identified through the relation of small trades versus large trades.

A simple approach to identify liquidity shocks would be to calculate price differences between small and large trades, where the cutoff between small and large is some chosen dollar value. This approach is problematic for several reasons. First, information in trades of different sizes is ignored. For example, if the cutoff is \$500,000, then any price differences between trades in the size range \$250,000–500,000 and \$0–100,000 are not used in inferring liquidity shocks. As Panels C and D in Table 2 show, there is a significant amount of information in those price differences. Second, maturity effects are ignored. When following the same bond over time, the maturity of the bond shortens, and this has an effect on the impact of liquidity shocks on price differences, as Panels C and D show. Third, many bonds trade infrequently, so when constructing the measure, there are many missing observations over time.

[8] Although not shown, the pattern for median price differences is very similar and in some cases more pronounced compared with the pattern for mean price differences, so the results are not due to a small number of extreme observations.

To overcome the limitations of the simple approach, I structurally estimate the search model in Section 2. In the structural approach, information is extracted from the whole cross-section of trade sizes. The longer maturity a bond has, the stronger is the price reaction to selling pressure. In the structural approach, this is taken into account when extracting information from bond trades. And missing observations are easy to handle.

In the estimation, I fit the model to demeaned prices. By demeaning, effects due to credit risk or other fundamental effects are "filtered out," while cross-sectional differences in trade prices identify liquidity effects. Any bid or ask prices for a given bond on a given day are demeaned with the average of all bid and ask prices for this bond on this day. All prices refer to trades that are part of IRTs. That is, if there are N_{tb} IRTs on bond b on day t, and A_{tbi} is the ith ask price and B_{tbi} the corresponding bid price, the demeaned ask price is defined as $A_{tbi} - \overline{AB}_{tb}$ and demeaned bid price as $B_{tbi} - \overline{AB}_{tb}$, where $\overline{AB}_{tb} = \frac{1}{2N_{tb}} \sum_{i=1}^{N_{tb}} (A_{tbi} + B_{tbi})$.

Let Θ be a vector with the parameters of the model, and s be a shock size between 0 and 1 defined in Definition 2.1. For day t and bond b, all demeaned bid and ask prices are denoted $P_{tb}^1, P_{tb}^2, \ldots, P_{tb}^{2N_{tb}-1}, P_{tb}^{2N_{tb}}$ (the sorting does not matter). With a shock size of s on day t, the demeaned fitted prices $\hat{P}_{tb}^1(\Theta, s_t), \hat{P}_{tb}^2(\Theta, s_t), \ldots, \hat{P}_{tb}^{2N_{tb}-1}(\Theta, s_t), \hat{P}_{tb}^{2N_{tb}}(\Theta, s_t)$ are calculated using Theorem 2.2. I assume that fitting errors are independent and normally distributed with zero mean and a standard deviation that depends on the maturity of the bond

$$P_{tb}^i - \hat{P}_{tb}^i(\Theta, s_t) \sim N(0, w_{tb}\sigma^2),$$
$$w_{tb} = \max(1, T_{tb}),$$

where T_{tb} is the maturity of bond b on day t. The choice of w_{tb} is motivated by the fact that pricing errors tend to increase with maturity, while at the same time excessive influence of prices for bonds with maturity close to zero is avoided. With this error specification, we have that

$$\epsilon_{tb}^i(\Theta, s_t) = \frac{P_{tb}^i - \hat{P}_{tb}^i(\Theta, s_t)}{\sqrt{w_{tb}}} \sim N(0, \sigma^2).$$

The likelihood function is given as

$$\begin{aligned} -2 \log L(\Theta | Y) = & \frac{1}{\sigma^2} \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} \epsilon_{tb}^i(\Theta, s_t(\Theta))^2 \\ & + \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} [\log(\sigma^2) + 2\pi], \end{aligned} \qquad (2)$$

where N_b is the number of bonds in the sample and $s_t(\Theta)$ is defined as

$$s_t(\Theta) = \arg\min_{\xi} \sum_{\substack{\text{all days } u \text{ that} \\ \text{belong to same} \\ \text{month as day } t}} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{ub}} \epsilon_{ub}^{i}(\Theta, \xi)^2. \tag{3}$$

That is, I assume that all days in a month experience the same liquidity shock, and for a given parameter vector Θ this shock is found to be minimizing the sum of squared pricing errors for that month's prices. The approach is similar to that of Jarrow, Li, and Zhao (2007), and a more detailed discussion about the estimation procedure can be found there. The estimation jointly estimates the parameter vector Θ and a time series of monthly liquidity shocks.

I use trade size as a proxy for investor sophistication. Specifically, there are six investor classes that differ in their search intensity ρ, and they trade in par values of \$0–10,000, \$10,000–50,000, \$50,000–100,000, \$100,000–500,000, \$500,000–1,000,000, and more than \$1,000,000.[9] Goldstein, Hotchkiss, and Sirri (2007) and Bessembinder, Kahle, Maxwell, and Xu (2009) find that trades smaller than \$100,000 are mainly retail trades and trades bigger than \$100,000 are predominantly institutional trades. So, one interpretation of a small trader is that of an unsophisticated retail investor, while a large trader is a sophisticated institutional investor. Lagos and Rocheteau (2009) and Gârleanu (2009) ease the restriction that asset holdings are zero or one. They find that there is a positive relationship between trade size and sophistication, as measured by search intensity. The restriction on asset holdings does not allow for such a positive relationship here, but I control for this empirically by using trade size as a proxy for investor sophistication. For future research, it would be interesting to exploit trade size information in the estimation even more by allowing for arbitrary asset holdings.

There are a number of parameters in the model for which historical estimates are available. The riskless rate is set to $r = 0.05$, which is close to the average ten-year swap rate of 4.94% in the estimation period. The bond coupon is set to seven, close to the average coupon rate in the sample period, and face value to $F = 100$. The default intensity is set to $\lambda_D = 0.012$, and the recovery rate on the bond in case of default is set to 42% such that $f = 0.58$. The last two are averages for the period 1994–2008 (see Exhibit 26 and 45 in Moody's 2009). I could let the riskless rate be time-varying in the estimation, allow for different default intensities across rating, and let the bond coupon reflect each bond's actual coupon. Since the effect on the estimation results of doing so is small because I fit to demeaned prices and not to price levels, I choose the more parsimonious approach. Finally, I set $\omega = 0.0001$. The parameters to estimate are $\Theta = (\delta, \lambda, \pi, z, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6)$.

9 Table 1 shows that average trade size decreases from \$180,000 to 200,000 to approximately \$150,000 during the subprime crisis (see Dick-Nielsen, Feldhütter, and Lando forthcoming for a further discussion). This might influence the results, but the effect is likely to be small because the differences in the trade size of investor classes are large.

5. Empirical Results

5.1 Parameter estimates and model fit

Table 3 shows the parameter estimates. We see that search intensities increase as trade size increases, so more sophisticated investors trade in larger sizes. The most unsophisticated investors (trading in sizes between 0 and \$5,000) have a search intensity of 40. This implies that they need around a week on average before they find a dealer with whom to trade with. This can be viewed as the time it takes a nonprofessional to learn how to trade in the corporate bond market, keep up to date about information relevant for trading, and find an alternative dealer in case his preferred one gives him uncompetitive prices. The most sophisticated investors (trading sizes of more than \$1,000,000) have a search intensity of 372, implying that it takes half a day to complete trades of large size. The product $\lambda\pi = 0.33$ implies that, without aggregate liquidity shocks, it is a rare event for a corporate bond investor to be hit by a liquidity shock; it occurs on average once every three years. A liquidity-shocked investor remains shocked for about three months since $\lambda = 3.58$. The estimated bargaining power of dealers of $z = 0.97$ shows that dealers are in a strong bargaining position relative to investors.

Panel A of Table 4 shows fitted round-trip costs. The model underestimates round-trip costs for the smallest trades, while round-trip costs for large trades are matched well. In particular, the strong negative relation between trade size and trading costs is captured. In Panel B, we see that the model replicates the positive relation between round-trip costs and bond maturity although costs are underestimated for long-maturity bonds. Chakravarty and Sarkar (2003) point to increased interest rate risk as a possible explanation for the positive relation between trading costs and maturity. This analysis shows that to a large extent the relation can be explained by better outside options of investors trading short maturity bonds.

Panels C and D of Table 4 show the change in price differences between small and large trades after the onset of the subprime crisis. Remember that these differences identify selling pressure in the model. Compared with actual changes in Panels C and D of Table 2, we see that the model largely captures the size of the change for both buy and sell transactions. Also, the model captures the relation between price difference changes and bond maturity and

Table 3
Parameter estimates

δ	λ	π	z	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
2.911	3.580	0.092	0.970	40	38	50	101	278	372
(0.003)	(0.090)	(0.013)	(0.001)	(1.1)	(1.0)	(0.9)	(1.7)	(23.7)	(8.5)

This table shows estimated parameters of the search model. Model parameters are estimated by maximum likelihood, and standard errors are calculated using the outer product of gradients estimator. Corporate bond data used in estimation are transactions from TRACE for the period October 1, 2004, to June 30, 2009.

Table 4
Estimated round-trip costs and price differences

Panel A: Trade size [Panel D in Table 1]

0–10K	11–50K	51–100K	101–500K	501–1000K	> 1000K
54	54	47	38	21	19

Panel B: Maturity [Panel E in Table 1]

0–2m	2m–4m	4–6m	6m–1y	1–5y	5–30y
18	30	37	43	50	52

Panel C: Buy-diff(early) - buy-diff(late) [Panel C in Table 2]

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−15	−22	−22	−24	−21
250K–500K	−21	−28	−27	−29	−27
500–1,000K	−32	−43	−42	−46	−41
> 1,000K	−33	−43	−44	−46	−42
Average	−26	−34	−34	−36	

Panel D: Sell-diff(early) - sell-diff(late) [Panel D in Table 2]

	0–2y	2–5y	5–7y	7–30y	Average
0–100K	0	0	0	0	0
100K–250K	−15	−22	−22	−24	−21
250K–500K	−20	−29	−28	−31	−27
500–1,000K	−32	−44	−43	−46	−41
> 1,000K	−33	−43	−44	−46	−42
Average	−25	−34	−34	−37	

This table reports model-fitted round-trip costs and price differences. Panel A compares with Panel D in Table 1, Panel B compares with Panel E in Table 1, Panel C compares with Panel C in Table 2, and Panel D compares with Panel D in Table 2.

trade size. Thus, the model captures how price differences change along bond maturity and trade size, and it does so for both buy and sell prices.

The following calculations provide an estimate of the additional cost due to search that investors in the corporate bond market incur compared with that of the Treasury market. The average maturity in the data sample is 5.5 years, so a five-year bond is the most representative bond for the corporate market. An estimate of the average bid-ask spread as a percentage of par value of a five-year bond in the Treasury market is 0.012% according to Fleming (2003). For an average investor, i.e. an investor with an average search intensity, the corresponding estimate for a five-year bond in the corporate bond market is 0.343% according to the parameter estimates and Equation (1). Thus, an estimate of the cost of search on a trade in the corporate bond market relative to the Treasury market is half the round-trip cost, 0.166%. The yearly trading volume in the corporate bond market was $4,284 billion in 2009.[10] So, an estimate of the

[10] Average daily volume in the U.S. corporate bond market was $16.8 billion according to the Securities and Financial Markets Association (www.sifma.org), so yearly was 255*$16.8 billion.

additional yearly costs investors bear in the corporate bond market compared with the Treasury market is $4,284 billion$*$0.166\% = \$7.1$ billion.

5.2 Selling pressure

In response to a liquidity shock, prices decrease and slowly return to their equilibrium level as time passes. In previous literature, selling pressure is identified through this pattern. However, it is difficult to disentangle price effects due to a liquidity shock from price effects due to changes in fundamentals. For example, a downgrade might lead to selling pressure, but there is also an informational effect of the downgrade.

In this article, selling pressure is identified by the cross-sectional variation in prices. For example, assume that the difference in bid prices in a bond between large trades and small trades in steady state is 20 cents. If this decreases to 10 cents one month, there is a liquidity shock that month. If it decreases to, say, -10 cents, there is an even larger liquidity shock. The same pattern in ask prices identifies liquidity shocks, and the estimation procedure uses the information in both bid and ask prices. Note that the shock size is identified only through multiple observations of bid and ask prices in a bond on a given day for investors with different search intensities. If investors were not sorted according to sophistication and there instead was a single representative investor, shocks could not be identified.

Figure 3 graphs the estimated selling pressure. A 95% confidence interval is bootstrapped according to Bradley (1981).[11] In the first part of the sample period, there is one modest shock occurring. GM and Ford were downgraded to junk bond status in May 2005, causing a test for the corporate bond market because of the amount of GM/Ford debt outstanding. To examine the effect of the downgrade on the corporate bond market more closely, Figure 4 shows the selling pressure for Ford bonds, GM bonds, and the rest of the corporate bond market around this period.

In late 2004, S&P downgraded GM to BBB–, the last rating notch before a junk rating, and the graph shows some selling pressure in this period consistent with evidence in Acharya, Schaefer, and Zhang (2008). Many bond investors and asset managers are restricted to invest in only investment-grade bonds, so they started to sell off GM bonds, anticipating the future downgrade to junk. BIS (2005) write, "The downgrade had long been anticipated and so asset managers had ample opportunity to adjust their portfolios. Since mid-2003, the automakers' spreads had been trading closer to speculative-grade issuers than those on other BBB-rated issuers." As it became increasingly likely that especially GM would be downgraded, selling pressure increased in the beginning of 2005. Interestingly, selling pressure temporarily decreased in February 2005. On January 24, 2005, Lehman announced that it would

[11] For each month, the bootstrapped standard errors are based on 500 simulated datasets.

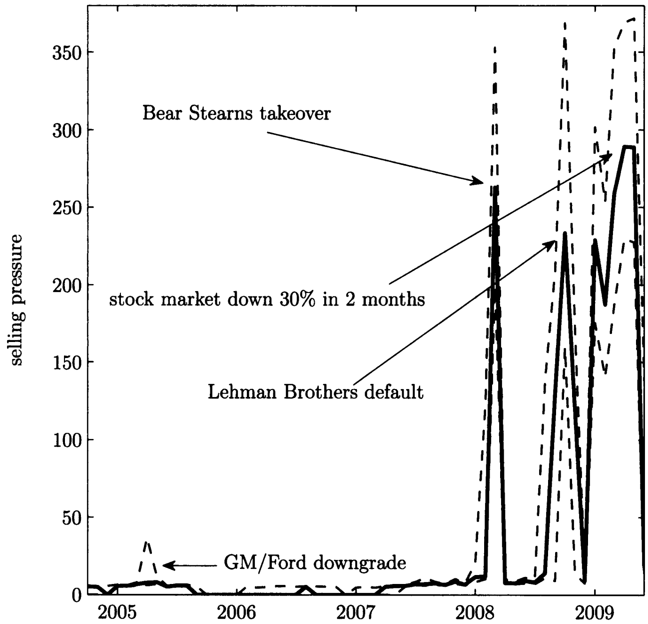

Figure 3
Selling pressure in the corporate bond market
This graph shows the estimated time variation of selling pressure in the corporate bond market. Higher values mean that more investors wish to sell. Selling pressure is defined as $\frac{s}{\omega}$, as explained in Section 2. A 95% confidence interval for selling pressure is bootstrapped and shown as dashed lines.

change methodology for computing its index rating for bonds where the rating agencies disagree on whether it is investment grade or junk (see Chen, Lookman, Schürhoff, and Seppi 2009). Before the announcement, it was the lower of S&P and Moody's rating. Beginning July 1, 2005, it would be the middle rating of S&P, Moody's, and Fitch. For many investment-grade investors, the Lehman investment-grade index is an important benchmark. This move made it less likely that Ford/GM bonds would drop out of the index since a downgrade from one of the two major rating agencies was now not sufficient to cause such a drop. This change likely caused a temporary ease in selling pressure. However, a steep profit warning from GM on March 16 reintensified selling pressure, and in May selling pressure peaked when GM was downgraded to junk by both Fitch and S&P. We see a different selling pressure pattern for Ford bonds, which had a peak in April and a decrease in May. In contrast to GM bonds, Ford bonds were downgraded by only S&P and were still classified as investment grade under the new Lehman index rule. We see from the figure that there was at best moderate selling pressure in other bonds, so the sell-off was concentrated in GM and Ford bonds.

A period with a large number of forced sellers according to Figure 3 began in fall 2007 when interbank markets froze and the "credit crunch" began.

