

## Geometry of Image: Coursework 2

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### Part A: Power & Phase Spectrum

In part of phase-randomized version of natural images, the phase spectrums are randomized while power spectrums are kept as before. On the contrary to phase-randomized version, whiten version of natural images keep the same phase spectrum as original ones whereas their power spectrums become flat, i.e. power spectrum matrix is filled by constant.

In the task, two images were tested totally and the results could be seen as Figure 1 and Figure 2. The first row shows the original natural image with its power and phase spectrum, the second row shows the phase-randomized version of input image with its power and phase spectrum and the third row shows the whiten version of input image with its power and phase spectrum.

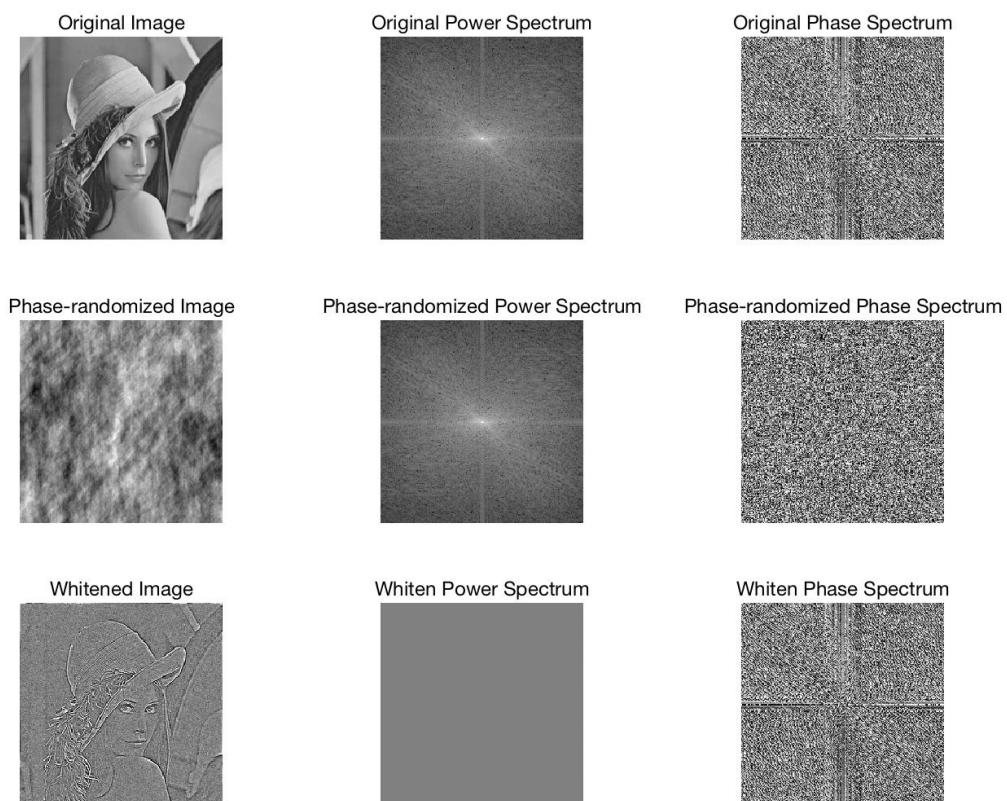


Figure 1: 'Lenna.jpg'

