CSIEB0080-Quiz 3 | Instructor: PaoLien Lai

Nov. 12, 2012 14:10 pm ~ 15:30 pm | Closed book

Student name:______________________ Student number: ____________

There are 7 questions and 100 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (15 points) Let $P(n)$ be the statement that $1^2+2^2+\ldots+n^2=n(n+1)(2n+1)/6$ for the positive integer n.
   (a) What is the statement $P(1)$?
   (b) Show that $P(1)$ is true, completing the basis step of the proof.
   (c) What is the inductive hypothesis?
   (d) What do you need to prove in the inductive step?
   (e) Complete the inductive step.

2. (15 points) Prove that $1^2+3^2+5^2+\ldots+(2n+1)^2= (n+1)(2n+1)(2n+3)/3$ whenever n is a nonnegative integer.

3. (15 points) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. Prove $P(n)$ is true for $n \geq 8$ by the following strong induction proof process.
   (a) Show that the statement $P(8)$, $P(9)$, and $P(10)$ are true, completing the basis step of the proof.
   (b) What is the inductive hypothesis of the proof?
   (c) What do you need to prove in the inductive step?
   (d) Complete the inductive step for $k \geq 10$.

4. (15 points) Find $f(2), f(3), f(4),$ if f is defined recursively by $f(0)=-1, f(1)=2$ and for $n=1,2, \ldots$
   (a) $f(n+1)=f(n)^2 f(n-1)$.
   (b) $f(n+1)=3f(n)^2-4f(n-1)^2$.
   (c) $f(n+1)=f(n-1)/f(n)$.

5. (10 points) Give a recursive definition of the sequence $\{a_n\}$, $n=1,2,3,\ldots$ if
   (a) $a_n=2n+1$ (b) $a_n=5$

6. (10 points) Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$

7. (20 points) Give a recursive algorithm for finding the sum of the first n odd positive integers.