

Filamentation of Femtosecond Vector Beams

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We numerically study the filamentation of femtosecond vector beams with spatially varying polarization profiles in air. The vector beams include azimuthal, radial, and spiral cylindrical vector beams (CVBs), as well as low-order full Poincaré beams (FPBs) with star and lemon polarization topologies. Comparing the filamentation of CVBs to that of a circularly polarized Laguerre-Gaussian (LG) beam, we found that CVBs undergo filamentation more easily due to their more effective nonlinear focusing, in contrast to the LG beam. In the case of low-order FPBs, we examined the role of the constituent orthogonally polarized spatial modes in their filamentation process. Our findings revealed that the Gaussian mode within these beams primarily contributes to their filamentation. Additionally, when compared to a linearly polarized Gaussian beam, star and lemon FPBs displayed greater resistance to filamentation. Furthermore, we investigated the evolution of polarization profiles of the beams during filamentation. We observed that the polarization profiles of CVBs remained largely unchanged. In contrast, the polarization profiles of the FPBs underwent significant non-uniform changes due to differences in accumulated nonlinear phase and Gouy phase among the constituent modes.

I. INTRODUCTION

Filamentation of femtosecond pulses is a captivating phenomenon that has emerged as a fascinating area of research in the field of ultrafast optics. This unique phenomenon involves the propagation of intense ultrashort laser pulses through transparent materials, where the pulse undergoes self-focusing and self-channeling, leading to the formation of long, stable, and self-guided filaments of light [1]. These filaments can stretch over significant distances, defying conventional diffraction limits and maintaining their spatial coherence [2]. Since the first experimental observations of self-channeling of high-peak-power femtosecond pulses in air [1, 2], filamentation has been under extensive investigation to explore the underlying physics or potential applications of this intriguing phenomenon [3]. Among several models for the description of the phenomenon, filamentation is often successfully described as a dynamic equilibrium involving several linear and nonlinear effects including Kerr self-focusing, plasma defocusing, diffraction, dispersion, and losses due to multiphoton absorption and absorption by plasma [4–7]. Filamentation has several important features, including intensity clamping (at $\sim 10^{13} \text{ W/cm}^2$) over an almost constant diameter persisting over remarkably long distances beyond several Rayleigh ranges, generation of underdense plasma channels, supercontinuum generation, and conical emission [3, 8].

Owing to these features, filamentation has been utilized in numerous applications such as white-light LIDAR [9], single-cycle pulse generation in near-[10], and mid-IR [11], high-harmonic generation [12],

micro-/nano-structuring in the bulk of transparent solids [13–15], cancer treatment [16], generation of plasma photonic devices [17], THz generation [18–20], and laser-guided lightning [21].

In spite of the extensive investigation of filamentation, simplified scalar models are often utilized for the description of filamentation [3], and the effect of polarization and its dynamics during nonlinear propagation have been mostly unexplored. Present reports on these aspects of filamentation are limited to optical beams with uniform linear, circular, or elliptical polarization [22, 23]. On the other hand, the generation of novel forms of structured light, including vector beams with space-varying polarization profiles, has led to intriguing possibilities for diverse applications and fundamental science [24, 25]. Cylindrical vector beams (CVBs) [26], and full Poincaré beams (FPBs) [27] are two interesting categories of vector beams. The polarization morphology of the CVBs, which possesses a cylindrical symmetry around the beam axis, is in the form of radial and azimuthal distributions or their superpositions. More interestingly, the polarization distribution in full Poincaré beams spans the entire surface of the Poincaré sphere. Both CVBs and FPBs can be expressed as a superposition of the Laguerre-Gaussian (LG) spatial eigenmodes with orthogonal circular polarizations [28, 29]. CVBs and FPBs have been used for various applications including optical manipulation [26, 30], optical microscopy [31], quantum entanglement of complex photon polarization patterns [32], single-shot polarimetry imaging [33], formation of polarization speckle [34] and laser-induced mass transport [35]. Moreover, nonlinear propagation of several continuous wave (CW) CVBs and FPBs, in a saturable Kerr medium (e.g. Rb vapor) has been studied in the past few years to investigate the effect of polarization shaping on optical collapse [28], polarization rotation

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of the FPBs [29], and suppression of the formation of the optical rogue waves with Poincaré beams [36]. Furthermore, electronic photocurrent shaping through polarization shaping of the incident vector beams for THz magnetic field generation [37, 38], and the generation of reconfigurable THz metasurfaces [39] has been recently demonstrated. Most importantly, investigation of the effect and evolution of the polarization distribution during the filamentation of vector beams are of critical importance for several novel applications of the vector beams involving laser-plasma interactions, such as THz generation, harmonic generation, generation of sculptured electronic photocurrent circuits, and also for polarization-sensitive filament-induced material structuring [40–43].

In this study, we perform numerical investigations on the filamentation of femtosecond vector beams, including azimuthal, radial, and spiral CVBs, as well as FPBs with “star” and “lemon” polarization topologies. Our aim is to explore how these polarization patterns affect the nonlinear propagation of the beams. We make comparisons between CVBs and a circularly polarized LG beam, as well as between FPBs and a linearly polarized Gaussian beam. Our findings reveal that, even in an isotropic medium like air, contributions from orthogonal polarization components significantly influence the filamentation of vector beams. Specifically, CVBs are more susceptible to filamentation compared to optical vortex beams, while FPBs exhibit greater resilience to filamentation when compared to equivalent Gaussian beams. Additionally, we investigate the evolution of the polarization morphologies of these beams and demonstrate that the polarization profile of CVBs remains unchanged, while FPBs undergo notable non-uniform polarization changes during their nonlinear propagation.

The structure of this paper is as follows: we begin with an introduction in Section I. In Section II, we provide a detailed account of our model and methodology. Section III is dedicated to presenting our results and discussions, starting with CVBs in Section III A and then addressing FPBs in Section III B. Finally, our conclusions are presented in Section IV.