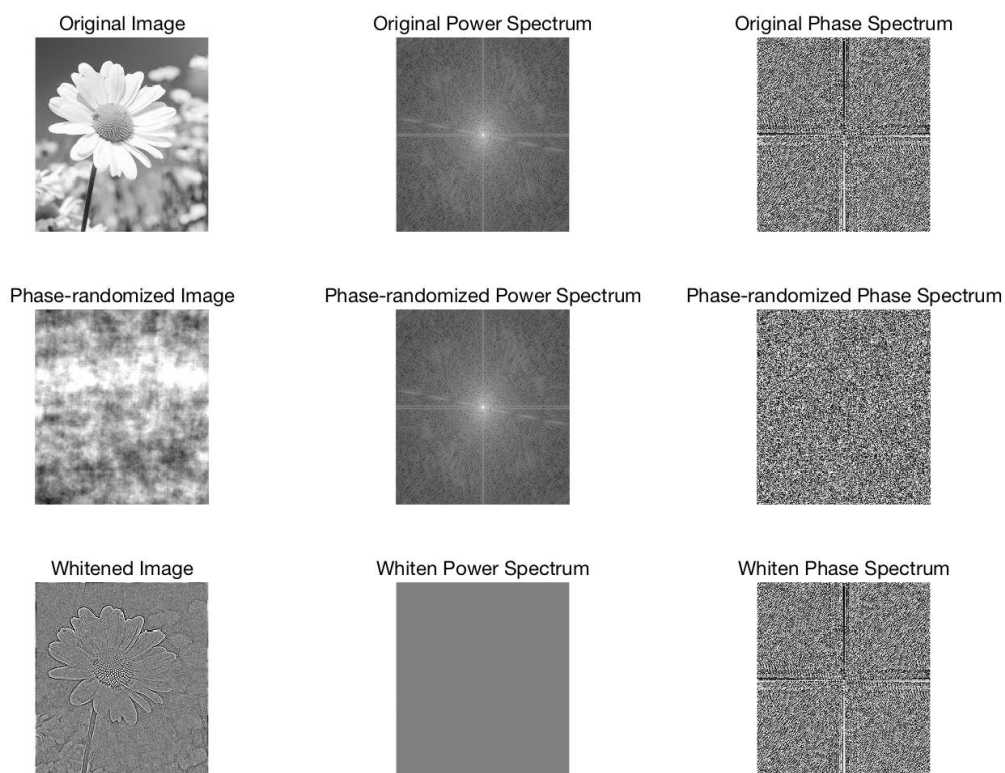


Figure 2: 'Flower.jpg'

From below results, we can find that reconstructed image in phase-randomized version loses almost content of the original one though it still keeps the same power spectrum as before. Phase spectrum is obtained based on image in frequency domain (via Fourier Transform) and it represents how sine waves with different frequency is positioned, while power spectrum represents how much energy there is in these sine waves used to build up the image. In this case, the original phase is randomized hence the reconstructed image is built up based on original power spectrum mostly and we cannot make any sense of the image even though the relative size of components is given. However, the reconstructed image based on phase spectrum includes most of original content except intensity, even though some details are unclear. Therefore, we could obtain a conclusion from above figures: phase spectrum could be used to restore some important image features, such as edges, while phase spectrum reflects the intensity of the input image.

## Part B: DtG filtering

In this part, we are required to implement DtGs to filtering images. Figure 3 shows the Derivatives of 2D Gaussian kernel. The first row shows the zero order, the second row shows the first order derivatives and the last row shows the second order derivatives of Gaussian kernel.

