University of Western Attica
Faculty of Engineering
Department of Informatics and Computer Engineering

Circuit Theory Lab Exercises

2nd EXERCISE

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Material revision, Editor 2021, Christos Kampouris

ATHENS
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Task 2 h – E RLC fittings , transient response

1.1 Theoretical part

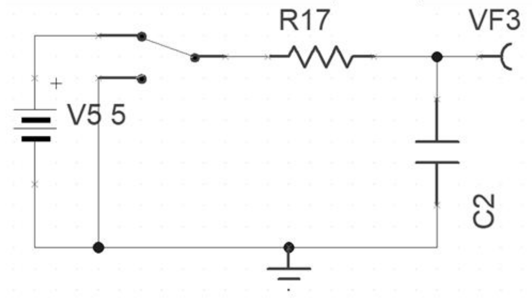
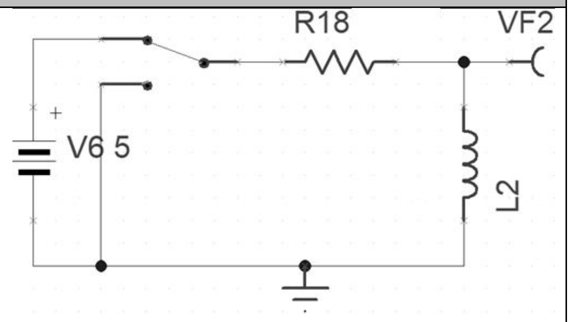
RLC circuits , we can have a complete picture of their behavior in the time domain, when we study **their time response** .

(note. there are of course other responses such as in the frequency field, or for specific inputs, or through transformations)

The total circuit response results from the sum (due to superposition) of the initial conditions and the circuit excitation)

	COMPLETE RESPONSE	
	↙ ↘	
TRANSIENT RESPONSE		FORCED RESPONSE
ZERO INPUT RESPONSE		ZERO STATE RESPONSE
NATURAL RESPONSE		ZERO STATE RESPONSE
Transient phenomena		Forced response
There are no sources or other stimuli		There are only excitements
There is only stored energy		Everything is "unloaded" with no warehouse. energy

In the lab we will study the transient response, but to achieve this we will first simulate a forced response to charge our capacitors in initial conditions etc. When we connect a circuit containing a capacitor and supply it with a constant voltage, then after it is fully charged (at its maximum capacity) it then behaves as an open circuit, i.e. it behaves like an open switch. Conversely, the coil behaves like a short circuit. Let's observe what happens when the circuit closes in each of the following circuits:

<table>
<tr><th>RC in series</th><th>RL in series</th></tr>
<tr><td>
</td><td>
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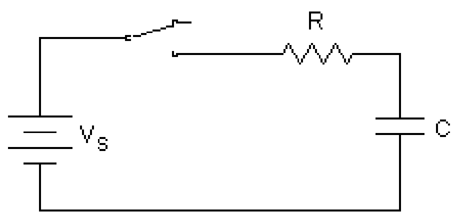
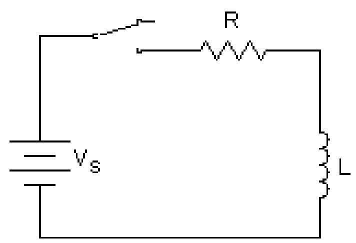
<tr><td>

Circuit 1a) in simplified form</td><td>

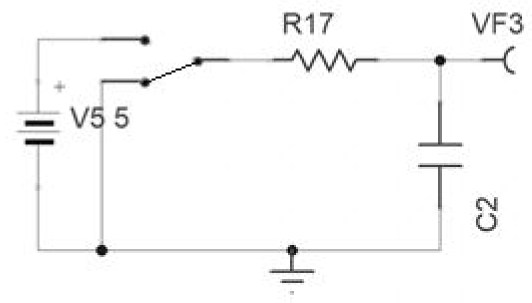
Circuit 1b) in simplified form</td></tr>
</table>

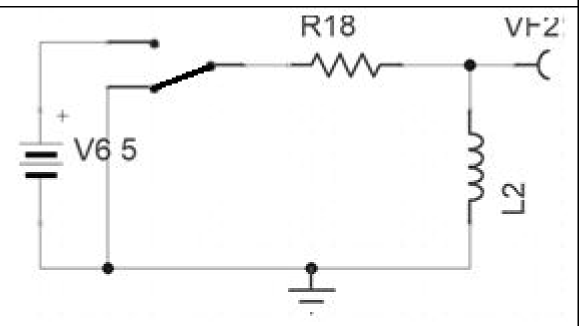
RC circuit study

Initial thoughts: the capacitor will be exposed to a potential difference, and will start to leak current through the resistor. Therefore it will start charging. For circuit 1a we have:

- Kirchoff $V_s - V_r - V_c = 0$'s law
- The voltage drop across the resistor and capacitor are given by the relations $\begin{cases} V_R(t) = RI(t) \\ V_c(t) = \frac{1}{C}\int I(t)dt \end{cases}$
- Combining the above relations we have: $V_s = RI(t) + \frac{1}{C}\int I(t)dt$. Differentiating both legs with respect to time we have
$$\frac{dI(t)}{dt} + \frac{1}{RC}I(t) = 0$$
- Solving the differential equation assuming that the capacitor charges through the source $V_s = V$ present in the circuit, we get as solutions for voltage and current:

$$
\begin{cases}
I(t) = \dfrac{V - V_c(0)}{R} e^{-\frac{1}{RC}t} \\
V_R(t) = V e^{-\frac{1}{RC}t} \\
V_c(t) = V\left(1 - e^{-\frac{1}{RC}t}\right), \begin{cases} \gamma\iota\alpha \ t \to 0, e^{-\frac{0}{RC}} = 1 \to V_c = 0 \\ \gamma\iota\alpha \ t \to RC, e^{-\frac{RC}{RC}} = e^{-1} \to V_c = 0.632V \\ \gamma\iota\alpha \ t \to 5RC, e^{-\frac{5RC}{RC}} = e^{-5} \to V_c = 0.993V \\ \gamma\iota\alpha \ t \to \infty, e^{-\frac{\infty}{RC}} = 0 \to V_c = V \end{cases}
\end{cases}
$$

<table>
<tr><td colspan="2">B. stage.
Disconnecting from the source and connecting the active component to ground through the resistor</td></tr>
<tr><td>

Circuit 1a)</td><td>

Circuit 1 b)</td></tr>
</table>

- Second stage: the capacitor will have stored energy which it will channel into the circuit through the resistance. Therefore it will start to discharge. For the circuit 1a we have: Solving the differential equation considering that the capacitor is charged ($V_c(0) = -V_s = -V$) and the circuit has removed the power source we get as solutions for voltage and current:

$$
\begin{cases}
I(t) = -\dfrac{V}{R} e^{-\frac{1}{RC}t} \\
V_R(t) = -V e^{-\frac{1}{RC}t} \\
V_c(t) = V e^{-\frac{1}{RC}t}, \begin{cases} \gamma\iota\alpha \ t \to 0, e^{-\frac{0}{RC}} = 1 \to V_c = V \\ \gamma\iota\alpha \ t \to RC, e^{-\frac{RC}{RC}} = e^{-1} \to V_c = 0.368V \\ \gamma\iota\alpha \ t \to 5RC, e^{-\frac{5RC}{RC}} = e^{-5} \to V_c = 0.007V \\ \gamma\iota\alpha \ t \to \infty, e^{-\frac{\infty}{RC}} = 0 \to V_c = 0 \end{cases}
\end{cases}
$$

So practically, the capacitor charges and discharges after a time equal to 5 RC.

The constant τ = RC is called the time constant of the circuit

Note: The voltage at the corresponding time constant, during charging and discharging cumulatively, is equal to the source voltage, i.e. to the total potential at the ends of the capacitor.

	1*RC	**2*RC**	**3*RC**	**4*RC**	**5*RC**
Charging	63.2 %V_{in}	86.5% V_{in}	95% V_{in}	98% V_{in}	99.3% V_{in}
Discharge	36.8% V_c	13.5% V_c	5% V_c	2% V_c	0.7%V_c
V_{c_ch} + V_{c_dis}	100% V_p	100% V_p	100% V_p	100% V_p	100% V_p

RL circuit study

For circuit 1b respectively we have:

- Kirchoff $V_s - V_r - V_L = 0$'s law
- The voltage drop across the resistor and the coil are given by the relations $\begin{cases} V_R(t) = RI(t) \\ V_L(t) = L\frac{I(t)}{dt} \end{cases}$
- Combining the above relations we have: $V_s = RI(t) + L\frac{I(t)}{dt}$. Differentiating both legs with respect to time we have
$$\frac{dI(t)}{dt} + \frac{R}{L}I(t) = \frac{V}{L}$$
- Solving the differential equation considering that the switch is initially open and at time t_0=0 the switch closes therefore the circuit flows from the source current:

$$\begin{cases} I(t) = \dfrac{V}{R}(1 - e^{-\frac{R}{L}t}), \begin{cases} \gamma\iota\alpha\ t \to \infty, e^{-\frac{R}{L}\infty} = 0 \to I = \dfrac{V}{R} \\ \gamma\iota\alpha\ t \to 0, e^{-\frac{R}{L}0} = e^0 \to I = 0A \\ \gamma\iota\alpha\ t \to \dfrac{L}{R}, e^{-\frac{RL}{RL}} = e^{-1} \to I = 0.632\dfrac{V}{R} \end{cases} \\ V_R(t) = V\left(1 - e^{-\frac{R}{L}t}\right) \\ V_L(t) = Ve^{-\frac{R}{L}t} \end{cases}$$

