# Interpreting the coefficients in Linear Models

This section applies only to Linear models (`LinearRegression`, `LogisticRegression`)

It may seem overly specialized, but since these models are used so often, we will spend some time.

Also, because we can assign a meaning to the coefficients, these models are highly interpretable.

# Numeric features

For Linear Regression

$\hat{\mathbf{y}} = \Theta^T \cdot \mathbf{x} = \sum_{j=1}^{n} \Theta_j * \mathbf{x}_j$

so for a *unit* change in $\mathbf{x}_j$:

$\Delta\hat{\mathbf{y}} = \Theta_j$

Thus $\Theta_j = \frac{\partial \hat{\mathbf{y}}}{\partial \mathbf{x}_j}$

That is, the coefficient $\Theta_j$ is the amount $\hat{\mathbf{y}}$ changes for a $1$ unit change in $\mathbf{x}_j$

Does this mean that, for two features $j, j'$ if

- $\Theta_j > \Theta_{j'}$
- that feature $j$ is more important than $j'$ ?

No.

Unless $\mathbf{x}_j$ and $\mathbf{x}_{j'}$ are on same scale (e.g., have been standardized) a change of $1$ unit isn't comparable.

For example

- $\mathbf{x}_j$ in miles, $\mathbf{x}_{j'}$ in inches

What $\Theta_j$ *does* tell you is

- the direction
- and magnitude

of how much the prediction changes when the feature changes

This can be useful for interpretation

- In our housing price premium example: premium *increases* with area since $\Theta_j > 0$

Remember that transformations change units of the variables (features or targets).

So the $\Theta_j$ reflects the sensitivity to the change in the *transformed* units, not the original.

Recall the target for Logistic Regression was transformed to log odds

$$\mathbf{y} = \log(\frac{\hat{p}}{1 - \hat{p}})$$

So a unit change in feature $\mathbf{x}_j$ with parameter $\Theta_j$ changes the *odds* $\frac{\hat{p}}{1-\hat{p}}$ in a *multiplicative* way

$$\log(\frac{\hat{p}}{1 - \hat{p}}) + \Theta_j = \log\left(\frac{\hat{p}}{1 - \hat{p}} * \exp \Theta_j\right)$$

-

## Examples

- Log transform of target:
  - $\log \mathbf{y} = \Theta_0 + \Theta_1 * \mathbf{x}_1$
  - $\theta_1 = \frac{\partial \log \mathbf{y}}{\partial \mathbf{x}_1} = \%$ change in $\mathbf{y}$ per unit change in $\mathbf{x}_1$

- Log transform of both target and feature:

  - $\log \mathbf{y} = \Theta_0 + \Theta_1 * \log \mathbf{x}_1$
  - $\Theta_1 = \frac{\partial \log \mathbf{y}}{\partial \log \mathbf{x}_1} = \%$ change in $y$ per $\%$ change in $\mathbf{x}_1$

- Standardize feature

  - Transform $\mathbf{x}$ into $z_{\mathbf{x}} = \frac{\mathbf{x} - \bar{\mathbf{x}}}{\sigma_{\mathbf{x}}}$
  - $\mathbf{y} = \Theta_0 + \Theta_1 * z_{\mathbf{x}}$
  - $\Theta_1 = \frac{\partial \log \mathbf{y}}{\partial z_{\mathbf{x}}}$ change in $\mathbf{y}$ per $1$ standard deviation change in $\mathbf{x}$
    - since $z$ is in units of "number of standard deviations"

**Remember**

- if you transform features in training, you must apply the same transformation to features in test
    - if the transformation is parameterized, the parameters are determined at **train** fit time, not test !
- if you transform the target, the prediction is in different units than the original
    - you can perform the inverse transformation to get a prediction in original units

# Categorical features

Consider when $x_j$ is a binary categorical feature: $x_j \in \{0, 1\}$

Then $\Theta_j$ is the increase in $\mathbf{y}$ associated with $\mathbf{x}_j$ being equal to $1$ rather than $0$.

Just like with numeric features.

Recall our discussion about the dummy variable trap for Linear Regression:

- if we have an intercept (with parameter $\Theta_0$)
    - then $\Theta_0$ is the *contribution* (not increment) to $\mathbf{y}$ when $\mathbf{x}_j = 0$
    - as you can see from equation $\mathbf{y} = \Theta_0 + \ldots + \Theta_j * \mathbf{x}_j$, when $\mathbf{x}_j = 0$

## What's wrong with representing multinomial categorical values as numbers ?

We have already given several reasons why representing Categorical features as numeric is a bad idea.

Now that we can interpret $\Theta_j$, we add another reason to the list.