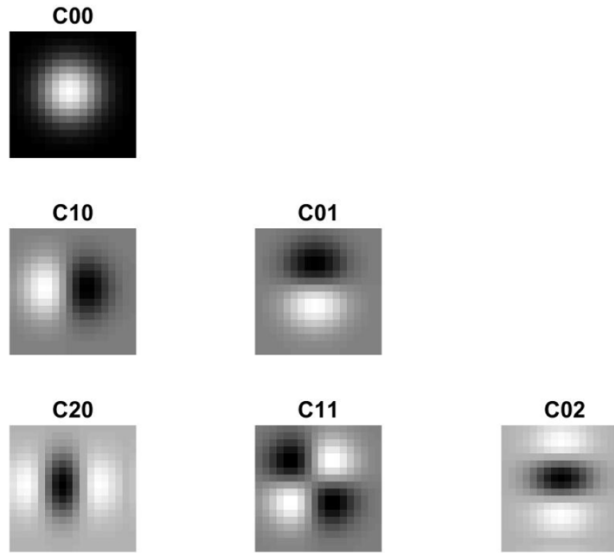


Figure 3: The Derivatives of 2D Gaussian kernel

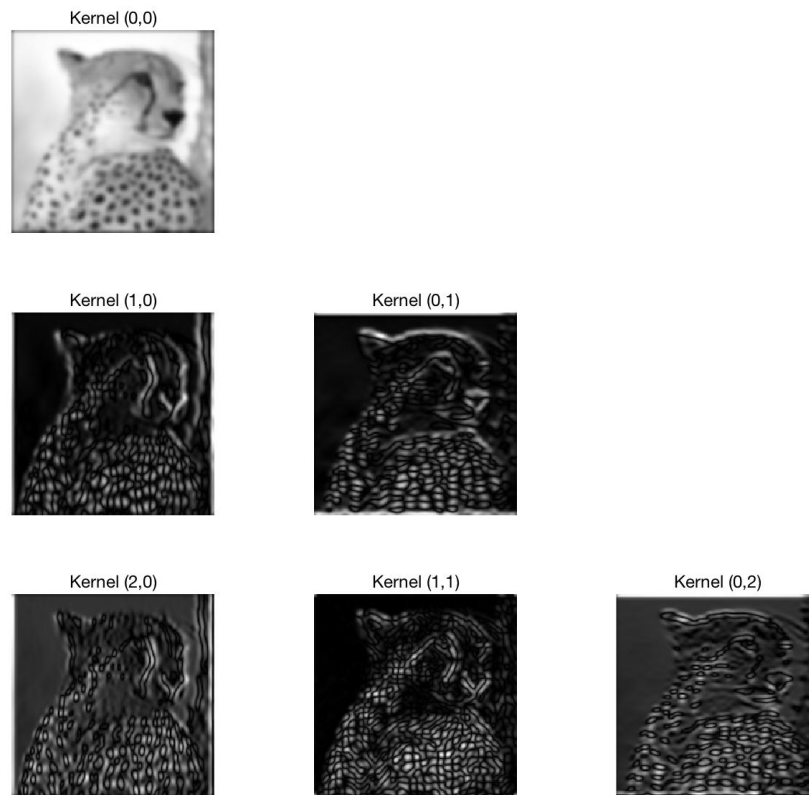
An image shows a cheetah was tested and corresponded results of is are displayed below.

The upper group of images (6 images totally) are images filtered by single filter from zero to second order. The figure shown in the first row is the one filtered by the filter in the zero order. The second row shows two images filtered by filters in first order from horizontal and vertical directions respectively. The third row shows the results filtered by three single filters included in second derivative in horizontal, diagonal and vertical direction respectively.

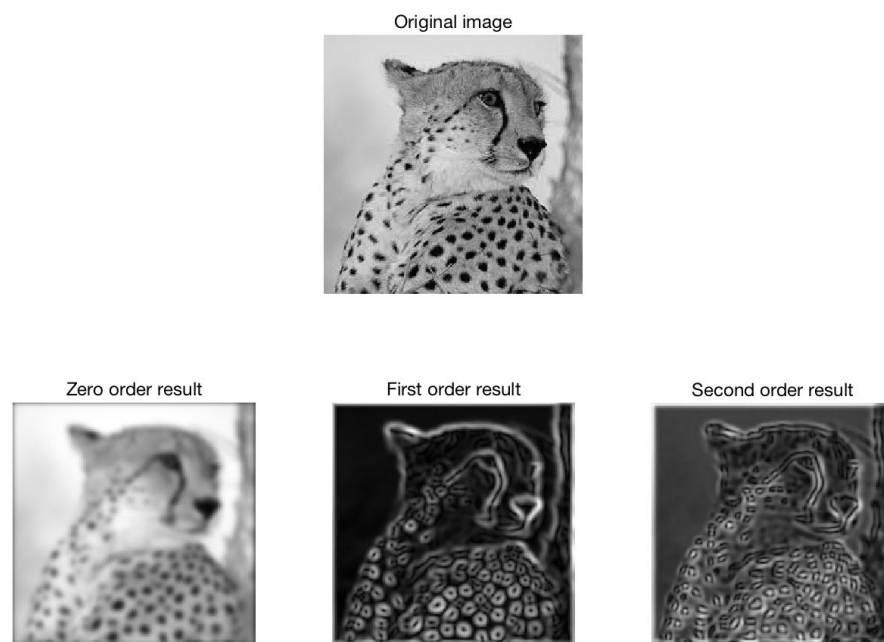
The bottom group of images (4 images totally) are results filtered by a family of filtered from zero to second order.

