Neutron diffusion

Criticial mass of atomic bomb

Student : Cristian Cioriia

Summary

1. Introduction
2. Results
3. Conclusion

Student : Cristian Cioriia

Assumptions

1. Quantum effects do not play an important role in this process
2. Neutrons can be treated as classical particles
3. Consider only the bulk behaviour of neutrons
4. Neutrons can undergo any of the following processes: generation via fission, scattering and absorption

The goal is to find the minimum amount of fissile material for which a runaway process can happen

Diffusion Equation

As the title suggests, neutrons behaviour is described by the diffusion equation.

$$\frac{\partial n}{\partial t} = \mu \nabla^2 n + \eta n \qquad (2.1)$$

$n(x,y,z,t)$ is the neutron number density
μ is the difussion constant
η is the neutron generation rate

$$IC : n(x, y, z, t=0) = f(x, y, z) \qquad (2.2)$$

On which we impose Dirichlet Boundary conditions (neutron number density is 0 at the surface of the fissile material) and Von Neumann Boundary conditions (at the very last case only)

Diffusion Equation

Solution is reached by imposing separable solution ansatz.

This works for all symmetry scenarios studied :

1. Cartesian
2. Cylindrical
3. Spherical

The derivation of the solution can be found in any differential equation textbook

e.g Cartesia symmetry:

$$n(x, y, z, t) = X(x)\,Y(y)\,Z(z)\,T(t) \tag{3.1}$$

1d scenario

The solution to the diffusion equation is the following :

$$n = \sum_{p=1}^{\infty} a_p exp\left(\left(\eta - \frac{\mu\, p^2 \pi^2}{L^2}\right)t\right)\sin\left(\frac{p\pi}{L}x\right) \quad (4.1)$$

$$a_p = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{p\pi}{L}x\right),\ p = 1, 2, 3.. \quad (4.2)$$

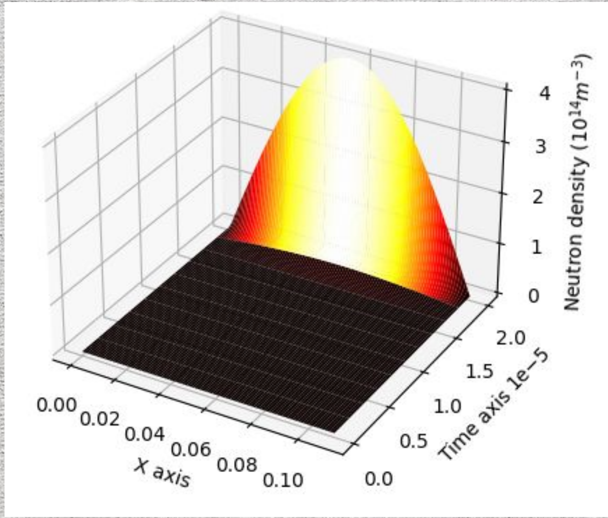
Thus the critical size below which there can be no runaway reaction is

$$L_{crit} = \pi\sqrt{\frac{\mu}{\eta}} = 11.05\ cm$$

1d scenario

By using a slightly large 11.1 cm as the operational length we can obtain a runaway process.

After 1e-5 seconds, the neutron number density increases exponentially.



2d scenario

The solution to the diffusion equation is the following :

$$n = \sum_{p,q}^{\infty} a_{pq} exp\left(\left(\eta - \mu\left(\frac{p^2\pi^2}{L_x^2} + \frac{q^2\pi^2}{L_y^2}\right)\right)t\right)\sin\left(\frac{p\pi}{L_x}x\right)\sin\left(\frac{q\pi}{L_y}y\right) \quad (6.1)$$

$$a_{pq} = \frac{4}{L^2}\int_0^L\int_0^L f(x,y)\sin\left(\frac{p\pi}{L}x\right)\sin\left(\frac{q\pi}{L}y\right),\ p,q = 1,2,3.. \quad (6.2)$$

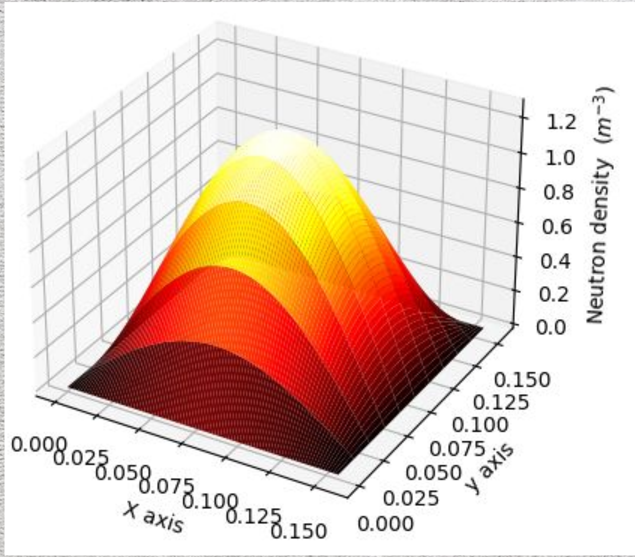
Thus the critical size below which there can be no runaway reaction is

$$L_{crit} = \pi\sqrt{\frac{2\mu}{\eta}} = 15.62\ cm$$

2d scenario

By using a slightly large 15.7 cm as the operational length we can obtain a runaway process.

