AM_10700-Quiz 1 | Instructor: PaoLien Lai

Oct. 21, 2013 14:10 pm ~ 15:00 pm | Closed book, no calculators

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Student name: ______________________________

Student number: ______________________________

There are 9 questions and 100 marks total.

1. (15 points) Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find
   a) $A \times B \times C$. b) $C \times B \times A$. c) $C \times A \times B$.
2. (10 points) Find $f \circ g$ and $g \circ f$, where $f(x) = 2x + 1$ and $g(x) = x^3 + 2$, are functions from R to R.
3. (10 points) Let $f(x) = ax + b$ and $g(x) = cx + d$, where *a*, *b*, *c*, and *d* are constants. Determine necessary and sufficient conditions on the constants *a*, *b*, *c*, and *d* so that $f \circ g = g \circ f$.
4. (10 points) Determine whether $f : Z \times Z \to Z$ is onto if
   a) $f(m, n) = m + n + 1$. b) $f(m, n) = |m| - |n|$.
5. (15 points) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with *n* cars made in the *n*th month.
   a) Set up a recurrence relation for the number of cars produced in the first *n* months by this factory.
   b) How many cars are produced in the first year?
   c) Find an explicit formula for the number of cars produced in the first *n* months by this factory.
6. (10 points) Show that $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}$.
7. (10 points) Let $A_i = \{-i, -i + 1, \ldots, -1, 0, 1, \ldots, i - 1, i\}$ for every positive integer i. Find
   a) $\bigcup_{i=1}^{\infty} A_i$ b) $\bigcap_{i=1}^{\infty} A_i$
8. (10 points) Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.
   a) $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$ b) $a_n = na_{n-1} + n^2 a_{n-2}$, $a_0 = 1$, $a_1 = 1$
9. (10 points) Compute each of these double sums.
   a) $\sum_{i=1}^{3}\sum_{j=1}^{2}(2i - j)$ b) $\sum_{i=0}^{3}\sum_{j=0}^{2} i^2 j^3$