

第1題.

題號	答	答
1.	1. five-digit falling numbers 0~9 number 10 numbers if we choose 5 不重複 there are C_5^{10} we can 按照大小順序排列 $C_5^{10} \times 1$ the falling numbers 一定是由大到小 $\Rightarrow C_5^{10}$ #	同理 choose n 種 there are C_n^{10}
2.	1-digit 10 2-digit $\frac{(1+9) \times 9}{2} = 45$ 3-digit C_3^{10} 4-digit C_4^{10} 5-digit C_5^{10} 6-digit C_6^{10} 7-digit C_7^{10}	8-digit: $C_8^{10} = 45$ 9-digit: $C_9^{10} = 10$
3.	the total number by 二項式定理 $(1+1)^{10} = C_0^{10} + \dots + C_{10}^{10} = 1024$ but there 沒有 0-digit C_0^{10} the total falling numbers <u>1023</u> #	$x^n y^{10-n}$

第2題.

(a) 取2: sum = 3~17 取7: sum = 28~42
取3: sum = 6~24 取8: sum = 36~44
取4: sum = 10~30 取9: sum = 45
取5: sum = 15~35
取6: sum = 21~39

total = 1+9+8+7+6+5+4+3 = 43 種 #

(b) $C_2^9 + C_3^9 + C_4^9 + C_5^9 + C_6^9 + C_7^9$
= 21 + 35 + 35 + 21 + 7 + 1
= 120

第3題.

if n 是奇 $P(n) = 2^{\frac{n+1}{2}}$ 種 n 是偶 $P(n) = 2^{\frac{n}{2}}$

$n=19$ 1 $P(19) = 2^{10} = 1024$ 合

20 2
21 3
22 4
23 5
24 6
25 7

$n=26$ 8 $P(26) = 2^{13} = 1024 \times 8$ 合

$n=27$ $P(27) = 2^{14} = 1024 \times 16 > 10000$ (不合)

8個 #

Diagram 1: n 奇, $\frac{n+1}{2}$ 種

Diagram 2: n 偶, $\frac{n}{2}$ 種

第4題.

n men n women

Diagram 1: n men and n women arranged in a circle, with n men grouped together.

Diagram 2: n women and n men arranged in a circle, with n women grouped together.

$n! \times n! + n! \times n!$

$= 2 \times n! \times n!$ #

第5題.

7 red 3 black

7個 red

8個位子

選3個位子 C_3^8

給 black 這樣黑球不會相鄰

卡

$\frac{8 \times 7 \times 6}{6}$

$C_3^8 = 56$ 種

#

第6題.

6. 小分 100000

same $x_1 + x_2 + x_3 + x_4 + x_5 = 17$ but $0 \leq x_i \leq 9$ for $i=1,2,3,4,5$

$10 \leq x_i$ $i=1,2,3,4,5$ 必須減掉

+15

$$H_{17}^5 - 5 \times H_7^5$$

$$= C_{17}^{21} - 5 \times C_7^{11} = C_4^{21} - 5 \times C_4^{11} \quad \text{重複個}$$

#

第7題.

7. $x_1 + x_2 + x_3 = 13$ where $0 \leq x_i \leq 6$ for $i=1,2,3$

+15

$$H_{13}^3 - 3 \times H_6^3$$

$$= C_{13}^{15} - 3 \times C_6^8 = C_{13}^{15} - 3 \times C_6^8 \quad \#$$

$$= 105 - 84$$

$$= 21 \quad \#$$

第8題.

設 答 最多 案

a) if there are only 19 freshmen

19 sophomores

19 juniors in the class

there are 57 students in the class

假設錯誤 有58個 students

there are at least 20 freshmen, at least 20 sophomores

or at least 20 juniors in the class. #

b) 設 if there are 最多 6 freshmen

38 sophomores

13 juniors

44

57 students

但是有58個 students \Rightarrow 假設錯誤

there are either at least seven freshmen at least 39 sophomores

, or at least 14 juniors in the class.