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YEAR : 2^{ND} (UG)

SEMESTER: 4^{TH}

SUBJECT: MATHEMATICS FOR IT-II

DATE: 31^{ST} MAY, 2021

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CO1

CO1:a) Sample space for a single die $= \{1,2,3,4,5,6\}$

Number of possible outcomes for 1 die $= 6$

Number of dice present $= 3$

$\therefore$ Total number of possible outcomes $= 6 \times 6 \times 6 = 216$

Number of outcomes in which 0 4's appear $= 5 \times 5 \times 5 = 125$

Number of outcomes in which exactly 1 '4' appears $= \binom{3}{1} \times 5^2$
$= 75$

[We have $\binom{3}{1}$ ways to pick the die on which '4' appears and 5^2 ways of picking the numbers on the other two dice]

Number of outcomes in which 2 '4''s appear $= \binom{3}{2} \times 5^1 = 3 \times 5 = 15$

[We have $\binom{3}{2}$ ways to pick the 2 dice on which 4 appears and 5 ways of picking the number for the third die]

Number of outcomes in which 3 '4's appear $= \binom{3}{3} \times 5^0 = 1$

Let us take a random variable X which is equal to the number of 4's appearing on the 3 dice.

Then, its probability distribution table:-

X	0	1	2	3
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

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CO1:a Let E = event of getting atleast 1 '4'

$P(E) = P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) = \frac{75}{216} + \frac{15}{216} + \frac{1}{216} = \frac{91}{216}$

$= 1 - P(X<1) = 1 - P(X=0) = 1 - \frac{125}{216} = \frac{91}{216} = 0.421$

$\therefore$ The probability of getting atleast 1 '4' when 3 dice are rolled

$= \frac{91}{216} \simeq 0.421$

CO1:b The probability distribution table for the given random variable X:

x	0.2	0.4	0.5	0.8	1
$P_X(X=x)$	0.1	0.2	0.2	0.3	0.2

a) Range of $X = R_X$ = set of possible values X can take

$\therefore R_X = \{0.2, 0.4, 0.5, 0.8, 1\}$

b) $P(X \leq 0.5) = P(X=0.2) + P(X=0.4) + P(X=0.5)$

$= 0.1 + 0.2 + 0.2 = 0.5$

c) $P(0.25 < X < 0.75) = P(X=0.4) + P(X=0.5)$

$= 0.2 + 0.2 = 0.4$

d) $P(X=0.2 | X<0.6) = \frac{P(X=0.2 \cap X<0.6)}{P(X<0.6)}$ [By Law of Conditional Probability]

CO1:b d) $P(X=0.2 \mid X<0.6) = \dfrac{P(X=0.2)}{P(X=0.2)+P(X=0.4)+P(X=0.5)}$

$= \dfrac{0.1}{0.1+0.2+0.2} = \dfrac{0.1}{0.5} = \dfrac{1}{5} = 0.2$

CO2

CO2:A Let X be a continuous random variable denoting the length of the required manufactured machine part.

Now, X is normally distributed with a mean $=\mu$ and standard deviation $=\sigma$

where,

$\mu = 11cm$

$\sigma = 2\,cm$

Probability density function of X: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Probability that length is between 10.6cm and 11.2cm

$= P(10.6 \leq X \leq 11.2)$

$= P\left(\dfrac{10.6-\mu}{\sigma} \leq \dfrac{x-\mu}{\sigma} \leq \dfrac{11.2-\mu}{\sigma}\right)$

$= P\left(\dfrac{10.6-11}{2} \leq \dfrac{x-11}{2} \leq \dfrac{11.2-11}{2}\right)$ [Converting to z-scores, where $z = \dfrac{x-\mu}{\sigma}$]

$= P(-0.2 \leq z \leq 0.1)$

$= P(-0.2 \leq z \leq 0) + P(0 \leq z \leq 0.1)$

P.T.O.

