

Student name: _____ Student number: _____

There are 6 questions and 120 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (30 points) For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.
 - (a) $\{(1, 1), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
 - (b) $\{(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
 - (c) $\{(2, 4), (4, 2)\}$
 - (d) $\{(1, 2), (2, 3), (3, 4)\}$
 - (e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
 - (f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$
2. (10 points) How many different relations are there from a set with n elements to a set with m elements?
3. (20 points) Let R be the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of positive integers. Find
 - (a) R^{-1} .
 - (b) \bar{R} .

4. (20 points) Let R be the relation represented by the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Find the matrix representing

- (a) R^2 .
 - (b) R^3 .
5. (25 points) Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

- (a) $R_1 \cup R_2$.
 - (b) $R_1 \cap R_2$.
 - (c) $R_2 \circ R_1$.
 - (d) $R_1 \circ R_2$.
 - (e) $R_1 \oplus R_2$.
6. (15 points) List the ordered pairs in the relations represented by the directed graph above.