In below figures, the result of zero order of DtG filters is the convolution result between original image and 2D Gaussian filter hence the output is blurred. From the output results of first order of DtG filters, we could find the edge information of images based on gradients of image in gray version, which are obtained by applying the first order derivatives of Gaussian filter in both horizontal (kernel  $(0,1)$ ) and vertical (kernel  $(1,0)$ ) direction. In the part of second order of DtG filters, there are three

kernels included which represent gradient in horizontal, diagonal and vertical direction respectively. Through applying kernels on input image, we can observe that the second order of DtG filters are able to extract some feature details of the image.



*Figure 4: The Gaussian filter from zero to second order for DtG filter family*



*Figure 5: DtG filtered results from zero to second order*

## Part C: Basic Image Feature Classification

In Part C, it is required to use derivatives obtained from Part B to compute a BIF classification of input image. In this case, the size of Gaussian filter is defined as 40. Figure 6 illustrates the BIF classification for the image *castle.jpg* and compares the output results for different  $\sigma$  and  $\epsilon$  value setting. It could be found that if  $\sigma$  increases, the pink area will decrease and areas of other colors will increase; if  $\epsilon$  increases, the pink area will increase while other colors' areas will reduce at the same time.

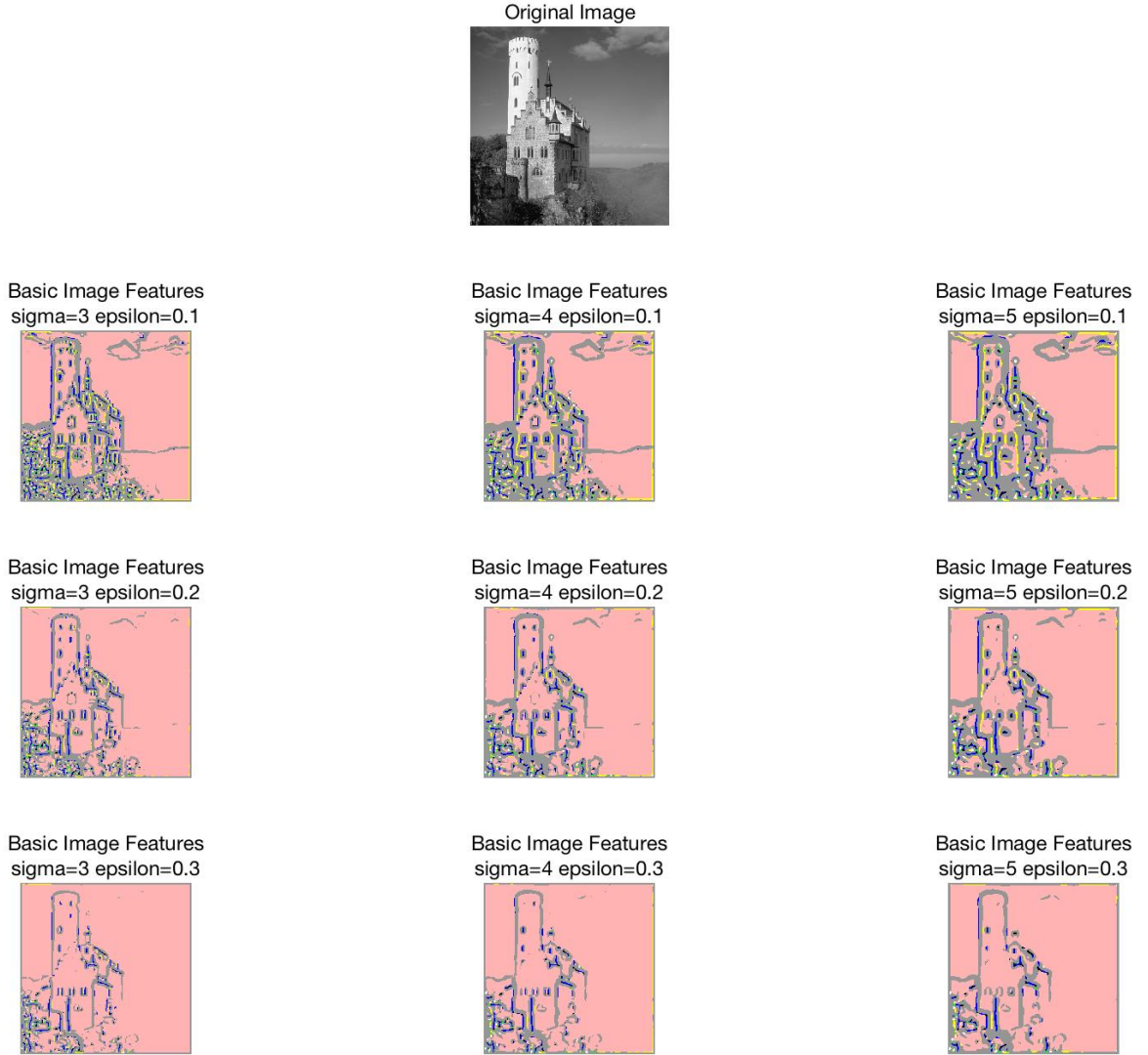


Figure 6: BIF classification results for 'castle.jpg'

Figure 7 compares the BIF of images in original and phase-randomized versions. We can observe that the BIF of phase-randomized image is not able to illustrate the basic image features successfully, which results from that phase-randomized image lost almost all of images features (except intensity) due to the randomized phase spectrum.

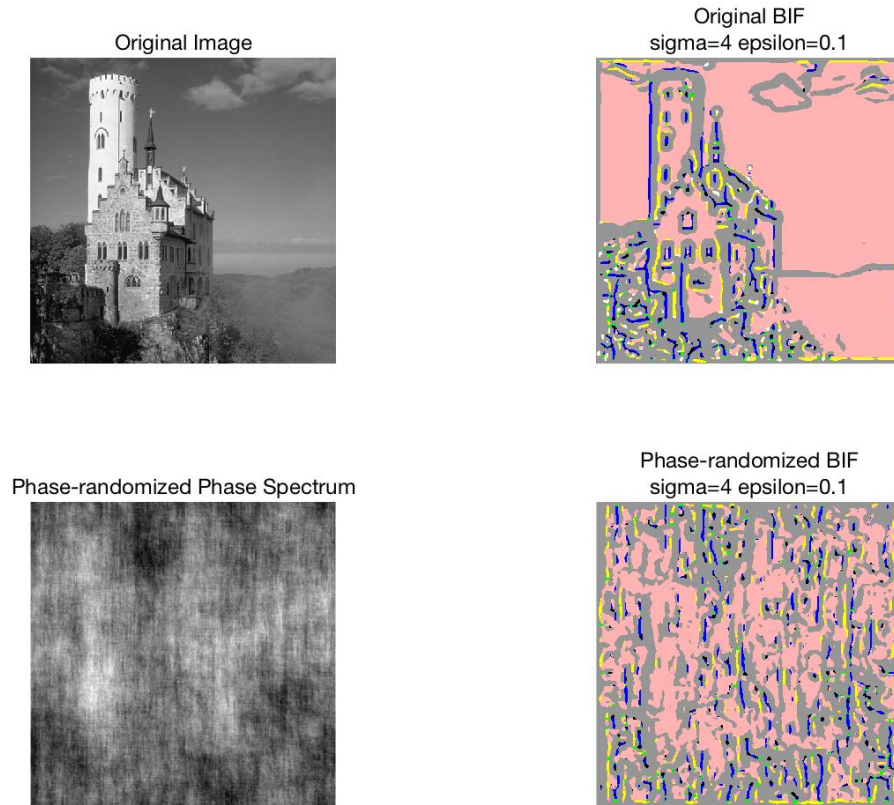


Figure 7: BIF classification in original and phase-randomized versions

The histogram of BIF in both versions are shown as Figure 8 and Figure 9. It is could be learned that the histogram of phase-randomized BIF is different from that of the original BIF, which means that the phase-randomized image losses most of basic image features contained in the original one.