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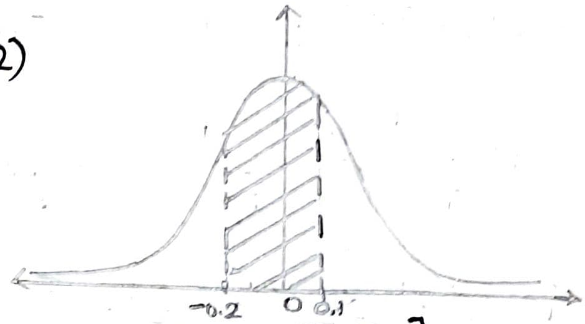
CO2:A Since the Normal distribution is symmetric about the mean and the Standard Normal Distribution has mean = 0

$$P(-0.2 \le z \le 0) = P(0 \le z \le 0.2)$$



$\therefore P(-0.2 \le z \le 0) + P(0 \le z \le 0.1)$

$= P(0 \le z \le 0.2) + P(0 \le z \le 0.1)$

$= 0.0793 + 0.0398$ [From the Standard Normal Table]

$= 0.1191$

$\therefore$ Proportion that will be accepted $= 0.1191$

Proportion that will be rejected $= 1 - 0.1191 = \underline{0.8809}$ (Ans.)

CO2:B Given:-

Population mean $(\mu) = 24$

Population variance $(\sigma^2) = 324$

Sample size $(n) = 81$

$\therefore$ Population standard deviation $(\sigma) = \sqrt{\sigma^2} = \sqrt{324} = 18$

Using Central Limit Theorem, for large enough sample size (approximately ≥ 30) the sampling distribution of means will be normally distributed.

Here sample size $(n) = 81$ so we can assume that sampling distribution of means is normally distributed.

Here $\frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{81}} = \frac{18}{9} = 2$

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CO2:B | $\therefore$ Probability that the sample mean lies between 23.9 and 24.2

$= P(23.9 \leq \bar{X} \leq 24.2)$ [Taking $\bar{x}$ = sample mean]

$$= P\left(\frac{23.9-\mu}{\frac{\sigma}{\sqrt{n}}} \leq \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \leq \frac{24.2-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(\frac{23.9-24}{2} \leq z \leq \frac{24.2-24}{2}\right)$$ [Calculating z-scores where $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$]

$= P(-0.05 \leq z \leq 0.1)$

$= P(-0.05 \leq z \leq 0) + P(0 \leq z \leq 0.1)$

Since the Standard Normal Distribution is symmetric about its mean 0,

$P(-0.05 \leq z \leq 0) = P(0 \leq z \leq 0.05)$

$P(-0.05 \leq z \leq 0) + P(0 \leq z \leq 0.1)$
$= P(0 \leq z \leq 0.05) + P(0 \leq z \leq 0.1)$

$= 0.0199 + 0.0398$ [From the Standard Normal Table]
$= \underline{0.0597}$

Ans:- The required probability is $\underline{0.0597}$.

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CO3

CO3: 3.A)

Value (x_i)	Frequency (f_i)	$f_i x_i$	Cumulative Frequency
5	6	30	6
6	8	48	14
7	14	98	28
8	22	176	50
9	28	252	78
10	36	360	114
	$\Sigma f_i = 114$	$\Sigma f_i x_i = 964$	

$\therefore$ Mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{964}{114} = \underline{8.456}$

For median,

$N = \Sigma f_i = 114$ (even)

For even N,

$$\text{Median} = \frac{\frac{N}{2}^{th}\text{ element} + (\frac{N}{2}+1)^{th}\text{ element}}{2} = \frac{57^{th}\text{ element} + 58^{th}\text{ element}}{2}$$

Looking at cumulative frequency column, the first cumulative frequency greater than 58 is 78 for $x_i = 9$

