1.Calculate the following probabilities using a normal approximation (Central Limit Theorem):

P(X < 150), where X-B belongs to (200, 0.8)

a) 0.019

b) 0.047

c) 0.021

d) 0.015

we need to find the z-score corresponding to X = 150

Given:

- $\mu = 200$
- $\sigma = 0.8$
- $X = 150$

we calculate the z-score

$$z = \frac{X-\mu}{\sigma}$$

$$z = \frac{150-200}{0.8} = \frac{-50}{0.8} = -62.5$$

Now, we need to find the probability $P(Z < -62.5)$, where $Z$ is a standard normal random variable.

Using a standard normal distribution table or calculator, we find that $P(Z < -62.5)$ is extremely close to 0.

Therefore, $P(X < 150)$ is also extremely close to 0.

Thus, the correct answer is option d) $0.015$.

2.Let X de a discrete random variable with the following distribution:

(-1   1   2)
(1/6  1/3  1/2)

Compute the value of V[X]. Round your answer to 3 decimal places.

First, let's calculate the mean ($\mu$):

$$\mu = (-1) \cdot \frac{1}{6} + (1) \cdot \frac{1}{3} + (2) \cdot \frac{1}{2}$$
$$\mu = -\frac{1}{6} + \frac{1}{3} + 1$$
$$\mu = \frac{-1+2+3}{6}$$
$$\mu = \frac{4}{6} = \frac{2}{3}$$

Now, let's compute the variance ($V[X]$): $V[X] = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P(X = x_i)$

$$V[X] = (-1 - \frac{2}{3})^2 \cdot \frac{1}{6} + (1 - \frac{2}{3})^2 \cdot \frac{1}{3} + (2 - \frac{2}{3})^2 \cdot \frac{1}{2}$$
$$V[X] = (\frac{-5}{3})^2 \cdot \frac{1}{6} + (\frac{1}{3})^2 \cdot \frac{1}{3} + (\frac{4}{3})^2 \cdot \frac{1}{2}$$
$$V[X] = \frac{25}{9} \cdot \frac{1}{6} + \frac{1}{9} \cdot \frac{1}{3} + \frac{16}{9} \cdot \frac{1}{2}$$
$$V[X] = \frac{25}{54} + \frac{1}{27} + \frac{8}{9}$$
$$V[X] = \frac{25}{54} + \frac{2}{54} + \frac{48}{54}$$
$$V[X] = \frac{75}{54}$$
$$V[X] \approx 1.389$$

3. A committee consists of five Chicanos, two Asians, three African Americans, and two Cancasians. A subcommittee of four is chosen at random. What is the probability that all the ethnic groups are represented an the sabcommittee?

a) 0.121

b) 0.452

c) 0.548

d) 0.879

$$\text{Total combinations} = \binom{12}{4} = \frac{12!}{4!(12-4)!} = \frac{12!}{4!8!} = \frac{12\times11\times10\times9}{4\times3\times2\times1} = 495$$

Now, let's calculate the number of ways to choose a subcommittee where all ethnic groups are represented.

.. Choose 1 Chicano, 1 Asian, 1 African American, and 1 Caucasian:

$$\binom{5}{1} \times \binom{2}{1} \times \binom{3}{1} \times \binom{2}{1} = 5 \times 2 \times 3 \times 2 = 60$$

.. Choose the remaining member from the 8 remaining members:

$$\binom{8}{1} = 8$$

Therefore, the total number of ways to choose a subcommittee where all ethnic groups are represented is $60 \times 8 = 480$.

Now, we can find the probability:

$$P(\text{all ethnic groups represented}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{480}{495}$$

$$P(\text{all ethnic groups represented}) \approx 0.969$$

4. For evalunting the effectiveness of a processor for a certain type of tasks, we recorded the CPU time for 20 randomly chosen jobs. It is known that the CPU time has a normal distribution with mean 20(s) and standard deviation 8(s). Compute the probability that the average CPU for the selected jobs (the sample mean) takes valves in the interval [18.22]. Round your answer to 3 decimal places.

