A Differential Privacy Encrypted Communication Method Based on Transmit Power Allocation

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In the field of machine learning, a new concept called differential privacy was introduced in recent years. By adding a specific distribution of noise to the data, the deterministic values of the data will be indistinguishable. Since the distribution characteristics of the noise are known and regular, the statistical characteristics of the original data, such as mean and variance, can still be estimated. This concept can be applied to wireless communication as well. In some wireless communication scenarios, users want their data to be confidential to the endpoint, while the endpoint requires access to the user's overall statistics for further calculation. In this study, we found that, by controlling the transmit power in a communication system, the data error caused by the channel and additive Gaussian white noise satisfies a specific distribution. Thus, the received data can be regarded as the sum of the original data and a specific distributed noise.

Typically, a critical evaluation metric in a wireless communication system is the bit error rate, or in other words, bit error probability. Usually, we define the bit error probability as P_e, and it is the consequence of the effects of the channel and additive Gaussian white noise. To be more specific, in a BPSK system where transmitted symbols are "a" and "-a", if we transmit a symbol "a" through a wireless channel, the receiving detector has the probability P_e to mistake it for the opposite symbol "-a".

Assume that a decimal digit is transmitted in a packet of 4 bits, i.e., one number is represented by four bits $\{x_4, x_3, x_2, x_1\}$ and the range of the digit is [0,15]. For example, If the transmitted number is 10, the actual bit stream transmitted by the system is 1010, and if the transmitted number is 0, the system will transmit 0000 instead of a single 0.

Define data error as $\left|Data_{tx} - Data_{rx}\right|$, the purpose of our work is to study the distribution of data errors and see if there is any possibility of controlling the error distribution manually. Since the digits are regenerated after the digital receiver, the error distribution is directly related to P_e. According to wireless communication theory, P_e is only related to the signal-to-noise ratio in a fixed-channel wireless communication system, and channel noise power usually remains constant over time. Therefore, the purpose of our study shifts to controlling the data error distribution by regulating the transmit power.

1 Modeling Methods and Measurements

In the first section, we clarify the system architecture and noise evaluation metrics used in this project.

1.1 Baseband Equivalent System Model

All the discussion below is based on the baseband equivalent wireless communication system model.

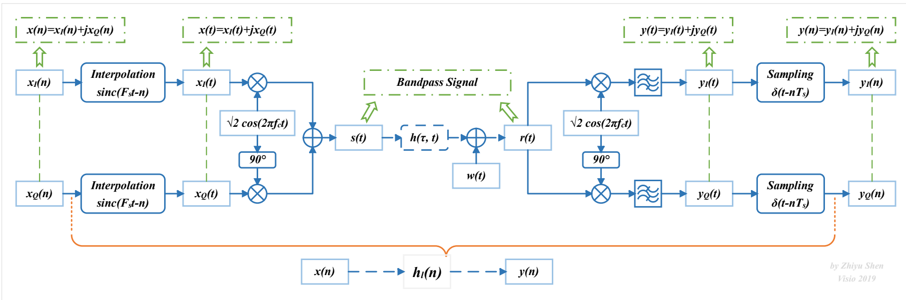


Figure 1 Wireless communication system architecture

As is shown in Figure 1, the baseband equivalent model directly investigates the relationship between the baseband signals at the transmit and receive ends, making it unnecessary to consider the RF characteristic of the system, i.e., the up-conversion and down-conversion parts, in the simulation.

By introducing the baseband equivalent model, the transmission characteristic function of the system turns into the equation below.

$$y(n) = \sum_{l} h_l(n) x(n-l) + w(n) \tag{1.1}$$

Where $h(n)$ is the channel coefficient matrix and $w(n)$ is a circular symmetry complex Gaussian variable, $w(n) \sim \mathcal{CN}(0, N_0)$.

1.2 How to Evaluate the Noise Level

In our system, we adopt E_b / N_0 to measure the noise figure. Usually, in digital communication systems, we use E_b / N_0 to replace P_S / P_N which is widely adopted in analog systems. The relationship between the two indicators is as follows.

$$\frac{E_b}{N_0} = \frac{P_S T_b}{P_N / W} = \frac{P_S}{P_N}\left(\frac{W}{R}\right) \tag{1.2}$$

Where *R* is the bitrate of the signal and *W* is the bandwidth of the signal. In most cases, $W = F_s$, where F_s is the sampling rate of the baseband signal.

It is also worth mentioning that an important measurement of a digital communication system is the $P_B - E_b / N_0$ curve.

1.3 Channel Model

1.3.1 AWGN Channel

To look into the relationship between data error distribution and transmission power, we need to clarify our channel model, which is the most critical factor that influences the BER of a communication system. As we have concluded in the section at the very beginning, the data error distribution is only related to the bit error probability P_e. Therefore, the only thing we have to focus on in channel modeling is the bit error probability P_e of a system with one particular channel model. Let us start with the AWGN channel model.

In a communication system with an AWGN channel, we can simplify Equation (1.1) to the following format:

$$y(n) = x(n) + w(n) \tag{1.3}$$

For a system with BPSK modulation and AWGN channel, the theoretical bit error probability is as follows.

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \approx \frac{1}{2}e^{-\frac{E_b}{N_0}} \tag{1.4}$$

1.3.2 Rayleigh Fading Channel

However, the channel model would be much more complex because of electromagnetic waves' reflection, diffraction, and scattering. Typically, channel fading is classified into large-scale fading and small-scale fading. The large-scale fading usually causes a logarithmic decrease in the amplitude of the transmission signal, and the small-scale fading brings about signal distortion both in amplitude and phase. Sometimes, the small-scale fading will also cause frequency shifts in the carrier wave, called Doppler Shift.

Researchers have built statistical models to describe small-scale fadings to better model and simulate the system. Some well-known models have been widely adopted, like the Rayleigh fading model, the Rician fading model, and the Nakagami fading model. The details of channel modeling for different statistical models are virtually the same, so here we look at the Rayleigh fading channel as an example.

In this project, we try not to make the channel modeling process too complicated. Assume that the fading channel model is a frequency non-selective fast fading channel, and the large-scale fading is temporarily ignored. On top of that, Equation (1.1) would turn into the following format:

$$y(n) = h(n)x(n) + w(n) \tag{1.5}$$

Where $h(n)$ is a random process following complex Gaussian distribution.

