答案僅供參考

第1題.

basis step: P(1)，n is positive integer，因為只有一塊
所以 0 moves，完成 ~~basis~~ step.

~~inductive step~~: 已知 basis step 得證，n is positive integer，
~~假設~~ P(2) 成立，當 P(2) 時 add 1塊 = 2-1 = 1 moves 個動作
P(3) 當 P(3) 時，有 add 1塊 做2次 或 2塊組好後 join
P(4) = 3-1 = 2 moves
當 P(4) 時，有 add 1塊做3次 塊 or 3塊 組好後 join
= 4-1 = 3 moves 完成 basis step

inductive step: basis step 成立 P(k) 假設成立，P(k) 共有 k-1 個動作
k≥4
P(k+1) 有 k+1-1 個動作，完成 induction hyphethesis.

藉由 strong induction 可得知 jigsaw puzzle of n pieces, it will always take n-1 moves to solve the puzzle 可得證

第2題.

Proof (of Theorem 1): We will prove this result using strong induction. Let $T(n)$ be the statement that every simple polygon with n sides can be triangulated into $n-2$ triangles.

BASIS STEP: $T(3)$ is true because a simple polygon with three sides is a triangle. We do not need to add any diagonals to triangulate a triangle; it is already triangulated into one triangle, itself. Consequently, every simple polygon with $n = 3$ has can be triangulated into $n - 2 = 3 - 2 = 1$ triangle.

INDUCTIVE STEP: For the inductive hypothesis, we assume that $T(j)$ is true for all integers j with $3 \leq j \leq k$. That is, we assume that we can triangulate a simple polygon with j sides into $j-2$ triangles whenever $3 \leq j \leq k$. To complete the inductive step, we must show that when we assume the inductive hypothesis, $P(k+1)$ is true, that is, that every simple polygon with $k+1$ sides can be triangulated into $(k+1)-2 = k-1$ triangles.

So, suppose that we have a simple polygon P with $k+1$ sides. Because $k+1 \geq 4$, Lemma 1 tells us that P has an interior diagonal ab. Now, ab splits P into two simple polygons Q, with s sides, and R, with t sides. The sides of Q and R are the sides of P, together with the side ab, which is a side of both Q and R. Note that $3 \leq s \leq k$ and $3 \leq t \leq k$ because both Q and R have at least one fewer side than P does (after all, each of these is formed from P by deleting at least two sides and replacing these sides by the diagonal ab). Furthermore, the number of sides of P is two less than the sum of the numbers of sides of Q and the number of sides of R, because each side of P is a side of either Q or of R, but not both, and the diagonal ab is a side of both Q and R, but not P. That is, $k+1 = s+t-2$.

We now use the inductive hypothesis. Because both $3 \leq s \leq k$ and $3 \leq t \leq k$, by the inductive hypothesis we can triangulate Q and R into $s-2$ and $t-2$ triangles, respectively. Next, note that these triangulations together produce a triangulation of P. (Each diagonal added to triangulate one of these smaller polygons is also a diagonal of P.) Consequently, we can triangulate P into a total of $(s-2)+(t-2) = s+t-4 = (k+1)-2$ triangles. This completes the proof by strong induction. That is, we have shown that every simple polygon with n sides, where $n \geq 3$, can be triangulated into $n-2$ triangles. ◁

答案僅供參考

證　　明 : 利用強歸納法。令命題 T(n)為每個有 n 個編的簡單多邊形，都能被切分成 n-2 個三角形

基礎步驟 : T(3)為真，因為有 3 個邊的多邊形就是三角形。不需要添加任何對角線來切分，就有 3-2=1 個三角形。

歸納步驟 : 歸納假說為假設對所有的正整數 j，$3 \le j \le k$， T(j)皆為真。我們將往證 T(k+1)為真，亦即有 k+1 個邊的簡單多邊形都能切分成 (k+1)-2=k-1 個三角形。

若有個多邊形 P，其有 k+1 個邊，根據 Lemma 1，一定有內部對角線 ab。連接此對角線可得到兩個多邊形 Q 與 R，令其分別有 s 個邊和 t 個邊(其中有一個共用的邊 ab，而其他邊都是多邊形 P 的邊)，所以 (s-1)+(t-1)=k+1，即 s+t=k+3。由於 $s, t \ge 3$，我們有 $3 \le s, t \le k$。根據歸納假說，多邊形 Q 能切分成 s-2 個三角形，而多邊形 R 能切分成 t-2 個三角形，所以我們證明出多邊形 P 能切分成(s-2)+(t-2)=(s+t)-4=(k+3)-4=k-1 個三角形，此等式證明了 T(k+1)為真。

完成兩個步驟，根據強歸納法，得證結果為真。

第3題.

