Triangulation and Trilateration

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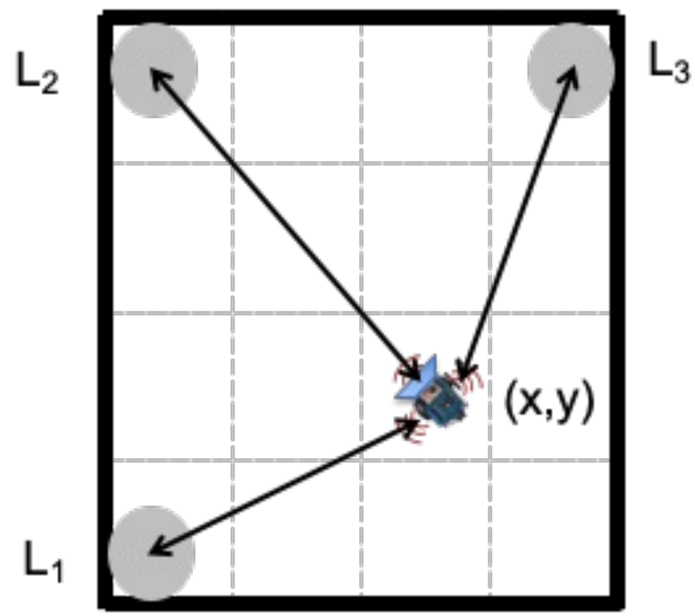
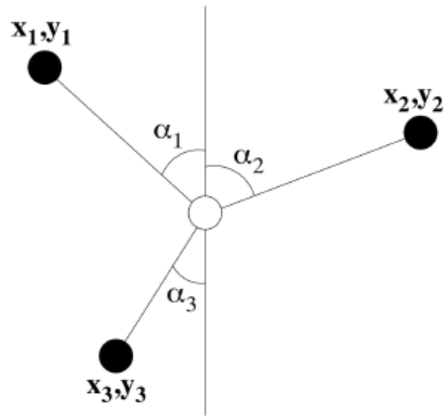
Triangulation and Trilateration

- Estimating location in 2D in relation to landmarks (L)
  - From measured ranges (distances)
  - From measured bearings (directions)
  - Both noiseless and noisy cases



Triangulation

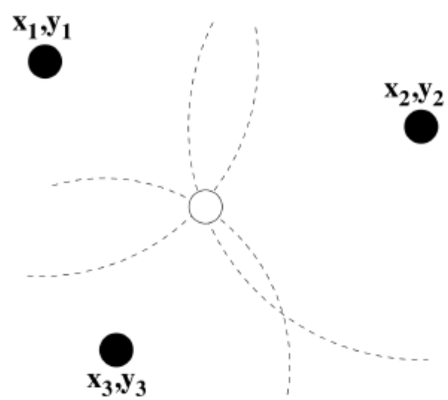
- Triangulation-based localization
  - Uses the geometric properties of triangles to estimate location
  - Relies on angle (bearing) measurements
  - Minimum of two bearing lines (and the locations of anchor nodes or the distance between them) are needed for two-dimensional space



*Fundamentals of Wireless Sensor Networks: Theory and Practice, W. Dargie and C. Poellabauer, 2010, Wiley

Trilateration

- Trilateration-based localization
  - Localization based on measured distances between a node and a number of anchor points with known locations
  - Basic concept: given the distance to an anchor, it is known that the node must be along the circumference of a circle centered at anchor and a radius equal to the node-anchor distance
  - In two-dimensional space, at least three non-collinear anchors are needed and in three-dimensional space, at least four non-coplanar anchors are needed



