Hotelling's Minimum Differentiation Principle in a Continuous-Time Experiment with Price and Location Choices *

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Abstract

Using a continuous-time experimental setup, we study how well Hotelling's principle of minimum differentiation holds in its original two-dimensional (price and location) setting. By having subjects choose their location on a two-dimensional plane, we observe that the principle of minimum differentiation largely holds. We also study how different adjustment rates affect price coordination and find that, contrary to recent literature, the ability to respond quickly does not always increase cooperation rates.

Keywords Spatial competition · experiment · continuous time

JEL Classification C72 · C92 · D21 · D43

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1 Introduction

Hotelling's seminal paper "Stability in Competition" (Hotelling, 1929) characterized the stylized fact that individuals buy commodities from different sellers despite modest differences in price. The model is often taught and discussed as a simple location model in which firms decide how to position their product in a linear product space and has been adapted to numerous phenomena; from voting habits (Downs, 1957) to entry deterrence (Schmalensee, 1978) to competition in various industries (Baum and Mezias, 1992 for hotels, Calem and Rizzo, 1995 for hospitals, or Iyer et al., 2014 for religions).

In the classic setup, firms face a constant price and a uniformly distributed mass of consumers who can buy at most one unit from one of the firms. The utility function of consumers decreases linearly with distance, so they purchase the homogeneous good from the closest vendor. This means that with two firms in the market, these will locate back-to-back at the midpoint (i.e., the two firms will produce identical products). This outcome is known as the *principle of minimum differentiation*.

However, Hotelling's location model for two firms is extremely sensitive to small changes in the set of assumptions. For example, Eaton and Lipsey (1975) detail equilibria for more than two players and show that minimum differentiation does not generalize easily, even if local clustering tends to emerge. In a similar vein, Hotelling suggested, but did not prove, that his spatial competition model would also lead to a *price equilibrium* between firms. This suggestion has since been proven incorrect, as firms can always improve profits by moving to a new position after a competitor moves, be it a new location or a new price point (see D'Aspremont et al., 1979).[1]

The absence of a pure strategy Nash equilibrium in the Hotelling (1929) original setup motivates our empirical work as we turn to the laboratory to test Hotelling's intuition. To do so, we design a continuous-time experiment in which pairs of anonymously matched subjects can adjust both their price and location during four-minute periods. Using a new interactive interface, subjects can instantly move across the whole price-location strategy, with payoffs depending on their instantaneous price-location position relative to their counterpart. This continuous-time setup allows us to adhere closely to the original model

[1]In fact, in this same paper D'Aspremont shows that in an environment with quadratic costs and a price-location strategy space, firms will tend to *maximize differentiation*.

and question whether Hotelling's Law — another name for the principle of minimum differentiation — holds.

A recent series of papers have underlined the differences between continuous- and discrete-time setups (e.g., Calford and Oprea, 2017) and have shown that, among other things, continuous-time setups have either cooperation rate effects (Friedman and Oprea, 2012) or endgame effects (Bigoni et al., 2015; Friedman et al., 2015). In fact, Kephart and Friedman (2015) and Huck et al. (2002) show that in a four-player Hotelling setup with no price differentiation, subjects converge to the Nash equilibrium in continuous-time sessions but not in discrete-time ones. The ability of firms to quickly adjust to changes in the market is not only important from a theoretical point of view but is also relevant beyond the laboratory, where different markets allow for very different response times.[2] To study such differences, in our experiment we introduce three different treatments that vary in how often subjects are allowed to adjust their position. The first treatment is the Continuous Instant (CI), where subjects can instantly change their price and location. The second is the Continuous Slow (CS), where any change on the price-location coordinates is slowly implemented. Finally, in the Discrete Slow (DS) subjects are only allowed to make changes every three seconds.

Our results indicate that despite the theoretical ambiguity suggesting otherwise, Hotelling's principle of minimum differentiation largely holds. As expected, subjects achieve higher price and space coordination levels in the continuous-time setup than in the two slower adjustment rate treatments. However, we also find that the relationship between colluding and the ability to collude is non-monotonic. While *a priori* collusion should be harder in the DS treatment (see Section 3 for treatment details), the CS treatment results in lower prices (i.e., a more competitive environment) and overall lower payoffs for both subjects. This result highlights how sensitive collusive markets are to their different dimensions.

From the experimental point of view, there have been several attempts to study Hotelling's model. Brown-Kruse and Schenk (2000) investigated a two-player uncertain endpoint model but focused on the effect of communication on collusion. As referenced above, Huck et al. (2002) were the first to test a four-person Hotelling game but found

[2]Take two extreme examples. Advances in algorithmic trading allow financial investors to react almost instantly to any news or price changes. Conversely, the toilet paper industry could not adjust to the sudden spike in demand that resulted from the first weeks of panic during the coronavirus pandemic.

little support for the equilibrium hypothesis. Kephart (2014) showed that the four-player Nash equilibrium emerged more quickly with the ability to adjust location instantly.[3]

However, few authors have included vertical differentiation in their experimental setting. To our knowledge, only three works have tested a setup with both price and location as choice variables. The first attempt was by Mangani and Patelli (2002), who specified their model with quadratic transport costs such that subjects should maximally differentiate in the location dimension to relax price competition. Kusztelak (2011) also used quadratic transport costs and allowed for limited communication between subjects. Barreda et al. (2011) tested the hypothesis that firms use product differentiation to relax price competition by focusing on a limited, discrete location decision. Specifically, subjects could only choose among either seven or eight location slots, depending on the treatment. In their most relevant treatments, the authors found less product differentiation than theory would predict and relatively few high prices.

This paper is the first to implement Hotelling's spatial competition setup with linear distance costs and price competition using a *continuous* bi-dimensional strategic space and a *continuous-time* setup.[4] Therefore, our framework allows us to make three contributions to the literature. First, our experimental design is very close to the original two-player Hotelling model with horizontal and vertical decision variables and linear transport costs, which allows us to study how Hotelling's seminal result fares in an ideal setting. Second, by having three different treatments, our experiment blurs the sharp distinction between the continuous choice model and sequential models, providing insight into how the ability to adjust quickly affects firm behavior. Third, we use an intuitive interface that allows subjects to interact using a fine grid of options, dissipating any concerns of participant apprehension skewing results in the competitive setting.[5]

The remainder of the paper proceeds as follows: In Section 2 we provide a brief overview of the theoretical setup of our experiment. In Section 3 describes the experi-

[3]The four-player location-only game has its own form of the principle of minimum differentiation as the equilibrium, with players located back-to-back on the first and third quartiles.

[4]While Kephart and Friedman (2015) implemented a continuous-time Hotelling experiment before we did, they only had one dimension (space) in their setup. Similarly, while Barreda et al. (2011) ran a two-dimensional Hotelling experiment, they did so using a discrete location and time setup.

[5]See Bosch-Rosa et al. (2018) or Bosch-Rosa and Meissner (2020) for the effects of experimental complexity in two very different setups.

mental setup. Section 4 reports the results of the experiment and we conclude in Section 5.

2 Model and Predictions

We begin by recalling Hotelling's assumptions following D'Aspremont et al. (1979)'s notation. In it, two firms (A and B) sell a homogeneous product with zero production cost to customers who are evenly distributed on a line of length l.[6] Each customer consumes one unit of the good and will buy from the seller who gives the least delivered price. Firms locate at points a and b, respectively, such that a is the distance from 0, b is the distance from l, $a + b \leq l$, with $a \geq 0$ and $b \geq 0$. For simplicity, we normalize l to be 1 in our experiment. Firms also set prices p_A and p_B, respectively. Transport costs are linear and are denoted by c.

