Self Energy effect in frequency dependent Vertex flow equation

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We are geniuses...

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I. INTRODUCTION

Introduction bla bla

II. FORMALISM

A. Model

The Hubbard model describes spin-$\frac{1}{2}$ fermions with a density-density interaction:

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} \tag{1}$$

where $c^{\dagger}_{i,\sigma}$ and $c_{i,\sigma}$ are, respectively, creation and annihilation operators for fermions with spin $\sigma =\uparrow,\downarrow$. We consider the two-dimensional case with square lattice and repulsive interaction $U > 0$. The hopping amplitude is restricted to $t_{ij} = t$ for nearest neighbors, $t_{ij} = t'$ for next-to-nearest neighbors and 0 otherwise.

B. Flow equations

In the following paragraph we will introduce the functional renormalization group in the implementation that we used, and we will clarify some notational issue about the vertex.

Generally speaking, the fRG allows to use the renormalization group idea to approach functional integrals. This is done by endowing the non-interacting propagator with an additional dependence on a scale parameter Λ, which generates an exact functional flow equation with known initial conditions.

We will apply this approach to the effective action, whose expansions into the fields generates the one-particle irreducible (1PI) functions. By expanding the functional flow equation one obtains a hierarchy of flow equations for the 1PI functions, involving vertexes of arbitrarily high orders. We will restrict ourselves to the level-two truncation by retaining only the two lowest nonvanishing orders in the expansion, i.e., we consider the flow of the (scale dependent) self-energy Σ^{Λ} and of the two-particle 1PI vertex V^{Λ}, neglecting the effects of higher order vertexes. Hence our approach becomes perturbative, and sums up efficiently, although approximately, the so-called parquet-diagrams.

, given a cutoff choice with $G_0^{\Lambda_0} = 0$We use the energy and momentum conservation to fix one of the arguments of the arguments of the self energy and of the vertex. Furthermore we restrict ourselves to the spin-symmetric phase. Hence for the self-energy we only need to consider one function depending on one frequency-momentum argument:

$$\Sigma^{\Lambda}_{\sigma\sigma'}(k) = \Sigma(k)\delta_{\sigma,\sigma'}, \tag{2}$$

where $\sigma = \{\uparrow, \downarrow\}$, and $k = (\nu, \mathbf{k})$, ν being a Matsubara frequency and $\mathbf{k}$ a momentum in the first Brillouin zone.

For the notation of the two-particle vertex we refer to Fig. (fig), where $k_i = (\nu_i, \mathbf{k_i})$, and $k_4 = (\nu_1 + \nu_2 - \nu_3, \mathbf{k_1} + \mathbf{k_2} - \mathbf{k_3})$ can be omitted. Furthermore SU(2)-symmetry guarantees that the vertex does not vanish only for six spin combinations, pairwise equal under spin inversion: $V^{\Lambda}_{\uparrow\uparrow\uparrow\uparrow} = V^{\Lambda}_{\downarrow\downarrow\downarrow\downarrow}$, $V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow} = V^{\Lambda}_{\downarrow\uparrow\downarrow\uparrow}$, and $V^{\Lambda}_{\uparrow\downarrow\downarrow\uparrow} = V^{\Lambda}_{\downarrow\uparrow\uparrow\downarrow}$. Finally, due to SU(2) symmetry and crossing relation one has:

$$\begin{aligned} V^{\Lambda}_{\uparrow\uparrow\uparrow\uparrow}(k_1,k_2,k_3) &= V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_3) \\ &\quad - V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_1+k_2-k_3), \qquad (3) \\ V^{\Lambda}_{\uparrow\downarrow\downarrow\uparrow}(k_1,k_2,k_3) &= -V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_1+k_2-k_3). \qquad (4) \end{aligned}$$

This allows us to consider for the vertex only one function of three arguments: $V^{\Lambda}_{\uparrow\downarrow\uparrow\downarrow}(k_1,k_2,k_3) = V^{\Lambda}(k_1,k_2,k_3)$, all the others spin components being obtained by symmetry.

With these considerations the flow equation for the self energy reads:

$$\frac{d}{d\Lambda}\Sigma^{\Lambda}(k) = -\int_q S^{\Lambda}(q)\left[2V^{\Lambda}(k,q,q) - V^{\Lambda}(k,q,k)\right], \tag{5}$$

with $q = (\omega, \mathbf{q})$ and $k = (\nu, \mathbf{k})$ and we use the notation $\int_q = T\sum_\omega \int_{\mathbf{q}}$, and $\int_{\mathbf{q}} = \int \frac{d\mathbf{q}}{4\pi^2}$ is the normalized integral over the first Brillouin zone.

$$S^{\Lambda} = \left.\frac{dG^{\Lambda}}{d\Lambda}\right|_{\Sigma=\text{const}} \tag{6}$$

is the single-scale propagator; $G^{\Lambda} = \left[(G_0^{\Lambda})^{-1} - \Sigma^{\Lambda}\right]^{-1}$ is the full propagator, G_0^{Λ} is the non-interacting Green's function.

