

Student name: _____ Student number: _____

There are 6 questions and 120 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (20 points) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

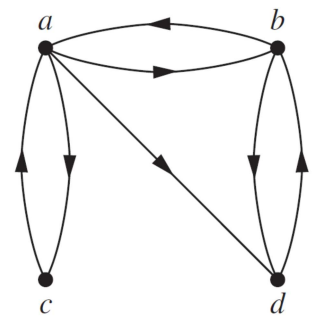
- (a) $x + y = 0$.
- (b) $x - y$ is a rational number.
- (c) $xy \geq 0$.
- (d) $x = 1$ or $y = 1$.

2. (25 points) Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

- (a) $R_1 \cup R_2$.
- (b) $R_1 \cap R_2$.
- (c) $R_2 \circ R_1$.
- (d) $R_1 \circ R_1$.
- (e) $R_1 \oplus R_2$.



3. (20 points) Find the directed graph of the smallest relation that is both **reflexive and symmetric** with directed graph above.

4. (20 points) What is the congruence class $[n]_7$ (that is, the equivalence class of n with respect to congruence modulo 7) when n is

- (a) 4?
- (b) -4?
- (c) 8?
- (d) 13?

5. (20 points) Draw the Hasse diagram for divisibility on the set $\{1, 2, 3, 4, 5, 6, 7, 8, 11, 13\}$.

6. (15 points) Determine whether the relation represented by the zero-one matrix is partial order.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$