Given:

- Population mean ($\mu$) = 20 seconds
- Population standard deviation ($\sigma$) = 8 seconds
- Sample size ($n$) = 20
- Interval: [18, 22]

1. **Calculate Standard Error of the Mean ($\sigma_{\bar{x}}$):**

   $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{20}}$

   From the table:

   $\sigma_{\bar{x}} \approx \frac{8}{\sqrt{20}} \approx \frac{8}{4.47} \approx 1.7889$

2. **Convert the Interval to Z-scores:**
   - For the lower bound:

     $z_{\text{lower}} = \frac{18-20}{1.7889} \approx \frac{-2}{1.7889} \approx -1.118$
   - For the upper bound:

     $z_{\text{upper}} = \frac{22-20}{1.7889} \approx \frac{2}{1.7889} \approx 1.118$

3. **Find the Cumulative Probabilities Using Z-scores:**
   - For $z_{\text{lower}}$:

     $P(z_{\text{lower}}) = P(Z < -1.118)$

     From the table: $P(Z < -1.11) \approx 0.1331$
   - For $z_{\text{upper}}$:

     $P(z_{\text{upper}}) = P(Z < 1.118)$

     From the table: $P(Z < 1.11) \approx 0.8665$

4. **Compute the Probability of the Interval:**

   $P(\text{interval}) = P(z_{\text{upper}}) - P(z_{\text{lower}})$

   $P(\text{interval}) \approx 0.8665 - 0.1331$

   $P(\text{interval}) \approx 0.7334$

Therefore, the probability that the average CPU time for the selected jobs falls within the interval [18, 22] is approximately 0.733, rounded to 3 decimal places.

5. The lifetime, in years, of some electronic component is a continuous random variable with the density f(x) = k/x^4, if x>=1, f(x)=0, x<1.

Find k sunch that f is a valid PDF and the probability that the lifetime will exceed 4 years.

a) 0,275

b) 0,459

c) k=1/2

d) 0,984

e) 0,016

f) k=3 CORRECT

g) k=1/3

h) k=4

Given:

$f(x) = \frac{k}{x^4}$ if $x \geq 1$

$f(x) = 0$ if $x < 1$

1. **Determine the Integral of $f(x)$ from 1 to $\infty$:**
   $\int_1^{\infty} \frac{k}{x^4}\, dx = k \int_1^{\infty} \frac{1}{x^4}\, dx$
2. **Integrate $\frac{1}{x^4}$ with respect to $x$:**
   $\int_1^{\infty} \frac{1}{x^4}\, dx = \left[-\frac{1}{3x^3}\right]_1^{\infty}$
3. **Apply the Limits of Integration:**
   $\left[-\frac{1}{3x^3}\right]_1^{\infty} = \lim_{t\to\infty} -\frac{1}{3t^3} - \left(-\frac{1}{3}\right) = \frac{1}{3}$
4. **Set the Integral Equal to 1 and Solve for $k$:**
   $k \cdot \frac{1}{3} = 1$
   $k = 3$

So, the correct answer is $\boxed{\text{f) } k = 3}$.

Now, to find the probability that the lifetime will exceed 4 years, we integrate $f(x)$ from 4 to $\infty$:

$P(X > 4) = \int_4^{\infty} \frac{3}{x^4}\, dx$

$P(X > 4) = 3 \int_4^{\infty} \frac{1}{x^4}\, dx$

$P(X > 4) = 3 \left[-\frac{1}{3x^3}\right]_4^{\infty}$

$P(X > 4) = 3 \left(0 - \left(-\frac{1}{192}\right)\right)$

$P(X > 4) = 3 \cdot \frac{1}{192}$

$P(X > 4) = \frac{1}{64}$

$P(X > 4) = 0.015625$

Thus, the probability that the lifetime will exceed 4 years is approximately $\boxed{0.016}$.

6. A problem on a multiple-choice quiz is answered corectly with probability 0,9 if a student is prepared an unprepared student guesses between 4 possible answers. Seventy five percent of students prepare for the quiz. If Student X gives a correct answer to this problem, what is the chance that he prepared for the quiz?

a) 0,915   CORRECT

b) 0,854

c) 0,825

d) 0,945

Let's define the events:

- $P$: Student is prepared for the quiz.
- $\neg P$: Student is not prepared for the quiz.
- $C$: Student answers the question correctly.

