答案僅供參考

第1題.

(1) $P(n) = 1^2+2^2+\dots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(a) $P(1) = \dfrac{1(1+1)(2\cdot1+1)}{6} = 1^2$

(b) $P(1) = \dfrac{2\cdot3}{6} = 1^2 : \dfrac{6}{6} = 1$

$1 = 1$

$\therefore$ P(1) is TRUE

(c) $P(k): 1^2+2^2+\dots k^2 = \dfrac{k(k+1)(2k+1)}{6}$

(d) $P(k+1): 1^2+2^2+\dots k^2+(k+1)^2 = \dfrac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$

$= \dfrac{(k+1)[(k+2)(2k+3)]}{6}$

(e) $1^2+2^2+\dots k^2+(k+1)^2 = k(k+1)(2k+1)/6 + (k+1)^2$

$= \dfrac{k(k+1)(2k+1)}{6} + 6(k+1)^2/6$

$= k+1[k(2k+1)+6(k+1)]/6$

$= k+1[2k^2+k+6k+6]/6$

$= k+1[2k^2+7k+6]/6$

$= k+1[(2k+3)(k+2)]/6$

第2題.

(2) $1^2+3^2+5^2+\dots+(2n+1)^2 = (n+1)(2n+1)(2n+3)/3$

$n=1$: $10 = 2\cdot\cancel{3}\cdot5/\cancel{3}$ is true

$n=k$: $1^2+3^2+5^2+\dots(2k+1)^2 = (k+1)(2k+1)(2k+3)/3$

$n=k+1$: $1^2+3^2+5^2+\dots(2k+1)^2+(2k+3)^2 = (k+2)(2k+3)(2k+5)/3$

$S_{k+1} = S_k + P(k+1) = \dfrac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2 \underline{\ 3\ }$

$\dfrac{(2k+3)((k+1)(2k+1)+3\cdot(2k+3))}{3} \Rightarrow$

$\Rightarrow \dfrac{(2k+3)(2k^2+k+2k+1+6k+9)}{3} = \dfrac{(2k+3)(2k^2+9k+10)}{3}$

$2k^2+9k+10 = (k+2)(2k+5)$

$\Rightarrow S_{k+1} = \dfrac{(2k+3)(k+2)(2k+5)}{3}$ /* it is the same */

答案僅供參考

第3題.

a) $P(8)$: one 3 cents, one 5 cents

$P(9)$: three 3 cents

$P(10)$: two 5 cents

b) the inductive is the statement that using 3 cent and 5 cent stamps we can postage for all j which $8 \le j \le k$, $k \geq 10$

c) we must show, assuming the inductive hypothesis, that we can form $k+1$ cents postage using 3 cents and 5 cents.

d) Since $k \geq 10$, we know that $P(k-2)$ is true that we can form $k-2$ cents of postage. Put one more 3 cents stamp on the envelope and we have formed $k+1$ cents of postage,

第4題.

a) $f(n+1) = f(n)^2 f(n-1)$ $\quad f(1) = (2) \quad f(0) = (-1)$

$f(2) = f(1+1) = f(1)^2 f(0) = 2^2 \cdot (-1) = -4$

$f(3) = f(2+1) = f(2)^2 f(1) = 16 \cdot 2 = 32$

$f(4) = f(3+1) = f(3)^2 \cdot f(2) = 32^2 \cdot (-4) = 1024 \cdot (-4) = -4096$

b) $f(n+1) = 3f(n)^2 - 4f(n-1)^2$

$f(2) = f(1+1) = 3 \cdot f(1)^2 - 4 \cdot f(0)^2 = 3 \cdot 2^2 - 4 \cdot (-1)^2 = 12 - 4 = 8.$

$f(3) = f(2+1) = 3 \cdot f(2)^2 - 4 \cdot f(1)^2 = 3 \cdot 8^2 - 4 \cdot 2^2 = 192 - 16 = 176$

$f(4) = f(3+1) = 3 \cdot f(3)^2 - 4 \cdot f(2)^2 = 3 \cdot 176^2 - 4 \cdot 8^2 = 92928 - 256 =$

$= 92672$

c) $f(n+1) = f(n-1)/f_n$

$f(2) = f(1+1) = f(0)/f_1 = -1/2 = \frac{-1}{2}$

$f(3) = f(2+1) = f(1)/f_2 = 2/-\frac{1}{2} = -4$

$f(4) = f(3+1) = f(2)/f(3) = -\frac{1}{2} : -4 = \frac{1}{8}.$

答案僅供參考

第5題.

(a) $f(n+1)=f(n)+2, a_1=3$ or $f(n)=f(n-1)+2, a_1=3$

(b) $f(n+1)=f(n), a_1=5$ or $f(n)=f(n-1), a_1=5$

第6題.

$F(n)=1+2+3+\ldots+n$

$F(1)=1$

$F(2)=3$

$F(3)=6$

$F(n)=F(n-1)+n$

第7題.

7) n th odd positive integer $(2n-1)$

procedure sum of (n: positive integer)

If $n=1$ then sum of odds(n) := 1

else sum of odds(n) := sum of odds(n-1) + (2n-1)