II. MODEL AND METHOD

We consider a pulse propagating along z -axis, whose electric field is expressed as $\mathbf{E}(r, t) = \text{Re}\{\mathcal{E}(r, t)e^{i(kz - \omega_0 t)}\}$, where $\mathcal{E}(r, t)$ is the slowly varying envelope, k is the wave number, and ω_0 is the central frequency. The field envelope vector can be expanded on an arbitrary orthogonal polarization basis (i.e. horizontal/vertical, or right-/left-circular polarizations). On the basis of circular polarizations, the field envelope vector can be written as

$$\mathcal{E} = \mathcal{E}^- \hat{\epsilon}_L + \mathcal{E}^+ \hat{\epsilon}_R, \quad (1)$$

where $\hat{\epsilon}_L$ and $\hat{\epsilon}_R$ are the unit vectors denoting the left-circular and right-circular polarizations, LCP and RCP, respectively; while \mathcal{E}^+ and \mathcal{E}^- are the complex amplitudes of the RCP and LCP components, respectively.

For the simulation of the nonlinear propagation, we use the vector-extended version of the standard model of filamentation [22], in which the nonlinear propagation equation of the field amplitudes of the two polarization components in an isotropic medium is given by

$$\begin{aligned} \frac{\partial \mathcal{E}^\pm}{\partial z} = & \frac{i}{2k} \Delta_\perp \mathcal{E}^\pm + i \frac{k''}{2} \frac{\partial^2 \mathcal{E}^\pm}{\partial t^2} \\ & - \frac{\sigma}{2} (1 + i\omega_0 \tau_c) \rho \mathcal{E}^\pm - \frac{\beta^{(K)}}{2} |\mathcal{E}|^{2(K-1)} \mathcal{E}^\pm \quad (2) \\ & + i \frac{2}{3} k n_2 (|\mathcal{E}^\pm|^2 + 2|\mathcal{E}^\mp|^2) \mathcal{E}^\pm, \end{aligned}$$

where Δ_\perp stands for transverse Laplacian operator, $|\mathcal{E}|^2 = |\mathcal{E}^+|^2 + |\mathcal{E}^-|^2$, $k'' = \frac{\partial^2 k}{\partial \omega^2}$ is the group velocity dispersion (GVD) coefficient, σ is the cross-section of inverse bremsstrahlung, τ_c is the electron collision time, $\beta^{(K)}$ is the K -photon absorption coefficient, ρ is the electron density and n_2 is the nonlinear Kerr index. The first and second terms on the right-hand side (RHS) stand for diffraction and dispersion, respectively, and the last five terms represent absorption by plasma, plasma defocusing, multiphoton absorption (MPA), and instantaneous Kerr effect, respectively.

For an arbitrary input polarization, the instantaneous Kerr effect is composed of two contributions: a “self”-Kerr term, and a “cross”-Kerr term. The self-Kerr term depends on the intensity of the identical polarization component, while the cross-Kerr term scales with the intensity of the orthogonal polarization component. Additionally, in the case of non-resonant nonlinear electronic response, the cross-Kerr term is twice as effective as the self-Kerr term in isotropic media [22, 23, 44]. Self-focusing due to optical Kerr nonlinearity can balance the diffraction only if the input beam power (P_{in}) exceeds the critical power of self-focusing, $P_{\text{cr}} \simeq \lambda_0^2 / 2\pi n_0 n_2$, where λ_0 is the central wavelength, and n_0 is the refractive index of the propagation medium [3].

Moreover, to account for the effect of plasma, Eq. (2) must be simultaneously solved with the rate equation of the electron density

$$\frac{\partial \rho}{\partial t} = \sigma_k |\mathcal{E}|^{2K} (\rho_{at} - \rho) + \frac{\sigma}{U_i} \rho |\mathcal{E}|^2 - a \rho^2, \quad (3)$$

where σ_k is the cross-section for multiphoton ionization (MPI), ρ_{at} is the density of neutral atoms, σ is the cross-section for inverse bremsstrahlung, U_i is the ionization potential of the medium, and a is the recombination rate. The first term of the RHS stands for multiphoton ionization (MPI), the second term represents avalanche ionization, and the last term denotes plasma recombination. It should be noted that for near-IR laser pulses with a pulse duration of a few

tens of femtoseconds, propagating in air, the last two terms in Eq. (3) can be neglected [22].

In order to investigate the effect of polarization on different features of filamentation, we consider the input beam to have a general form of a Poincaré beam whose field envelope is constructed by a vector superposition of two optical angular momentum (OAM)-carrying spatial transverse eigenmodes with different topological charges and orthogonal circular polarizations [28, 29]

$$\mathcal{E} = \cos(\gamma) \text{LG}_{m,l} \hat{\epsilon}_L + \sin(\gamma) e^{i\beta} \text{LG}_{m',l'} \hat{\epsilon}_R. \quad (4)$$

The complex amplitudes of the LCP and RCP components are in the form of the Laguerre-Gaussian modes, $\text{LG}_{m,l}$ [45] where m and l are the radial and azimuthal indices, respectively. The amplitudes of the polarization components are determined by parameter γ , while their phase difference is denoted by β . The temporal envelope of the pulse is considered to have a Gaussian shape in the form of $\exp[-(t/t_p)^2]$ with a width of t_p . Most generally, the LCP and RCP components may have different azimuthal and radial indices with arbitrary relative amplitudes and phases.