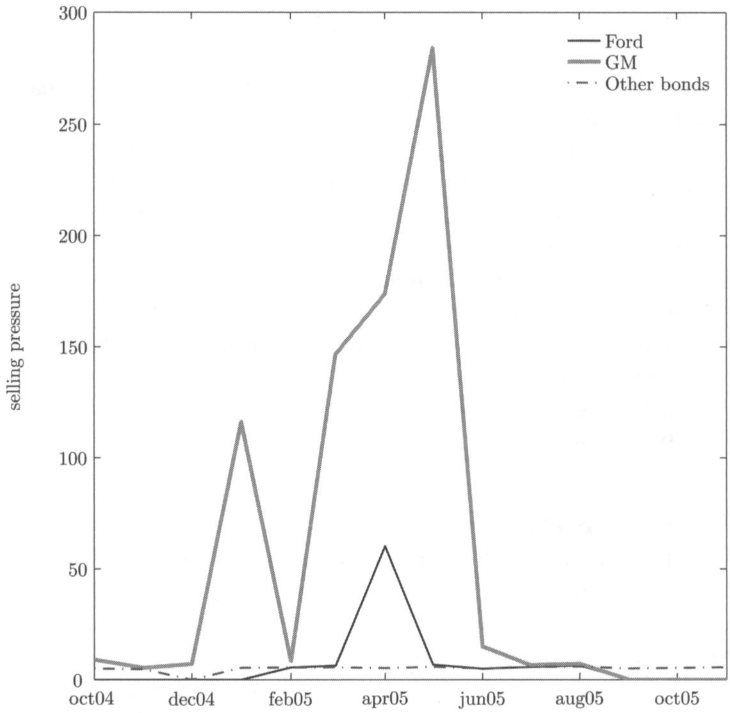

Figure 4
Selling pressure in GM and Ford bonds around their downgrade to junk
This graph shows the time variation in selling pressure in GM bonds, Ford bonds, and the rest of the corporate bond market around the downgrade of GM and Ford to junk in 2005. The y-axis shows selling pressure, and higher values correspond to more sellers.

However, the first signs of selling pressure appeared already in April 2007 when the subprime mortgage crisis spilled over into the corporate bond market (Brunnermeier 2009). Figure 3 shows a large shock in March 2008. BIS (2008) write, “Turmoil in credit markets deepened in early March...tightening repo haircuts caused a number of hedge funds and other leveraged investors to unwind existing positions. As a result, concerns about a cascade of margin calls and forced asset sales accelerated the ongoing investor withdrawal from various financial markets. In the process, spreads on even the most highly rated assets reached unusually wide levels, with market liquidity disappearing across most fixed income markets.” A liquidity squeeze on Bear Stearns caused a takeover by JPMorgan on March 17. The Federal Reserve cut the policy rate by 75 basis points, and “(t)hese developments appeared to herald a turning point in the market...with investors increasingly adopting the view that various central bank initiatives aimed at reliquifying previously dysfunctional markets were gradually gaining traction” (BIS 2008). The selling pressure in May 2008 is very low compared with a few months earlier. According to BIS (2008), “By the end of the period in late May, the process of disorderly deleveraging had come to a halt, giving way to more orderly credit market conditions. Market liquidity had improved and risk appetite increased, luring investors back into the market." However, this rebound of the corporate bond market was short-lived, and the model-implied liquidity shocks peaked again in September and October 2008. Lehman Brothers filed for bankruptcy on September 2008, one of the biggest credit events in history, triggering a new and intensified stage of the credit crisis. At the end of 2008, there was a brief halt in selling pressure, but in the first three months of 2009, selling pressure intensified again. In this period, stock markets lost more than 30% in value. This loss likely worsened funding conditions, leading to a loss in liquidity across asset classes as predicted by Brunnermeier and Pedersen (2009). Finally, the second quarter of 2009 saw a decrease in the selling pressure consistent with credit spreads tightening in the period.

5.3 The credit spread puzzle

One of the most widely employed frameworks of credit risk, structural models, was developed in the seminal work of Merton (1974). Structural models take as given the dynamics of the value of a firm and value corporate bonds as contingent claims on the firm value. In structural models, the spread between the yield on a corporate bond and the riskless rate goes to zero as maturity shortens. However, yield spreads are typically positive, also at very short maturities. This has given rise to the "credit spread puzzle," namely that corporate yield spreads are too high to be explained by the corporate bond issuer's default risk (see, for example, Huang and Huang 2003; Chen, Collin-Dufresne, and Goldstein 2009). The puzzle is particularly severe at very short maturities. Consistent with this evidence, among others, Longstaff (2004), Longstaff, Mithal, and Neis (2005), and Feldhütter and Lando (2008) find a large non-default component. In this section, I examine to what extent search frictions and selling pressures can explain this non-default component.[12]

I define the search premium for an investor as the midyield paid by this investor in steady state minus the yield in steady state of an investor who can instantly find a trading partner ($\rho = \infty$).[13] This mimics a trade in the corporate bond market versus a trade in the liquid Treasury market. I do this for an "average" corporate bond investor, where the search intensity

[12] The model in this article is related to reduced-form models of credit risk, where there is an intensity process governing the risk of default. Thus, it does not predict a near-zero contribution of default risk to spreads at very short maturities as structural models. Nevertheless, the implications of search costs can be examined in the model.

[13] It is easy to show that the discounted present value of the promised payments of a bond using discount rate y is $E(\int_0^{\tau_T} Ce^{-yt}dt + e^{-y\tau_T} F) = \frac{C+\lambda_T F}{y+\lambda_T}$ where τ_T is the (stochastic) maturity. Therefore the yield y on a bond with price P is $y = \frac{C+\lambda_T F}{P} - \lambda_T$, which is the formula used to convert prices to yields.

is the average of all the estimated search intensities in Table 3.[14] For the same "average" investor, I define the selling pressure premium as the average estimated midyield across all months minus the average midyield across all months where all liquidity shocks are set to zero.

Figure 5 graphs the term structure of search premia and selling pressure premia. The figure shows that search costs affect primarily the short end of the yield curve. The premium is more than 100 basis points for bonds with very short maturities (less than three weeks). For bonds with maturities of more than two years, the effect of search costs is in the single-digit range. There are two reasons for the downward-sloping costs of search. First, if a liquidity-shocked investor owns a short-maturity bond, he is likely to be liquidity-shocked during the life of the bond. This means that he values the bond almost as a bond paying $C - \delta$ in coupons. So, the yield cost of holding the bond is δ. If a liquidity-shocked investor owns a long-maturity bond, he values it higher than a bond paying coupons $C - \delta$ because he will likely switch type during the life of the bond. Therefore, the average yield cost of holding the bond is less than δ. Second, if an investor owns a short-maturity bond, he has fewer trading opportunities during the life of the bond. So, for a short-maturity bond, the alternative to trade is essentially to let the bond mature. For a long-maturity bond, the additional alternative is to look for another counterparty.

Turning to the impact of selling pressure, Figure 5 shows that the average effect decreases as a function of maturity. The yield spread due to selling pressure at a given maturity can be viewed as the average of future expected excess returns. The initial returns at the beginning of a liquidity shock are high, so for short maturity bonds all future expected excess returns are high. For long maturity bonds, the average is over initial high returns and subsequent lower returns when the economy has fully recovered. Therefore, the effect is stronger for short maturity bonds.[15]

For a five-year bond, the selling pressure effect is 40 basis points. Huang and Huang (2003) and Longstaff, Mithal, and Neis (2005) find the average non-default component of the five-year AAA-Treasury spread to be 50–55 basis points. The combined effect of selling pressure and search costs is 45 basis points in my data sample. Such comparisons should be interpreted with care due to differences in sample periods, but the comparison does suggest that the estimated premium is close to but underestimates that reported in the literature. One reason might be that investors in the model do not recognize the possibility of a future liquidity shock. To examine this, I solve the model in the case where investors anticipate future liquidity shocks. This is done in Appendix B.

14 Specifically, when I calculate the yields for an "average" investor, I set ρ_4 to 147 instead of 101 and look at yields for investor 4. When I calculate yields for an investor with $\rho = \infty$, I set $\rho_6 = \infty$ instead of 372 and look at yields for investor 6.

15 The premium due to selling pressure is hump-shaped at very short maturities. This is because the rate of price increases for a while is higher when markets become integrated compared with immediately after the shock.

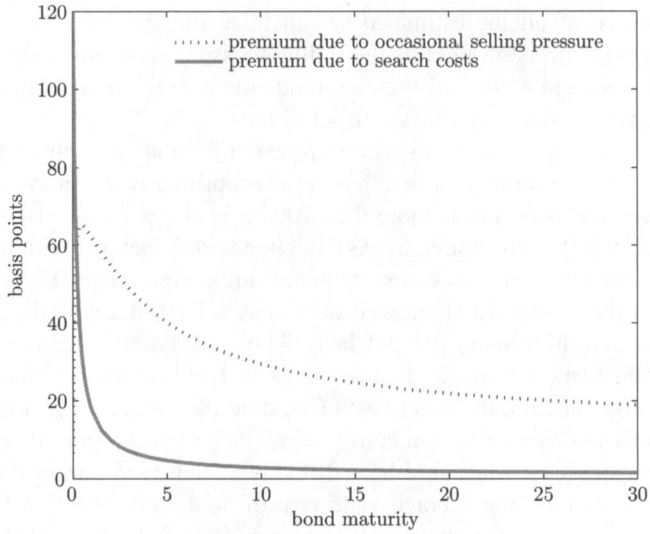

Figure 5
Premium in yields due to search costs and occasional selling pressures
This graph shows the premium in yields across bond maturity due to search costs and occasional selling pressures. The search premium is the average yield at which a corporate bond investor transacts in steady state minus the steady-state yield an investor who can trade instantly trades at. The selling pressure premium is the estimated yield—averaged over the sample period—paid by a corporate bond investor minus the average yield that would have prevailed in absence of any selling pressure shocks.

To keep the model tractable, I assume that when an aggregate liquidity shock occurs all investors become low investors. This is a severe shock. An aggregate shock happens with intensity λ_l. Figure 6 shows the impact on steady-state yields when investors anticipate aggregate shocks.[16] The figure shows that the impact on steady-state yields is increasing in maturity. Steady-state prices are the long-run prices in absence of liquidity shocks happening. Again, we can think of the yield spread as an average of future expected excess returns. If the economy is hit by an aggregate shock, returns are negative. Prices rebound subsequently, but if an investor needs to sell the bond before prices have recovered, he has a loss. To be compensated for this, steady-state prices are lower (and yields higher). The amount of compensation depends on the probability of the market being under selling pressure when selling. For interpretation, assume that we have discrete time periods Δ_t. Since we look at steady-state prices, it is implicitly assumed that it is a long time since a shock has happened. For a one-period bond, the probability of the price being low in period one is the probability of an aggregate shock happening, $\Delta_t\lambda_l$. Assume

[16] The impact of aggregate shocks on the selling pressure premium and search cost premium is small for the values of λ_l shown in Figure 6.

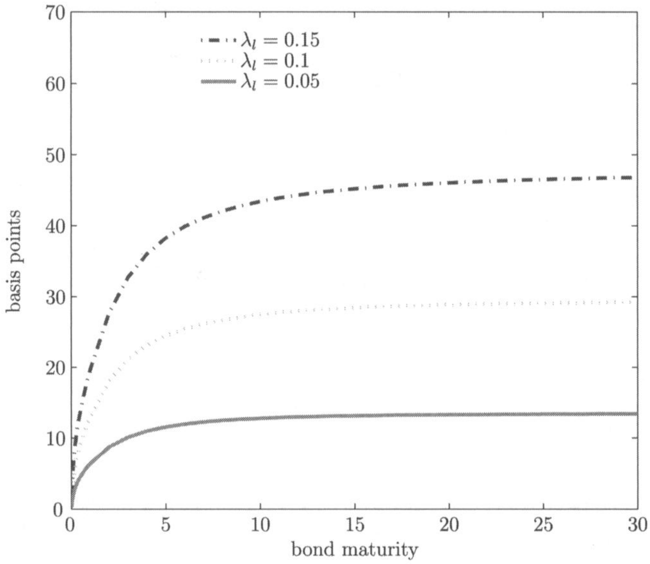

Figure 6
Yield spread due to future expected selling pressure
Assume that investors anticipate that future liquidity shocks leading to selling pressure might occur. This graph shows how yields in steady state are affected relative to the case in which investors do not anticipate future liquidity shocks. λ_l is the intensity of aggregate liquidity shocks.

now that a liquidity shock lasts for several periods. For a two-period bond, the probability of the price being low in period one is $\Delta_t \lambda_l$. The probability in period two is $2\Delta_t \lambda_t$ because the shock might have occurred in period one or two. Therefore, the compensation in a two-period bond is higher than in a one-period bond. Obviously, the compensation for all maturities increases as the frequency of aggregate shocks, λ_l, increases.

The presence of aggregate shocks potentially influences the identification of liquidity shocks. To examine this, we look at two different investors, the average corporate bond investor with a search intensity of $\rho_1 = 147$ and the most sophisticated investor with $\rho_2 = 372$. Figure 7 shows for different bond maturities the price differences between the investors in steady state and after an aggregate shock. More specifically, the y-axis shows

$$P(\rho_1, \text{shock}) - P(\rho_2, \text{shock}) - [P(\rho_1, \text{steady state}) - P(\rho_2, \text{steady state})],$$

where $P(\rho_1, \text{state})$ is the bid or ask price of an investor with search intensity ρ_1 in state equal to "shock" or "steady state." As the figure shows, the difference in price differences are only modestly influenced by aggregate liquidity shocks. So, the selling pressure measure is robust to investors anticipating future aggregate shocks.

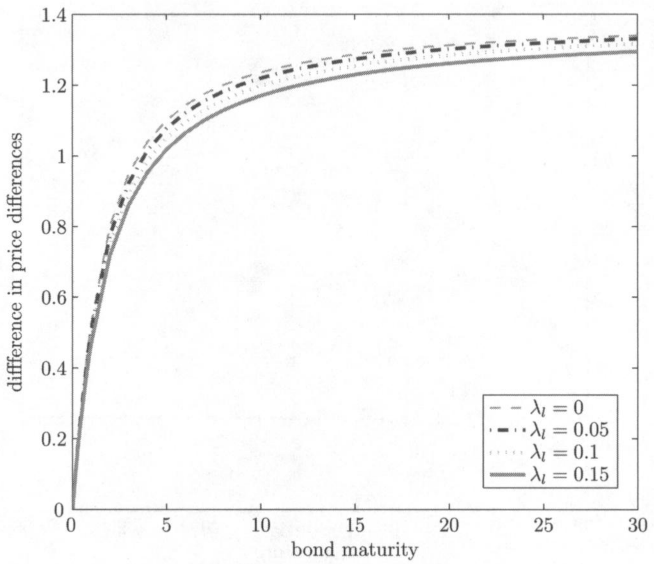

Figure 7
Identification of selling pressure in presence of aggregate liquidity shocks
This graph shows how selling pressure can still be identified in presence of aggregate liquidity shocks. Price differences are graphed for an average corporate bond investor (search intensity $\rho_1 = 147$) and the most sophisticated investor in the sample ($\rho_2 = 372$). On the y-axis, we have the price difference between the two investors after a shock minus the price difference in steady state. λ_l is the intensity of aggregate liquidity shocks.

6. Conclusion

For over-the-counter traded assets, I propose a measure that identifies when asset prices are affected by selling pressure—namely, the price difference between small and large trades. In a model capturing the search-and-bargaining features in over-the-counter markets, the connection between the measure and selling pressure is derived. I structurally estimate the model using U.S. corporate bond transaction data from October 2004 to June 2009. The estimation provides new insights into two periods of selling pressure: the downgrade of GM/Ford and the subprime crisis. Also, the effect of trading frictions and selling pressures on the term structure of corporate bond illiquidity premia is examined.

The analysis raises a number of questions. Here, the U.S. corporate bond market is examined on an aggregate level. An extension is to understand the cross-sectional variation in selling pressure across bonds. The U.S. municipal bond market is another illiquid over-the-counter market with transaction data available, and the nature of its illiquidity can be studied using the approach in this article. Even in highly liquid markets, search frictions matter, as Ashcraft and Duffie (2007) show for the federal funds market, and the Treasury market can be viewed through the lens of a search model. In late 2008 and early 2009, there was selling pressure in the corporate bond market, while short-term yields in the Treasury market were close to and on a few occasions below zero, indicating strong demand. My model addresses selling pressure, but with a few modifications it can be used to examine buying pressure in the Treasury market.

Appendix A: Equilibrium Allocations and Prices

In this Appendix, the results in the text are derived. To solve the model, I adopt the analytical methods developed in Lagos and Rocheteau (2009), Lagos and Rocheteau (2007), and Lagos, Rocheteau, and Weill (2009). These articles contain more intuition with respect to the derivations. All Poisson processes are independent.

