

Project Summary

Five people work together on a task that includes two randomly selected spies and three good guys. To prevent the spies from sabotaging all the tasks, the supervisor develops a strategy. In odd-numbered rounds of the game, three people are selected to carry out the task, while in even-numbered rounds, only two people are selected. Some tasks are real, while others are decoys. As spies lack knowledge about the authenticity of the current task, they will try to vote against as much as possible in each task to increase the probability of affecting the true tasks. However, there is one exception: when a spy suspects they may be arousing suspicion. For example, if a spy voted against the previous round, causing the task to fail, they are likely to vote in favor in the next round if selected once more."

There is no limit to the number of tasks, but the success of each task requires the unanimous consent of all participants. The spies can choose to vote for or against the task, while the good guys can only vote in favor of the task. In addition, each person may not participate in the task for more than two consecutive rounds, and spies will ultimately be determined based on the results of each round.

Propositions

j : player name $j \in \{\text{Alice, Bob, Chris, David, Eric}\}$

i : round number - Integer greater than or equal to 1

K_i : True if task round ' i ' succeeds, False if task round ' i ' fails;

M_{ij} : In task ' i ', True if player ' j ' votes to accept, False if player ' j ' votes to reject.

R_{ij} : player ' j ' attend task round ' i '

G_j : True if player ' j ' is good person, False if not

P_i : round ' i ' is playing.

S_j : Represents suspicion that a player with the name "name" is a spy.

D_j : player ' j ' has been identified as a spy.

Constraints

3 players in odd-numbered rounds, 2 players in even-numbered rounds

$$[i \pmod{2} + 2]: (R_{ij(Alice)} \wedge R_{ij(Bob)}) \text{ or } (R_{ij(Alice)} \wedge R_{ij(Bob)} \wedge R_{ij(Chris)})$$

The success of the task requires the acceptance of all members participating in the task

$$M_{ij(Alice)} \wedge M_{ij(Bob)} \wedge M_{ij(Chris)} \rightarrow K_i \text{ or } M_{ij(Alice)} \wedge M_{ij(Bob)} \rightarrow K_i$$

Failure of the task means that there must be a spy among the members participating in the task

$$\neg K_i \rightarrow \neg M_{ij(Alice)} \vee \neg M_{ij(Bob)} \vee \neg M_{ij(Chris)}$$

Good people can only vote for acceptance.

$$G_j \rightarrow M_{ij}$$

Spies can vote to accept or reject.

$$(\neg G_j \rightarrow M_{ij}) \vee (\neg G_j \rightarrow \neg M_{ij})$$

No one can participate in more than two tasks in a row

$$\neg(R_{ij} \wedge R_{(i+1)j} \wedge R_{(i+2)j})$$

The spy cannot vote “accept” two consecutive rounds of task

$$\neg G_j \rightarrow \neg (M_{ij} \wedge M_{(i+1)j})$$

If player ‘j’ has been identified as a spy, player ji’ can not attend the following task.

$$\neg(D_j \wedge P_{(i+1)})$$

If the task the player is on fails, they will be suspected

$$\neg K_i \wedge R_{ij} \rightarrow S_j$$

Suspect identified as a good guy, no longer suspected after three consecutive successful tasks

$$S_j \wedge K_i \wedge K_{i+1} \wedge K_{i+2} \rightarrow G_j$$

If the suspect is included in the voting in the next round, but the result of this round of voting is mission success, then this suspect will be temporarily cleared of suspicion

$$K_{i+1} \wedge R_{(i+1)j} \rightarrow \neg S_j$$

(pending) players being suspected will be immediately asked to vote 3 consecutive rounds, in order to determine whether there is one spy

Count the number of rounds. If both spies are found, the game terminates; Vice Versa.

Model Exploration

To clarify the game mechanics, we've instituted a rule: once a player is identified as a spy, they're sidelined for the remainder of the game, effectively being ousted from the team.

When we began crafting our game dynamics, the primary method to differentiate between a good player and a spy was by observing mission failures. The idea was that spies, when paired with varying players for votes, would yield different outcomes, enabling others to pinpoint the spies over time.

To enhance the identification process, our strategy was to place the player under suspicion in a list, requiring them to participate in three consecutive rounds. By incorporating this rule into our model and testing it against several scenarios, we noticed an inconsistency. Despite the suspicion, the player could still choose to vote 'accept' or 'reject'. This vote influenced the overall results, confirming the presence of a spy in that round but not conclusively revealing their identity.

We decided to use the exclusion method to find out the spies, because with the spies messing around probably everyone would be the object of suspicion. However, spies try to fail missions as much as possible, so among the suspects if that player participates in three consecutive successful missions, he can be recognised as a good person and no longer be suspected.

Jape Proof Ideas

1. Good people can only vote accept, so those who vote reject must be Spy. (Because jape cannot recognize M, I use E instead.)

$$(G \rightarrow M) \vdash (\neg M \rightarrow \neg G)$$

$G \rightarrow E \vdash (\neg E \rightarrow \neg G)$	
1: $G \rightarrow E$	premise
2: $\neg E$	assumption
3: G	assumption
4: E	\rightarrow elim 1,3
5: \perp	\neg elim 4,2
6: $\neg G$	\neg intro 3-5
7: $(\neg E \rightarrow \neg G)$	\rightarrow intro 2-6

2. Suspect identified as a good guy, no longer suspected after three consecutive successful tasks

$P \wedge P \wedge P, P \rightarrow G \vdash G$

1: $P \wedge P \wedge P, P \rightarrow G$	premises
2: $P \wedge P$	\wedge elim 1.1
3: P	\wedge elim 2
4: G	\rightarrow elim 1.2,3
5: P	\wedge elim 1.1
6: G	hyp 4

Requested Feedback

1. Determine if the spy's method is appropriate?
2. Do we need to take into account the number of rounds it requires to finish the game?

- We deeply appreciate any advice you've provided, Thank you!

First-Order Extension

For example, to define that there must be a spy in a failed task, we can use \exists to represent "there exists" a spy. In a successful task, all players must vote to accept. We can use \forall to indicate that all players have voted to accept.

Useful Notation

$\wedge \vee \neg \rightarrow \forall \exists$