For different values of γ , β , m , m' , l , and l' various polarization morphologies can be generated. We generate CVBs with azimuthal, radial, and spiral polarization profiles along with two low-order FPBs with “star”, and “lemon” polarization topologies for different values of the parameters γ , β , l' and l , all with $m = m' = 0$. In our numerical investigations, we consider an initial pulse width of $t_p = 35$ fs and a beam width of $w_0 = 0.5$ mm (both, half-widths at e^{-1} of the maximum amplitude), and a central wavelength of $\lambda_0 = 800$ nm. The propagation medium is air for which $P_{\text{cr}} = 3.2$ GW, for a linearly polarized Gaussian beam. The values of the physical parameters used in the model are given in Table I [46].

TABLE I. Physical parameters for $\lambda_0 = 800$ nm and a pulse width of 35 fs used in our simulations.

Quantity	Value and unit
n_0	1
n_2	$3.2 \times 10^{-19} \text{ cm}^2/\text{W}$
k''	$0.2 \text{ fs}^2/\text{cm}$
K	8
$\beta^{(K)}$	$4 \times 10^{-95} \text{ cm}^{13}/\text{W}^7$
σ_K	$4 \times 10^{-96} \text{ s}^{-1} \text{ cm}^{16}/\text{W}^8$
σ	$5.5 \times 10^{-20} \text{ cm}^2$
ρ_{at}	$5 \times 10^{18} \text{ cm}^{-3}$
τ_c	350 fs

For numerically solving the nonlinear propagation equation, Eq. (2), we use alternate direction implicit (ADI) scheme based on the Crank-Nicolson method, which is an implicit unconditionally stable finite difference method for solving partial differential equations (PDEs) in the form of the paraxial beam propagation equation [47]. The nonlinear terms in Eq.

(2) were implemented in the numerical simulation using the first-order Adams-Bashforth scheme [47]. The use of the ADI scheme enables numerical simulations in four dimensions (x, y, z, t), without assuming any sort of symmetry.

III. RESULTS AND DISCUSSION

A. Filamentation of cylindrical vector beams

Different CVBs can be generated by setting $m' = m = 0$, and $l' = -l = 1$ in Eq. (4), such that the superposition of the orthogonally circular polarized LG components carries zero net OAM, and cylindrically symmetric polarization morphology is determined by the relative amplitude (i.e. through the parameter γ) and the phase difference (β) of the constituting LCP and RCP LG components. Here, we compare the filamentation of LCP $\text{LG}_{0,1}$ beam, and three CVBs with different parameters and polarization morphologies:

- (a) ($\gamma = 0$): LCP $\text{LG}_{0,1}$ beam
- (b) ($\gamma = \pi/4$, $\beta = \pi$): Azimuthal CVB
- (c) ($\gamma = \pi/4$, $\beta = 0$): Radial CVB
- (d) ($\gamma = \pi/4$, $\beta = \pi/2$): Spiral CVB.

The transverse intensity profiles of the beams and their polarization morphologies at the initial propagation distance ($z = 0$) are shown in Fig. 1. The polarization profiles on arbitrary transverse planes are mapped using the Stokes parameters [29, 48]

$$S_0 = \mathcal{I} = |\mathcal{E}^+|^2 + |\mathcal{E}^-|^2, \quad S_1 = 2 \operatorname{Re}\{\mathcal{E}^+{}^* \mathcal{E}^-\}, \quad (5)$$

$$S_2 = 2 \operatorname{Im}\{\mathcal{E}^+{}^* \mathcal{E}^-\}, \quad S_3 = |\mathcal{E}^+|^2 - |\mathcal{E}^-|^2,$$

and the local polarization ellipticity, χ , and polarization orientation, ψ :

$$\chi = \frac{1}{2} \sin^{-1} \left(\frac{S_3}{S_0} \right), \quad \psi = \frac{1}{2} \tan^{-1} \left(\frac{S_2}{S_1} \right). \quad (6)$$

As mentioned earlier, filamentation can occur only if the input beam power exceeds the critical power, P_{cr} . The critical power depends on the input beam profile and the values given for P_{cr} in the literature are usually associated with either Gaussian or flat-top beams, while it is shown that the optical vortex beams (i.e. OAM-carrying optical beams, such as the LG beams) are more resilient to self-focusing and collapse [49, 50], and the critical power of self-focusing for optical vortex beams depends on their topological charge, l . Particularly, for $l = \pm 1$ it is shown that the critical power is 3.85 times higher than that of a corresponding Gaussian beam, $P_{\text{cr}}^{(1)} = 3.85 P_{\text{cr}}$ [49]. Therefore, in this investigation, the input power of all four beams is set to

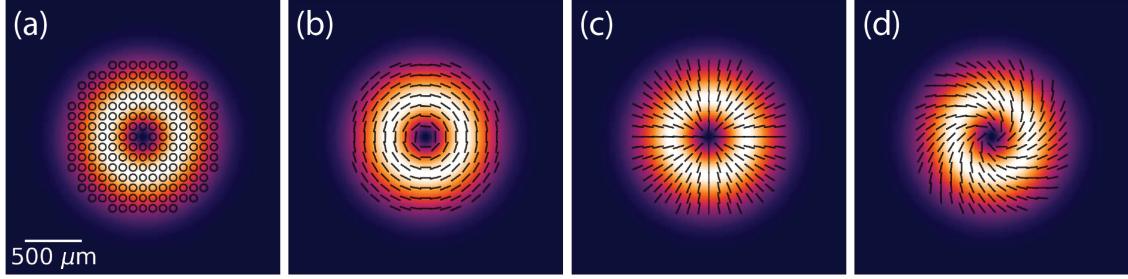


FIG. 1. Transverse intensity and polarization profiles of the input beams at $z = 0$. (a) LCP $\text{LG}_{0,1}$ beam, (b) azimuthal, (c) radial, and (d) spiral CVBs.

