Self Energy effect in frequency dependent Vertex flow equation

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We are geniuses...

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I. INTRODUCTION

Introduction bla bla

II. FORMALISM

A. Model

The Hubbard model describes spin-$\frac{1}{2}$ fermions with a density-density interaction:

$$H = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} \tag{1}$$

where $c^{\dagger}_{i,\sigma}$ and $c_{i,\sigma}$ are creation and annihilation operators, respectively, for fermions with spin $\sigma =\uparrow,\downarrow$. We consider 2-dimensional case with square lattice and repulsive interaction $U > 0$. We use the hopping amplitude $t_{ij} = t$ for nearest sites while $t_{ij} = t'$ for next-to-nearest sites.

B. Flow equation

- Flow equation for vertex and self energy: Diagrams

- Two ways.
  1) Introduce flow equation with full spin component such as $\dot{V}_{\sigma\sigma'} = ...$ and discuss in the next section spin relations.
  2) Introduce in the effective action only the $V_{\uparrow\downarrow}$ component and show flow equation only for that one.

- Cutoff discussion

C. Flow equations

- General consideration about functional renormalizaton group: 1PI, truncation,

- vertex: notation, momentum and energy conservation, spin. Spin symmetry

In the following paragraph we will introduce the functional renormalization group in the implementation that we used, and we will clarify some notational issue about the vertex.

Generally speaking, the fRG allows to use the renormalization group idea to approach functional integrals. This is done by endowing the non-interacting propagator with an additional dependence on a scale parameter Λ, which generates an exact functional flow equation with known initial conditions.

We will apply this approach to the effective action, whose expansions into the fields generates the one-particle irreducible (1PI) functions. By expanding the functional flow equation one obtains a hierarchy of flow equations for the 1PI functions, involving vertexes of arbitrarily high orders. We will restrict ourselves to the level-two truncation by retaining only the two lowest nonvanishing orders in the expansion, i.e., we consider the flow of the (scale dependent) self-energy Σ^{Λ} and of the two-particle 1PI vertex V^{Λ}, neglecting the effects of higher order vertexes. Hence our approach becomes perturbative, and sums up efficiently, although approximately, the so-called parquet-diagrams.

Symmetry considerations We use the energy and momentum conservation to fix one of the arguments of the arguments of the self energy and of the vertex. Furthermore we restrict ourselves to the spin-symmetric phase. Hence for the self-energy we only need to consider one function depending on one frequency-momentum argument:

$$\Sigma^{\Lambda}_{\sigma\sigma'}(k) = \Sigma(k)\delta_{\sigma,\sigma'}, \tag{2}$$

where $\sigma = \{\uparrow,\downarrow\}$, and $k = (\nu, \mathbf{k})$, ν being a Matsubara frequency and $\mathbf{k}$ a momentum in the first Brillouin zone.

For the notation of the two-particle vertex we refer to Fig. (fig), where $k_i = (\nu_i, \mathbf{k_i})$, and $k_4 = (\nu_1 + \nu_2 - \nu_3, \mathbf{k_1} + \mathbf{k_2} - \mathbf{k_3})$ can be omitted. Furthermore SU(2)-symmetry guarantees that the vertex does not vanish only for six spin combinations, pairwise equal under spin inversion: $V^{\Lambda}_{\uparrow\uparrow\uparrow\uparrow} = V^{\Lambda}_{\downarrow\downarrow\downarrow\downarrow}$, $V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow} = V^{\Lambda}_{\downarrow\uparrow\downarrow\uparrow}$, and $V^{\Lambda}_{\uparrow\downarrow\downarrow\uparrow} = V^{\Lambda}_{\downarrow\uparrow\uparrow\downarrow}$. Finally, due to SU(2) symmetry and crossing relation one has:

$$\begin{aligned} V^{\Lambda}_{\uparrow\uparrow\uparrow\uparrow}(k_1,k_2,k_3) &= V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_3) \\ &\quad - V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_1+k_2-k_3), \qquad (3) \\ V^{\Lambda}_{\uparrow\downarrow\downarrow\uparrow}(k_1,k_2,k_3) &= -V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_1+k_2-k_3). \qquad (4) \end{aligned}$$

This allows us to consider for the vertex only one function of three arguments: $V^\Lambda_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_3) = V^\Lambda(k_1,k_2,k_3)$, all the others spins components being obtained by symmetry.

With these considerations the flow equation for the self energy reads:

$$\frac{d}{d\Lambda}\Sigma^\Lambda(k) = -\int_q S^\Lambda(q)\left[2V^\Lambda(k,q,q) - V^\Lambda(k,q,k)\right], \tag{5}$$

with $q=(\omega,\mathbf{q})$ and $k=(\nu,\mathbf{k})$ and we use the notation $\int_q = T\sum_\omega \int_{\mathbf{q}}$, and $\int_{\mathbf{q}} = \int \frac{d\mathbf{q}}{4\pi^2}$ is the normalized integral over the first Brillouin zone.

$$S^\Lambda = \left.\frac{dG^\Lambda}{d\Lambda}\right|_{\Sigma=\mathrm{const}} \tag{6}$$

is the single-scale propagator; $G^\Lambda = \left[(G_0^\Lambda)^{-1} - \Sigma^\Lambda\right]^{-1}$ is the full propagator, G_0^Λ is the non-interacting Green's function.

