

Theoretical Computer Science Cheat Sheet					
$\pi \approx 3.14159,$		$e \approx 2.71828,$	$\gamma \approx 0.57721,$	$\phi = \frac{1+\sqrt{5}}{2} \approx 1.61803,$	$\hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -.61803$
i	2^i	p_i	General	Probability	
1	2	2	Bernoulli Numbers ($B_i = 0$, odd $i \neq 1$):	Continuous distributions: If	
2	4	3	$B_0 = 1, B_1 = -\frac{1}{2}, B_2 = \frac{1}{6}, B_4 = -\frac{1}{30},$	$\Pr[a < X < b] = \int_a^b p(x) dx,$	
3	8	5	$B_6 = \frac{1}{42}, B_8 = -\frac{1}{30}, B_{10} = \frac{5}{66}.$	then p is the probability density function of X . If	
4	16	7	Change of base, quadratic formula:	$\Pr[X < a] = P(a),$	
5	32	11	$\log_b x = \frac{\log_a x}{\log_a b}, \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$	then P is the distribution function of X . If P and p both exist then	
6	64	13	Euler's number e :	$P(a) = \int_{-\infty}^a p(x) dx.$	
7	128	17	$e = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \cdots$	Expectation: If X is discrete	
8	256	19	$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x.$	$E[g(X)] = \sum_x g(x) \Pr[X = x].$	
9	512	23	$\left(1 + \frac{1}{n}\right)^n < e < \left(1 + \frac{1}{n}\right)^{n+1}.$	If X continuous then	
10	1,024	29	$\left(1 + \frac{1}{n}\right)^n = e - \frac{e}{2n} + \frac{11e}{24n^2} - O\left(\frac{1}{n^3}\right).$	$E[g(X)] = \int_{-\infty}^{\infty} g(x)p(x) dx = \int_{-\infty}^{\infty} g(x) dP(x).$	
11	2,048	31	Harmonic numbers:	Variance, standard deviation:	
12	4,096	37	$1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}, \frac{7129}{2520}, \dots$	$\text{VAR}[X] = E[X^2] - E[X]^2,$	
13	8,192	41	$\ln n < H_n < \ln n + 1,$	$\sigma = \sqrt{\text{VAR}[X]}.$	
14	16,384	43	$H_n = \ln n + \gamma + O\left(\frac{1}{n}\right).$	For events A and B :	
15	32,768	47	Factorial, Stirling's approximation:	$\Pr[A \vee B] = \Pr[A] + \Pr[B] - \Pr[A \wedge B]$	
16	65,536	53	$1, 2, 6, 24, 120, 720, 5040, 40320, 362880, \dots$	$\Pr[A \wedge B] = \Pr[A] \cdot \Pr[B],$	
17	131,072	59	$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right).$	iff A and B are independent.	
18	262,144	61	Ackermann's function and inverse:	$\Pr[A B] = \frac{\Pr[A \wedge B]}{\Pr[B]}$	
19	524,288	67	$a(i, j) = \begin{cases} 2^j & i = 1 \\ a(i-1, 2) & j = 1 \\ a(i-1, a(i, j-1)) & i, j \geq 2 \end{cases}$	For random variables X and Y :	
20	1,048,576	71	$\alpha(i) = \min\{j \mid a(j, j) \geq i\}.$	$E[X \cdot Y] = E[X] \cdot E[Y],$	
21	2,097,152	73		if X and Y are independent.	
22	4,194,304	79		$E[X + Y] = E[X] + E[Y],$	
23	8,388,608	83		$E[cX] = c E[X].$	
24	16,777,216	89	Binomial distribution:	Bayes' theorem:	
25	33,554,432	97	$\Pr[X = k] = \binom{n}{k} p^k q^{n-k}, \quad q = 1 - p,$	$\Pr[A_i B] = \frac{\Pr[B A_i] \Pr[A_i]}{\sum_{j=1}^n \Pr[A_j] \Pr[B A_j]}.$	
26	67,108,864	101	$E[X] = \sum_{k=1}^n k \binom{n}{k} p^k q^{n-k} = np.$	Inclusion-exclusion:	
27	134,217,728	103	Poisson distribution:	$\Pr\left[\bigvee_{i=1}^n X_i\right] = \sum_{i=1}^n \Pr[X_i] +$	
28	268,435,456	107	$\Pr[X = k] = \frac{e^{-\lambda} \lambda^k}{k!}, \quad E[X] = \lambda.$	$\sum_{k=2}^n (-1)^{k+1} \sum_{i_1 < \dots < i_k} \Pr\left[\bigwedge_{j=1}^k X_{i_j}\right].$	
29	536,870,912	109	Normal (Gaussian) distribution:	Moment inequalities:	
30	1,073,741,824	113	$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad E[X] = \mu.$	$\Pr[X \geq \lambda E[X]] \leq \frac{1}{\lambda},$	
31	2,147,483,648	127	The "coupon collector": We are given a random coupon each day, and there are n different types of coupons. The distribution of coupons is uniform. The expected number of days to pass before we to collect all n types is	$\Pr[X - E[X] \geq \lambda \cdot \sigma] \leq \frac{1}{\lambda^2}.$	
32	4,294,967,296	131	$nH_n.$	Geometric distribution:	
Pascal's Triangle				$\Pr[X = k] = pq^{k-1}, \quad q = 1 - p,$	
1				$E[X] = \sum_{k=1}^{\infty} kpq^{k-1} = \frac{1}{p}.$	
1 1					
1 2 1					
1 3 3 1					
1 4 6 4 1					
1 5 10 10 5 1					
1 6 15 20 15 6 1					
1 7 21 35 35 21 7 1					
1 8 28 56 70 56 28 8 1					
1 9 36 84 126 126 84 36 9 1					
1 10 45 120 210 252 210 120 45 10 1					