First, consider the case that price and location are chosen simultaneously. The payoff functions for A and B are given by

$$\pi_A(p_A, p_B, a, b) = \begin{cases} ap_A + \frac{1}{2}(l-a-b)p_A + \frac{1}{2c}p_A p_B - \frac{1}{2}p_A^2 & \text{if } |p_A - p_B| \leq c(l-a-b) \\ lp_A & \text{if } p_A < p_B - c(l-a-b) \\ 0 & \text{if } p_A > p_B + c(l-a-b) \end{cases}$$

$$\pi_B(p_A, p_B, a, b) = \begin{cases} bp_B + \frac{1}{2}(l-a-b)p_B + \frac{1}{2c}p_A p_B - \frac{1}{2}p_B^2 & \text{if } |p_A - p_B| \leq c(l-a-b) \\ lp_B & \text{if } p_B < p_A - c(l-a-b) \\ 0 & \text{if } p_B > p_A + c(l-a-b). \end{cases}$$

Figure 1 shows a graphical representation of each of the three cases. On the horizontal axis, the firms change their location, while in the vertical axis they can change their prices. The red and blue dots represent the location of firm A and B (respectively), and the diagonal dotted lines represent the cost of consumers to purchase their product. In the leftmost example $P_a = P_b$, and both firms share (unevenly) the mass of consumers. In the center case, A sells to all consumers since P_b is so high that consumers at B's location

[6] Our experiment only examines the two-player game, but the setup can be generalized to n sellers. See Brenner (2005) for a derivation of the model with more than two players.

(b) will prefer to purchase from A. In the last case, P_a is too high, and all consumers, even those located at a, will prefer to consume B's product, leaving A with no payoffs.

Figure 2 presents a specific example of the profit function faced by firms in our setup. The right panel shows the profit function of A when it varies its price (p_A) between 0 and 1 while located at $a = 0.25$, with firm B at $0.6 \rightarrow b = (1 - 0.6) = 0.4$ with a fixed price of $p_B = 0.5$ and $l = c = 1$ (see left panel in 2). As shown in the right panel, as p_a increases, A transitions from controlling the entire territory ($p_a \in [0, 0.15)$) to possessing a portion of the market jointly with B ($p_a \in [0.15, 0.85]$) to finally holding no market ($p_a \in (0.85, 1]$).

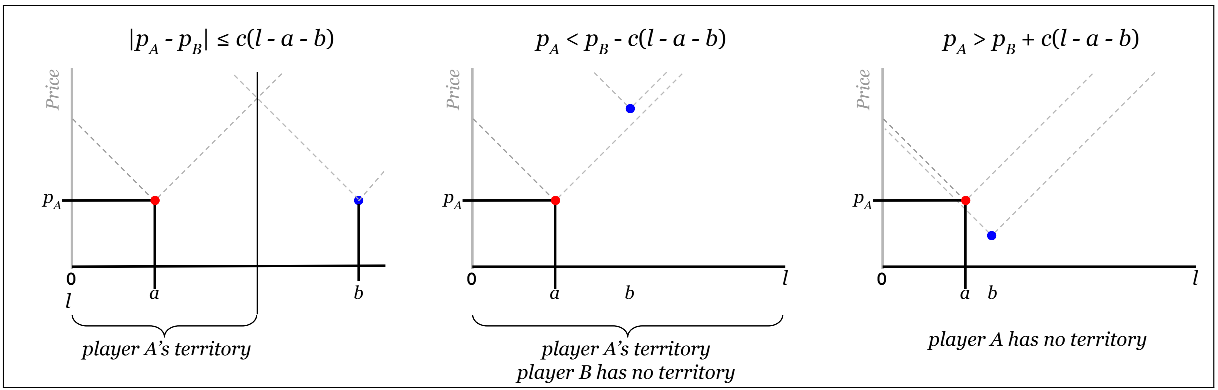


Figure 1: Three examples of how A and B can share the mass of consumers. In all cases, the dots represent the firms' location, and the dashed lines emanating from each firm's location represent the total cost (transport costs plus price) to consumers along the horizontal axis. In the leftmost case, A and B have the same price and share the consumers. In the middle case, B has set the price (p_b) so high that not even consumers at B's location (b) purchase goods from B. In the rightmost case, B has set the prices so low that even consumers at A's location (a) consume only products from B.

These profit functions are discontinuous at the points where the delivered price of one firm is equal to the price of a rival at the rival's location. At these points, a whole group of consumers will be indifferent between the two firms. D'Aspremont et al. (1979) show that there is a Nash-Cournot equilibrium point only if sellers are sufficiently far from each other or such that

$$\left(l + \frac{a - b}{3}\right)^2 \geq \frac{4}{3}(a + 2b)l, \tag{1}$$

$$\left(l + \frac{b - a}{3}\right)^2 \geq \frac{4}{3}(b + 2a)l. \tag{2}$$

Take as an example any case of symmetric positioning where the sufficient conditions (1) and (2) simplify down to $a = b \leq l/4$—i.e., players are located outside of the first and third quartiles, respectively—-and the Nash-Cournot equilibrium price for both players is

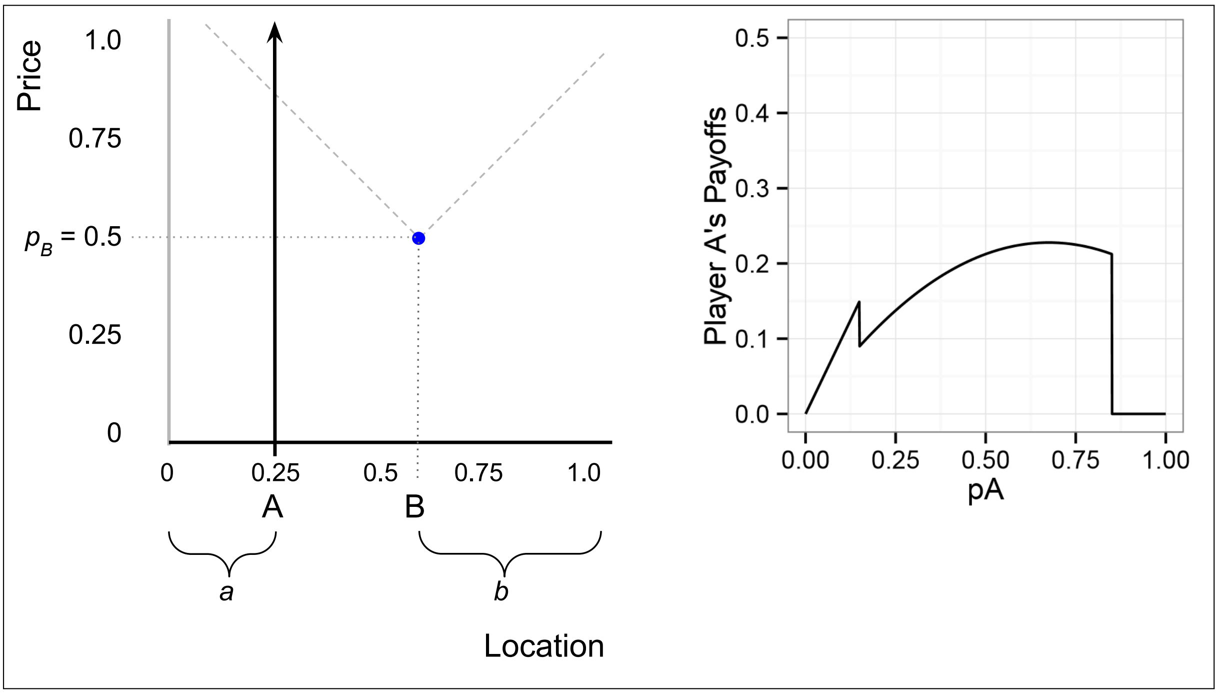


Figure 2: Payoff function for A. On the left panel, we fix A at location a=0.25 and B at b=0.6 with a price $p_b = 0.5$. In the right panel we plot the payoff of A as P_a fluctuates between 0 and 1, given the conditions of the left panel. The vertical drops are discontinuities in the payoff function of A.