The vertex flow equation can be written as:

$$\frac{d}{d\Lambda}V(k1,k2,k3)=\mathcal{T}^{\Lambda}_{\mathrm{pp}}(k_1,k_2,k_3)+\mathcal{T}^{\Lambda}_{\mathrm{ph}}(k_1,k_2,k_3)+\mathcal{T}^{\Lambda}_{\mathrm{phc}}(k_1,k_2,k_3), \tag{7}$$

where:[1]

$$\begin{aligned}\mathcal{T}^{\Lambda}_{\mathrm{pp}}(k_1,k_2,k_3) &= -\frac{1}{2}\int_q P^{\Lambda}(q,k_1+k_2-q)\Big\{V^{\Lambda}(k_1,k_2,k_1+k_2-q)V^{\Lambda}(k_1+k_2-q,q,k_3) \\ &\quad +V^{\Lambda}(k_1,k_2,q)V^{\Lambda}(q,k_1+k_2-q,k_3)\Big\}; \end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{T}^{\Lambda}_{\mathrm{ph}}(k_1,k_2,k_3) &= -\int_q P^{\Lambda}(q,k_3-k_1+q)\Big\{2V^{\Lambda}(k_1,k_3-k_1+q,k_3)V^{\Lambda}(q,k_2,k_3-k_1+q) \\ &\quad -V^{\Lambda}(k_1,k_3-k_1+q,q)V^{\Lambda}(q,k_2,k_3-k_1+q)-V^{\Lambda}(k_1,k_3-k_1+q,k_3)V^{\Lambda}(k_2,q,k_3-k_1+q)\Big\}; \end{aligned} \tag{9}$$

$$\mathcal{T}^{\Lambda}_{\mathrm{phc}}(k_1,k_2,k_3) = \int_q P^{\Lambda}(q,k_2-k_3+q)V^{\Lambda}(k_1,k_2-k_3+q,q)V^{\Lambda}(q,k_2,k_3). \tag{10}$$

Here we have defined the quantity:

$$P^{\Lambda}(q,q')=G^{\Lambda}(q)S^{\Lambda}(q')+G^{\Lambda}(q')S^{\Lambda}(q). \tag{11}$$

For any cutoff choice with $G_0^{\Lambda_0}=0$ the initial condition for the self-and the vertex are, respectively, $\Sigma^{\Lambda_0}=0$ and $V^{\Lambda_0}=U$.

III. VERTEX APPROXIMATION

We start by decomposing the vertex as follows:

$$V^{\Lambda}(k_1,k_2,k_3)=U-\phi^{\Lambda}_{\mathrm{p}}(k_1+k_2,k_1,k_3)+\phi^{\Lambda}_{\mathrm{m}}(k_3-k_1,k_1,k_3)+\frac{1}{2}\phi^{\Lambda}_{\mathrm{m}}(k_2-k_3,k_1,k_2)-\frac{1}{2}\phi^{\Lambda}_{\mathrm{c}}(k_2-k_3,k_1,k_2), \tag{12}$$

where the physical meaning of each ϕ channel is defined by its flow equations:

$$\dot{\phi}^{\Lambda}_{\mathrm{p}}(q,k_1,k_3)=-\mathcal{T}^{\Lambda}_{\mathrm{pp}}(k_1,q-k_1,k_3), \tag{13}$$

$$\dot{\phi}^{\Lambda}_{\mathrm{c}}(q,k_1,k_2)=-2\mathcal{T}^{\Lambda}_{\mathrm{ph}}(k_1,k_2,k_2-1)+\mathcal{T}^{\Lambda}_{\mathrm{phc}}(k_1,k_2,q+k_1), \tag{14}$$

$$\dot{\phi}^{\Lambda}_{\mathrm{m}}(q,k_1,k_2)=\mathcal{T}^{\Lambda}_{\mathrm{phc}}(k_1,k_2,q+k_1). \tag{15}$$

It should be emphasized that each channel can be associated with a possible instabily of the system. ϕ_{p} is associated to a pairing instability, ϕ_{m} to magnetic instabilities and ϕ_{c} to charge instabilities.

We address first the momentum dependence. To this end, we introduce a decomposition of the unity by means of a set of orthonormal form factors for the two fermionic momenta $\{f_l(\mathbf{k})\}$ obeying the completeness relation:

$$\int_{\mathbf{k}} f_l(\mathbf{k})f_m(\mathbf{k})=\delta_{l,m} \tag{16}$$

We can then project each channel on a subset of form factors whose choice is physically motivated. Let us stress that the full form factor expansion is exact but the truncation introduces an approximation.

For the pairing channel we keep only $f_s(\mathbf{k})=1$ and $f_d(\mathbf{k})=\cos k_x-\cos k_y$:

$$\phi^{\Lambda}_{\mathrm{p}}(q,k_1,k_3)=\mathcal{S}^{\omega,\nu_1,\nu_3}_{\mathbf{q}}+f_d\left(\frac{\mathbf{q}}{2}-\mathbf{k}_1\right)f_d\left(\frac{\mathbf{q}}{2}-\mathbf{k}_3\right)\mathcal{D}^{\omega,\nu_1,\nu_3}_{\mathbf{q}}, \tag{17}$$

with the shorthand notation $q=(\omega,\mathbf{Q})$ and $k_i=(\nu_i,\mathbf{k}_i)$.