Let's consider the Passenger Class ( `PClass` ) variable from the Titanic example:

$$\text{PClass} \in \{1, 2, 3\}$$

- The difference in prediction for
  - $(PClass = 1)$ vs $(PClass = 2)$,
  - or $(PClass = 2)$ vs $(PClass = 3)$
  - is $\Theta_{\text{PClass}}$
- BUT the difference in prediction for $PClass = 1$ vs $(PClass = 3)$ is $2 \times \Theta_{\text{Pclass}}$
  - Is $\mathbf{y}$ (log odds of Survival) impacted by a factor of 2 ?
  - That's what you are saying by representing `PClass` as the integer range $[1, 3]$

- What if $\text{Pclass} \in \{100, 200, 300\}$ ?
    - Does changing one class impact log odds of Survival by $100 * \Theta_{\text{PClass}}$ ?

Categorical variables have neither order nor magnitude; integers have both.

Beware of representing Categorical variables with numbers.

# Bucketing/Binning re-visited

Suppose $\mathbf{x}_j$ is a continous numeric feature (e.g., `Age` ).

Then a 1 year change in `Age` means that $\mathbf{y}$ changes by $\Theta_j$.

Do you think that 1 year of increase is as important for an adult as for an infant ?

- That's what the regression equation is telling you
- If not, consider
    - Bucketing/Binning continous variables
    - Percent changes

# Interpreting the MNIST classifier: template matching

The $\Theta$ produced by a linear classifier can be viewed as templates

- the strength of $\Theta_j$ tells you how strongly feature $\mathbf{x}_j$ influences the target

So we can interpret $\Theta$ as a "template" for what a model is looking for.

Let's look at the template for

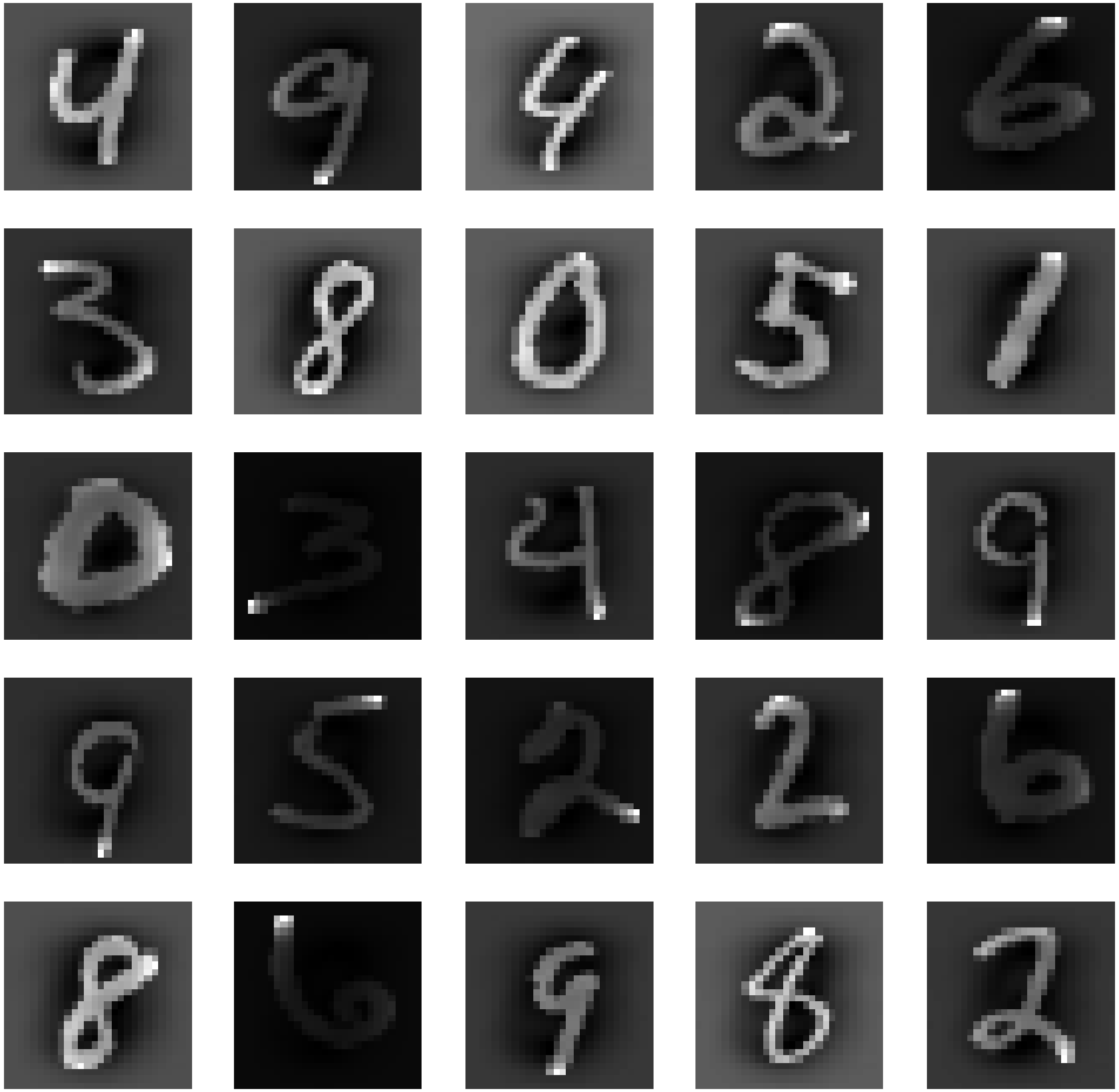
- The 10 separate, single-digit binary MNIST classifiers
- Or similarly: each *row* of $\Theta$ for the multinomial 10 class MNIST classifier

