Drawing in 3D

(viewing, projection, and the rest of the pipeline)

CS559 – Spring 2017

Lecture 6

February 2, 2017

The first 4 Key Ideas...

1. Work in convenient coordinate systems. Use transformations to get from where you want to be to where you need to be. Hierarchical modeling lets us build things out of pieces.
2. Use homogeneous coordinates and transformations to make common operations easy. Translation, projections, coordinate system shift all become simple matrix multiplies.
3. Create viewing transformations with projection. The geometry of imaging (pinhole camera model) leads to linear transformations in homogeneous coordinates.
4. Implement primitive-based rendering (interactive graphics) with a pipeline. The abstractions map nicely onto hardware, and let you do things like visibility computations easily. Be aware that there are other paradigms for drawing.

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Last 2 Weeks

This Week

(and next)

ReCap: Basic Idea

To draw we need a coordinate system

Transformations can

- Move between coordinate systems
- Move objects around

Useful transformations as matrices

ReCap: Math

Points, Vectors, Matrices

Transformations as linear operators

- Matrices (homogeneous coordinates)

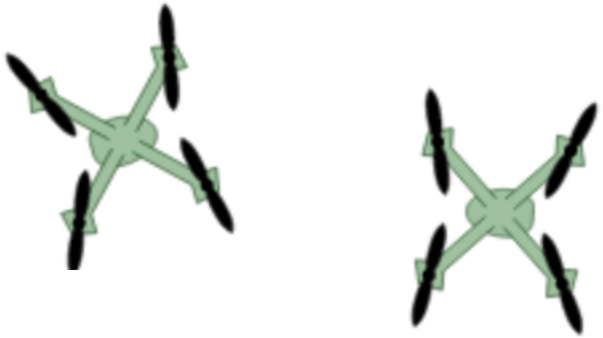
3D Coordinate Systems

- cross product, right hand rule

ReCap: Code

Matrix stack

- Save
- Transform (concat)
- Draw (current trans)
- Restore



```
this.context.save();
this.context.translate(50,50);
this.context.rotate(this.frontPropAngle);
this.drawProp();
this.context.restore();
```

What about 3D?

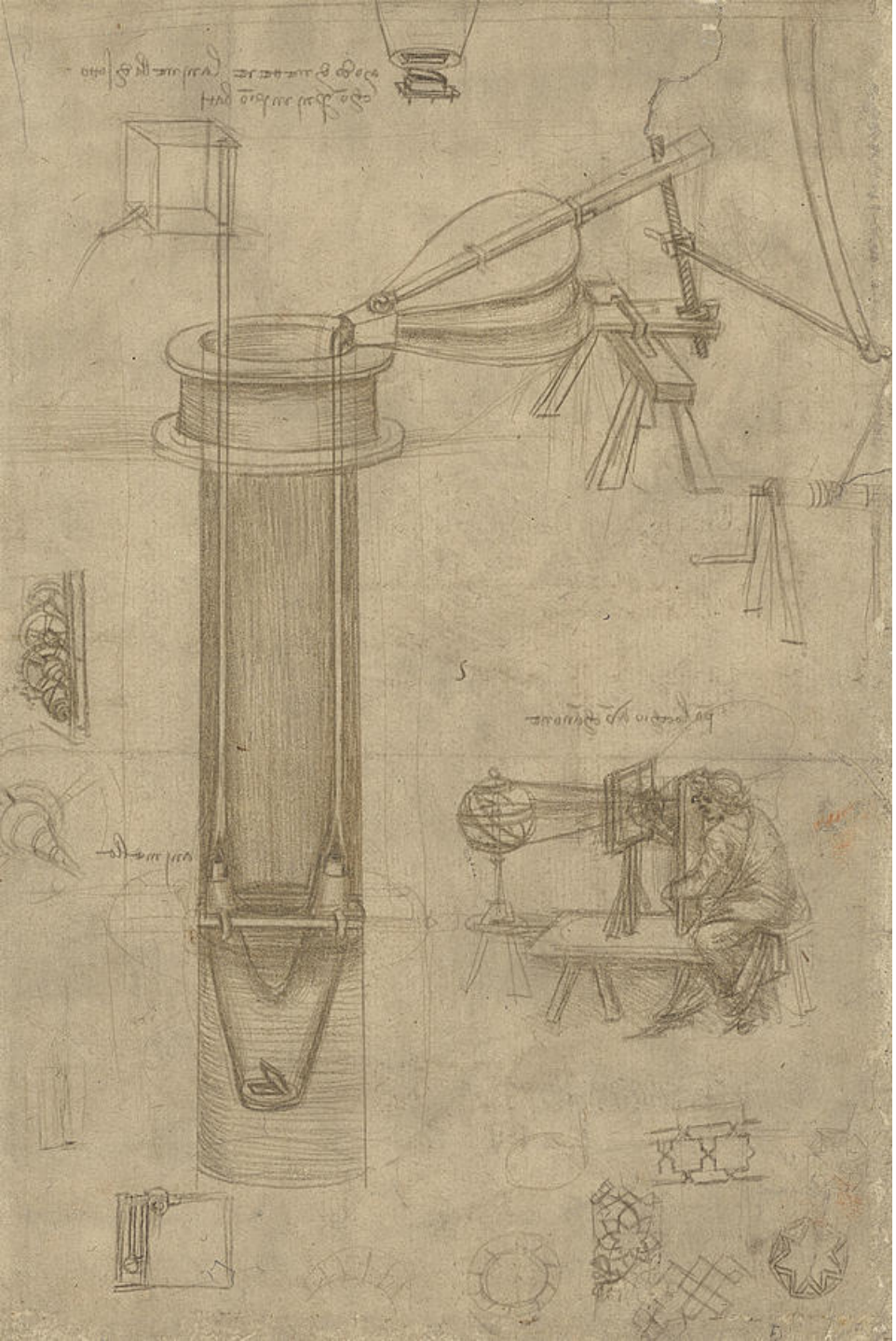
How to draw in 3D?

We are making a 2D picture!

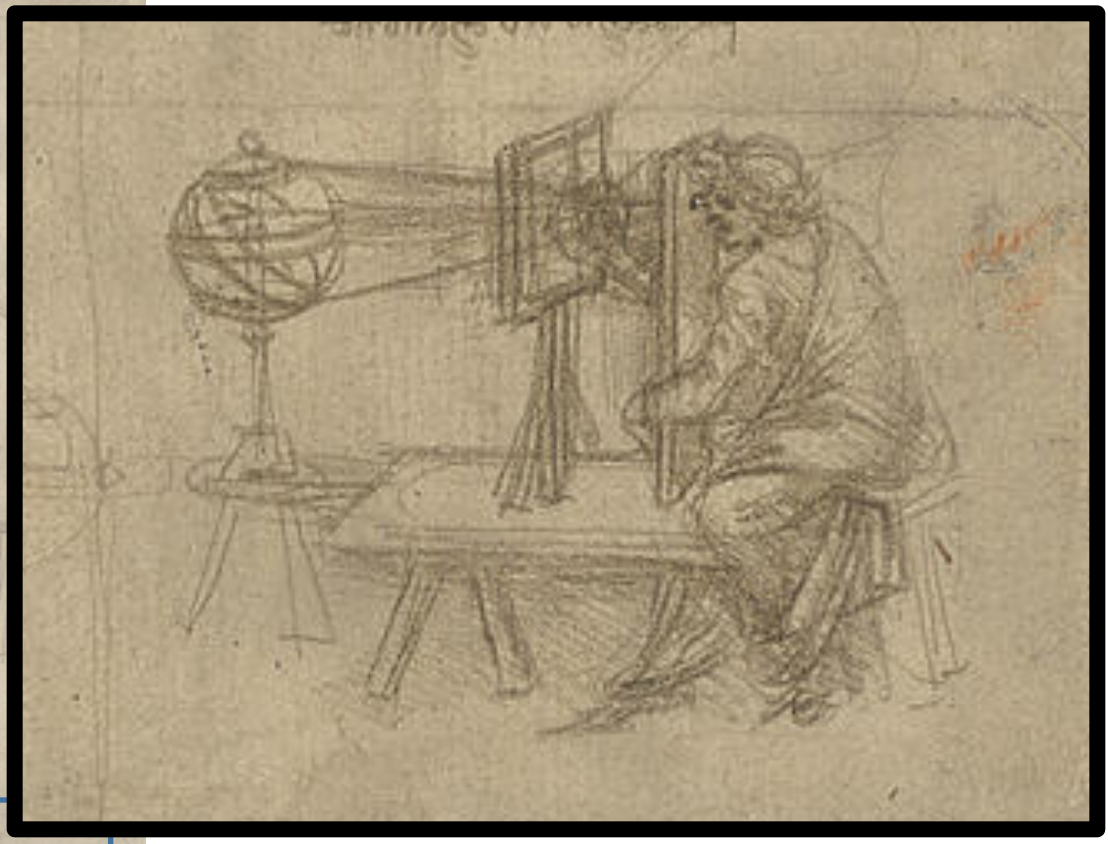
Do it ...

Like nature does

Like artists do



Do it like Da Vinci!



Primitive-based Rendering

Draw 2D Objects in the image

- Paint strokes on canvas
- Lines / Triangles on screen

Map (Transform)

3D primitives (world) to 2D primitives (screen)

What does it take to do this?

1. Put a 3D **primitive** in the **World**
2. Figure out what **color** it should be
3. Position relative to the **Eye**
4. Get rid of stuff behind you/offscreen
5. Figure out where it goes on **screen**
6. Figure out if something else blocks it
7. Draw the **2D primitive**

In terms of the readings…

A lot of the pieces are there

Not necessarily in any particular order

Good for the details

Today we'll try for the big picture

What does it take to do this?

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1. Put a 3D primitive in the World
   Modeling
2. Figure out what color it should be
   Shading
3. Position relative to the Eye
   Viewing / Camera Transformation
4. Get rid of stuff behind you/offscreen
   Clipping
5. Figure out where it goes on screen
   Projection (sometimes called Viewing)
6. Figure out if something else blocks it
   Visibility / Occlustion
7. Draw the 2D primitive
   Rasterization (convert to Pixels)

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Did some, will do more

A little for P4, lots later

Today (FCG 7)

Not much to say (FCG 8)

Today (FCG 7)

This week (FCG 8)

Not much to say (FCG 8)

In case you're wondering...
(We'll come back to this. It's a detail)

For the kinds of projection we will use...

3D Points map to 2D Points

3D Lines map to 2D Lines

3D Triangles map to 2D Triangles

Doesn't work for curves (even ellipses)

Viewing

How to get from the object to the screen?

A **transformation** between coord systems

A little weird…

3D to 2D

Do we lose a dimension?