The instability in the channel $\mathcal{S}$ is associated to s-wave superconductivity, while $\mathcal{D}$ is associated to d-wave superconductivity.

For the charge and magnetic channels we restrict ourselves to $f_s(\mathbf{k})=1$ only:

$$\phi_{\mathrm{c}}(q,k_1,k_2)=\mathcal{C}^{\omega,\nu_1,\nu_2}_{\mathbf{q}}, \tag{18}$$

$$\phi_{\mathrm{c}}(q,k_1,k_2)=\mathcal{M}^{\omega,\nu_1,\nu_2}_{\mathbf{q}} \tag{19}$$

corresponding to instabilities in the charge and magnetic channels, respectively. For now on, for notation simplicity we omit the Λ-dependences of channels $\mathcal{S}$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{M}$.

Let us stress that for each channel we have used a different frequency notation. This consists of one frequency corresponding to the frequency transfered in the specific channel and two remaining independent frequencies. At finite temperature the frequency transfer, beeing a sum or a difference of two fermionic Matsubara frequencies, is a bosonic Matsubara frequency.
The choice of the mixed notation is the most natural since the transferred momentum and frequency plays a special role in the diagrammatics. Indeed, on the one hand, it is the only dependence generated in second order perturbation theory and the main dependence in finite order perturbation theory. On the other hand, this notation is convenient in the Bethe-Salpeter equations (cite).

In the fRG literature (cite), where the fermionic frequency dependence is neglected the channel functions above are often losely intepreted as mediators of bosonic interactions. Such an interpretation is missing in the presence of all frequencies.

Although one expects a leading dependence in the bosonic frequency, in particular in the weak coupling regime, we will see that in some case the dependence on fermionic frequencies can become strong and not negligible.

The flow equations for the channels $\mathcal{S}$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{M}$ can be derived from the projection into form factors of eq. (cite):

$$\dot{\mathcal{S}}_{\mathbf{q}}^{\omega,\nu_1,\nu_3} = -\int_{\mathbf{k}_1}\int_{\mathbf{k}_2} \mathcal{T}_{\mathrm{pp}}(k_1, q-k_1, k_3) \tag{20}$$

$$\dot{\mathcal{D}}_{\mathbf{q}}^{\omega,\nu_1,\nu_3} = -\int_{\mathbf{k}_1}\int_{\mathbf{k}_2} f_{\frac{\mathbf{q}}{2}-\mathbf{k}_1}\mathcal{T}_{\mathrm{pp}}(k_1, q-k_1, k_3) f_{\frac{\mathbf{q}}{2}-\mathbf{k}_3} \tag{21}$$

$$\dot{\mathcal{C}}_{\mathbf{q}}^{\omega,\nu_1,\nu_2} = \int_{\mathbf{k}_1}\int_{\mathbf{k}_2} \mathcal{T}_{\mathrm{phc}}(k_1, k_2, q+k_1) - 2\mathcal{T}_{\mathrm{ph}}(k_1, k_2, k_2-q) \tag{22}$$

$$\dot{\mathcal{M}}_{\mathbf{q}}^{\omega,\nu_1,\nu_2} = \int_{\mathbf{k}_1}\int_{\mathbf{k}_2} \mathcal{T}_{\mathrm{phc}}(k_1, k_2, q+k_1) \tag{23}$$

IV. RESULTS

A. Frequency dependence of Vertex

- Forward scattering problem seen by Salmhofer
- Show phase diagram, Λ_{cri} vs $x = 1-n$, with and without Σ (for differenct t')
- Self energy "solve" the problem of charge instability.
- Suggestion: The charge problem exists also at van Hove filling where, according to the literature, the Σ has no effect when Karrasch approximation is taken into account.
- Colorplots: Mag and Charge channel

B. Forward scattering problem

- Introduce perpendicular ladder (PL) for charge.
- Colorplot of charge in PL.
- Discuss the role of the Bubble at $\boldsymbol{Q} = (0,0)$ and plot it as a function of ν.

C. Self energy effects

- With self energy feedback, we didn't find any charge instability problem for any parameters range studied.
- Plot of the Fermi surface based patch scheme.
- Plot of $\Sigma(i\omega)$ at $\boldsymbol{k} = (\pi, 0)$, $\boldsymbol{k} = \boldsymbol{k}_{HS}$ and $\boldsymbol{k} = (\pi/2, \pi/2)$ in frequency space.
- Plot of $Z_{\boldsymbol{k}}$
- Plot of occupation with and without Σ

V. CONCLUSIONS

Conclusions...

Acknowledgments

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VI. APPENDICES

Appendices...

[1] The equation for the particle-particle channel is slightly different from the one usually reported in fRG. This is because we took $V^\Lambda = V^\Lambda_{\uparrow\downarrow\uparrow\downarrow}$ instead of $V^\Lambda = V^\Lambda_{\uparrow\downarrow\downarrow\uparrow}$.