The vertex flow equation can be written as:

$$\frac{d}{d\Lambda}V(k1,k2,k3) = \mathcal{T}^\Lambda_{\mathrm{pp}}(k_1,k_2,k_3) + \mathcal{T}^\Lambda_{\mathrm{ph}}(k_1,k_2,k_3) + \mathcal{T}^\Lambda_{\mathrm{phc}}(k_1,k_2,k_3), \tag{7}$$

where:[1]

$$\begin{aligned}
\mathcal{T}^\Lambda_{\mathrm{pp}}(k_1,k_2,k_3) &= -\frac{1}{2}\int_q P^\Lambda(q,k_1+k_2-q)\Big\{V^\Lambda(k_1,k_2,k_1+k_2-q)V^\Lambda(k_1+k_2-q,q,k_3) \qquad (8)\\
&\quad + V^\Lambda(k_1,k_2,q)V^\Lambda(q,k_1+k_2-q,k_3)\Big\};\\
\mathcal{T}^\Lambda_{\mathrm{ph}}(k_1,k_2,k_3) &= -\int_q P^\Lambda(q,k_3-k_1+q)\Big\{2V^\Lambda(k_1,k_3-k_1+q,k_3)V^\Lambda(q,k_2,k_3-k_1+q) \qquad (9)\\
&\quad - V^\Lambda(k_1,k_3-k_1+q,q)V^\Lambda(q,k_2,k_3-k_1+q) - V^\Lambda(k_1,k_3-k_1+q,k_3)V^\Lambda(k_2,q,k_3-k_1+q)\Big\};\\
\mathcal{T}^\Lambda_{\mathrm{phc}}(k_1,k_2,k_3) &= \int_q P^\Lambda(q,k_2-k_3+q)V^\Lambda(k_1,k_2-k_3+q,q)V^\Lambda(q,k_2,k_3). \qquad (10)
\end{aligned}$$

Here we have defined the quantity:

$$P^\Lambda(q,q') = G^\Lambda(q)S^\Lambda(q') + G^\Lambda(q')S^\Lambda(q). \tag{11}$$

III. VERTEX APPROXIMATION

- Discuss spin symmetry of the vertex with frequencies relations. It's important because those relations become trivial in case of Karrasch approximation while in this case they become crucial. Point out: $V_{\uparrow\downarrow}$ as the only "complete" component.
- Decomposition:

  We use frequency dependent vertex decomposition:

$$V^{\omega_1,\omega_2,\omega_3}_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3} = U + \phi^{\omega_1,\omega_2,\omega_3}_{pp,\boldsymbol{k}_1+\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3} + \phi^{\omega_1,\omega_2,\omega_3}_{m,\boldsymbol{k}_3-\boldsymbol{k}_1} + \frac{1}{2}\phi^{\omega_1,\omega_3,\omega_2}_{m,\boldsymbol{k}_2-\boldsymbol{k}_3} - \frac{1}{2}\phi^{\omega_1,\omega_2,\omega_3}_{c,\boldsymbol{k}_2-\boldsymbol{k}_3} \tag{12}$$

- Form factor decomposition for PP channel:

$$\phi^{\omega_1,\omega_2,\omega_3}_{pp,\boldsymbol{Q},\boldsymbol{k}_1,\boldsymbol{k}_3} = \phi^{\omega_1,\omega_2,\omega_3}_{sw,\boldsymbol{Q}} + f_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}_1} f_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}_3} \phi^{\omega_1,\omega_2,\omega_3}_{dw,\boldsymbol{Q}} \tag{13}$$

- Comment: The channels are defined by the possibility to build an instability only for the exchange momentum $\boldsymbol{Q}$
- Channel division:

$$\phi_{sw} = \int \tau \tag{14}$$

IV. RESULTS

A. Frequency dependence of Vertex

- Forward scattering problem seen by Salmhofer
- Show phase diagram, Λ_{cri} vs $x = 1-n$, with and without Σ (for differenct t')
- Self energy "solve" the problem of charge instability.
- Suggestion: The charge problem exists also at van Hove filling where, according to the literature, the Σ has no effect when Karrasch approximation is taken into account.

- Colorplots: Mag and Charge channel

B. Forward scattering problem

- Introduce perpendicular ladder (PL) for charge.
- Colorplot of charge in PL.
- Discuss the role of the Bubble at $\boldsymbol{Q} = (0,0)$ and plot it as a function of ν.

C. Self energy effects

- With self energy feedback, we didn't find any charge instability problem for any parameters range studied.
- Plot of the Fermi surface based patch scheme.
- Plot of $\Sigma(i\omega)$ at $\boldsymbol{k} = (\pi, 0)$, $\boldsymbol{k} = \boldsymbol{k}_{HS}$ and $\boldsymbol{k} = (\pi/2, \pi/2)$ in frequency space.
- Plot of $Z_{\boldsymbol{k}}$
- Plot of occupation with and without Σ

V. CONCLUSIONS

Conclusions...

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VI. APPENDICES

Appendices...

[1] The equation for the particle-particle channel is slightly different from the one usually reported in fRG. This is because we took $V^\Lambda = V^\Lambda_{\uparrow\downarrow\uparrow\downarrow}$ instead of $V^\Lambda = V^\Lambda_{\uparrow\downarrow\downarrow\uparrow}$.