After 1e-7 seconds, the neutron number density increases exponentially.



3d scenario Cartesian

The solution to the diffusion equation is the following :

$$n = \sum_{p,q,r}^{\infty} a_{pqr} exp\left(\left(\eta - \mu\left(\frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2}\right)\right)t\right)\sin\left(\frac{p\pi}{L_x}x\right)\sin\left(\frac{q\pi}{L_y}y\right)\sin\left(\frac{r\pi}{L_z}z\right) \quad (8.1)$$

$$a_{pqr} = \frac{8}{L^3}\int_0^L\int_0^L\int_0^L f(x,y,z)\sin\left(\frac{p\pi}{L}x\right)\sin\left(\frac{q\pi}{L}y\right)\sin\left(\frac{r\pi}{L}z\right),\ p,q,r = 1,2,3.. \quad (8.2)$$

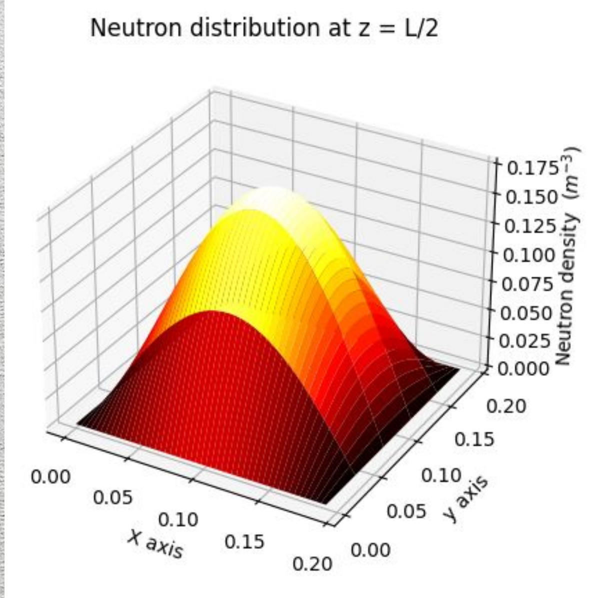
Thus the critical size below which there can be no runaway reaction is

$$L_{crit} = \pi\sqrt{\frac{3\mu}{\eta}} = 19.14\ cm$$

3d scenario Cartesian

By using a slightly large 19.2 cm as the operational length we can obtain a runaway process.

After 1e-7 seconds, the neutron number density increases exponentially.



The solution to the diffusion equation is the following :

$$n = \sum_{q=1}^{\infty} a_{1q} exp\left(\left(\eta - \mu\left(\frac{\alpha_q^2 L^2 + \pi^2 r_1^2}{r_1^2 L^2}\right)\right)t\right)\sin\left(\frac{\pi}{L}z\right) J_0\left(\frac{\alpha_q r}{r_1}\right) \quad (10.1)$$

$$a_{1q} = \frac{4}{L\, r_1^2 J_1^2(\alpha_q)} \int_0^L \int_0^{r_1} J_0\left(\frac{\alpha_q r}{r_1}\right) \sin^2\left(\frac{\pi}{L}z\right) r\left(1 - \frac{r^2}{r_1^2}\right),\ q = 1, 2, 3.. \quad (10.2)$$

Thus the critical size below which there can be no runaway reaction is

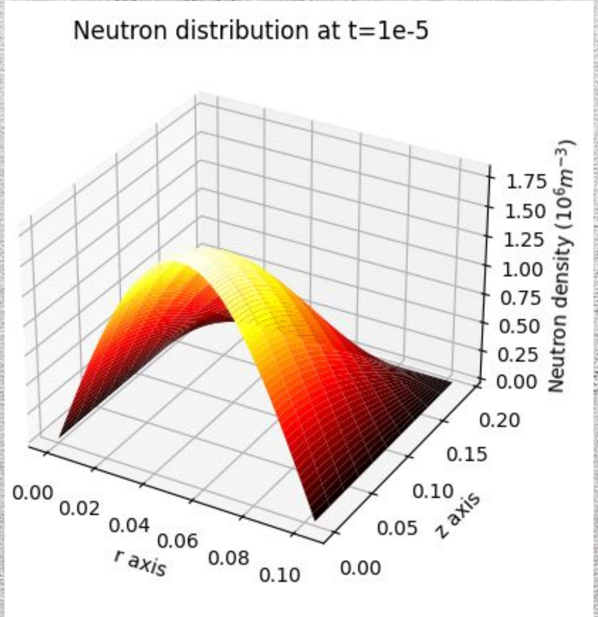
$$L_{crit} = \pi\sqrt{\frac{3\mu}{\eta}} = 19.14\ cm$$

