

Student name: _____ Student number: _____

There are 9 questions and 150 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (20 points)
 - (a) (10 points) Find a recurrence relation for the number of bit strings of length n that do not contain **two** consecutive 0s.
 - (b) (5 points) What are the initial conditions? *two*
 - (c) (5 points) How many bit strings of length seven do not contain ~~three~~ *two* consecutive 0s?
2. (20 points) A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the ~~three~~ *two* previous years.
 - (a) (10 points) Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n , under the assumption for this model. *and*
 - (b) (10 points) Find L_n if 1,000 lobsters were caught in year 1, ~~2,000 were caught in year 2, and 3,000 were caught in year 3.~~ *2,000 were caught in year 2, and 3,000 were caught in year 3.*
3. (15 points) Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 14$, $a_1 = -8$, and $a_2 = 16$.
4. (15 points)
 - (a) (5 points) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n$.
 - (b) (5 points) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 5$.
5. (15 points) Use generating functions to determine the number of different ways **12** identical balloons can be given to **five** children if each child receives at least two balloons.
6. (15 points) Use generating functions to find the number of ways to make change for **\$10** using
 - (a) (5 points) \$1, \$2, and \$5 bills.
 - (b) (5 points) \$1, \$2, \$5, and \$10 bills.
 - (c) (5 points) \$1, \$2, and \$5 bills if at least one bill of each denomination is used.
7. (15 points) How many students are enrolled in a course either in **calculus, discrete mathematics, data structures, or programming languages** at a school if there are **605, 292, 213, and 443** students in these courses, respectively; **16** in both calculus and data structures; **200** in both calculus and programming languages; **200** in both discrete mathematics and data structures; **43** in both discrete mathematics and programming languages; and no student may take calculus and discrete mathematics, or data structures and programming languages, concurrently?
8. (15 points) Find the number of positive integers not exceeding **8000** that are either the square or the cube of an integer.
9. (20 points) Messages are transmitted over a communications channel using **two signals**. The transmittal of one signal requires **1 microsecond**, and the transmittal of the other signal requires **2 microseconds**.
 - (a) (10 points) Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in **n microseconds**.
 - (b) (5 points) What are the initial conditions?
 - (c) (5 points) How many different messages can be sent in **10 microseconds** using these two signals?