In fact all numbers from 51^{st} element to 78^{th} element are 9

$\therefore$ Median $= \frac{9+9}{2} = \frac{18}{2} = \underline{9}$

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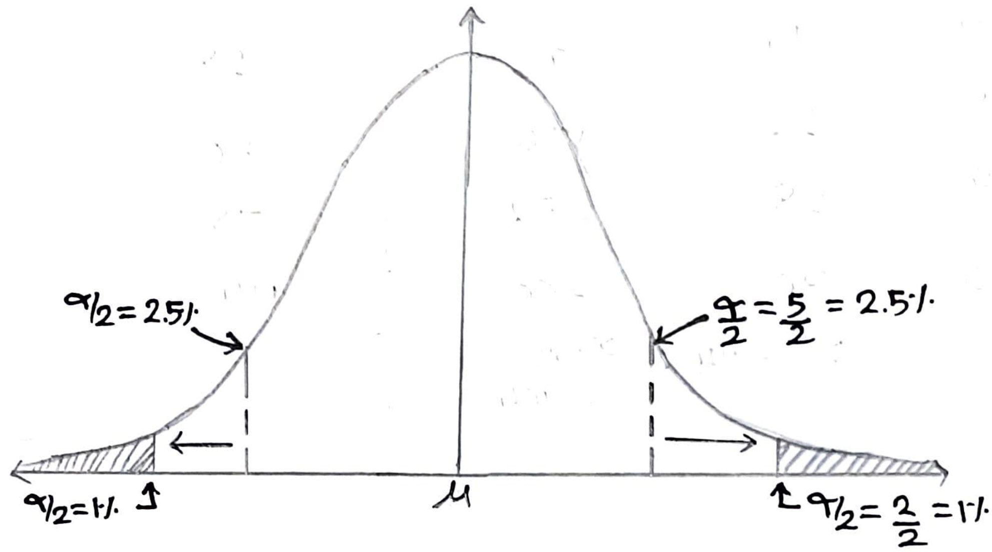
CO3: 3A) Mode = x_i with maximum frequency = 10 ($f_i = 36$)

∴ Mean = 8.456
Median = 9
Mode = 10

CO3: 3.B)



Change in area enclosed when confidence level increases from 95% to 98%.

As the confidence level increases while no change takes place in mean or standard deviation of the normal distribution, the value of α decreases and thus the area under the curve within the confidence interval increases. Mathematically, this also means that the probability of parameter to be found within this interval in future trials increases.

With mean and standard deviation fixed, the shape of the curve doesn't change but with decrease in α, area under curve bounded by confidence interval increases.

∴ The width of the confidence interval must increase.

Correct answer: (c) becomes wider

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CO4

CO4:A

Our population here is the temperatures recorded in Glens Falls for the month of February.

Population mean $(\mu) = 23$ degrees

Population standard deviation $(\sigma) = 4.2$ degrees

Our recorded temperature $(x) = 17$ degrees

$$\text{z-score } (z) = \frac{x-\mu}{\sigma}$$

$$\therefore \text{ z-score } (z) = \frac{17-23}{4.2} = -1.42857 \simeq \underline{-1.429}$$

Ans:- The z-score is $= -1.429$.

CO4:B The distribution of z-scores will always have a standard deviation of 1.

- True

Proof:-

Let, the standard deviation of z-scores be σ_z

population mean be μ

population standard deviation be σ

population size be n

Then,

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - \left(\frac{\Sigma z}{n}\right)^2$$

$$\Rightarrow \sigma_z^2 = \frac{\Sigma\left(\frac{x-\mu}{\sigma}\right)^2}{n} - \left\{\frac{\Sigma\left(\frac{x-\mu}{\sigma}\right)}{n}\right\}^2$$

CO4.B $\Rightarrow \sigma_z^2 = \dfrac{\frac{1}{\sigma^2}\Sigma(x-\mu)^2}{n} - \left\{\dfrac{\frac{1}{\sigma}(\Sigma x - n\mu)}{n}\right\}^2$

$\Rightarrow \sigma_z^2 = \dfrac{\frac{1}{\sigma^2}\Sigma(x^2+\mu^2-2x\mu)}{n} - \dfrac{1}{\sigma^2}\left(\dfrac{n\mu - n\mu}{n}\right)^2$ $[\because \Sigma x = n\mu]$

$\Rightarrow \sigma_z^2 = \dfrac{1}{\sigma^2}\cdot\dfrac{\Sigma(x^2+\mu^2-2x\mu)}{n} - 0$

$\Rightarrow n\sigma^2\sigma_z^2 = \Sigma x^2 - 2\mu\Sigma x + n\mu^2$

$\Rightarrow n\sigma^2\sigma_z^2 - n\mu^2 = \Sigma x^2 - 2n\mu^2$ $[\because \Sigma x = n\mu]$

