答案僅供參考

第1題.

(a) $a_n = a_{n-1} + a_{n-2} + a_{n-5}$

(b) $a_0 = 1,\ a_1 = 1,\ a_2 = 2,\ a_3 = 3,\ a_4 = 5$

(c) $a_{10} = a_9 + a_8 + a_5$

$= 75 + 44 + 9$

$= 119 + 9$

$= 128$ #

$a_5 = 5 + 3 + 1 = 9$

$a_6 = 9 + 5 + 1 = 15$

$a_7 = 15 + 9 + 2 = 26$

$a_8 = 26 + 15 + 3 = 44$

$a_9 = 44 + 26 + 5$

$= 75$

第2題.

(a) $a_n=a_{n-1}+a_{n-2}+a_{n-3}$

(b) $a_0=1,\ a_1=1,\ a_2=2$

(c) $a_8=a_7+a_6+a_5=81$

第3題.

3. $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$

15

$\Rightarrow r^3 + 3r^2 + 3r + 1 = 0$

$\Rightarrow (r+1)^3 = 0$

$\Rightarrow r = -1$

$a_n = \alpha_1(-1)^n + \alpha_2 n(-1)^n + \alpha_3 n^2(-1)^n$

$\Rightarrow 10 = \alpha_1,\ -18 = -\alpha_1 - \alpha_2 - \alpha_3,\ 30 = \alpha_1 + 2\alpha_2 + 4\alpha_3$

$\Rightarrow 2\alpha_2 + 4\alpha_3 = 20,\ \alpha_2 + \alpha_3 = 8 \Rightarrow \alpha_3 = 2,\ \alpha_2 = 6$

$\Rightarrow a_n = 10\cdot(-1)^n + 6n\cdot(-1)^n + 2n^2\cdot(-1)^n$ #

答案僅供參考

第4題.

$a_n = 2a_{n-1} + 3\cdot 2^n$

15

$a_n^{(h)} = \alpha 2^n$

$a_n^{(p)} = Cn\cdot 2^n$

$= 2\left(C\cdot(n-1)\cdot 2^{n-1}\right) + 3\cdot 2^n$

$= C\cdot(n-1)\cdot 2^n + 3\cdot 2^n$

$= \left(C(n-1)+3\right)\cdot 2^n$

$\Rightarrow C(n-1)+3 = Cn$

$\Rightarrow \cancel{Cn} - C + 3 = \cancel{Cn}$

$\Rightarrow C = 3$

$\Rightarrow a_n = \alpha 2^n + 3n\cdot 2^n$ #

第5題.

解法 1:

$(x^3+x^4+x^5+x^6)^5$ 取 x^{20} 係數

$\Rightarrow (x^{12}+4x^{13}+10x^{14}+20x^{15}+31x^{16}+40x^{17}+44x^{18}+40x^{19}+31x^{20})(x^3+x^4+x^5+x^6)$ 取 x^{20} 係數

$x \quad x+10x^{20}+20x^{20}+31x^{20}+40x^{20}$ $\quad 10+20+31+40=101$ $\quad$ ANS: 101種

解法 2:

$x_1+x_2+x_3+x_4+x_5=20$

$3\le x_1\le 6$, $3\le x_2\le 6$, $3\le x_3\le 6$, $3\le x_4\le 6$, $3\le x_5\le 6$

$(3,3,3,5,6) \Rightarrow \frac{5!}{3!1!1!} = 20$

$(3,3,4,5,5) \Rightarrow \frac{5!}{2!1!2!} = 30$

$(3,3,4,4,6) \Rightarrow \frac{5!}{2!2!1!} = 30$

$(3,4,4,4,5) \Rightarrow \frac{5!}{1!3!1!} = 20$

$(4,4,4,4,4) \Rightarrow 1$

$20+30+30+20+1=101$ 種.

解法 3:

$x_1+x_2+x_3+x_4+x_5=5$

$C^9_4 - (C^5_1 \times 5)$

$= 126-25 = 101$ #

第6題.

~~10~~ 15

$(x^2+x^3+\cdots) = \frac{1}{x^2}(1+x+x^2+\cdots) = \frac{1}{x^2}\left(\frac{1}{1-x}\right)$

$(x^2+x^3+x^4+x^5) = \frac{1}{x^2}(1+x+x^2+x^3) = \frac{1}{x^2}\cdot\frac{x^4-1}{x-1}$

$(x+x^2+x^3+x^4) = \frac{1}{x}(1+x+x^2+x^3) = \frac{1}{x}\left(\frac{x^4-1}{x-1}\right)$

$(1+x+x^2+\cdots) = \frac{1}{1-x}$

$\frac{1}{x^5}\times\frac{1}{(1-x)^4}$

$a_k = C(n+3+5,3)$

$= C(n+8,3)$

$= \frac{1}{x^5}\times\frac{(x^4-1)^2}{(1-x)^4}$

$\frac{1}{(1-x)^4} \Rightarrow A_k = C(4+k-1,k)$

(a) $= \frac{x^8-2x^4+1}{x^5}\times\frac{1}{(1-x)^4}$

$\Rightarrow A_n = C(n+3,3)$

$\Rightarrow a_n = C(n-5+3,3)$

$= C(n-2,3)$

(b) 10

第7題.

1698+999+543

=3220

3220−876−321−290

=1733

1733+198

=1931

2405−1931=474 #

答案僅供參考

第8題.

題目補充

1.數字不可重複使用

2.分別解出(1)、(2)、(3)

8 ← (數字不可重複的狀況下)

15

satisfy only (1)

9876 _ _ _ _ _ _ = 6!

satisfy only (2)

_ _ _ _ 34 _ _ _ _ = $C^8_4 \times 4! + C^4_4 \times 4!$

satisfy only (3)

_ _ _ _ _ _ _ _ 12 = 8!

satisfy (1) and (2)

987634 _ _ _ _ = 4!

satisfy (2) and (3)

_ _ _ _ 34 _ _ 12 = $C^6_4 \times 4! + C^2_2 \times 2!$

satisfy (1) and (3)

9876 _ _ _ _ 12 = 4!

satisfy (1) and (2) and (3)

987634 _ _ 12 = 2!

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