

Self Energy effect in frequency dependent Vertex flow equation

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I. INTRODUCTION

Introduction bla bla

II. FORMALISM

A. Model

The Hubbard model¹ describes spin- $\frac{1}{2}$ fermions with a density-density interaction:

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} \quad (1)$$

where $c_{i,\sigma}^\dagger$ and $c_{i,\sigma}$ are, respectively, creation and annihilation operators for fermions with spin $\sigma = \uparrow, \downarrow$. We consider the two-dimensional case with square lattice and repulsive interaction $U > 0$. The hopping amplitude is restricted to $t_{ij} = t$ for nearest neighbors, $t_{ij} = t'$ for next-to-nearest neighbors.

B. Flow equations

In the following paragraph we will introduce the functional renormalization group^{2,3} in the implementation that we used, and we will clarify some notation issue about the vertex.

Generally speaking, the fRG allows to use the renormalization group idea in the functional integral formalism. This is done by endowing the non-interacting propagator with an additional dependence on a scale parameter Λ , which generates an exact functional flow equation with known initial conditions.

We will apply this approach to the effective action, whose expansions into the fields generates the one-particle irreducible (1PI) functions. By expanding the functional flow equation one obtains a hierarchy of flow equations for the 1PI functions, involving vertexes of arbitrarily high orders. We will restrict ourselves to the level-two truncation by retaining only the two lowest nonvanishing orders in the expansion, i.e., we consider the flow of the (scale dependent) self-energy Σ^Λ and of the two-particle 1PI vertex V^Λ , neglecting the effects of higher order vertexes. Hence our approach becomes

perturbative, and sums up efficiently, although approximately, the so-called parquet-diagrams⁴⁻⁶.

We use the energy and momentum conservation to fix one of the arguments of the self energy and of the vertex. Furthermore we restrict ourselves to the spin-symmetric phase, at finite temperature using discrete Matsubara frequencies. Hence for the self-energy we only need to consider one function depending on one frequency-momentum argument:

$$\Sigma_{\sigma\sigma'}^\Lambda(k) = \Sigma(k) \delta_{\sigma,\sigma'}, \quad (2)$$

where $\sigma = \{\uparrow, \downarrow\}$, and $k = (\nu, \mathbf{k})$, ν being a Matsubara frequency and \mathbf{k} a momentum in the first Brillouin zone.

For the notation of the two-particle vertex we refer to Fig. (fig), where $k_i = (\nu_i, \mathbf{k}_i)$, and $k_4 = k_1 + k_2 - k_3$. Furthermore SU(2)-symmetry guarantees that the vertex does not vanish only for six spin combinations⁷: $V_{\uparrow\uparrow\uparrow}^\Lambda = V_{\downarrow\downarrow\downarrow}^\Lambda$, $V_{\uparrow\downarrow\uparrow}^\Lambda = V_{\downarrow\uparrow\uparrow}^\Lambda$, and $V_{\uparrow\downarrow\downarrow}^\Lambda = V_{\downarrow\uparrow\downarrow}^\Lambda$. Finally, due to SU(2) symmetry and crossing relation one has:

$$V_{\uparrow\uparrow\uparrow}^\Lambda(k_1, k_2, k_3) = V_{\uparrow\downarrow\downarrow}^\Lambda(k_1, k_2, k_3) - V_{\uparrow\downarrow\uparrow}^\Lambda(k_1, k_2, k_1 + k_2 - k_3), \quad (3)$$

$$V_{\uparrow\downarrow\uparrow}^\Lambda(k_1, k_2, k_3) = -V_{\uparrow\downarrow\downarrow}^\Lambda(k_1, k_2, k_1 + k_2 - k_3). \quad (4)$$

This allows us to consider only one function of three arguments for the vertex: $V_{\uparrow\downarrow\uparrow}^\Lambda(k_1, k_2, k_3) = V^\Lambda(k_1, k_2, k_3)$, all the others spin components being obtained by symmetry.