- Solving the differential equation considering that the commutator is initially turned to the source and at time t_0 =0 the commutator is switched to earth, therefore the value of the current intensity changes and an inductance voltage appears in the coil:

$$
\begin{cases}
I(t) = \frac{V}{R} e^{-\frac{R}{L}t}, \begin{cases} \gamma\iota\alpha\ t \to \infty, e^{-\frac{R}{L}\infty} = 0 \to I = \ 0A \\ \gamma\iota\alpha\ t \to 0, e^{-\frac{R}{L}0} = e^{0} \to I = \frac{V}{R} \\ \gamma\iota\alpha\ t \to \frac{L}{R}, e^{-\frac{RL}{RL}} = e^{-1} \to I = 0.368\frac{V}{R} \end{cases} \\
V_R(t) = Ve^{-\frac{R}{L}t} \\
V_L(t) = \ -Ve^{-\frac{R}{L}t}
\end{cases}
$$

Practically, therefore, the intensity of the current reaches its maximum value and correspondingly goes to zero, after a time t =5 L / R .

The constant τ= L / R is called the time constant of the circuit

Note: The sum of the coil currents in the same study time (e.g. at 3m), when switching the switch to both source and ground, equals the total current of the V/R circuit.

	1*L/R	2* L/R	3* L/R	4* L/R	5* L/R
Cl. Dik.	63.2 %I max	86.5% I max	95% I max	98% I max	99.3% I max
Ad.	36.8% I max	13.5% I max	5% I max	2% I max	0.7% I max
I openSw +I closedSw	100% I max	100% I max	100% I max	100% I max	100% I max

1.2 Laboratory part

V square pulse voltage source and the A and B channels of the oscilloscope to the input and output of each circuit. Record your measurements in the tables below for the transition from 0 V →12 V . Then repeat the measurements for the transition from 12 V →0 V . Finally you displayed the data graphically, separately but also superimposed. Comment your measurements.

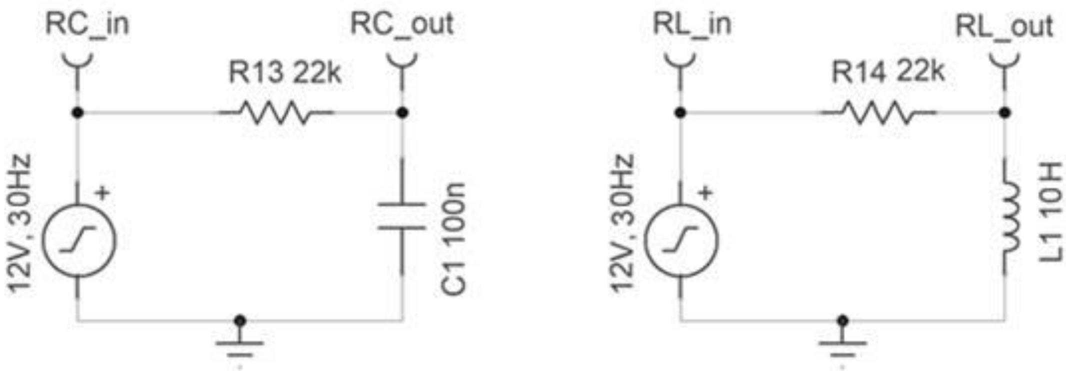


Figure 6

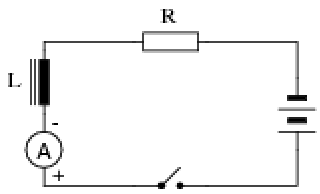
V = 12 V				**Source frequency 3 0H z**		
RC				**time constant= ?**		
R (Ω)	**τ=RC** **Sec**	**V C (1t)** **Volts**	**V C (2t)** **Volts**	**V C (3t)** **Volts**	**V C (4m)** **Volts**	**V C (5m)** **Volts**
1k						
4k7						
12k						
22k						

V = 12 V	**Source frequency 3 0H z**
RL	**time constant= ?**

R (Ω)	τ=L/R	V_L (1 m)	V_L (2m)	V_L (3m)	V_L (4m)	V_L (5m)
3k3						
4k7						
12k						
22k						

1.3 Questions

- When an engineer needs a circuit to provide a time delay, he almost always chooses an RC circuit over an RL circuit . Explain why.
- Describe the maximum value of the current, as well as what will be observed in the current when the switch is closed in the circuit below:



- What value of resistance is required in an RC circuit with a capacitor value of 50μF in order to have a time delay of one second?