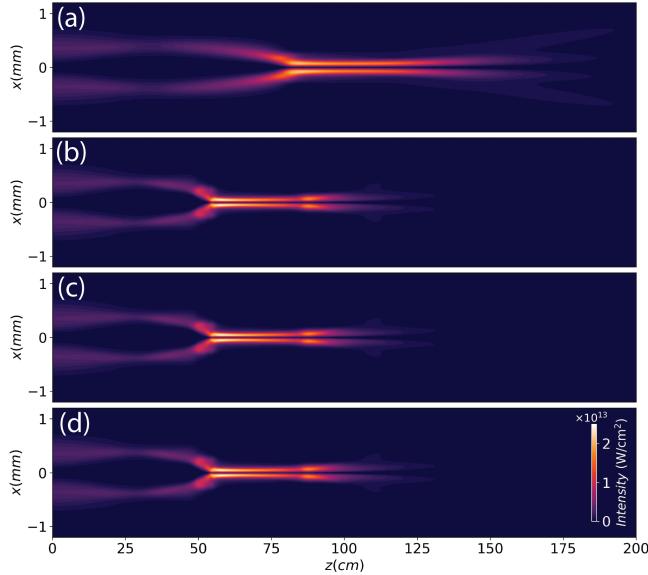


FIG. 2. Time-averaged cross-sectional nonlinear propagation profiles of the LG beam and the CVBs. (a) LCP $\text{LG}_{0,1}$ beam, (b) azimuthal, (c) radial, and (d) spiral CVBs. The corresponding initial transverse intensity and polarization profiles of the beams are shown in Fig. 1(a)-(d), respectively.

39 GW, corresponding to $3.16P_{\text{cr}}^{(1)}$, to ensure that the polarization components in the form of $\text{LG}_{0,\pm 1}$ modes can form filaments.

Equation (2) intuitively implies that during the nonlinear propagation of the pulse, the spatiotemporal profiles as well as the polarization profile of the beams may evolve because of the combined actions of different linear and nonlinear effects. Starting from the evolution of the intensity of the beams during propagation in the filamentation regime, Fig. 2 illustrates the time-averaged cross-sectional propagation profiles, $I(x, z)$, of the LCP $\text{LG}_{0,1}$ beam and the three CVBs; where z is the propagation distance and x is a transverse Cartesian coordinate. The time averaging is performed over the temporal extension of the pulse. The nonlinear propagation profiles presented in Fig. 2(a-d), correspond to the initial profiles illustrated in Fig. 1(a-d),

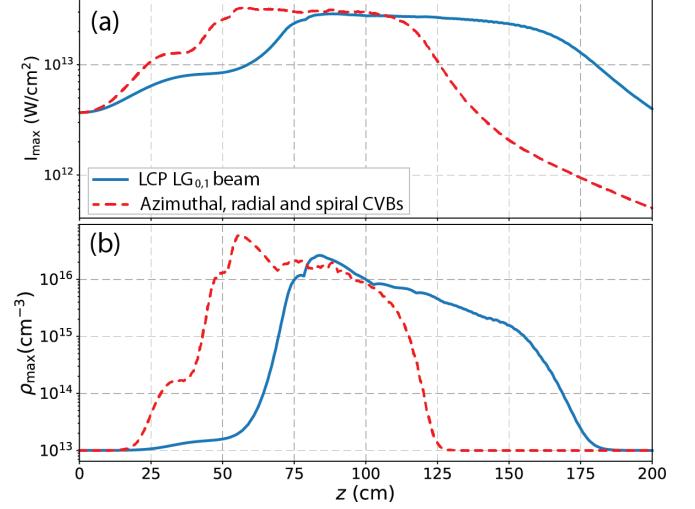


FIG. 3. Evolution of maximum intensity (a), and maximum electron density (b) during the filamentation of LCP $\text{LG}_{0,1}$ beam and the three CVBs. Solid (blue) curve: LCP $\text{LG}_{0,1}$; dashed (red) curve: azimuthal, radial, and spiral CVBs.

respectively. In all cases, the maximum time-averaged intensity slightly exceeds $2 \times 10^{13} \text{ W/cm}^2$. Nevertheless, the onset position of the filamentation, its length, and the transverse dimensions of the filamentary structures are different in some cases. The LCP $\text{LG}_{0,1}$ beam (Fig. 2(a)) begins filamentary propagation from the farthest distance, and it generates largest transverse filamentary structures. Also, in this case, the intensity clamping occurs for a longer propagation distance in comparison to the other cases. Moreover, it is clearly seen that the azimuthal, radial, and spiral CVBs (Fig. 2(b-d), respectively) have an identical propagation profile. The CVBs form smaller filamentary structures, beginning from a closer propagation distance, and with a shorter filamentation length, in comparison to the LCP $\text{LG}_{0,1}$ beam.

In addition to the time-averaged propagation profiles, further quantitative analysis can be performed by investigating the evolution of the maximum intensity and the maximum electron density values achieved throughout the spatiotemporal extensions of the pulse at

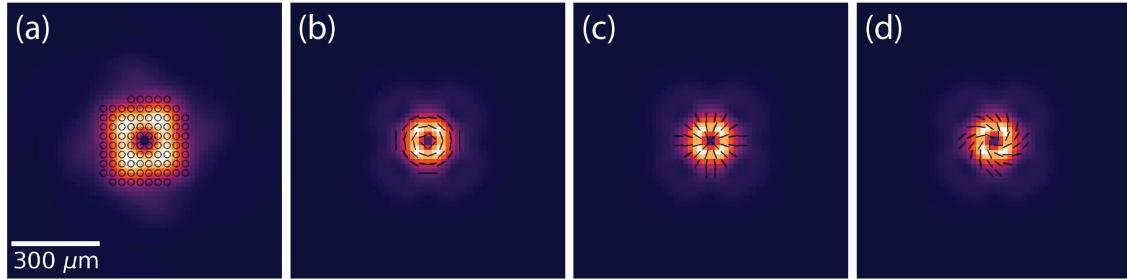


FIG. 4. Transverse intensity and polarization profiles of the LG beam, and the CVBs at the middle of their filamentary propagation range. (a) LCP $\text{LG}_{0,1}$ beam (at $z = 90$ cm); (b-d) azimuthal, radial, and spiral CVBs, respectively (all at $z = 70$ cm).