No – we actually need to keep it
Yes – but we'll just ignore Z

The screen as a fishtank

Let's build a viewing transform

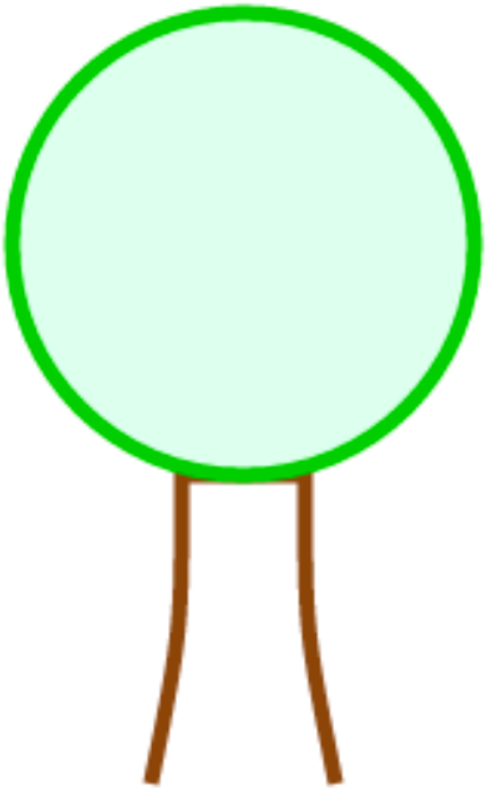
A toy example (with code)

My Neighbor …

A (simple) bird

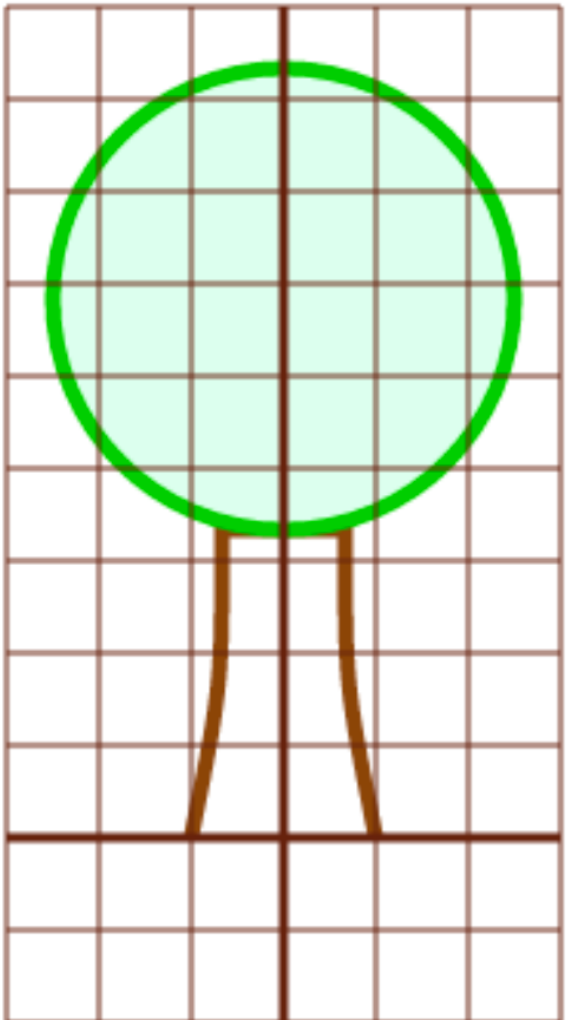
```
function birdGeometry(ct) {
    "use strict";
    ct.fillStyle = "#88C";
    ct.beginPath();
    ct.arc(0,0,20,   0, 2 * Math.PI, false);
    ct.arc(15,15,10, 0, 2 * Math.PI, false);
    ct.fill();
    ct.fillStyle = "#CC0";
    ct.beginPath();
    ct.moveTo(24,10);
    ct.lineTo(24,20);
    ct.lineTo(32,15);
    ct.closePath();
    ct.fill();
}
```

A Tree for the Bird



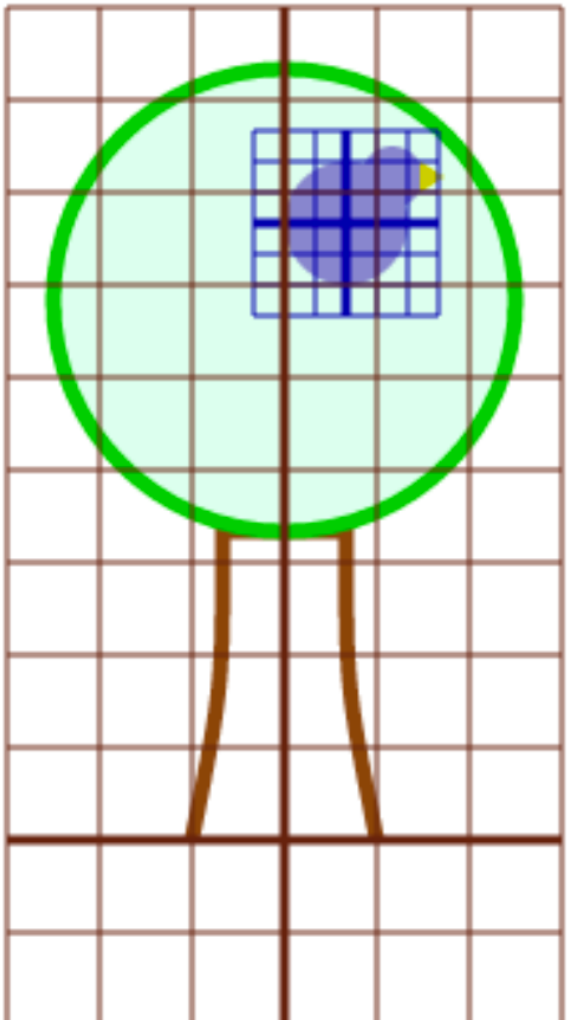
In it's coordinate system

```
function treeGeometry(ct) {
    "use strict";
    ct.strokeStyle = "#8B4513";
    ct.lineWidth=5;
    ct.beginPath();
    ct.moveTo(30,0);
    ct.bezierCurveTo(20,50,20,50,20,100);
    ct.lineTo(-20,100);
    ct.bezierCurveTo(-20,50,-20,50, -30,0);
    ct.stroke();
    ct.strokeStyle = "#0C0";
    ct.fillStyle = "#DFE";
    ct.beginPath();
    ct.arc(0,175,75,0, 2 * Math.PI, false);
    ct.fill();
    ct.stroke();
}
```

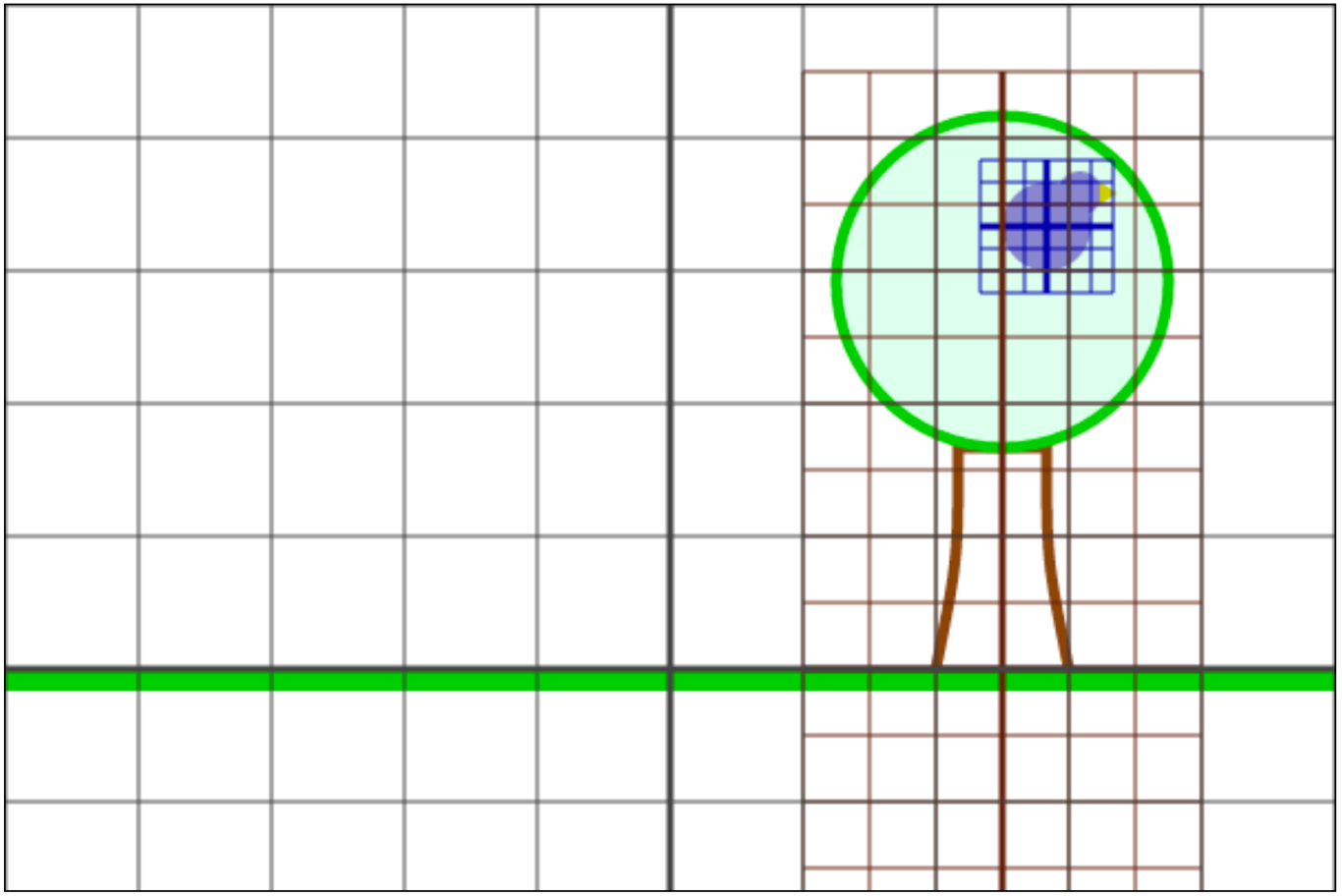


Bird in Tree

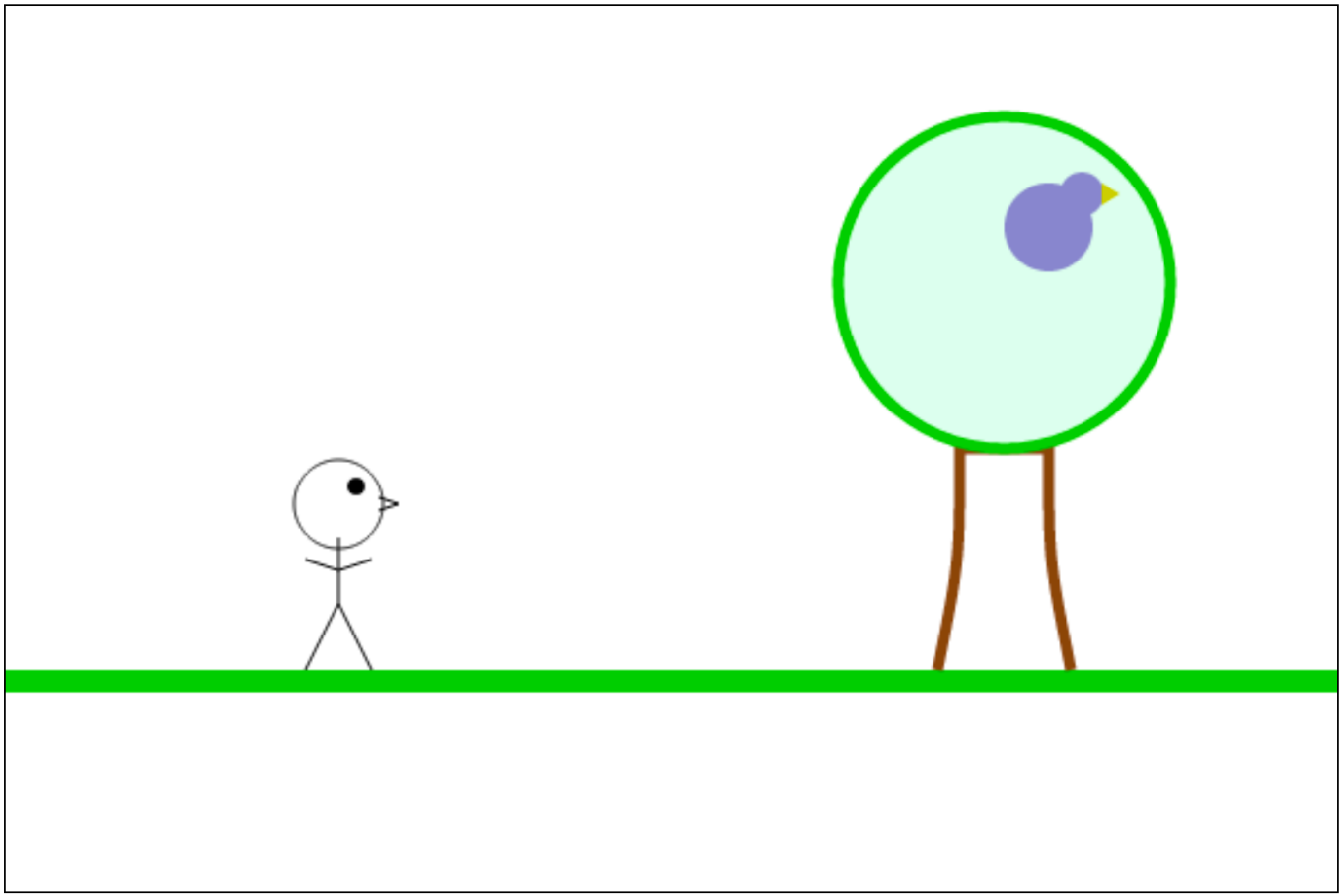
```
function drawTree(ct) {
    treeGeometry(ct);
    ct.save();
    birdInTree.apply(ct); // transform
    drawBird(ct);
    ct.restore();
}
```