With these considerations the flow equation for the self energy reads:

$$\frac{d}{d\Lambda} \Sigma^\Lambda(k) = - \int_p S^\Lambda(p) [2V^\Lambda(k, p, p) - V^\Lambda(k, p, k)], \quad (5)$$

with $p = (\omega, \mathbf{p})$ and $k = (\nu, \mathbf{k})$. We use the notation $\int_p = T \sum_\omega \int_{\mathbf{p}}$, $\int_{\mathbf{q}} = \int \frac{d\mathbf{q}}{4\pi^2}$ being the normalized integral over the first Brillouin zone.

$$S^\Lambda = \frac{dG^\Lambda}{d\Lambda} \Big|_{\Sigma=\text{const}} \quad (6)$$

is the single-scale propagator; $G^\Lambda = [(G_0^\Lambda)^{-1} - \Sigma^\Lambda]^{-1}$ is the full propagator, G_0^Λ is the non-interacting Green's function.

The vertex flow equation can be written as:

$$\frac{d}{d\Lambda} V(k_1, k_2, k_3) = \mathcal{T}_{\text{pp}}^\Lambda(k_1, k_2, k_3) + \mathcal{T}_{\text{ph}}^\Lambda(k_1, k_2, k_3) + \mathcal{T}_{\text{phc}}^\Lambda(k_1, k_2, k_3), \quad (7)$$

where:⁸

$$\begin{aligned} \mathcal{T}_{\text{pp}}^\Lambda(k_1, k_2, k_3) = & -\frac{1}{2} \int_p \mathcal{P}^\Lambda(p, k_1 + k_2 - p) \left\{ V^\Lambda(k_1, k_2, k_1 + k_2 - p) V^\Lambda(k_1 + k_2 - p, p, k_3) \right. \\ & \left. + V^\Lambda(k_1, k_2, p) V^\Lambda(p, k_1 + k_2 - p, k_3) \right\}; \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{T}_{\text{ph}}^\Lambda(k_1, k_2, k_3) = & -\int_p \mathcal{P}^\Lambda(p, k_3 - k_1 + p) \left\{ 2V^\Lambda(k_1, k_3 - k_1 + p, k_3) V^\Lambda(p, k_2, k_3 - k_1 + p) \right. \\ & \left. - V^\Lambda(k_1, k_3 - k_1 + p, p) V^\Lambda(p, k_2, k_3 - k_1 + p) - V^\Lambda(k_1, k_3 - k_1 + p, k_3) V^\Lambda(k_2, p, k_3 - k_1 + p) \right\}; \end{aligned} \quad (9)$$

$$\mathcal{T}_{\text{phc}}^\Lambda(k_1, k_2, k_3) = \int_p \mathcal{P}^\Lambda(p, k_2 - k_3 + p) V^\Lambda(k_1, k_2 - k_3 + p, p) V^\Lambda(p, k_2, k_3). \quad (10)$$

Here we have defined the quantity:

$$\mathcal{P}^\Lambda(p, p') = G^\Lambda(p) S^\Lambda(p') + G^\Lambda(p') S^\Lambda(p). \quad (11)$$

For any cutoff choice with $G_0^{\Lambda_0} = 0$ the initial condition for the self-and the vertex are, respectively, $\Sigma^{\Lambda_0} = 0$ and $V^{\Lambda_0} = U$.

III. VERTEX APPROXIMATION

We start by decomposing the vertex as follows:

$$V^\Lambda(k_1, k_2, k_3) = U - \phi_{\text{p}}^\Lambda(k_1 + k_2; k_1, k_3) + \phi_{\text{m}}^\Lambda(k_3 - k_1; k_1, k_3) + \frac{1}{2} \phi_{\text{m}}^\Lambda(k_2 - k_3; k_1, k_2) - \frac{1}{2} \phi_{\text{c}}^\Lambda(k_2 - k_3; k_1, k_2), \quad (12)$$

where the physical meaning of each ϕ channel can be understood by its flow equations:

$$\dot{\phi}_{\text{p}}^\Lambda(Q; k_1, k_3) = -\mathcal{T}_{\text{pp}}^\Lambda(k_1, Q - k_1, k_3), \quad (13)$$

$$\begin{aligned} \dot{\phi}_{\text{c}}^\Lambda(Q; k_1, k_2) = & -2\mathcal{T}_{\text{ph}}^\Lambda(k_1, k_2, Q + k_1) \\ & + \mathcal{T}_{\text{phc}}^\Lambda(k_1, k_2, Q + k_1), \end{aligned} \quad (14)$$

$$\dot{\phi}_{\text{m}}^\Lambda(Q; k_1, k_2) = \mathcal{T}_{\text{phc}}^\Lambda(k_1, k_2, Q + k_1), \quad (15)$$

controllare ph where $Q = (\Omega, \mathbf{Q})$ is a frequency and momentum transfer.