A.1 Investor's value function

Let $U_i^j(a, t, W_t)$ be the maximum expected discounted utility attainable by an investor who has preference type i, search intensity ρ_j, and wealth W_t, and who holds a of the bond. The preference type is 1 for "high" or 2 for "low." The bond holding a is 1 or 0, and t indicates time. Each investor's utility for future consumption depends on his current type, search intensity, asset holding, and wealth W_t in his bank account. More specifically,

$$U_i^j(a, t, W_t) = \sup_{\zeta,\theta} E_t \int_0^{\infty} e^{-rs} d\zeta_{t+s}, \tag{4}$$

subject to

$$dW_t = rW_t dt - d\zeta_t + \theta_t (C - \delta 1_{\{k(t)=2\}})dt - \hat{P}_t^j d\theta_t, \tag{5}$$

where ζ is a cumulative consumption process, $\theta_t \in \{0, 1\}$ is a feasible holding process, k is the preference type process, and at the time of a possible holding change, $\hat{P}^j$ is the "trade price." The trade price $\hat{P}_t^j \in \{A_t^j, B_t^j, F, (1-f)F\}$ can be the bid or ask price paid in a transaction, the face value if the bond matures, or the fraction of face value if the bond defaults. From (4) and (5), we have that lifetime utility is $W(t) + V_i^j(a, t)$, where

$$V_i^j(a, t) = \sup_{\theta} E_t [\int_t^{\infty} e^{-r(t-s)} \theta_s (C - \delta 1_{\{k(s)=2\}})ds - e^{-r(s-t)} \hat{P}_s^j d\theta_s].$$

Let T_M be the time the asset matures, T_D the next time the asset defaults, T_ρ^j the next time a dealer is met, and $T_{\rho MD}^j = \min(T_M, T_D, T_\rho^j)$. Then, the value function satisfies

$$\begin{aligned} V_i^j(0, t) = E_i\Big[e^{-r(T_\rho^j - t)} \{ V^j_{k(T_\rho^j)}(a_{k(T_\rho^j)}(T_\rho^j), T_\rho^j) - p^j(T_\rho^j) a_{k(T_\rho^j)}(T_\rho^j) \\ -\phi^j_{k(T_\rho^j)}(0, T_\rho^j) \} \Big] \end{aligned} \tag{6}$$

$$\begin{aligned} V_i^j(1, t) = E_i\Big[\int_t^{T_{\rho MD}^j} e^{-r(s-t)} (C - \delta 1_{\{k(s)=2\}})ds \\ + 1_{\{T_{\rho MD}^j = T_\rho^j\}} e^{-r(T_\rho^j - t)} \{ V^j_{k(T_\rho^j)}(a_{k(T_\rho^j)}(T_\rho^j), T_\rho^j) \end{aligned} \tag{7}$$

$$
\begin{aligned}
&-p^j(T_\rho^j)(a_{k(T_\rho^j)}(T_\rho^j)-1)-\phi^j_{k(T_\rho^j)}(1,T_\rho^j)\} \\
&+1_{\{T^j_{\rho MD}=T_D\}}e^{-r(T_D-t)}\{V^j_{k(T_D)}(0,T_D)+(1-f)F\} \\
&+1_{\{T^j_{\rho MD}=T_M\}}e^{-r(T_M-t)}\{V^j_{k(T_M)}(0,T_M)+F\}\Big],
\end{aligned}
$$

where the expectation is with respect to T_ρ^j, T_D, T_M, and $k(s)$. The expectation is indexed by i to indicate that it is conditional on $k(t) = i$. When the investor meets a dealer at time T_ρ^j, he chooses bond holding $a_{k(T_\rho^j)}(T_\rho^j)$ and the holding depends on his preference type k at this time. The bid and ask prices are decomposed into the price at which the dealer unloads the bond in the interdealer market, p^j, and the intermediation fee that the investor pays the dealer, ϕ_i^j. The intermediation fee $\phi_i^j(a,t)$ at time t depends on the investor's preference type i, search intensity ρ^j, and bond holding a before the possible trade. There is a superscript j on the interdealer price because interdealer markets in a liquidity crisis are assumed to be segmented according to investor sophistication, as will be explained later. Without the segmentation assumption, there would be one interdealer price at any time t and no need for the j-superscript.

Consider a meeting at time t between an investor and a dealer. The investor holds a of the bond before the meeting and a' after the meeting (where a and a' are 0 or 1). If a' is different from a, the investor traded and the dealer gains an intermediation fee ϕ. Let the pair (a',ϕ) be the outcome corresponding to the Nash solution to a bargaining problem with the dealer having bargaining power $z \in [0,1]$. If the investor trades, his utility is $V_i^j(a',t)-p^j(t)(a'-a)-\phi$, and it is $V_i^j(a,t)$ otherwise. His utility gain from trading is therefore $V_i^j(a',t)-V_i^j(a,t)-p^j(t)(a'-a)-\phi$. The bargaining outcome is

$$
[a_i^j(t),\phi_i^j(a,t)] = \underset{(a',\phi)}{\arg\max}[V_i^j(a',t)-V_i^j(a,t)-p^j(t)(a'-a)-\phi]^{1-z}\phi^z,
$$

where $a' \in \{0,1\}$. The solution can be written as

$$
a_i^j(t) = \underset{a'\in\{0,1\}}{\arg\max}[V_i^j(a',t)-p^j(t)a'] \tag{8}
$$

$$
\phi_i^j(a,t) = z\Big(V_i^j(a_i^j(t),t)-V_i^j(a,t)-p^j(t)[a_i^j(t)-a]\Big). \tag{9}
$$

Substituting (8) and (9) into (6) and (7) yields

$$
\begin{aligned}
V_i^j(0,t) = E_i\Big[&e^{-r(T_\rho^j-t)}\{(1-z)\max_{a'\in\{0,1\}}[V^j_{k(T_\rho^j)}(a',T_\rho^j)-p^j(T_\rho^j)a'] \\
&+zV^j_{k(T_\rho^j)}(0,T_\rho^j)\}\Big]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
V_i^j(1,t) = E_i\Big[&\int_t^{T^j_{\rho MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds \\
&+1_{\{T^j_{\rho MD}=T^j_\rho\}}e^{-r(T_\rho^j-t)}\{(1-z)\max_{a'\in\{0,1\}}[V^j_{k(T_\rho^j)}(a',T_\rho^j)-p^j(T_\rho^j)(a'-1)] \\
&+zV^j_{k(T_\rho^j)}(0,T_\rho^j)\}
\end{aligned} \tag{11}
$$

$$+1_{\{T^j_{\rho MD}=T_D\}} e^{-r(T_D-t)}\{V^j_{k(T_D)}(0,T_D)+(1-f)F\}$$
$$+1_{\{T^j_{\rho MD}=T_M\}} e^{-r(T_M-t)}\{V^j_{k(T_M)}(0,T_M)+F\}\Big].$$

From the investor's standpoint, (10) and (11) show that the stochastic trading process and the bargaining solution are payoff-equivalent to an alternative trading arrangement in which he has all bargaining power but meets only dealers with rate $\kappa^j = \rho^j(1-z)$. Let T^j_κ be the next time the investor meets a dealer in this economy. We can rewrite (10) and (11) as

$$V^j_i(0,t) = E_i\Big[e^{-r(T^j_\kappa - t)} \max_{a'\in\{0,1\}} [V^j_{k(T^j_\kappa)}(a', T^j_\kappa) - p^j(T^j_\kappa)a']\Big] \tag{12}$$

$$V^j_i(1,t) = E_i\Big[\int_t^{T^j_{\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds \tag{13}$$
$$+1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa - t)} \max_{a'\in\{0,1\}} [V^j_{k(T^j_\kappa)}(a', T^j_\kappa) - p^j(T^j_\kappa)(a'-1)]$$
$$+1_{\{T^j_{\kappa MD}=T_D\}} e^{-r(T_D-t)}\{V^j_{k(T_D)}(0,T_D)+(1-f)F\}$$
$$+1_{\{T^j_{\kappa MD}=T_M\}} e^{-r(T_M-t)}\{V^j_{k(T_M)}(0,T_M)+F\}\Big],$$

where $T^j_{\kappa MD} = min(T^j_\kappa, T_D, T_M)$. For $T_x, x = D, M$, we can use the law of iterated expectations to show that

$$E_i\Big[1_{\{T^j_{\kappa MD}=T_x\}} e^{-r(T_x-t)} V^j_{k(T_x)}(0,T_x)\Big]$$
$$= E_i\Big[1_{\{T^j_{\kappa MD}=T_x\}} e^{-r(T_x-t)} E_{k(T_x)}\Big[e^{-r(T^j_\kappa - T_x)} \max_{a'\in\{0,1\}} [V^j_{k(T^j_\kappa)}(a', T^j_\kappa) - p^j(T^j_\kappa)a']\Big]\Big]$$
$$= E_i\Big[1_{\{T^j_{\kappa MD}=T_x\}} e^{-r(T^j_\kappa - t)} \max_{a'\in\{0,1\}} [V^j_{k(T^j_\kappa)}(a', T^j_\kappa) - p^j(T^j_\kappa)a']\Big]. \tag{14}$$

Furthermore,

$$E_i[1_{\{T^j_{\kappa MD}=T_D\}} e^{-r(T_D-t)}] = \int_t^\infty \int_t^\infty 1_{\{x<y\}} e^{-r(x-t)} \lambda_D e^{-\lambda_D(x-t)}(\lambda_T+\kappa^j)e^{-(\lambda_T+\kappa^j)(y-t)} dy dx$$
$$= \int_t^\infty \lambda_D e^{-(r+\lambda_D)(x-t)} \int_x^\infty (\lambda_T+\kappa^j)e^{-(\lambda_T+\kappa^j)(y-t)} dy dx$$
$$= \int_t^\infty \lambda_D e^{-(r+\lambda_D)(x-t)} e^{-(\lambda_T+\kappa^j)(x-t)} dx$$
$$= \frac{\lambda_D}{r+\lambda_D+\lambda_T+\kappa^j}, \tag{15}$$

and the same calculations show that

$$E_i[1_{\{T^j_{\kappa MD}=T_M\}} e^{-r(T_M-t)}] = \frac{\lambda_T}{r+\lambda_D+\lambda_T+\kappa^j}. \tag{16}$$

Substituting (14), (15), and (16) into (13) yields

$$V^j_i(1,t) = E_i\Big[\int_t^{T^j_{\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds$$

$$
\begin{aligned}
&+1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} \max_{a'\in\{0,1\}}[V^j_{k(T^j_\kappa)}(a',T^j_\kappa)-p^j(T^j_\kappa)a']+1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} p^j(T^j_\kappa)\\
&+1_{\{T^j_{\kappa MD}=T_D\}} e^{-r(T^j_\kappa-t)} \max_{a'\in\{0,1\}}[V^j_{k(T^j_\kappa)}(a',T^j_\kappa)-p^j(T^j_\kappa)a']+\frac{\lambda_D(1-f)F}{r+\lambda_D+\lambda_T+\kappa^j}\\
&+1_{\{T^j_{\kappa MD}=T_M\}} e^{-r(T^j_\kappa-t)} \max_{a'\in\{0,1\}}[V^j_{k(T^j_\kappa)}(a',T^j_\kappa)-p^j(T^j_\kappa)a']+\frac{\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j}\Bigg]\\
&=E_i\Big[\int_t^{T^j_{\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds\Big]+E_i\Big[1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} p^j(T^j_\kappa)\Big]\\
&+E_i\Big[e^{-r(T^j_\kappa-t)} \max_{a'\in\{0,1\}}[V^j_{k(T^j_\kappa)}(a',T^j_\kappa)-p^j(T^j_\kappa)a']\Big]+\frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j},
\end{aligned}
\tag{17}
$$

where I have used $1_{\{T^j_{\kappa MD}=T_M\}}+1_{\{T^j_{\kappa MD}=T_D\}}+1_{\{T^j_{\kappa MD}=T^j_\kappa\}}=1$. Rewriting (12) and (17) gives

$$
\begin{aligned}
V^j_i(a,t)&=\overline{U}^j_i(a)+E_i\Big[e^{-r(T^j_\kappa-t)} \max_{a'\in\{0,1\}}[V^j_{k(T^j_\kappa)}(a',T^j_\kappa)-p^j(T^j_\kappa)a']\Big]\\
&\quad+a\left\{E_i\Big[1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} p^j(T^j_\kappa)\Big]+\frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j}\right\}\\
\overline{U}^j_i(a)&=aE_i\Big[\int_t^{T^j_{\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds\Big].
\end{aligned}
\tag{18}
$$

A.2 Prices

By combining (8) and (18), the problem of an investor who meets a dealer at time t is found to be

$$
\begin{aligned}
\max_{a'\in\{0,1\}}\Big[\overline{U}^j_i(a')-\{p^j(t)-E_i\Big[1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} p^j(T^j_\kappa)\Big]\\
-\frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j}\}a'\Big].
\end{aligned}
\tag{19}
$$

Next, the two terms $\overline{U}^j_i(a')$ and $E_i\Big[1_{\{T^j_{\kappa MD}=T^j_\kappa\}} e^{-r(T^j_\kappa-t)} p^j(T^j_\kappa)\Big]$ in (19) are rewritten. Let T_λ be the next time the investor is hit by a preference shock, and let $T^j_{\lambda\kappa}=min(T_\lambda,T^j_\kappa)$, $T_{MD}=min(T_M,T_D)$, and $T^j_{\lambda\kappa MD}=min(T^j_{\lambda\kappa},T_{MD})$. Obviously, we have that $\overline{U}^j_i(0)=0$. Furthermore,

$$
\begin{aligned}
\overline{U}^j_i(1)&=E_i\Big[\int_t^{T^j_{\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds\Big]\\
&=E_i\Big[\int_t^{T^j_{\lambda\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds\\
&\quad+1_{\{T^j_{\lambda\kappa}<T_{MD}\}}1_{\{T_\lambda<T^j_\kappa\}}e^{-r(T_\lambda-t)}\overline{U}^j_{k(T_\lambda)}(1)+1_{\{T^j_{\lambda\kappa}>T_{MD}\}}e^{-r(T_{MD}-t)}\overline{U}^j_i(0)\Big]\\
&=E_i\Big[\int_t^{T^j_{\lambda\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{i=2\}})ds+1_{\{T^j_{\lambda\kappa}<T_{MD}\}}1_{\{T_\lambda<T^j_\kappa\}}e^{-r(T_\lambda-t)}\overline{U}^j_{k(T_\lambda)}(1)\Big].
\end{aligned}
\tag{20}
$$

We have that

$$
E_i\Big[\int_t^{T^j_{\lambda\kappa MD}} e^{-r(s-t)}(C-\delta 1_{\{i=2\}})ds
$$

$$= (C - \delta 1_{\{i=2\}}) \int_t^{\infty} \Big[\int_t^x e^{-r(s-t)} ds \Big] (\lambda + \kappa^j + \lambda_D + \lambda_T) e^{-(\lambda+\kappa^j+\lambda_D+\lambda_T)(x-t)} dx$$
$$= \frac{C - \delta 1_{\{i=2\}}}{r + \kappa^j + \lambda + \lambda_D + \lambda_T}. \tag{21}$$

Let us denote $\pi_1 = (1 - \pi)$ and $\pi_2 = \pi$. Then,

$$E_i \Big[1_{\{T^j_{\lambda\kappa} < T_{MD}\}} 1_{\{T_\lambda < T^j_\kappa\}} e^{-r(T_\lambda - t)} \overline{U}^j_{k(T_\lambda)}(1) \Big]$$
$$= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} 1_{\{\tau_\lambda < \tau_{MD}\}} 1_{\{\tau_\lambda < \tau^j_\kappa\}} e^{-r\tau_\lambda} \Big[\sum_{k=1}^{2} \pi_k \overline{U}^j_k(1) \Big]$$
$$\times \lambda e^{-\lambda \tau_\lambda} \kappa^j e^{-\kappa^j \tau^j_\kappa} (\lambda_D + \lambda_T) e^{-(\lambda_D+\lambda_T)\tau_{DM}} d\tau_\lambda d\tau^j_\kappa d\tau_{DM}$$
$$= \Big[\sum_{k=1}^{2} \pi_k \overline{U}^j_k(1) \Big] \int_0^{\infty} \int_0^{\tau_{DM}} \int_{\tau_\lambda}^{\infty} e^{-r\tau_\lambda}$$
$$\times \kappa^j e^{-\kappa^j \tau^j_\kappa} \lambda e^{-\lambda\tau_\lambda} (\lambda_D + \lambda_T) e^{-(\lambda_D+\lambda_T)\tau_{DM}} d\tau^j_\kappa d\tau_\lambda d\tau_{DM}$$
$$= \frac{\lambda \Big[\sum_{k=1}^{2} \pi_k \overline{U}^j_k(1) \Big]}{r + \lambda_D + \lambda_T + \lambda + \kappa^j}. \tag{22}$$