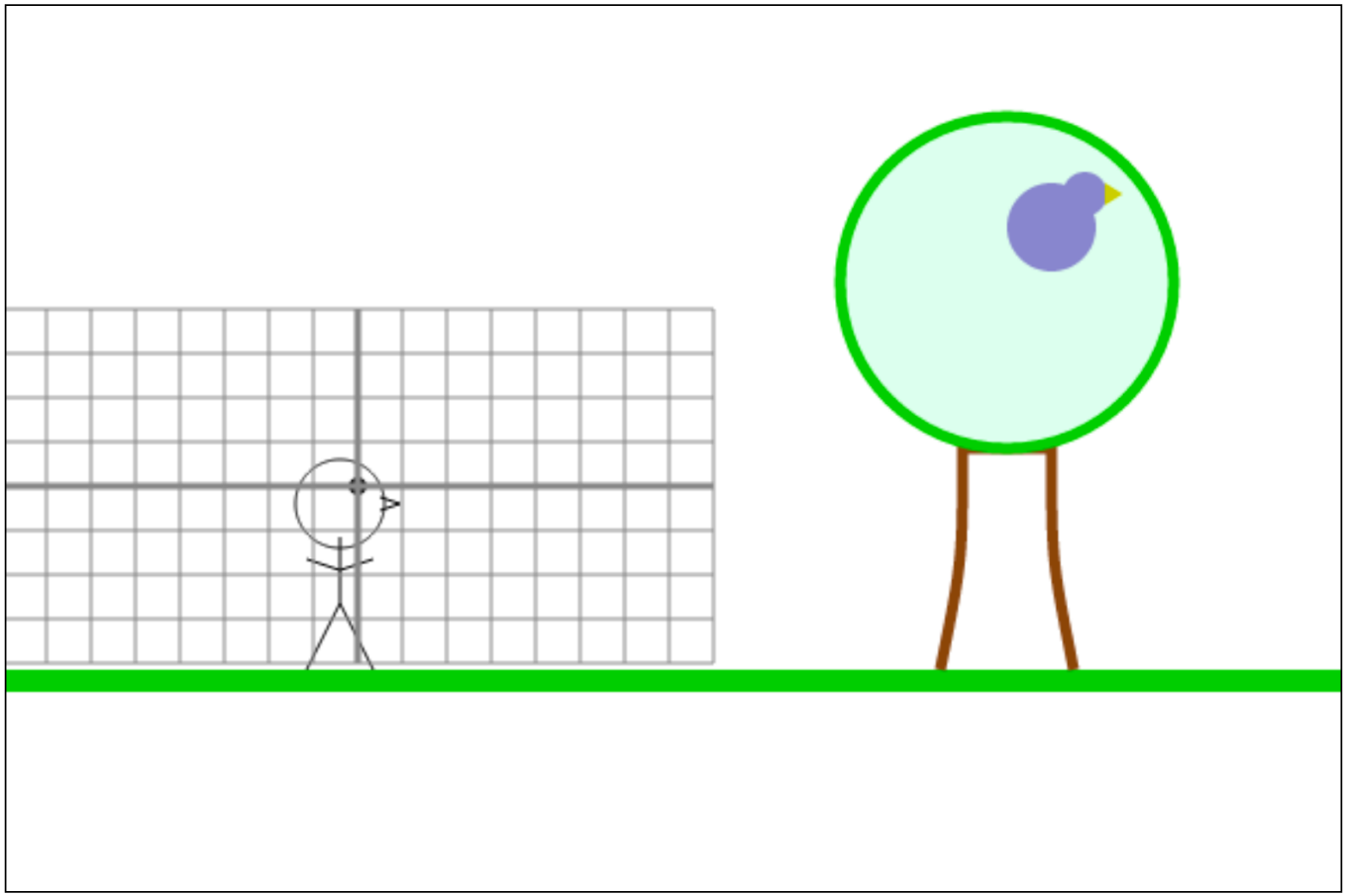
Bird in Tree, Tree in Park



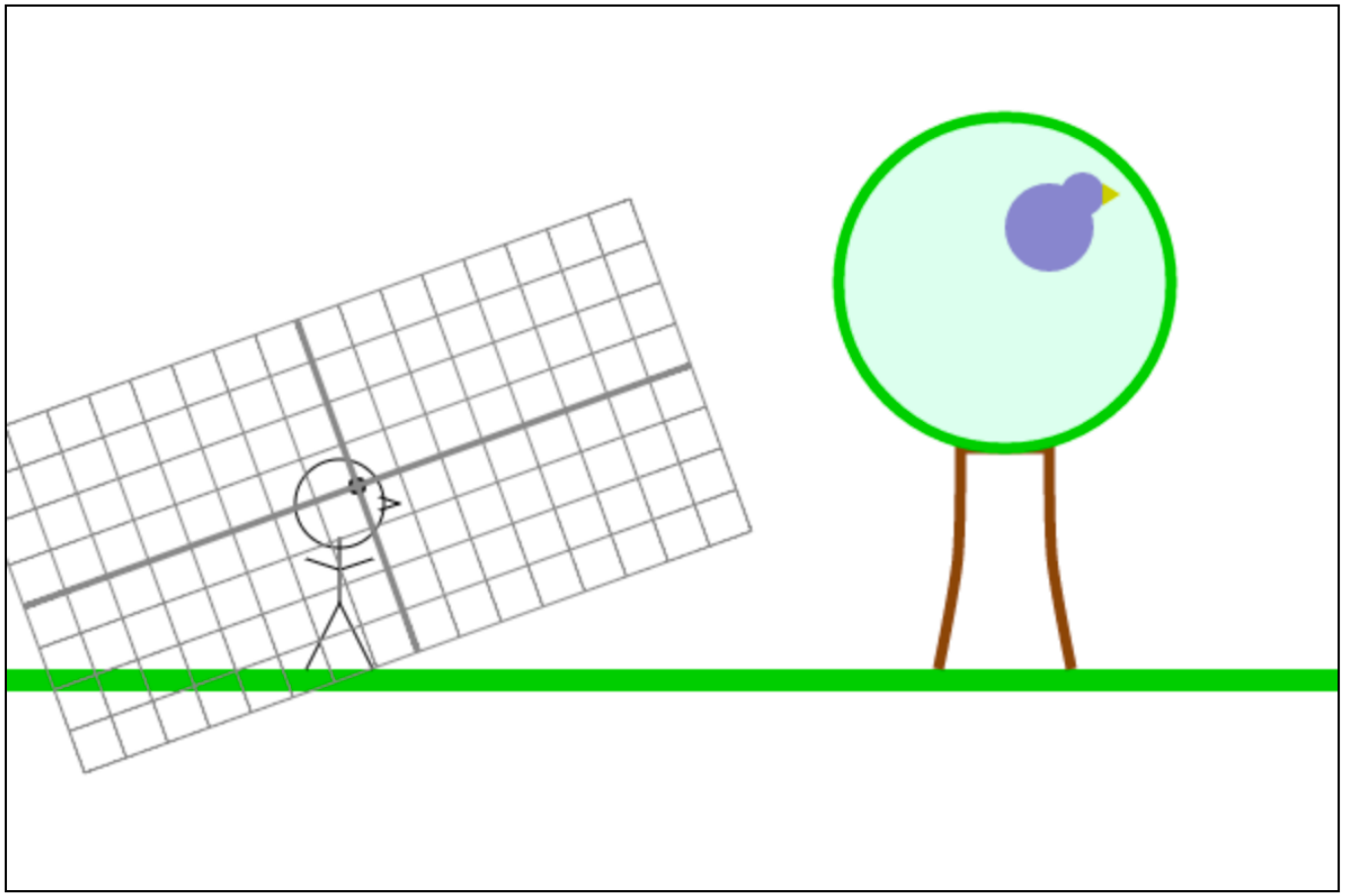
And a person to look at the bird...



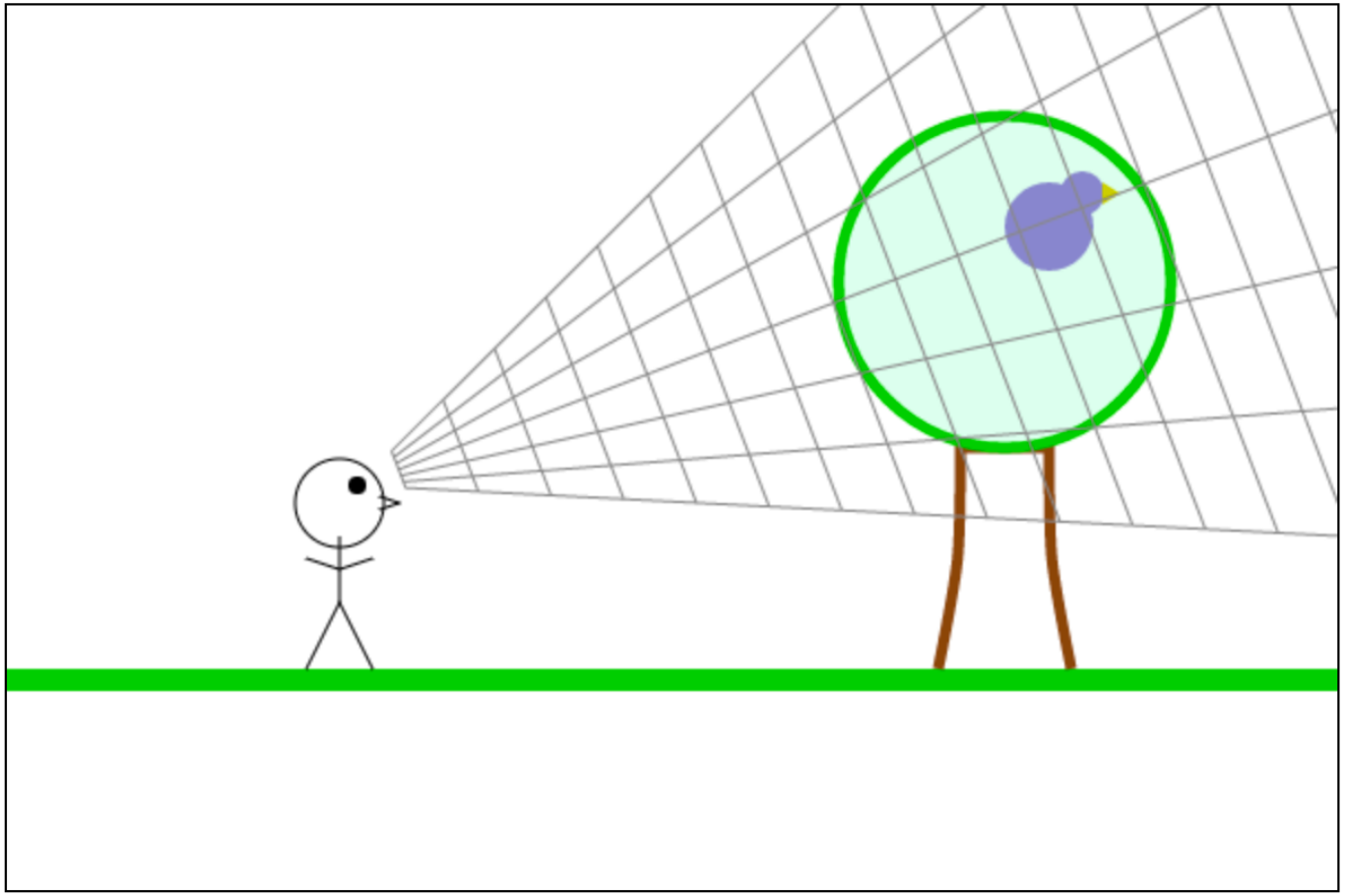
The Eye Coordinate System (or camera)