For a system with BPSK modulation and AWGN channel, the theoretical bit error probability is as follows:

$$P_e \approx \frac{1}{2}\left(1 - \sqrt{\frac{E_b / N_0}{1 + E_b / N_0}}\right) \tag{1.6}$$

When E_b / N_0 is quite large, usually larger than 10dB, Equation (1.6) turns into:

$$P_e \approx \frac{1}{4E_b / N_0} \tag{1.7}$$

2 Discussion on Data Error Distribution

Before planning a power allocation strategy, it is necessary to clarify the data error distribution of the communication system under study.

Apparently, for a system whose packet size is N_p, the possible data error is different when different digits are transmitted, and the error distribution seems to be complicated. So, let us start with a simple case where the transmitted digits are uniformly distributed between the widest digit range, e.g., [0,15] for a 4-bit packet.

2.1 Theoretical Foundation

Before simulating with MATLAB, it is necessary to figure out the theoretical basis of our work.

2.1.1 The Error Distribution is Symmetrical

In the discussion below, we adopt data with a packet size of 4, for example.

Define the probability of digit *a* being sent at the transmitter as $P(TX = a)$. Since the transmitted digits are uniformly distributed between the widest range, different numbers within the range have the same probability of occurring and let it be P_0. We can easily calculate that

$P(TX = 0) = P(TX = 1) = \cdots = P(TX = 15) = P_0 = \frac{1}{16}$.

Define the probability of data error *b* when transmitted digit *a* as $P(e = b \mid a)$ and the probability of data error *b* in general is $P(e = b)$. According to the Bayesian formula:

$$P(e = b) = \sum_{i=0}^{15} P(e = b \mid a) P(TX = i) = \sum_{i=0}^{15} P(e = b \mid a) P_0 \tag{2.1}$$

If we send two digits that are 1's complement to each other, the error distribution at the receiving end is symmetrical because, under our assumption, the probability of the receiver misclassifying bit 0 as bit 1 is equal to that of misclassifying bit 1 as bit 0. For example, numbers 0 and 15 are 1's complement to each other in binary format: 0000 and 1111. The possible data error that will occur when transmitting digit 0 can be $\{0,1,\ldots,15\}$ and the possible data error that will occur when transmitting digit 15 can be $\{0,-1,\ldots,-15\}$. We can quickly get that $P(e=k\,|\,0)=P(e=-k\,|\,15)$ and it is the same with other digits with the same feature. The conclusion can be written as follows:

$$P(e=k\,|\,a)=P(e=-k\,|\,\overline{a}) \tag{2.2}$$

Where $\overline{a}$ represents 1's complement of digit a.

On this basis, we can summarize a law of data error distribution as follows.

$$P(e=b)=\sum_{i=0}^{15}P(e=b\,|\,a)P_0=\sum_{i=0}^{15}P(e=-b\,|\,\overline{l}\,)P_0=P(e=-b) \tag{2.3}$$

The derivation above proves the first half of the earlier conclusion, while the second half is obvious.

If we set a different transmit power for each bit in a pack individually, we can get different bit error probability for different bits. Let us suppose the bit error probabilities for different bits in a pack of 4 are Pe_1, Pe_2, Pe_3, Pe_4 and the probability of data error 0 should be

$$P(e=0)=\prod_{i=1}^{4}(1-P_{e_i})$$

In a general communication system, unless the signal-to-noise ratio is extremely low, in most cases, the bit error probability or BER (bit error rate) is not too large, usually much less than 1. Therefore, the value of the expression above is very close to one in a high SNR case and at least larger than the probability of other data errors in most cases.

Conclusion 2.1: The data error distribution is symmetrical

The distribution of data errors is symmetric about zero, with the peak at zero point.

$$P(e=b)=P(e=-b) \tag{2.4}$$

$$\max\left\{P(e=b)\right\}=P(e=0) \tag{2.5}$$

Where $b=0,1,2,\ldots,2^{N_p}-1$ *and* N_p *is the number of bits in a packet.*

2.2 Recursive Expression of Data Errors

In last section, we defined the bit error probability as P_e. At the transmission end, uniformly distributed data is transmitted, the probability of transmitting 0 and 1 are both 1/2. Thus, at the receiving end, the error could be -1, 0, 1and their occurrence probabilities are as follow:

$$P(Error_{bit}=x)=\begin{cases}\frac{1}{2}p_e, & x=-1\\ 1-p_e, & x=0\\ \frac{1}{2}p_e, & x=1\end{cases} \tag{2.6}$$

Assume the system transmits data in a pack of N_p. The decimal format of transmitted and received data can be expressed by their binary format in the following way:

$$Data_{tx}=2^{N_p-1}T_{N_p-1}+2^{N_p-2}T_{N_p-2}+\cdots+2^1T_1+2^0T_0 \tag{2.7}$$

$$Data_{rx}=2^{N_p-1}R_{N_p-1}+2^{N_p-2}R_{N_p-2}+\cdots+2^1R_1+2^0R_0 \tag{2.8}$$

Where T_i is the *i*th transmitted bit, R_i is the *i*th received bit.

The data error is the difference between Equation (2.7) and (2.8):

$$\begin{aligned}Error_{data}&=Data_{rx}-Data_{tx}\\&=2^{N_p-1}\left(R_{N_p-1}-S_{N_p-1}\right)+\cdots+2\left(R_1-S_1\right)+\left(R_0-S_0\right)\\&=2^{N_p-1}X_{N_p-1}+\cdots+2X_1+X_0\end{aligned} \tag{2.9}$$

For each bit, it follows the error probability derived in Equation (2.6).

Next, look at each term in the expression of $Error_{data}$. Let random variables $2^i X_i = Y_i \ (i = 0,1,\ldots,N_p - 1)$, then the expression of data error in Equation (2.9) can be written as:

$$Error_{data} = \sum_{i=0}^{N_p - 1} Y_i \tag{2.10}$$

To rewrite the above equation into a recursive form, first define random variables:

$$Z_k = \sum_{i=0}^{k} Y_i \tag{2.11}$$

The recursive expression of Z_k is:

$$Z_k = \begin{cases} Z_{k-1} + Y_k, & k \geq 1 \\ Y_0, & k = 0 \end{cases} \tag{2.12}$$

Therefore, the distribution of random variable x_i can be derived as:

$$P(Y_i = y_i) = \begin{cases} \frac{1}{2} p_{e_i}, & y_i = -2^i \\ 1 - p_{e_i}, & y_i = 0 \\ \frac{1}{2} p_{e_i}, & y_i = 2^i \end{cases} \tag{2.13}$$

