

第2題.

Proof (of Theorem 1): We will prove this result using strong induction. Let T(n) be the statement that every simple polygon with n sides can be triangulated into n-2 triangles.

BASIS STEP: T(3) is true because a simple polygon with three sides is a triangle. We do not need to add any diagonals to triangulate a triangle; it is already triangulated into one triangle, itself. Consequently, every simple polygon with n = 3 has can be triangulated into n - 2 = 3 - 2 = 1 triangle.

INDUCTIVE STEP: For the inductive hypothesis, we assume that T(j) is true for all integers j with $3 \le j \le k$. That is, we assume that we can triangulate a simple polygon with j sides into j-2 triangles whenever $3 \le j \le k$. To complete the inductive step, we must show that when we assume the inductive hypothesis, P(k+1) is true, that is, that every simple polygon with k+1 sides can be triangulated into (k+1)-2=k-1 triangles.

So, suppose that we have a simple polygon P with k+1 sides. Because $k+1 \ge 4$, Lemma 1 tells us that P has an interior diagonal ab. Now, ab splits P into two simple polygons Q, with s sides, and R, with t sides. The sides of Q and R are the sides of P, together with the side ab, which is a side of both Q and R. Note that $3 \le s \le k$ and $3 \le t \le k$ because both Q and R have at least one fewer side than P does (after all, each of these is formed from P by deleting at least two sides and replacing these sides by the diagonal ab). Furthermore, the number of sides of P is two less than the sum of the numbers of sides of Q and the number of sides of R, because each side of P is a side of either Q or of R, but not both, and the diagonal ab is a side of both Q and R, but not P. That is, k+1=s+t-2.

We now use the inductive hypothesis. Because both $3 \le s \le k$ and $3 \le t \le k$, by the inductive hypothesis we can triangulate Q and R into s-2 and t-2 triangles, respectively. Next, note that these triangulations together produce a triangulation of P. (Each diagonal added to triangulate one of these smaller polygons is also a diagonal of P.) Consequently, we can triangulate P into a total of (s-2)+(t-2)=s+t-4=(k+1)-2 triangles. This completes the proof by strong induction. That is, we have shown that every simple polygon with P0 sides, where P1 sides, can be triangulated into P2 triangles.

答案僅供參考

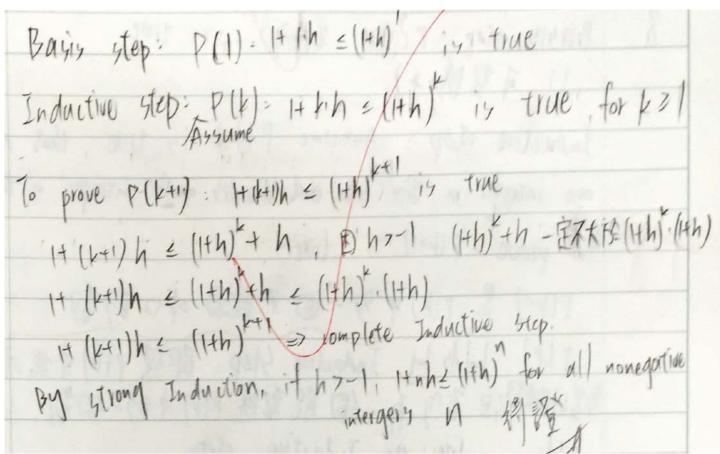
證 明:利用強歸納法。令命題 T(n)為每個有 n 個編的簡單多邊形,都能被切分成 n-2 個三角形基礎步驟: T(3)為真,因為有 3 個邊的多邊形就是三角形。不需要添加任何對角線來切分,就有 3-2=1 個三角形。

