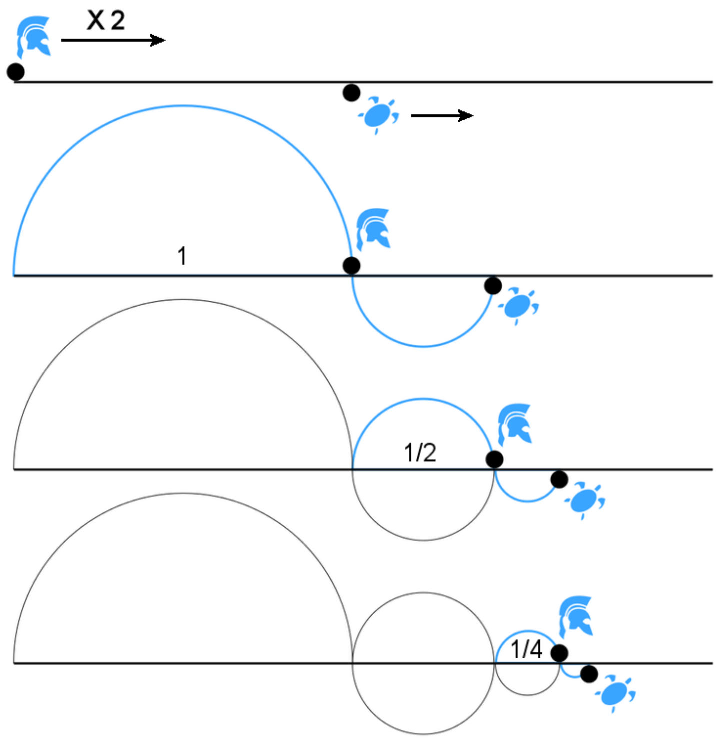
Aplicatii nostime ale seriilor de numere

Paradoxul lui Zeno



$$1 + \frac{1}{2} + \frac{1}{4} + \ldots = \sum_{n=0}^{\infty} \frac{1}{2^n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1 - \frac{1}{2}} = 2$$

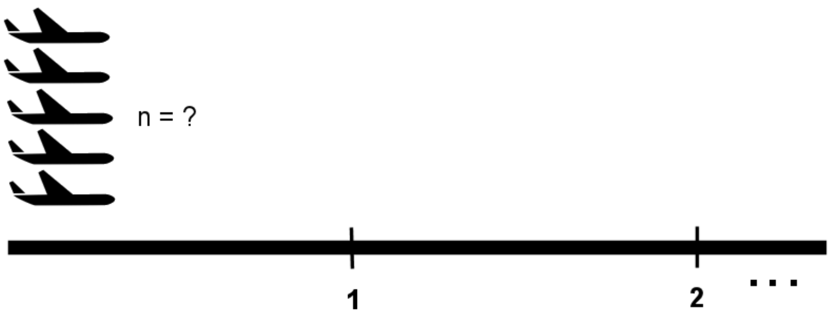
O problema de logistica militara

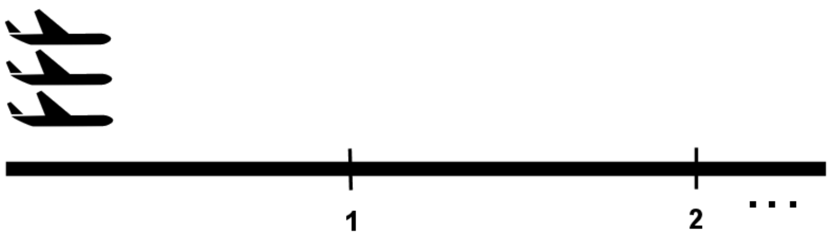
- "n" aparate de zbor pornesc simultan in aceeasi directie cu scopul de a acoperi o distanta maxima