$p_a^* = p_b^* = 1$. However, as Hotelling remarked, if conditions (1) and (2) hold, then both $\partial\pi_A(p_a^*, p_b^*)/\partial a$ and $\partial\pi_B(p_a^*, p_b^*)/\partial b$ are strictly positive, implying that each firm should move closer to its rival.[7]

In other words, when sellers locate close to each other, it is optimal for them to undercut each other's price and capture the entire market. But if (1) and (2) hold (i.e., when sellers are far enough), then each firm has the temptation to move closer to her rival to capture their space. However, circularly, once the firms are relatively close to one another, (1) and (2) are violated, implying a Nash equilibrium does not exist. This lack of an equilibrium translates into subjects following each other closely in the action space, with frequent adjustments to their price and location, and a large volatility in profits. Of course, subjects gain higher profits from collusion; however, there are always incentives to cheat.[8] This behavior mirrors the classic prisoner's dilemma, albeit with far more

[7] D'Aspremont et al. (1979) show that for any p_2^* to be an equilibrium strategy against p_2^* given a and b, then $\pi_A(p_1^*, p_2^*) = \frac{c}{2}(l + \frac{a-b}{3})^2 \geq l(p_2^* - c(l - a - b) - \varepsilon)$, which can be rewritten as condition (1), with a complementary expression for condition (2). If these are verified, then $\partial\pi_A(p_A^*, p_B^*)/\partial a > 0$ and $\partial\pi_B(p_A^*, p_B^*)/\partial b > 0$.

[8] A possible evasion of this problem is to characterize the setup as a two-stage game, where firms first simultaneously choose a location and then simultaneously choose a price (with full information).

intermediate outcomes. Friedman and Oprea (2012) showed that continuous-time treatments greatly increase cooperation; as such, we would predict successful non-competitive behavior to be much more prevalent with fewer restrictions on adjustment. More generally, moving from discrete to continuous time can substantially affect strategic interaction (Simon and Stinchcombe, 1989), but cooperation is more likely to be sustainable when it can be supported by an equilibrium (Dal Bó and Fréchette, 2018).

3 Experimental Design

The experiment was performed in sessions differing only in the timing of the game. We study three treatments: DS, CI, and CS. Sessions included only one of the treatments and consisted of 2 practice periods followed by 12 potentially paid periods. Subjects were randomly matched into two-person pairs and were re-matched with a new counterpart each period. To avoid endgame effects, periods lasted four to five minutes with random endings. Sessions contained six participants, and subjects could be re-matched to any other subject at the start of another period.

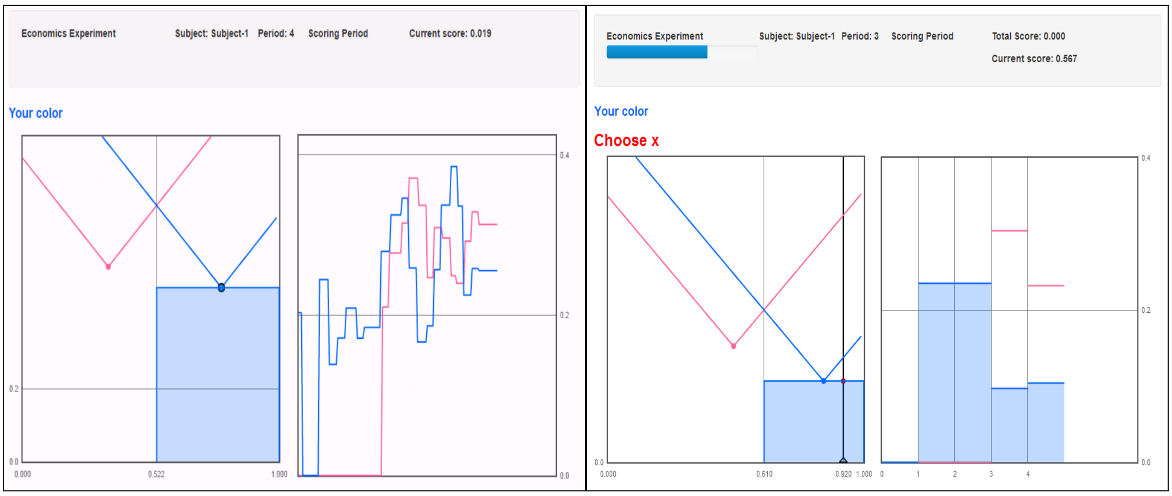
Figure 3 presents the user interface for the continuous (panel 3a) and discrete cases (panel 3b). In all treatments, participants chose their location and price by clicking in the x-y action space, represented at the left of the user interface. The action selections could be made with pixel precision. In some previous laboratory investigations, action selection grids have been limited from the single digits to several dozen discrete actions available. Our implementation—with several hundred thousand available x-y coordinates available to participants—approximates continuous action selection far more closely.[9]

In the DS treatment, subjects played an n-stage game in which location is selected first, followed by price with full information about location decisions. Subjects were given three seconds to choose their location, indicated by a progress bar on the top of their computer screen.[10] The screen then adjusted to reflect the location the subject and her counterpart chose, and subjects were given three seconds to choose a price, again indicated

Dasgupta and Maskin (1986) prove that each price-setting stage has an equilibrium in mixed strategies, however Osborne and Pitchik (1987) cannot characterize them.

[9]We implement an action space of 425 square pixels, resulting in about 180,000 potential location and price combinations available to subjects.

[10]Across all treatments, subjects could select their starting at the beginning of each period.



(a) Continuous Time

(b) Discrete Time

Figure 3: User Interfaces for the continuous and discrete treatments.

by a progress bar. We define these three-second intervals as subperiods. Subjects had four subperiods of price decisions before they were allowed to readjust the location.[11]

Panel 3b shows the user interface for the DS treatment. "Flow" payoffs are shown as bars on the right of the interface and are updated after every subperiod. The blue dot indicates the subject's position in the last subperiod, while the red dot indicates the counterpart's position. The black line shows the subject's current choice for that subperiod, while the gray line simply follows the mouse. The user interface included the linear transport costs running away from their position, the cutoff that determined the edge of the area they control, and a shaded region showing the area they control.

In the CI treatment, subjects chose both location and price freely and instantaneously.[12] Panel 3a shows the user interface for this treatment. Flow payoffs are shown to the right of the interface and are updated continuously. The blue dot indicates the subject's current position, and the pink dot shows her counterpart's current position. The gray crosshairs simply follow the mouse.

[11]In pilot sessions, we also ran treatments in which location- and price-setting subperiods alternated, with no discernible difference in subject behavior. These pilot sessions also allowed us to calibrate the length of time subjects were given for adjustment decisions, with three seconds deemed long enough to process and move their position in the action space without disengaging during subperiods.

[12]The latency between a subject's click and seeing the action on the computer screen was around 50 milliseconds, or far faster than human reaction time. This latency did increase slightly during periods of very frequent position adjustment by subjects but not above tolerable levels that would disrupt subject behavior.

