

Project Summary

There are five people in a team performing a mission together. The manager knows through secret intelligence that they include two spies and three good guys, but it is not currently known which two are spies. In order to help the captain find out the real two spies, he planned to issue a virtual mission to these five people.

The following is the mission information released: 'This mission contains 18 small tasks. You need to succeed in at least more than half of the tasks (10) to be considered successful. In odd-numbered rounds of the game, three people are randomly selected to perform the mission, while in even-numbered rounds, only two people are randomly selected to perform the mission. The success of each task requires the unanimous consent of all participants, and tasks are voted on anonymously. In addition, each person may not participate in the task more than two consecutive rounds. '

Spies can choose to vote 'Accept' or 'Reject' the mission, while the good guys can only vote 'Accept' the mission. Based on the information the spies know, they have enough incentive to vote 'Reject' as much as possible in every task to ensure that at least 10 tasks are destroyed and the overall task fails. But they are likely to vote to disguise themselves when they think they are about to be exposed. Only the managers know that this is a virtual mission, and the actual goal is to test out spies.

The manager will record each failed round of task participants as being suspicious once, and the number of suspicions will gradually increase with the number of failed tasks. After the 6th and 12th rounds of the game, the manager will vote out the person with the highest number of suspicions so that he will not be able to participate in the next 6 rounds of games (if the number of suspicions is the same, choose any one, and the players voted out in the sixth round will participate the 12th round of voting), which can better observe the results of the remaining four players. After all 18 rounds of the game, the manager needs to vote out two spies based on the number of suspicions. If the real spy is caught, the real purpose will be successful, otherwise it will fail.

Propositions

j : player name $j \in \{\text{Alice, Bob, Chris, David, Eric}\}$

i : round number - Integer greater than or equal to 1

K_i : True if task round ' i ' succeeds, False if task round ' i ' fails;

M_{ij} : In task ' i ', True if player ' j ' votes to accept, False if player ' j ' votes to reject.

R_{ij} : player 'j' attend task round 'i'

G_j : True if player 'j' is good person, False if not

P_i : round 'i' is playing.

S_j : Represents one suspicion that player 'j' is a spy.

V_{ij} : After round i, player j is voted on.

F: represent the whole mission final success, False if fail.

Constraints

3 players in odd-numbered rounds, 2 players in even-numbered rounds

$[i \pmod{2} + 2]$: $(R_{ij(Alice)} \wedge R_{ij(Bob)})$ or $(R_{ij(Alice)} \wedge R_{ij(Bob)} \wedge R_{ij(Chris)})$

The success of the task requires the acceptance of all members participating in the task

$M_{ij(Alice)} \wedge M_{ij(Bob)} \wedge M_{ij(Chris)} \rightarrow K_i$ or $M_{ij(Alice)} \wedge M_{ij(Bob)} \rightarrow K_i$

Failure of the task means that there must be a spy among the members participating in the task

$\neg K_i \rightarrow \neg M_{ij(Alice)} \vee \neg M_{ij(Bob)} \vee \neg M_{ij(Chris)}$

Good people can only vote for acceptance.

$G_j \rightarrow M_{ij}$

Spies can vote to accept or reject.

$(\neg G_j \rightarrow M_{ij}) \vee (\neg G_j \rightarrow \neg M_{ij})$

No one can participate in more than two tasks in a row

$\neg(R_{ij} \wedge R_{(i-1)j} \wedge R_{(i-2)j})$

If the task the player is on fails, they will be suspected

$(R_{ij} \rightarrow \neg K_i) \rightarrow S_j$

At the end of rounds 6 and 12, the voted member cannot participate in the next 6 rounds of the game

$V_{6j} \rightarrow \neg(R_{(i+1)j} \wedge R_{(i+2)j} \wedge R_{(i+3)j} \wedge R_{(i+4)j} \wedge R_{(i+5)j} \wedge R_{(i+6)j})$

$V_{12j} \rightarrow \neg(R_{(i+1)j} \wedge R_{(i+2)j} \wedge R_{(i+3)j} \wedge R_{(i+4)j} \wedge R_{(i+5)j} \wedge R_{(i+6)j})$

If the number of suspects is the same, one person will be randomly selected to vote.

$(S_j \vee S_{(j+1)}) \rightarrow V_{ij}$

At the end of round 18, the manager votes for the two final suspected spies, if the true spy is voted for the whole mission will succeed, otherwise it will fail.

$$\neg(G_j \wedge G_{(j+1)}) \rightarrow (V_{18j} \wedge V_{18(j+1)}) \rightarrow F$$

Model Exploration

To clarify the game mechanics, we've instituted a rule: once a player is identified as a spy, they're sidelined for the remainder of the game, effectively being ousted from the team.

When we began crafting our game dynamics, the primary method to differentiate between a good player and a spy was by observing mission failures. The idea was that spies, when paired with varying players for votes, would yield different outcomes, enabling others to pinpoint the spies over time. To enhance the identification process, our strategy was to place the player under suspicion in a list, requiring them to participate in three consecutive rounds. By incorporating this rule into our model and testing it against several scenarios, we noticed an inconsistency. Despite the suspicion, the player could still choose to vote 'accept' or 'reject'. This vote influenced the overall results, confirming the presence of a spy in that round but not conclusively revealing their identity. We decided to use the exclusion method to find out the spies, because with the spies messing around probably everyone would be the object of suspicion. However, spies try to fail missions as much as possible, so among the suspects if that player participates in three consecutive successful missions, he can be recognised as a good person and no longer be suspected.