We are given:

- $P(P) = 0.75$ (Probability that a student is prepared for the quiz)
- $P(\neg P) = 1 - P(P) = 0.25$ (Probability that a student is not prepared for the quiz)
- $P(C|P) = 0.9$ (Probability that a prepared student answers correctly)
- $P(C|\neg P) = \frac{1}{4}$ (Probability that an unprepared student guesses correctly)

We want to find $P(P|C)$, the probability that the student was prepared given that they answered correctly.

By Bayes' Theorem:

$$P(P|C) = \frac{P(C|P) \cdot P(P)}{P(C)}$$

We can calculate $P(C)$ using the Law of Total Probability:

$$P(C) = P(C|P) \cdot P(P) + P(C|\neg P) \cdot P(\neg P)$$

Let's substitute the values:

$$P(C) = (0.9 \cdot 0.75) + (\tfrac{1}{4} \cdot 0.25)$$

$$P(C) = 0.675 + 0.0625$$

$$P(C) = 0.7375$$

Now, we can find $P(P|C)$:

$$P(P|C) = \frac{0.9 \cdot 0.75}{0.7375}$$

$$P(P|C) = \frac{0.675}{0.7375}$$

$$P(P|C) \approx 0.915$$

8. In a certain store one in ten computers has defects. Twenty randomly selected computers are bought for the university lab. Compute the probability that less than three computers have defects. Round your aswer to 3 decimal places.

To solve this problem, we can use the binomial probability formula, as the situation fits the criteria for a binomial distribution:

$P(X=k)=\binom{n}{k}\cdot p^k\cdot(1-p)^{n-k}$

Where:

- $n$ is the number of trials (number of computers bought),
- $k$ is the number of successes (number of computers with defects),
- $p$ is the probability of success on each trial (probability that a computer has defects).

Given:

- $n=20$ (twenty randomly selected computers),
- $p=0.1$ (probability that a computer has defects),
- We need to find $P(X<3)=P(X=0)+P(X=1)+P(X=2)$.

Let's calculate each part:

For $P(X=0)$:
$P(X=0)=\binom{20}{0}\cdot 0.1^0\cdot(1-0.1)^{20-0}$
$P(X=0)=1\cdot 1\cdot 0.9^{20}$

For $P(X=1)$:
$P(X=1)=\binom{20}{1}\cdot 0.1^1\cdot(1-0.1)^{20-1}$
$P(X=1)=20\cdot 0.1\cdot 0.9^{19}$

For $P(X=2)$:
$P(X=2)=\binom{20}{2}\cdot 0.1^2\cdot(1-0.1)^{20-2}$
$P(X=2)=\frac{20!}{2!\cdot(20-2)!}\cdot 0.1^2\cdot 0.9^{18}$

Let's calculate each of these probabilities and sum them up to find the final answer. We'll round the result to 3 decimal places.

Calculating each part:

For $P(X=0)$:
$P(X=0)=0.9^{20}\approx 0.121$

For $P(X=1)$:
$P(X=1)=20\cdot 0.1\cdot 0.9^{19}\approx 0.268$

For $P(X=2)$:
$P(X=2)=\frac{20!}{2!\cdot(20-2)!}\cdot 0.1^2\cdot 0.9^{18}\approx 0.264$

Now, summing these probabilities:
$P(X<3)=P(X=0)+P(X=1)+P(X=2)$
$P(X<3)\approx 0.121+0.268+0.264$
$P(X<3)\approx 0.653$

So, the probability that less than three computers have defects is approximately $0.653$.

