
Student name: _____ Student number: _____

There are 10 questions and 100 marks total. Please write an answer and the detailed calculation to each of the following questions.

1. (10 points) What are the quotient and remainder when
(a) 789 is divided by 23? (b) -202 is divided by 87?
2. (10 points) Decide whether each of these integers is congruent to 5 modulo 17
(a) 101 (b) -112
3. (10 points) What sequence of pseudorandom numbers is generated using the pure multiplicative generator $x_{n+1} = 3x_n \bmod 11$ with seed $x_0 = 2$?
4. (10 points) Find the prime factorization of each of these integers.
(a) 627 (b) 9099
5. (10 points) Determine whether the integers in each of these sets are pairwise relatively prime.
(a) {12, 17, 31, 35} (b) {7, 8, 19, 111}
6. (10 points) What are the greatest common divisors of these pairs of integers?
(a) $3^7 \cdot 5^3 \cdot 7^3$, $2^7 \cdot 3^3 \cdot 5^9$ (b) 111, 99
7. (10 points) Find an inverse of 144 modulo 233.
8. (10 points) Find all solutions, if any, to the system of congruences.
$$\begin{aligned}x &\equiv 1 \pmod{2} \\x &\equiv 2 \pmod{3} \\x &\equiv 3 \pmod{5} \\x &\equiv 4 \pmod{11}\end{aligned}$$
9. (10 points) Use the extended Euclidean algorithm to express $\gcd(144, 89)$ as a linear combination of 144 and 89.
10. (10 points) Show that if a , b , c , and d are integers such that $a|c$ and $b|d$, then $ab|cd$.