答案僅供參考

第1題.

1. (a) $-123 = 19 \times (-7) + 10$

$a = dq + r,\ q = -7,\ r = 10$

(b) $777 = 21 \times 37 + 0$

$a = dq + r,\ q = 37,\ r = 0$

第2題.

2. (a) $-122 = 17 \times (-8) + 14$, $-122 \equiv 14 \pmod{17}$

$-122 \not\equiv 5 \pmod{17}$

(b) $29 = 17 \times 1 + 12$, $29 \equiv 12 \pmod{17}$

$29 \not\equiv 5 \pmod{17}$

第3題.

3. $x_0 = 3$

$x_1 = (7 \times 3 + 4) \bmod 9 = 7$

$x_2 = (7 \times 7 + 4) \bmod 9 = 8$

$x_3 = (7 \times 8 + 4) \bmod 9 = 6$

$x_4 = (7 \times 6 + 4) \bmod 9 = 1$

$x_5 = (7 \times 1 + 4) \bmod 9 = 2$

$x_6 = (7 \times 2 + 4) \bmod 9 = 0$

$x_7 = (7 \times 0 + 4) \bmod 9 = 4$

$x_8 = (7 \times 4 + 4) \bmod 9 = 5$

$x_9 = (7 \times 5 + 4) \bmod 9 = 3$ (repeat as x_0)

$\vdots$

答案僅供參考

第4題.

4\. (a) $2\,|\,998$, 499 $\quad 998 = 2^1 \times 499^1$

(b) $11\,|\,122221$, $41\,|\,11111$, 271 $\quad 122221 = 11^1 \times 41^1 \times 271^1$

第5題.

5 (a)
$21 = 3\times 7$
$34 = 2\times 17$
$47 = 1\times 47$
$55 = 5\times 11$
$\Rightarrow$ yes, they are pairwise relatively prime

(b)
$17 = 1\times 17$
$18 = 2\times 3^2$
$19 = 1\times 19$
$25 = 5^2$
$\Rightarrow$ yes, they are pairwise relatively prime.

第6題.

6\. (a) $\operatorname{lcm}(3^7\cdot 5^3\cdot 7^3,\ 2^7\cdot 3^3\cdot 5^9) = 2^7\cdot 3^7\cdot 5^9\cdot 7^3$

(b)
$11111 = 41\times 271$
$9999 = 3^2\times 11\times 101$
$\operatorname{lcm}(11111, 9999) = 3^2\times 11^1\times 41^1\times 101^1\times 271^1$

第7題.

7\.

$$\begin{aligned}
141 &= 19\cdot 7 + 8 \\
19 &= 8\cdot 2 + 3 \\
8 &= 3\cdot 2 + 2 \\
3 &= 2\cdot 1 + 1 \\
2 &= 1\cdot 2
\end{aligned}
\qquad
\begin{aligned}
1 &= 3 - 2\cdot 1 \\
&= 3 - (8 - 3\cdot 2) \\
&= 3\cdot 3 - 8 \\
&= (19 - 8\cdot 2)\cdot 3 - 8 \\
&= 19\cdot 3 - 7\cdot 8 \\
&= 19\cdot 3 - 7\cdot(141 - 19\cdot 7) \\
&= 52\cdot 19 - 7\cdot 141
\end{aligned}$$

$\Rightarrow$ 52 is an inverse of 19 modulo 141.

答案僅供參考

第 8 題.

$x \equiv 5 \pmod 6$

$x \equiv 3 \pmod{10}$

$x \equiv 8 \pmod{15}$

$x = 6t+5 \equiv 3 \pmod{10}$

find an inverse $t=3$

$t = 10u+3$

$\Rightarrow x = 6(10u+3)+5$

$= 60u + 23 \equiv 8 \pmod{15}$

find any integer is inverse

$\Rightarrow$ solution: $x \equiv 23 \pmod{60}$

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第 9 題.

$669 = 457 \times 1 + 212$

$457 = 212 \times 2 + 33$

$212 = 33 \times 6 + 14$

$33 = 14 \times 2 + 5$

$14 = 5 \times 2 + 4$

$5 = 4 \times 1 + 1$

$4 = 1 \times 4$

$1 = 5 - 4 \times 1$

$= 5 - (14 - 5 \times 2)$

$= 5 \times 3 - 14$

$= (33 - 14 \times 2) \times 3 - 14$

$= 33 \times 3 - 14 \times 7$

$= 33 \times 3 - (212 - 33 \times 6) \times 7$

$= 33 \times 45 - 212 \times 7$

$= (457 - 212 \times 2) \times 45 - 212 \times 7$

$= 457 \times 45 - 212 \times 97$

$= 457 \times 45 - (669 - 457 \times 1) \times 97$

$= 457 \times 142 - 669 \times 97$

$1 = 457 \times 142 - 669 \times 97$ is a linear combination of 457 and 669.

第 10 題.

$n^2 \equiv 1 \pmod 8$

$\Rightarrow$ n^2-1 可被 8 整除

$\Rightarrow$ $8 \mid n^2-1$

$\Rightarrow$ $8 \mid (n+1)(n-1)$

if $n=1$

$(2 \times 0) \div 8 = 0$

if $n=3$

$(4 \times 2) \div 8 = 1$

if $n=5$

$(6 \times 4) \div 8 = 3$

if $n=7$

$(8 \times 6) \div 8 = 6$

if $n=9$

$(10 \times 8) \div 8 = 10$

$\vdots$

if $n = 2K+1$ $(K \in N)$

$(2K+2)(2K)$

$= 4K(K+1)$

連續2个正整數,其中1个必為2的倍數,故

$8 \mid 4K(K+1)$

得証