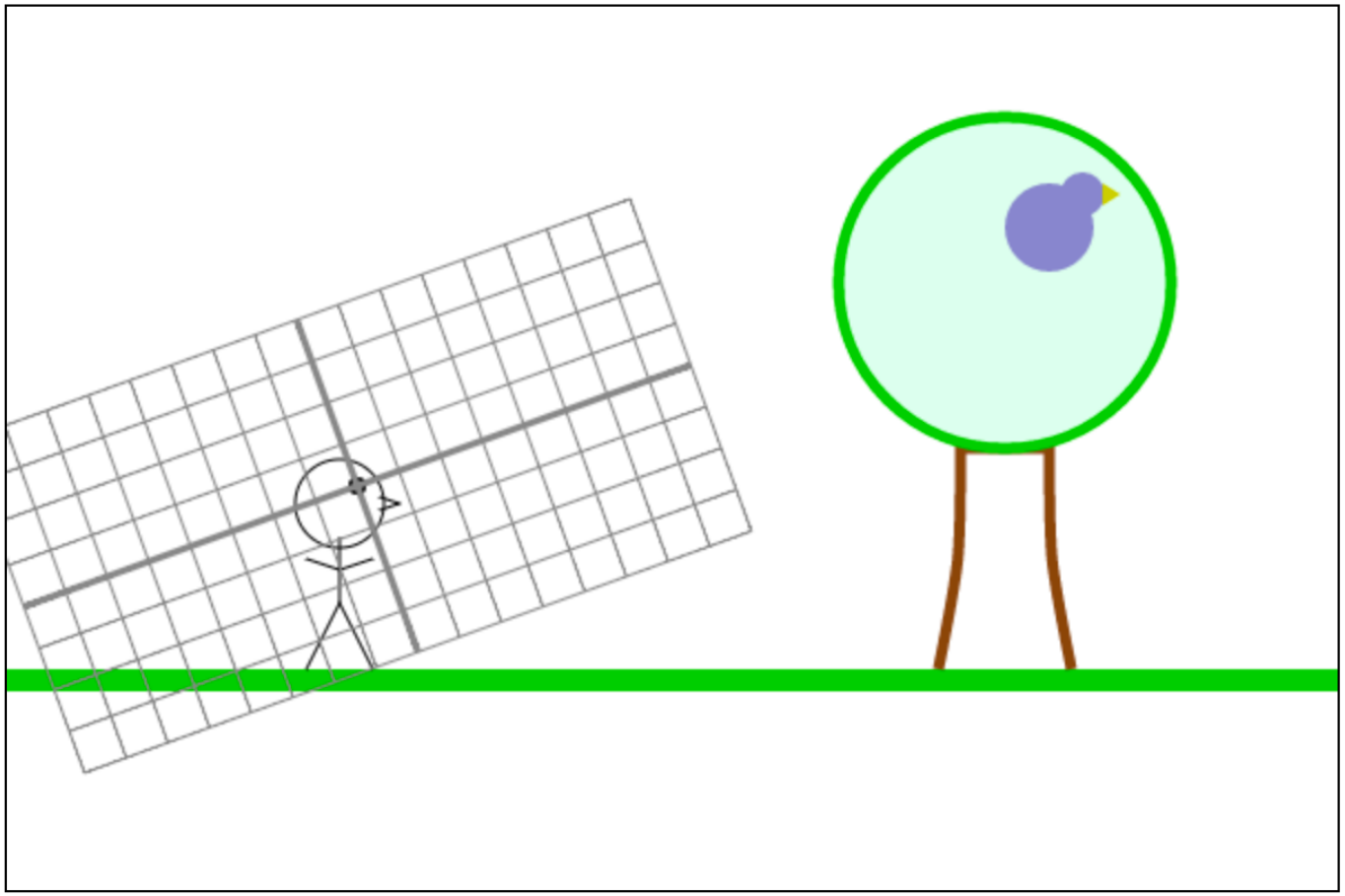
Look At (the bird)



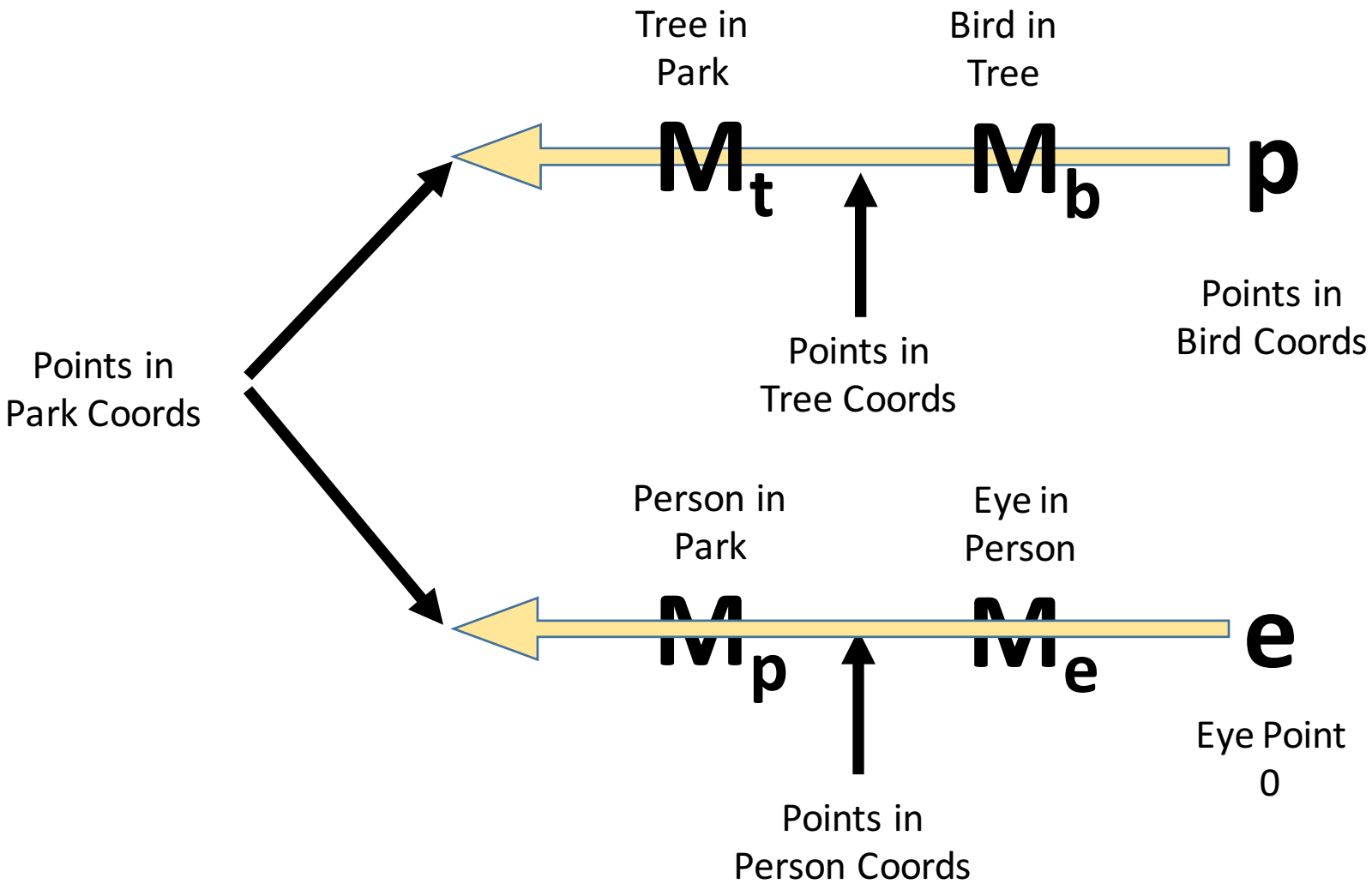
Perspective (just wait)



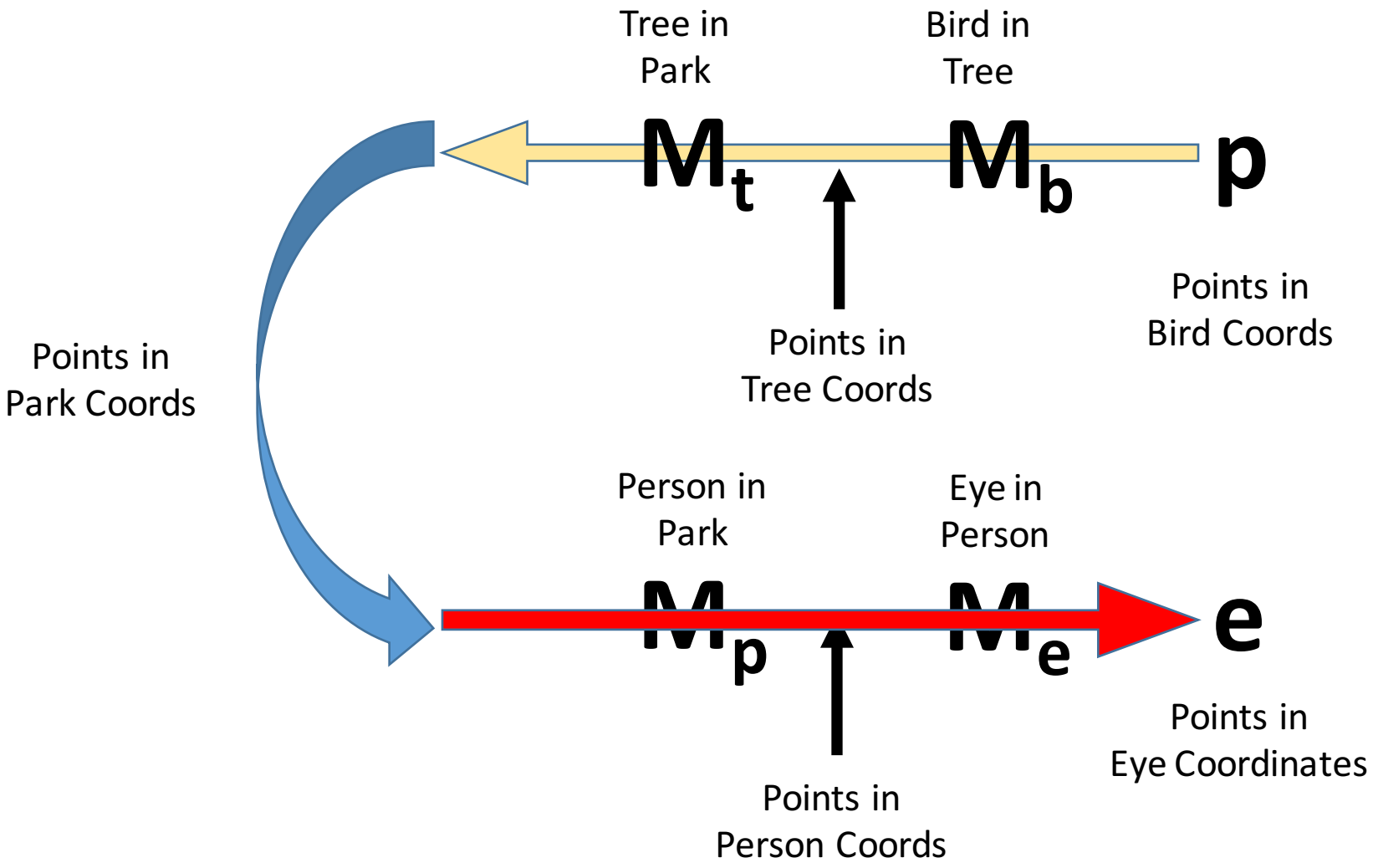
Look At (the bird)