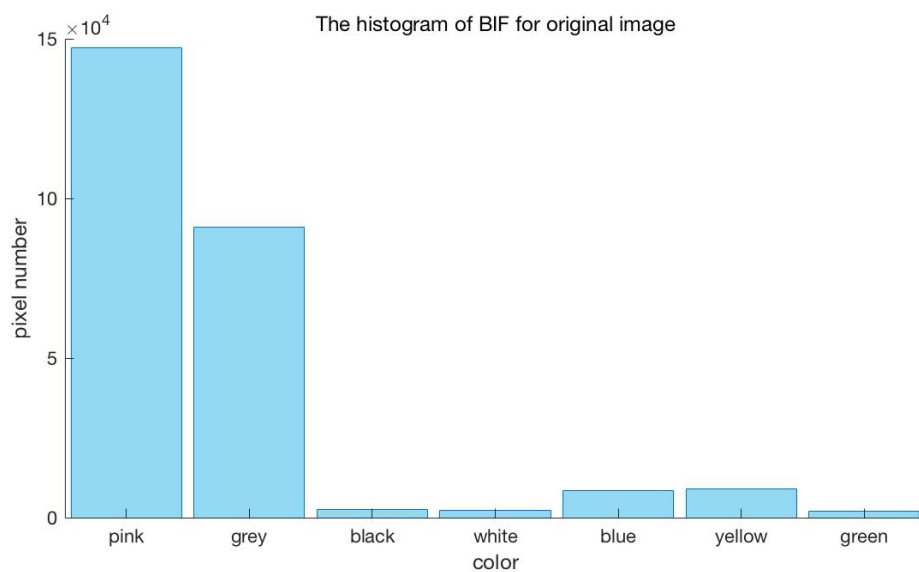
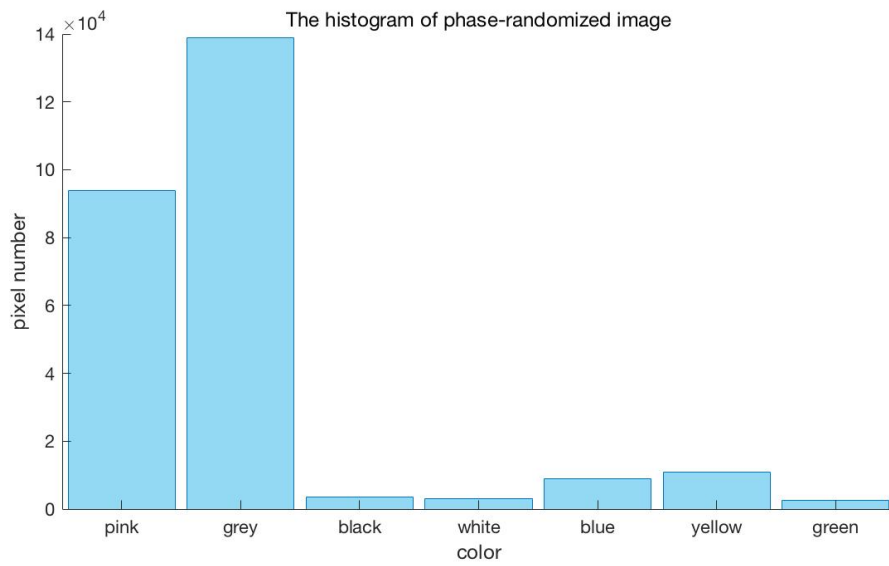


Figure 8: The histogram of BIF for original natural image



*Figure 9: The histogram of BIF for phase-randomized image*