The CS treatment is identical to the CI treatment except for a "speed limit" on subject movement in the action space. When a subject chose a new location and price coordinate, a gray dot appeared at that location while her actual position adjusted slowly to that point. If a subject wanted to change direction while her position was adjusting, a new gray dot appeared, and her position immediately began to adjust to the new target. As an analog to the discrete-time treatment, the subject position could be adjusted four times quicker on the price dimension than on the location dimension.

Our subject pool comprised 72 undergraduates from all major disciplines at the University of California Santa Cruz.[13] Subjects were invited through ORSEE (Greiner, 2015), and the sessions were programmed and run using ConG (Pettit et al., 2014). None of the subjects had previous experience with Hotelling experiments in our lab. Upon entering, subjects were randomly seated and the instructions were read aloud. Subjects then saw a short silent instructional video with on-screen text, which was read aloud as it appeared. Pink noise—a full-frequency audio process used to mask ambient noises—was played in the background to prevent subjects from hearing the mouse clicking of other subjects. Sessions lasted 80–90 minutes each, and subjects were paid their point total multiplied by \$20 for two periods, which were decided by an overt dice roll by one of the participants. Average earnings across all types of sessions were \$16.21 (a breakdown by treatment is available in Table 7 of Appendix A).[14]

4 Results

4.1 Subject Price and Location Decisions

Figure 4 shows heat maps of all players' price and location decisions by treatment, respectively. In these figures, "hotter" colors mean players spent more time in these positions, while "cooler" colors indicate little time was spent in that area of the action space. The most striking feature of these figures is the heat distribution between continuous- and

[13]This is in line with the numbers used in other comparable continuous-time experimental setups. For example, Friedman et al. (2015) also have 72 subjects, Kephart and Friedman (2015) uses 52 subjects, Bosch-Rosa (2018) 92, and Magnani and Munro (2020) has 80.

[14]Due to a computer glitch, we are missing the data for group 2 in period 9 of session 4 of the CI treatment. This would represent around 5,000 observations out of a total of 1,500,000 (i.e., 0.33 percent of the data).

		Discrete (DS)	Continuous Slow (CS)	Continuous (CI) Instant
Price	Mean	0.575	0.468	0.588
	SD	0.281	0.271	0.266
Payoff	Mean	0.255	0.215	0.273
	SD	0.218	0.199	0.219
Location	Mean	0.542	0.509	0.534
	SD	0.247	0.170	0.186

Table 1: Mean and standard Deviation of the price, payoff, and location for each treatment.

discrete-time treatments. Subject's positions are clearly more concentrated in continuous-time treatments, with discrete-time positions more evenly distributed in the action space. At the same time, in continuous-time treatments players tended to be centrally located on the x-dimension, while price positions vary more by treatment. It is also clear from Figure 4 that prices were highest in the CI treatment and lowest in the CS treatment.

To analyze the data of the continuous-time sessions we follow Kephart and Friedman (2015) and use 100 milliseconds as our unit of time (tick), which leaves us with 600 observations for each minute of play. Table 1 provides basic summary statistics of the mean instant payoff ($payoff_{i,p,t}$) for subject i in period p at time t and price ($price_{i,p,t}$), and location ($location_{i,p,t}$) across treatments. As is clear in Figure 4, in the CI treatment subjects have the highest average prices and profits of any treatment. When subjects can adjust price quickly but not location, prices are lower than with instant adjustment, and median payoffs are the lowest of any treatment. For comparison, if both subjects exhibited joint profit-maximizing behavior, prices would be equal to 1 and profits would be equal to 0.5 for each pair's subject.

In Table 2 we present the results of an OLS regression of the price choices of subjects at each tick ($price_{i,p,t}$) (columns (1) and (2)), the instantaneous payoffs of each subject at each tick ($payoff_{i,p,t}$) (columns (3) and (4)), and the location of subjects at each tick ($location_{i,p,t}$) (columns (5) and (6)) on a dummy for each of the three treatments where CI is the baseline treatment. The results show that both price and payoffs tend to be lower in the CS and DS treatments than in the CI treatments, although these differences are only statistically significant at the 5 percent level for the CS treatment. This result confirms what we saw in Figure 4 and Table 1: subjects achieve higher levels of collusion (i.e.,

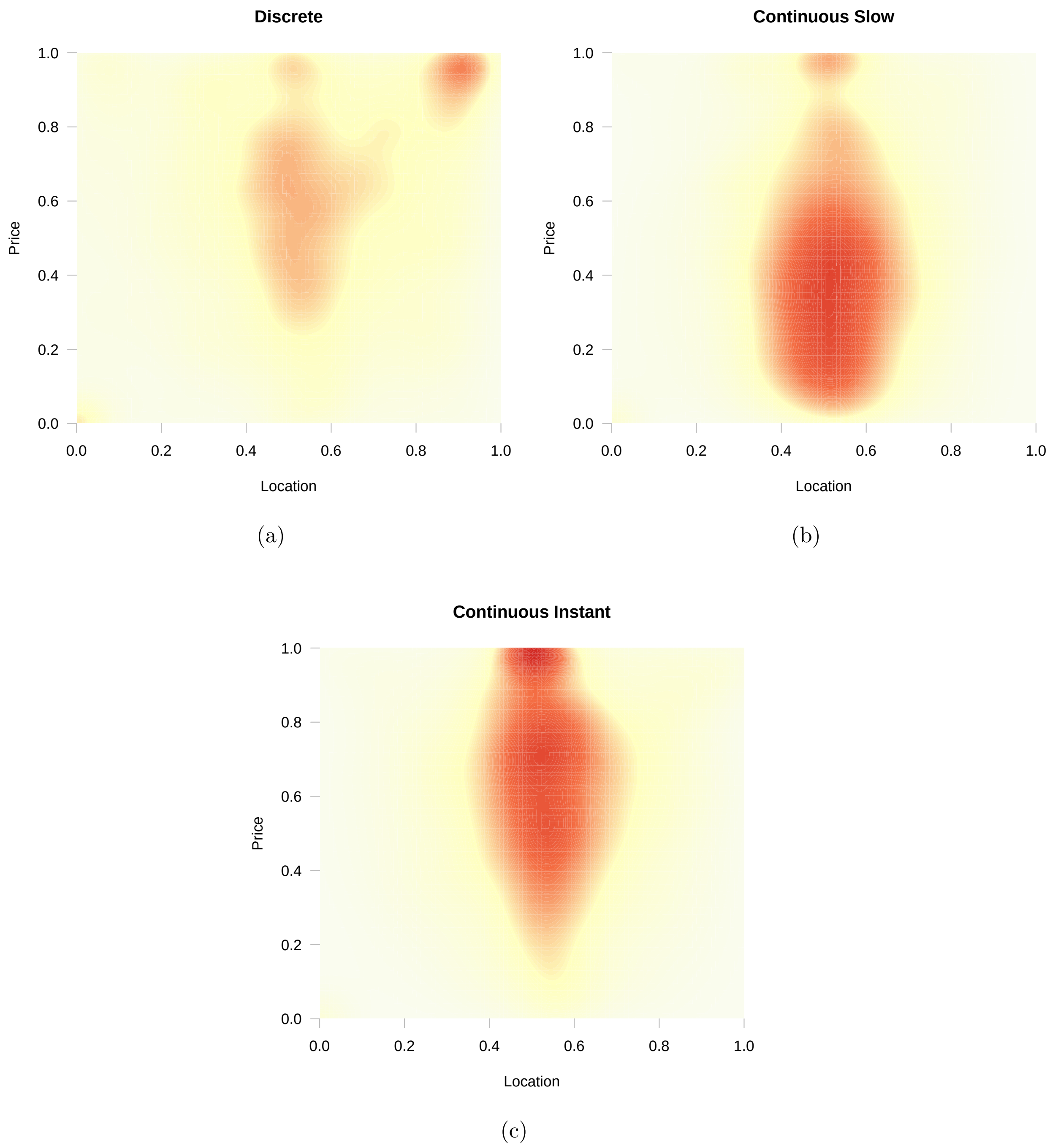


Figure 4: Heat Maps of Subject Price and Location Decisions by Treatment. The figure shows price and location decisions by treatment. The heat maps run from cool to hot colors, with "hotter" colors indicating that players spent more time in those positions.