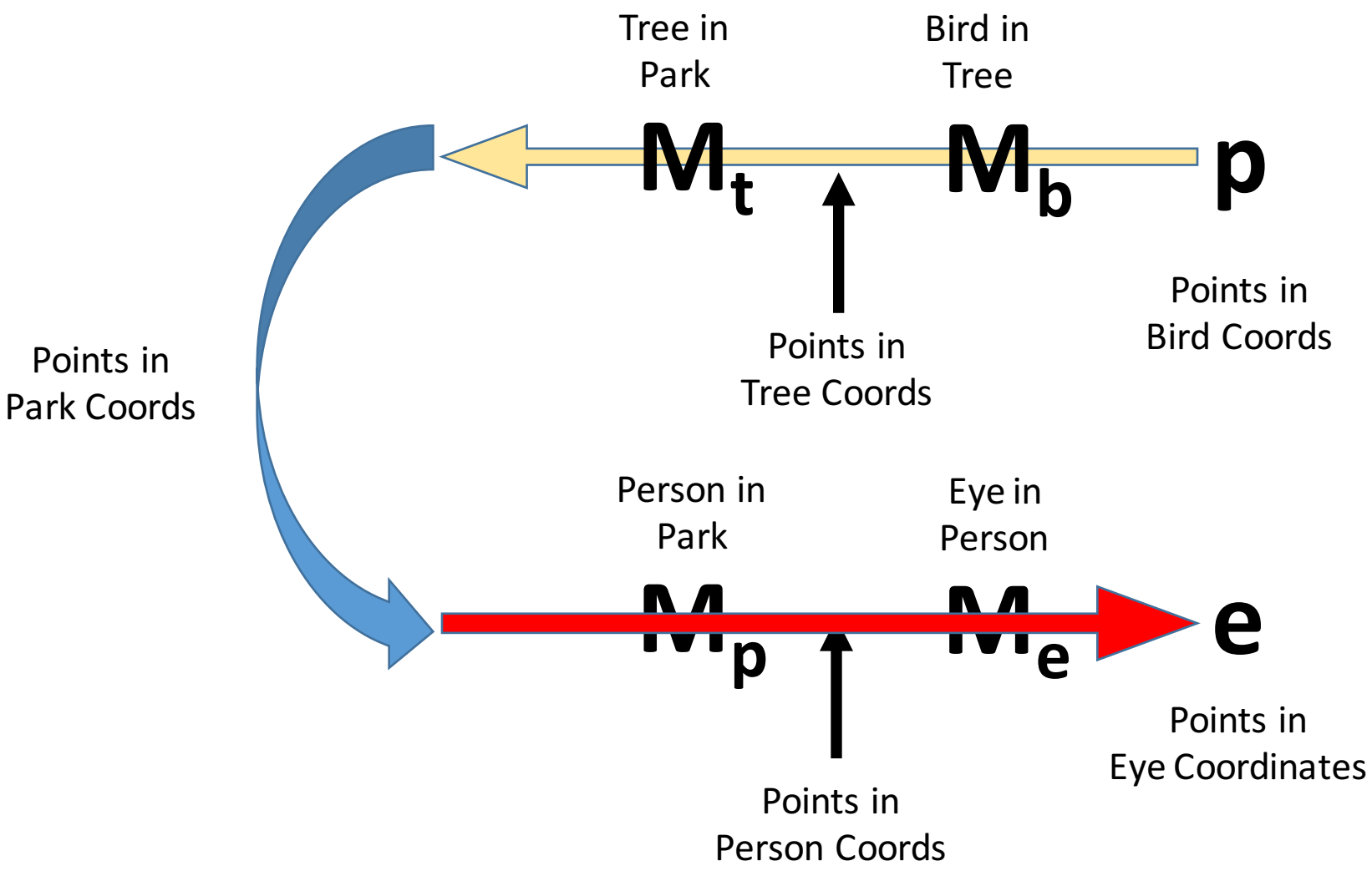
Where is everything in the Park?



Bird in Eye (Camera) Coordinates

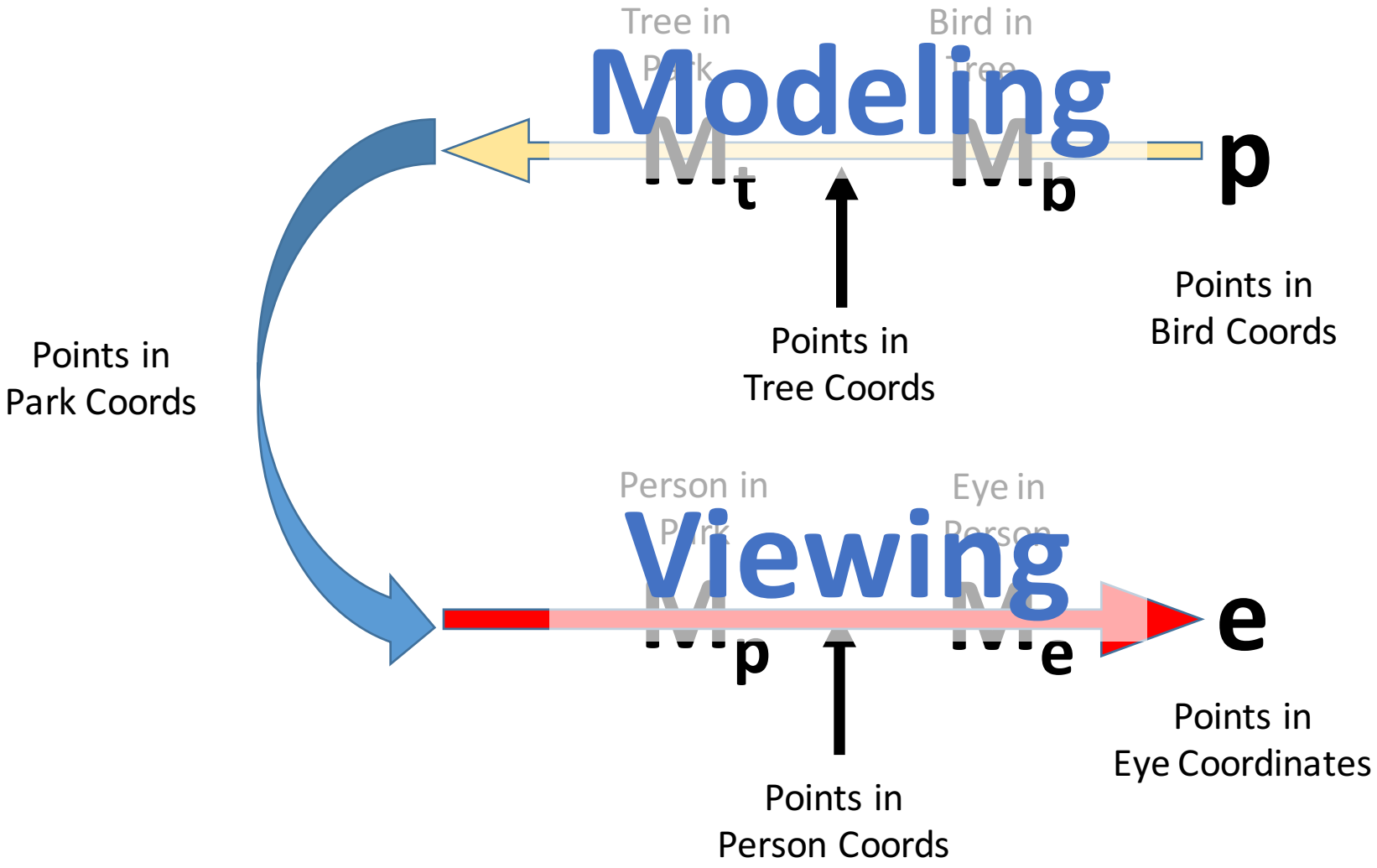


$$e = M_e^{-1} M_p^{-1} M_t M_b p$$



Viewing Matrix Model Matrix

$$e = M_e^{-1} M_p^{-1} M_t M_b p$$



From object to eye: ModelView

Modeling matrix: object to world

Viewing matrix: world to eye / camera

Rigid Transformation (rotate/translate)

Invert the camera's model matrix

Build a "LookAt / LookFrom" matrix

Next Problem: Projection

Convert 3D (eye coordinates) to 2D (screen)

A transformation

Types:

- Orthographic
- Perspective
- some others we won't talk much about

Orthographic Projection

Scale X and Y to fit things on screen

Note: we can look in any direction

we are already in camera coodinates!

Orthographic Projections

Simple

Preserves Distances

Objects far away same size as close

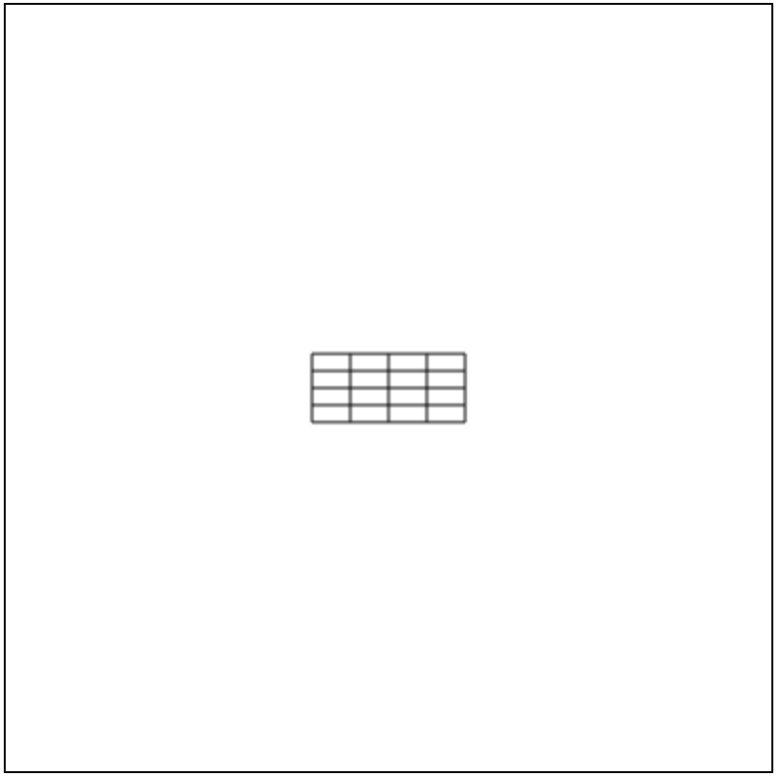
Looks weird

Perspective Projections

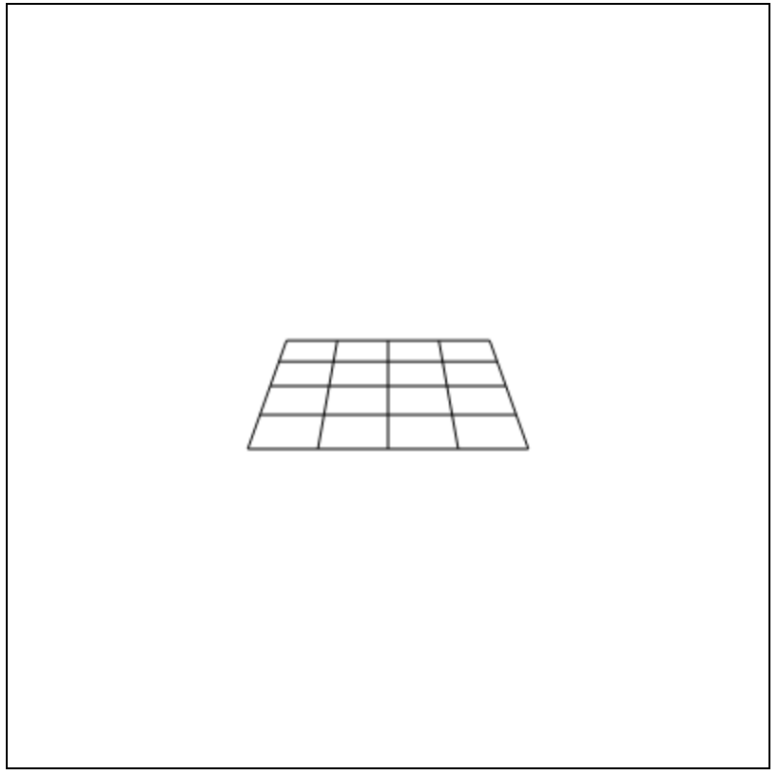
Objects that are far away look smaller

My sample P3 . . .

