答案僅供參考

第1題.

$$(x^{-3}+x^{-2}+x^{-1}+x^{0}+x^{1}+\cdots)(x^{-3}+x^{-2}+x^{-1}+x^{0}+x^{1}+\cdots)(x^{-5}+x^{-4}+\cdots+x^{0}+x^{1}+\cdots+x^{4}+x^{5})(x^{0}+x^{1}+x^{2}+x^{3}+\cdots)$$

generating function ↑ the equation $c_1+c_2+c_3+c_4=25$ 的答案為 x^{25} 的係數

第2題.

$$G(x)\underset{-1}{=}\sum_{n=1}^{\infty} C_n^{n-\frac{1}{2}}\times 4^n x^n$$

$$(1-4x)^{-\frac{1}{2}} = \frac{1}{(1-4x)^{\frac{1}{2}}} = 1 + C_1^{\frac{1}{2}}\,4x + \overbrace{C_2^{\frac{1}{2}+1}(4x)^2}^{n} + \cdots$$

$$C_n^{\frac{1}{2}+n-1} = C_n^{n-\frac{1}{2}} = \frac{\frac{2n-1}{2}\times\frac{2n-3}{2}\times\cdots\times\frac{1}{2}}{n!}\times 4^n$$

$$C_n^{2n} = \frac{2n\times(2n-1)\times\cdots\times(n+1)}{n!} \Rightarrow \frac{(2n-1)(2n-3)\times\cdots\times 1}{2^n\, n!}\times 2^{2n} \Rightarrow \frac{(2n-1)(2n-3)\times\cdots\times 1}{n!}\times 2^n$$

同 乘上 $n!$ 乘上 $n!$

$$\frac{2n\times(2n-1)\times\cdots\times(n+1)\times n!}{n!} = \frac{(2n-1)(2n-3)\times\cdots\times 1}{n!}\times 2^n \times \frac{(n\times(n-1)\times(n-2)\times\cdots)}{2n\times(2n-2)\cdots\times 2}$$

左右相等 整理

$$\frac{2n\times(2n-1)(2n-3)\times(2n-4)\times\cdots\times 2\times}{n!}$$

$$C_n^{2n}\times n! = C_n^{n-\frac{1}{2}}\times 4^n\times n!$$

$$C_n^{2n} = C_n^{n-\frac{1}{2}}\times 4^n \quad \#$$

$$G(x)\underset{-1}{=}\sum_{n=1}^{\infty} C_n^{n-\frac{1}{2}}\times 4^n x^n$$

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第3題.

$$\frac{1}{(x-3)(x-2)^2} = \frac{-1}{(3-x)(2-x)^2} = \frac{-1}{3\times4\,(1-\frac{x}{3})[1-(\frac{x}{2})]^2}$$

$$= \frac{-1}{12}\left(1+\frac{x}{3}+\left(\frac{x}{3}\right)^2+\cdots\right)\left(1+2\left(\frac{x}{2}\right)+3\left(\frac{x}{2}\right)^2+\cdots\right)$$

$$(n+1)\left(\frac{x}{2}\right)^n$$

1	$\times 9(\frac{x}{2})^8$
$\frac{x}{3}$	$\times 8(\frac{x}{2})^7$
$(\frac{x}{3})^2$	$\times 7(\frac{x}{2})^6$
$(\frac{x}{3})^3$	$\times 6(\frac{x}{2})^5$
$(\frac{x}{3})^4$	$\times 5(\frac{x}{2})^4$
$(\frac{x}{3})^5$	$\times 4(\frac{x}{2})^3$
$(\frac{x}{3})^6$	$\times 3(\frac{x}{2})^2$
$(\frac{x}{3})^7$	$\times 2(\frac{x}{2})^1$
$(\frac{x}{3})^8$	$\times 1$

$$= \frac{-1}{12}\left[9\times\left(\frac{1}{2}\right)^8+\left(\frac{1}{3}\right)\right.$$

$$= \frac{-1}{12}\sum_{n=0}^{8}\left(\frac{1}{3}\right)^n\times\left(\frac{1}{2}\right)^{8-n}\times(9-n)$$

#

第4題.

