
Student name: _____ Student number: _____

There are 8 questions and 130 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (20 points) A vending machine dispensing books of stamps accepts only **one-dollar coins, \$2 bills, and \$5 bills**.
 - (a) (10 points) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.
 - (b) (5 points) What are the initial conditions?
 - (c) (5 points) How many ways are there to deposit **\$10** for a book of stamps?
2. (20 points)
 - (a) (10 points) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take **one stair, two stairs, or three stairs at a time**.
 - (b) (5 points) What are the initial conditions?
 - (c) (5 points) In how many ways can this person climb a flight of eight stairs?
3. (15 points) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with **$a_0 = 10$, $a_1 = -18$, and $a_2 = 30$** .
4. (15 points) Find the solution of the recurrence relation $a_n = 2a_{n-1} + 3 \cdot 2^n$.
5. (15 points) In how many ways can **20** identical donuts be distributed to **five** police officers so that each officer gets **at least three but no more than six donuts**?
6. (15 points)
 - (a) (10 points) What is the generating function for $\{a_k\}$, where a_k is the number of solutions of $x_1 + x_2 + x_3 + x_4 = k$ when x_1, x_2, x_3 , and x_4 are integers with **$x_1 \geq 2$, $2 \leq x_2 \leq 5$, $1 \leq x_3 \leq 4$, and $x_4 \geq 0$** ?
 - (b) (5 points) Use your answer of part (a) to find a_7 .
7. (15 points) There are **2405** computer science students at a school. Of these, **1678** have taken a course in Java, **999** have taken a course in Linux, and **543** have taken a course in C. Further, **876** have taken courses in both Java and Linux, **321** have taken courses in both Linux and C, and **290** have taken courses in both Java and C. If **198** of these students have taken courses in Linux, Java, and C, how many of these **2405** students have not taken a course in any of these three programming languages?
8. (15 points) How many permutations of the 10 digits satisfy at least one of the following three conditions:
(1) **begin with the 4 digits 9876**, (2) contain the **digits 34 in the fifth and sixth positions**, and (3) **end with the 2 digits 12**?