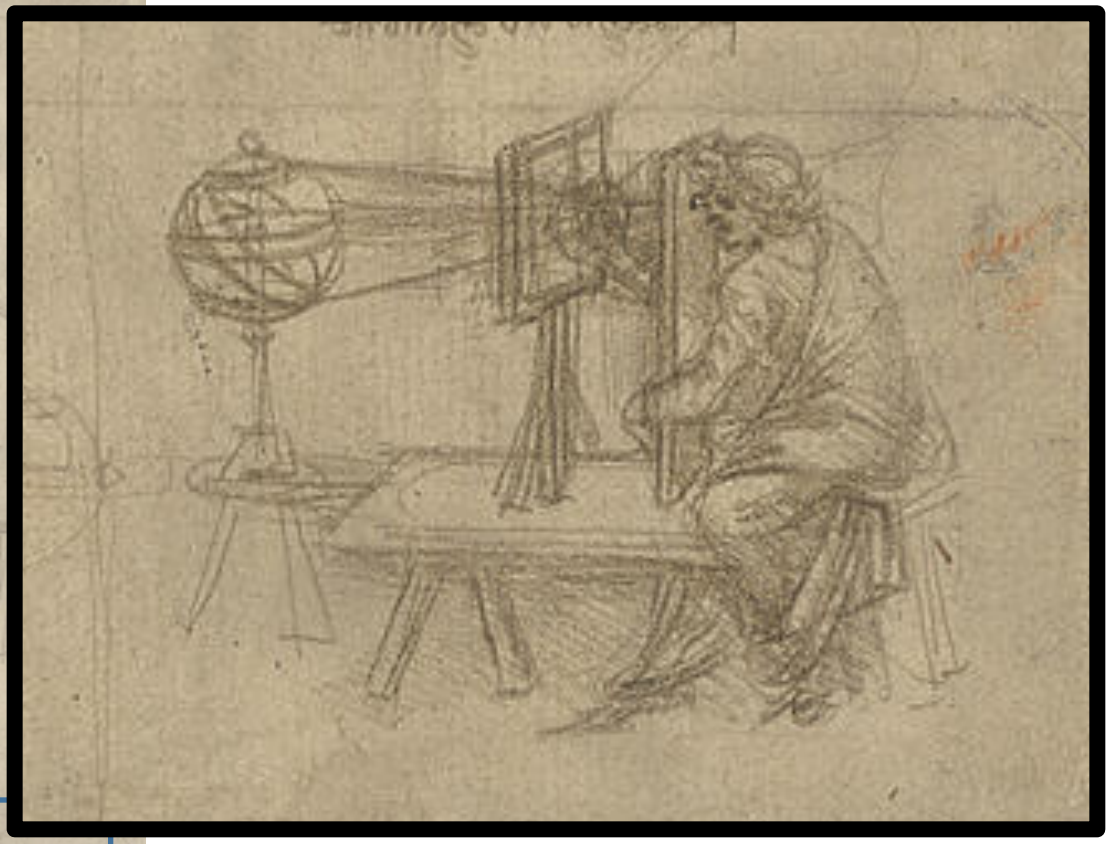
Orthographic

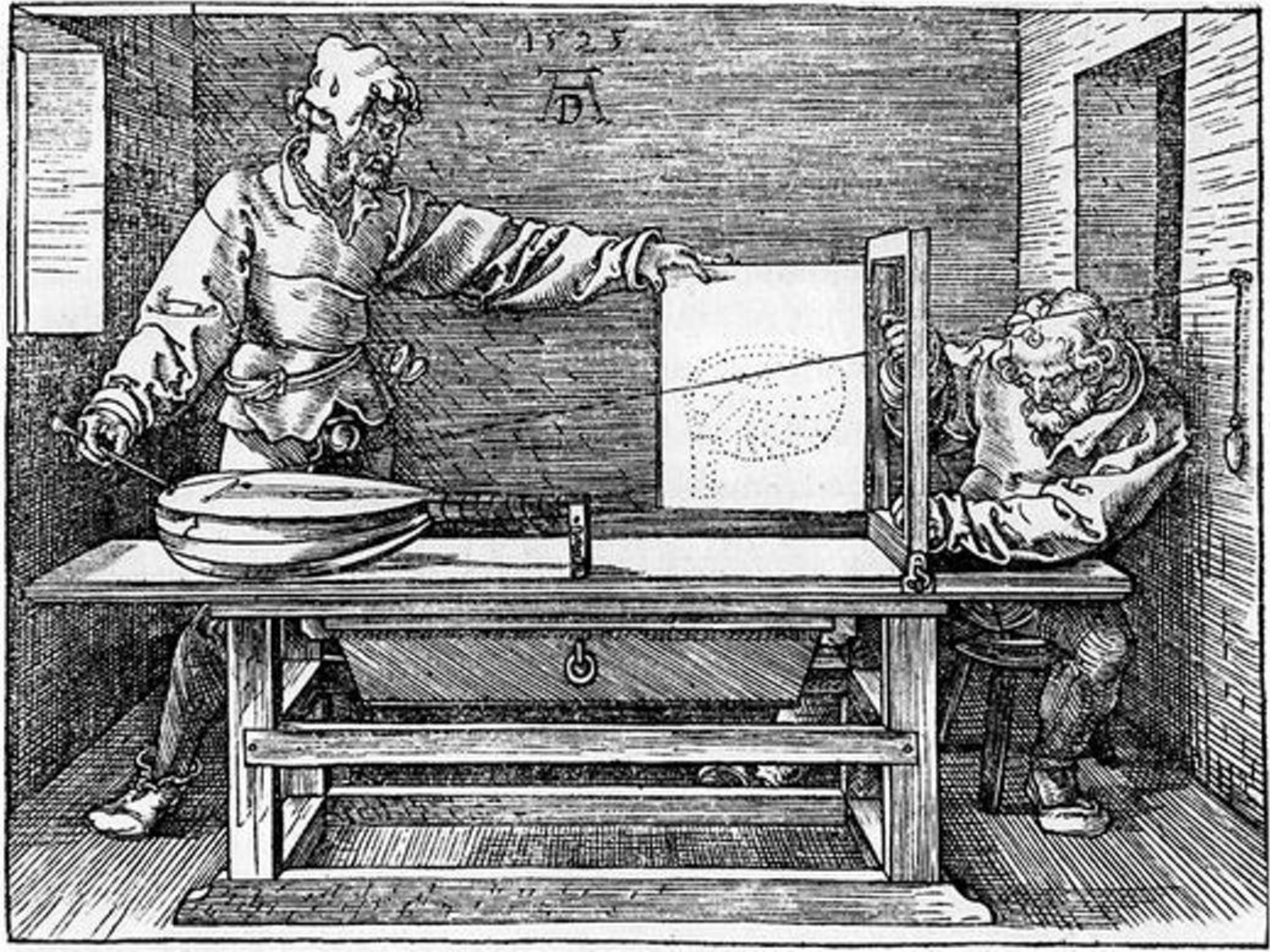


Perspective



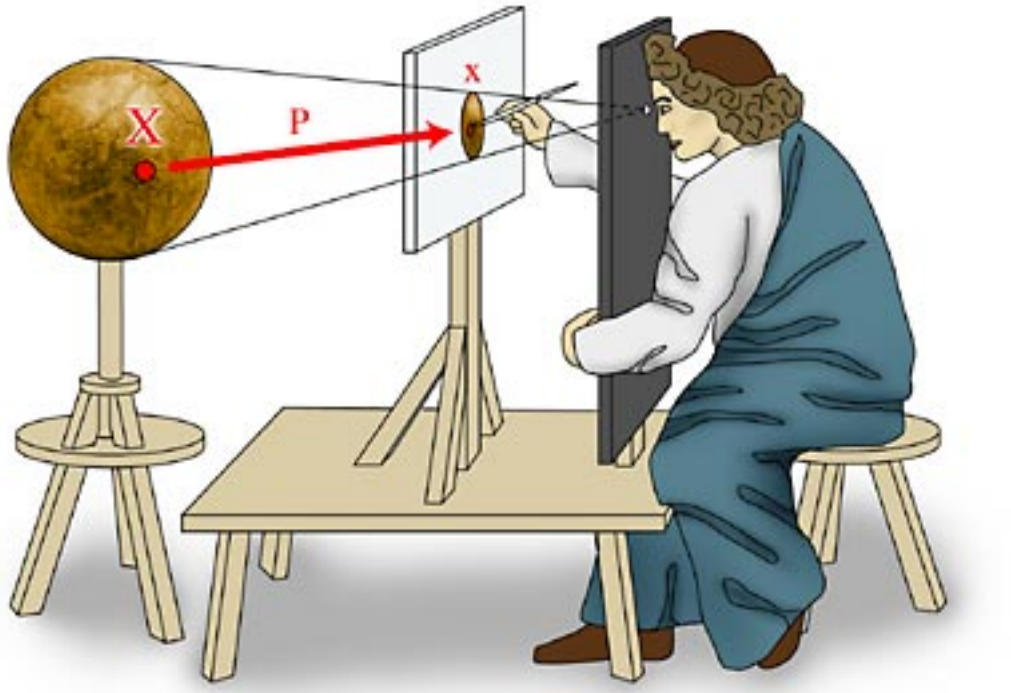
Drawing in Perspective




1525

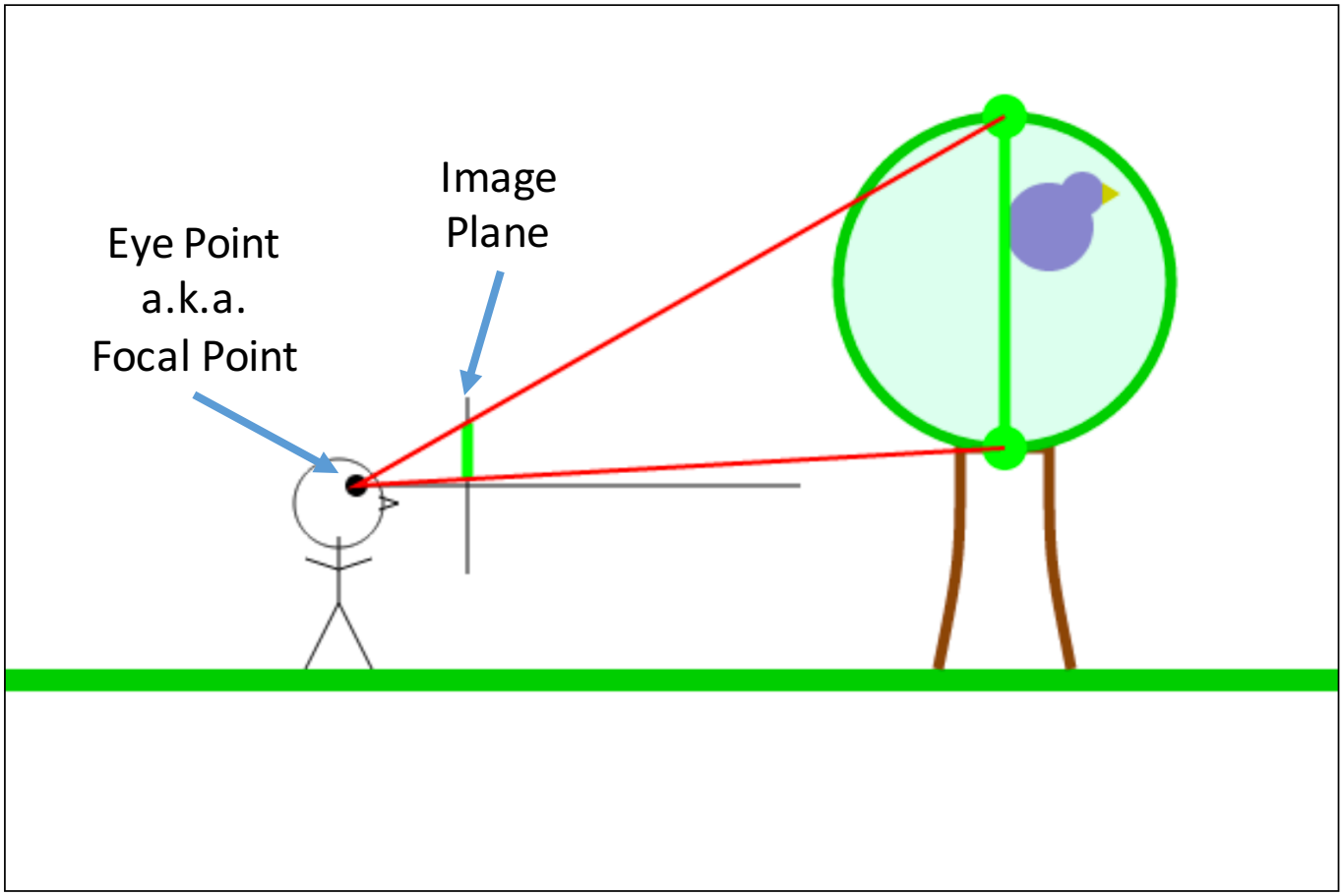
A draughtsman drawing a portrait, Albrecht Dürer, 1532. E.44-1894