Inserting (21) and (22) into (20) gives

$$\overline{U}^j_i(1) = \frac{C - \delta 1_{\{i=2\}} + \lambda \Big[\sum_{k=1}^{2} \pi_k \overline{U}^j_k(1) \Big]}{r + \kappa^j + \lambda + \lambda_D + \lambda_T}. \tag{23}$$

Multiply (23) by π_i, add over i, solve for $\sum_{i=1}^{2} \overline{U}_i(a)\pi_i$, and substitute this expression back into (23) to obtain

$$\overline{U}^j_i(1) = \frac{1}{r + \kappa^j + \lambda + \lambda_D + \lambda_T} \Big[C - \delta 1_{\{i=2\}} + \frac{(C - \delta\pi_2)\lambda}{r + \kappa^j + \lambda_D + \lambda_T} \Big]. \tag{24}$$

The term $E_i \Big[1_{\{T^j_{\kappa MD} = T^j_\kappa\}} e^{-r(T^j_\kappa - t)} p^j(T^j_\kappa) \Big]$ in (19) can be simplified as

$$E_i \Big[1_{\{T^j_{\kappa MD} = T^j_\kappa\}} e^{-r(T^j_\kappa - t)} p^j(T^j_\kappa) \Big] = \int_t^{\infty} \int_s^{\infty} e^{-r(s-t)} p^j(s) (\lambda_T + \lambda_D)$$
$$\times e^{-(\lambda_T+\lambda_D)(y-t)} \kappa^j e^{-\kappa^j (s-t)} dy ds$$
$$= \kappa^j \int_0^{\infty} e^{-(r+\kappa^j+\lambda_T+\lambda_D)s} p^j(t+s) ds. \tag{25}$$

If we substitute (24) and (25) into (19), the problem of an investor who meets a dealer at time t is

$$\max_{a' \in \{0,1\}} \Big[\overline{U}^j_i(a') - q^j(t) a' \Big], \tag{26}$$

where

$$\overline{U}^j_i(a) = \frac{a}{r + \kappa^j + \lambda + \lambda_D + \lambda_T} \Big[C - \delta 1_{\{i=2\}} + \frac{(C - \delta\pi_2)\lambda}{r + \kappa^j + \lambda_D + \lambda_T} \Big]$$
$$q^j(t) = p^j(t) - \kappa^j \int_0^{\infty} e^{-(r+\kappa^j+\lambda_T+\lambda_D)s} p^j(t+s) ds - \frac{\lambda_D(1-f)F + \lambda_T F}{r + \lambda_D + \lambda_T + \kappa^j}. \tag{27}$$

To obtain the relationship between $q^j(t)$ and $p^j(t)$, we rewrite

$$p^j(t) - q^j(t) = \kappa^j e^{(r+\kappa^j+\lambda_T+\lambda_D)t} \int_t^\infty e^{-(r+\kappa^j+\lambda_T+\lambda_D)s} p^j(s)ds + \frac{\lambda_D(1-f)F + \lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j}.$$

This is useful because $f(t) := p^j(t) - q^j(t)$ is continuous, as seen from Equation (27), so we can differentiate with respect to t and obtain

$$\kappa^j q^j(t) + (\lambda_D(1-f)+\lambda_T)F = (r+\lambda_D+\lambda_T)f(t) - \dot{f}(t).$$

Integrating this forward yields

$$p^j(t) = q^j(t) + \int_t^\infty e^{-(r+\lambda_D+\lambda_T)(s-t)}[\kappa^j q^j(s) + (\lambda_D(1-f)+\lambda_T)F]ds. \quad (28)$$

A.3 Asset Holdings

Next, we find the clearing condition in the asset market. Irrespective of asset holding, investors with search intensity ρ_j have the same probability of meeting a dealer. Let $A_j(t)$ be the total amount of the asset held by these investors, so the total amount of bonds outstanding at time t is $A(t) = \sum_{j=1}^N A_j(t)$. According to the law of large numbers, the instantaneous quantity of assets supplied in the interdealer market by these investors is $\rho^j A_j(t)$.[17] The supplied quantity by all investors is $\sum_{j=1}^N \rho^j A_j(t)$. Because firms issue new bonds at intensity $(\lambda_T + \lambda_D)A(t)$, the total supplied quantity is

$$(\lambda_T+\lambda_D)A(t) + \sum_{j=1}^N \rho^j A_j(t).$$

Let $n_i^j(t)$ be the measure of investors with preference type i and search intensity ρ_j at time t. The process for preference shocks implies that

$$n_i^j(t) = e^{-\lambda t} n_i^j(0) + (1-e^{-\lambda t})\frac{\pi_i}{N}, \quad (29)$$

since the total mass of investors with search intensity ρ^j is $\frac{1}{N}$. The instantaneous quantity of investors with search intensity ρ_j and preference state i who meet a dealer is $\rho^j n_i^j(t)$. Let $\rho^j n_i^j(t)\overline{a}_i^j(t)$ be their equilibrium demand. $\overline{a}_i^j(t)$ is equal to $a_i^j(t)$ except if the investor is indifferent between buying or selling. In this case, $\overline{a}_i^j(t)$ is the number between zero and one such that the following clearing condition for the asset market is satisfied:

$$(\lambda_T+\lambda_D)A(t) + \sum_{j=1}^N \rho^j A_j(t) = \sum_{j=1}^N \sum_{i=1}^2 \rho^j n_i^j(t)\overline{a}_i^j(t). \quad (30)$$

We now find the distribution of investors' states. Let $H_t^j(a,i)$ be the time-t measure of investors with search intensity ρ^j and preference type i, and whose asset holding is a. Let $n_{ki}^{0,j}(a,t)$ be the

[17] Over a short time period dt, the measure of bond-owning investors who meet with a dealer is $\rho^j A_j(t)dt$, of which a fraction $1-(\lambda_M+\lambda_D)dt$ do not have their bond maturing or defaulting during the same period. Thus, $\lim_{dt\to 0} \frac{\rho^j A_j(t)dt[1-(\lambda_M+\lambda_D)dt]}{dt} = \rho^j A_j(t)$.

time-t measure of investors with search intensity ρ^j who started off with preference type k, whose preference type is i and asset holding is a at time t and who have never met a dealer. The measure of investors with asset holding a who started as type i and who have never met a dealer is for $a = 1$ equal to $e^{-(\rho^j+\lambda_T+\lambda_D)t} H_0^j(1,i)$, while for $a = 0$ it is $(1 - e^{-(\lambda_T+\lambda_D)t})e^{-\rho^j t} H_0^j(1,i) + e^{-\rho^j t} H_0^j(0,i)$. The fraction of investors who were of preference type j at time 0 and are of type i at time t is $(1 - e^{-\lambda t})\pi_i + e^{-\lambda t} 1_{\{j=i\}}$. Thus,

$$\begin{aligned} n_{ki}^{0,j}(1,t) &= [(1 - e^{-\lambda t})\pi_i + e^{-\lambda t} 1_{\{j=i\}}] e^{-(\rho^j+\lambda_T+\lambda_D)t} H_0^j(1,i) \\ n_{ki}^{0,j}(0,t) &= [(1 - e^{-\lambda t})\pi_i + e^{-\lambda t} 1_{\{j=i\}}](1 - e^{-(\lambda_T+\lambda_D)t})e^{-\rho^j t} H_0^j(1,i) + e^{-\rho^j t} H_0^j(0,i). \end{aligned} \tag{31}$$

Let $n_{ki}^j(a_t, a_{t-\tau}, \tau, t)$ be the time-t density of investors with search intensity ρ^j, who have bond holding a_t and preference type i at time t, and whose last time they met a dealer was at time $t - \tau$ when their preference type was k and they chose to hold $a_{t-\tau}$ of the bond. The density measure of investors who last met a dealer at time $t - \tau$ is $\rho^j e^{-\rho^j \tau}$. For $a_k^j(t - \tau) = 1$, they still hold the bond if it does not mature/default between $t - \tau$ and t. The bond does not default/mature between $t - \tau$ and t with probability $e^{-\tau(\lambda_T+\lambda_D)}$, so

$$\begin{aligned} n_{ki}^j(1,1,\tau,t) &= \rho^j e^{-\rho^j \tau} e^{-\tau(\lambda_T+\lambda_D)} \Big[(1 - e^{-\lambda\tau})\pi_i + e^{-\lambda\tau} 1_{\{k=i\}}\Big] n_k^j(t-\tau)\overline{a}_k^j(t-\tau) \\ n_{ki}^j(0,1,\tau,t) &= \rho^j e^{-\rho^j \tau}(1 - e^{-\tau(\lambda_T+\lambda_D)}) \Big[(1 - e^{-\lambda\tau})\pi_i + e^{-\lambda\tau} 1_{\{k=i\}}\Big] n_k^j(t-\tau)\overline{a}_k^j(t-\tau) \\ n_{ki}^j(a,0,\tau,t) &= 1_{\{a=0\}} \rho^j e^{-\rho^j \tau} \Big[(1 - e^{-\lambda\tau})\pi_i + e^{-\lambda\tau} 1_{\{k=i\}}\Big] n_k^j(t-\tau)[1 - \overline{a}_k^j(t-\tau)]. \end{aligned}$$

Now we have

$$H_t^j(a,i) = \sum_{k=1}^{2} \Big[n_{ki}^{0,j}(a,t) + \int_0^t n_{ki}^j(a,1,\tau,t) + n_{ki}^j(a,0,\tau,t) d\tau \Big]. \tag{32}$$

The first term $\sum_{k=1}^2 n_{ki}^{0,j}(a,t)$ are those ρ^j-investors with preference i and bond holding a at time t who have never met a dealer. The time-t measure of ρ^j-investors with preference i and asset holding a who chose to hold the bond at the last time they met with a dealer, given that their preference type at that time was k, is $\int_0^t n_{ki}^j(a,1,\tau,t)d\tau$. The time-$t$ measure of ρ^j-investors with preference i and asset holding a who chose not to hold the bond at the last time they met with a dealer, given that their preference type at that time was k, is $\int_0^t n_{ki}^j(a,0,\tau,t)d\tau$.

A.4 Proof of Theorem 2.1

We now find the prices prevailing in steady state. In the notation, dependence on time is ignored because we are looking at steady-state quantities. We have that $lim_{t\to\infty} n_i^j(t) = \frac{\pi_i}{N}$ according to (29). In steady state, the interdealer price is constant and the same for all investors, so (27) gives us

$$q^j = \Big(\frac{r + \lambda_T + \lambda_D}{r + \kappa^j + \lambda_T + \lambda_D} \Big) p_{ss} - \frac{\lambda_D(1-f)F + \lambda_T F}{r + \lambda_D + \lambda_T + \kappa^j}. \tag{33}$$

Using (24), we have

$$\begin{aligned}\overline{U}_i^j(1) - q^j &= \frac{1}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}\Big[C - \delta 1_{\{i=2\}} + \frac{(C-\delta\pi_2)\lambda}{r+\kappa^j+\lambda_D+\lambda_T}\Big] \\ &\quad -\Big(\frac{r+\lambda_T+\lambda_D}{r+\kappa^j+\lambda_T+\lambda_D}\Big)p_{ss} + \frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j} \\ &= \frac{C+\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j} - \Big(\frac{r+\lambda_T+\lambda_D}{r+\kappa^j+\lambda_T+\lambda_D}\Big)p_{ss} \\ &\quad - \frac{\delta}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}\Big[1_{\{i=2\}} + \frac{\pi_2\lambda}{r+\kappa^j+\lambda_D+\lambda_T}\Big]. \end{aligned} \tag{34}$$

According to (26), if $\overline{U}_i^j(1) - q^j < 0$ then $a_i^j = 0$ (selling), while $a_i^j = 1$ (buying) if $\overline{U}_i^j(1) - q^j > 0$. The bond market clearing condition (30) implies that there is at least one class of investors with preference m and search intensity κ^n who have $0 < \overline{a}_m^n < 1$ and are indifferent between buying or selling.[18] For this class, we have $\overline{U}_m^n(1) - q^n = 0$, and the interdealer price is given as

$$p_{ss} = \frac{C+\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T} - \delta\Big[\frac{1_{\{m=2\}}(r+\kappa^n+\lambda_D+\lambda_T)+\lambda\pi_2}{(r+\kappa^n+\lambda+\lambda_D+\lambda_T)(r+\lambda_D+\lambda_T)}\Big]. \tag{35}$$

Combining (34) and (35), we get

$$\begin{aligned}\overline{U}_i^j(1) - q^j &= \frac{\delta}{r+\kappa^j+\lambda_T+\lambda_D}\Big[\frac{1_{\{m=2\}}(r+\kappa^n+\lambda_T+\lambda_D)+\lambda\pi_2}{r+\kappa^n+\lambda+\lambda_T+\lambda_D} \\ &\quad - \frac{1_{\{i=2\}}(r+\kappa^j+\lambda_T+\lambda_D)+\lambda\pi_2}{r+\kappa^j+\lambda+\lambda_T+\lambda_D}\Big] \\ &= C_1\Big[(\kappa^j-\kappa^n)\lambda\pi_2 + \lambda(1_{\{m=2\}}\kappa^n - 1_{\{i=2\}}\kappa^j) + (1_{\{m=2\}} - 1_{\{i=2\}})C_2\Big],\end{aligned}$$

where $C_1 = \frac{\delta}{(r+\kappa^j+\lambda_T+\lambda_D)(r+\kappa^j+\lambda+\lambda_T+\lambda_D)(r+\kappa^n+\lambda+\lambda_T+\lambda_D)}$ and $C_2 = \lambda(r+\lambda_T+\lambda_D)+(r+\kappa^n+\lambda_T+\lambda_D)(r+\kappa^j+\lambda_T+\lambda_D)$ are positive for any parameter values. Assume that $m = 1$. This implies that the marginal buyer is a high type. For $i = 1$, we have $\overline{U}_1^j(1) - q^j = (\kappa^j - \kappa^n)C_1\lambda\pi_2$. Thus, high types with lower search intensities than the marginal buyer sell, while high types with higher search intensities buy. For $i = 2$, we have $\overline{U}_2^j(1) - q^j = -C_1[\lambda\kappa^j(1-\pi_2)+\lambda\pi_2\kappa^n+C_2] < 0$. Since $\overline{U}_2^j(1) - q^j < 0$, low types sell. Assume now that $m = 2$. For $i = 2$, we have $\overline{U}_2^j(1) - q^j = (\kappa^n - \kappa^j)C_1\lambda(1-\pi_2)$. So, when the marginal buyer is a low type, investors with lower search intensities than the marginal buyer buy while those with higher search intensities sell. For $i = 1$, we have $\overline{U}_1^j(1) - q^j = C_1[\kappa^j\lambda\pi_2 + (1-\pi_2)\lambda\kappa^n + C_2] > 0$, so all high-type investors buy.

The above shows that when bond supply is low the marginal buyer is a high type, and low types and more unsophisticated high types are sellers. When bond supply is high, the marginal buyer is a low type and more sophisticated low types are sellers while all other investors are buyers. In particular, when the most unsophisticated investors are marginal buyers, we see both buys and sells from more sophisticated investors (investors with higher search intensities). If the most unsophisticated investors are not marginal buyers, then unsophisticated investors always buy or never trade in steady state.