Since $\left\{X_1, X_2, \ldots, X_{N_p - 1}\right\}$ are independent, $\left\{Y_1, Y_2, \ldots, Y_{N_p - 1}\right\}$ are independent random variables. According to the Convolution Theorem, the probability distribution of Z_k can be written as:

$$\begin{aligned} P(Z_k = z_k) &= \sum_{l=-\infty}^{\infty} P\left(Y_k = l\right) P\left(Z_{k-1} = z_k - l\right) \\ &= \sum_{l=-2^k, 0, 2^k} P\left(Y_k = l\right) P\left(Z_{k-1} = z_k - l\right) \\ &= \frac{1}{2} p_{e_k} \left[P\left(Z_{k-1} = z_k + 2^k\right) + P\left(Z_{k-1} = z_k - 2^k\right) \right] + \left(1 - p_{e_k}\right) P\left(Z_{k-1} = z_k\right) \end{aligned} \tag{2.14}$$

To simplify the equation above, we first look at the valid value of random variable Z_{k-1}:

$$Z_{k-1} \in \left\{-2^k + 1, -2^k + 2, \ldots, -1, 0, 1, \ldots, 2^k - 2, 2^k - 1\right\}$$

$$P(Z_{k-1}=z_{k-1})=0, \quad (z_{k-1}<-2^k+1, z_{k-1}>2^k+1)$$

From the previous derivation, we have already known that the probability of any random variable Z_i is symmetric about 0. Thus, we can only look at the probability of $Z_k \le 0$.

First, let us deal with the case when $z_k=0$, it can be simply written as:

$$P(Z_k=0)=(1-p_{e_k})(1-p_{e_{k-1}})\cdots(1-p_{e_1})(1-p_{e_0})=\prod_{i=1}^{k}(1-p_{e_i})$$

Then it comes to the case when $z_k<0$. Since $z_k-2^k \le -2^k < -2^k+1$, then $P(Z_{k-1}=z_k-2^k)=0$. The minimal valid value of z_k, satisfies the equation $z_k+2^k=-2^k+1$, i.e., $z_k=-2^{k+1}+1$.

From the above derivation, it can be verified that: When $-2^{k+1}+1 \le z_k \le -1$, $P(Z_{k-1}=z_k-2^k)>0$; When $z_k \le -2^{k+1}$, $P(Z_{k-1}=z_k-2^k)=0$.

Similarly, when $z_k \le -2^k$, $P(Z_{k-1}=z_k)=0$; When $-2^k+1 \le z_k \le -1$, $P(Z_{k-1}=z_k)>0$.

Conclusion 2.2: The data error distribution can be expressed in a recursive form

A communication system transmits data in a pack of N_p *and the error probability of each bit in a pack is* $\left\{p_{e_{N_p}}, p_{e_{N_p-1}}, \ldots, p_{e_2}, p_{e_1}\right\}$. *The data error distribution can be derived from the following recursive expression:*

$$P(Z_k=z_k)=\begin{cases} \frac{1}{2}p_{e_k}P(Z_{k-1}=z_k+2^k), & -2^{k+1}+1 \le z^k \le -2^k \\ \frac{1}{2}p_{e_k}P(Z_{k-1}=z_k+2^k)+(1-p_{e_k})P(Z_{k-1}=z_k), & -2^k+1 \le z_k \le -1 \\ \prod_{i=1}^{k}(1-p_{e_i}), & z_k=0 \end{cases} \quad (2.15)$$

The cases where $z_k \ge 0$ *and the cases where* $z_k \le 0$ *are symmetric:*

$$P(Z_k = z_k) = \begin{cases} \frac{1}{2} p_{e_k} P(Z_{k-1} = z_k - 2^k), & 2^k \le z^k \le -2^{k+1} - 1 \\ \frac{1}{2} p_{e_k} P(Z_{k-1} = z_k - 2^k) + (1 - p_{e_k}) P(Z_{k-1} = z_k), & 1 \le z_k \le 2^k - 1 \\ \prod_{i=1}^{k} (1 - p_{e_i}), & z_k = 0 \end{cases} \quad (2.16)$$

2.3 Zero-Centred Decreasing Distribution

This project aims to adjust the transmission power of different bits so that the data error forms a zero-centred decreasing distribution, which means the probability of smaller data errors should be higher than that of larger ones. A better scenario is to find a transmit power allocation scheme such that the received data error has a Laplace or Gaussian distribution.

3 A Power Allocation Strategy to Form Laplace Distributed Data Error

3.1 Relationship between Transmit Power and Data Error Distribution

In our research, we specify the transmit gain of the highest bit in a pack as the unit gain and calculate the noise power with it. To be more specific, if we define the ratio between the transmit power of MSB and the ith bit as k_i, the transmit gain of each bit in a pack can be expressed as:

$$\begin{aligned} G_{TX} &= \left\{ \alpha k_1, \alpha k_2, \ldots, \alpha k_{N_p - 1}, \alpha \right\} \\ G_{TX} &= \alpha k_i, \quad i = 1, 2, \ldots, N_p \end{aligned} \quad (3.1)$$

And the transmit power of each bit in a pack can be expressed as:

$$\begin{aligned} P_{TX} &= \left\{ \alpha k_1^2, \alpha k_2^2, \ldots, \alpha k_{N_p - 1}^2, \alpha \right\} \\ P_{TX_i} &= \alpha k_i^2, \quad i = 1, 2, \ldots, N_p \end{aligned} \quad (3.2)$$

Since the additive noise is introduced into the transmission system by the environment, the noise power can be viewed as constant in a certain period of time:

$$P_N = \rho \quad (3.3)$$

To simplify the calculation, we do not introduce baseband shaping process in this model, which means the bitrate of the signal is the same with its bandwidth, i.e., $R = W$. The E_b / N_0 of the ith bit can be calculated according to Equation (1.2):

$$\left(\frac{E_b}{N_0}\right)_i = \frac{S_i}{N_i}\left(\frac{W_i}{R_i}\right) = \frac{P_{TX_i}}{P_N} = \frac{\alpha k_i^2}{\rho} \tag{3.4}$$

In Section 2.2, we obtained a recursive expression for the data error distribution with the BER for each bit as the independent variable, which is shown in Equation (2.15) and (2.16). In Section 1.3, we derived the relation between E_b / N_0 and BER for both AWGN channel and Rayleigh fading channel, which is expressed by Equation (1.4) and (1.6). In theory, by combining these formulas with Equation (3.4), we can obtain the data error's PDF (probability density function) with transmit power as independent variable. However, the derivation is complex. Instead, in our project, a MATLAB program is written to calculate the value of data error's PDF, and by adjusting the transmit power of each bit, we attempted to find a power allocation strategy that can form Laplacian or Gaussian distributed data error.