higher prices and payoffs) when they can respond rapidly to their counterpart's actions.[15] Also confirming what we saw in Figure 4: there seems to be no effects of the treatments

[15]The differences in payoffs are available in Figure 7 in Appendix A, where we plot the cumulative density functions of the final payoffs for each treatment.

	price		payoff		location	
	(1)	(2)	(3)	(4)	(5)	(6)
Continuous Slow (CS)	–0.120**	–0.119**	–0.0589**	–0.0587**	–0.0254	–0.0253
	(0.0517)	(0.0518)	(0.0262)	(0.0262)	(0.0168)	(0.0168)
Discrete (DS)	–0.0131	–0.0127	–0.0184	–0.0182	0.00815	0.00819
	(0.0628)	(0.0628)	(0.0330)	(0.0331)	(0.0150)	(0.0150)
Constant	0.589***	0.626***	0.274***	0.291***	0.535***	0.532***
	(0.0411)	(0.0379)	(0.0216)	(0.0204)	(0.00710)	(0.00802)
N	1,545,168	1,545,168	1,545,168	1,545,168	1,545,168	1,545,168
Period FE	No	Yes	No	Yes	No	Yes

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Least Square with Fixed Effects. In the first two columns, we regress price for each subject at each tick (*price*) on dummies for the different treatments. In columns (3) and (4) we regress the the instantaneous payoffs of each subject at each tick (*payoff*), while in columns (5) and (6) the dependent variable is the location of each subject at each tick *location*. All standard errors are robust and clustered at the session level.

on $location_{i,p,t}$, with the intercept being around 0.5 (i.e., minimum differentiation).

To study how subjects coordinate in each treatment, we define three different measures of distance between subjects.The first one is based on the distance between subjects on the location axis ($LocationDistance_{g,p,t}$) for each group g in each period p at time t. The second measure is the distance on the price axis ($PriceDistance_{g,p,t}$) for each group g in each period p at time t, and the last one is the Euclidean distance between subjects ($EuclideanDistance_{g,p,t}$) for each group g in each period p at time t.[16] Both axes are scaled to one such that a distance of 0.1 is very close to the other player, while a distance of 0.5 is quite far from the counterpart.

Table 3 presents summary statistics on different measures of distance between subject pairs. Subjects were much closer together on all measures of distance in the continuous-time treatments such that the discrete stage game tended to push subjects apart in the action space. This can be seen easily in Figure 4's heat maps. Price distance is consistently smaller across treatments, even in the continuous-time treatment that did not inhibit location adjustment in any way. Appendix B shows and discusses the vector fields resulting from subjects' movement decisions.

[16]The Euclidean distance is the linear distance (on the plane) between both subjects, which, by Pythagoras (Yanney and Calderhead, 1896), is defined as $EuclideanDistance_{g,p,t} = \sqrt{PriceDistance^2_{g,p,t} + PriceDistance^2_{g,p,t}}$.

		Discrete (DS)	Continuous Slow (CS)	Continuous Instant(CI)
Location Distance	Mean	0.252	0.128	0.161
	SD	0.218	0.123	0.166
Price Distance	Mean	0.171	0.112	0.108
	SD	0.180	0.116	0.130
Euclidean Distance	Mean	0.343	0.190	0.214
	SD	0.235	0.147	0.191

Table 3: Comparison to Counterpart Statistics by Treatment. The table shows the mean and standard deviation of distances on specified dimension by treatment. Axes are scaled such that maximum differentiation would give a distance of one.

Finally, using a linear regression with fixed effects for each period, we study the effect of each treatment on the Euclidean distance between players, the distance between their location, and the distance between prices. Table 4 presents the results, showing that all of the distances are larger in both CS and DS than in the CI baseline. As expected, such distances are bigger in the DS treatment than in the CS treatment.

Overall, Tables 2 and 4 are in line with the results of Friedman et al. (2015) and Bigoni et al. (2015), who show that continuous- and long-time horizons foster collaboration between experimental subjects. To have a better understanding of how subjects reach such collusive states, in Section 4.2 we study the dynamic behavior of subjects under all three treatments.

4.2 Non-Competitive Behavior

As detailed in Section 2, subjects have an ever-present incentive to undercut on either dimension. However, given that the mean and median prices are between 0.5 and 0.6, it is clear that subjects do not undercut each other. To study this collusive behavior more in depth, we define non-competitive behavior as the *fraction of time* each pair of subjects refrains from immediately undercutting each other. We first define such non-competitive behavior as a situation where two players can settle into a position where the payoffs of each subject are within 20% of each other.[17]

[17]Although this is a rather conservative and *ad hoc* threshold, our results are robust to a range of changes to this threshold.

	loc_dist		price_dist		euc_dist	
	(1)	(2)	(3)	(4)	(5)	(6)
Continuous Slow (CS)	–0.0339	–0.0338	0.00447	0.00446	–0.0242	–0.0242
	(0.0214)	(0.0214)	(0.0159)	(0.0158)	(0.0291)	(0.0290)
Discrete (DS)	0.0910***	0.0910***	0.0633***	0.0633***	0.129***	0.129***
	(0.0174)	(0.0174)	(0.0149)	(0.0149)	(0.0184)	(0.0184)
Constant	0.162***	0.180***	0.108***	0.113***	0.214***	0.236***
	(0.0119)	(0.0135)	(0.0107)	(0.0124)	(0.0170)	(0.0145)
N	772,584	772,584	772,584	772,584	772,584	772,584
Period FE	No	Yes	No	Yes	No	Yes

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Least Square with Fixed Effects. In the first two columns the dependent variable is the distance between subjects in the location axis (loc_dist) and the independent variables the dummies for the different treatments (CS and DS). In columns (3) and (4), the dependent variables is the distance in the price axis (price_dist), while in and in columns (5) and (6) the dependent variable is the Euclidean distance between both subjects. All standard errors are robust and clustered at the session level.

	Discrete (DS)	Continuous Slow (CS)	Continuous Instant (CI)
Payoff20pct (difference of 20% or less between both payoffs)			
Mean	0.167	0.218	0.253
SD	0.138	0.147	0.191
Steady Positive Payoffs (both subjects have positive payoffs)			
Mean	0.599	0.539	0.608
SD	0.147	0.184	0.154

Table 5: Non-Competitive Behavior Rates by Treatment.