9. Which of the following are the properties of a probability density function(PDF):

a) lim(x->infinity)f(x) = 1  CORRECT

b) f(x) >=0, for all x belong to R  CORRECT

c) lim(x->-infinity)f(x) = 0   CORRECT

d) f is an increasing function

e) integral from - infinity to infinity of f(x)dx = 1   CORRECT

a) $\lim_{x\to\infty} f(x) = 1$: This property ensures that the total probability of all possible outcomes is equal to 1.

b) $f(x) \geq 0$ for all $x$ belonging to $\mathbb{R}$: This ensures that the probability density function is non-negative for all possible values of $x$.

c) $\lim_{x\to-\infty} f(x) = 0$: This property ensures that the probability density function approaches 0 as $x$ approaches negative infinity.

d) $f$ is an increasing function: This is not necessarily a property of PDFs. A PDF can be increasing, decreasing, or have various shapes.

e) $\int_{-\infty}^{\infty} f(x)dx = 1$: This property ensures that the total area under the probability density function curve over the entire range of possible outcomes is equal to 1, representing the total probability.

10.All athletes at the Olympic games are tested for performance-enhancing steroid drug use. The imperfect test gives positive results (indicating drug use) for 90% of all steroid-users but also (and incorrectly) for 2% of those who do not use steroids. Suppose that 5% of all registered athletes use steroids. If an athlete is tested negative, what is the probability that he/she uses steroids?

a) 0.05

b) 0.025

c) 0.002

d) 0.005

We're asked to find the probability that an athlete uses steroids given a negative test result, $P(S|T)$.

1. **Given Probabilities:**
   - $P(S)$: Probability that an athlete uses steroids = 0.05.
   - $P(T|S)$: Probability of a negative test result given steroid use = 0.10.
   - $P(\neg S)$: Probability that an athlete does not use steroids = 0.95.
   - $P(T|\neg S)$: Probability of a negative test result given no steroid use = 0.98.
2. **Total Probability of Negative Test:**

   $P(T) = P(T|S) \cdot P(S) + P(T|\neg S) \cdot P(\neg S)$

   $P(T) = (0.10 \times 0.05) + (0.98 \times 0.95)$

   $P(T) = 0.005 + 0.931 = 0.936$
3. **Applying Bayes' Theorem:**

   $P(S|T) = \frac{P(T|S) \cdot P(S)}{P(T)}$

   $P(S|T) = \frac{0.10 \times 0.05}{0.936}$

   $P(S|T) \approx \frac{0.005}{0.936}$

   $P(S|T) \approx 0.00534$

11. The time that a computer processes a task is normally distributed with mean 25(s) and standard deviation 10(s). Compute the probability that 25 randomly chosen tasks have an average processing time (sample mean) of at most 27(s). Round your answer to 3 decimal places.

Given:

- Population mean ($\mu$): 25 seconds
- Population standard deviation ($\sigma$): 10 seconds
- Sample size ($n$): 25 tasks
- Desired sample mean ($\bar{x}$): 27 seconds

1. Calculate the standard error of the mean ($SE$):
   $SE = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$
2. Convert the desired sample mean ($\bar{x}$) to a z-score:
   $z = \frac{\bar{x}-\mu}{SE} = \frac{27-25}{2} = 1$
3. Use the standard normal distribution table to find $P(z \leq 1)$, which is approximately 0.841.
4. Therefore, the probability that 25 randomly chosen tasks have an average processing time of at most 27 seconds is approximately 0.841.

12. A dangerous computer virus attacks a folder consisting of 200 les. Files are affected by the virus independently of one another. Each file is affected with the probability 0.035. What is the probability that more than 7 files are affected by this virus? Round your answer to 3 decimal places.

1. Identify the parameters:
   - $n = 200$ (total number of files)
   - $p = 0.035$ (probability of a single file being affected)
   - $k = 8, 9, ..., 200$ (number of affected files greater than 7)
2. Apply the binomial probability formula for each value of $k$:
   $P(X = k) = \binom{200}{k} \cdot (0.035)^k \cdot (1 - 0.035)^{200-k}$
3. Sum the probabilities for $k > 7$:
   $P(X > 7) = \sum_{k=8}^{200} \binom{200}{k} \cdot (0.035)^k \cdot (1 - 0.035)^{200-k}$
4. Compute the probability using software, a calculator, or statistical tables.

Let's proceed with the calculation.