various propagation distances (Fig. 3(a-b), respectively). The peak values of the maximum intensity (I_{\max}), for the LCP $\text{LG}_{0,1}$ beam and the CVBs are $(2.89 \text{ and } 3.28) \times 10^{13} \text{ W/cm}^2$, respectively. Although these values are comparable to some extent, their differences indicate distinctions in the filamentation of the beams, since the nonlinear terms in the governing equations of the propagation (i.e. Eqs. (2-3)) scale with the powers of the pulse intensity. These graphs also show that the filamentation length and its starting position differ for different cases. By defining the filamentation length as the axial distance over which the intensity exhibits small deviations (i.e. $<13.5\%$) around the respective mean value of the intensity in the intensity clamping region, we find that the filamentary propagation of the LCP $\text{LG}_{0,1}$ beam begins from $z = 74.9$ cm, and the filamentation length is 74.5 cm, while for the CVBs, filamentation starts from and $z = 51.7$ cm, with a length of 58 cm.

Additionally, the maximum electron density (ρ_{\max}) graphs, shown in Fig. 3(b), reveal that the CVBs yield higher peak electron density than LCP $\text{LG}_{0,1}$ beam. The peak maximum electron densities achieved for the CVBs and the $\text{LG}_{0,1}$ beam are $(6, \text{ and } 2.4) \times 10^{16} \text{ cm}^{-3}$, respectively.

It is interesting to note that although all four beams had the same initial power and similar intensity profiles, their filamentary propagation is different. The difference in the filamentation of the beams can be interpreted in terms of the distinct action of the optical Kerr effect on the evolution of the intensity of the beams that is consequently mapped onto the higher-order nonlinear effects, as the following: In the case of the LCP $\text{LG}_{0,1}$ beam, the cross-Kerr effect totally vanishes since the RCP component does not exist and the whole input power is carried by the LCP component. Hence, the beam is nonlinearly focused solely due to the self-Kerr effect, and MPA kicks in as the local intensity builds up to produce plasma. Also, an increase in the plasma density leads to plasma defocusing and absorption of the pulse energy by the plasma through inverse bremsstrahlung, though MPA is more effective as it grows with a higher power of intensity. In this case, the farther onset position of the filamentation, and also larger transverse

dimensions of the beam at the nonlinear focus are both due to the weaker nonlinear focusing. Conversely, for the three CVBs, since both RCP and LCP components have equal amplitudes, both the self-, and cross-Kerr effects contribute to the nonlinear focusing of each beam, and the phase difference between the components, which determines the polarization morphology, has no effect. Therefore, the nonlinear propagation profiles of all three beams are identical. The equal contributions of the polarization components in the nonlinear focusing of the whole beam, in these cases, result in filamentation onset at a shorter propagation distance, and the formation of smaller filamentary structures (in transverse planes) leading to a higher peak intensity at the focus, and consequently higher electron densities originating from MPI, which scales with the K^{th} power of the intensity. Furthermore, higher peak intensities due to stronger nonlinear focusing also lead to more energy loss through MPA for the CVBs, and consequently, the filamentation length of the CVBs is shorter than that of the LCP $\text{LG}_{0,1}$ beam.

Figure 4(a-d) illustrates the time-averaged transverse intensity, and polarization profiles of the beams, at specific propagation distances within their filamentary propagation range. The intensity profiles show that all four beams form ring-like structures each with four intense lobes. Additionally, it is seen that the filamentary structures of the CVBs are smaller than those of the LCP $\text{LG}_{0,1}$ beam. Moreover, a comparison of the polarization profile of each beam with its respective initial polarization profile, shown in Fig. 1(a-d), reveals that the polarization distributions of the beams are preserved upon filamentation because of the absence of RCP polarization component in the case of the LCP $\text{LG}_{0,1}$ beam, and the balance between the two polarization components in the case of the CVBs.

B. Filamentation of full Poincaré beams

We also examine the filamentation of the lowest-order full Poincaré beams (FPBs), with polarization topologies known as “star” and “lemon” [28]. The star polarization

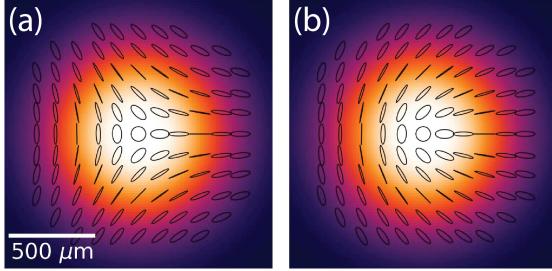


FIG. 5. Transverse intensity, and polarization profiles of the star (a), and lemon (b) full Poincaré beams at the initial propagation distance ($z = 0$).

topology can be generated by setting $\gamma = \pi/4$, $\beta = 0$, $m' = m = 0$, $l = 0$, and $l' = +1$, while the lemon polarization topology is generated by setting $l' = -1$, with otherwise the same parameters. Figures 5(a, b) displays the transverse intensity profiles of the star and lemon FPBs, respectively, along with their corresponding polarization topologies at $z = 0$. The intensity profiles of both beams closely resemble a Gaussian distribution and therefore, we compare the filamentary propagation of both beams with that of a linearly polarized (LP) Gaussian beam with equal input power (i.e. $17 \text{ GW} \simeq 5.3P_{\text{cr}}$), and the same width (i.e. $w_0 = 0.5 \text{ mm}$).