http://www.vam.ac.uk/content/articles/d/drawing-techniques/



http://hans.wyrdweb.eu/about-perspective/

Perspective Imaging

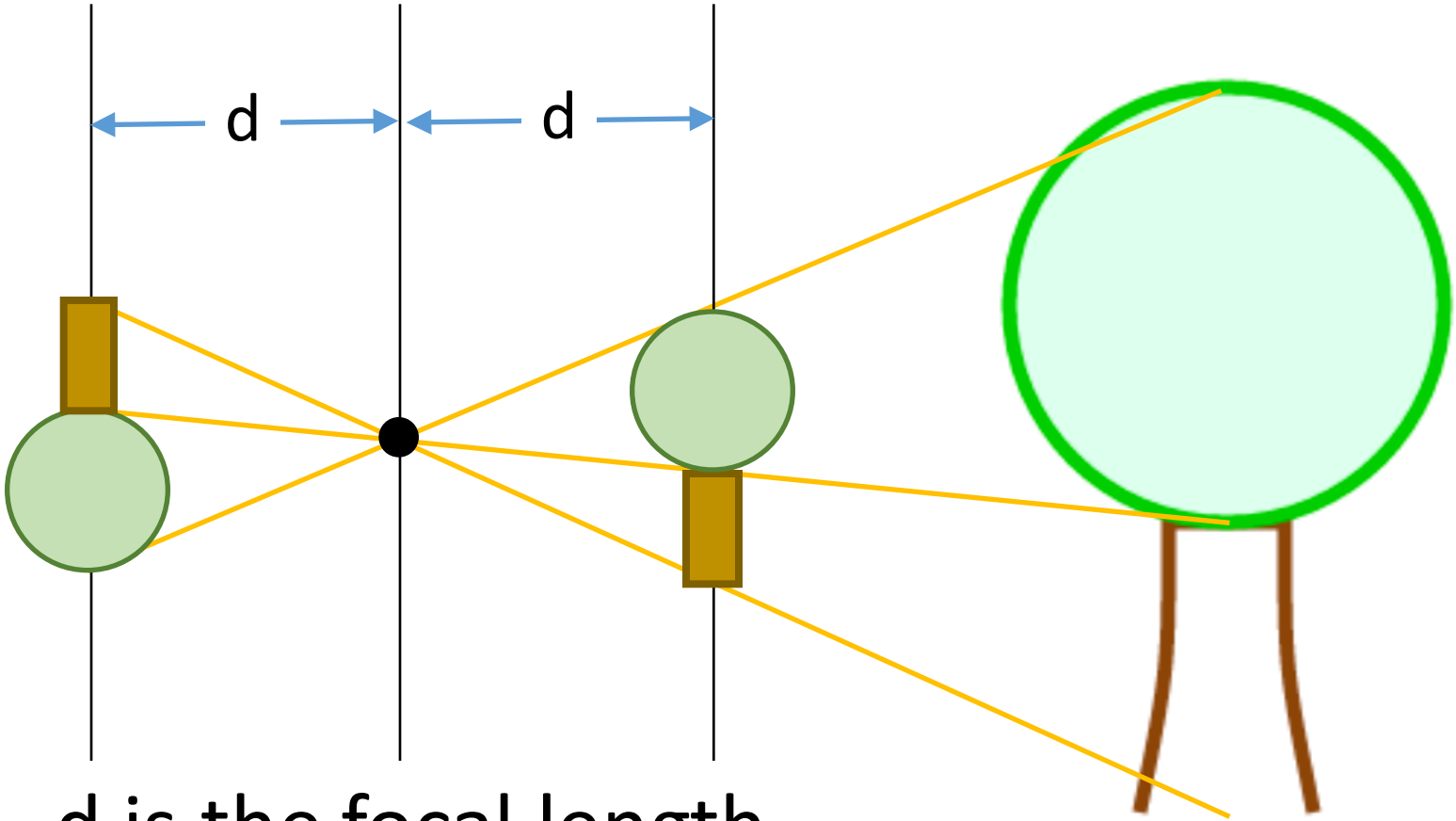


Perspective Assumptions

There is a single focal point

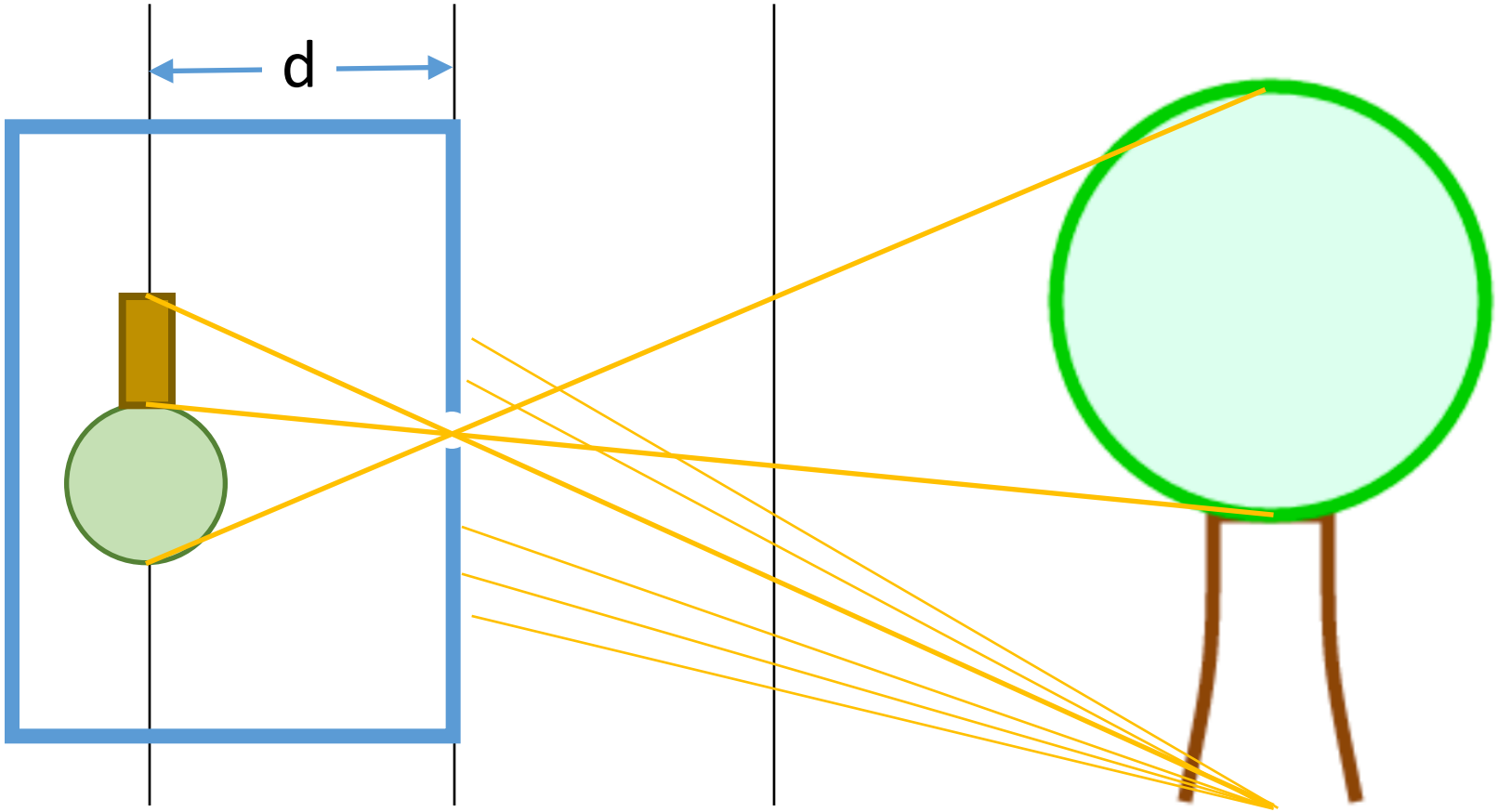
Simplifying Assumptions: (not required)
Image plane orthogonal to view direction
Image plane centered on view direction