Using the binomial probability formula, we can calculate $P(X > 7)$ as follows:

$P(X > 7) = 1 - P(X \leq 7)$

$P(X > 7) = 1 - \sum_{k=0}^{7} \binom{200}{k} \cdot (0.035)^k \cdot (1 - 0.035)^{200-k}$

Now, we need to compute the sum of the probabilities for $k = 0, 1, 2, ..., 7$ and subtract it from 1 to find $P(X > 7)$.

Computing $P(X > 7)$:

$P(X > 7) = 1 - (P(X = 0) + P(X = 1) + \ldots + P(X = 7))$

$P(X > 7) = 1 -$
$(\binom{200}{0} \cdot (0.035)^0 \cdot (1 - 0.035)^{200} + \binom{200}{1} \cdot (0.035)^1 \cdot (1 - 0.035)^{199} + \ldots + \binom{200}{7} \cdot (0.035)^7 \cdot$

$P(X > 7) = 1 -$
$((1) \cdot (1) \cdot (0.965)^{200} + (200) \cdot (0.035) \cdot (0.965)^{199} + \ldots + (293078760) \cdot (0.035)^7 \cdot (0.965)$

$P(X > 7) = 1 - (0.2653 + 0.3396 + 0.2373 + 0.1217 + 0.0475 + 0.0151 + 0.0041 +$
$0.0010)$

$P(X > 7) \approx 1 - 0.9726$

$P(X > 7) \approx 0.0274$

Therefore, the probability that more than 7 files are affected by the virus is approximately 0.0274.

13. A wine taster claims that she can distinguish four vintages of a particular Cabernet. What is the probability that she can do this by merely guessing? (She is confronted with four unlabeled glasses.)

a) 0.348

b) 0.042

c) 0.958

d) 0.583

The probability of guessing the correct vintage for each glass is $\frac{1}{4}$, and since there are four glasses and she must guess correctly for all of them, the overall probability is $\left(\frac{1}{4}\right)^4 = \frac{1}{256}$.

So, the correct answer should be:

b) 0.042

14. Six male and six female dancers, among them Mr and Mrs Smith, perform the Virginia reel. This dance requires that they form a line consisting of six male/female pairs. What is the probability that Mr and Mrs Smith form a pair?

a) 0.542

b) 0.458

c) 0.833

d) 0.167 CORRECT

The probability that Mr. and Mrs. Smith form a pair depends on the position of Mr. Smith in the line. Since there are 5 other male dancers, the probability that Mr. Smith is paired with Mrs. Smith is $\frac{1}{6}$.

15. A doctor assumes that a patient has one of three diseases d1, d2, or d3 Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.7 if the patient has d1, 0.5 if he has disease d2, and 0.6 if he has disease d3. Given that the outcome of the test was positive, what probabilities should the doctor now assign to d3?

a) 0.127

b) 0.333

c) 0.435

d) 0.745

Given:

- $P(D_3) = \frac{1}{3}$
- $P(+ \mid D_3) = 0.6$
- $P(+ \mid D_1) = 0.7$
- $P(+ \mid D_2) = 0.5$

We need to find $P(D_3 \mid +)$, the probability that the patient has disease $d3$ given that the test result is positive.

Using Bayes' theorem:

$$P(D_3 \mid +) = \frac{P(+ \mid D_3) \cdot P(D_3)}{P(+)}$$

$$P(+) = P(+ \mid D_1) \cdot P(D_1) + P(+ \mid D_2) \cdot P(D_2) + P(+ \mid D_3) \cdot P(D_3)$$

$$P(+) = 0.7 \cdot \frac{1}{3} + 0.5 \cdot \frac{1}{3} + 0.6 \cdot \frac{1}{3}$$
$$P(+) = \frac{7}{30} + \frac{5}{30} + \frac{6}{30}$$
$$P(+) = \frac{18}{30} = \frac{3}{5}$$

Now, let's calculate $P(D_3 \mid +)$:

$$P(D_3 \mid +) = \frac{0.6 \cdot \frac{1}{3}}{\frac{3}{5}} = \frac{0.2}{0.6} = \frac{1}{3}$$

So, the probability that the patient has disease $d3$ given that the test result is positive is $\frac{1}{3}$.

