答案僅供參考

第1題.

(a) $P(2) \Rightarrow 1+\frac{1}{4} < 2-\frac{1}{2}$

(b) basis step : $P(2) = \frac{5}{4} < \frac{6}{4}$ 符合

(c) 假設 $1+\frac{1}{4}+\frac{1}{9}+\dots+\frac{1}{k^2} < 2-\frac{1}{k}$

(d) 證明 $1+\frac{1}{4}+\frac{1}{9}+\dots+\frac{1}{k^2}+\frac{1}{(k+1)^2} < 2-\frac{1}{(k+1)}$

e) $1+\frac{1}{4}+\frac{1}{9}+\dots+\frac{1}{k^2}+\frac{1}{(k+1)^2} < 2-\frac{1}{k}+\frac{1}{(k+1)^2} = 2-\frac{(k+1)^2}{k(k+1)^2}+\frac{k}{k(k+1)^2}$

$= 2+\frac{k-k^2-2k-1}{k(k+1)^2} = 2+\frac{-k^2-k-1}{k(k+1)^2} < 2-\frac{1}{(k+1)}$

第2題.

Let $P(n) = 3^n < n!$, $n > 6$

basis step $P(7) = 3^7 < 7! \Rightarrow 2187 < 5040$ true

Suppose $P(k) = 3^k < k!$ true

prove $P(k+1) = 3^{k+1} < (k+1)!$

$\Rightarrow P(k+1) = 3^k \cdot 3 < k! \cdot (k+1)$

because $k > 6$, and $3^k < k!$ is true

$\Rightarrow P(k+1) = 3^{k+1} < (k+1)!$ true $\Rightarrow P(n)$ is true, for $n > 6$

第3題.

basis step : 4 : 2 × two-dollar 5 : 1 × five-dollar

let $P(n)$ = the number can be formed using two-dollar and five-dollar

$P(4)$, $P(5)$ is true , let $k \geq 5$

$P(k)$ is true $\Rightarrow P(k-1)$ is true

$\Rightarrow P(k-1) + 1 \times$ two-dollar $= P(k+1)$ is true

$\Rightarrow P(n)$ is true for $n \geq 4$

答案僅供參考

第4題.

(a) $f(0)=1$　　$f(2)=f(1)f(0)$　　$f(4)=f(3)f(2)$

$f(1)=1$　　$=1$　　$=1$

$f(3)=f(2)f(1)$

$=1$

(b) $f(2)=f(1)^2+f(0)^3$　　$f(3)=f(2)^2+f(1)^3$　　$f(4)=f(3)^2+f(2)^3$

$=2$　　$=5$　　$=33$

(c) $f(2)=\frac{f(1)}{f(0)}=1$　　$f(3)=\frac{f(2)}{f(1)}=1$　　$f(4)=\frac{f(3)}{f(2)}=1$

第5題.

(a) $a_1=2,\ a_2=6,\ a_3=10,\ a_4=14$

$\Rightarrow a_n=a_{n-1}+4,\ n\geq 2,\ a_1=2.$

(b) $a_1=2\quad a_2=6,\ a_3=12,\ a_4=20$

$\Rightarrow a_n=a_{n-1}+2n,\ n\geq 2,\ a_1=2.$

第6題.

(a) Let S be the set of positive integers congruent to 2 modulo 3

$\Rightarrow 2\in S \Rightarrow$ every $2+3n\in S,\ n\geq 0$

(b) let S be the set of positive integers not divisible by 5

$\Rightarrow 1\in S,\ 2\in S,\ 3\in S,\ 4\in S$

$\Rightarrow$ every $1+5n\in S,\ n\geq 0$

$2+5n\in S,\ n\geq 0$

$3+5n\in S,\ n\geq 0$

$4+5n\in S,\ n\geq 0$

第7題.

findmod (n, m)

basis step

if n=1 , return m;

recursive step

ans = n·findmod(n-1, m) mod m;