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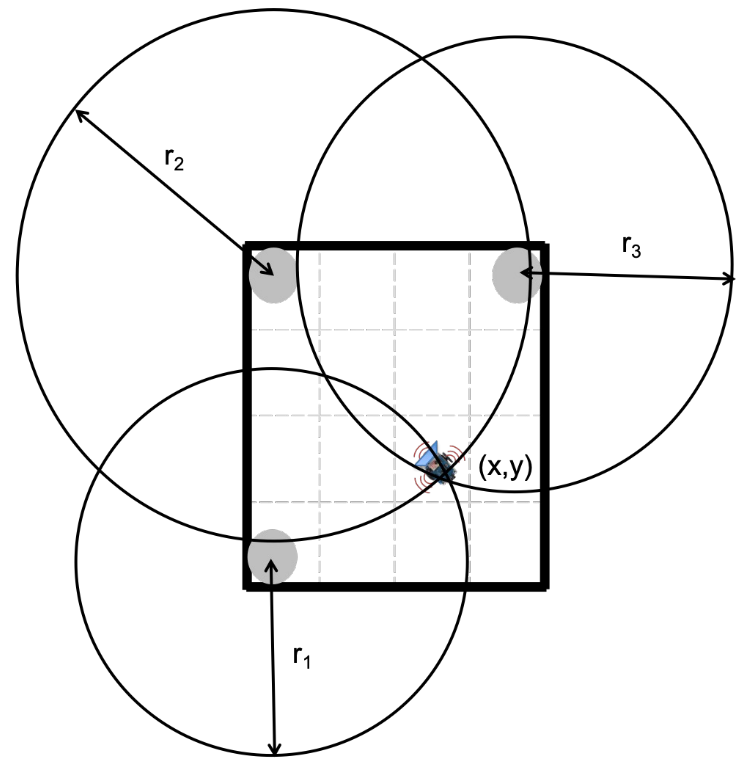
Trilateration

- In the case of unreliable angle calculations, robot localization (*x,y*) can be computed using trilateration methods based on the determination of absolute or relative measured distances to at least 3 known beacon or landmark locations.
- Intersection of 3 circles formed by the distances r_1, r_2, r_3, of the robot to the 3 beacons.
- Intersection at robot position may not be a single point since distance readings to beacons or landmarks may be noisy.