Following Refs. ??, we address first the momentum dependence. To this end, we introduce a decomposition of the unity by means of a set of orthonormal form factors for the two fermionic momenta $\{f_l(\mathbf{k})\}$ obeying the completeness relation:

$$\int_{\mathbf{k}} f_l(\mathbf{k}) f_m(\mathbf{k}) = \delta_{l,m}. \quad (16)$$

The procedure outlined here is described in detail, e.g., in Ref. ?

We can then project each channel on a subset of form factors, whose choice is physically motivated⁷. Let us

stress that the if one could keep all the form factors the expansion would be exact.

For the pairing channel we keep only $f_s(\mathbf{k}) = 1$ and $f_d(\mathbf{k}) = \cos k_x - \cos k_y$:

$$\begin{aligned} \phi_{\text{p}}^\Lambda(Q; k_1, k_3) = & \mathcal{S}_{\mathbf{Q}}^{\Omega; \nu_1, \nu_3} \\ & + f_d\left(\frac{\mathbf{q}}{2} - \mathbf{k}_1\right) f_d\left(\frac{\mathbf{q}}{2} - \mathbf{k}_3\right) \mathcal{D}_{\mathbf{Q}}^{\Omega; \nu_1, \nu_3} \end{aligned} \quad (17)$$

The instability in the channel \mathcal{S} is associated to s -wave superconductivity, while \mathcal{D} to d -wave superconductivity.

For the charge and magnetic channels we restrict ourselves to $f_s(\mathbf{k}) = 1$ only:

$$\phi_{\text{c}}^\Lambda(Q; k_1, k_2) = \mathcal{C}_{\mathbf{Q}}^{\Omega; \nu_1, \nu_2}, \quad (18)$$

$$\phi_{\text{m}}^\Lambda(Q; k_1, k_2) = \mathcal{M}_{\mathbf{Q}}^{\Omega; \nu_1, \nu_2} \quad (19)$$

corresponding to instabilities in the charge and magnetic channels, respectively (for notation simplicity we omit the Λ -dependences of the channel functions \mathcal{S} , \mathcal{D} , \mathcal{C} and \mathcal{M}).

Let us stress that for each channel we have used its own frequency notation: we use one frequency corresponding

to the frequency transferred in the specific channel and two remaining independent frequencies. At finite temperature the frequency transfer, being a sum or a difference of two fermionic Matsubara frequencies, is a bosonic Matsubara frequency.

The choice of the mixed notation is the most natural[?] since the transferred momentum and frequency play a special role in the diagrammatics. Indeed, on the one hand, it is the only dependence generated in second order perturbation theory and the main dependence in finite order perturbation theory. On the other hand, this notation is convenient in the Bethe-Salpeter equations⁷.

In the fRG literature^{???}, where the fermionic frequency dependence is neglected, the channel functions above are sometimes interpreted as bosonic exchange propagators. Such an interpretation is missing in the presence of all frequencies.

Although one expects a leading dependence in the bosonic frequency, in particular in the weak coupling regime, we will see that in some cases the dependence on fermionic frequencies can become strong and not negligible.

The flow equations for the channels \mathcal{S} , \mathcal{D} , \mathcal{C} and \mathcal{M} can be derived from the projection onto form factors of Eq. (8)-(10):

$$\dot{\mathcal{S}}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_3} = - \int_{\mathbf{k}_1, \mathbf{k}_3} \mathcal{T}_{pp}(k_1, q - k_1, k_3); \quad (20)$$

$$\dot{\mathcal{D}}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_3} = - \int_{\mathbf{k}_1, \mathbf{k}_3} f_d \left(\frac{\mathbf{q}}{2} - \mathbf{k}_1 \right) f_d \left(\frac{\mathbf{q}}{2} - \mathbf{k}_3 \right) \mathcal{T}_{pp}(k_1, q - k_1, k_3); \quad (21)$$

$$\dot{\mathcal{C}}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_2} = \int_{\mathbf{k}_1, \mathbf{k}_2} \mathcal{T}_{phc}(k_1, k_2, q + k_1) - 2\mathcal{T}_{ph}(k_1, k_2, k_2 - q); \quad (22)$$

$$\dot{\mathcal{M}}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_2} = \int_{\mathbf{k}_1, \mathbf{k}_2} \mathcal{T}_{phc}(k_1, k_2, q + k_1). \quad (23)$$

