AM_10700-Quiz 5 Instructor: PaoLien Lai

Jan. 6, 2014 14:10 pm ~ 16:00 pm Closed book, no calculators

Student name: ____

Student number: ____

There are 10 questions and 150 marks total.

1. (20%) Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 25$ where $-3 \leq c_1$, $-3 \leq c_2$, $-5 \leq c_3 \leq 5$, and $0 \leq c_4$.
2. (10%) Show that $(1-4x)^{-1/2}$ generates sequence $\binom{2n}{n}, n \in N$.
3. (10%) Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.

$1000 - \lfloor \frac{1000}{7} \rfloor + \lfloor \frac{1000}{11} \rfloor - \lfloor \frac{1000}{77} \rfloor$

220.

4. (10%) How many positive integers not exceeding 1000 are divisible by 7 or 11?
5. (15%) Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length six?
6. (15%) For $n \geq 2$, suppose that there are n people at a party and that each of these people shakes hands (exactly one time) with all of the other people there (and no one shakes hands with himself or herself). How many handshakes occur among n people?
7. (15%) How many ways are there to assign five different jobs to four different employees if every employee is assigned at least one job?
8. (15%) Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.
   a). Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.
   b). What are the initial conditions?
   c). How many different messages can be sent in 12 microseconds using these two signals?
9. (20%) For $n \geq 0$, let $S = \{1, 2, 3, \ldots, n\}$ (when $n = 0$, $S = \emptyset$), and let a_n denote the number of subsets of S that contain no consecutive integers. Find and solve a recurrence relation for a_n.
10. (20%) For n distinct objects, let $a(n, r)$ denote the number of ways we can select, without repetition, r of the n objects when $0 \leq r \leq n$. Here $a(n, r) = 0$ when $r > n$. Use the recurrence relation $a(n, r) = a(n-1, r-1) + a(n-1, r)$, where $n \geq 1$ and $r \geq 1$, to show that $f(x) = (1 + x)^n$ generates $a(n, r)$, $r \geq 0$.