1000	7 ~ 994	11 ~ 990	77 ~ 924
	142	90	12

$142+90-12 = \underline{220}$ #

第5題.

lenth 1: 0、1 → 2

lenth 2: 11、10、01 → 3

lenth 3: 111, 110, 011, 010, 101 → 5.

lenth 4: 8

lenth 5: 13

lenth 6: 21

$6 = 5+4$

$= (4+3)+(3+2)$

$= (3+2)$

relation → $a_n = a_{n-1} + a_{n-2}$ #

initial → $a_1 = 2,\ a_2 = 3$ #

$\underline{a_6 = 21}$ #

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第6題.

$a_2 = 1 \quad a_3 = 3 \quad \cancel{a_n =}$

$(1 \cdots n-1)\ n$

$a_{n-1} + n - 1 = a_n$

$a_n = \frac{(n-1)\times n}{2}$

$a_n = a_{n-1} \quad r = 1$

$a_n = p_1 n + p_0$

$p_1 n + p_0 = p_1 (n-1) + p_0 + n - 1$

$p_1 = n-1$

$\cancel{a_n = (n-1)\times n}$

$a_n = \alpha 1^n + \beta \frac{(n-1)\times n}{\cancel{\ }} + p_0$

$a_2 = \alpha + \beta \times 2 + p_0 = 1$

$a_3 = \alpha + \beta \times 6 + p_0 = 3$

$\beta = \frac{1}{2}$

$a_2 = \alpha + p_0 + \beta \times 2 = 1$

$= 0 + 1 = 1$

$\Rightarrow \alpha + p_0 = 0$

$a_n = \frac{1}{2}(n-1)\times n \qquad n \geq 2$

#

第7題.

1 2 3 4 5

A

B

C

D . . . .

$4^5 - C^4_1 3^5 + C^4_2 2^5 - C^4_3 1^5$

$= 1024 - 972 + 192 - 4$

$= 240$ #

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第8題.

8. a) a_{n-1} ⌋ 1 — a_{n-1} $\quad a_n = a_{n-1} + a_{n-2} \quad n \geq 2$ #

⌋ 2 a_{n-2}

b) $a_0 = 1 \quad a_1 = 1$ #

1	1	2	3	5	8	13	21	34	55	89	144	233
	1	2	3	4	5	6	7	8	9	10	11	12

c) find $a_{12} = 233$ #

第9題.

(1, 2,) 下 正 $\quad a_0 = 1$

$a_1 = 2 \quad$ 3.

(1, 2, 3) $\quad a_2 = 3$

(1 2 3 4) 正 正 $\quad a_3 = 5$

$a_4 = 8.$

$r^2 - r - 1 = 0 \quad a_n = a_{n-1} + a_{n-2}$

$r^2 = r + 1, \quad r = \frac{1 \pm \sqrt{5}}{2}$

$$a_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$a_0 = 1 = \alpha_1 + \alpha_2 \qquad \alpha_1 = \frac{5+3\sqrt{5}}{10}$

$a_1 = 2 = \frac{1+\sqrt{5}}{2}\alpha_1 + \frac{1-\sqrt{5}}{2}\alpha_2 \qquad \alpha_2 = \frac{5-3\sqrt{5}}{10}$

$$a_n = \frac{5+3\sqrt{5}}{10} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-3\sqrt{5}}{10} \cdot \left(\frac{1-\sqrt{5}}{2}\right)^n$$ #

答案僅供參考

第10題.

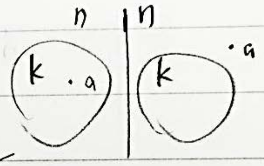
$$(1+x)^n$$

$$= \sum_{k=0}^{n} C^n_k \, 1^{n-k} \, x^k$$

$$= \sum_{k=0}^{n} C^n_k x^k$$

取 不取

$$C^n_k = C^{n-1}_{k-1} + C^{n-1}_k$$



由題目得知

$a(n,r) = C^n_r$, $a(n-1,r-1) = C^{n-1}_{r-1}$, $a(n-1,r) = C^{n-1}_r$

又 $C^n_k = C^{n-1}_{k-1} + C^{n-1}_k$

$\Rightarrow a(n,r) = a(n-1,r-1) + a(n-1,r)$