AM_10700-Quiz 4 Instructor: PaoLien Lai

Dec. 16, 2013 13:10 pm ~ 15:00 pm Closed book, no calculators

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There are 8 questions and 120 marks total.

1. (15 points) A ***falling*** number is an integer whose decimal representation has the property that each digit except the units digit is larger than the one to its right. For example, 96520 is a falling number but 89642 is not. **How many five-digit falling numbers are there? How many n-digit falling numbers are there, for n = 1,2,3,4,5,6,7,8, and 9? What is the total number of falling numbers of all sizes?**

2. (20 points) Counting sums of subset members.
   (a) How many numbers can be expressed as a sum of two or more distinct members of the set {1,2,3,4,5,6,7,8,9}?
   (b) How many integers can be expressed as a sum of two or more different members of the set {0,1,2,4,8,16,32}?

3. (15 points) A palindrome on the alphabet {H,T} is a sequence of H's and T's which reads the same from left to right as from right to left. Thus HTH, HTTH, HTHTH, and HTHHTH are palindromes of lengths 3, 4, 5, and 6, respectively. Let P(n) denote the number of palindromes of length n over {H,T}. How many values of n is $1000 < P(n) < 10000$?

4. (10 points) A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate.

5. (10 points) Seven identical red cards and three identical black cards are laid down in a row on a table. How many distinguishable arrangements are possible if no two black cards are allowed to be adjacent to each other?

6. (15 points) How many positive integers less than 100,000 have the sum of their digits equal to 17?

7. (15 points) How many solutions to the equation $x_1 + x_2 + x_3 = 13$, where $0 \le x_i \le 6$ for i = 1, 2, 3?

8. (20 points) Suppose that every student in a discrete mathematics class of 58 students is a freshman, a sophomore, or a junior.
   (a) Show that there are at least 20 freshmen, at least 20 sophomores, or at least 20 juniors in the class.
   (b) Show that there are either at least seven freshmen, at least 39 sophomores, or at least 14 juniors in the class.