The final equations are then obtained by substituting the decomposition (12) into the equations above, and using trigonometric eventualities. As an example we report here the equation for the magnetic channel, while the expression for the other channels are reported in the Appendix:

$$\dot{\mathcal{M}}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_2} = \sum_{\nu} L_{\mathbf{Q}}^{\Omega;\nu_1,\nu} P_{\mathbf{Q}}^{\Omega,\nu} L_{\mathbf{Q}}^{\Omega;\nu,\Omega-\nu_2}, \quad (24)$$

(the Λ -dependence is implicit), with:

$$P_{\mathbf{Q}}^{\Omega;\nu} = \int_{\mathbf{p}} G_{\mathbf{p}}^{\Lambda}(\nu) S_{\mathbf{Q}+\mathbf{p}}^{\Lambda}(\Omega+\nu) + G_{\mathbf{Q}+\mathbf{p}}^{\Lambda}(\Omega+\nu) S_{\mathbf{p}}^{\Lambda}(\nu), \quad (25)$$

and:

$$\begin{aligned} L_{\mathbf{Q}}^{\Omega;\nu_1,\nu_2} &= U + \mathcal{M}_{\mathbf{Q}}^{\Omega;\nu_1,\nu_2} \\ &+ \int_{\mathbf{p}} \left\{ -\mathcal{S}_{\mathbf{p}}^{\Omega+\nu_1+\nu_2;\nu_1,\nu_1+\Omega} \right. \\ &- \mathcal{D}_{\mathbf{p}}^{\Omega+\nu_1+\nu_2;\nu_1,\nu_1+\Omega} [\cos(Q_x) + \cos(Q_y)] \\ &\left. + \frac{1}{2} \mathcal{M}_{\mathbf{p}}^{\nu_1-\nu_2;\nu_2+\Omega,\nu_2} \right\} \end{aligned} \quad (26)$$

Let us notice that after the momentum integrals in P and L are performed, the right hand side can be expressed as a matrix-matrix multiplication in frequency space, where Ω and \mathbf{Q} appear as parameters.

A. Cutoff choice

We have used the cutoff blabla.

IV. RESULTS

A. Frequency dependence of Vertex

- Forward scattering problem seen by Salmhofer
- Show phase diagram, Λ_{cri} vs $x = 1 - n$, with and without Σ (for different t')
- Self energy "solve" the problem of charge instability.
- Suggestion: The charge problem exists also at van Hove filling where, according to the literature, the Σ has no effect when Karrassch approximation is taken into account.
- Colorplots: Mag and Charge channel

B. Forward scattering problem

- Introduce perpendicular ladder (PL) for charge.
- Colorplot of charge in PL.
- Discuss the role of the Bubble at $\mathbf{Q} = (0,0)$ and plot it as a function of ν .

C. Self energy effects

- With self energy feedback, we didn't find any charge instability problem for any parameters range studied.
- Plot of the Fermi surface based patch scheme.
- Plot of $\Sigma(i\omega)$ at $\mathbf{k} = (\pi,0)$, $\mathbf{k} = \mathbf{k}_{HS}$ and $\mathbf{k} = (\pi/2, \pi/2)$ in frequency space.

- Plot of $Z_{\mathbf{k}}$
- Plot of occupation with and without Σ

VI. APPENDICES

V. CONCLUSIONS

Conclusions...

Acknowledgments

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Appendices...

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- ⁸ The equation for the particle-particle channel is slightly different from the one usually reported in fRG, see, e.g., Ref. ? . This is because we took $V^{\Lambda} = V_{\uparrow\downarrow\uparrow\downarrow}^{\Lambda}$ instead of $V^{\Lambda} = V_{\uparrow\downarrow\downarrow\uparrow}^{\Lambda}$.