3.2 Exponentially Decaying Transmit Power

Through our simulation and analysis, it is found that if the transmit gain decays exponentially with the number of bits, the data error distribution will approximate a Laplace distribution. The transmit gain can be expressed in the following way:

$$\begin{aligned} G_{TX} &= \left\{\alpha e^{-(N_p-1)}, \alpha e^{-(N_p-2)}, \ldots, \alpha e^{-2}, \alpha e^{-1}, \alpha\right\} \\ G_{TX_i} &= \alpha e^{-(N_p-i)}, \quad i = 1, 2, \ldots, N_p \end{aligned} \tag{3.5}$$

The PDF of data error approximates the following expression:

$$y = \frac{1}{2b} e^{-\frac{|z_{N_p} - \mu|}{b}}, \quad z_{N_p} = -2^{N_p}+1, -2^{N_p}+2, \ldots, -1, 0, 1, \ldots, 2^{N_p}-2, 2^{N_p}-1 \tag{3.6}$$

Where $\mu = 0$ and b satisfies the following expression:

$$\frac{1}{2b} = P(Z_{N_p} = 0) = \sum_{i=0}^{N_p} (1 - p_{e_i}) \tag{3.7}$$

3.2.1 Curve Fitting Method

In theory, as is described in last section, we assume that the data error PDF fits the Laplace distribution with parameter μ_t and b_t defined by Equation (3.7). In practice, the parameters of Laplace distribution μ_m and b_m can be estimated from the statistical characteristics of measured data error. μ_m is the median of data error and b_m is the mean value of $|z_{N_p} - \mu_m|$.

3.2.2 Power Saving

To receive the accurate data at the receiver end, conventionally, the transmit power of each bit should be the same as that of MSB:

$$\begin{aligned} &G_0 = \{\alpha, \alpha, \ldots, \alpha\} \\ &G_{0_i} = \alpha, \quad i = 1, 2, \ldots, N_p \end{aligned} \tag{3.8}$$

Therefore, it is possible to calculate transmit power savings of our exponentially distributed power allocation strategy compared to the conventional strategy. The saved power can be calculated as:

$$\begin{aligned} P_{saved} &= \sum_{i=1}^{N_p} \left(P_{0_i} - P_{TX_i}\right) = \sum_{i=1}^{N_p} \left(G_{0_i}^2 - G_{TX_i}^2\right) \\ &= \alpha^2 \left(N_p - \frac{1 - e^{-2N_p}}{1 - e^{-2}} \right) (Watt) \end{aligned} \tag{3.9}$$

In practice, the number of bits in a package satisfies the following condition:

$$N_p \in \mathbb{N}^+, \quad N_p \geq 2 \tag{3.10}$$

An approximation can be made under such condition: $1 - e^{-2N_p} \approx 1$. Thus, Equation (3.9) can be simplified as:

$$P_{saved} \approx \alpha^2 \left(N_p - \frac{1}{1 - e^{-2}} \right) \approx \alpha^2 \left(N_p - 1.1565 \right) (Watt) \tag{3.11}$$

4 Experiments on Transmit Power Allocation

4.1 Experiment 1: Experiments in Systems with AWGN Channel

In this experiment, we transmit data through a system with an AWGN channel and BPSK modulation. Experiments will be conducted to compare conventional power allocation strategy and exponentially decaying power allocation strategy.

4.1.1 Experiment Conditions

The physical layer and baseband parameters are shown in Table 1. The noise power is set to be constant.

Channel Model	AWGN
Modulation	BPSK
Bitrate	100,000
Number of Data	1,000,000
Package Size	4
Noise Power	30 dBm (1 W)

Table 1 Physical layer and baseband parameters of Experiment in systems with AWGN channel

4.1.2 Experiment Results

Table 2 shows the transmit power, signal-to-noise ratio and BER of each bit when transmitting bits with constant power. Table 3 shows the transmit power, signal-to-noise ratio and BER of each bit when transmitting bits with constant power.

Bit Index	Transmit Power	E_b/N_0	Theoretical BER	Measured BER
1 (LSB)	30.00 dBm	0 dB	7.865×10^{-2}	7.881×10^{-2}
2	30.00 dBm	0 dB	7.865×10^{-2}	7.900×10^{-2}
3	30.00 dBm	0 dB	7.865×10^{-2}	7.889×10^{-2}
4 (MSB)	30.00 dBm	0 dB	7.865×10^{-2}	7.861×10^{-2}

Table 2 Parameters of constant transmit power experiment with AWGN channel

Bit Index	Transmit Power	E_b/N_0	Theoretical BER	Measured BER

1 (LSB)	30.00 dBm	0 dB	7.865×10^{-2}	7.872×10^{-2}
2	21.31 dBm	-17.37 dB	3.014×10^{-2}	3.014×10^{-2}
3	12.63 dBm	-34.74 dB	42.41×10^{-2}	42.39×10^{-2}
4 (MSB)	3.94 dBm	-52.12 dB	47.19×10^{-2}	47.23×10^{-2}

Table 3 Parameters of exponentially decaying transmit power experiment with AWGN channel

Figure 2 shows the PDF of data error of both transmit power allocation strategy.