$\Rightarrow n\sigma^2\sigma_z^2 - n\mu^2 + 2n\mu^2 = \Sigma x^2$

Now, we know,

$\sigma^2 = \dfrac{\Sigma x^2}{n} - \dfrac{(\Sigma x)^2}{n^2}$

$\Rightarrow \Sigma x^2 = n\left(\sigma^2 + \dfrac{(\Sigma x)^2}{n^2}\right) = n\sigma^2 + \dfrac{(\Sigma x)^2}{n}$

$\Rightarrow n\sigma^2\sigma_z^2 + n\mu^2 = \dfrac{(\Sigma x)^2}{n} + n\sigma^2$

Multiplying both sides by n,

$\Rightarrow n^2\sigma^2\sigma_z^2 + n^2\mu^2 = (\Sigma x)^2 + n^2\sigma^2$

$\Rightarrow n^2\sigma^2\sigma_z^2 + \cancel{n^2\mu^2} = \cancel{n^2\mu^2} + n^2\sigma^2$ $[\because \Sigma x = n\mu \Rightarrow (\Sigma x)^2 = n^2\mu^2]$

$\Rightarrow n^2\sigma^2(\sigma_z^2 - 1) = 0$

As $n^2\sigma^2 > 0$,

$\sigma_z^2 - 1 = 0$

$\Rightarrow \sigma_z^2 = 1$

$\Rightarrow \sigma_z = 1$ ~~$[\because \sigma_z \geq 0]$~~ $[\because \sigma_z$ is non-negative$]$

CO4:B Thus proved, standard deviation of distribution of z-scores $= 1$.

CO4:C For linear regression using least squares method,

$$e_i = y_i - \hat{y}_i$$

where

e_i = error of y at x_i

y_i = observed value of y at x_i

$\hat{y}_i$ = predicted value of y at x_i

Now $E(e)$

$$= \frac{1}{n}\sum e_i$$

$$= \frac{1}{n}\sum (y_i - \hat{y}_i)$$

$$= \frac{1}{n}\sum \left\{ y_i - \bar{y} - r\frac{S_y}{S_x}(x_i - \bar{x}) \right\}$$ [r = correlation coefficient, S_x and S_y are standard deviations of x and y]

$$= \frac{1}{n}\sum (y_i - \bar{y}) - r\frac{S_y}{S_x} \times \frac{1}{n}\sum (x_i - \bar{x})$$

$$= \frac{1}{n}\sum y_i - \frac{1}{n} \times n\bar{y} - r\frac{S_y}{S_x}\left[\frac{1}{n}\sum x_i - \frac{1}{n} \times n\bar{x}\right]$$

$$= (\bar{y} - \bar{y}) - r\frac{S_y}{S_x}[\bar{x} - \bar{x}]$$

$$= 0 - 0$$

$$= 0$$

$\therefore$ Option (a) is false

Option (b): variance of error term $(Var(e)) = S_y^2(1 - r^2)$ [constant for x]

$\therefore$ (b) is true ~~[independent of x]~~

Option (c) is true

Option (d) is true

CO4:c ∴ The assumption that is not required is (a).

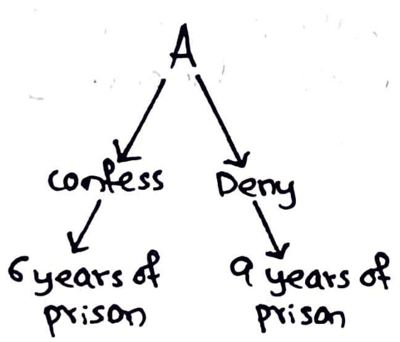
CO5

CO5. Payoff matrix:-

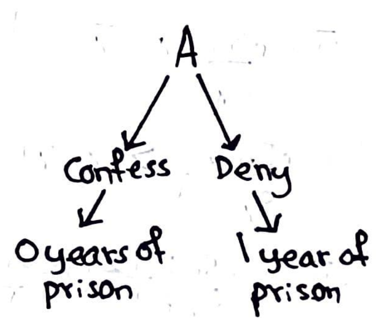
A \ B	Silent	Confess
Silent	-1,-1	-9,0
Confess	0,-9	-6,-6