Basis step: $P(1): 1+1\cdot h \le (1+h)^1$ is true

Inductive step: $P(k): 1+k\cdot h \le (1+h)^k$ is true, for $k \ge 1$
Assume

To prove $P(k+1): 1+(k+1)h \le (1+h)^{k+1}$ is true

$1+(k+1)h \le (1+h)^k + h$, 因 $h > -1$　$(1+h)^k + h$ 一定不大於 $(1+h)^k \cdot (1+h)$

$1+(k+1)h \le (1+h)^k + h \le (1+h)^k \cdot (1+h)$

$1+(k+1)h \le (1+h)^{k+1}$ ⇒ complete Inductive step.

By strong Induction, if $h > -1$, $1+nh \le (1+h)^n$ for all nonnegative intergers n 得證

答案僅供參考

第4題.

Basis step: P(0): $1^2 = \frac{1\cdot 1\cdot 3}{3} = 1$ is true.

Inductive step: Assum P(k): $1^2+3^2+\cdots+(2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$

is true, $k \geq 0$

To prove $P(k+1) = 1^2+3^2+\cdots+(2k+1)^2+(2k+3)^2 = \frac{(k+2)(2k+3)(2k+5)}{3}$ is true

$P(k+1) = 1^2+3^2+\cdots+(2k+1)^2+(2k+3)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2$

$= (2k+3)\cdot\left(\frac{2k^2+3k+1}{3} + \frac{6k+9}{3}\right) = \frac{(k+2)(2k+3)(2k+5)}{3}$ complete Inductive step

By Mathematical Induction, $1^2+3^2+\cdots(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

for all nonnegative interger n. #

第5題.

5.1

recursive definition

$\max(a_1, a_2) = a_1$ if $a_1 \geq a_2$

$\max(a_1, a_2) = a_2$ if $a_2 < a_1$

$\min(a_1, a_2) = a_1$ if $a_1 \leq a_2$

$\min(a_1, a_2) = a_2$ if $a_1 > a_2$

+10.

$\max(a_1 \cdot a_2 \cdot a_3 \cdots a_n) = \max(\max(a_1 \cdot a_2\, a_3 \cdots a_{n-1}), a_n)$

$\min(a_1 \cdot a_2 \cdot a_3 \cdots a_n) = \min(\min(a_1 \cdot a_2 \cdot a_3 \cdots a_{n-1}), a_n)$ #

答案僅供參考

第6題.

a) $\max(-a_1, -a_2, \dots -a_n) = -\min(a_1, a_2, \dots a_n)$

$\max(-a_1, -a_2) = -\min(a_1, a_2) = -a_1$ if $a_1 < a_2$ holds

$\max(-a_1, -a_2) = -\min(a_1, a_2) = -a_2$ if $a_1 > a_2$

$\max(-a_1, -a_2 \dots -a_n) = -\min(a_1, a_2, \dots a_n)$，因max裡加了負號，使得max找出的是原本$a_n$中最小的加負號，等於 $-\min(a_1, a_2, \dots a_n)$

b)

$\max(a_1+b_1, a_2+b_2, \dots a_n+b_n) \le \max(a_1, a_2 \dots a_n) + \max(b_1, b_2 \dots b_n)$

後項找出了最大的a_n項，b_n項相加，而前項找出的是最大的a_n+b_n項，前項並未是最大的a，b相加，若是最大之項相加也不超過 $\max(a_1, a_2 \dots a_n) + \max(b_1, b_2 \dots b_n)$

第7題.

$\max(a_1, a_2 \dots a_n)$, the first is a_n

if $n=1$ return a_1;

else return $\max(\max(a_1, a_2, a_3, \dots a_{n-1}), a_n)$

第8題.

The base case is $n=1$. If we are given a set of two elements from {1, 2}, then indeed one of them divides the other. Assume the inductive hypothesis, and consider a set A of $n+2$ elements from $\{1, 2, ..., 2n, 2n+1, 2n+2\}$. We must show that at least one of these elements divides another. If as many as $n+1$ of the elements of A are less than $2n + 1$, then the desired conclusion follows immediately from the inductive hypothesis.

Therefore we can assume that both $2n+1$ and $2n+2$ are in A, together with n smaller elements. If $n+1$ is one of these smaller elements, then we are done, since $n+1 \mid 2n+2$. So we can assume that $n+1 \notin A$.

Now apply the inductive hypothesis to $B = A - \{2n+1, 2n+2\} \cup \{n+1\}$. Since B is a collection of $n+1$ numbers from $\{1, 2, \ldots, 2n\}$, the inductive hypothesis guarantees that one element of B divides another. If $n+1$ is not one of these two numbers, then we are done. So we can assume that $n+1$ is one of these two numbers. Certainly $n+1$ can't be the divisor, since its smallest multiple is too big to be in B, so there is some $k \in B$ that divides $n+1$. But now k and $2n+2$ are numbers in A, with k dividing $n+2$, and we are done. An alternative proof of this theorem is given in Example 11 of Section 6.2.