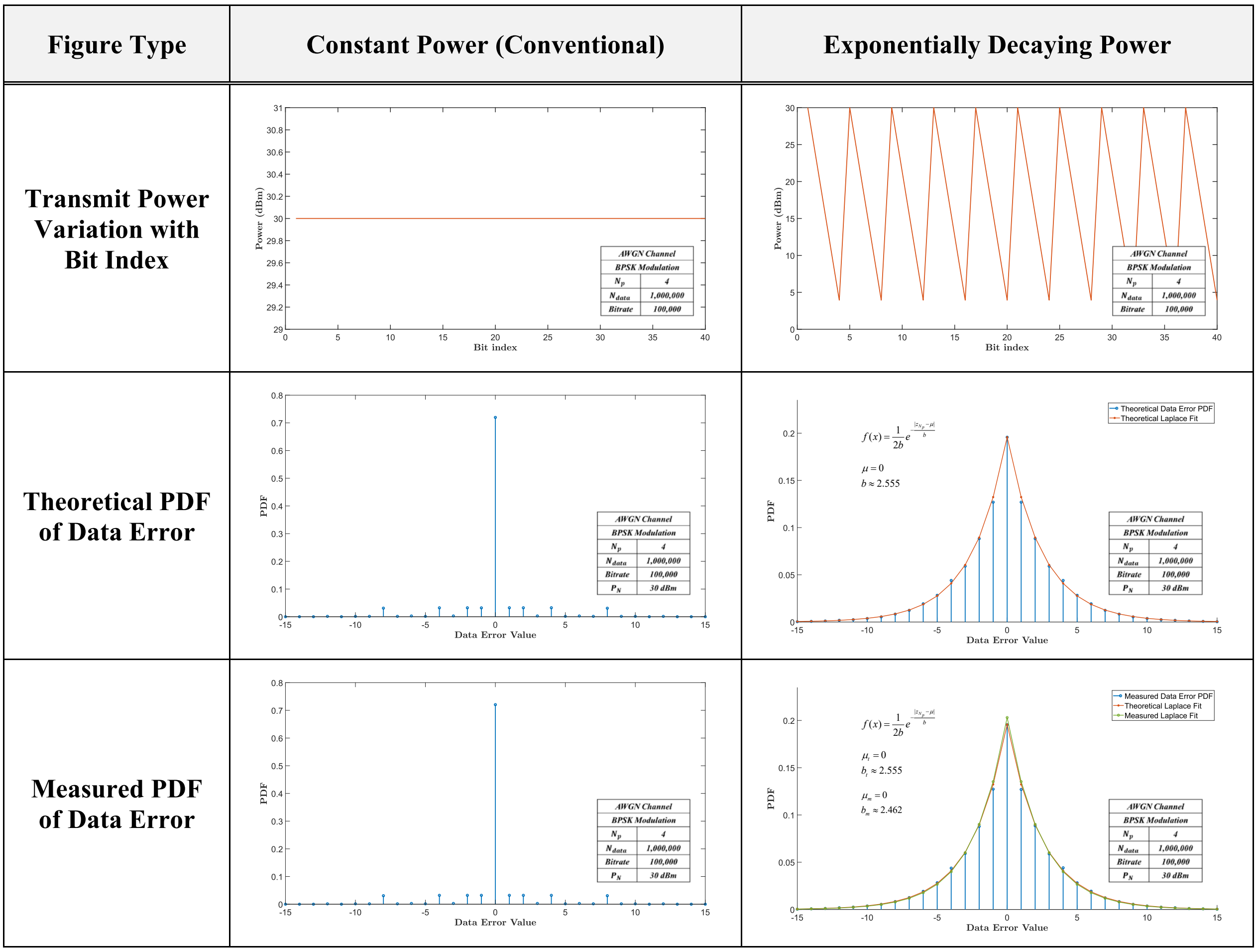


Figure 2 Comparison between two transmit power allocation strategy with AWGN channel

When the transmit power remains constant with varying bit, the data error is concentrated at zero, which is in line with the data error distribution in general communication system. When the transmit power decays exponentially in a data package, the PDF of data error approximates a Laplace distribution.

4.2 Experiment 2: Experiments in Systems with Rayleigh Fading Channel

In this experiment, we transmit data through a system with a Rayleigh Fading channel and BPSK modulation. Like what is shown in last section, experiments will be conducted to compare conventional power allocation strategy and exponentially decaying power allocation strategy.

4.2.1 Experiment Conditions

The physical layer and baseband parameters are shown in Table 4. The noise power is set to be constant.

Channel Model	Rayleigh
Modulation	BPSK
Bitrate	100,000
Number of Data	1,000,000
Package Size	4
Noise Power	30 dBm (1 W)

Table 4 Physical layer and baseband parameters of Experiment in systems with Rayleigh fading channel

To construct a Rayleigh fading channel, we adopt the Jakes model, and its parameters are shown in Table 5. **Error! Reference source not found.** shows the distribution of Rayleigh channel coefficient.

$\boldsymbol{N_w}$	98
$\boldsymbol{f_m}$	50 Hz
$\boldsymbol{t_0}$	0 s
$\boldsymbol{\varphi_N}$	0 rad

Table 5 Parameters of Jakes model constructing Rayleigh channel

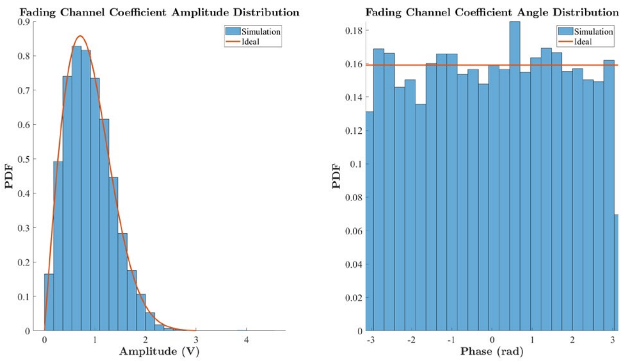


Figure 3 Amplitude and angle distribution of Rayleigh channel coefficient

4.2.2 Experiment Results

Table 6 shows the transmit power, signal-to-noise ratio and BER of each bit when transmitting bits with constant power. Table 7 shows the transmit power, signal-to-noise ratio and BER of each bit when transmitting bits with constant power.

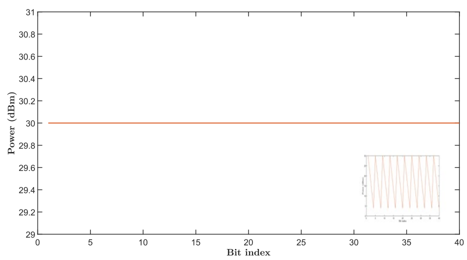
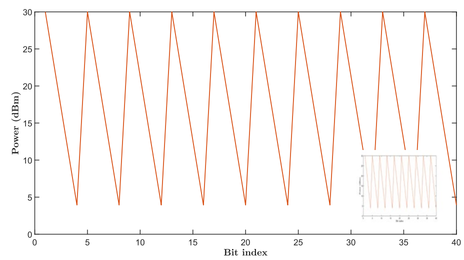
Bit Index	**Transmit Power**	$\mathbf{E_b/N_0}$	**Theoretical BER**	**Measured BER**
1 (LSB)	30.00 dBm	0 dB	1.464×10^{-1}	1.452×10^{-1}
2	30.00 dBm	0 dB	1.464×10^{-1}	1.458×10^{-1}
3	30.00 dBm	0 dB	1.464×10^{-1}	1.470×10^{-1}
4 (MSB)	30.00 dBm	0 dB	1.464×10^{-1}	1.466×10^{-1}

Table 6 Parameters of constant transmit power experiment with Rayleigh fading channel

Bit Index	**Transmit Power**	$\mathbf{E_b/N_0}$	**Theoretical BER**	**Measured BER**
1 (LSB)	30.00 dBm	0 dB	1.464×10^{-1}	1.669×10^{-1}
2	21.31 dBm	-17.37 dB	3.274×10^{-1}	3.307×10^{-1}
3	12.63 dBm	-34.74 dB	4.329×10^{-1}	4.294×10^{-1}
4 (MSB)	3.94 dBm	-52.12 dB	4.751×10^{-1}	4.758×10^{-1}

Table 7 Parameters of exponentially decaying transmit power experiment with Rayleigh fading channel

Figure 4 shows the PDF of data error of both transmit power allocation strategy.