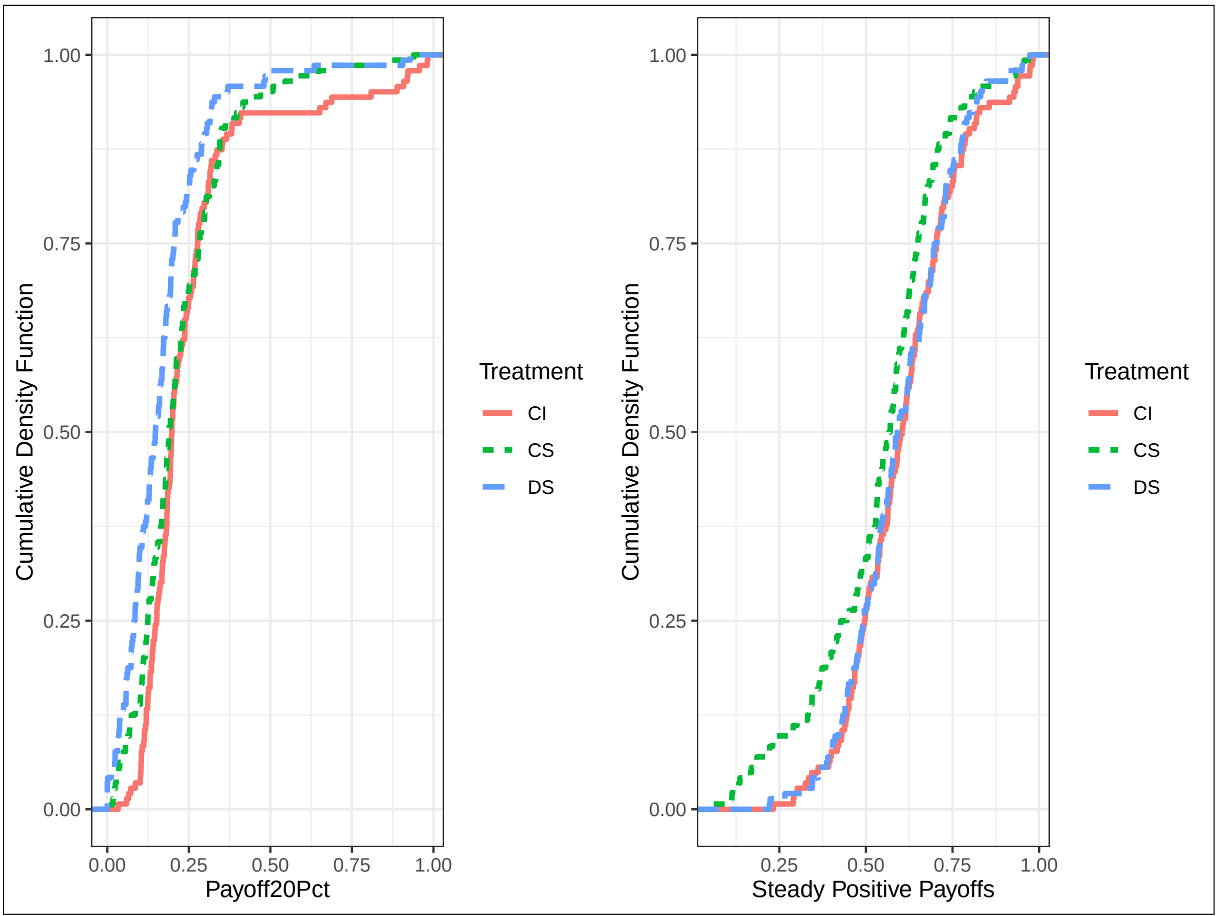
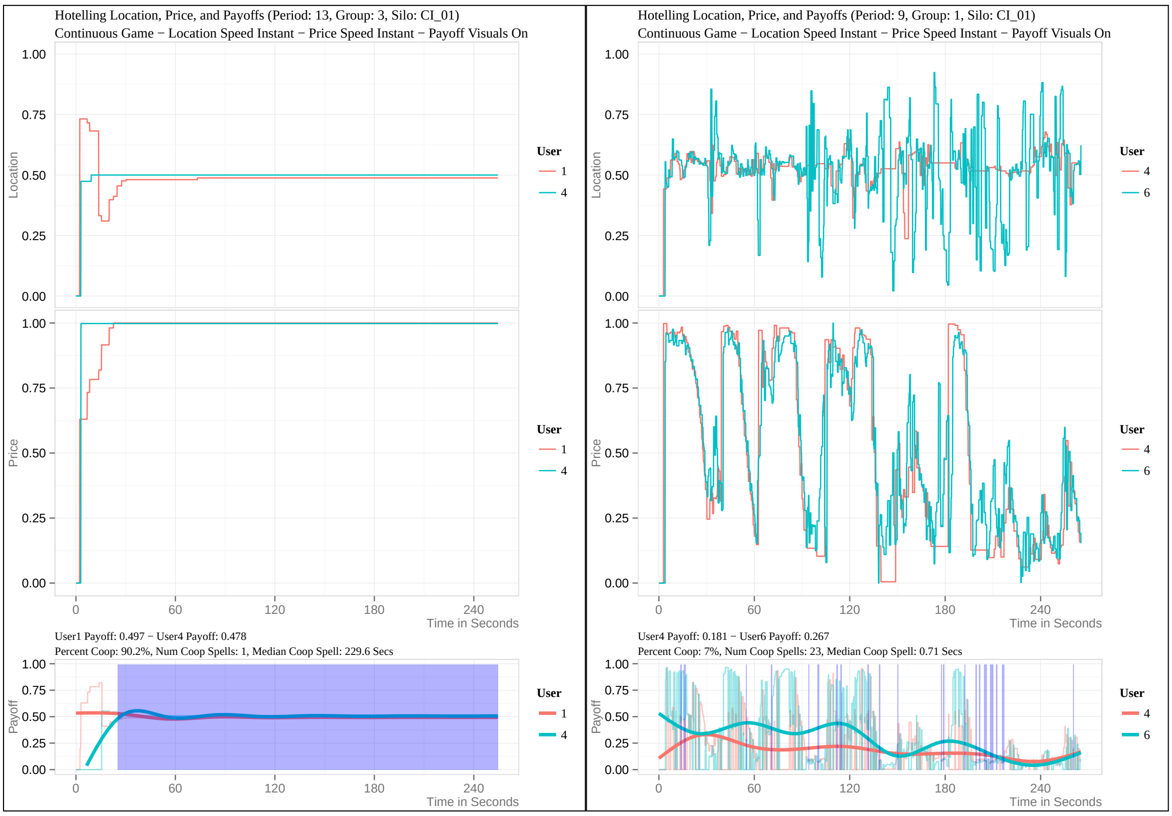


Figure 5: Cumulative density functions. The figure shows the Cumulative density functions' plotting percentage of time in each non-competitive behavior across all periods and treatments.

Another way of defining non-competitive behavior is what we call *Steady Positive Payoffs*. This concept abstracts away from a specific threshold, with $\rho_{g,p}$ being any time when both subjects have positive payoffs. The subject pair's spell in Steady Positive Payoffs is broken if one of the players undercuts her counterpart. The rates that come from this definition are reported in Table 5, which shows that subjects carved out some portion of the market well over 50% of the time. As expected, the CI treatment had the least intense competition. However, CS treatments had lower non-competitive rates by this measure and were even lower than the DS treatment.

In Table 6 we regress both measures of cooperation on dummies for the different treatments and for each period.[18] The results show that the differences between treatments heavily depend on the definition we use for *cooperation*. When looking at the differences

[18] 6 has 431 and not 432 observations, because as reported in footnote 16, we lost the data for period 9 of Group 2 of session 4 of the CI sessions due to a computer glitch.



(a) Collusion (b) No Collusion

Figure 6: Two very different pairs. The left panel shows two subjects who successfully colluded for the majority of a period in the Continuous Time (CI) treatment. The top panel is the subject pairs' location decisions over time, the middle panel is their price decisions over time, and the bottom panel is their flow payoff. Note that player 4 was an "aggressive colluder," as she willingly took losses at the beginning of the period while waiting for her counterpart to conform. The right panel shows two subjects who were unable to collude in a period in the CI treatment. The top panel is the subject pairs' location decisions over time, the middle panel is their price decisions over time, and the bottom panel is their flow payoff. Note that player 4 is the same subject shown in Figure 6a but with a more aggressive counterpart.

in the time spent in positions where the payoffs are within 20% of each other, we see negative values for the parameters of both treatments, but only the DS coefficient is significant at a 10% level. However, if we define non-competitive behavior as the amount of time in which both subjects of a pair get a positive payoff, then the results change, as pairs in the CS treatment spend significantly less time in a collusive state than those in the CI treatment, while we observe no significant differences for the DS treatment. Such a flipping of differences in collusive behavior is clear in Figure 5, where we plot the cumulative density functions of time spent in a non-competitive behavior (for each definition) for each treatment.