However, based on the suggestions of the TA and students, we found that the method of identifying spies was not applicable and that the number of mission rounds should be limited. The previous method of identifying spies was to have the suspected member participate in three missions again to make sure that if they succeeded in all three missions, they must be a good person, which is using the elimination method. But as TA and classmates said, taking three missions in a row conflicts with the restriction that 'no member can take more than two missions in a row'. And with an unlimited number of tasks spies have no incentive to vote no in three consecutive tests. So in the final game setup we added a game goal and a limit on the number of missions, which gave the spies an incentive to sabotage more than half of the small tasks to make the whole mission fail. while it is reasonable to infer a spy by the number

of suspicions, it will not always identify the spy, depending on the random selection of participating missions and the spy's plan to sabotage the mission.

We set up two testing opportunities, in rounds 6 and 12, where the manager could vote for a person with the highest number of suspicions to be unable to participate in the next 6 rounds of the mission. If the highest number of suspicions is the same for several people one will be chosen at random, which allows for a better view of the outcome of the game for the remaining 4 people.

Jape Proof Ideas

1. Good people can only vote accept, so those who vote reject must be Spy. (Because jape cannot recognize M, I use E instead.)

$$(G \rightarrow M) \vdash (\neg M \rightarrow \neg G)$$

G → E ⊢ (¬E → ¬G)	
1: G → E	premise
2: ¬E	assumption
3: G	assumption
4: E	→ elim 1,3
5: ⊥	¬ elim 4,2
6: ¬G	¬ intro 3-5
7: (¬E → ¬G)	→ intro 2-6

2. If the task the player is on fails, they will be suspected (Because jape cannot recognize K, I use P instead), so the player will not be suspected when the task succeeds.

$$(R \rightarrow \neg K) \rightarrow S \vdash \neg S \rightarrow \neg(R \rightarrow \neg K)$$

$(R \rightarrow \neg P) \rightarrow S \vdash \neg S \rightarrow \neg(R \rightarrow \neg P)$	
1: $(R \rightarrow \neg P) \rightarrow S$	premise
2: $\neg S$	assumption
3: $R \rightarrow \neg P$	assumption
4: S	\rightarrow elim 1,3
5: \perp	\neg elim 4,2
6: $\neg(R \rightarrow \neg P)$	\neg intro 3-5
7: $\neg S \rightarrow \neg(R \rightarrow \neg P)$	\rightarrow intro 2-6

3 If the number of suspects is the same, one person will be randomly selected to vote.
 (Because jape cannot recognize M, I use P instead)
 $(S \vee S) \rightarrow M \vdash \neg M \rightarrow \neg(S \vee S)$

$(S \vee S) \rightarrow P \vdash \neg P \rightarrow \neg(S \vee S)$	
1: $(S \vee S) \rightarrow P$	premise
2: $\neg P$	assumption
3: $\neg \neg(S \vee S)$	assumption
4: $S \vee S$	Theorem $\neg \neg P \vdash P$ 3
5: P	\rightarrow elim 1,4
6: \perp	\neg elim 5,2
7: $\neg(S \vee S)$	contra (classical) 3-6
8: $\neg P \rightarrow \neg(S \vee S)$	\rightarrow intro 2-7

First-Order Extension

First, Extension our current game can use predicate logic.
 For example, in constraint:

1.The success of the task requires the acceptance of all members participating in the task

$$\forall j. M(j) \rightarrow K$$

Set up a model, domain = {Alice, Bob, Chris}; M = {(Alice), (Bob), (Chris)} K = T. Where M represents the list of voting acceptances. When all participants are included in M, K is True and the task is successful.

2.Failure of the task means that there must be a spy among the members participating in the task

$$\exists j. \neg M(j) \rightarrow \neg K$$

Set a model, domain = {Alice, Bob, Chris}; M = {(Alice), (Bob)} K = T. where M represents the list of voting acceptances. When there is at least one participant who did not vote in favor, ' $\neg K$ ' mission failed.

3.No one can participate in more than two tasks in a row

$$\forall j. \neg (R(j) \wedge R_{i-1}(j) \wedge R_{i-2}(j))$$

Set a model, domain = {Alice, Bob, Chris, David, Eric}; R = {}, which means that all members cannot participate in more than 2 tasks in a row. When R = {} the formula is true.

In addition to using predicate logic to expand, game theory logic can also be added to the game settings. Managers and spies can make optimal choices based on members' mission results. For example, if the cost of exposing themselves is too high for spies, they will choose to vote 'accept' to disguise themselves so as not to be discovered. Suppose that the payoff to the manager for not finding the spy is -5 because of the time and effort he expended. So for him, even if he votes for the three people with the highest number of suspicions, finding the spy and permanently kicking a good guy out of the team may be more profitable than -5. Adding these settings may make the game more realistic, but it will also increase a certain degree of difficulty.

Useful Notation

$$\wedge \quad \vee \quad \neg \quad \rightarrow \quad \forall \quad \exists$$