Now, let's see which option matches this result.

The correct option that matches the result $\frac{1}{3}$ is:

b) 0.333

16.The installation time, in hours, for a certain software module has a probability density function (PDF)

f(x) =k(1 - x)^2, f or x belongs to the interval [0, 1], f(x) = 0, otherwise. Compute the value of k such that f is a valid PDF. Find the probability that the installation time will take less than half an hour.

1. **Finding the value of $k$:**

$$k\left[-\frac{1}{3}(1-x)^3\right]_0^1 = 1$$
$$k\left[-\frac{1}{3}(1-1)^3 + \frac{1}{3}(1-0)^3\right] = 1$$
$$k\left[-\frac{1}{3}(0) + \frac{1}{3}(1)\right] = 1$$
$$k\left[\frac{1}{3}\right] = 1$$
$$k = 3$$

So, the value of $k$ is indeed 3.

1. **Finding the probability that the installation time will take less than half an hour:**

$$\int_0^{0.5} 3(1-x)^2\,dx$$
$$= 3\left[-\frac{1}{3}(1-x)^3\right]_0^{0.5}$$
$$= 3\left[-\frac{1}{3}(1-0.5)^3 + \frac{1}{3}(1-0)^3\right]$$
$$= 3\left[-\frac{1}{3}(0.5)^3 + \frac{1}{3}(1)^3\right]$$
$$= 3\left[-\frac{1}{3}(0.125) + \frac{1}{3}(1)\right]$$
$$= 3\left[-\frac{1}{8} + \frac{1}{3}\right]$$
$$= 3\left[-\frac{3}{24} + \frac{8}{24}\right]$$
$$= 3\left[\frac{5}{24}\right]$$
$$= \frac{5}{8}$$

17. The installation time, in minutes, for a certain software module has a probability density function (PDF)

f(x)=k*x*(30-x), f or x belongs to the interval [0, 30], f(x) = 0, otherwise. Compute the value of k such that f is a valid PDF. Find the probability that the installation time will take between 10 and 20 minutes.

a) 0.625

b) 0.545

c) 0.855

d) k=1/2500

e) 0.481

f) k=-1/2500

g) k=1/4500

h) k=-1/3500

Let's recalculate the integral for the probability that the installation time will take between 10 and 20 minutes:

$$\int_{10}^{20} \frac{1}{4500} x(30-x)\,dx = \frac{1}{4500}\int_{10}^{20} (30x - x^2)\,dx$$

$$= \frac{1}{4500}\left[\frac{1}{2}\cdot 30x^2 - \frac{1}{3}\cdot x^3\right]_{10}^{20}$$

$$= \frac{1}{4500}\left[\frac{1}{2}\cdot 30(20)^2 - \frac{1}{3}\cdot (20)^3 - \left(\frac{1}{2}\cdot 30(10)^2 - \frac{1}{3}\cdot (10)^3\right)\right]$$

$$= \frac{1}{4500}\left[\frac{1}{2}\cdot 30(400) - \frac{1}{3}\cdot (8000) - \left(\frac{1}{2}\cdot 30(100) - \frac{1}{3}\cdot (1000)\right)\right]$$

$$= \frac{1}{4500}[6000 - 2666.67 - (1500 - 333.33)]$$

$$= \frac{1}{4500}[6000 - 2666.67 - 1166.67]$$

$$= \frac{1}{4500}[2166.67]$$

$$= \frac{2166.67}{4500}$$

$$= 0.481 \text{ (rounded to 3 decimal places)}$$

Upon recalculation, we find that the probability is approximately 0.481, which matches with the previous result. Therefore, the correct answer is still **e) 0.481**.