We can now calculate the intermediation fee. If we plug (18) into (9), we can calculate the fee:

$$\phi_i^j(a) = z[\overline{U}_i^j(a') - \overline{U}_i^j(a) - q^j(a'-a)], \tag{36}$$

[18] Except in the knife-edge case, where the clearing condition is satisfied with all a's being 0 or 1. In this case, the price is not uniquely identified.

where a' is the solution to (26) and a is the old bond holding. Any investor class with search intensity κ^j where the investors both buy and sell must have that high types buy and low types sell. According to (36), the ask price is $p_{ss} + z(\overline{U}_1^j(1) - q^j)$, the bid price is $p_{ss} + z(\overline{U}_2^j(1) - q^j)$, and the bid-ask spread is given as

$$\phi_1^j(0) - \phi_2^j(1) = z[\overline{U}_1^j(1) - \overline{U}_2^j(1)] = \frac{\delta z}{r + \kappa^j + \lambda + \lambda_D + \lambda_T}.$$

A.5 Bond Supply

We saw in the previous section that when the most unsophisticated investors are marginal buyers in steady state, there are both buy and sell transactions by investors with search intensity ρ^j for $j = 1, ..., N$. If the most unsophisticated investors are not marginal buyers in steady state, this is not the case. To find the bond supply for which the investors with the lowest search intensity are marginal buyers, we note again that (29) gives that $n_i^j(t) = \frac{\pi_i}{N}$. Also, from (31), $n_{ki}^{0,j} = 0$. Equation (32) gives us the measure of ρ^j-investors who own bonds in steady state as

$$\begin{aligned}
A_j = \sum_{i=1}^{2} H^j(1,i) &= \sum_{i=1}^{2}\sum_{k=1}^{2} \overline{a}_k^j \rho^j \frac{\pi_k}{N}\left(\frac{\pi_i}{\rho^j + \lambda_T + \lambda_D} + \frac{1_{\{k=i\}} - \pi_i}{\lambda + \rho^j + \lambda_T + \lambda_D}\right) \\
&= \frac{1}{N}\rho^j \sum_{i=1}^{2}\left(\frac{\pi_i[\sum_{k=1}^2 \overline{a}_k^j \pi_k]}{\rho^j + \lambda_T + \lambda_D} + \frac{\overline{a}_i^j \pi_i - [\sum_{k=1}^2 \overline{a}_k^j \pi_k]\pi_i}{\lambda + \rho^j + \lambda_T + \lambda_D}\right) \\
&= \frac{1}{N}\rho^j \left(\frac{[\sum_{k=1}^2 \overline{a}_k^j \pi_k]}{\rho^j + \lambda_T + \lambda_D} + \frac{[\sum_{k=1}^2 \overline{a}_k^j \pi_k] - [\sum_{k=1}^2 \overline{a}_k^j \pi_k]}{\lambda + \rho^j + \lambda_T + \lambda_D}\right) \\
&= \frac{1}{N}\frac{\rho^j}{\rho^j + \lambda_T + \lambda_D}\left[\sum_{k=1}^{2} \overline{a}_k^j \pi_k\right].
\end{aligned}$$

The clearing condition (30) gives

$$(\lambda_T + \lambda_D)A + \frac{1}{N}\sum_{j=1}^{N}\rho^j\left[\frac{\rho^j[\sum_{k=1}^2 \overline{a}_k^j \pi_k]}{\rho^j + \lambda_T + \lambda_D}\right] = \sum_{j=1}^{N}\sum_{i=1}^{2}\rho^j \frac{\pi_i}{N}\overline{a}_i^j$$

so

$$\frac{1}{N}\sum_{j=1}^{N}\frac{\rho^j}{\rho^j + \lambda_T + \lambda_D}\sum_{k=1}^{2}\overline{a}_k^j \pi_k = A. \tag{37}$$

Assume that investors with lowest search intensity are marginal buyers. This is particularly convenient because this is the only case where we see trades by investors with search intensity ρ^j for all j. We have $\overline{a}_1^j = 1$ for $j > 1$ because high investors with higher search intensities always buy. We also have $\overline{a}_2^j = 0$ for $j > 1$ because low investors with higher search intensities always sell. The bond supply is, according to (37), between $\frac{\pi_1}{N}\sum_{j=2}^{N}\frac{\rho^j}{\rho^j+\lambda_T+\lambda_D}$ and $\frac{\pi_1}{N}\sum_{j=2}^{N}\frac{\rho^j}{\rho^j+\lambda_T+\lambda_D} + \frac{1}{N}\frac{\rho^1}{\rho^1+\lambda_T+\lambda_D}$. If we assume that $\overline{a}_1^1 = 1 - \omega$ and $\overline{a}_2^1 = 0$ such that a high investor with the lowest search intensity is the marginal buyer, the bond supply is

$$\frac{\pi_1}{N}\left(\sum_{j=2}^{N}\frac{\rho^j}{\rho^j + \lambda_T + \lambda_D} + (1-\omega)\frac{\rho^1}{\rho^1 + \lambda_T + \lambda_D}\right). \tag{38}$$

When high investors are marginal buyers, the midprice—the average price of a buy and sell—of investors with high search intensities is higher than the midprice of investors with low search intensities. This case essentially corresponds to Condition 1 in Duffie, Gârleanu, and Pedersen (2005). The reverse is the case if low investors are marginal buyers. Because the empirical section shows that the midprice of large trades is higher than the midprice of small trades, I assume that high investors are marginal buyers in steady state.

A.6 Proof of Theorem 2.2

When a liquidity shock of size s occurs, a high investor becomes a low investor with probability s. In steady state, the mass of high investors with search intensity ρ^j is $\frac{\pi_1}{N}$, so immediately after the liquidity shock it is $\frac{(1-s)\pi_1}{N}$. Denote the time the shock occurs as time 0 and the mass of high investors as $n_1(0)$ (we can ignore a superscript j because $n_1(0)$ is the same for all j). There is a subscript s on variables to indicate that they at a given time depend on the size of the shock, but occasionally I drop the subscript. We have that

$$n_1(0) = (1-s)\frac{\pi_1}{N},$$

and according to (29),

$$n_1(t) = \frac{\pi_1}{N}\left(1 - se^{-\lambda t}\right).$$

Assume also that the bond supply is as in (38) and that markets are segmented until the time t, where $n_1(t) = \frac{\pi_1}{N}(1-\omega)$. If $n_2(0) \geq \frac{\pi_1}{N}(1-\omega)$ markets never become segmented, and this is the case if $s \leq \omega$. According to (28), we have that

$$\int_t^{\infty} e^{-(r+\lambda_D+\lambda_T)s}[\kappa^j q_s^j(u) + (\lambda_D(1-f)+\lambda_T)F]du = e^{-(r+\lambda_D+\lambda_T)t}[p_s(t) - q_s^j(t)],$$

while

$$\int_0^t e^{-(r+\lambda_D+\lambda_T)u}[\kappa^j q_s^j(u) + (\lambda_D(1-f)+\lambda_T)F]du$$
$$= \left(1 - e^{-(r+\lambda_D+\lambda_T)t}\right)\left(\frac{\kappa^j q_s^j(0) + (\lambda_D(1-f)+\lambda_T)F}{r+\lambda_D+\lambda_T}\right),$$

so

$$\begin{aligned}
p_s^j(0) &= q^j(0) + e^{-(r+\lambda_D+\lambda_T)t}[p(t) - q^j(t)] + \left(1 - e^{-(r+\lambda_D+\lambda_T)t}\right) \\
&\quad \times \left(\frac{\kappa^j q^j(0) + (\lambda_D(1-f)+\lambda_T)F}{r+\lambda_D+\lambda_T}\right) \\
&= e^{-(r+\lambda_D+\lambda_T)t}[p(t) + q^j(0) - q^j(t)] + \left(1 - e^{-(r+\lambda_D+\lambda_T)t}\right) \\
&\quad \times \left(\frac{r+\kappa^j+\lambda_D+\lambda_T}{r+\lambda_D+\lambda_T}q^j(0) + \frac{(\lambda_D(1-f)+\lambda_T)F}{r+\lambda_D+\lambda_T}\right).
\end{aligned} \tag{39}$$

Assume first that the $s \leq \omega$ so that markets do not become segmented. Immediately after the liquidity shock, the clearing condition is, according to (30),

$$(\lambda_T + \lambda_D)A(t) + \sum_{j=1}^{N} \rho^j A_j(t) = \sum_{j=1}^{N}\sum_{i=1}^{2} \rho^j n_i^j(t)\overline{a}_i^j(t). \tag{40}$$

Because $A_j = \frac{1}{N}\frac{\rho^j \pi_1}{\rho^j+\lambda_T+\lambda_D}$, $j = 2, ..., N$ and $A_1 = \frac{1}{N}\frac{(1-\omega)\pi_1\rho^1}{\rho^1+\lambda_T+\lambda_D}$, in steady state the left-hand side of (40) immediately after the shock is $\frac{\pi_1}{N}(\sum_{j=2}^N \rho^j + (1-\omega)\rho^1)$. If the marginal buyer after the shock is still a (ρ^1)-high investor, there is no price reaction and prices are equal to steady state prices. This is the case if in (40) we have

$$(\lambda_T + \lambda_D)A(t) + \sum_{j=1}^N \rho^j A_j(t) \le \sum_{j=1}^N \rho^j n_1^j(t). \tag{41}$$

Because the left-hand side immediately after the shock is $\frac{\pi_1}{N}(\sum_{j=2}^N \rho^j + (1-\omega)\rho^1)$, we have

$$\frac{\pi_1}{N}\left(\sum_{j=2}^N \rho^j + (1-\omega)\rho^1\right) \le \sum_{j=1}^N \rho^j (1-s)\frac{\pi_1}{N}, \tag{42}$$

so for $s \le \omega\frac{\rho^1}{\sum_{j=1}^N \rho^i}$ there is no price reaction. Calculations as before (now with $\overline{a}_2^1 = 1$) show that for $\frac{\rho^1(1-\pi_1)+\omega\rho^1}{\pi_1\sum_{j=2}^N \rho^j} \ge s > \omega\frac{\rho^1}{\sum_{j=1}^N \rho^i}$ the marginal buyer is a low investor with search intensity ρ^1. Assume that

$$\omega \le \frac{\rho^1(1-\pi_1)}{\pi_1\sum_{j=2}^N \rho^j}. \tag{43}$$

Because markets are integrated when $s \le \omega$, (43) implies that the marginal buyer is a low ρ^1-investor for $\omega \ge s > \omega\frac{\rho^1}{\sum_{j=1}^N \rho^i}$. In this case, according to (29), the marginal buyer becomes a high ρ^1-investor and prices return to steady-state prices at time $t_1(s)$, where t_1 solves

$$e^{-\lambda t_1}(1-s)\frac{\pi_1}{N} + (1-e^{-\lambda t_1})\frac{\pi_1}{N} = \left(1-\omega\frac{\rho^1}{\sum_{j=1}^N \rho^i}\right)\frac{\pi_1}{N}.$$

Rearranging, we get $t_1(s) = \log\left(\frac{s}{\omega}\frac{\sum_{j=1}^N \rho^j}{\rho^1}\right)/\lambda$. For $\omega \ge s > \omega\frac{\rho^1}{\sum_{j=1}^N \rho^i}$, we can calculate prices for any $0 \le t < t_1(s)$. Before $t_1(s)$, the marginal buyer is a low ρ^1-investor, so $q_s^1(0) = \overline{U}_2^1(1)$. At $t_1(s)$, the marginal buyer becomes a high ρ^1-investor, so $q_s^1(t_1(s)) = \overline{U}_1^1(1)$. Insert $q_s^1(t_1(s))$ and $q_s^1(0)$ in (39) to get

$$p_s(t) = p_{ss} - V(t_1(s) - t) \tag{44}$$

$$V(t) = \delta\frac{r + \lambda_D + \lambda_T + \kappa^1\left(1 - e^{-(r+\lambda_D+\lambda_T)t}\right)}{(r + \kappa^1 + \lambda + \lambda_D + \lambda_T)(r + \lambda_D + \lambda_T)}, \tag{45}$$

where p_{ss} is the steady-state price in (35). According to (27),

$$\begin{aligned} q_s^j(t) &= p_s(t) - \kappa^j \int_0^\infty e^{-(r+\kappa^j+\lambda_T+\lambda_D)u} p_s(t+u)du - \frac{\lambda_D(1-f)F + \lambda_T F}{r + \lambda_D + \lambda_T + \kappa^j} \\ &= p_s(t) - \kappa^j \int_0^{t_1(s)-t} e^{-(r+\kappa^j+\lambda_T+\lambda_D)u} p_s(t+u)du - \kappa^j \int_{t_1(s)-t}^\infty e^{-(r+\kappa^j+\lambda_T+\lambda_D)u} p_{ss}du \\ &\quad - \frac{\lambda_D(1-f)F + \lambda_T F}{r + \lambda_D + \lambda_T + \kappa^j}. \end{aligned}$$

Because

$$\int_0^{t_1(s)-t} e^{-(r+\kappa^j+\lambda_T+\lambda_D)u} e^{-(r+\lambda_D+\lambda_T)(t_1(s)-t-u)} du = \frac{e^{-(r+\lambda_D+\lambda_T)(t_1(s)-t)} - e^{-(r+\kappa^j+\lambda_T+\lambda_D)(t_1(s)-t)}}{\kappa^j},$$

calculations show that

$$\begin{aligned} q_s^j(t) &= \frac{r+\lambda_T+\lambda_D}{r+\kappa^j+\lambda_T+\lambda_D} p_{ss} - \delta \frac{1}{(r+\kappa^1+\lambda+\lambda_T+\lambda_D)(r+\kappa^j+\lambda_T+\lambda_D)} \times \\ &\quad \left[r+\kappa^1+\lambda_T+\lambda_D+e^{-(r+\kappa^j+\lambda_T+\lambda_D)(t_1(s)-t)}(\kappa^j-\kappa^1)\right] - \frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j} \\ &= \frac{1}{r+\kappa^j+\lambda_T+\lambda_D}\left[C - \delta \frac{r+\kappa^1+\lambda\pi_2+\lambda_T+\lambda_D+e^{-(r+\kappa^j+\lambda_T+\lambda_D)(t_1(s)-t)}(\kappa^j-\kappa^1)}{r+\kappa^1+\lambda+\lambda_D+\lambda_T}\right], \end{aligned}$$

and the ask price is $p_s(t) + z(\overline{U}_1^j(1) - q_s^j(t))$, while the bid price is the ask price minus $\frac{\delta z}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}$. As shown in (33), the steady-state value of q is $q_{ss}^j = \frac{r+\lambda_T+\lambda_D}{r+\kappa^j+\lambda_T+\lambda_D} p_{ss} - \frac{\lambda_D(1-f)F+\lambda_T F}{r+\lambda_D+\lambda_T+\kappa^j}$, so

$$q_s^j(t) = q_{ss}^j - \delta \frac{r+\kappa^1+\lambda_T+\lambda_D+e^{-(r+\kappa^j+\lambda_T+\lambda_D)(t_1(s)-t)}(\kappa^j-\kappa^1)}{(r+\kappa^1+\lambda+\lambda_T+\lambda_D)(r+\kappa^j+\lambda_T+\lambda_D)}.$$

Therefore, we have that $\overline{U}_1^j(1) - q_s^j(t) = R^j + S^j(t_1(s)-t)$, where

$$R^j = \overline{U}_1^j(1) - q_{ss}^j = \delta \frac{(\kappa^j-\kappa^1)\pi_2\lambda}{(r+\kappa^1+\lambda+\lambda_T+\lambda_D)(r+\kappa^j+\lambda_T+\lambda_D)(r+\kappa^j+\lambda+\lambda_T+\lambda_D)} \tag{46}$$

$$S^j(t) = \delta \frac{r+\kappa^1+\lambda_T+\lambda_D+e^{-(r+\kappa^j+\lambda_T+\lambda_D)t}(\kappa^j-\kappa^1)}{(r+\kappa^1+\lambda+\lambda_T+\lambda_D)(r+\kappa^j+\lambda_T+\lambda_D)}. \tag{47}$$

Combining this result with (44), we find the ask price to be

$$\begin{aligned} A_s^j(t) &= p_s(t) + z[\overline{U}_1^j(1) - q_s^j(t)] \\ &= p_{ss} - V(t_1(s)-t) + z[\overline{U}_1^j(1) - q_{ss}^j + S^j(t_1(s)-t)] \\ &= A_{ss}^j - V(t_1(s)-t) + zS^j(t_1(s)-t). \end{aligned} \tag{48}$$

Now we find the price for a liquidity shock of size $s > \omega$. In this case, the market is segmented for a time period of $t_2(s) = log(\frac{s}{\omega})/\lambda$ after the shock (t_2 is found in the same way as t_1). Assume that while markets are segmented, newly issued bonds are sold in the same proportion to different investors as in steady state. (If firms issue bonds with intensity ϵ to ρ^j-investors in steady state, they issue bonds with the same intensity to ρ^j-investors while markets are segmented.) For investor j, the clearing condition while markets are segmented is

$$(\rho^j+\lambda_T+\lambda_D)A_j = \sum_{i=1}^{2} \rho^j n_i(t)\overline{a}_i^j(t),$$

so

$$\frac{N(\rho^j+\lambda_T+\lambda_D)}{\rho^j} A_j = [1-\pi_2-s(1-\pi_2)e^{-\lambda t}]\overline{a}_1^j(t) + [\pi_2+s(1-\pi_2)e^{-\lambda t}]\overline{a}_2^j(t).$$

In steady state, $A_j = \frac{1}{N}\frac{\rho^j}{\rho^j+\lambda_T+\lambda_D}[\sum_{k=1}^{2}\pi_k\overline{a}_k^{j,ss}]$, so the clearing condition implies that

$$(1-\pi_2)(\overline{a}_1^j(t)-\overline{a}_1^{j,ss})+\pi_2(\overline{a}_2^j(t)-\overline{a}_2^{j,ss}) = s(1-\pi_2)e^{-\lambda t}[\overline{a}_1^j(t)-\overline{a}_2^j(t)]. \tag{49}$$

