AM_10700-Quiz 3 Instructor: PaoLien Lai
Dec. 02, 2013 13:10 pm ~ 15:00 pm Closed book, no calculators

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Student name:______________________________

Student number:______________________________

There are 8 questions and 120 marks total.

1. (15 points) A jigsaw puzzle consists of a number of pieces. Two or more pieces with matched boundaries can be put together to form a "big" piece. To be more precise, we use the term *block* to refer to either a single piece or a number of pieces with matched boundaries that are put together to form a "big" piece. Thus, we can simply say that blocks with matched boundaries can be put together to form another block. Finally, when all pieces are put together as one single block, the jigsaw puzzle is solved. Putting 2 blocks together with matched boundaries is called one move. **Please prove (using strong induction) that for a jigsaw puzzle of n pieces, it will always take n-1 moves to solve the puzzle.**
2. (15 points) A **polygon** is a closed geometric figure consisting of a sequence of line segments $s_1, s_2, \ldots, s_n$, called **sides**. Each pair of consecutive sides, s_i and s_{i+1}, $i = 1, 2, \ldots, n-1$, as well as the last side s_n and the first side s_1, of the polygon meet at a common endpoint, called a **vertex**. A polygon is called **simple** if no two nonconsecutive sides intersect. **Please prove that a simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles.**
3. (15 points) Prove that if $h > -1$, then $1 + nh \leq (1 + h)^n$ for all nonnegative integers n. This is called **Bernoulli's inequality**.
4. (15 points) Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$ whenever n is a nonnegative integer.
5. (10 points) Give a recursive definition of the functions max and min so that $\max(a_1, a_2, \ldots, a_n)$ and $\min(a_1, a_2, \ldots, a_n)$ are the maximum and minimum of the n numbers $a_1, a_2, \ldots, a_n$, respectively.
6. (20 points) Let $a_1, a_2, \ldots, a_n$, and $b_1, b_2, \ldots, b_n$ be real numbers. Use the recursive definitions that you gave in Problem 5 to prove these.
   a) $\max(-a_1, -a_2, \ldots, -a_n) = -\min(a_1, a_2, \ldots, a_n)$
   b) $\max(a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n) \leq \max(a_1, a_2, \ldots, a_n) + \max(b_1, b_2, \ldots, b_n)$
7. (10 points) Give a recursive algorithm for finding the maximum of a finite set of integers, making use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first $n - 1$ integers in the list.
8. (20 points) Use mathematical induction to show that given a set of $n + 1$ positive integers, none exceeding $2n$, there is at least one integer in this set that divides another integer in the set.