

第 1 題.

Via truth tables:

p	q	$p \vee q$	$\neg p \vee q$	$p \vee \neg q$	$\neg p \vee \neg q$	result
T	T	T	T	T	F	F
T	F	T	F	T	T	F
F	T	T	T	F	T	F
F	F	F	T	T	T	F

Via equivalences:

$$\begin{aligned}
 & (p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q) \\
 \equiv & ((p \wedge \neg p) \vee q) \wedge ((p \wedge \neg p) \vee \neg q) \\
 \equiv & (F \vee q) \wedge (F \vee \neg q) \\
 \equiv & q \wedge \neg q \\
 \equiv & F
 \end{aligned}$$

第 2 題.

$$\begin{aligned}
 f \circ g &= f(g(x)) = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5, \\
 g \circ f &= g(f(x)) = g(x^2 + 1) = x^2 + 1 + 2 = x^2 + 3.
 \end{aligned}$$

第 3 題.

$$(a) f(Z) = 2Z$$

$$(b) f(R) = R$$

第 4 題.

$$f^{-1}(y) = (y-1)^{1/3}$$

第 5 題.

$$(a) \bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{\dots, -2, -1, 0, 1, 2, \dots, n\}$$

$$(b) \bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{\dots, -2, -1, 0, 1\}$$

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$$(a) 00000111000$$

$$(b) 10001001010$$

第7題.

A	B	C	$A \oplus B$	$B \oplus C$	$A \oplus (B \oplus C)$	$(A \oplus B) \oplus C$
T	T	T	F	F	T	T
F	T	T	T	F	F	F
T	F	T	T	T	F	F
F	F	T	F	T	T	T
T	T	F	F	T	F	F
F	T	F	T	T	T	T
T	F	F	T	F	T	T
F	F	F	F	F	F	F

By Truth Table

$$\Rightarrow A \oplus (B \oplus C) = (A \oplus B) \oplus C \quad *$$

第8題.

$$\begin{aligned}
 (a) & \sum_{i=1}^n \sum_{j=1}^3 (-i + 4j) \\
 &= \sum_{i=1}^3 [(-i + 4) + (-i + 8) + (-i + 12)] \\
 &= \sum_{i=1}^3 (-3i + 24) \\
 &= (13 + 24) + (-6 + 24) + (-9 + 24) + (-12 + 24) + (-15 + 24) + (-18 + 24) + (-21 + 24) \\
 &= 84 \quad *
 \end{aligned}$$

$$\begin{aligned}
 (b) & \sum_{i=0}^2 \sum_{j=0}^2 (i^2 j^3) \\
 &= \sum_{i=0}^2 [(i^2 \cdot 0^3) + (i^2 \cdot 1) + (i^2 \cdot 8) + (i^2 \cdot 27)] \\
 &= \sum_{i=0}^2 36i^2 \\
 &= (36 \cdot 0^2) + (36 \cdot 1^2) + (36 \cdot 4) \\
 &= 180 \quad *
 \end{aligned}$$

第9題.

$$\begin{aligned}
 (a) & \text{Yes} \quad * \\
 f &= Z \times Z \rightarrow Z \\
 (b) & \text{No} \quad * \\
 f &= Z \times Z \rightarrow Z^+ \cdot 0
 \end{aligned}$$