Because $\overline{a}_1^j(t)-\overline{a}_2^j(t)\geq 0$ (high types always buy before low types), we have that

$$(1-\pi_2)(\overline{a}_1^j(t)-\overline{a}_1^{j,ss})+\pi_2(\overline{a}_2^j(t)-\overline{a}_2^{j,ss})\geq 0. \tag{50}$$

If $\overline{a}_1^{j,ss}=1$, then $\overline{a}_1^j(t)=1$ (for $\overline{a}_1^j(t)<1$ would imply $\overline{a}_2^j(t)=0$ and then (50) cannot be true). Furthermore, $\overline{a}_2^j(t)>0$ (else (49) is not true) and low types are marginal buyers. In the proof of Theorem 2.1, we showed that $\overline{a}_1^{j,ss}=1$ for $j\geq 2$, so the marginal buyer for $j\geq 2$ is a low type for $t<t_2(s)$ after the shock because $\overline{a}_2^j(t)>0$, as just shown. For investors with search intensity ρ^1, we need to check whether high investors are marginal buyers at some point while markets are segmented. The clearing condition (49) gives us that after a liquidity shock, the marginal ρ^1-buyer is a low investor until time $t=\frac{\log(s/\omega)}{\lambda}$. This is precisely the time when markets become integrated, so the marginal buyer for ρ^1-investors is also a low type while markets are segmented. Thus, for any j,

$$\begin{aligned} q_s^j(t)=\overline{U}_2^j(1) &= \frac{1}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}\left[C-\delta+\frac{(C-\delta\pi_2)\lambda}{r+\kappa^j+\lambda_D+\lambda_T}\right]\\ &= \frac{C}{r+\kappa^j+\lambda_D+\lambda_T}-\delta\frac{r+\kappa^j+\pi_2\lambda+\lambda_D+\lambda_T}{(r+\kappa^j+\lambda+\lambda_D+\lambda_T)(r+\kappa^j+\lambda_D+\lambda_T)}\end{aligned}$$

for $0\leq t\leq t_2(s)$, and the price immediately after a shock of size s is, according to (39),

$$\begin{aligned} p_s^j(0) &= e^{-(r+\lambda_D+\lambda_T)t_2(s)}[p_s^j(t_2(s))+U_2^j(1)-q_s^j(t_2(s))]\\ &+(1-e^{-(r+\lambda_D+\lambda_T)t_2(s)})\Big(\frac{C+(\lambda_D(1-f)+\lambda_T)F}{r+\lambda_D+\lambda_T}-\delta\frac{r+\kappa^j+\pi_2\lambda+\lambda_D+\lambda_T}{(r+\kappa^j+\lambda+\lambda_D+\lambda_T)(r+\lambda_D+\lambda_T)}\Big).\end{aligned}$$

Because markets become integrated at time $t_2(s)$, we have that $p_s^j(t_2(s)) = p_\omega(t_1(\omega))$ and $q_s^j(t_2(s))=q_\omega^j(t_1(\omega))$. Therefore,

$$\begin{aligned} p_s^j(t_2(s))+U_2^j(1)-q_s^j(t_2(s)) &= p_\omega(t_1(\omega))+U_2^j(1)-q_\omega^j(t_1(\omega))\\ &= B_\omega^j(t_1(\omega))+(1-z)[U_2^j(1)-q_\omega^j(t_1(\omega))]\\ &= B_\omega^j(t_1(\omega))+(1-z)\left[U_1^j(1)-q_\omega^j(t_1(\omega))-\frac{\delta}{r+\kappa^j+\lambda+\lambda_T+\lambda_D}\right]\\ &= B_\omega^j(t_1(\omega))+(1-z)\left[R+S^j(t_1(\omega))-\frac{\delta}{r+\kappa^j+\lambda+\lambda_T+\lambda_D}\right],\end{aligned}$$

where R and S^j are given in (46) and (47). Since the bid price is $B_s^j(0)=p_s^j(0)$ while the ask price is $A_s^j(0)=p_s^j(0)+\frac{\delta z}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}$, we have

$$\begin{aligned} B_s^j(0) &= e^{-(r+\lambda_D+\lambda_T)t_2(s)}\left(B_\omega^j(t_1(\omega))+(1-z)\left[R+S^j(t_1(\omega))-\frac{\delta}{r+\kappa^j+\lambda+\lambda_T+\lambda_D}\right]\right)\\ &+(1-e^{-(r+\lambda_D+\lambda_T)t_2(s)})\Big(\frac{C+(\lambda_D(1-f)+\lambda_T)F}{r+\lambda_D+\lambda_T}-\delta\frac{r+\kappa^j+\pi_2\lambda+\lambda_D+\lambda_T}{(r+\kappa^j+\lambda+\lambda_D+\lambda_T)(r+\lambda_D+\lambda_T)}\Big)\\ &+\frac{\delta z}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}\end{aligned}$$

and the ask price is $A_s^j(0)=B_s^j(0)+\frac{\delta z}{r+\kappa^j+\lambda+\lambda_D+\lambda_T}$.

A.7 Proof of Theorem 2.3

To prove Theorem 2.3 for $s > \omega$, we rewrite the expression for $p_s^j(0)$

$$p_s^j(0) = U_2^j(1) - q_\omega^j(t_1(\omega)) + e^{-(r+\lambda_D+\lambda_T)t_2(s)} p_\omega(t_1(\omega)) + (1 - e^{-(r+\lambda_D+\lambda_T)t_2(s)})$$
$$\times \left(q_\omega^j(t_1(\omega)) - U_2^j(1) + \frac{C + (\lambda_D(1-f) + \lambda_T)F}{r + \lambda_D + \lambda_T} - \delta \frac{r + \kappa^j + \pi_2\lambda + \lambda_D + \lambda_T}{(r + \kappa^j + \lambda + \lambda_D + \lambda_T)(r + \lambda_D + \lambda_T)}\right).$$

To prove the theorem, we need to show that $\frac{\partial [p_s^i(0) - p_s^j(0)]}{\partial s} > 0$ for any $\kappa^i < \kappa^j$, and this amounts to showing $\frac{\partial^2 p_s^j(0)}{\partial s \partial \kappa^j} < 0$.[19] Since $U_2^j(1)$ and $q_\omega^j(t_1(\omega))$ do not depend on s, $e^{-(r+\lambda_D+\lambda_T)t_2(s)}$ and $p_\omega(t_1(\omega))$ do not depend on κ^j, and $\frac{\partial(1-e^{-(r+\lambda_D+\lambda_T)t_2(s)})}{\partial s} > 0$ since $\frac{\partial t_2(s)}{\partial s} > 0$, we have to show that

$$\frac{\partial}{\partial \kappa^j}\left[\delta \frac{r + \kappa^j + \pi_2\lambda + \lambda_D + \lambda_T}{(r + \kappa^j + \lambda + \lambda_D + \lambda_T)(r + \lambda_D + \lambda_T)} + U_2^j(1) - q_\omega^j(t_1(\omega))\right] > 0.$$

If we define $\Delta = r + \lambda_D + \lambda_T$, calculations show that

$$\delta \frac{\kappa^j + \pi_2\lambda + \Delta}{(\kappa^j + \lambda + \Delta)\Delta} + U_2^j(1) - q_\omega^j(t_1(\omega))$$
$$= \frac{\delta}{\kappa^j + \Delta}\left(\frac{(\kappa^j + \pi_2\lambda + \Delta)\kappa^j}{(\kappa^j + \lambda + \Delta)\Delta} + \frac{\kappa^1 + \lambda\pi_2 + \Delta + e^{-(\kappa^j+\Delta)t_1(\omega)}(\kappa^j - \kappa^1)}{\kappa^1 + \lambda + \Delta}\right).$$

Since

$$\frac{\partial}{\partial \kappa^j}\left(\frac{(\kappa^j + \pi_2\lambda + \Delta)\kappa^j}{\kappa^j + \lambda + \Delta}\right) = \frac{(1 - \pi_2)\lambda\kappa^j}{(\kappa^j + \lambda + \Delta)^2} + \frac{\kappa^j + \pi_2\lambda + \Delta}{\kappa^j + \lambda + \Delta},$$

we have that

$$\frac{\partial}{\partial \kappa^j}\left[\frac{1}{\kappa^j + \Delta}\left(\frac{(\kappa^j + \pi_2\lambda + \Delta)\kappa^j}{(\kappa^j + \lambda + \Delta)\Delta} + \frac{\kappa^1 + \lambda\pi_2 + \Delta + e^{-(\kappa^j+\Delta)t_1(\omega)}(\kappa^j - \kappa^1)}{\kappa^1 + \lambda + \Delta}\right)\right]$$
$$= -\frac{1}{(\kappa^j + \Delta)^2}\left(\frac{(\kappa^j + \pi_2\lambda + \Delta)\kappa^j}{(\kappa^j + \lambda + \Delta)\Delta} + \frac{\kappa^1 + \lambda\pi_2 + \Delta + e^{-(\kappa^j+\Delta)t_1(\omega)}(\kappa^j - \kappa^1)}{\kappa^1 + \lambda + \Delta}\right)$$
$$+ \frac{1}{\kappa^j + \Delta}\left(\frac{(1 - \pi_2)\lambda\kappa^j}{(\kappa^j + \lambda + \Delta)^2\Delta} + \frac{\kappa^j + \pi_2\lambda + \Delta}{(\kappa^j + \lambda + \Delta)\Delta} + \frac{1 - t_1(\omega)(\kappa^j - \kappa^1)}{\kappa^1 + \lambda + \Delta} e^{-(\kappa^j+\Delta)t_1(\omega)}\right)$$
$$= \frac{\kappa^j + \pi_2\lambda + \Delta}{(\kappa^j + \lambda + \Delta)(\kappa^j + \Delta)} + \frac{\Delta e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)} + \frac{\kappa_1 e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)^2}$$
$$- \frac{t_1(\omega)(\kappa^j - \kappa^1)e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)} - \frac{\kappa^1 + \pi_2\lambda + \Delta}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)^2} + \frac{(1 - \pi_2)\lambda\kappa^j}{(\kappa^j + \lambda + \Delta)^2(\kappa^j + \Delta)\Delta}$$
$$\geq \frac{\kappa^j + \pi_2\lambda + \Delta}{(\kappa^j + \lambda + \Delta)(\kappa^j + \Delta)} + \frac{\Delta e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)} + \frac{\kappa_1 e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)^2}$$
$$- \frac{e^{-(\Delta+\kappa^1)t_1(\omega)-1}}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)} - \frac{\kappa^1 + \pi_2\lambda + \Delta}{(\kappa^1 + \lambda + \Delta)(\kappa^j + \Delta)^2} + \frac{(1 - \pi_2)\lambda\kappa^j}{(\kappa^j + \lambda + \Delta)^2(\kappa^j + \Delta)\Delta}$$

[19] This statement is strictly correct when assuming $\frac{\partial t_1(s)}{\partial \kappa^j} = 0$. I assume this in the following.

$$= \frac{(1-\pi_2)\lambda\kappa^j}{(\kappa^j+\lambda+\Delta)^2(\kappa^j+\Delta)\Delta} + \frac{[\Delta(\kappa^j+\Delta)+\kappa^1]e^{-(\Delta+\kappa^j)t_1(\omega)}}{(\kappa^1+\lambda+\Delta)(\kappa^j+\Delta)^2}$$
$$+ \frac{(\kappa^j+\Delta)(\kappa^j+\pi_2\lambda+\Delta)-(\kappa^j+\Delta)e^{-(\Delta+\kappa^1)t_1(\omega)-1}-(\kappa^1+\pi_2\lambda+\Delta)}{(\kappa^1+\lambda+\Delta)(\kappa^j+\Delta)^2},$$

where I have used that the maximum of $f(\kappa^j) = t_1(\omega)(\kappa^j-\kappa^1)e^{-(\Delta+\kappa^j)t_1(\omega)}$ is $\kappa^j = \frac{1}{t_1(\omega)} + \kappa^1$. Since

$$\begin{aligned}
&(\kappa^j+\Delta)(\kappa^j+\pi_2\lambda+\Delta)-(\kappa^j+\Delta)e^{-(\Delta+\kappa^1)t_1(\omega)-1}-(\kappa^1+\pi_2\lambda+\Delta)\\
&\quad\geq (\kappa^j+\Delta)(\kappa^j+\pi_2\lambda+\Delta)-(\kappa^j+\Delta)e^{-(\Delta+\kappa^1)t_1(\omega)-1}-(\kappa^j+\pi_2\lambda+\Delta)\\
&\quad= \pi_2\lambda(\kappa^j+\Delta-1)+(\kappa^j+\Delta)(\kappa^j+\Delta-1-e^{-(\Delta+\kappa^1)t_1(\omega)-1}),
\end{aligned}$$

a sufficient condition for $\frac{\partial^2 p^j(0)}{\partial s\partial\kappa^j} < 0$ is that $\kappa^j+\Delta > 1+e^{-(\Delta+\kappa^1)t_1(\omega)-1}$.

Appendix B: Aggregate liquidity shocks

In this section, I derive prices when investors anticipate that aggregate liquidity shocks might occur in the future. The derivations follow those in the previous section, so they are less detailed here and in some cases omitted.

Assume that an aggregate liquidity shock of size 1 defined in Definition 2.1 occurs with a Poisson intensity λ_l (independent of all other random variables). That is, when a liquidity shock occurs, all investors become low investors. As in Duffie, Gârleanu, and Pedersen (2007) we denote time $t = 0$ as the time where an aggregate liquidity shock occurs, and when a liquidity shock occurs again the time is reset to 0. Knowledge of the times at which shocks occur allows a translation of the solution to calendar time. A price at time t is the price prevailing when a liquidity shock last happened t years ago. Let T_ρ^j be the next time a dealer is met, T_D the next time the bond defaults, T_M the next time the bond matures, T_l the next time an aggregate liquidity shock occurs, $T_{\rho l}^j = \min(T_\rho^j, T_l)$, and $T_{\rho MDl}^j = \min(T_\rho^j, T_M, T_D, T_l)$. The value function at time t is

$$\begin{aligned}
V_i^j(0,t) = E_i\Big[&1_{\{T_{\rho l}^j=T_l\}}e^{-r(T_l-t)}V_2^j(0,0) &(51)\\
&+1_{\{T_{\rho l}^j=T_\rho^j\}}e^{-r(T_\rho^j-t)}\{V^j_{k(T_\rho^j)}(a_{k(T_\rho^j)}(T_\rho^j),T_\rho^j)-p^j(T_\rho^j)a_{k(T_\rho^j)}(T_\rho^j)\\
&-\phi^j_{k(T_\rho^j)}(0,T_\rho^j)\}\Big] &(52)
\end{aligned}$$

$$\begin{aligned}
V_i^j(1,t) = E_i\Big[&\int_t^{T_{\rho MDl}^j} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds + 1_{\{T_{\rho MDl}^j=T_l\}}e^{-r(T_l-t)}V_2^j(1,0) &(53)\\
&+1_{\{T_{\rho MDl}^j=T_\rho^j\}}e^{-r(T_\rho^j-t)}\{V^j_{k(T_\rho^j)}(a_{k(T_\rho^j)}(T_\rho^j),T_\rho^j)-p^j(T_\rho^j)(a_{k(T_\rho^j)}(T_\rho^j)-1)\\
&-\phi^j_{k(T_\rho^j)}(1,T_\rho^j)\}\\
&+1_{\{T_{\rho MDl}^j=T_D\}}e^{-r(T_D-t)}\{V^j_{k(T_D)}(0,T_D)+(1-f)F\}\\
&+1_{\{T_{\rho MDl}^j=T_M\}}e^{-r(T_M-t)}\{V^j_{k(T_M)}(0,T_M)+F\}\Big],
\end{aligned}$$

so

$$V_i^j(0,t) = \frac{\lambda_l}{r+\rho^j+\lambda_l} V_2^j(0,0) + E_i\Big[1_{\{T_{\rho l}^j = T_\rho^j\}} e^{-r(T_\rho^j - t)} \{V_{k(T_\rho^j)}^j (a_{k(T_\rho^j)}(T_\rho^j), T_\rho^j) - p^j(T_\rho^j) a_{k(T_\rho^j)}(T_\rho^j) - \phi_{k(T_\rho^j)}^j(0, T_\rho^j)\}\Big] \tag{54}$$

$$\begin{aligned} V_i^j(1,t) = {} & \frac{\lambda_l}{r+\rho^j+\lambda_D+\lambda_T+\lambda_l} V_2^j(1,0) + E_i\Big[\int_t^{T_{\rho MDl}^j} e^{-r(s-t)}(C - \delta 1_{\{k(s)=2\}})ds \\ & + 1_{\{T_{\rho MDl}^j = T_\rho^j\}} e^{-r(T_\rho^j - t)} \{V_{k(T_\rho^j)}^j (a_{k(T_\rho^j)}(T_\rho^j), T_\rho^j) - p^j(T_\rho^j)(a_{k(T_\rho^j)}(T_\rho^j) - 1) - \phi_{k(T_\rho^j)}^j(1, T_\rho^j)\}\Big] \\ & + 1_{\{T_{\rho MDl}^j = T_D\}} e^{-r(T_D - t)} \{V_{k(T_D)}^j(0, T_D)\} + 1_{\{T_{\rho MDl}^j = T_M\}} e^{-r(T_M - t)} \{V_{k(T_M)}^j(0, T_M)\}\Big] \\ & + \frac{\lambda_D(1-f)F + \lambda_T F}{r+\rho^j+\lambda_D+\lambda_T+\lambda_l}. \end{aligned} \tag{55}$$

