

Student name: _____ Student number: _____

There are 9 questions and 100 marks total.

1. (15 points) Let $S=\{0,1,2,3\}$, $T=\{1,x,y\}$, and $V=\{0,w,z\}$.

Find (a) $S \times V$ (b) $S - T - V$ (c) $S \cap T \cap V$

2. (10 points) **Translate** each of these quantifications into English and **determine** its truth value.

(a) $\forall x \in \mathbf{R} (x^2 \neq -1)$ (b) $\exists x \in \mathbf{Z} (x^2 = 2)$

3. (10 points) **Find** these values. (a) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$ (b) $\left\lfloor -\frac{7}{8} + \left\lceil -\frac{3}{4} \right\rceil \right\rfloor$

4. (15 points) **Determine** whether each of these functions is a bijection from \mathbf{R} to \mathbf{R} .

(a) $f(x) = -3x + 5$ (b) $f(x) = -5x^2 + 6$ (c) $f(x) = (x^2 - 1)/(x + 1)$

5. (10 points) **Compute** each of these double sums.

(a) $\sum_{i=1}^2 \sum_{j=1}^3 (i + j)$ (b) $\sum_{i=0}^2 \sum_{j=0}^3 (ij)$

6. (10 points) **Give an example** of a *decreasing function* with the set of real numbers as its domain and codomain that is not one-to-one.

7. (10 points) Let $S = \{-1, 0, 2, 4, 7\}$. **Find** $f(S)$ if (a) $f(x) = 1$ (b) $f(x) = 2x + 1$.

8. (10 points) **Find** $f+g$ and fg for f and g , where $f(x) = x^2 + 1$ and $g(x) = x + 2$ are two functions from \mathbf{R} to \mathbf{R} .

9. (10 points) Let $A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$. **Find** (a) $\bigcup_{i=1}^n A_i$ (b) $\bigcap_{i=1}^n A_i$