

Student name: _____ Student number: _____

There are 10 questions and 100 marks total. Please write a detailed answer to each of the following questions.

1. (10 points) What are the quotient and remainder when
(a) -123 is divided by 19? (b) 777 is divided by 21?
2. (10 points) Decide whether each of these integers is congruent to 5 modulo 17
(a) -122 (b) 29
3. (10 points) What sequence of pseudorandom numbers is generated using the following generator
 $x_{n+1} = (7x_n + 4) \bmod 9$ with seed $x_0 = 3$?
4. (10 points) Find the prime factorization of each of these integers.
(a) 998 (b) $122,221$
5. (10 points) Determine whether the integers in each of these sets are pairwise relatively prime.
(a) $\{21, 34, 47, 55\}$ (b) $\{17, 18, 19, 25\}$
6. (10 points) What are the least common multiples of these pairs of integers?
(a) $3^7 \cdot 5^3 \cdot 7^3$, $2^7 \cdot 3^3 \cdot 5^9$ (b) 11111 , 9999
7. (10 points) Find an inverse of 19 modulo 141.
8. (10 points) Find all solutions, if any, to the system of congruences.
$$x \equiv 5 \pmod{6}$$
$$x \equiv 3 \pmod{10}$$
$$x \equiv 8 \pmod{15}$$
9. (10 points) Use the extended Euclidean algorithm to express $\gcd(457, 669)$ as a linear combination of 457 and 669.
10. (10 points) Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.