Figure Type	**Constant Power (Conventional)**	**Exponentially Decaying Power**
Transmit Power Variation with Bit Index		

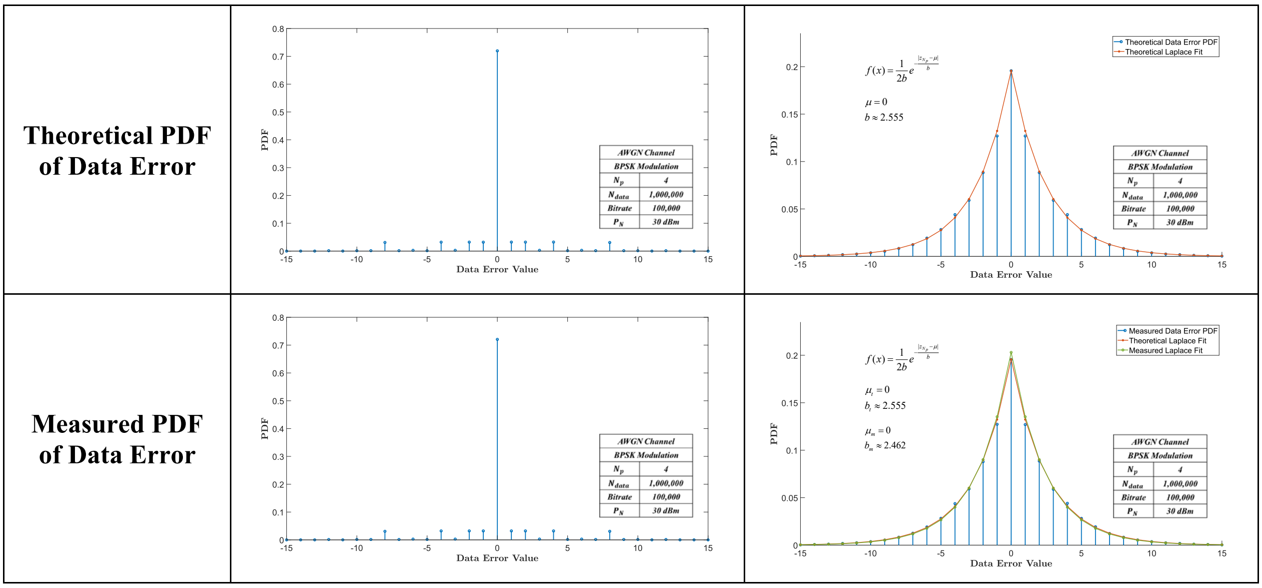


Figure 4 Comparison between two transmit power allocation strategy with Rayleigh fading channel

The data error distribution is similar to that of system with AWGN channel. When the transmit power remains constant with varying bit, the data error is concentrated at zero. When the transmit power decays exponentially in a data package, the PDF of data error approximates a Laplace distribution.

4.3 Experiment 3: Experiments on Laplace Distribution Parameter with AWGN Channel

Experiment 1 and 2 proves that, in both theory and practice, exponentially decaying power allocation strategy can make the data error distributed in the Laplace form. In this section, experiments are done in system with AWGN channel to see the variation of Laplace parameter b with changing package size N_p and signal power P_S. In other words, we try to find a way to control the shape of the Laplace distribution.

4.3.1 Experiment Conditions

The physical layer and baseband parameters of these experiments are shown in Table 8.

Channel Model	AWGN
Modulation	BPSK
Bitrate	100,000
Number of Data	1,000,000
Noise Power	30 dBm (1 W)

Table 8 Physical layer and baseband parameters of Experiment 3

4.3.2 Experiment with N_p as an Independent Variable

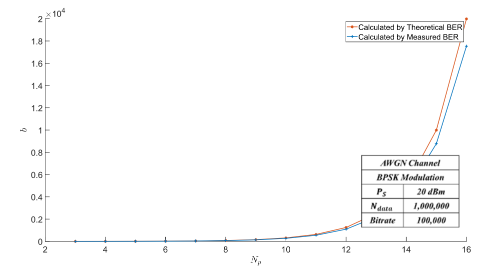
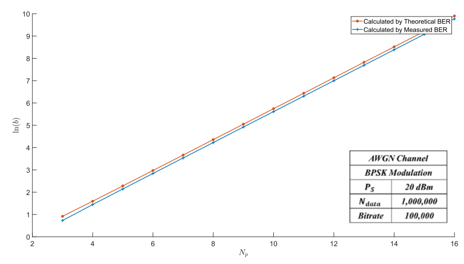
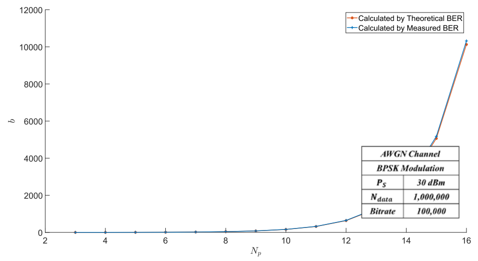
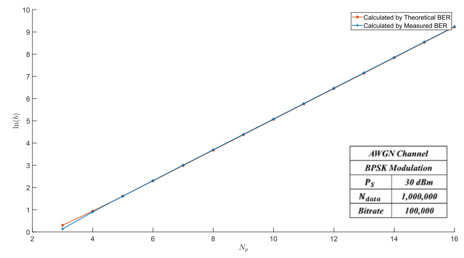
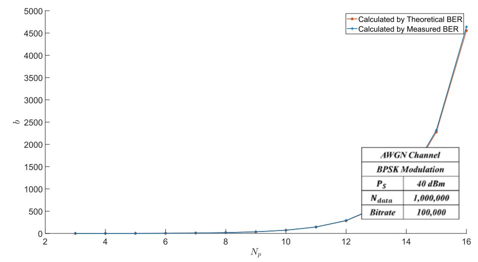
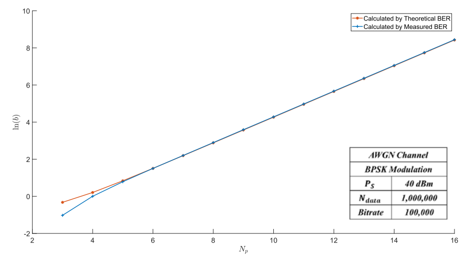
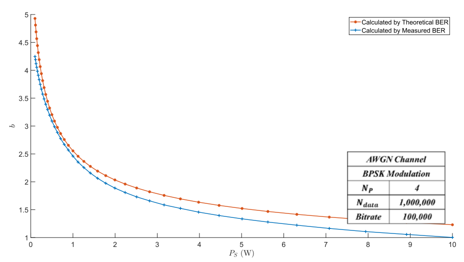
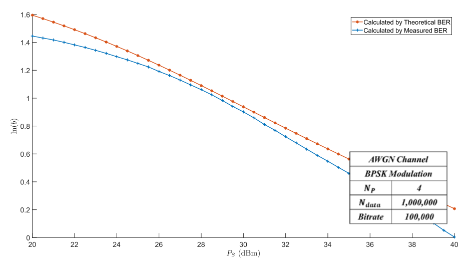
P_S	$b - N_p$ Curve	$\ln b - N_p$ Curve
20 dBm		
30 dBm		
40 dBm		