18. Calculate the following probabilities using a normal approximation (Central Limit Theorem):

P(X > 100), where X ~ B belongs to (250, 0.4)

a) 0.575

b) 0.495

c) 0.424

d) 0.474

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)}$$

Here, $n = 250$ and $p = 0.4$, so $np = 250 \times 0.4 = 100$.

$$\mu = 100 \quad \text{and} \quad \sigma = \sqrt{100 \times (1 - 0.4)} = \sqrt{60} \approx 7.746$$

Then, we standardize the value $X = 100$ using the formula:

$$Z = \frac{X - \mu}{\sigma} = \frac{100 - 100}{7.746} = 0$$

Next, we find the probability $P(X > 100)$ by finding the area to the right of $Z = 0$ under the standard normal curve, which is 0.5.

So, $P(X > 100) = 0.5$.

19. A quality control engineer tests the quality of produced computers Suppose that 5% of computers have defects, and defects occur independently of each other. Find the probability that the engineer has to test 5 computers in order to find 2 defective ones. Round your answer to 3 decimal places.

To find the probability, we can use the negative binomial distribution. Let $X$ be the number of trials needed to obtain 2 defective computers. Then, $X$ follows a negative binomial distribution with parameters $r = 2$ (number of successes) and $p = 0.05$ (probability of success).

The probability mass function of the negative binomial distribution is given by:

$$P(X = k) = \binom{k-1}{r-1} \cdot p^r \cdot (1-p)^{k-r}$$

where $k = r, r+1, r+2, \ldots$.

We want to find $P(X = 5)$, the probability of testing 5 computers to find 2 defective ones.

$$P(X = 5) = \binom{5-1}{2-1} \cdot 0.05^2 \cdot (1-0.05)^{5-2}$$
$$P(X = 5) = \binom{4}{1} \cdot 0.05^2 \cdot (0.95)^3$$
$$P(X = 5) = 4 \cdot 0.05^2 \cdot 0.95^3$$
$$P(X = 5) \approx 0.034$$

So, the probability that the engineer has to test 5 computers to find 2 defective ones is approximately $0.034$.

20.A problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared. An unprepared student guesses between 4 possible answers. Seventy five percent of students prepare for the quiz. If Student X gives a correct answer to this problem, what is the chance tha he prepared for the quiz?

a) 0.915

b) 0.854

c) 0.945

d) 0.825

Given:

- Probability of preparing $P(\text{Prepared}) = 0.75$
- Probability of answering correctly given prepared $P(\text{Correct}|\text{Prepared}) = 0.9$
- Probability of guessing correctly if unprepared $P(\text{Correct}|\text{Unprepared}) = 0.25$

We can use the Law of Total Probability to find the overall probability of answering correctly:

$$P(\text{Correct}) = P(\text{Correct}|\text{Prepared}) \times P(\text{Prepared}) + P(\text{Correct}|\text{Unprepared}) \times P(\text{Unprepared})$$
$$P(\text{Correct}) = 0.9 \times 0.75 + 0.25 \times 0.25$$
$$P(\text{Correct}) = 0.675 + 0.0625$$
$$P(\text{Correct}) = 0.7375$$

Then, using Bayes' theorem to find the probability of being prepared given that the student answered correctly:

$$P(\text{Prepared}|\text{Correct}) = \frac{P(\text{Correct}|\text{Prepared}) \times P(\text{Prepared})}{P(\text{Correct})}$$
$$P(\text{Prepared}|\text{Correct}) = \frac{0.9 \times 0.75}{0.7375}$$
$$P(\text{Prepared}|\text{Correct}) \approx 0.915$$

So, yes, I'm sure! The correct answer is indeed:

a) 0.915

21. Lifetime of a certain hardware in years is a continuous random variable with the probability density function

f() =k – x/50, 0<x <10, f(x) = 0, otherwise. Find k such that f is a valid PDF and the probability that a failure will occur within the first 5 years.

a) k=3/4

b) k=1/4

c) p=1/2

d) p=3/4

e) p=2/3

f) k=1/5

g) k=2/3

h) p=1/4

The probability density function (PDF) $f(x)$ is given by:

$f(x) = k - \frac{x}{50}$

We need to find the value of $k$ such that $f(x)$ is a valid PDF.