A's decision tree:-

If B confesses

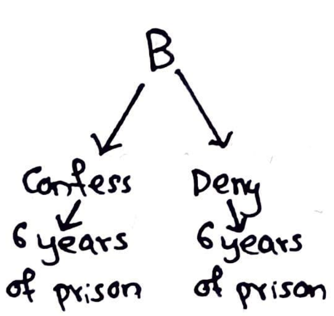


If B denies (remains silent)

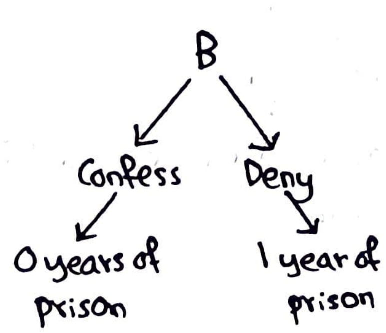


B's decision tree:-

If A confesses



If A denies



CO5: To find Nash equilibrium for best possible solution for both parties which would be optimal, we compare and analyze each state of the payoff matrix.

A \ B	Silent	Confess
Silent	1 -1, -1	2 -9, 0
Confess	3 0, -9	4 -6, -6

In state 1, if one party changes strategy while the other remained fixed, the person who changes strategy would gain.

If A denies and B decides to confess then B gets no prison time and vice versa. Thus, state 1 is not Nash equilibrium.

Similarly in state 2, so long as A and B both remain fixed on their strategies, B gets no prison time while A gets 9 years. But if A decides to confess, A's sentence reduces to 6 years while B's increases to 6 years. Thus, it is not in Nash equilibrium.

Similary to state 2, in state 3 change in strategy for B while A remains constant would help B and hence it is not in Nash equilibrium.

However, in state 4, where both confess, change in strategy by either prisoner while the other remains fixed, will only result in increase in prison ~~staten~~ term for the person who changes to denial.

Thus, no change in strategy will improve their condition and state 4 is in Nash equilibrium.

So confessing for both would be their ideal choice.

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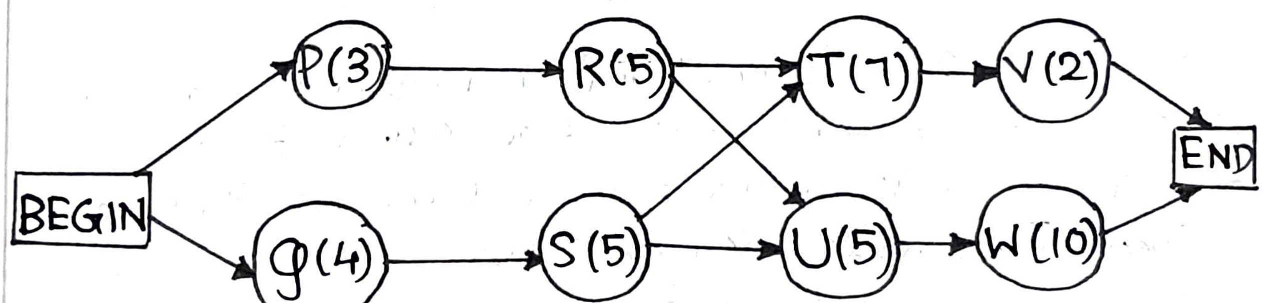
CO6:

Activity	Precedence	Duration (days)
P	—	3
Q	—	4
R	P	5
S	Q	5
T	R, S	7
U	R, S	5
V	T	2
W	U	10

Activity on Node (AON) Diagram



Path	Duration (in days)
P→R→T→V	3+5+7+2 = 17
P→R→U→W	3+5+5+10 = 23
Q→S→U→W	4+5+5+10 = 24
Q→S→T→V	4+5+7+2 = 18

Path with maximum cost = ~~Q→S→T→V~~ Q→S→U→W (24 days)

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CO6:- Ans: The critical path is = Q → S → U → W (duration = 24 days)

Final AON diagram: