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The calculation of standard coordinates in Schmidt plate astrometry

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Starting from an exact definition of the tangential point on Schmidt plates the three azimuthal projections used in Schmidt plate astrometry are compared and plate-tilt effects, which differ from those on astrographic plates, are discussed. It is shown that the azimuthal equidistant projection should be used preferably in Schmidt plate reduction. Practical recommendations are given to minimize the number of significant terms in the reduction model.

Ausgehend von einer exakten Definition des Tangentialpunkts auf Schmidtplatten werden die drei Azimuthalprojektionen verglichen, die in der Astrometrie mit Schmidt-Teleskopen Anwendung finden, sowie die Effekte der Plattenneigung diskutiert, die sich anders als auf Astrographenplatten äußern. Es wird gezeigt, daß die mittelabstandstreue Azimuthalprojektion bevorzugt für die Reduktion von Schmidtplatten verwendet werden sollte. Zur Minimierung der Zahl der signifikanten Glieder im Reduktionsmodell werden praktische Empfehlungen gegeben.

Key words: astrometry – Schmidt telescope – standard coordinates

AAA subject classification: 041

1. Introduction

In spite of the experience collected for 35 years with Schmidt telescopes as astrometric instruments, a number of unresolved problems still remain. The central question is how to model the rather complicated geometry of Schmidt plates. To decrease the number of terms in the reduction polynomials and thus to decrease the transformation errors, two suggestions were made recently: HIRTE et al. (1990) proposed to use a statistically strong method of stepwise regression, whereas TAFF (1989) offered a subplate overlap technique to avoid global modelling.

We will show here that, in connection with stepwise regression, a certain decrease in the number of significant terms can be reached in the pre-modelling steps of reduction, i.e. the calculation of standard coordinates and/or the introduction of corrections to the measured coordinates. For this purpose we will rediscuss the definition of the tangential point, the choise of the projection, and the plate-tilt effects. These questions may seem to be rather elementary, but some details have to be clarified because they are treated by several authors in different ways. This way, on the one hand, we hope to make a certain contribution to the theoretical foundations of Schmidt plate astrometry, and, on the other hand, we will try to make some recommendations for practical work.

To illustrate the recommendations we will demonstrate examples of reduction models on plates of the Tautenburg Schmidt Telescope (134/200/400). Ordinary solutions to second-order polynomials (POL2) and stepwise-regression solutions to second- and third-order polynomials (STR2 and STR3) are presented. In the latter non-significant terms are excluded. The stepwise regression method described by HIRTE et al. (1990) was used.

2. The tangential point on Schmidt plates

In the following analysis we will distinguish the "intrinsic" tangential point T_I on the plate itself and the "assumed" tangential point T_A on the celestial sphere, which is used for the calculation of standard coordinates from catalogue positions. It is well known that in astrographs T_I is the point where the optical axis intersects the photographic plate. Ideally, the objective of the telescope produces a gnomonic projection of the celestial sphere onto the plate relative to T_I.

In the Schmidt telescope, the mirror conformly projects the celestial sphere onto the spherical focal surface on which no distinguished point exists (see, e.g., MURRAY 1983). This led to some confusion about the definition of the tangential point on the Schmidt plate which is constrained to the focal surface during exposition. This point seems to be a not well-defined concept for Schmidt systems (TAFF et al. 1990). It was regarded to be mainly a computational convenience (DIECKVOSS 1960, ANDERSEN 1971) or it was introduced as the "plate centre" without any exact definition (DIXON 1962, WAYMAN 1978).

Beside this, another point has been defined on the Schmidt plate, which "remains at rest during the process of bending in the plateholder" (DIECKVOSS 1960). This "bending point" – or "neutral point" (MURRAY 1983) – is regarded as the zero point of the so-called tangent correction, which has the form of a cubic radial distortion.

The problem of the nature of the intrinsic tangential point can be solved by recognizing how the projection of the celestial sphere onto the flat plate is realized in the Schmidt telescope. In contrast to astrographs, this projection is not performed by the objective, but after exposure by the plate itself when it is restored to its planar state. Thus, the intrinsic tangential point and the neutral point are identical. However, this point is not well-defined in the sense that its position on the plate is not a characteristic of the optical system, but depends on the structure of the individual glass plate. Therefore, it is not useful to develop methods for finding a general position of the tangential point relative to the plate centres as in the case of astrographs.

3. Projections for Schmidt plate astrometry

As usual, we consider a celestial sphere with centre 0 and radius r (Fig. 1). At the tangential point T with equatorial coordinates (A, D) a tangent plane touches the sphere. S' is the projection of an arbitrary point S with spherical coordinates (α, δ) onto the plane. A rectangular system of standard coordinates ξ and η in the tangent plane with origin at T is introduced, so that the η axis is directed to the projection P' of the celestial pole P. For the angles $\theta = T\hat{O}S$ and $\psi = S'\hat{T}P'$ we have

$$\cos\theta = \sin\delta \sin D + \cos\delta \cos D \cos(\alpha - A)\,, \tag{1}$$

$$\sin\theta \cos\psi = \cos\delta \sin(\alpha - A)\,, \tag{2}$$

$$\sin\theta \cos\psi = \sin\delta \cos D - \cos\delta \sin D \cos(\alpha - A)\,. \tag{3}$$

For simplicity we provide the standard coordinates expressed in units of the radius r. Then

$$\xi = TS' \sin\psi\,, \qquad \eta = TS' \cos\psi\,. \tag{4}$$

Three different azimuthal projections were suggested for the use in Schmidt plate astrometry. In all these projections the great circles through T are depicted as straight lines. (For projections in general see, e.g., Richardus and Adler (1972)).

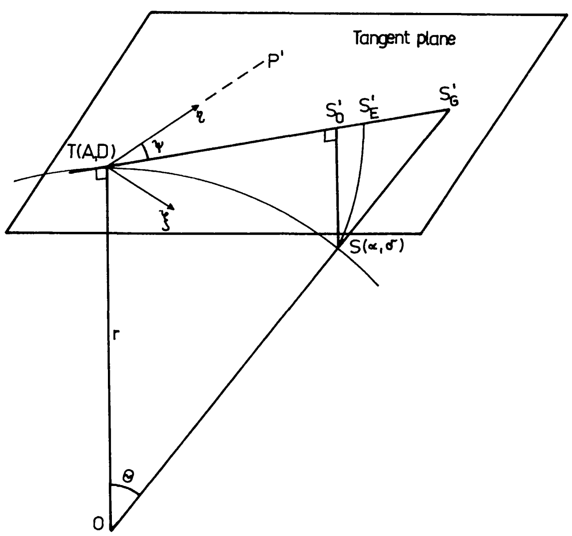

Fig. 1. The three azimuthal projections of the celestial sphere onto a tangent plane

3.1 The gnomonic projection

The gnomonic or central projection has been widely used in Schmidt plate astrometry because workers are familiar with it from astrographic astrometry. The projection is achieved through rays from the centre 0. The distance of the projected point S'_G from T is

$$TS'_G = \tan\theta\,. \tag{5}$$

From eqs. (2) to (5) we obtain the well-known mapping equations of the gnomonic projection

$$\xi_G = \frac{\cos\delta\sin(\alpha - A)}{\cos\theta} \tag{6}$$

$$\eta_G = \frac{\sin\delta\cos D - \cos\delta\sin D\cos(\alpha - A)}{\cos\theta}, \tag{7}$$

where $\cos\theta$ is defined by eq. (1) and their inversions

$$\tan(\alpha - A) = \frac{\xi_G}{\cos D - \eta_G \sin D}, \tag{8}$$

$$\tan\delta = \frac{\eta_G\cos D + \sin D}{\cos D - \eta_G\sin D}\cos(\alpha - A). \tag{9}$$

3.2. The azimuthal equidistant projection

In this projection the distance from the projected point S'_E to T is equal to the arc length from S to T:

$$TS'_E = \theta. \tag{10}$$

(In the stronger sense this is not a projection, because the mapping is not achieved through rays. However, this distinction is usually made only in German cartographic literature.)

The mapping equations are

$$\xi_E = \frac{\theta}{\sin\theta}\cos\delta\sin(\alpha - A), \tag{11}$$

$$\eta_E = \frac{\theta}{\sin\theta}(\sin\delta\cos D - \cos\delta\sin D\cos(\alpha - A), \tag{12}$$

where θ and $\sin\theta$ must be obtained from eq. (1). Only in such cases when double precision is not available, this may be a problem in computer calculations.

The inverted mapping equations are

$$\theta^2 = \xi_E^2 + \eta_E^2, \tag{13}$$

$$\sin\delta = \eta_E\cos D\frac{\sin\theta}{\theta} + \sin D\cos\theta, \tag{14}$$

$$\sin(\alpha - A) = \frac{\xi_E}{\cos\delta}\frac{\sin\theta}{\theta}. \tag{15}$$

In first approximation, the Schmidt plate realizes this projection when returning to its planar state. Obviously, most of the problems with modelling the geometry of Schmidt plates lie in the deviations from this approximation.

The use of the azimuthal equidistant projection (but without referring to this term) for Schmidt plate astrometry was suggested independently from each other by STOCK (1965, unpublished; cited by PENALOZA 1981), LUYTEN and LA BONTE (1972), LEUPIN (1972) and WAYMAN (1978). PENALOZA (1981) called it concentric projection.

Neglecting very small terms, standard coordinates ξ_E, η_E from equidistant projection can be transformed into gnomonic standard coordinates ξ_G, η_G by

$$\xi_G = \xi_E + \tfrac{1}{3}\xi_E(\xi_E^2 + \eta_E^2) \tag{16}$$

$$\eta_G = \eta_E + \tfrac{1}{3}\eta_E(\xi_E^2 + \eta_E^2) \tag{17}$$

Thus, using the gnomonic projection for Schmidt plate astrometry an apparent cubic distortion is introduced into the standard coordinates relative to the measured coordinates. To avoid the appearance of these terms in the plate model, the measured coordinates x, y should be corrected for this apparent distortion with the help of eqs. (16) and (17) replacing ξ and η by x and y (see eqs. (31) and (32)). This transformation is known as tangent or Schmidt correction. When x and y are not expressed in units of the focal length f, the distortion coefficient in eqs. (31) and (32) is $\frac{1}{3f^2}$.

3.3 The orthographic projection

This projection is obtained through parallel rays perpendicular to the tangent plane. The distance of the projected point S'_0 from T is

$$TS'_0 = \sin\theta\,. \tag{18}$$

Thus, the mapping equations are

$$\xi_0 = \cos\delta\sin(\alpha - A)\,, \tag{19}$$

$$\eta_0 = \sin\delta\cos D - \cos\delta\sin D\cos(\alpha - A)\,, \tag{20}$$

and their inversions

$$\sin^2\theta = \xi_0^2 + \eta_0^2\,, \tag{21}$$

$$\sin\delta = \eta_0\cos D + \cos\theta\sin D\,, \tag{22}$$

$$\sin(\alpha - A) = \frac{\xi_0}{\cos\delta} \tag{23}$$

The use of this projection was suggested by DIXON (1962). He called the standard coordinates defined by eqs. (19) and (20) "Schmidt coordinates". This naming was rather arbitrary since the orthographic standard coordinates ξ_0 and η_0 deviates from the equidistant standard coordinates ξ_E and η_E (and thus from the measured coordinates on a Schmidt plate) in a similar way as the gnomonic standard coordinates but with the distortion coefficient $-\frac{1}{6}$ (or $-1/6f^2$).

In the following we will not discuss this projection, because there is no reason to use it in praxis instead of the preceding ones. However, the considerations concerning gnomonic projection are analogously valid for orthographic projection. Only the terms for plate tilt differ (DIXON 1962).

4. Plate tilt

Considering the geometry of the system of image centres, hereafter called "plate geometry", we have to distinguish three cases: (1) the "absolute" geometry of measured coordinates relative to standard coordinates, (2) the differential geometry of two exposures on two plates, and (3) the differential geometry of two exposures on one plate.

4.1 Absolute geometry

Let the celestial sphere coincide with the spherical surface of the Schmidt plate in its constraint state during exposure. Then the plate in its planar state represents a plane tangent to the sphere at the intrinsic tangential point T_I with the equatorial coordinates (A_I, D_I). The coordinates (A_A, D_A) of the assumed tangential point T_A differ by the small values p and q from (A_I, D_I):

$$p = (A_I - A_A)\cos D_I\,, \qquad q = D_I - D_A\,. \tag{24}$$

It is well known that this so-called plate tilt (or centring error) leads in first approximation to the following relations between gnomonic standard and measured coordinates (the latter are assumed to be related to the intrinsic tangential point):

$$\xi_G = x_G + p + L_x + x_G(px_G + qy_G)\,, \tag{25}$$

$$\eta_G = y_G + q + L_y + y_G(px_G + qy_G)\,, \tag{26}$$

where L_x and L_y are linear terms expressing an orthogonal rotation of the coordinate axis. For the azimuthal equidistant projection the corresponding relationships are

$$\xi_E = x_E + p + L_x + \tfrac{1}{3}\,y_E(qx_E - py_E)\,, \tag{27}$$

$$\eta_E = y_E + q + L_y - \tfrac{1}{3}\,x_E(qx_E - py_E)\,. \tag{28}$$

These equations were derived by LEUPIN (1972), but his paper remained unnoticed by workers in this field. LEUPIN used spherical trigonometry for the derivation. We will show that eqs. (27) and (28) may be derived from eqs. (25) and (26) by a formal substitution of coordinates.

Replacing in

$$\xi_E = \xi_G - \tfrac{1}{3}\,\xi_G(\xi_G^2 + \eta_G^2) \tag{29}$$

ξ_G and η_G by x_G and y_G according to eqs. (25) and (26) and neglecting higher-order terms, we get

$$\xi_E = x_G + p + L_x - \tfrac{1}{3} x_G(x_G^2 + y_G^2) + \tfrac{1}{3} q x_G y_G - \tfrac{1}{3} p y_G^2 \,. \tag{30}$$

Substituting herein x_G and y_G by

$$x_G = x_E + \tfrac{1}{3} x_E(x_E^2 + y_E^2) \tag{31}$$

and

$$y_G = y_E + \tfrac{1}{3} y_E(x_E^2 + y_E^2) \,, \tag{32}$$

and neglecting again higher-order terms, we finally obtain eq. (27). Similarly, we may derive eq. (28).

Thus, an error in the coordinates of the assumed tangential point has on Schmidt plates an influence on the standard coordinates which is three times smaller than on astrographic plates and gives rise to other quadratic terms when the azimuthal equidistant projection is used.

For the case when the gnomonic projection is used and the measured coordinates are corrected for the apparent distortion, the plate-tilt terms depend on how this correction is made. Note that small errors u and v in the zero points of x_E and y_E, respectively, change eqs. (31) and (32) to

$$x'_G = x_E + u + \tfrac{1}{3}(3u x_E^2 + 2v x_E y_E + u y_E^2) + \tfrac{1}{3} x_E(x_E^2 + y_E^2) \,, \tag{33}$$

$$y'_G = y_E + v + \tfrac{1}{3}(v x_E^2 + 2u x_E y_E) + 3v y_E^2) + \tfrac{1}{3} y_E(x_E^2 + y_E^2) \,. \tag{34}$$

When x'_G andy'_G are used instead of x_G and y_G. i.e., when the zero point of the measured coordinates does not coincide with the intrinsic tangential point, the errors introduced according to eqs. (33) and (34) have to be taken into account in the relations between measured and standard coordinates. Eqs. (25) and (26) have to be modified:

$$\xi_G = x_G + p + L_x + x_G(p x_G + q y_G) - u - \tfrac{1}{3}(3u x_G'^2 + 2v x'_G y'_G + u y_G'^2) \,, \tag{35}$$

$$\eta_G = y_G + q + L_y + y_G(p x_G + q y_G) - v - \tfrac{1}{3}(v x_G'^2 + 2u x'_G y'_G + 3v y_G'^2) \,. \tag{36}$$

If the zero point of the measured coordinates could be really put into the intrinsic tangential point, so that $u = 0$ and $v = 0$, the plate tilt terms in the plate model would be that of eqs. (25) and (26), i.e., as on astrographic plates. This is naturally only a theoretical case.

When the zero point of the measured coordinates coincides with the assumed tangential point, i.e., $u = p$ and $v = q$, eqs. (35) and (36) become identical with eqs. (27) and (28), except the absolute terms being not essential.

In all other cases, the plate model is rather complicated. In other words, when the zero point of the measured coordinates does not coincide with the assumed tangential point, more quadratic terms may appear as significant in the general reduction polynomial. This is especially disadvantageous when we try to reduce the number of terms by means of stepwise regression or if we use a special polynomial with plate-tilt terms only.

It can easily be shown that the same is true for the case when the gnomonic projection is used without applying the tangent correction.

4.2. Differential geometry of two plates

Even in the case when the geometrical plate centres of two plates have the same equatorial coordinates, their intrinsic tangential points may have a distance of several arcminutes. Designing the (unknown) differences of the equatorial coordinates of the two intrinsic tangential points in analogy to eqs. (24) by p and q, and recognizing that the rectangular coordinates on the plates are produced by an azimuthal equidistant projection, we again may use eqs. (27) and (28) replacing ξ_E and η_E by the measured coordinates of the second plate.

The introduction of tangent corrections to the measured coordinates makes no sense in the consideration of differential geometry, e.g., when we derive proper motions by the differential method. It would only give rise to additional quadratic terms according to eqs. (35) and (36), which, however, could be taken into account in advance because the u and v, which are here the differences in the coordinates of the zero points used, are principially known.

4.3 Differential geometry of two exposures on one plate

The intrinsic tangential point on a plate is the same for all exposures on this plate, because it is produced by the plate itself. However, this point has different equatorial coordinates for different exposures. The differences p and q of these equatorial coordinates may be determined with sufficient accuracy by measuring the distance of two images of the same object near the plate centre. The measured coordinates of the second system of images can then be corrected for plate-tilt terms according to eqs. (27) and (28) before fitting them to the coordinates of the first system of images.

5. Practical considerations

According to the theoretical considerations made above the following practical recommendations can be given:
1) When using gnomonic projection in Schmidt plate reduction, the measured coordinates should be corrected for the apparent cubic distortion to avoid-order terms in the plate model (Tab. 1). The zero point of this correction should coincide with the assumed tangential point in order to avoid additional quadratic terms (cf. below, Tab. 2). To fulfil this, the equatorial coordinates of the zero point of the measured coordinates, which are usually identical with the zero point of the tangent correction, have to be determined before the reduction procedure.

One may argue that the apparent cubic distortion introduced by the gnomonic projection can be accomodated by appropriate terms in the plate models, as was, e.g., done in the plate reductions for the Guide Star Catalogue (Russel et al. 1989). But our experience with the Tautenburg Schmidt Telescope has shown that in many cases intrinsic cubic terms did not appear as significant in the reduction model, so that by avoiding the apparent terms which are always significant, the model could be simplified.

2) Note that the plate-tilt terms are different for Schmidt telescopes than for astrographs even when the gnomonic projection is used. This has to be taken into account when one wants to apply a special polynomial with plate-tilt terms only or to calculate the coordinates of the intrinsic tangential point from the quadratic terms.

3) In some cases it is possible to exclude quadratic terms from the reduction polynomial by a two-step procedure: in the first step approximate values for the coordinates of the tangential point are used when calculating the standard coordinates. After the fitting of measured and standard coordinates, p and q can be determined from the corresponding terms in the polynomials and added to the assumed equatorial coordinates of the tangential point. Now the assumed tangential point comes very near to the intrinsic tangential point, so that after the determination of new standard coordinates the plate-tilt terms disappear (Tab. 2). This procedure also allows the use of rather rough values for the coordinates of the plate centre in first approximation, e.g. the readings of the pointing mechanism of the telescope.

As other effects than plate-tilt may influence the quadratic terms, it is not recommended to use a common polynomial for the x and y coordinates with plate-tilt terms only. Preferably, the p and q should be calculated from full second-order polynomials, separately for x and y coordinates, and then be averaged.

Table 1
Example for recommendation 1. (Plate 4492; 82 AGK3 stars; STR3 solutions; standard deviation of the residuals: 0.26″)

Term	Gnomonic projection				Gnomonic projection + tangent. correction or azimuthal equidistant projection			
	in x		in y		in x		in y	
x^2	–		0.15	(2.0)	–		0.14	(2.0)
xy	−0.17	(2.3)	–		−0.15	(2.3)	–	
y^2	–		–		–		–	
x^3	1.05	(8.7)	–		–		–	
x^2y	–		0.91	(8.6)	–		–	
xy^2	1.05	(8.4)	–		–		–	
y^3	–		1.14	(8.1)	–		–	

Plate constants are given in arcseconds for coordinates in units of 5000″ (i.e. values at a distance of about 100 mm from the plate centre). In parantheses constant-to-error ratios are presented. The zero point of the measured coordinates coinciding with the assumed tangential point was put into the field centre.

Table 2. Example for recommendation 3. (Plate 4492, cf. Tab. 1)

Term	Both projections POL2				Azim. equid. proj. after correcting A, D for p, q STR2		Gnomonic projection after correcting A, D for p, q STR2			
	in x		in y		in x	in y	in x		in y	
x^2	0.06	(0.8)	0.14	(2.0)	–	–	0.15	(2.2)	0.14	(1.9)
xy	−0.14	(2.2)	−0.05	(0.8)	–	–	0.28	(4.3)	–	
y^2	0.01	(0.1)	−0.03	(0.4)	–	–	–		0.39	(5.0)

Units as in Tab. 1

When using the gnomonic projection, the zero-point of the tangent correction has to be moved together with the assumed tangential point. Otherwise quadratic terms according to eqs. (35) and (36) will appear in the reduction polynomial (Tab. 2, last column).

4) It is possible that due to other effects than plate tilt the formally calculated p and q are very different for the two coordinates. In this case the formal approach described above may even enlarge the number of significant terms (Tab. 3). The following non-formally heuristic approach can be used to avoid this difficulty:

Table 3
Example for recommendation 4. (Plate 6507; 52 AGK3 stars; azimuthal equidistant projection; standard deviation of the residuals: about 0.50″)

Term	Without corrections							
	POL2				STR2			
	in x		in y		in x		in y	
x^2	−0.33	(1.6)	0.13	(0.6)	−0.36	(1.8)	–	
xy	0.58	(3.2)	0.30	(1.7)	0.54	(3.2)	0.32	(1.8)
y^2	−0.21	(1.0)	0.15	(0.8)	–		–	
	After formally correcting A, D for p, q							
x^2	−0.33	(1.6)	0.36	(1.7)	−0.32	(1.6)	0.36	(1.8)
xy	0.35	(2.0)	0.05	(0.3)	0.36	(2.1)	–	
y^2	0.05	(0.2)	0.15	(0.8)	–		–	
	After non-formally correcting A, D for p, q							
x^2	−0.33	(1.6)	0.21	(1.0)	−0.32	(1.6)	–	
xy	0.49	(2.8)	0.04	(0.2)	0.53	(3.1)	–	
y^2	0.05	(0.2)	0.15	(0.8)	–		–	

Units as in Tab. 1

We consider the terms ay^2 in the fitting polynomial of ξ and x and bxy in the polynomial of η and y. According to eqs. (27) and (28),

$$p = \tfrac{3}{2}\,(b - a) \tag{34}$$

gives an estimation of the plate-tilt effect in right ascension. Using this value as a correction to $A_A \cos D_A$, the new values of a and b after the calculation of new standard coordinates will be approximately

$$a' = b' = \tfrac{1}{2}\,(a + b)\,. \tag{36}$$

If $|a'|$ is greater than $|a|$ and greater than its error, it may give rise to a significant term $a'y^2$, even in the case when ay^2 was not significant. To avoid this, we decrease the module of p so, that a' will not be greater than its error. We take the estimated standard deviation s_a of a as an approximation to the standard deviation of a' and calculate a new value p' from p as

$$p' = p - 3(\tfrac{1}{3}\,|a + b| - s_a)\,\mathrm{sign}\,p\,. \tag{37}$$

The same has to be done if $|b'|$ is greater than $|b|$ and similarly we may obtain a new value for q.

5) The most appropriate way to calculate standard coordinates in Schmidt plane reduction is to use the azimuthal equidistant projection. Its application avoids the introduction of non-linear corrections to the measured coordinates which may disturb the error distribution, although this effect is very small (cf. HIRTE and NEUMANN 1984). But what is more important, it avoids paying attention to the coincidence of the assumed tangential point and the zero point of the measured coordinates.

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