AM_10700-Quiz 1 Instructor: PaoLien Lai

Oct. 18, 2012 14:10 pm ~ 15:00 pm Closed book, no calculators

---

Student name:________________________________

Student number: ________________________________

There are 9 questions and 100 marks total.

1. (15 points) **Show** that the compound proposition below is a **contradiction**:

$$(\mathbf{p} \vee \mathbf{q}) \wedge (\sim \mathbf{p} \vee \mathbf{q}) \wedge (\mathbf{p} \vee \sim \mathbf{q}) \wedge (\sim \mathbf{p} \vee \sim \mathbf{q})$$

2. (10 points) **Find** $\mathbf{f} \circ \mathbf{g}$ and $\mathbf{g} \circ \mathbf{f}$, where $f(x)=x^2+1$ and $g(x)=x+2$, are functions from $\mathbf{R}$ to $\mathbf{R}$.

3. (10 points) Let $f(x)=2x$. What is (a) $f(Z)$? (b) $f(R)$?

4. (10 points) **Find the inverse function** of $f(x)=x^3+1$ where $x \in R$.

5. (10 points) Let $A_i=\{\ldots,-2,-1,0,1,2,\ldots,i\}$ for $i=1,2,3,\ldots$. **Find** (a) $\bigcup_{i=1}^{n} A_i$ (b) $\bigcap_{i=1}^{n} A_i$

6. (10 points) Suppose that the universal set is $U=\{1,2,3,4,5,6,7,8,9,10\}$. **Express each of these sets with bit strings** where the ith bit in the string is 1 if i is in the set and 0 otherwise. (Note, the elements of a bit string are numbered from zero up to the number of bits in the string less one, in *right to left order*, (the rightmost bit is numbered zero).)

   (a) $\{3,4,5\}$ (b) $\{1,3,6,10\}$

7. (15 points) The ***symmetric difference*** of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B. **Determine whether the symmetric difference is associative**; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?

8. (10 points) **Compute** each of these double sums.

   (a) $\sum_{i=1}^{7} \sum_{j=1}^{3} (-i + 4j)$ (b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (i^2 j^3)$

9. (10 points) **Determine** whether the function $f: Z \times Z \rightarrow Z$ is **onto** if

   (a) $f(m,n)=m+n$ (b) $f(m,n)=m^2+n^2$