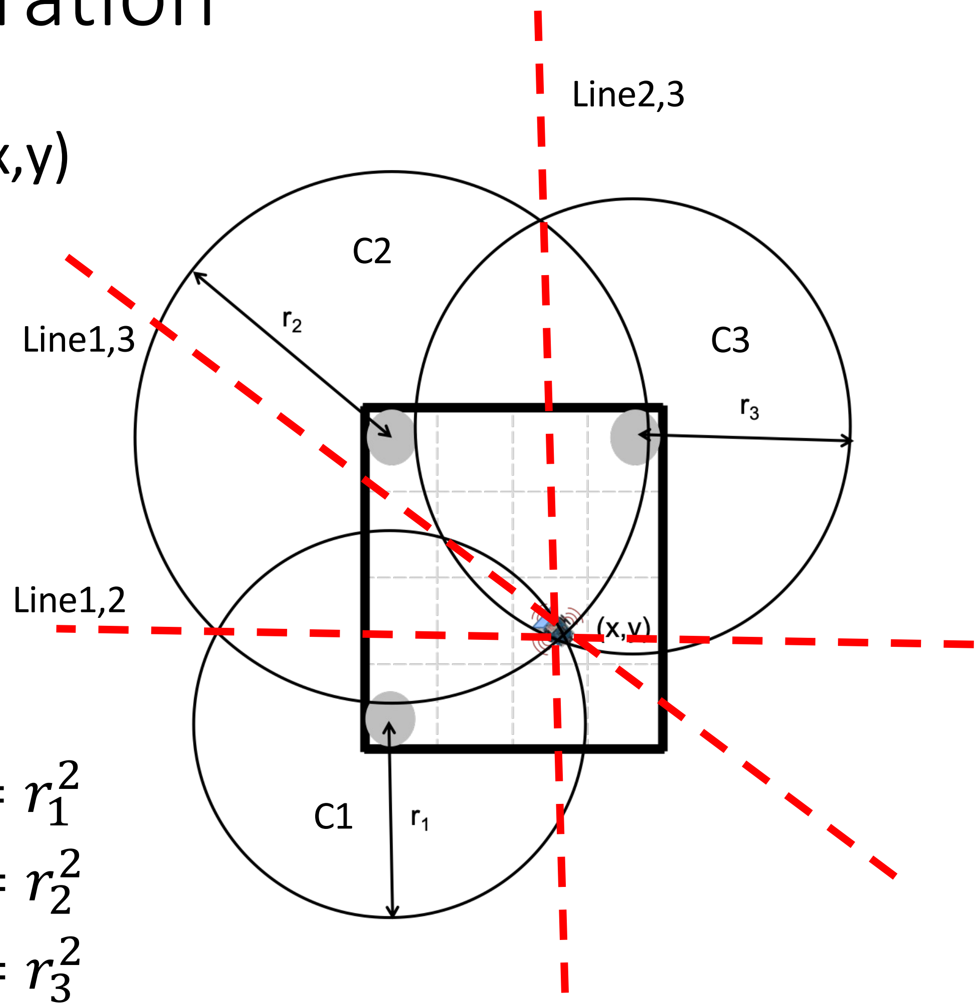
Trilateration

- Circles intersections to compute robot (x,y) location.
- The circles equations are given by:

$$C1: (x - x_1)^2 + (y - y_1)^2 = r_1^2$$
$$C2: (x - x_2)^2 + (y - y_2)^2 = r_2^2$$
$$C3: (x - x_3)^2 + (y - y_3)^2 = r_3^2$$

- The expanded circle equations are:

$$C1: x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 = r_1^2$$
$$C2: x^2 - 2x_2x + x_2^2 + y^2 - 2y_2y + y_2^2 = r_2^2$$
$$C3: x^2 - 2x_3x + x_3^2 + y^2 - 2y_3y + y_3^2 = r_3^2$$



Trilateration

- Subtracting two circle equations generates two lines:

$$\text{C1-C2: } (-2x_1 + 2x_2)\, x + (-2y_1 + 2y_2)\, y = r_1^2 - r_2^2 - x_1^2 + x_2^2 - y_1^2 + y_2^2$$

$$\text{C2-C3: } (-2x_2 + 2x_3)\, x + (-2y_2 + 2y_3)\, y = r_2^2 - r_3^2 - x_2^2 + x_3^2 - y_2^2 + y_3^2$$

- These two lines will intersect at (*x,y*) to generate an approximate robot location (without orientation information) given by the following two lines (note that a third line may generate a different (*x,y*) intersection due to trilateration computation imprecisions):

$$\text{C1-C2: } Ax + By = C$$

$$\text{C2-C3: } Dx + Ey = F$$

$$\text{C1-C2: } A = (-2x_1 + 2x_2),\ B = (-2y_1 + 2y_2),\ C = r_1^2 - r_2^2 - x_1^2 + x_2^2 - y_1^2 + y_2^2$$

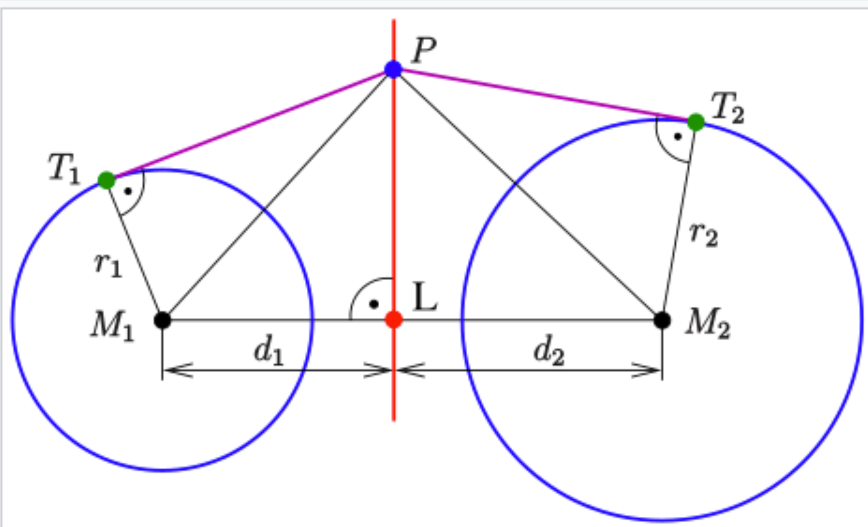
$$\text{C2-C3: } D = (-2x_2 + 2x_3),\ E = (-2y_2 + 2y_3),\ F = r_2^2 - r_3^2 - x_2^2 + x_3^2 - y_2^2 + y_3^2$$