- autonomie de zbor : o unitate de lungime

- abilitate transfer combustibil intre aparate

- se cere numarul initial "n" pentru a putea acoperi distanta de 10 unitati





In general, cele "n" aparate parcurg distanta maxima

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \rightarrow +\infty \qquad (n \rightarrow +\infty)$$

$$H_n = 10 \Longrightarrow n = ?$$

```
N[HarmonicNumber[12 366], 10]
N[HarmonicNumber[12 367], 10]
```

```
9.999962148
10.00004301
```

$$\Longrightarrow n = 12\,367$$

<u>Fractalul lui Koch</u>

Se porneste cu un triunghi echilateral de latura unitate

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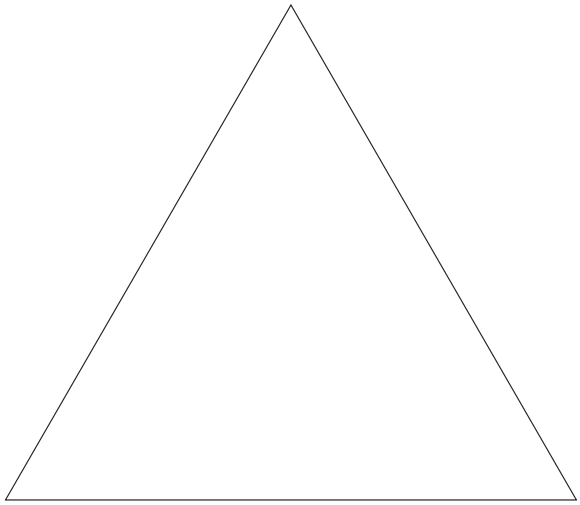
KochSnowflake[0]
```



Latura = 1

$$\textbf{Aria} = \frac{\sqrt{3}}{4} \overset{\text{not}}{=} \alpha$$

Perimetrul = 3

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KochSnowflake[1]
```
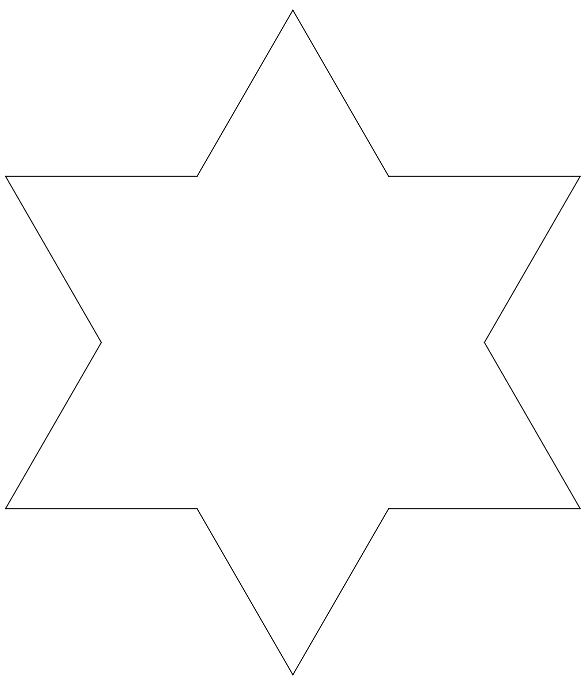
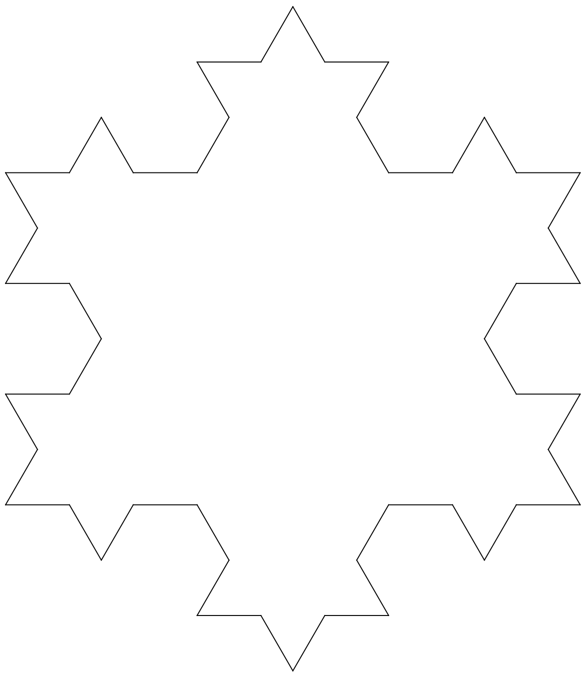