The time-averaged intensity profiles, $I(x, z)$, of the nonlinear propagation of the LP Gaussian beam, and star, and lemon FPBs are presented in Fig. 6(a-c), respectively. Obviously, the intensity evolution during the propagation is identical for the star and lemon beams, since the ratio of the magnitudes of the constituting LCP and RCP components (i.e. $|\mathcal{E}^+|/|\mathcal{E}^-|$) is equal for the two beams. Moreover, the time-averaged intensity for all three beams is similarly clamped at $\sim 2 \times 10^{13} \text{ W/cm}^2$. It is observed that the filamentation of the LP Gaussian beam begins at a shorter distance from the initial plane with a filamentation length about twice longer than that of the star and lemon Poincaré beams. Additionally, the mean diameters of the filaments formed by the LP Gaussian beam, and the Poincaré beams are about $120 \mu\text{m}$ and $175 \mu\text{m}$, respectively.

The evolution of the maximum intensity and the maximum electron density during the nonlinear propagation of the three beams are shown in Fig. 7(a-b), respectively. For all three beams, the maximum intensity is clamped at $\sim 3 \times 10^{13} \text{ W/cm}^2$. For the LP Gaussian beam, filamentation begins at $z = 20.85 \text{ cm}$ with a length of 114.59 cm . Conversely, for the Poincaré beams filamentation onsets at $z = 69.41 \text{ cm}$ and the filamentation length is 56.63 cm , about 2 times shorter than that of the LP Gaussian beam. Moreover, the maximum electron density during the filamentation of the Gaussian beam reaches $3.3 \times 10^{16} \text{ cm}^{-3}$, and a slightly lower value of $2.85 \times 10^{16} \text{ cm}^{-3}$ for both FPBs.

The differences in the filamentation of the LP Gaussian beam and FPBs can be understood based on a similar

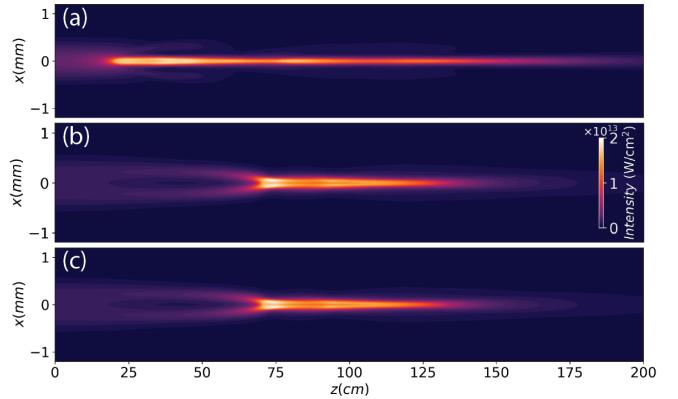


FIG. 6. Cross-sections of the intensity evolution of the beams during filamentation. (a) LP Gaussian beam; (b) and (c) Poincaré beams with the star and lemon polarization topologies.

description presented for the filamentation of CVBs in Section III A, with certain differences due to distinct mode structures of the LCP and RCP components of the FPBs, and their interactions during the nonlinear propagation. The LP Gaussian beam can be considered to be composed of two Gaussian beams with orthogonal circular polarizations, but otherwise identical properties. Therefore, the nonlinear propagation of each circular polarization component of the LP Gaussian beam is similarly affected by the orthogonal polarization component through the cross-Kerr effect. On the other hand, for the star and lemon Poincaré beams, the total input power of the beam is equally divided between their LCP Gaussian and RCP LG_{0,±1} components. The power carried by the LCP Gaussian component corresponds

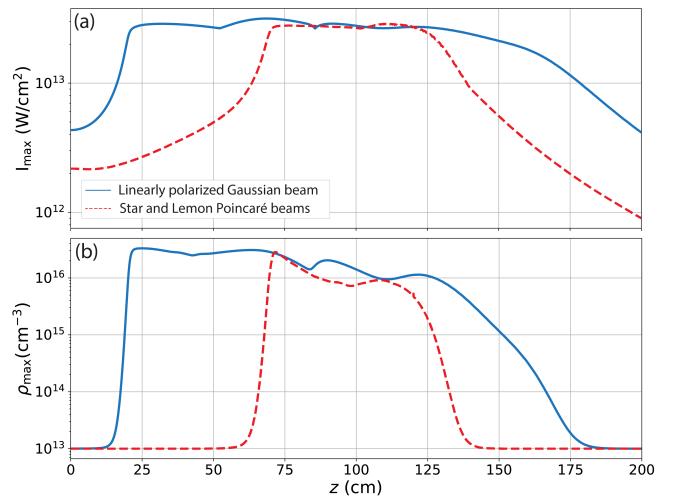


FIG. 7. Evolution of the maximum intensity (a), and maximum electron density (b) during the filamentation of the LP Gaussian beam, and Poincaré beams with star and lemon polarization topologies.

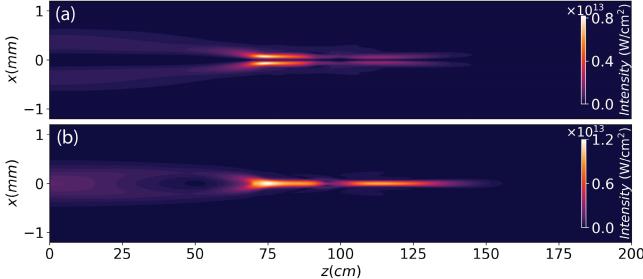


FIG. 8. Cross-sections of the intensity evolution of the orthogonal polarization modes of the FPBs during filamentation. (a): RCP $\text{LG}_{0,\pm 1}$ mode, and (b): LCP Gaussian mode.