Here's the training data

In [5]:

```
mnh.setup()
mnh.visualize()
```

```
Retrieving MNIST_784 from cache
```



Let's fit a `LogisticRegression` model and examine the templates (coefficients $\Theta$)

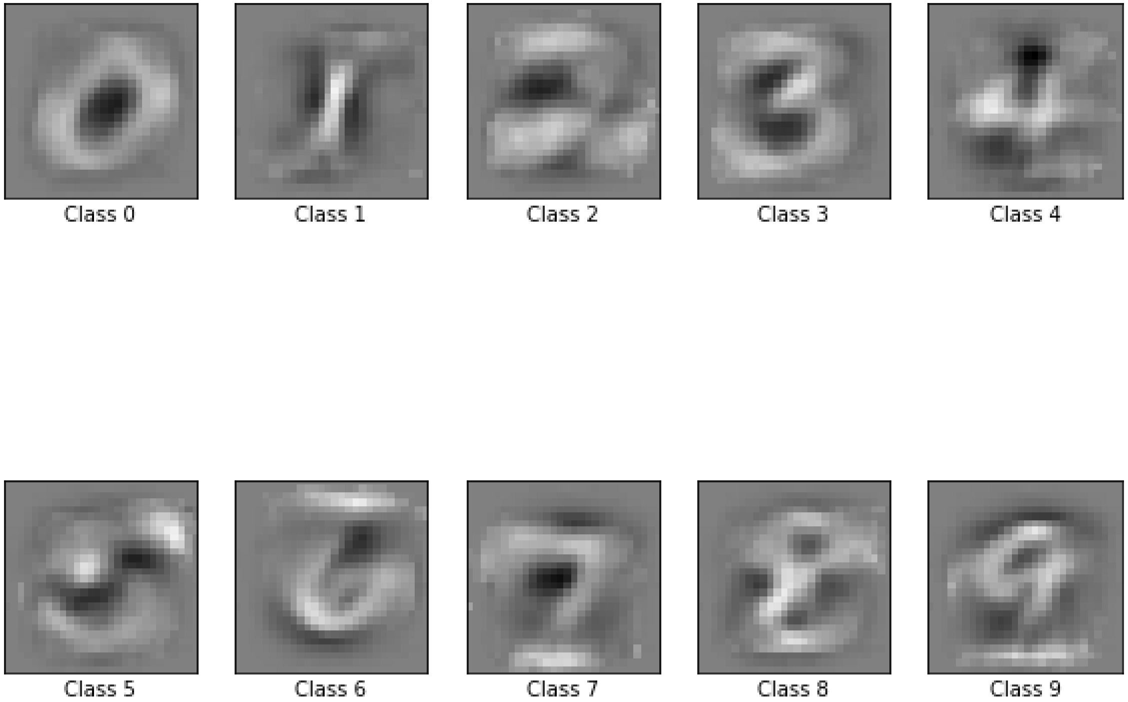
In [6]:
```
mnh.fit()
mnist_fig, mnist_ax = mnh.plot_coeff()
```

```
Example run in 3.128 s
```

Out[6]:
```
LogisticRegression(C=0.01, class_weight=None, dual=False, fit_intercept=True,
          intercept_scaling=1, max_iter=100, multi_class='multinomial',
          n_jobs=None, penalty='l2', random_state=None, solver='saga',
          tol=0.1, verbose=0, warm_start=False)
```

```
/home/kjp/anaconda3/lib/python3.7/site-packages/matplotlib/figure.py:445: User
Warning: Matplotlib is currently using module://ipykernel.pylab.backend_inlin
e, which is a non-GUI backend, so cannot show the figure.
  % get_backend())
```

Parameters for...



Recall

- There is one parameter per pixel
- The parameters are ordered in the same way as the linearization of the pixels
    - from $(28 \times 28)$ grid to a vector of 784 numbers.
- We can display the 784 parameters in a $(28 \times 28)$ image to show the intensity of parameter associated with a pixel
- White is high parameter value; Black is low (or negative)

- The template for $0$ emphasizes small values (absence of bright pixels) in the center of the image
- The template for $1$ emphasizes bright vertical pixels
- The template for $8$ emphasizes the absence of bright pixels
    - in the two circles
    - in the pinched waist

You can now imagine how these templates might lead to misclassification

What is the classification of

- a "7" with a strong vertical line in the center (that's what the "1" template tries to match)
- a thin "0" (the "0" template is looking for a large donut)

So interpretation is a very powerful diagnostic tool for both understanding and improving your models.

In [7]:

```
print("Done")
```

```
Done
```