Figure 5 Variation of Laplace parameter b with variable Np under different signal power in Experiment 3

It can be concluded from Figure 5 that, when the package size N_p increases linearly, the Laplace distribution parameter b increased exponentially.

4.3.3 Experiment with P_S as an Independent Variable

N_p	$b - P_S\ (W)$ Curve	$\ln b - P_S\ (dBm)$ Curve
4		

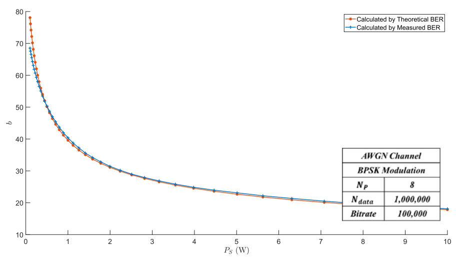
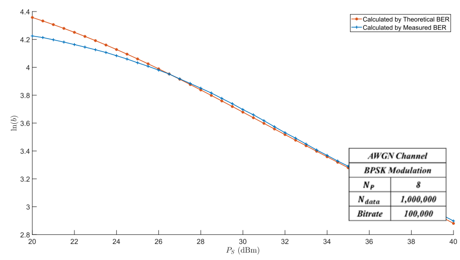
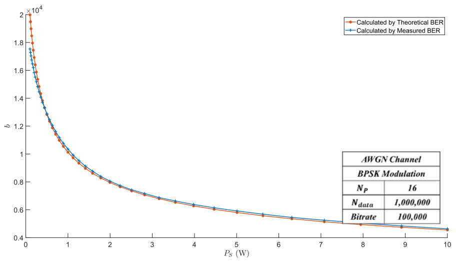
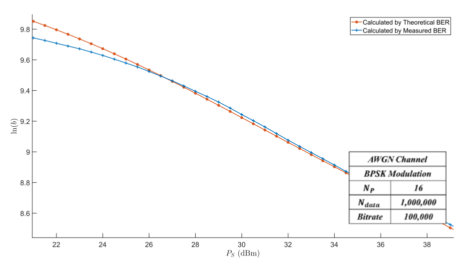
8		
16		

Figure 6　Variation of Laplace parameter b with variable P_S under different signal power in Experiment 3

As is shown in Figure 6, the $b - P_S\ (dBm)$ curves approximate linear ones. Let $Ps(dBm) = m$ and $Ps(W) = x$, then $m = 10\log_{10}(1000x)$. The relationship between b and $P_S(W)$ can then be derived:

$$
\begin{aligned}
&\ln b = -km + t \\
&\Rightarrow \ln b = -10k\log_{10}(1000x) + t \\
&\Rightarrow b = \exp[-10k\log_{10}(1000x) + t] = e^t\{\exp[\log_{10}(1000x)]\}^{-10k} \\
&\Rightarrow b = e^t(1000x)^{-\frac{10k}{\ln 10}} = e^t 1000^{-\frac{10k}{\ln 10}} x^{-\frac{10k}{\ln 10}} = px^{-q}
\end{aligned}
$$

$$
b = p\left[P_S(W)\right]^{-q} = \frac{p}{\left[P_S(W)\right]^q} \tag{4.1}
$$

In Equation (4.1), $p > 0$ and $q > 0$.

4.4 Experiment 4: Experiments on Laplace Distribution Parameter with Rayleigh Channel

In this section, experiments are done in system with Rayleigh fading channel to see the variation of Laplace parameter b with changing package size N_p and signal power P_S.

4.4.1 Experiment Conditions

The physical layer and baseband parameters of these experiments are shown in Table 9.

Channel Model	Rayleigh
Modulation	BPSK
Bitrate	100,000
Number of Data	1,000,000
Noise Power	30 dBm (1 W)