to $2.66P_{\text{cr}}$, while the power contained in the RCP $\text{LG}_{0,\pm 1}$ component corresponds to $0.7P_{\text{cr}}^{(1)}$. Moreover, at $z = 0$, the peak intensity of the LCP Gaussian mode is 2.72 times higher than that of the RCP $\text{LG}_{0,\pm 1}$ modes. Therefore, the LG modes undergo nonlinear focusing solely due to the cross-Kerr effect, while the RCP Gaussian component undergoes nonlinear focusing both due to the self-, and cross-Kerr effects although the self-Kerr focusing is slightly stronger. The resilience of the LG modes of the FPBs to self-focusing results in a weaker nonlinear focusing of the whole FPB and consequently the filamentation of the FPBs begins from a longer propagation distance and with larger transverse dimensions of the filamentary structures. It is also important to note that in the case of the FPBs, different mode shapes of the two polarization components lead to mode reshaping during the nonlinear propagation under the action of the Kerr effect. The time-averaged intensity cross-section profiles of the RCP LG mode and LCP Gaussian mode of the FPBs during their nonlinear propagation are presented in Fig. 8(a, b), respectively. The cross-Kerr effect of the LG mode acting on the LCP Gaussian mode reshapes the beam to a hollow beam at the initial stages of the propagation, before the onset of filamentation (Fig. 8(b)). On the other hand, the cross-Kerr effect of the LCP Gaussian mode acting on the LG mode results in a uniform nonlinear focusing of the LG mode that leads to an increase in the nonlinear refractive index over the doughnut-shaped profile of the beam, which in turn acts as a waveguide for the Gaussian mode that reshapes the Gaussian mode back to a bell-shape distribution at the nonlinear focus and beyond. Moreover, Fig. 8 also shows that the peak intensity of the RCP LG mode reaches a maximum value of $\sim 8 \times 10^{12} \text{ W/cm}^2$ while the peak intensity of the LCP Gaussian mode exceeds 10^{13} W/cm^2 . Therefore, it is inferred that the MPA is about 25 times more effective for the LCP Gaussian mode than the RCP LG mode. Hence, the main contribution to the filamentation of the FPBs is from their Gaussian mode. Furthermore, the power carried by the Gaussian mode of FPBs is twice lower than that of the LP Gaussian beam and thus the

filamentation length of the FPBs is shorter than that of the LP Gaussian beam by a comparable factor. This is in agreement with the estimated relation between the filamentation length and the pulse energy [51].

We also investigate the evolution of the polarization profiles of the star and lemon FPBs during their filamentation. The time-averaged transverse intensity and polarization profiles of the beams at several propagation distances are shown in Fig. 9(a) and Fig. 9(c), respectively. Moreover, transverse profiles of the local orientation of polarization, ψ , of the beams at the same propagation distances are presented in Fig. 9(b), and Fig. 9(d), respectively.

A comparison of the polarization orientation profiles, especially the trace of the jump between $-\pi/2$ and $\pi/2$, at different propagation distances with the ones at $z = 0$, reveals that the polarization profile does not rotate uniformly on the transverse planes, in contrast to the linear propagation where a uniform π rotation of the polarization profile is expected to occur when the beam propagates over many Rayleigh ranges from one side of the waist plane to another, due to the Gouy phase difference between the constituent Gaussian and LG components [29]. Here, the polarization of the most intense central part of the beam, undergoes a $\sim 3\pi/4$ rotation in opposite directions for the star and lemon beams (counter-clockwise and clockwise, respectively), from $z = 0$ to $z = 150$ cm. Interestingly, no drastic change in the polarization orientation profiles is seen after $z = 100$ cm, which is the middle of the filamentary propagation range. It should be mentioned that in the case of the nonlinear propagation of CW star and lemon Poincaré beams in a saturable Kerr medium the average polarization rotation, over the transverse extension of the beam, is shown to monotonically increase by increasing the propagation length inside the medium, at least up to three Rayleigh ranges from the initial plane, for different input powers [29]. In both cases of the filamentation of femtosecond Poincaré beams, and in the nonlinear propagation of the CW Poincaré beams in a saturable Kerr medium, the polarization rotation is due to the difference between the accumulated nonlinear phases of the constituting spatial RCP and LCP eigenmodes during the nonlinear propagation, combined with the phase change due to diffraction (i.e. the Gouy phase). In the latter case, the nonlinear phase difference arises from different actions of the self-, and cross-Kerr effects for the two eigenmodes (due to their different intensities), which leads to a change in their spatial overlap, and hence to a difference in the cross-phase modulation, and also to a difference in the diffraction of the components. The collective effect of these phenomena results in a non-uniform rotation of the polarization profile and its distortion during the nonlinear propagation. On the other hand, in the case of filamentation, the wavefront of each eigenmode undergoes a dynamic change due to the additional effects of the plasma defocusing and temporal evolution of the pulse, and losses. In fact, it has been