As shown in the previous section, we can rewrite the value functions as

$$V_i^j(0,t) = \frac{\lambda_l}{r+\kappa^j+\lambda_l} V_2^j(0,0) + E_i\Big[1_{\{T_{\kappa l}^j = T_\kappa^j\}} e^{-r(T_\kappa^j - t)} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j) a']\}\Big]$$

$$\begin{aligned} V_i^j(1,t) = {} & \frac{\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l} V_2^j(1,0) + E_i\Big[\int_t^{T_{\kappa MDl}^j} e^{-r(s-t)}(C - \delta 1_{\{k(s)=2\}})ds \\ & + 1_{\{T_{\kappa MDl}^j = T_\kappa^j\}} e^{-r(T_\kappa^j - t)} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j)(a'-1)]\} \\ & + 1_{\{T_{\kappa MDl}^j = T_D\}} e^{-r(T_D - t)} \{V_{k(T_D)}^j(0, T_D)\} + 1_{\{T_{\kappa MDl}^j = T_M\}} e^{-r(T_M - t)} \{V_{k(T_M)}^j(0, T_M)\}\Big] \\ & + \frac{\lambda_D(1-f)F + \lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}, \end{aligned}$$

where the investor meets dealers with speed $\kappa^j = \rho^j(1-z)$ and T_κ^j is the next time the investor meets a dealer in this economy. Letting $t = 0$ on the left-hand side in the above to find the value function at time $t = 0$ and entering that in the value function at time t gives

$$\begin{aligned} V_i^j(0,t) = {} & \frac{\lambda_l}{r+\kappa^j} E_2\Big[1_{\{T_{\kappa l}^j = T_\kappa^j\}} e^{-rT_\kappa^j} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j) a']\}\Big] \\ & + E_i\Big[1_{\{T_{\kappa l}^j = T_\kappa^j\}} e^{-r(T_\kappa^j - t)} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j) a']\}\Big] \end{aligned}$$

$$\begin{aligned} V_i^j(1,t) = {} & E_i\Big[\int_t^{T_{\kappa MDl}^j} e^{-r(s-t)}(C - \delta 1_{\{k(s)=2\}})ds \\ & + 1_{\{T_{\kappa MDl}^j = T_\kappa^j\}} e^{-r(T_\kappa^j - t)} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j)(a'-1)]\} \\ & + 1_{\{T_{\kappa MDl}^j = T_D\}} e^{-r(T_D - t)} \{V_{k(T_D)}^j(0, T_D)\} + 1_{\{T_{\kappa MDl}^j = T_M\}} e^{-r(T_M - t)} \{V_{k(T_M)}^j(0, T_M)\}\Big] \\ & + \frac{\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\Big(E_2\Big[\int_0^{T_{\kappa MDl}^j} e^{-rs}(C - \delta 1_{\{k(s)=2\}})ds \\ & + 1_{\{T_{\kappa MDl}^j = T_\kappa^j\}} e^{-rT_\kappa^j} \{\max_{a' \in \{0,1\}} [V_{k(T_\kappa^j)}^j(a', T_\kappa^j) - p^j(T_\kappa^j)(a'-1)]\}\Big]\Big) \\ & + \frac{\lambda_D(1-f)F + \lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T}. \end{aligned}$$

We can use the same approach as in the previous section to find that the problem of an investor who meets a dealer at time t is

$$\begin{aligned} \max_{a' \in \{0,1\}} \Big[U_i^j(a) - \{p^j(t) - E_i\Big[1_{\{T_{\kappa MDl}^j = T_\kappa^j\}} e^{-r(T_\kappa^j - t)} p^j(T_\kappa^j)\Big] \\ - \frac{\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T} E_2\Big[1_{\{T_{\kappa MDl}^j = T_\kappa^j\}} e^{-rT_\kappa^j} p^j(T_\kappa^j)\Big]\}a'\Big] - \frac{\lambda_D(1-f)F + \lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T}, \end{aligned} \tag{56}$$

where

$$U_i^j(a) = \overline{U}_i^j(a) + \frac{\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\overline{U}_2^j(a)$$

$$\overline{U}_i^j(a) = aE_i\Big[\int_t^{T^j_{\kappa MDl}} e^{-r(s-t)}(C-\delta 1_{\{k(s)=2\}})ds.$$

The calculations in the previous section show that

$$\overline{U}_i^j(1) = \frac{C}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l} - \delta\frac{1_{\{i=2\}}+\frac{\pi_2\lambda}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}}{r+\kappa^j+\lambda+\lambda_D+\lambda_T+\lambda_l},$$

and furthermore show that the problem of an investor is

$$\max_{a'\in\{0,1\}}\Big[U_i^j(a') - q^j(t)a'\Big], \tag{57}$$

where

$$U_i^j(a) = a\Big(\frac{C}{r+\kappa^j+\lambda_D+\lambda_T} - \delta\frac{\pi_2\lambda+1_{\{i=2\}}(\kappa^j+r+\lambda_D+\lambda_T)+\lambda_l}{(r+\kappa^j+\lambda_D+\lambda_T)(r+\kappa^j+\lambda+\lambda_D+\lambda_T+\lambda_l)}\Big) \tag{58}$$

$$\begin{aligned} q^j(t) = p^j(t) &- \frac{\kappa^j\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s}p^j(s)ds \\ &-\kappa^j\int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s}p^j(s+t)ds - \frac{\lambda_D(1-f)F+\lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T}. \end{aligned}$$

To obtain the relationship between $q^j(t)$ and $p^j(t)$, we rewrite

$$\begin{aligned} p^j(t) - q^j(t) &= \kappa^j e^{(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t}\int_t^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s}p^j(s)ds \\ &+ \frac{\kappa^j\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s}p^j(s)ds + \frac{\lambda_D(1-f)F+\lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T} \end{aligned}$$

and differentiate $f(t) := p^j(t) - q^j(t)$ with respect to t to obtain

$$\begin{aligned} (r+\lambda_l+\lambda_D+\lambda_T)f(t) - \dot{f}(t) &= \kappa^j q^j(t) \\ &+ \frac{(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)\kappa^j\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s}p^j(s)ds \\ &+ \frac{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}\Big(\lambda_D(1-f)F+\lambda_T F\Big). \end{aligned}$$

Integrating this forward yields

$$\begin{aligned} p^j(t) &= q^j(t) + \int_t^\infty e^{-(r+\lambda_D+\lambda_T+\lambda_l)(s-t)}\kappa^j q^j(s)ds \\ &+ \frac{(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)}{(r+\kappa^j+\lambda_D+\lambda_T)(r+\lambda_D+\lambda_T+\lambda_l)}[\kappa^j\lambda_l z^j+\lambda_D(1-f)F+\lambda_T F] \\ &= q^j(t) + \int_t^\infty e^{-(r+\lambda_D+\lambda_T+\lambda_l)(s-t)} \\ &\times\Big[\kappa^j q^j(s) + \frac{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}[\kappa^j\lambda_l z^j+\lambda_D(1-f)F+\lambda_T F]\Big]ds, \end{aligned} \tag{59}$$

where

$$z^j = \int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)u} p^j(u)du.$$

We use (59) to rewrite

$$p^j(t) = q^j(t) + e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_u-t)}[p^j(t_u) - q^j(t_u)] + \int_t^{t_u} e^{-(r+\lambda_D+\lambda_T+\lambda_l)(s-t)} \times \Big[\kappa^j q^j(s) + \frac{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}[\kappa^j\lambda_l z^j + \lambda_D(1-f)F + \lambda_T F]\Big]ds \tag{60}$$

for any $t_u \geq t$.

We now find the prices prevailing in steady state (the price prevailing as $t \to \infty$ in absence of aggregate liquidity shocks). In the notation, dependence on time is ignored because we are looking at steady-state quantities. In steady state, the interdealer price is constant and the same for all investors, so (58) gives us

$$q^j = \Big(\frac{r+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}\Big)p - \frac{\kappa^j\lambda_l z^j + \lambda_D(1-f)F + \lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T}.$$

Using (58), we have

$$U_i^j(1) - q^j = \Big(\frac{C}{r+\kappa^j+\lambda_D+\lambda_T} - \delta\frac{\pi_2\lambda + 1_{\{i=2\}}(\kappa^j + r + \lambda_D + \lambda_T) + \lambda_l}{(r+\kappa^j+\lambda_D+\lambda_T)(r+\kappa^j+\lambda+\lambda_D+\lambda_T+\lambda_l)}\Big) - \Big[\Big(\frac{r+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}\Big)p - \frac{\kappa^j\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}z^j - \frac{\lambda_D(1-f)F+\lambda_T F}{r+\kappa^j+\lambda_D+\lambda_T}\Big].$$

Assume that the marginal buyer in steady state is a high investor with the lowest search intensity ($i = 1, j = 1$). This implies that $\overline{U}_1^1(1) - q^1 = 0$, and the interdealer price is given as

$$p_{ss} = \frac{r+\kappa^1+\lambda_D+\lambda_T+\lambda_l}{r+\lambda_D+\lambda_T+\lambda_l}\Big[\frac{C+\lambda_D(1-f)F+\lambda_T F}{r+\kappa^1+\lambda_D+\lambda_T} + \frac{\kappa^1\lambda_l}{r+\kappa^1+\lambda_D+\lambda_T}z^1 - \delta\frac{\pi_2\lambda+\lambda_l}{(r+\kappa^1+\lambda_D+\lambda_T)(r+\kappa^1+\lambda+\lambda_D+\lambda_T+\lambda_l)}\Big]. \tag{61}$$

The buy price is given as $p + z(U_1^j(1) - q^j)$, and the sell price is $p + z(U_2^j(1) - q^j)$. Assume now that as in the previous section, all high investors buy in steady state and all low investors sell in steady state apart from those with the lowest search intensity (for them, low types sell and high types both buy and sell). When a shock occurs, markets become integrated at time $t_2 = -\log(\omega)/\lambda$. At any time $t < t_2$, we have for any j that

$$q^j(t) = \overline{U}_2^j(1) = \frac{C}{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l} - \delta\frac{r+\kappa^j+\pi_2\lambda+\lambda_D+\lambda_T+\lambda_l}{(r+\kappa^j+\lambda+\lambda_D+\lambda_T+\lambda_l)(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)},$$

and the price at time t after an aggregate liquidity is, according to (60),

$$\begin{aligned} p^j(t) &= q^j(t) + e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2-t)}[p^j(t_2) - q^j(t_2)] + \int_t^{t_2} e^{-(r+\lambda_D+\lambda_T+\lambda_l)(s-t)} \\ &\quad\times \Big[\kappa^j q^j(0) + \frac{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}[\kappa^j\lambda_l z^j + \lambda_D(1-f)F+\lambda_T F]\Big]ds \\ &= q^j(0) + e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2-t)}[p(t_2) - q^j(t_2)] + \frac{1-e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2-t)}}{r+\lambda_D+\lambda_T+\lambda_l} \\ &\quad\times \Big[\kappa^j q^j(0) + \frac{r+\kappa^j+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^j+\lambda_D+\lambda_T}[\kappa^j\lambda_l z^j + \lambda_D(1-f)F+\lambda_T F]\Big]. \end{aligned} \tag{62}$$

At time t_2, markets become integrated and the marginal buyer is a low-type ρ^1 investor until time t_2+t_1 with $t_1=\log(\frac{\sum_{j=1}^N \rho^j}{\rho^1})/\lambda$. Thereafter, the marginal buyer is a high-type ρ^1-investor. So, $q^1(t)=U_2^1(1)$ for $t_2 \le t < t_2+t_1$ and $q^1(t)=U_1^1(1)$ for $t \ge t_2+t_1$. Insert these q^1's into (60) to get

$$\begin{aligned}
p(t) &= U_2^1(1)+e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-t)}[p_{ss}-U_1^1(1)]+\int_t^{t_2+t_1} e^{-(r+\lambda_D+\lambda_T+\lambda_l)(s-t)} \\
&\quad \times \Big[\kappa^1 U_2^1(1)+\frac{r+\kappa^1+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^1+\lambda_D+\lambda_T}[\kappa^1\lambda_l z^1+\lambda_D(1-f)F+\lambda_T F]\Big]ds \\
&= U_2^1(1)+e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-t)}[p_{ss}-U_1^1(1)]+\frac{1-e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-t)}}{r+\lambda_D+\lambda_T+\lambda_l} \\
&\quad \times \Big[\kappa^1 U_2^1(1)+\frac{r+\kappa^1+\lambda_D+\lambda_T+\lambda_l}{r+\kappa^1+\lambda_D+\lambda_T}[\kappa^1\lambda_l z^1+\lambda_D(1-f)F+\lambda_T F]\Big] \\
&= U_2^1(1)+e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-t)}[p_{ss}-U_1^1(1)]+\Big(1-e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-t)}\Big)C_2(\kappa^1,z^1)
\end{aligned}$$

for $t_2 \le t < t_2+t_1$, where

$$\begin{aligned}
C_1(\kappa^1,z^1) &= \frac{\kappa^1\lambda_l z^1+\lambda_D(1-f)F+\lambda_T F}{r+\kappa^1+\lambda_D+\lambda_T} \\
C_2(\kappa^1,z^1) &= \frac{1}{r+\lambda_D+\lambda_T+\lambda_l}\Big[\kappa^1 U_2^1(1)+(r+\kappa^1+\lambda_D+\lambda_T+\lambda_l)C^1(\kappa^1,z^1)\Big].
\end{aligned}$$

In particular, we have

$$p(t_2)=U_2^1(1)+e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_1}[p_{ss}-U_1^1(1)]+\Big(1-e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_1}\Big)C_2(\kappa^1,z^1).$$

According to (58), we have for $t_2 \le t < t_2+t_1$

$$\begin{aligned}
q^j(t_2) &= p(t_2)-\kappa^j\int_0^\infty e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)u}p(t_2+u)du-C_1(\kappa^j,z^j) \\
&= p(t_2)-\kappa^j\int_0^{t_1} e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)u}p(t_2+u)du \\
&\quad -\kappa^j\int_{t_1}^\infty e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)u}p_{ss}du-C_1(\kappa^j,z^j).
\end{aligned}$$

Because

$$\begin{aligned}
&\int_0^{t_1} e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)u}e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_1-u)}du \\
&= \frac{e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_1}-e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)t_1}}{\kappa^j},
\end{aligned}$$

calculations show that

$$\begin{aligned}
q^j(t_2) &= p(t_2)-\kappa^j[U_2^1(1)+C_2(\kappa^1,z^1)]\frac{1-e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)t_1}}{r+\kappa^j+\lambda_T+\lambda_D+\lambda_l} \\
&\quad -[p_{ss}-U_1^1(1)-C_2(\kappa^1,z^1)](e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_1}-e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)t_1}) \\
&\quad -\frac{\kappa^j p_{ss}}{r+\kappa^j+\lambda_T+\lambda_D+\lambda_l}e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_1}-C_1(\kappa^j,z^j),
\end{aligned}$$

and therefore

$$p(t_2) - q^j(t_2) = \kappa^j [U_2^1(1) + C_2(\kappa^1, z^1)] \frac{1 - e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)t_1}}{r + \kappa^j + \lambda_T + \lambda_D + \lambda_l}$$
$$+ [p_{ss} - U_1^1(1) - C_2(\kappa^1, z^1)](e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_1} - e^{-(r+\kappa^j+\lambda_T+\lambda_D+\lambda_l)t_1})$$
$$+ \frac{\kappa^j p_{ss}}{r + \kappa^j + \lambda_T + \lambda_D + \lambda_l} e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_1} - C_1(\kappa^j, z^j). \tag{63}$$