Table 9   Physical layer and baseband parameters of Experiment 4

4.4.2 Experiment with N_p as an Independent Variable

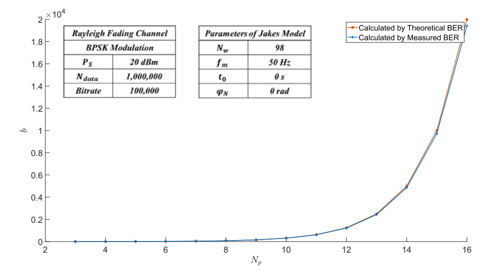
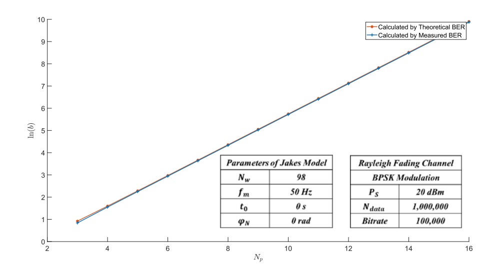
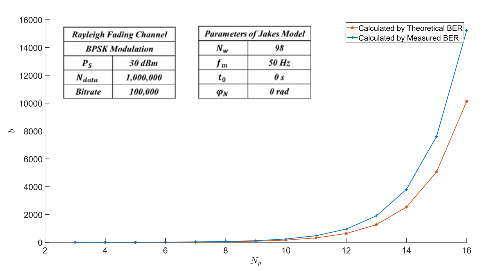
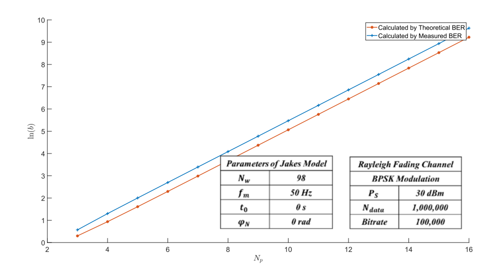
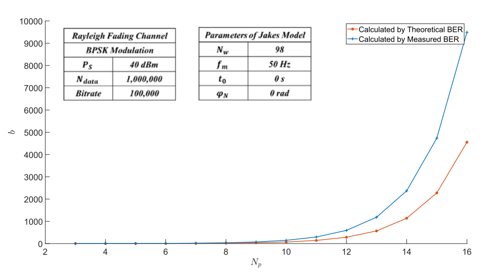
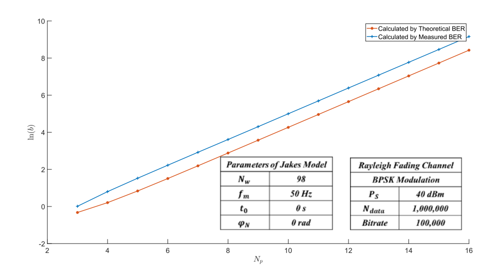
P_S	$b - N_p$ Curve	$\ln b - N_p$ Curve
20 dBm		
30 dBm		
40 dBm		

Figure 7   Variation of Laplace parameter b with variable Np under different signal power in Experiment 4

4.4.3 Experiment with P_S as an Independent Variable

N_p	$b - P_S\ (W)$ Curve	$\ln b - P_S\ (dBm)$ Curve

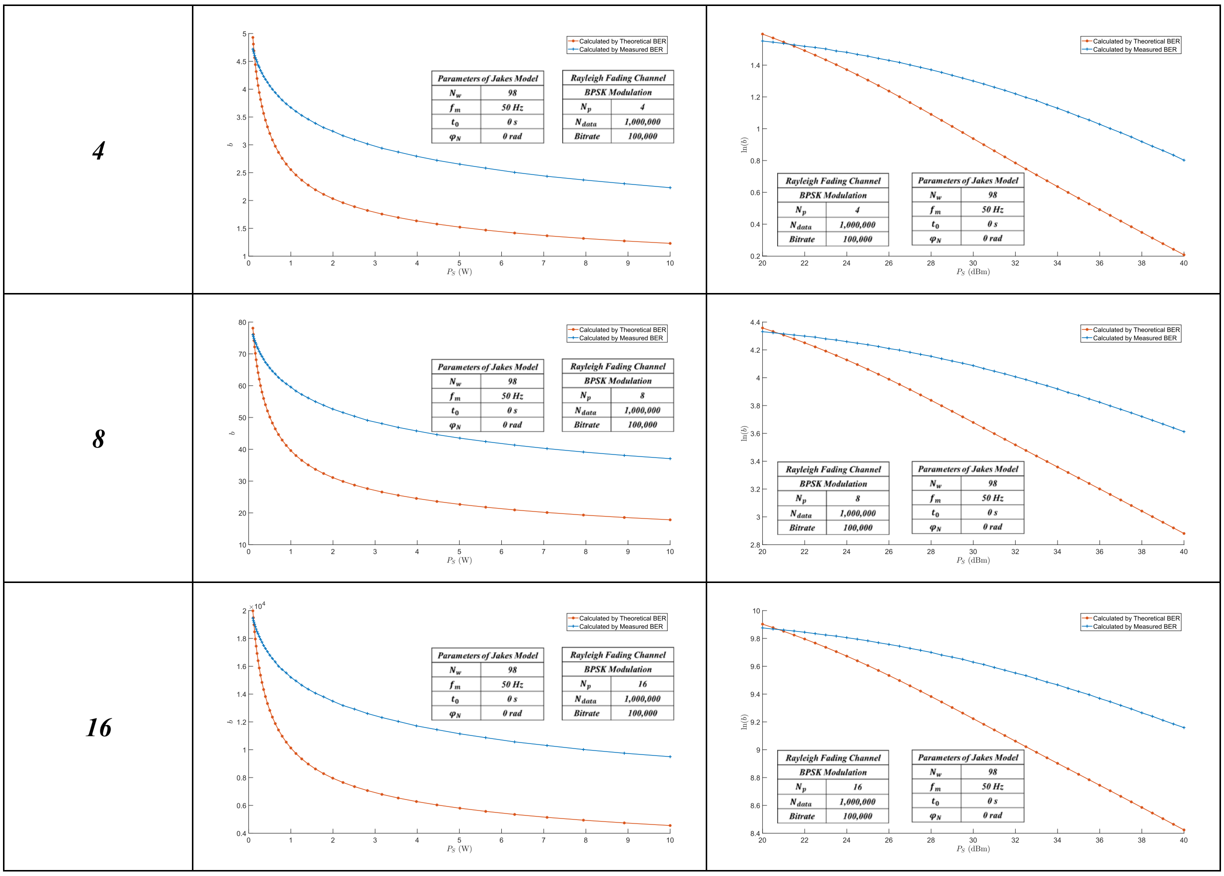


Figure 8 Variation of Laplace parameter b with variable P_S under different signal power in Experiment 4

From Figure 7 and Figure 8, we can draw conclusions similar to Experiment 3.

5 Problems to Be Solved

We have generated a series of useful conclusions from the discussion above. However, the research still needs to be continued since there are a lot of theoretical derivations to be done.

Firstly, the Laplace distributed data error has been observed in practice and the recursive form expression, but the complete formula of data error PDF is needed to prove our conclusion in theory.

Also, we have noticed that the Laplace distribution curves we have currently fitted vary under different fitting methods, especially when Rayleigh fading channel model is adopted. Once we fetch the theoretical formula, we would find a better way to fit the data error distribution with a Laplace distribution curve.

After all these theoretical works are finished, we are then able to describe our power allocation strategy in a more accurate and precise way.