Finally, as an illustration of how non-competitive states are reached, Figure 6a provides

	Payoff20pct		Steady Positive Payoffs	
	(1)	(2)	(3)	(4)
Continuous Slow (CS)	–0.0349	–0.0350	–0.0689**	–0.0689**
	(0.0473)	(0.0479)	(0.0252)	(0.0255)
Discrete (DS)	–0.0861*	–0.0862*	–0.00889	–0.00890
	(0.0449)	(0.0455)	(0.0362)	(0.0366)
Constant	0.253***	0.223***	0.608***	0.610***
	(0.0405)	(0.0390)	(0.0231)	(0.0351)
N	431	431	431	431
Period FE	No	Yes	No	Yes

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Least Square with Fixed Effects. In the first two columns the dependent variable is the fraction of time in each round in which the payoffs of subjects are within 20% of each other (Payoff20pct) and the independent variables the dummies for each treatment (CS and DS). In columns (3) and (4) the dependent variable is the fraction of time in each round in which both subjects have positive payoffs (Steady Positive Payoffs). All standard errors are robust and clustered at the session level.

circumstantial evidence of how players were able to coordinate. It shows a subject pair in the CI treatment, with shaded regions indicating non-competitive behavior between subjects. In the bottom panel, the thick lines are smoothed flow payoffs for each subject, while the actual flow payoffs are shown in the background. At the very beginning of the period, player 4 (the orange player) immediately adjusts her price to the maximum allowed (normalized to one) and her location to the middle. Notice that doing this reduced her payoff to be lower than her counterpart's while she waited for her counterpart to fall in line with her strategy. The subject pair colluded for almost the entire period, as indicated by the blue bars in the payoff figure, resulting in higher than average payoffs for the period.

On the other hand, Figure 6b shows a typical case of players following each other in the action space throughout the period. Player 4 is *the same player* who aggressively pushed for a collusive state in Figure 6a but is now matched with a more competitive player. Notice that she repeatedly attempts to increase prices, thus taking a momentary loss. But player 4's counterpart immediately undercuts her, forcing her to be drawn into tight competition. At the end of the period, player 4's payoffs are much lower than her counterpart's due to her attempts to ease competition. This kind of behavior was typical in the game, as "aggressive colluders" could only coax anti-competitive behavior out of a relatively low number of counterparts.

5 Conclusion

Hotelling's principle of minimum differentiation in one-dimensional spaces is both intuitive and elegant. However, this principle has proven to be sensitive to modifications in the model assumptions. Take, for example, changing the number of firms; while there is a Nash equilibrium in pure strategies for the case of two firms, there is not one for three firms but there is for four. Furthermore, as shown in D'Aspremont et al. (1979), Hotelling was wrong in asserting that the principle of minimum differentiation applied to his model, as, contrary to the well-known one-dimensional case, his original model has firms deciding on a *two-dimensional strategy space*. So, not only is that ice cream cart deciding where on the beach to set up shop, but it is also deciding at what price to sell the ice cream. This second dimension is crucial, as it creates a tension between the forces that draw firms to minimum differentiation (maximizing demand) and those that draw them to maximal differentiation (avoiding price competition), resulting in the lack of a pure Nash equilibrium.

The lack of a pure strategy Nash equilibrium in Hotelling's original setup motivates our work. To test whether Hotelling's intuition was right, we ran a continuous-time experiment where subjects could adjust their price and location. A laboratory setting seems ideal for this setup since the attribute space across products is clearly defined and pricing is not affected by second-order effects such as state regulation or unrelated brand strategic positioning. Additionally, our experimental setup also allows us to introduce different treatments that vary in how often subjects are allowed to adjust their price and position, contributing to the literature that studies the strategic differences between continuous- and discrete-time setups (e.g., Friedman and Oprea, 2012; Bigoni et al., 2015; Benndorf et al., 2016).

Our principal findings can be summarized briefly. First, subjects tended to locate close together in the middle of the action space, especially in continuous-time treatments. In the CI and CS treatment, subjects were heavily concentrated in the center and only 10% of the action space away from their counterpart, supporting Hotelling‚Äôs principle of minimum differentiation. Second, non-competitive behavior was higher in continuous-time treatments. Our results show that the free and instantaneous adjustment leads to the least intense competition, and we give circumstantial evidence that subjects aggressively push for collusive states at the expense of short-term payoffs. This is in line with previous laboratory experiments showing that the ability to respond quickly can increase cooperation (e.g., Friedman and Oprea, 2012).

Additionally, we find an interesting effect by which the CS treatment results in lower prices (i.e., more competitive environment) and overall lower payoffs for both subjects than in the DS treatment, an environment in which collusion should be harder to sustain. We believe that these effects come from the cost of retaliations across markets. In the CS treatment, a price war could be amended (relatively) fast, so deviations were tempting; however, in the DS treatment, any deviation from collusion could result in a very costly war. Similar to the "mutually assured destruction" doctrine of the Cold War, the fear of irreparable losses pushed subjects into relatively high rates of collusion in the CS treatment.

To conclude, our results show that Hotelling was (intuitively) right, even if (formally) wrong. In an oligopolistic setup where two firms can decide over two dimensions of their product, the principle of minimal differentiation will largely hold with firms colluding at high prices. However, how long this collusion can be sustained or (if ever) achieved will depend dramatically on their environment. This means that many applications of Hotelling's model, from voting theory to gas station placement, should be viewed with extreme caution in light of the instability — especially on the price dimension — shown in our experiment.

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A Additional Tables and Figures

Table 7

Table 7: Subjects and Payouts by Treatment

Treatment	Number of Subjects	Average Payout
Continuous Instant	24	\$18.39
Continuous Slow	24	\$14.67
Discrete	24	\$16.71
Total	72	\$16.59

Notes: "Average Payout" includes the \$5 show-up fee.