$$r_{crit} = \alpha_1\sqrt{\frac{3\mu}{2\eta}} = 10.36\ cm$$

3d scenario Cylindrical

By using a slightly larger height 19.2 cm and radius 10.4 as the operational lengths we can obtain a runaway process.

After 1e-5 seconds, the neutron number density increases exponentially.



The solution to the diffusion equation is the following :

$$n = \sum_{p=1}^{\infty} \frac{a_p}{r} exp\left(\left(\eta - \frac{\mu\, p^2\, \pi^2}{r_1^2}\right)t\right)\sin\left(\frac{p\pi}{r_1} r\right)$$

(12.1)

$$a_p = \frac{2}{r_1}\int_0^{r_1} r\left(1 - \frac{r^2}{r_1^2}\right)\sin\left(\frac{p\pi}{r_1} r\right),\ p = 1, 2, 3 \ldots$$

(12.2)

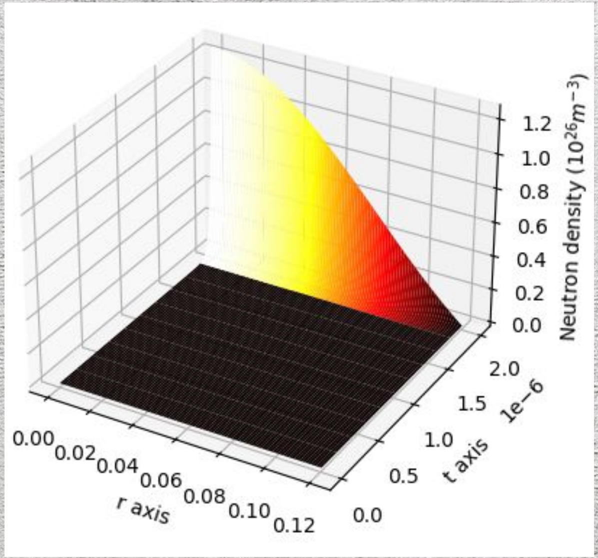
Thus the critical size below which there can be no runaway reaction is

$$r_{crit} = \pi\sqrt{\frac{\mu}{\eta}} = 11.05\ cm$$

3d scenario Spherical

By using a slightly larger radius 11.5 cm as the operational radius we can obtain a runaway process.

After 2e-6 seconds, the neutron number density increases exponentially.



3d scenario Spherical - Neumann

This time we use Neumann boundary conditions

The main assumption behind these conditions is that some neutrons are reflected back at the Surface of the fission material.

$$n = A\exp(-\alpha t)\,\frac{\sin(kr)}{r} \tag{14.1}$$

The criticality conditions is:

$$-1 + kr\cot(kr) + \frac{3r}{2\lambda_t} = 0 \tag{14.2}$$

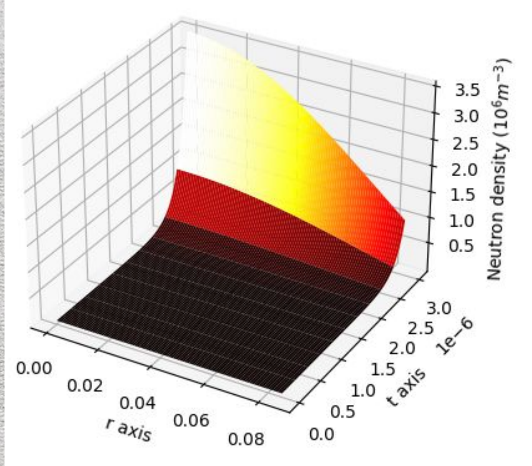
Where $k = \sqrt{\frac{\eta + \alpha}{\mu}}$

Set $\alpha = 0$ to get $R_{crit} = 8.369\ cm$

3d scenario Spherical - Neumann

By using a slightly larger radius 8.5 cm as the operational radius we can obtain a runaway process.

After 3e-6 seconds, the neutron number density increases exponentially.



Conclusion

Using empirical values for diffusion, neutron generation rate for U-235 we then get that the more realistic Von Neumann boundary condition provides the most economical estimate of size.

By plugging in some number we get a critical mass of 45.9 kg of U-235.
Given that weapons grade uranium reaches 80% U-235 purity this gives an approximate 57.4 kg.

The fissile material of the Fatman bomb was 64 kg