$$\text{Latura} = \frac{1}{3}$$

$$\text{Aria} = \alpha + 3 \times \frac{1}{3^2}\,\alpha$$

$$\text{Perimetrul} = 3 \times 4 \times \frac{1}{3}$$

```
KochSnowflake[2]
```

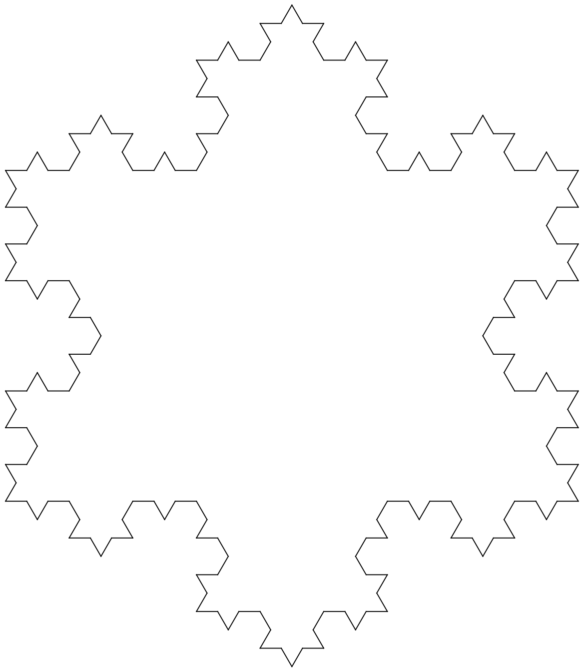


$$\text{Latura} = \frac{1}{3^2}$$

$$\text{Aria} = \alpha + 3 \times \frac{1}{3^2}\,\alpha + 3 \times 4 \times \frac{1}{3^4}\,\alpha$$

$$\text{Perimetrul} = 3 \times 4^2 \times \frac{1}{3^2}$$

```
KochSnowflake[3]
```

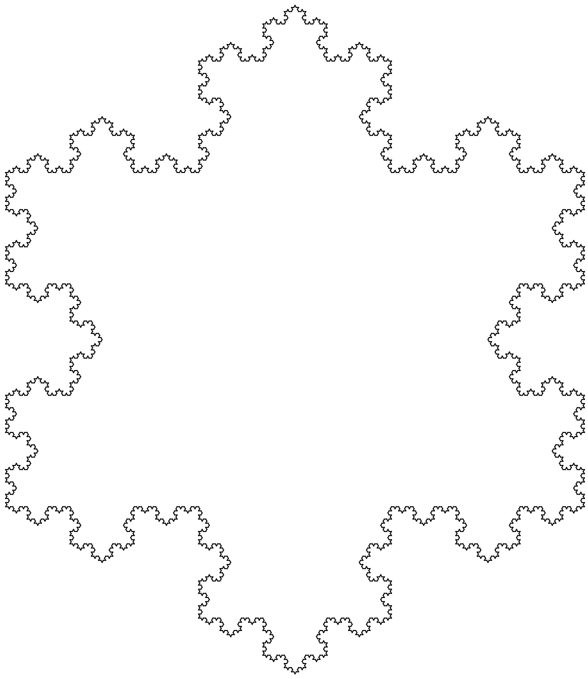


Latura $= \dfrac{1}{3^3}$

Aria $= \alpha + 3 \times \dfrac{1}{3^2}\,\alpha + 3 \times 4 \times \dfrac{1}{3^4}\,\alpha + 3 \times 4^2 \times \dfrac{1}{3^6}\,\alpha$

Perimetrul $= 3 \times 4^3 \times \dfrac{1}{3^3}$

```
KochSnowflake[5]
```



In general, dupa "n" iteratii avem

$$\text{Latura} = \frac{1}{3^n}$$

$$\text{Aria} = \alpha + 3 \times \frac{1}{3^2}\alpha + 3 \times 4 \times \frac{1}{3^4}\alpha + \ldots + 3 \times 4^{n-1} \times \frac{1}{3^{2n}}\alpha =$$

$$= \alpha + \frac{3}{4}\sum_{k=1}^{n}\left(\frac{4}{9}\right)^k \alpha$$

$$\text{Perimetrul} = 3\left(\frac{4}{3}\right)^n$$

La limita se obtine fractalul Koch ($n \to \infty$)

$$\text{Aria} \to \alpha + \frac{3}{4} \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n \alpha =$$

$$= \alpha + \frac{3}{4} \left[-1 + \sum_{n=0}^{\infty} \left(\frac{4}{9}\right)^n\right] \alpha = \alpha + \frac{3}{4}\left[-1 + \frac{1}{1 - \frac{4}{9}}\right] \alpha = \frac{8}{5} \alpha$$

Perimetrul $\to \infty$

Dati un alt exemplu de figura geometrica plana care sa fie simultan

- **marginita**
- **de arie finita (eventual nula)**
- **de perimetru infinit**

(fractalul Vicsek - vezi Wikipedia)