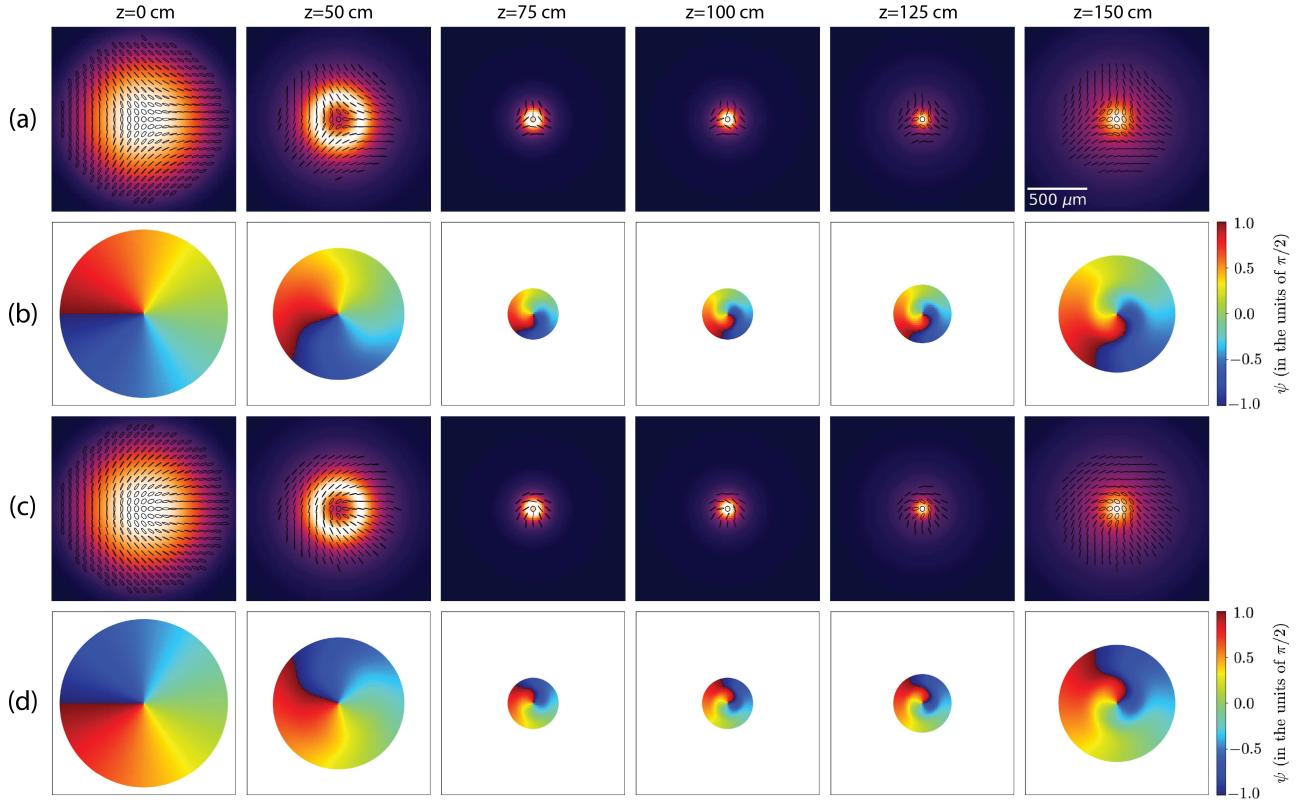


FIG. 9. Evolution of the transverse intensity and polarization topologies of the star (a)-(b), and lemon (c)-(d) Poincaré beams during their filamentation. (a) and (c): Transverse intensity and polarization profiles at several propagation distances. (b) and (d): Transverse profiles of the local orientation of polarization, ψ , at the same propagation distances.

shown that the wavefront of a femtosecond Gaussian beam is clamped during filamentation [52]. Therefore, the change in the amount of polarization rotation or the evolution of the polarization profile between the two cases can be attributed to the difference between the underlying nonlinear propagation phenomena for each case.

IV. CONCLUSION

Through numerical simulations, we investigated the filamentation of fs vector beams. The vector beams are in the form of azimuthal, radial, and spiral CVBs, and also star and lemon FPBs. All vector beams are considered as vector superpositions of orthogonal circularly polarized two spatial modes. In the case of the CVBs, the polarization components are in the form of LG modes with opposite topological charges, while for the FPBs the components are an LCP Gaussian mode and an RCP LG mode with the topological charge of ± 1 for the star and lemon polarization topologies, respectively. Our results revealed that the filamentation of all three CVBs is identical since they have equal amplitudes of the constituent components, and the phase difference between the components that determine the

polarization profiles of the CVBs has no effect on their filamentation. We also compared the filamentation of the CVBs with a circularly polarized LG beam. All beams exhibited well-known characteristics of filamentation in atmospheric pressure air such as intensity clamping beyond 10^{13} W/cm^2 , and electron densities of more than 10^{16} cm^{-3} over the light filaments. Interestingly, we demonstrated that due to the simultaneous action of both the self- and cross-Kerr effects during the nonlinear propagation of the CVBs, they are more prone to filamentation than the circularly polarized LG beam where only the self-Kerr effect is present.

We also investigated the filamentation of star and lemon FPBs, and the mutual interaction of their LCP Gaussian and RCP LG modes during the nonlinear propagation and their contribution to the filamentation of the whole beams. Our results indicate that the LCP Gaussian component of the FPBs has the main contribution to the filamentation of these beams since their LG modes are more resilient to the formation of light filaments. We compared the filamentation of these FPBs with an equivalent linearly polarized Gaussian beam and demonstrated that the LP Gaussian beam more easily forms filament starting from a shorter propagation length, a longer length, and a smaller transverse size in comparison to FPBs.

Furthermore, we studied the evolution of the polarization profile of the vector beams during their filamentation. We demonstrated that the polarization profiles of the CVBs remain substantially unchanged, while the polarization morphologies of the FPBs experience remarkable nonuniform changes because of the difference between the accumulated nonlinear phases, which is nonuniform due to nonlinear mode reshaping, and the difference between the Gouy phases of the two modes. This is in contrast to the evolution of the profile during the linear propagation where a uniform rotation of the polarization pattern is expected to occur solely due to the variation of the Gouy phase difference between the two polarization components.

Our findings may have the potential to contribute to advancements in the field of ultrashort pulse filamentation and its applications. These results

provide new insights into the application of intense ultrashort pulses, especially in situations where the polarization state of the intense light filaments is crucial such as in THz generation, high-harmonic generation, molecular alignment with ultrashort pulses, and laser-induced material structuring, where the spatially varying polarization profiles of the intense light filaments can be utilized to form sculptured transient currents, achieve non-uniform or cylindrically symmetric molecular alignment patterns, induce engineered material modification patterns, etc. Additionally, we propose that the polarization shaping of the initial femtosecond beam could enable precise control over light filament characteristics like peak intensity, plasma density, and diameter, which are of key importance for numerous applications of femtosecond filamentation.

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