Plugging (63) and $q^j(0) = \overline{U}_2^j(1)$ into (62) (and setting $t = 0$) gives the price immediately after the shock. However, we need to find z^j for any j. We have

$$\begin{aligned}
z^j &= \int_0^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s} p^j(s) ds \\
&= \int_{t_2+t_1}^\infty e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s} p_{ss} ds \\
&+ \int_{t_2}^{t_2+t_1} e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s} \\
&\times \left[U_2^1(1) + e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-s)}[p_{ss} - U_1^1(1)] + \left(1 - e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1-s)}\right) C_2(\kappa^1, z^1)\right] ds \\
&+ \int_0^{t_2} e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)s} \\
&\times \left[U_2^j(1) + e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2-s)}[p(t_2) - q^j(t_2)]\right. \\
&+ \left.\left(1 - e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2-s)}\right) \frac{\kappa^j U_2^j(1) + (r + \kappa^j + \lambda_D + \lambda_T + \lambda_l) C_1(\kappa^j, z^j)}{r + \lambda_D + \lambda_T + \lambda_l}\right] ds
\end{aligned} \tag{64}$$

$$\begin{aligned}
&= \frac{e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)}}{r + \kappa^j + \lambda_D + \lambda_T + \lambda_l} p_{ss} + \frac{e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_2} - e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)}}{r + \kappa^j + \lambda_D + \lambda_T + \lambda_l} [U_2^1(1) + C_2(\kappa^1, z^1)] \\
&+ e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)} \int_{t_2}^{t_2+t_1} e^{-\kappa^j s} [p_{ss} - U_1^1(1) - C_2(\kappa^1, z^1)] ds \\
&+ \frac{1 - e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_2}}{r + \kappa^j + \lambda_D + \lambda_T + \lambda_l} \left[U_2^j(1) + \frac{\kappa^j U_2^j(1) + (r + \kappa^j + \lambda_D + \lambda_T + \lambda_l) C_1(\kappa^j, z^j)}{r + \lambda_D + \lambda_T + \lambda_l}\right] \\
&+ e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_2} \int_0^{t_2} e^{-\kappa^j s} \left[p(t_2) - q^j(t_2) - \frac{\kappa^j U_2^j(1) + (r + \kappa^j + \lambda_D + \lambda_T + \lambda_l) C_1(\kappa^j, z^j)}{r + \lambda_D + \lambda_T + \lambda_l}\right] ds \\
&= \frac{e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)}}{r + \kappa^j + \lambda_D + \lambda_T + \lambda_l} p_{ss} + \frac{e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_2} - e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)}}{r + \kappa^j + \lambda_D + \lambda_T + \lambda_l} [U_2^1(1) + C_2(\kappa^1, z^1)] \\
&+ e^{-(r+\lambda_D+\lambda_T+\lambda_l)(t_2+t_1)} \frac{e^{-\kappa^j t_2} - e^{-\kappa^j (t_2+t_1)}}{\kappa^j} [p_{ss} - U_1^1(1) - C_2(\kappa^1, z^1)] \\
&+ \frac{1 - e^{-(r+\kappa^j+\lambda_D+\lambda_T+\lambda_l)t_2}}{r + \lambda_D + \lambda_T + \lambda_l} \left[U_2^j(1) + C_1(\kappa^j, z^j)\right] \\
&+ e^{-(r+\lambda_D+\lambda_T+\lambda_l)t_2} \frac{1 - e^{-\kappa^j t_2}}{\kappa^j} \left[p(t_2) - q^j(t_2) - \frac{\kappa^j U_2^j(1) + (r + \kappa^j + \lambda_D + \lambda_T + \lambda_l) C_1(\kappa^j, z^j)}{r + \lambda_D + \lambda_T + \lambda_l}\right].
\end{aligned} \tag{65}$$

We have that p_{ss} is a function of z^1, given in (61), and $p(t_2) - q^j(t_2)$ is a function of z^j and z^1 in (63). If we plug p_{ss} and $p(t_2) - q^1(t_2)$ into (65), we can solve for z^1. We can then insert z^1 into the expression for p_{ss}. Finally, we can plug $p(t_2) - q^j(t_2)$ into (65) and solve for z^j. This gives us z^j for any j, p_{ss}, and setting $t = 0$ in (62) gives us the price immediately after an aggregate liquidity shock.

Table 5
IRTs and liquidity shocks versus bid/ask/interdealer transactions

Panel A: IRT vs buy/interdealer/sell

	B–S	B–D	D–S	B	S	D
All	4.0%	19.2%	70.5%	0.2%	5.3%	0.8%
Trade size $\leq 10K$	3.2%	19.1%	75.8%	0.2%	1.3%	0.5%
$10K$ < trade size $\leq 50K$	2.0%	19.3%	76.0%	0.1%	1.9%	0.6%
$50K$ < trade size $\leq 100K$	3.1%	19.7%	71.8%	0.1%	4.3%	0.9%
$100K$ < trade size $\leq 500K$	5.9%	19.7%	58.0%	0.1%	14.8%	1.5%
$500K$ < trade size $\leq 1,000K$	16.9%	18.3%	30.3%	0.4%	32.4%	1.7%
$1,000K$ < trade size	24.8%	15.6%	21.7%	0.5%	35.6%	1.8%

Panel B: Correlation between price differences and liquidity shocks

	Small ≤ 100K Large ≥ 100K	Small ≤ 100K Large ≥ 1,000K	Small ≤ 24K Large ≥ 100K
Weekly			
Bid	18%	33%	24%
Ask	33%	34%	34%
Monthly			
Bid	36%	44%	41%
Ask	57%	42%	62%
Observations			
Bid	35,164	39,447	25,338
Ask	49,434	64,722	37,516

The sample period in the paper is October 2004–June 2009. For the subperiod November 2008–June 2009, TRACE has a bid/ask/interdealer indicator for each transaction. In the subperiod, Panel A shows what percentage of IRTs are buy-sell, buy-dealer, sell-dealer, sell-sell, buy-buy, interdealer-interdealer transactions. Panel B shows how estimated liquidity shocks are correlated with the average difference between bid prices of small and large trades and ask prices of small and large trades. First column uses $100,000 in face value as a cutoff between small and large prices. In the second column, small trades have face value smaller than $100,000, large trades greater than or equal to $1,000,000, and trades in between are discarded. In the third column, small trades have face value smaller than $24,000, large trades greater than $100,000, and trades in between are discarded. This is done both on a monthly (8 months) and a weekly (35 weeks) basis.

Appendix C: Robustness checks

IRTs are implicit measures of round-trip costs, the difference in price between a buy and a sell, and used because TRACE provides no buy/sell indicators for most of the data sample. For the last eight months, buy/sell/interdealer indicators are available, and I use this subsample to examine IRTs closer.

Table 5, Panel A, shows the percentage of IRTs that includes a buy and sell transaction, a buy and interdealer transaction, etc. Overall, 4% of IRTs include a buy and sell, 89.7% include an interdealer transaction together with an investor buy or sell transaction, and 6.3% include only sells, buys, or interdealer trades. This evidence suggests that a more appropriate interpretation of IRTs is that they represent the half-spread, since most IRTs reflect either buy-interdealer or interdealer-sell transactions. Panel A also shows that the percentage of IRTs representing full round-trip costs increases in trade size. The increase is accompanied by a corresponding increase in purely one-sided IRTs (only buys, sells, or interdealer trades), so IRTs are reasonable measures of the half-spread for both small and large trade sizes.

I assume in the empirical analysis that IRTs represent full round-trip costs. Since half-spread is a more appropriate interpretation of IRTs, estimated search intensities ρ_i's are upward biased. An alternative explanation is that the holding cost δ is downward biased, since a higher δ yields higher bid-ask spreads, as Equation (1) shows. Most importantly, however, the relation between estimated search intensities of different investors is unlikely to be influenced by the bias in IRT, since IRTs of different trade sizes have similar biases.

The identification of liquidity shocks is not affected by the fact that IRTs can be interpreted as a measure of the half-spread. The following example illustrates this. We can find the size of a liquidity shock by looking at, say, the "difference-in-bidprice-differences" γ_*, where

$$\gamma_*(s) = [B_*(s)^S - B_*(s)^L] - [B^S - B^L],$$

where S denotes "small trader," L denotes "large trader," $*$ marks prices under a liquidity shock of size s, and an absence of stars marks prices in equilibrium. Theorem 2.3 tells us that γ_* is increasing in s. Assume that we are mistakenly looking at sell transactions for the large trader and buy for the small trader, so instead of γ_* we are investigating

$$\begin{aligned}\gamma_{**}(s) &= [B_*(s)^S + \omega_*^S(s) - B_*(s)^L] - [B^S + \omega^S - B^L] \\ &= [B_*(s)^S - B_*(s)^L] - [B^S - B^L] + [\omega_*^S(s) - \omega^S],\end{aligned}$$

where ω^S is the bid-ask spread for a small trader in equilibrium and $\omega_*^S(s)$ is the bid-ask spread under a shock of size s. According to Theorem 2.3, $\omega_*^S(s)$ and ω^S are the same, so $\gamma_* = \gamma_{**}$ for every s. Therefore, liquidity shocks can be identified by looking at either γ_* or γ_{**}. In fact, any combination of bid, ask, or interdealer prices of small versus large traders can be used to identify liquidity shocks.

Liquidity shocks can be identified by looking at either bid or ask prices. To test separately on bid and ask prices, I do the following. I sort all bid prices in the period November 2008–June 2009 into small and large bid prices. In this robustness check, I use all available straight coupon bullet bond prices with a maturity less than 30 years (I have bid/ask indicators and therefore do not need to filter bid and ask prices out of the data using imputed round-trip trades). For robustness, I use three different cutoffs between small and large prices. Trade size of \$100,000 to distinguish between institutional and retail investors is one cutoff, prices smaller than \$100,000 and larger than \$1,000,000 is another (throwing in-between prices away), and prices smaller than \$24,000 (median trade size) and larger than \$100,000 is a third (throwing in-between prices away). For a given bond on a given day, if I have both a small and a large bid, I have an observation of the difference in bids (if I have several small respectively large bids, I take the average). I average all the differences in bids during a month to get a monthly average and find the correlation between monthly averages and estimated monthly liquidity shocks. I repeat this for ask prices to get a correlation between monthly average ask differences and liquidity shocks. Finally, I repeat this exercise on a weekly basis to provide a further robustness check. Panel B in Table 5 shows the results. Across different specifications, average correlation between estimated liquidity shocks and bid differences is 40%, while the corresponding average correlation is 52% for ask differences. Thus, differences in prices of small versus large trades, whether it is bid or ask prices, are correlated with estimated liquidity shocks.

References

Acharya, V., S. Schaefer, and Y. Zhang. 2008. Liquidity Risk and Correlation Risk: A Clinical Study of the General Motors and Ford Downgrade of May 2005. Working Paper, London Business School.

Ambrose, B. W., K. N. Cai, and J. Helwege. 2009. Fallen Angels and Price Pressure. Working Paper.

Ashcraft, A., and D. Duffie. 2007. Systemic Dynamics in the Federal Funds Market. *American Economic Review, Papers and Proceedings* 97:221–25.

Bao, J., J. Pan, and J. Wang. 2011. The Illiquidity of Corporate Bonds. *Journal of Finance* 66:911–46.

Bessembinder, H., K. M. Kahle, W. F. Maxwell, and D. Xu. 2009. Measuring Abnormal Bond Performance. *Review of Financial Studies* 22:4219–58.

Bessembinder, H., and W. Maxwell. 2008. Transparency and the Corporate Bond Market. *Journal of Economic Perspectives* 22:217–34.

Bessembinder, H., W. Maxwell, and K. Venkaraman. 2006. Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds. *Journal of Financial Economics* 82:251–88.

BIS. 2005. BIS Quarterly Review, June. *International Banking and Financial Market Developments* 1–103.

———. 2008. BIS Quarterly Review, June. *International Banking and Financial Market Developments* 1–93.

Bradley. 1981. Nonparametric Estimates of Standard Error: The Jackknife, the Bootstrap, and Other Methods. *Biometrika* 68:589–99.

Brunnermeier, M. K. 2009. Deciphering the Liquidity and Credit Crunch 2007–08. *Journal of Economic Perspectives* 23:77–100.

Brunnermeier, M. K., and L. H. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22:2201–38.

Chakravarty, S., and A. Sarkar. 2003. Trading Costs in Three U.S. Bond Markets. *Journal of Fixed Income* 13:39–48.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein. 2009. On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle. *Review of Financial Studies* 22:3367–409.

Chen, Z., A. A. Lookman, N. Schürhoff, and D. Seppi. 2009. Why Ratings Matter: Evidence from Lehman's Index Rating Rule Change. Working Paper.

Chen, H., G. Noronha, and V. Singhal. 2004. The Price Response to S&P 500 Index Additions and Deletions: Evidence of Asymmetry and a New Explanation. *Journal of Finance* 59:1901–29.

Coval, J., and E. Stafford. 2007. Asset Fire Sales (and Purchases) in Equity Markets. *Journal of Financial Economics* 86:479–512.

Dick-Nielsen, J. 2009. Liquidity Biases in TRACE. *Journal of Fixed Income* 19:43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando. Forthcoming. Corporate Bond Liquidity Before and After the Onset of the Subprime Crisis. *Journal of Financial Economics*.

Duffie, D. 2010. Asset Price Dynamics with Slow-moving Capital. *Presidential Address to the American Finance Association*.

Duffie, D., N. Gârleanu, and L. H. Pedersen. 2005. Over-the-counter Markets. *Econometrica* 73:1815–47.

———. 2007. Valuation in Over-the-counter Markets. *Review of Financial Studies* 20:1865–900.

Edwards, A. K., L. E. Harris, and M. S. Piwowar. 2007. Corporate Bond Market Transaction Cost and Transparency. *Journal of Finance* 62:1421–51.

Ellul, A., C. Jotikasthira, and C. T. Lundblad. 2011 . Regulatory Pressure and Fire Sales in the Corporate Bond Markets. *Journal of Financial Economics* 101:596–620.

Feldhütter, P., and D. Lando. 2008. Decomposing Swap Spreads. *Journal of Financial Economics* 88:375–405.

Fleming, M. J. 2003. Measuring Treasury Market Liquidity. *Federal Reserve Bank of New York Economic Policy Review* 9:83–108.

Gârleanu, N. 2009. Portfolio Choice and Pricing in Illiquid Markets. *Journal of Economic Theory* 144:532–64.

Goldstein, M. A., E. S. Hotchkiss, and E. R. Sirri. 2007. Transparency and Liquidity: A Controlled Experiment on Corporate Bonds. *Review of Financial Studies* 20:235–73.

Green, R., B. Hollifield, and N. Schürhoff. 2007. Financial Intermediation and the Costs of Trading in an Opaque Market. *Review of Financial Studies* 20:275–314.

Hong, G., and A. Warga. 2000. An Empirical Study of Bond Market Transactions. *Financial Analysts Journal* 56:32–46.

Huang, J., and M. Huang. 2003. How Much of the Corporate-Treasury Yield Spread Is Due to Credit Risk? Working Paper, Stanford University.

Jarrow, R., H. Li, and F. Zhao. 2007. Interest Rate Caps "Smile" Too! But Can the LIBOR Market Models Capture It? *Journal of Finance* 62:345–82.

Kraus, A., and H. R. Stoll. 1972. Price Impacts of Block Trading on the New York Stock Exchange. *Journal of Finance* 27:569–88.

Kyle, A. 1985. Continuous Auctions and Insider Trading. *Econometrica* 53:1315–35.

Lagos, R., and G. Rocheteau. 2007. Search in Asset Markets: Market Structure, Liquidity, and Welfare. *American Economic Review, Papers and Proceedings* 97:198–202.

———. 2009. Liquidity in Asset Markets with Search Frictions. *Econometrica* 77:403–26.

Lagos, R., G. Rocheteau, and P.-O. Weill. 2009. Crises and Liquidity in Over-the-counter Markets. Working Paper.

Longstaff, F. 2004. The Flight-to-liquidity Premium in U.S. Treasury Bond Prices. *Journal of Business* 77:511–26.

Longstaff, F., S. Mithal, and E. Neis. 2005. Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit-default Swap Market. *Journal of Finance* 60:2213–53.

Merton, R. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29:449–70.

Mitchell, M., L. H. Pedersen, and T. Pulvino. 2007. Slow-moving Capital. *American Economic Review, Papers and Proceedings* 97:215–20.

Moody's. 2009. Corporate Default and Recovery Rates, 1920–2008. *Moody's investors Service* 1–55.

Newman, Y. S., and M. A. Rierson. 2004. Illiquidity Spillovers: Theory and Evidence from European Telecom Bond Issuance. Working Paper, Stanford.

Schultz, P. 2001. Bond Trading Costs: A Peak Behind the Curtain. *Journal of Finance* 56:677–98.