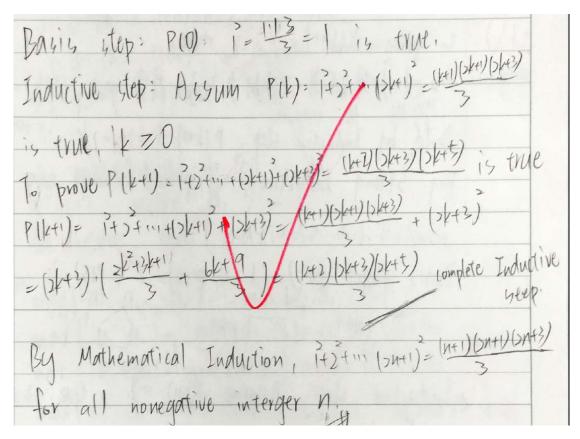
歸納步驟 : 歸納假說為假設對所有的正整數 j , $3 \le j \le k$, T(j)皆為真。我們將往證 T(k+1)為真,亦即有 k+1 個邊的簡單多邊形都能切分成 (k+1)-2=k-1 個三角形。

若有個多邊形 P,其有 k+1 個邊,根據 $Lemma\ 1$,一定有內部對角線 ab。連接此對角線可得到兩個多邊形 Q 與 R,令其分別有 s 個邊和 t 個邊(其中有一個共用的邊 ab,而其他邊都是多邊形 P 的邊),所以 (s-1)+(t-1)=k+1,即 s+t=k+3。由於 s, $t \geq 3$,我們有 $3 \leq s$, $t \leq k$ 。根據歸納假說,多邊形 Q 能切分成 s-2 個三角形,而多邊形 R 能切分成 t-2 個三角形,所以我們證明出多邊形 P 能切分成(s-2)+(t-2)=(s+t)-4=(k+3)-4=k-1 個三角形,此等式證明了 T(k+1)為真。

完成兩個步驟,根據強歸納法,得證結果為真。

第3題.





第5題.

| a) max (-a, -a,an) = - min (a, a) - an) |
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| $max(-a_1.a_2) = -min(a_1.a_2) = a_1 \text{ if } a_1 = a_2 $ $max(-a_1-a_2) = -min(a_1.a_2) = a_2 \text{ if } a_1 = a_2 $ $max(-a_1-a_2) = -min(a_1.a_2) = a_2 \text{ if } a_1 = a_2 $ |
| max (-a, -a, -a, -an) = -min (a, d, -a, -a,), @ max 2 to 3 g |
| 一品点使到mox 找出的是原生加中最小的协会活。另外 |
| g min (a., as, an) |
| max (a,+b, B+b2, "a,+bn) & max (a, a, a, -a,)+ max (b, a, "bn) |
| 线项式出了新的 dung, bu 项相切, 而前项式出的是最大 |
| 的 anthora 新旗旗 大克勒大的 a.b 和力D. 若是都长文及相加 |
| サス起流 max (aa.) + max (bi.bz. · bn) |

第7題.

第8題.

The base case is n=1. If we are given a set of two elements from $\{1, 2\}$, then indeed one of them divides the other. Assume the inductive hypothesis, and consider a set A of n+2 elements from $\{1, 2, ..., 2n, 2n+1, 2n+2\}$. We must show that at least one of these elements divides another. If as many as n+1 of the elements of A are less than 2n + 1, then the desired conclusion follows immediately from the inductive hypothesis.

Therefore we can assume that both 2n+1 and 2n+2 are in A, together with n smaller elements. If n+1 is one of these smaller elements, then we are done, since $n+1 \mid 2n+2$. So we can assume that $n+1 \notin A$.

Now apply the inductive hypothesis to $B = A - \{2n+1, 2n+2\} \cup \{n+1\}$. Since B is a collection of n+1 numbers from $\{1, 2, ..., 2n\}$, the inductive hypothesis guarantees that one element of B divides another. If n+1 is not one of these two numbers, then we are done. So we can assume that n+1 is one of these two numbers. Certainly n+1 can't be the divisor, since its smallest multiple is too big to be in B, so there is some $k \in B$ that divides n+1. But now k and 2n+2 are numbers in A, with k dividing n+2, and we are done. An alternative proof of this theorem is given in Example 11 of Section 6.2.