AM_10700-Quiz 3 Instructor: PaoLien Lai
Nov. 15, 2012 14:10 pm ~ 15:30 pm Closed book, no calculators

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Student name:_____ Student number: _____

There are 7 questions and 100 marks total. Please write a detailed answer to each of the following questions.

1. (15 points) Let $P(n)$ be the statement that $1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$, where n is an integer greater than 1.
   (a) What is the statement $P(2)$?
   (b) Show that $P(2)$ is true, completing the basis step of the proof.
   (c) What is the inductive hypothesis?
   (d) What do you need to prove in the inductive step?
   (e) Complete the inductive step.

2. (15 points) Prove that $3^n < n!$ if n is an integer greater than 6.

3. (15 points) Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction.

4. (15 points) Find $f(2)$, $f(3)$, and $f(4)$ if f is defined recursively by $f(0)=f(1)=1$ and for $n=1,2,\ldots$
   (a) $f(n+1)=f(n)\,f(n-1)$.
   (b) $f(n+1)= f(n)^2+f(n-1)^3$.
   (c) $f(n+1)=f(n)/f(n-1)$.

5. (10 points) Give a recursive definition of the sequence $\{a_n\}$, $n=1,2,3,\ldots$ if
   (a) $a_n=4n-2$ (b) $a_n=n(n+1)$

6. (10 points) Give a recursive definition of
   (a) the set of positive integers congruent to 2 modulo 3.
   (b) the set of positive integers not divisible by 5.

7. (20 points) Give a recursive algorithm for finding $n!$ **mod** m whenever n and m are positive integers.