- Coordinate (*x,y*) is given by the intersection of the two lines (Note that there is an exception when EA=BD):

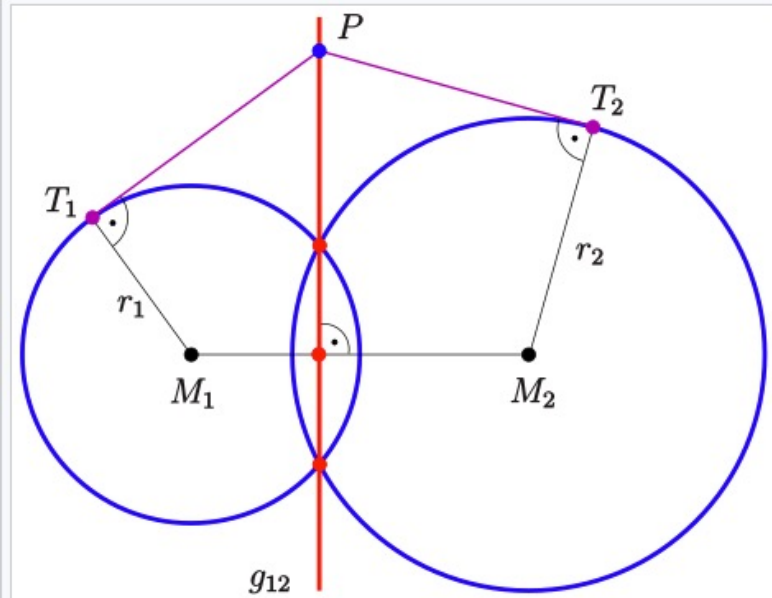
$$x = \frac{(\mathrm{CE} - \mathrm{FB})}{(\mathrm{EA} - \mathrm{BD})},\ y = \frac{(\mathrm{CD} - \mathrm{AF})}{(\mathrm{BD} - \mathrm{AE})}$$

Radical Axis or Power Line

- The **radical axis** (**power line** or **power bisector**) of two non-concentric circles is the set of points whose *power* with respect to the circles are equal, i.e. a real number that reflects the relative distance of a given point from a given circle.



Definition and calculation of d_1, d_2



Two circles, centered at M_1, M_2
Radical axis, with sample point P
Tangential distances from both circles to P
The tangent lines must be equal in length for any point on the radical axis: $|PT_1| = |PT_2|$. If P, T_1, T_2 lie on a common tangent, then P is the midpoint of $\overline{T_1T_2}$.

Ref: https://en.wikipedia.org/wiki/Radical_axis

Radical Axis or Power Line

- Given two circles A(B) and C(D), with centers at A and C and passing through points B and D, respectively.
- The radical axis of two circles (in red) is the locus of points (P) from which the tangents to the two circles are equal (|PR|=|PS|).
- To construct a radical axis, draw any circle E(F) that intersects both A(B) and C(D).
- Intersection points are G,H, and I,J, respectively. Let K be the intersection of GH and IJ.
- The radical axis of A(B) and C(D) is (in red) the line through K perpendicular to (in blue) the line of centers AC.