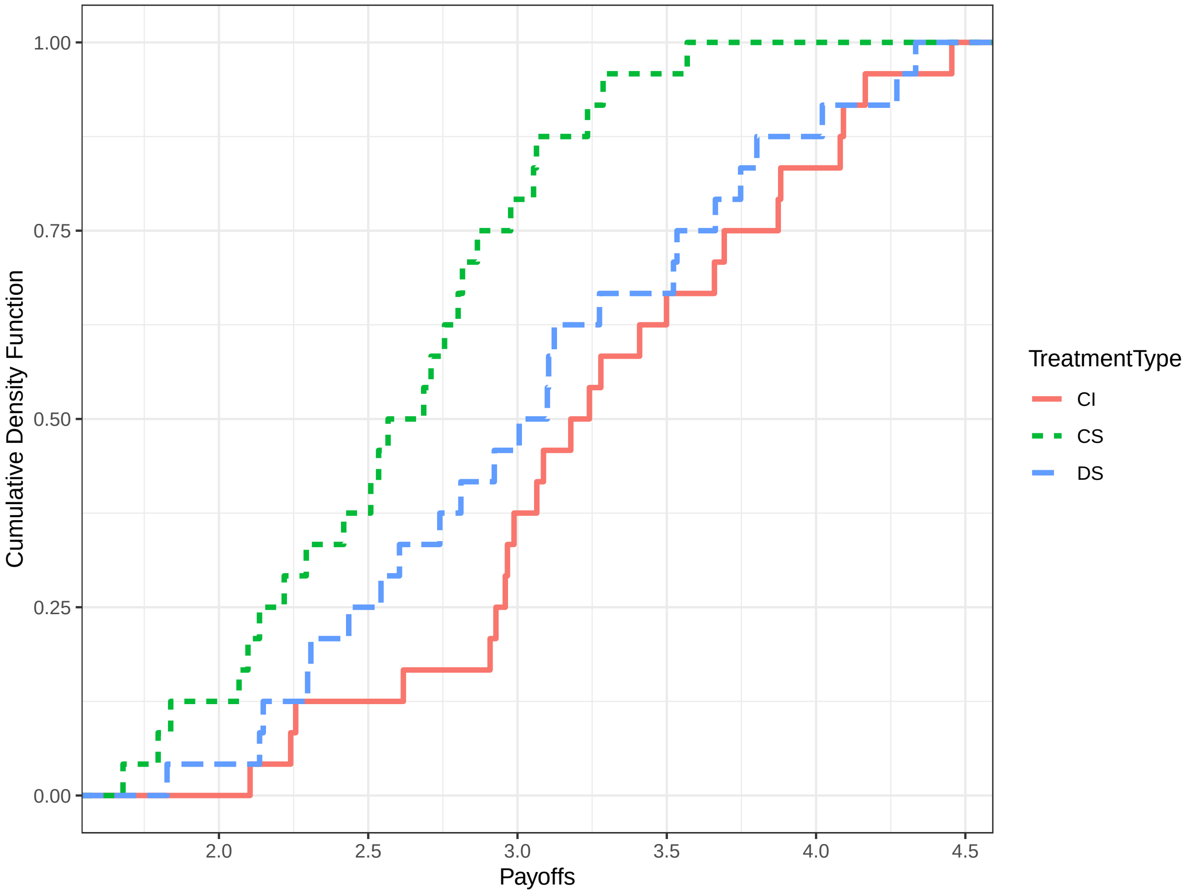


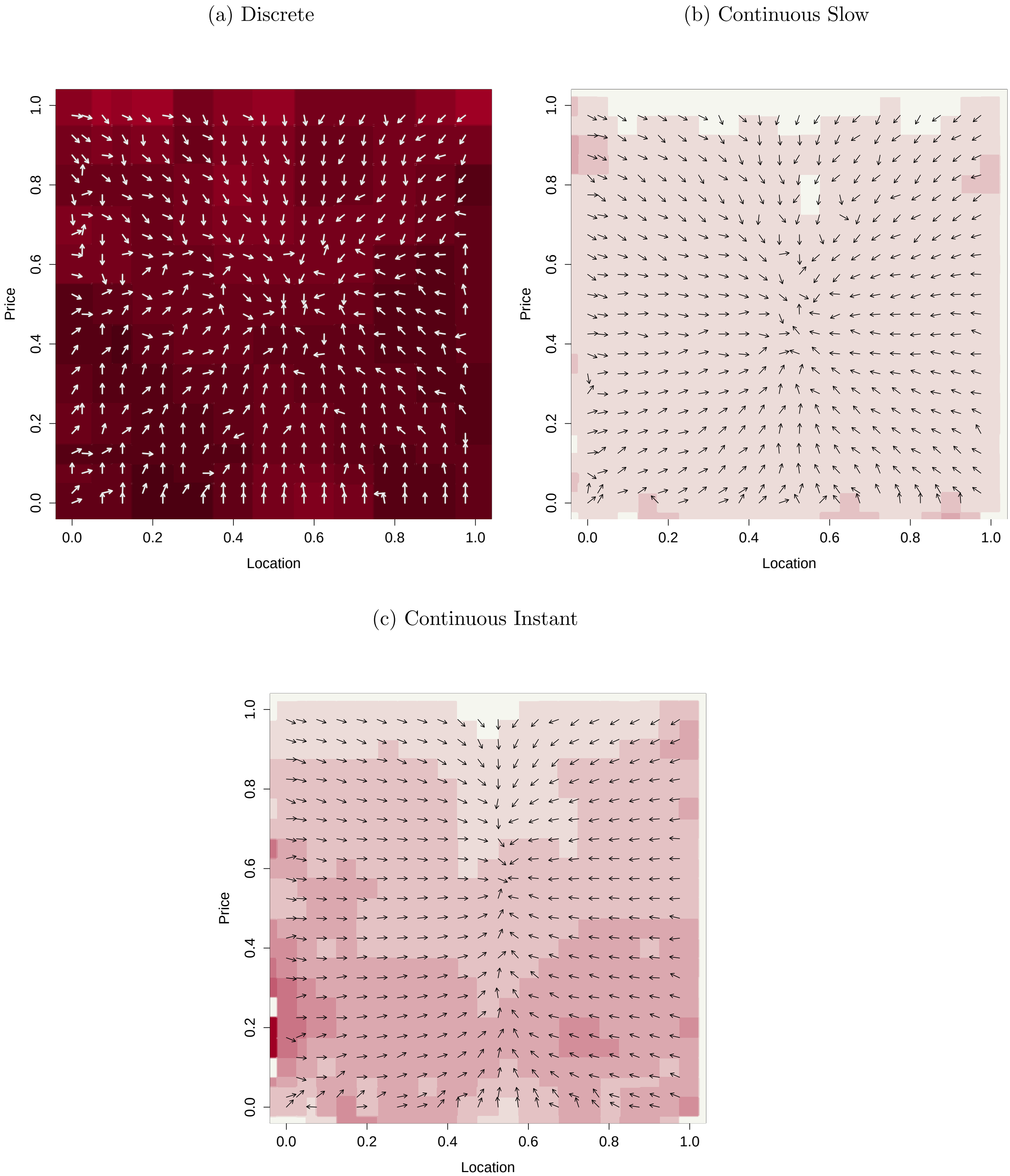
Figure 7: Subject Final Payoff by Treatment.

B Movement of Subjects

We have documented where subjects tended to locate in both dimensions, but we also wanted to characterize their movement when they did make adjustments. For this, we present in Figure 8 a form of an empirical vector field in which average subject movement from a given position is shown. Here, vectors show the average direction that subjects moved starting from that neighborhood. In the background, colors map to the percentage of observations in that neighborhood for which players changed their action set. Darker colors indicate that subjects tended to change their price/location decision in that area more often, with the direction of the change following the overlaid vector, on average.

Subject adjustments vary greatly by treatment. In the *Discrete* treatment, subjects tend to lower high prices, raise low prices, and tend to change their actions no matter where they are placed making the behavior is somewhat erratic. Movement in the *Continuous Slow* treatment is slightly clearer, with subjects tending to adjust toward the center. The heatmap for *Continuous Slow* is a bit deceptive since action changes were rate limited by the "slow" speed limit. The heatmap for this treatment shows changes in players *target* location and prices, which were far less frequent than in the *Discrete* or *Continuous Instant* treatments. But the clearest story emerges from the *Continuous Instant* treatment. Here, the lower edges of the figure are darker as subjects made more frequent adjustments to avoid being "boxed in" by their counterpart. Prices tend to adjust upward until about 0.6 — the median in this treatment — and downward above that. Central locations with medium to high prices tended to be the most stable action sets.

Figure 8: Vector Fields of Subject Position Adjustments by Treatment



Notes: These figures show all players' price and location decisions by treatment. Arrows indicate the average direction of action set changes starting from the arrow's neighborhood. The heat maps run from cool to hot colors, with "hotter" colors indicating that players were more likely to change their action set while in that neighborhood.