

Student name: \_\_\_\_\_ Student number: \_\_\_\_\_

There are 9 questions and 100 marks total. Please write a detailed answer to each of the following questions.

1. (10 points) Suppose that a large family has 15 children, including two sets of identical triplets, three sets of identical twins, and three individual children. How many ways are there to seat these children in a row of chairs if the identical triples or twins cannot be distinguished from one another?
2. (15 points) Fifteen people on a softball team show up for a game
  - (a) How many ways are there to choose 10 players to take the field?
  - (b) How many ways are there to assign the 10 positions by selecting players from the 15 people who show up?
  - (c) Of the 15 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?
3. (10 points) Show that  $\binom{n}{k} \leq 2^n$  for all positive integers  $n$  and all integers  $k$  with  $0 \leq k \leq n$ .
4. (10 points) Suppose that a department contains 7 men and 12 women. How many ways are there to form a committee with seven members if it must have more women than men?
5. (10 points) How many positive integers less than 1,000,000 have exactly one digit equal to 8 and have a sum of digits equal to 15?
6. (10 points) What is the minimum number of students, each of whom comes from one of the 22 counties, who must be enrolled in a university to guarantee that there are at least 100 who come from the same county?
7. (10 points) How many ways are there to seat eight people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table?
8. (15 points) How many strings of five decimal digits
  - (a) do not contain the same digit twice?
  - (b) end with an odd digit?
  - (c) have exactly three digits that are 8s?
9. (10 points) There are 9 questions on the discrete mathematics quiz. How many ways are there to assign scores to the questions if the sum of the scores is 100 and each question is worth at least 5 points?