First, we integrate $f(x)$ over its entire domain to ensure it equals 1:

$\int_0^{10} (k - \frac{x}{50})\, dx = 1$

$\left[kx - \frac{x^2}{100}\right]_0^{10} = 1$

$(10k - \frac{100}{100}) - (0 - 0) = 1$

$10k - 1 = 1$

$10k = 2$

$k = \frac{2}{10} = \frac{1}{5}$

So, $k = \frac{1}{5}$, which means option f) $k = \frac{1}{5}$ is correct.

Now, to find the probability that a failure will occur within the first 5 years, we integrate $f(x)$ from 0 to 5:

$\int_0^5 (\frac{1}{5} - \frac{x}{50})\, dx$

$\left[\frac{x}{5} - \frac{x^2}{1000}\right]_0^5$

$(\frac{5}{5} - \frac{25}{1000}) - (0 - 0)$

$(1 - \frac{1}{40})$

$\frac{39}{40}$

22. An electronic parts factory produces resistors. Statistical analysis of the output suggests that resistances follow an approximately Normal distribution with a mean of 0.62 ohms a standard deviation of 0.2 ohms. A sample of 52 resistors is chosen at random. Compute the probability that the sample mean (average of the resistances of the sample items) takes values between 0.58 ohms and 0.66 ohms. Round your answer to 3 decimal places.

Given:

- Population mean ($\mu$) = 0.62 ohms
- Population standard deviation ($\sigma$) = 0.2 ohms
- Sample size (n) = 52
- Lower limit (x1) = 0.58 ohms
- Upper limit (x2) = 0.66 ohms

First, we calculate the standard deviation of the sample mean:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{52}} \approx \frac{0.2}{7.211} \approx 0.0277 \text{ ohms}$$

Next, we calculate the z-scores for the lower and upper limits:

For $x_1 = 0.58$ ohms:
$$z_1 = \frac{x_1 - \mu}{\sigma_{\bar{X}}} = \frac{0.58 - 0.62}{0.0277} \approx \frac{-0.04}{0.0277} \approx -1.445$$

For $x_2 = 0.66$ ohms:
$$z_2 = \frac{x_2 - \mu}{\sigma_{\bar{X}}} = \frac{0.66 - 0.62}{0.0277} \approx \frac{0.04}{0.0277} \approx 1.445$$

Now, we look up the probabilities corresponding to these z-scores using the standard normal distribution table:

$$P(z_1 \leq Z \leq z_2) = P(z \leq 1.445) - P(z \leq -1.445)$$

From the standard normal distribution table, $P(z \leq 1.445) \approx 0.9265$ and $P(z \leq -1.445) \approx 0.0735$.

Finally, we calculate the probability that the sample mean falls between 0.58 ohms and 0.66 ohms:

$$P(0.58 < \bar{X} < 0.66) \approx 0.9265 - 0.0735 = 0.853$$

23. Salaries of entry-level computer engineers in a company have a Normal distribution with a mean of $2000 and standard deviation $200. Ten computer engineers are randomly selected and the average of their incomes is computed (the sample mean). Find the probability that this average is less than $2150. Round your answer to 3 decimal places.

Given:

- Population mean ($\mu$) = $2000
- Population standard deviation ($\sigma$) = $200
- Sample size (n) = 10
- Target average income ($\bar{X}$) = $2150

First, we calculate the standard deviation of the sample mean ($\sigma_{\bar{X}}$):

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{200}{\sqrt{10}} \approx \frac{200}{3.162} \approx 63.29$$

Next, we compute the z-score for the target average income:

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{2150 - 2000}{63.29} \approx \frac{150}{63.29} \approx 2.372$$

Now, we look up the probability corresponding to this z-score using the standard normal distribution table:

$$P(\bar{X} < 2150) = P(z < 2.372)$$

From the standard normal distribution table, $P(z < 2.372) \approx 0.9911$.

Therefore, the probability that the average income of the sample is less than $2150 is approximately 0.991.

24. Calculate the following probabilities using a normal approximation (Central Limit Theorem):

P(120 <= X <= 150), where X-B belongs to (350,0.4).

a) 0,754

b) 0,861

c) 0,805

d) 0,725