Image plane in front of eye
Image plane behind eye

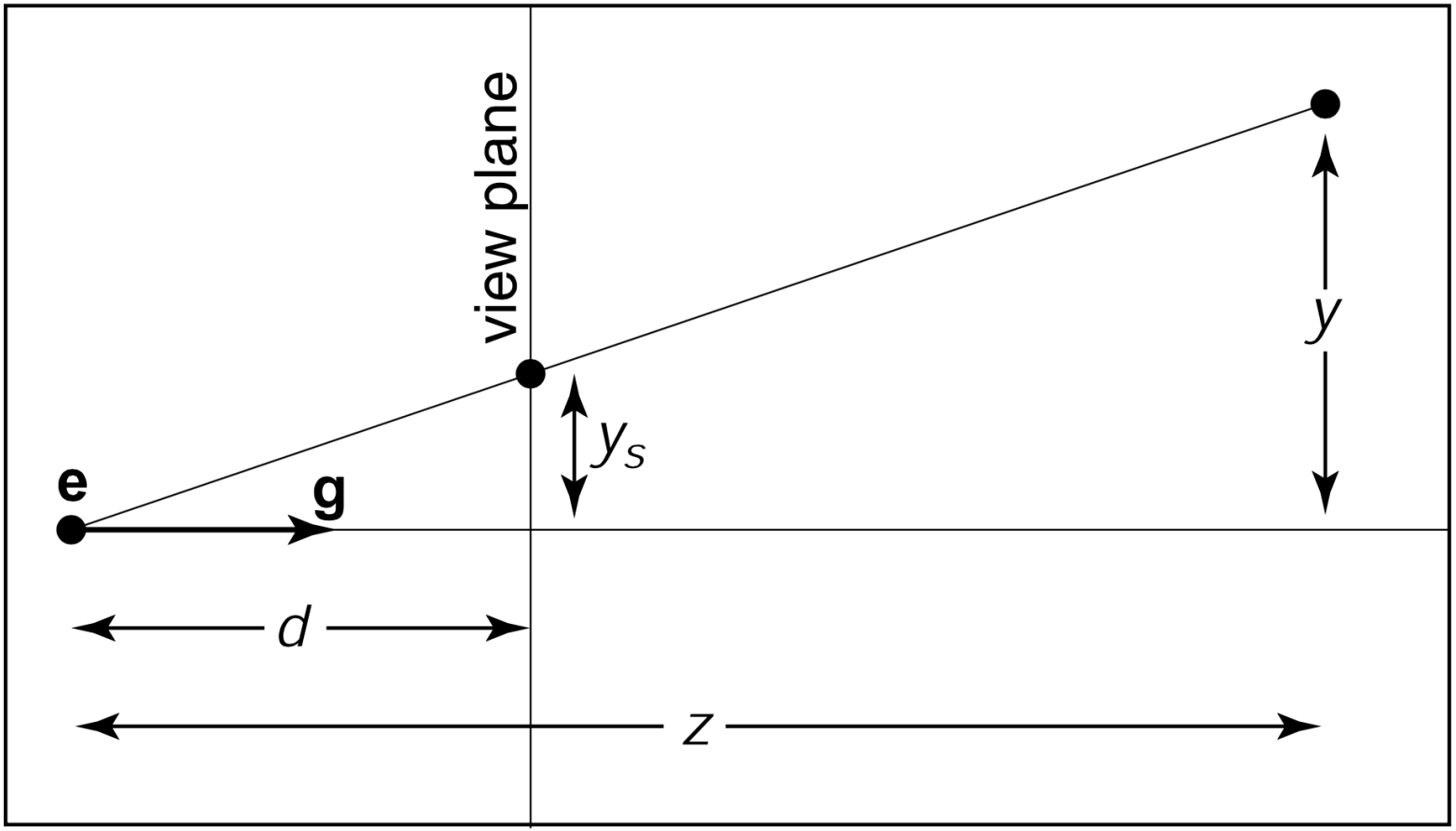


d is the focal length

Pinhole Camera

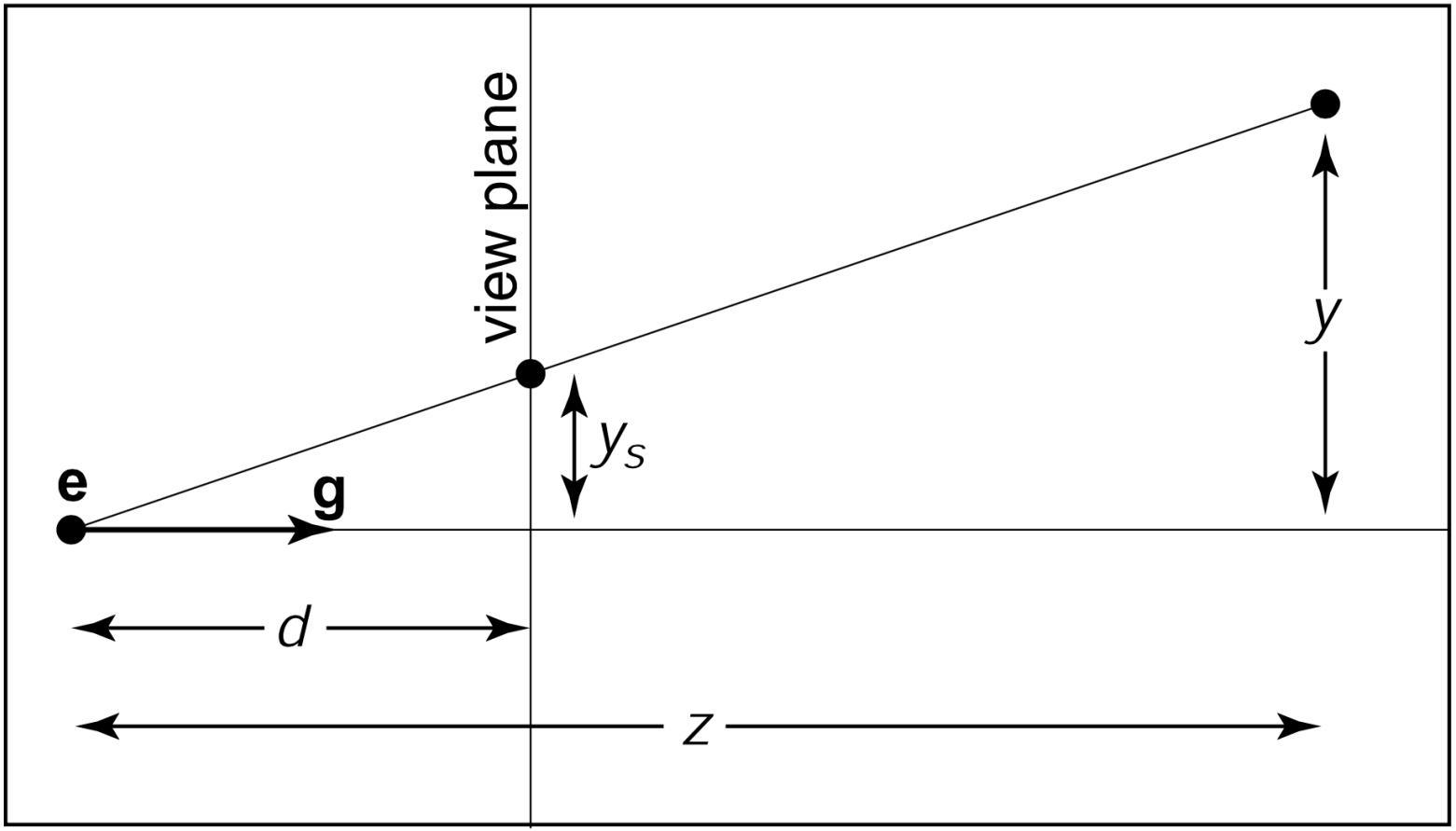


Perspective math



Perspective math

$$y_s = \frac{d}{z} y$$



Perspective Projection

This is linear in homogeneous coordinates

$$x_s = \frac{d}{z}x \qquad y_s = \frac{d}{z}y$$

Rotate so w=z, the divide by w does the job

Details in the book (or next class)

Homogeneous Coordinates

What does this **w** axis do?

So far, it's always been:

1 (for points)

0 (for vectors)

Projective Equivalence

In homogeneous coordinates, points correspond to rays through the origin.

In 3D...

The point x,y,z

Has a set of homogeneous coordinates:

[x,y,z,1] [2x, 2y, 2z, 2], ...

The divide by w

For a 3D point x,y,z

It can be anything of the form:

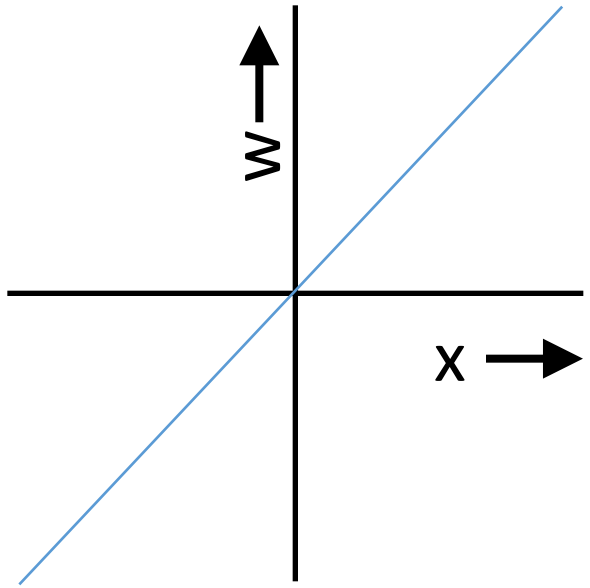
$$[\ \alpha x, \alpha y, \alpha z, \alpha\ ]$$

We consider the “hyperplane” [w=1] to be our “regular” 3D space. So we can convert:

$$[\ \frac{x}{\alpha}, \frac{y}{\alpha}, \frac{z}{\alpha}, 1\ ]$$ - divide by w

Projective coodinates for 1D

1D (x) becomes (x,w)



Simplest Projective Transform

$$\begin{bmatrix} dx \\ dy \\ 1 \\ z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

After the divide by w…

Note that this is dx/z, dy/z (as we want)

Note that z’ is 1/z (we can’t keep Z)

Fancier forms scale things correctly

All the coordinate systems

Window (Screen) – in pixels

Normalized Device – [-1 1]

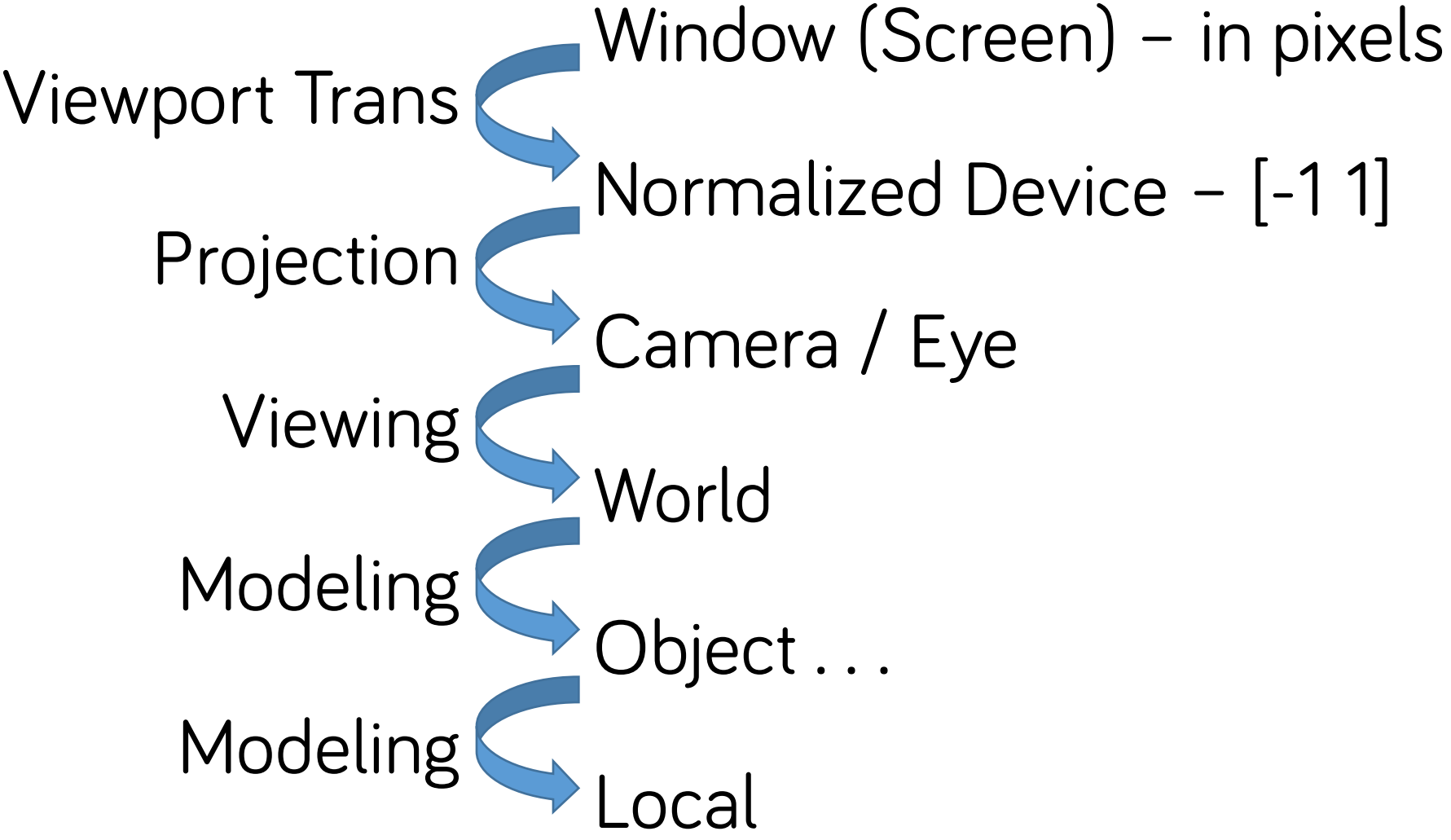
Camera / Eye

World

Object . . .

Local

Transformations between each



Program 3 Hints

All the transforms (even projection) are matrices.

The matrix library has them implemented
You don't have to implement them yourself
but it's important to understand them

TWGL's "screen" coordinates are [-1 , 1]
You have to convert to Canvas Pixels

Demos